Exercise-II-1

1)

Denote V as % of denomination

$$
\begin{aligned}
V &= e^{r(T-\widetilde{T})}E^{Q}[e^{-rT}H(S_T)] \\
&= e^{r(T-\widetilde{T})}E^{Q}[e^{-rT}(1-\frac{1}{K}(K-\frac{S_T}{S_0})^{+})] \\
&= e^{r(T-\widetilde{T})}E^{Q}[e^{-rT}(1-\frac{1}{K'}](K'-S_T)^{+})] \\
&= e^{r(T-\widetilde{T})}[e^{-rT}-\frac{1}{K'}E^{Q}[e^{-rT}(K'-S_T)^{+}]] \\
&= e^{-r\widetilde{T}}-\frac{e^{r(T-\widetilde{T})}}{K'}[e^{-rT}K'N(-d_2)-e^{-(q+repo)T}S_0N(-d_1)]
\end{aligned}
$$

Where

T is the interval between final valuation date and trade date, $\widetilde{T}$ is the interval between maturity date and issue date.

$$K' = K \times S_0$$

$$d_2 = d = \frac{ln\frac{S_0}{K} + (r-q-repo-\frac{1}{2}\sigma_S^2)T}{\sigma_S\sqrt{T}}$$

$$d_1 = d_2 + \sigma_S\sqrt{T}$$

The interest rate is transformed into continuous one(act/365):

$$(1+1.35\% \times \frac{44}{360}) = e^{r_c \times \frac{45}{365}}$$

$$r_c = \frac{365}{45}ln(1+1.35\% \times \frac{44}{360})$$

2,3,4) Delta,Vega and Rho are calculated as below:

$$Delta = \frac{Contract\_Value(S_0 * 1.01) - Contract\_Value(S_0 * 0.99)}{(2 * S_0 * 0.01)} * 1\%$$

$$Vega = \frac{Contract\_Value(Vol + 1.0\%) - Contract\_Value(Vol - 1.0\%)}{2\%} * 1\%$$

$$Rho = \frac{Contract\_Value(r + 0.05\%) - Contract\_Value(r - 0.05\%)}{(2 * 0.05\%)} * 0.01\%$$

Exercise-II-2:

Interest rate can be converted into continuous (Act/365) from (30/360) in the same way as Exercise-II-1. For simplifying, I convert with the longest tenor to get the continuous interest rate and use it for the following calculation.

$$(1+1.6\%\times\frac{194}{360}) = e^{r_c\frac{196}{365}}$$

$$r_c = \frac{365}{196}ln(1+1.6\%\times\frac{194}{360})$$

1) According to quanto prewashing technique, we have the price dynamic of stock price under USD risk neutral measure.

$$\frac{dS_B(t)}{S_B(t)} = (r_{JPY} + \rho_{x,B}\sigma_X\sigma_B)dt + \sigma_B dZ^{USD}_{B,t}$$

$$\frac{dS_A(t)}{S_A(t)} = r_{USD}dt + \sigma_A dZ^{USD}_{A,t}$$

$$\frac{S_B(t_i)}{S_B(0)} = e^{(r_{JPY}+\rho_{X,B}\sigma_X\sigma_B-\frac{1}{2}\sigma_B^2)t_i+\sigma_B\sum_{j=1}^{i}\sqrt{\delta t_j}\epsilon_B^j}$$

$$\frac{S_A(t_i)}{S_A(0)} = e^{(r_{USD}-\frac{1}{2}\sigma_A^2)t_i+\sigma_A\sum_{j=1}^{i}\sqrt{\delta t_j}\epsilon_A^j}$$

Where

$$\delta t_j = t_j - t_{j-1}$$

Denote V as % of denomination

$$V = e^{r(T-\widetilde{T})}E^Q[e^{-rT}(1-\frac{1}{K}(K-min(\frac{S_A(T)}{S_A(0)},\frac{S_B(T)}{S_B(0)}))^+)]$$

$$= e^{r\widetilde{T}} - \frac{e^{r(T-\widetilde{T})}}{K}e^{-rT}E^Q[e^{-rT}(K-min(\frac{S_A(T)}{S_A(0)},\frac{S_B(T)}{S_B(0)}))^+]$$

When $\frac{S^*(T)}{S^*(0)} < K$, the terminal payoff would be:

$$V = e^{r(T-\widetilde{T})}E^Q[e^{-rT}\frac{1}{KS^*(0)}S^*(T)]$$

Where

S^* is the worst performing asset

2)

$$SE = \sqrt{\frac{1}{N(N-1)}[\sum_{i=1}^{N}(Y_T^{(i)} - \overline{Y_T})^2]}$$

Where $\overline{Y_T}$ is the average discounted terminal payoffs and $Y_T^{(i)}$ denotes the discounted terminal payoff of the ith path, that is, $Y_T^{(i)} = e^{-rT}V_T$.

The 95% confidence interval of the contract value is given by:

$$[\overline{Y_T} - 1.96(SE), \overline{Y_T} + 1.96(SE)]$$