Solution Manual

to accompany the textbook

Fixed Income Securities:

Valuation, Risk, and Risk Management

by Pietro Veronesi

Chapters 2 - 8

Version 1

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Solutions to Chapter 2

Exercise 1.

Compute the discount factors implied by the STRIPS:

$$Z(0,3) = \exp(-3 \times 0.1) = 0.741 \tag{1}$$
$$Z(0,5) = \exp(-5 \times 0.05) = 0.779. \tag{2}$$

Since $Z(0,3) < Z(0,5)$, there is an arbitrage opportunity due to the violation of the positive time discount rate. To exploit it: buy the 3-year bond and sell the 5-year bond.

Exercise 2.

Compute the quoted price P of the T-bill as:

$$P = 100 \times \left[1 - \frac{n}{360} \times d\right], \tag{3}$$

using the discount rate given, d. The simple (bond equivalent) yield measures your annualized return as:

$$BEY = \frac{100 - P}{P} \times \frac{365}{n}. \tag{4}$$

Let $\tau = \frac{n}{365}$ be the time to maturity expressed as fraction of a year, and let T denote the maturity date of a given T-bill. The continuously compounded yield follows as:

$$r(t,T) = -\frac{1}{\tau} \ln \frac{P}{100}. \tag{5}$$

Finally, to obtain the semi-annually compounded yield for the 1-year T-bill, use:

$$r_2(0,1) = 2 \times \left(\frac{1}{(P/100)^{1/2}} - 1 \right) \tag{6}$$

	Maturity	n	$T-t$	Discount, d	Price, P	BEY	Cont. comp. yield	Semi-annual comp.	Date
a.	4-week	28	0.083	3.48%	99.7293	3.5379%	3.53%	–	12/12/2005
b.	4-week	28	0.083	0.13%	99.9899	0.13%	0.13%	–	11/6/2008
c.	3-month	90	0.25	4.93%	98.7675	5.06%	5.03%	–	7/10/2006
d.	3-month	90	0.25	4.76%	98.8100	4.88%	4.86%	–	5/8/2007
e.	3-month	90	0.25	0.48%	99.8800	0.49%	0.49%	–	11/4/2008
f.	6-month	180	0.5	4.72%	97.6400	4.90%	4.84%	–	4/21/2006
g.	6-month	180	0.5	4.75%	97.6250	4.93%	4.87%	–	6/6/2007
h.	6-month	180	0.5	0.89%	99.5550	0.91%	0.90%	–	11/11/2008
i.	360-day	360	1	1.73%	98.2700	1.78%	1.77%	1.75%	9/30/2008
j.	360-day	360	1	1.19%	98.8100	1.22%	1.21%	1.20%	11/5/2008

Exercise 3.

Compute respective discount factors taking into account the convention on which the interest rate is given:

$$\begin{aligned}
1.\ \ Z(t, t+1.5) &= \exp(-0.02 \times 1.5) = 0.97045 \\
2.\ \ Z(t, t+1.5) &= \exp(-0.03) = 0.97045 \\
3.\ \ Z(t, t+1.5) &= \frac{1}{(1+0.021)^{1.5}} = 0.96931 \\
4.\ \ Z(t, t+1.5) &= \frac{1}{(1+0.0201/2)^{2\times 1.5}} = 0.97045
\end{aligned}$$

Bond 3 is mispriced.

Exercise 4.

Using Table 2.4, obtain the discount factor $Z(t,T)$ for each maturity $T-t$ from 0.25 to 7.5 years:

$$Z(t,T) = \frac{1}{\left(1 + \frac{r_2(t,T)}{2}\right)^{2(T-t)}}. \tag{7}$$

Use Z to price each bond:

a. $P_z(0,5) = 100 \times Z(0,5) = 72.80$

b. $P_{c=15\%,n=2}(0,7) = \frac{15}{2} \times \sum_{i=1}^{14} Z(0,i/2) + 100 \times Z(0,7) = 151.23$

c. $P_{c=7\%,n=4}(0,4) = \frac{7}{4} \times \sum_{i=1}^{16} Z(0,i/4) + 100 \times Z(0,4) = 101.28$

d. $P_{c=9\%,n=2}(0,3.25) = \frac{9}{2} \times \sum_{i=1}^{7} Z(0,i/2-0.25) + 100 \times Z(0,3.25) = 108.55$

e. 100 (see Fact 2.11)

f. $P_{FR,n=1,s=0} = Z(0,0.5) \times 100 \times (1 + \frac{6.8\%}{1}) = 103.44$, where we assume that $r_1(0) = 6.8\%$

g. $P_{FR,n=4,s=0.35\%}(0,5.5) = 100 + \frac{0.35}{4} \sum_{i=1}^{22} Z(0,i/4) = 101.6$

h. $P_{FR,n=2,s=0.40\%} = Z(0,0.25) \times 100 \times (1 + \frac{6.4\%}{2}) + \frac{0.40}{2} \sum_{i=1}^{15} Z(0,i/2-0.25) = 104$, where we assume that $r_2(0) = 6.4\%$

Exercise 5.

a. When coupon c is equal to the yield to maturity y the bond trades at par; when coupon is below (above) the yield to maturity the bond trades above (below) par. Obtain bond prices given yield and the coupon using:

$$P_c(0,T) = \sum_{i=1}^{20} \frac{c/2 \times 100}{(1+y/2)^i} + \frac{100}{(1+y/2)^{20}} \tag{8}$$

It follows:

c	y	P
5%	6%	107.79
6%	6%	100
7%	6%	92.89

b. Figure 1 plots bond prices implied by different yields to maturity.

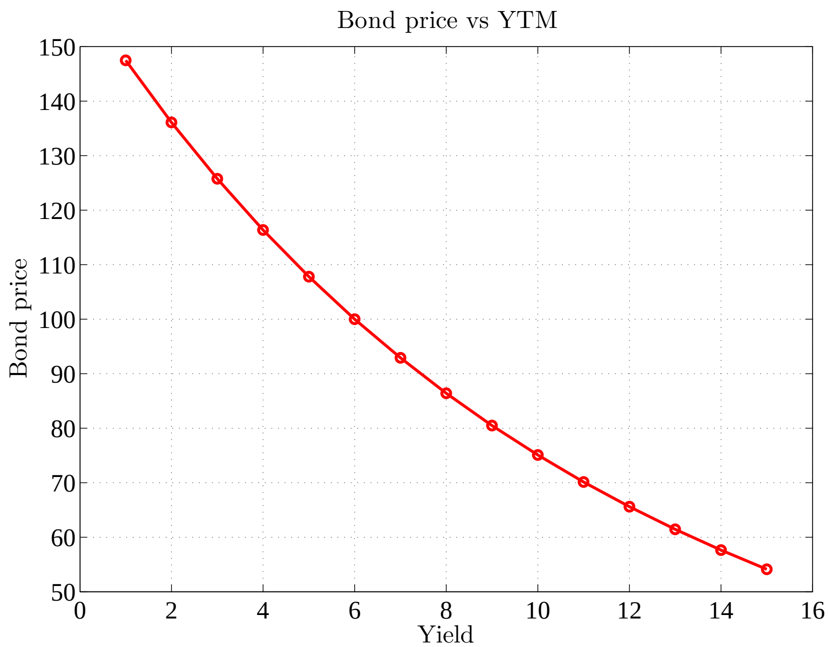

Fig. 1. Bond price as function of yield to maturity

Exercise 6.

a. To obtain bond prices use the expression:

$$P_c(0,T) = \frac{c}{2} \times \sum_{i=1}^{2T} Z(0, i/2) + 100 \times Z(0,T), \tag{9}$$

To compute the yield to maturity, solve equation (8) for y using a numerical solver.

c	$T-t$	P	y
15%	7	151.2306	5.9461%
3%	7	84.3482	5.7474%

b. The yields to maturity are different since bonds have different coupons, despite having the same time to maturity. Both bonds are priced using a no arbitrage discount curve. Therefore, their prices are fair.

Exercise 7.

a. Bootstrap the discount factors $Z(t,T)$ using the expression (9), and substituting recursively for the 6-month, 1-year, 1.5-year, and 2-year bonds. E.g., given $Z(0,0.5)$ and $Z(0,1)$, for the 1.5-year bond you have:

$$Z(0,1.5) = \frac{100.86 - \frac{7.5}{2}(Z(0,0.5) + Z(0,1))}{100 + \frac{7.5}{2}}. \tag{10}$$

This yields:

$T - t$	Coupon, c	Price, P	Issued	$Z(t,T)$
0.5	0.00%	\$96.80	5/15/2000	0.9680
1	5.75%	\$99.56	5/15/1998	0.9407
1.5	7.50%	\$100.86	11/15/1991	0.9032
2	7.50%	\$101.22	5/15/1992	0.8740

b. Compute the no-arbitrage price of the two bonds given the discount function obtained above. The prices of the two bonds are:

$$P_{c=8\%} = \$101.71 \tag{11}$$
$$P_{c=13.13\%} = \$106.60, \tag{12}$$

i.e. both are higher than the market prices. There is an arbitrage opportunity. You could make riskless profit by buying the underpriced bond at the traded price and selling the corresponding portfolio of zeros that replicates the cash flows from the bond.

Exercise 8.

The quotes are obtained on May 15, 2000. Use the mid bid-ask quote to compute the price. You want to obtain the semi-annual curve. The provided maturities of bonds are spaced semi-annually. We can assume that the clean (quoted) price is equal to the dirty (invoice) price, i.e. the accrual is zero. For each maturity, bootstrap the discount factors $Z(t,T)$ as in Exercise 7.a. The continuously compounded zero coupon yield is given as:

$$r(t,T) = -\frac{1}{T-t} \ln Z(t,T). \tag{13}$$

Cusip	$T-t$	Mid bid-ask quote	Coupon, c	Accrual	$Z(t,T)$	Yield, $r(t,T)$
912827ZN	0.5	100.9063	8.500%	0	0.96793	6.520%
912810CU	1	105.9961	13.125%	0	0.93508	6.713%
912810CX	1.5	112.4102	15.750%	0	0.90312	6.793%
912827F4	2	101.2188	7.500%	0	0.87418	6.724%
912810DA	2.5	110.6836	11.625%	0	0.84387	6.790%
912810DD	3	110.3438	10.750%	0	0.81638	6.762%
912810DG	3.5	115.3242	11.875%	0	0.78928	6.761%
912810DH	4	118.9141	12.375%	0	0.76267	6.773%
912810DM	4.5	118.3125	11.625%	0	0.73951	6.706%
912810DQ	5	121.6289	12.000%	0	0.71544	6.697%
912827V8	5.5	96.0000	5.875%	0	0.69440	6.631%
912827X8	6	100.6211	6.875%	0	0.67229	6.618%
912827Z6	6.5	98.7656	6.500%	0	0.65080	6.609%
9128272U	7	99.5781	6.625%	0	0.63152	6.566%
9128273X	7.5	93.1484	5.500%	0	0.61225	6.542%
9128274F	8	93.8008	5.625%	0	0.59478	6.494%
9128274V	8.5	87.9922	4.750%	0	0.57640	6.482%
9128275G	9	92.8398	5.500%	0	0.56151	6.413%

Solutions to Chapter 3

Exercise 1.

a. 3; equal to the maturity of the zero bond

b. 2.9542; the duration of the coupon bond is the weighted average of the coupon payment times

c. 0.9850

d. 0.5; equal to the time left to the next coupon payment

e. 0.5111; obtain the price P_{FR} of the floating rate bond (see Chapter 2, equation (2.39)). In analogy to a coupon bond, the duration is computed as:

$$D_{FR} = \frac{100}{P_{FR}} \times 0.5 + \frac{0.5s \times \sum_{t=0.5}^{3} Z(0,t) \times t}{P_{FR}}. \tag{14}$$

f. 0.2855; proceed as in point e. above but recognize that the valuation is outside the reset date. Assume that the coupon applying to the next reset date has been set at $r_2(0) = 6.4\%$.

Exercise 2.

Portfolio A:

Security	Duration, D	Weight, w	$D \times w$
4.5yr @ 5% semi	3.8660	40%	1.55
7yr @ 2.5% semi	6.4049	25%	1.60
1.75 fl + 30bps semi	0.2540	20%	0.05
1yr zero	1.0000	10%	0.10
2yr @ 3% quart	1.9530	5%	0.10
		Port. D	**3.40**

Portfolio B:

Security	Duration, D	Weight, w	$D \times w$
7yr @ 10% semi	5.4262	40%	2.17
4.25yr @ 3% quart	3.9838	25%	1.00
90 day zero	0.2500	20%	0.05
2yr fl semi	0.5000	10%	0.05
1.5yr @ 6% semi	1.4564	5%	0.07
		Port. D	**3.34**

The investors would select the shorter duration portfolio B.

Exercise 3.

Obtain yield to maturity y for each security. Compute modified and Macaulay duration accodring to equation (3.19) and (3.20) in the book.

	Yield	Duration	Modified	Macaulay
a.	6.95%	3	3	2.8993
b.	6.28%	2.9542	2.9974	2.9061
c.	6.66%	0.9850	0.9850	0.9689
d.	0.00%	0.5	0.5	0.5
e.	6.82%	0.5111	0.5111	0.4943
f.	6.76%	0.2855	0.2855	0.2761

Exercise 4.

Compute the duration of each asset and use the fact that the dollar duration is the bond price times its duration.

	Price	Duration	$ Duration
a.	$89.56	4.55	$407.88
b.	$67.63	-7.00	($473.39)
c.	$79.46	3.50	$277.74
d.	$100.00	0.5	$50.00
e.	$100.00	-0.25	($25.00)
f.	$102.70	-0.2763	($28.38)

Exercise 5.

a. Compute number of units of each security (N) in the portfolio and apply Fact 3.5.

Portfolio A:

Security	Price	Duration	Weight	N	$D \times P \times N$
4.5yr @ 5% semi	94.03	3.8660	40%	0.43	154.64
7yr @ 2.5% semi	81.56	6.4049	25%	0.307	160.12
1.75 fl + 30bps semi	102.09	0.2540	20%	0.20	5.08
1yr zero	93.61	1.0000	10%	0.11	10.00
2yr @ 3% quart	92.54	1.9530	5%	0.05	9.76
				$\$D$	**339.61**

Portfolio B :

Security	Price	Duration	Weight	N	$D \times P \times N$
7yr @ 10% semi	123.36	5.4262	40%	0.32	217.05
4.25yr @ 3% quart	86.83	3.9838	25%	0.29	99.59
90 day zero	98.45	0.2500	20%	0.20	5.00
2yr fl semi	100.00	0.5000	10%	0.10	5.00
1.5yr @ 6% semi	98.83	1.4564	5%	0.05	7.28
				$\$D$	**333.92**

b. For 1 bps increase, we have (see Definition 3.5):

Portfolio A: $339.61 \times 0.01/100 = -\$0.0340$
Portfolio B: $333.92 \times 0.01/100 = -\$0.0334$

c. Yes.

Exercise 6.

After the reshuffling of the portfolio, its value becomes \$50 mn.

a. Short -0.307 units of long bond in portfolio A, and -0.081 units in portfolio B.

b. New dollar durations are: 19.36 and 62.61 for portfolio A and B, respectively.

c. The conclusion reverses.

	N LT bond	New weight LT bond	New $\$D$
Port. A	-0.307	-25%	19.36
Port. B	-0.081	-10%	62.61

Exercise 7.

a. \$10 mn

b. Compute the dollar duration of the cash flows in each bond, and then the dollar duration of the portfolio:

Security	Position	\$ (mn)	Price	N	$\$D$	$\$D \times N$
6yr IF @ 20% - fl quart	Long	20.00	146.48	0.137	1,140.28	155.69
4yr fl 45bps semi	Long	20.00	101.62	0.197	53.54	10.54
5yr zero	Short	(30.00)	76.41	-0.393	382.052	-150.00
	Port. value	**\$10.00 mn**			Port. $\$D$	**16.23**

Exercise 8.

a. The price of the 3yr @ 5% semi bond is \$97.82. You want the duration of the hedged portfolio to be zero. You need to short 0.058 units of the 3-year bond, i.e. the short position is -\$5.69.

b. The total value of the portfolio is: \$4.31 mn.

Exerxise 9.

Compute the new value of the portfolio assuming the term structure of interest rates as of May 15, 1994.

	Original	Now	Δ value
Unhedged port.	\$10.00	\$8.97	(\$1.03)
Hedge	(\$5.69)	(\$5.44)	\$0.25
Total	\$4.31	\$3.53	(\$0.78)

a. \$8.97 mn

b. \$3.53 mn

c. The immunization covered part of the loss. The change in the value of the portfolio is both due to (i) the passage of time (coupon) and (ii) the increase in interest rates.

Exercise 10.

Use the curve given on May 15, 1994, but keep the times to maturity unchanged from the initial ones. The change in value is due to the change in interest rates only.

	Original	Now	Δ value
Unhedged port.	\$10.00	\$9.97	(\$0.03)
Hedge	(\$5.69)	(\$5.43)	\$0.26
Total	\$4.31	\$4.54	\$0.23

Exercise 11.

Use the curve given on February 15, 1994, but change the times to maturity to those on May 15, 1994. The change in value is due to coupon only.

	Original	Now	Δ value
Unhedged port.	\$10.00	\$9.12	(\$0.88)
Hedge	(\$5.69)	(\$5.68)	\$0.01
Total	\$4.31	\$3.45	(\$0.87)

Exercise 12.

a.,b. Loss of \$0.87 mn.

c. Gain of \$0.08 mn.

	Original	Now	ΔC	Δr	Total
	Ex.8	**Ex.9**	**Ex.11**	**(9)-(8)-(11)**	**(9)-(8)**
Unhedged port.	\$10.00	\$8.97	(\$0.88)	(\$0.15)	(\$1.03)
Hedge	(\$5.69)	(\$5.44)	\$0.01	\$0.24	\$0.25
Total	\$4.31	\$3.53	(\$0.87)	\$0.08	(\$0.78)

Solutions to Chapter 4

Exercise 1.

a. 16

b. 4.7 (see Fact 4.3)

c. 3.85

d. 0.28 = 0.25 (due to floater with zero spread) + 0.03 (due to the spread)

e. 0.14 = 0.06 (due to floater with zero spread) + 0.08 (due to the spread). Assume $r_2(0) = 6.40\%$.

Exercise 2.

Portfolio A:

Security	Price	Weight, w	Duration, D	Convexity, C	$w \times D$	$w \times C$
5yr @ 4% quart	89.20	30%	4.5510	21.9703	1.37	6.59
4.25yr @ 6% semi	99.15	25%	3.7535	15.2807	0.94	3.82
90-day zero	98.45	20%	0.25	0.0625	0.05	0.01
2.5 yr float quart	100.00	15%	0.25	0.0625	0.04	0.01
6yr zero	69.54	10%	6.0	36	0.60	3.60
Port.		100%			**2.99**	**14.03**

Portfolio B:

Security	Price	Weight, w	Duration, D	Convexity, C	$w \times D$	$w \times C$
7yr @ 2% semi	78.77	40%	6.5071	44.3923	2.60	17.76
3.25yr float 50 bps	103.16	30%	0.2717	0.1194	0.08	0.04
4yr @ 3.5% semi	89.00	20%	3.7506	14.6406	0.75	2.93
90-day zero	98.45	10%	0.25	0.0625	0.03	0.01
Port.		100%			**3.46**	**20.73**

Assume the interest rates move up by 1% in a parallel fashion, i.e. $dr = 1\%$. The change in the value of the portfolios is:

$$\frac{dP_A}{P_A} = -2.99 \times 0.01 + \frac{1}{2} \times 14.03 \times (0.01)^2 = -0.0292 \tag{15}$$

$$\frac{dP_B}{P_B} = -3.46 \times 0.01 + \frac{1}{2} \times 20.73 \times (0.01)^2 = -0.0336 \tag{16}$$

	Δ due to D	Δ due to C	Total ($ mn)
Port. A	-0.0299	0.0007	-0.0292
Port. B	-0.0346	0.0010	-0.0336

The portfolio A is less sensitive to the parallel shifts in interest rates.

Exercise 3.

It follows that: $Var(dr) = E(dr^2) = 3.451e^{-6}$. Therefore:

$$E\left(\frac{dP_A}{P_A}\right) = \frac{1}{2} \times C_A \times E(dr^2) = 2.42e^{-5} \times 252 = 0.61\% \text{ annualized} \tag{17}$$

$$E\left(\frac{dP_B}{P_B}\right) = \frac{1}{2} \times C_B \times E(dr^2) = 3.58e^{-5} \times 252 = 0.90\% \text{ annualized} \tag{18}$$

Exercise 4.

Follow the steps in Example 4.3 using a 2-year bond instead of a 10-year bond.

Duration Hedging				
Spot Curve Shift	$P_c(0,10)$	$P_z(0,2)$	Position	Change in Portfolio Value
Initial Values	103.58	91.39	-4.5507	
$dr = .1\%$	102.75	91.21	-4.5507	0.0030
$dr = 1\%$	95.63	89.58	-4.5507	0.2880
$dr = 2\%$	88.38	87.81	-4.5507	1.1087
$dr = -.1\%$	104.41	91.58	-4.5507	0.0030
$dr = -1\%$	112.29	93.24	-4.5507	0.3113
$dr = -2\%$	121.84	95.12	-4.5507	1.2957

The hedge performs better in that for any scenario the change in the value of the portfolio is positive.

Exercise 5.

Take into account the fact that the time to maturity of both securities is changing: a. by 1/252, b. by 1/52 and c. by 1/12 fraction of a year. Price the 10-year coupon bond and the 2-year zero bond for each of those cases and for the different levels of interest rates. Finally, compute the change in the value of the portfolio relative to the initial one.

a. Day Trade				
Spot Curve Shift	$P_c(0,10)$	$P_z(0,2)$	Position	Change in Portfolio Value
Initial Values	103.58	91.39	-4.5507	
$dr = .1\%$	102.77	91.23	-4.5507	-0.0540
$dr = 1\%$	95.65	89.60	-4.5507	0.2199
$dr = 2\%$	88.40	87.83	-4.5507	1.0284
$dr = -.1\%$	104.43	91.59	-4.5507	-0.0515
$dr = -1\%$	112.30	93.25	-4.5507	0.2680
$dr = -2\%$	121.86	95.13	-4.5507	1.2648

b. Week Trade

Spot Curve Shift	$P_c(0,10)$	$P_z(0,2)$	Position	Change in Portfolio Value
Initial Values	103.58	91.39	-4.5507	
$dr = .1\%$	102.84	91.29	-4.5507	-0.2734
$dr = 1\%$	95.73	89.68	-4.5507	-0.0422
$dr = 2\%$	88.49	87.92	-4.5507	0.7194
$dr = -.1\%$	104.50	91.65	-4.5507	-0.2614
$dr = -1\%$	112.36	93.30	-4.5507	0.1013
$dr = -2\%$	121.90	95.17	-4.5507	1.1461

c. Month Trade

Spot Curve Shift	$P_c(0,10)$	$P_z(0,2)$	Position	Change in Portfolio Value
Initial Values	103.58	91.39	-4.5507	
$dr = .1\%$	103.14	91.56	-4.5507	-1.1966
$dr = 1\%$	96.07	89.99	-4.5507	-1.1455
$dr = 2\%$	88.86	88.29	-4.5507	-0.5817
$dr = -.1\%$	104.79	91.91	-4.5507	-1.1443
$dr = -1\%$	112.62	93.51	-4.5507	-0.6000
$dr = -2\%$	122.10	95.32	-4.5507	0.6470

Exercise 6.

Level is computed as the average term structure, slope is the difference between the 10-year and the 1-month rate, and the butterfly (curvature) is obtained as:

$$\text{Curvature} = -1\text{M yield} + 2 \times 5\text{Y yield} - 10\text{Y yield} \tag{19}$$

Date	Level	Slope	Curvature	Δ Slope	Δ Curvature
9/26/2008	2.14%	3.64%	2.04%	1.57%	0.20%
9/10/2008	2.40%	2.07%	0.59%	-0.06%	-0.14%
8/25/2008	2.51%	2.13%	0.63%	-0.09%	-0.20%
8/11/2008	2.69%	2.22%	0.78%	-0.19%	-0.14%
7/25/2008	2.75%	2.41%	1.05%	0.06%	0.30%
7/10/2008	2.54%	2.35%	0.89%	-0.28%	-0.29%
6/25/2008	2.82%	2.63%	1.47%	0.52%	0.01%
6/10/2008	2.92%	2.11%	0.97%	-	-

a. 9/26/2008

b. 9/26/2008

c. See Figure 2

d. 9/10/2008–9/26/2008

e. 7/10/2008–7/25/2008

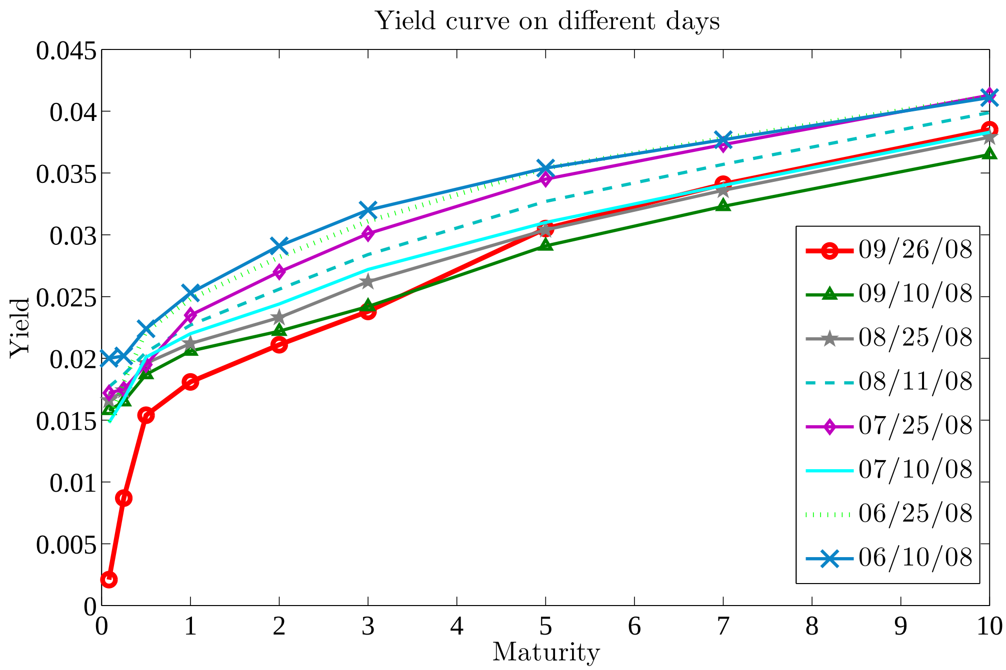

Fig. 2. Term structures 6/10/2008–9/26/2008 from CRSP

Exercise 7.

You need factor sensitivities for yields with maturities from 3 months to 4.25 years with a 0.25-year spacing. To obtain the 0.25-year grid of sensitivities, interpolate linearly the β's between available maturities. To compute factor durations, use Fact 4.5.

	Security	Price	Level D	Slope D	Curvature D
a.	4 yr zero	81.45	4.10	0.20	0.99
b.	2.5 yr @ 3% semi	96.24	2.50	-0.61	0.77
c.	3.25 yr float 0 bps	100.71	0.25	-0.06	-0.08
d.	4.25 yr float 35 bps	102.12	0.29	-0.07	-0.07

Exercise 8.

Use the assumptions of Chapter 3 (Exercise 7) for the pricing of the inverse floater.

a. i. $40 mn

ii. Refer to Exercise 7 in Chapter 3 for the pricing of the respective securities.

Security	Price, P	\$ (mn)	N (mn)	\$$D$	\$$D \times N$
6yr IF @ 20% - fl quart	\$146.48	\$30.00	0.205	1,140.28	233.541
4yr fl 45bps semi	\$101.62	\$30.00	0.295	53.54	15.805
3yr @ 4% semi	\$97.82	(\$20.00)	-0.204	279.16	-57.076
				Port. \$$D$	**192.27**

b. You need to choose the number of 3-month and 6-year bonds, k_S and k_L, such that the variation of the portfolio plus the two bonds is approximately zero.

For each bond in the portfolio compute the factor duration (see Fact 4.4):

	Price, P	D_1	D_2	weight, w	$D_1 \times w$	$D_2 \times w$
6yr IF						
coupon	\$174.72	4.27	1.01	1.19	5.09	1.21
float	(\$100.00)	0.25	-0.06	-0.68	-0.17	0.04
fix	\$71.76	6.09	2.29	0.49	2.98	1.12
total	\$146.48			1	**7.90**	**2.38**
4yr fl						
coupon	\$1.62	2.21	-0.37	0.02	0.04	-0.01
float	\$100.00	0.48	-0.16	0.98	0.47	-0.16
total	\$101.62			1	**0.50**	**-0.17**
3yr						
total	\$97.82	2.93	-0.43	1	**2.93**	**-0.43**

In the same fashion, compute the factor durations of the Long/Long/Short portfolio:

Security	Position	P	\$ mn	w	N (mn)	D_1	D_2	$w \times D_1$	$w \times D_2$
6yr IF @ 20% - fl quart	Long	\$146.48	\$30.00	0.75	0.205	7.90	2.38	5.93	1.78
4yr fl 45bps semi	Long	\$101.62	\$30.00	0.75	0.295	0.50	-0.17	0.38	-0.12
3yr @ 4% semi	Short	\$97.82	(\$20.00)	-0.5	-0.204	2.93	-0.43	-1.47	0.22
	Total		\$40.00	1				**4.84**	**1.87**

Obtain the factor durations for the short and long zero bonds:

	Portfolio	Short Bond	Long Bond
Price	\$40	\$98.24	\$71.76
D_1	4.84	0.48	6.09
D_2	1.87	-0.16	2.29

Use expression (4.29)–(4.30) to compute the weights in each zero bond:

	$P/P_i, i = \{S, L\}$	numerator	denominator	weight
	(A)	(B)	(C)	(A)×(B)/(C)
k_S	-0.41	-0.33	2.08	0.06
k_L	-0.56	-1.68	-2.08	-0.45

The investor has to establish the following hedge:

Hedge			
Position	Security	Amount	N
Long	0.5 zero	\$6.36	0.065
Short	6yr zero	(\$32.27)	-0.450

The value of the portfolio is: $40 + 6.36 - 32.27 = \$14.08$ mn.

c. i. \$36.82

   ii. \$13.03

   iii. The hedge covered part of the portfolio losses. The change in the value of the portfolio is not just due to coupon payment since the term structure has shifted as well.

	Original	Now	Δ value
Unhedged port.	\$40.00	\$36.82	(\$3.18)
Hedge	(\$25.92)	(\$23.79)	\$2.13
Total	\$14.08	\$13.03	(\$1.05)

d. The hedge would have worked better if it was not for the passage of time and the coupon effect.

   i. \$37.71

   ii. \$14.26

	Original	Now	Δ value
Unhedged port.	\$40.00	\$37.71	(\$2.29)
Hedge	(\$25.92)	(\$23.45)	\$2.46
Total	\$14.08	\$14.26	\$0.17

Solutions to Chapter 5

Exercise 1.

(t, T)	yield	$Z(t, T)$	$F(t, T-\Delta, T)$	$f(t, T-\Delta, T)$
0.25	6.17%	0.9847	0.9847	6.17%
0.50	6.52%	0.9679	0.9830	6.87%
0.75	6.32%	0.9537	0.9853	5.92%
1.00	6.71%	0.9351	0.9805	7.88%
1.25	6.76%	0.9190	0.9828	6.96%
1.50	6.79%	0.9032	0.9828	6.94%
1.75	6.77%	0.8883	0.9835	6.65%
2.00	6.72%	0.8742	0.9842	6.37%
2.25	6.72%	0.8597	0.9833	6.72%
2.50	6.79%	0.8439	0.9816	7.42%
2.75	6.78%	0.8299	0.9834	6.68%
3.00	6.76%	0.8164	0.9838	6.54%
3.25	6.77%	0.8025	0.9829	6.89%
3.50	6.76%	0.7893	0.9836	6.63%
3.75	6.63%	0.7799	0.9880	4.81%
4.00	6.77%	0.7628	0.9781	8.87%
4.25	6.77%	0.7500	0.9832	6.77%
4.50	6.71%	0.7394	0.9859	5.69%
4.75	6.66%	0.7288	0.9857	5.76%
5.00	6.70%	0.7153	0.9815	7.46%
5.25	6.71%	0.7031	0.9829	6.91%
5.50	6.63%	0.6944	0.9877	4.95%
5.75	6.69%	0.6807	0.9802	8.01%
6.00	6.62%	0.6722	0.9876	5.01%
6.25	6.63%	0.6608	0.9830	6.87%
6.50	6.61%	0.6507	0.9848	6.11%
6.75	6.58%	0.6414	0.9856	5.80%
7.00	6.57%	0.6313	0.9844	6.30%

Exercise 2.

Use the facts that:

$$F(0, T-\Delta, T) = e^{-f(0,T-\Delta,T)\Delta} \tag{20}$$

$$Z(0, T_i) = Z(0, T_{i-1}) \times F(0, T_{i-1}, T_i). \tag{21}$$

(t, T)	$f(t, T-\Delta, T)$	$F(t, T-\Delta, T)$	yield	$Z(t, T)$
0.25	3.53%	0.9912	3.53%	0.9912
0.50	3.58%	0.9911	3.55%	0.9824
0.75	4.19%	0.9896	3.77%	0.9721
1.00	3.99%	0.9901	3.82%	0.9625
1.25	4.54%	0.9887	3.97%	0.9516
1.50	5.00%	0.9876	4.14%	0.9398
1.75	4.76%	0.9882	4.23%	0.9287
2.00	5.88%	0.9854	4.43%	0.9151
2.25	5.30%	0.9868	4.53%	0.9031
2.50	4.92%	0.9878	4.57%	0.8921
2.75	6.09%	0.9849	4.71%	0.8786
3.00	5.29%	0.9869	4.76%	0.8670
3.25	6.48%	0.9839	4.89%	0.8531
3.50	6.20%	0.9846	4.98%	0.8400
3.75	6.34%	0.9843	5.07%	0.8268
4.00	6.00%	0.9851	5.13%	0.8145
4.25	5.99%	0.9851	5.18%	0.8024
4.50	6.58%	0.9837	5.26%	0.7893
4.75	6.26%	0.9845	5.31%	0.7770
5.00	6.69%	0.9834	5.38%	0.7641
5.25	6.12%	0.9848	5.42%	0.7525
5.50	5.70%	0.9859	5.43%	0.7419
5.75	6.81%	0.9831	5.49%	0.7294
6.00	6.50%	0.9839	5.53%	0.7176
6.25	6.59%	0.9837	5.57%	0.7059
6.50	7.06%	0.9825	5.63%	0.6935
6.75	6.87%	0.9830	5.68%	0.6817
7.00	6.37%	0.9842	5.70%	0.6709

Exercise 3.

Compute the forward discount factor:

$$F(0, 1, 2.5) = Z(0, 2.5)/Z(0, 1) = 0.9024 \tag{22}$$

Obtain the forward rate consistent with the current yield curve:

$$f_2(0, 1, 2.5) = 2 \times \left[\frac{1}{F(0, 1, 2.5)^{\frac{1}{1.5 \times 2}}} - 1 \right] = 6.96\% \tag{23}$$

The current forward rate on the market is lower than the one offered by the bank at 7.56%. There is an arbitrage opportunity.

Exercise 4.

a. $f_2(0, 0.5, 1) = 7.02\%$

b. The value of the forward at inception is zero as no money changes hands.

Exercise 5.

a. At inception, you buy

$$M = \frac{Z(0, 0.5)}{Z(0, 1)} = 1.0351 \tag{24}$$

T-bills maturing in 1 year. Using the term structure on August 15, 2000, the value of the FRA is:

$$FRA(0.25) = 100 \times [M \times Z(0, 0.75) - Z(0, 0.25)] = \$0.2156 \text{ mn} \tag{25}$$

b. i. \$0.3221 mn

ii. $r_2(0.5, 1) = 6.36\%$

iii. Net payment at $T_2 = \$100\text{mn} \times 0.5 \times [f_2(0, 0.5, 1) - r_2(0.5, 1)] = \$0.3323$ mn

Exercise 6.

a. $P^{fwd} = 100 \times F(0, 0.5, 2) = \$90.3210$

b. $M = \frac{\$50 \text{ mn}}{P^{fwd}} = 0.554$ mn

Exercise 7.

a. Payoff $= M \times (P(0.5, 2) - P^{fwd}) = \$0.64$ mn

b. You make money on the long forward.

Exercise 8.

a. i. $M = \$100\text{mn}/100.5 = 0.995$ mn

ii. From Fact 5.9 the forward price of a coupon bond is:

$$P_c^{fwd}(0, T, T^*) = \frac{c}{2} \times \sum_{i=1}^{n} P_z^{fwd}(0, T, T_i) + P_z^{fwd}(0, T, T_n). \tag{26}$$

Thus, $P_c^{fwd}(0, 0.25, 2) = 100.932$.

(t,T)	Yield	$Z(t,T)$	$F(t,T,T_n)$	P_z^{fwd}	P^{fwd} of CF
0.25	1.71%	0.9957	1.0000	100.00	
0.50	2.09%	0.9896	0.9939	99.39	1.4287
0.75	2.29%	0.9830	0.9872	98.72	
1.00	2.37%	0.9766	0.9808	98.08	1.4099
1.25	2.32%	0.9715	0.9756	97.56	
1.50	2.38%	0.9649	0.9690	96.90	1.3930
1.75	2.48%	0.9576	0.9617	96.17	
2.00	2.61%	0.9492	0.9533	95.33	96.7000
				P^{fwd} note	**100.932**

b. i., ii. Implied repo rate $= \frac{P_z^{fwd}(0,0.25,2)}{P_z(0,2)} - 1 = 0.43\%$, or 1.72% annualized. It is lower than the repo rate on the market.

iii. To exploit the arbitrage opportunity, implement the following strategy:

	Lend at repo rate		
(1)	At $t = 0$	(\$100.50)	- Buy Treasury, enter into repo
(2)	At $t = 0.25$	\$101.02	- In 3 months resell Treasury, receive 2.05% interest

	Borrow at implied repo rate		
(3)	At $t = 0$	\$100.50	- Sell Treasury
(4)	At $t = 0.25$	(\$100.93)	- Enter into forward contract to purchase Treasury in 3 months

	Payoff	
(3)+(1)	At $t = 0$	\$0.0000
(4)+(2)	At $t = 0.25$	\$0.0835

iv. Yes, the implied repo rate equals the rate of return on investing in the 3-month bond.

c. The implied repo rate is the same across all maturities, T, because:

$$\frac{P_z^{fwd}(0,0.25,T)}{P_z(0,T)} - 1 = \frac{P_z(0,T)}{P_z(0,0.25)} \times \frac{1}{P_z(0,T)} - 1 = \frac{1}{P_z(0,0.25)} - 1. \tag{27}$$

Exercise 9.

Use fact 5.11 to obtain the swap rate $c(t,T)$:

(t,T)	yield	$Z^L(t,T)$	$c(t,T), n=2$
0.50	2.76%	0.9863	2.77%
1.00	2.95%	0.9710	2.96%
1.50	2.98%	0.9564	2.99%
2.00	3.20%	0.9383	3.20%

Exercise 10.

a. On January 2, 2008, the 1-year swap rate is $c(0,1) = 4.21\%$.

b. 0

c. Compute the value of the fixed and floating leg at each date. For the floater take into account the value at and outside the reset dates.

					Value of the Fixed Leg (1 yr coupon bond @ 4.21% quart)					
Mat/Date	2-Jan	1-Feb	3-Mar	1-Apr	1-May	2-Jun	1-Jul	1-Aug	1-Sep	1-Oct
0.08			1.05			1.05			1.05	
0.17		1.05			1.05			1.05		
0.25	1.04			1.05			1.05			100.01
0.33			1.04			1.04			100.07	
0.42		1.04			1.04			99.80		
0.50	1.03			1.04			99.49			
0.58			1.04			99.33				
0.67		1.03			99.11					
0.75	1.02			99.17						
0.83			98.82							
0.92		98.46								
1.00	96.91									
	100.00	101.58	101.95	101.26	101.20	101.43	100.53	100.85	101.12	100.01
					Value of the Floating Leg (reset in Jan, Apr, Jul, Oct)					
$r_4(0)$	4.68%	4.68%	4.68%	2.68%	2.68%	2.68%	2.79%	2.79%	2.79%	4.15%
	100.00	100.65	100.91	**100.00**	100.21	100.47	**100.00**	100.25	100.49	**100.00**
					Value of the Swap = Float − Fixed					
	0.00	-0.94	-1.04	-1.26	-0.99	-0.96	-0.53	-0.60	-0.63	-0.01

Exercise 11.

On that day, the net spread was strongly negative:

$$
\begin{aligned}
SS &= c - ytm = -0.33\% \\
LRS &= \text{LIBOR} - \text{repo} = 1.71\% \\
SS - LRS &= -2.04\%
\end{aligned}
$$

You could envision the following strategy to exploit the large negative spread:

1. Buy Treasury through a repo transaction: get coupon, pay repo
2. Enter the floating-for-fixed swap: pay fixed, get LIBOR

Exercise 12.

a. To swap the payments based on different floating rates, the company will effectively enter into two swap contracts:

Basis swap:

1. Floating-for-fixed swap on LIBOR: receive c_1 & pay LIBOR
2. Fixed-for-floating swap on Treasuries: receive 6mo Treasury & pay c_2

= + 6mo Treasury + spread $(c_1 - c_2)$ − LIBOR

At inception the value of the contract is zero. Given the Treasury curve, obtain the respective discount factors, and compute the fair c_2 rate, $c_2 = 1.67\%$. It is immediate from the swap curve provided in the exercise that the 5-year swap rate is $c_1 = 2.58\%$. The spread is $c_1 - c_2 = 0.91\%$.

b. Compute the value of contracts 1. and 2. six month later:

$$\begin{aligned} \text{swap1} - 100 &= -6.45 \\ 100 - \text{swap2} &= 5.84 \end{aligned}$$

The value of the position is: −\$0.61.

c. Assume that the company has \$100 mn on the balance sheet:

	Market/Assets	CF	Basis Swap	CF	Total CF
Receive	3.13%/2	1.5667	(2.06%+0.91%)/2	1.4860	3.0527
Pay	2.06%/2	1.0321	3.13%/2	1.5667	2.5988
				P/L	**\$0.4539 mn**

Solutions to Chapter 6

Exercise 1.

From semi-annually compounded yields obtain (i) the discount factors, and (ii) the forward discount factors. Use Fact 5.9 in Chapter 5 to compute the forward price of a coupon bond.

a. 98.85

b. 0

c. Compute the time-t value of the forward as:

$$V(t) = Z(t,T) \times \left[P_c^{fwd}(t,T,T^*) - K\right],\ K = P_c^{fwd}(0,T,T^*) \tag{28}$$

e. Obtain the daily overnight discount factor $Z(o/n)$, then:

$$\text{Total P/L}(t_i) = \text{P/L}(t_i) + \frac{\text{Total P/L}(t_{i-1})}{Z_{o/n}(t_{i-1})} \tag{29}$$

		2/15/1994	2/16/1994	2/17/1994	2/18/1994	2/22/1994	2/23/1994
		t_0	t_1	t_2	t_3	t_4	t_5
	P_c^{fwd}	98.85	98.73	98.23	97.77	98.13	97.84
b., c.	$V(t)$	0	-0.1156	-0.5691	-0.9913	-0.6623	-0.9206
d.	P/L		-0.1262	-0.4964	-0.4616	0.3588	-0.2846
	rate o/n		3.54%	3.78%	3.90%	4.44%	2.88%
	$Z(o/n)$		0.9999	0.9999	0.9998	0.9998	0.9999
e.	Total P/L		-0.1262	-0.6227	-1.0844	-0.7257	$\boxed{-1.0105}$

Exercise 2.

a. For each day, compute the P&L as:

$$\text{Daily P\&L} = \$1\text{mn} \times 0.25 \times \frac{P^{fut}(t+dt,T) - P^{fut}(t,T)}{100} \times 100 \text{ contracts} \tag{30}$$

b. Total P&L $= \$1\text{mn} \times 0.25 \times (6.37\% - 6.41\%) \times 100$ contracts $= -\$11\,250$ mn

c. In both situations i. and ii. the cash-flow is -\$112.50 per contract.

			a.	b.	c.i.		c.ii.	
Date	Price	fut	daily P/L	add P/L	CF	Margin	CF	Margin
21-Mar-07	93.6350	6.37%			-1,485.00	1,485.00	-1,485.00	1,485.00
22-Mar-07	93.6850	6.32%	125.00	125.00	0.00	1,610.00	125.00	1,485.00
23-Mar-07	93.5700	6.43%	-287.50	-162.50	0.00	1,322.50	0.00	1,197.50
26-Mar-07	93.5000	6.50%	-175.00	-337.50	0.00	1,147.50	-462.50	1,485.00
27-Mar-07	93.3200	6.68%	-450.00	-787.50	-787.50	1,485.00	-450.00	1,485.00
28-Mar-07	93.3300	6.67%	25.00	-762.50	0.00	1,510.00	25.00	1,485.00
29-Mar-07	93.3300	6.67%	0.00	-762.50	0.00	1,510.00	0.00	1,485.00
...	...	...	...	...	...	...	...	...
9-May-07	93.2450	6.76%	175.00	-975.00	0.00	1,947.50	175.00	1,297.50
10-May-07	93.0950	6.91%	-375.00	-1,350.00	0.00	1,572.50	-562.50	1,485.00
11-May-07	92.8250	7.18%	-675.00	-2,025.00	-587.50	1,485.00	-675.00	1,485.00
14-May-07	92.9350	7.07%	275.00	-1,750.00	0.00	1,760.00	275.00	1,485.00
15-May-07	92.8700	7.13%	-162.50	-1,912.50	0.00	1,597.50	0.00	1,322.50
16-May-07	92.8950	7.11%	62.50	-1,850.00	0.00	1,660.00	62.50	1,322.50
17-May-07	93.0100	6.99%	287.50	-1,562.50	0.00	1,947.50	287.50	1,322.50
18-May-07	93.0900	6.91%	200.00	-1,362.50	0.00	2,147.50	200.00	1,322.50
21-May-07	93.1200	6.88%	75.00	-1,287.50	0.00	2,222.50	75.00	1,322.50
...	...	...	...	...	...	...	...	...
14-Sep-07	93.6200	6.38%	325.00	-37.50	0.00	3,472.50	325.00	1,285.00
17-Sep-07	93.5400	6.46%	-200.00	-237.50	0.00	3,272.50	-400.00	1,485.00
18-Sep-07	93.4200	6.58%	-300.00	-537.50	0.00	2,972.50	0.00	1,185.00
19-Sep-07	93.4850	6.52%	162.50	-375.00	0.00	3,135.00	162.50	1,185.00
20-Sep-07	93.4950	6.51%	25.00	-350.00	0.00	3,160.00	25.00	1,185.00
21-Sep-07	93.5900	6.41%	237.50	**-112.50**	0.00	3,397.50	237.50	1,185.00
Total			**-112.50**		**-112.50**		**-112.50**	

Exercise 3.

a. Yes, the rates did converge: Libor equals the futures rate at expiry.

b. Total profit from futures: $0.475 = 0.25 \times (97.2912 - 95.3900)$

c. Total profit from forward: $0.547 = 100 \times (0.9933 - 0.9878)$

d.,e.,f. See Figure 3.

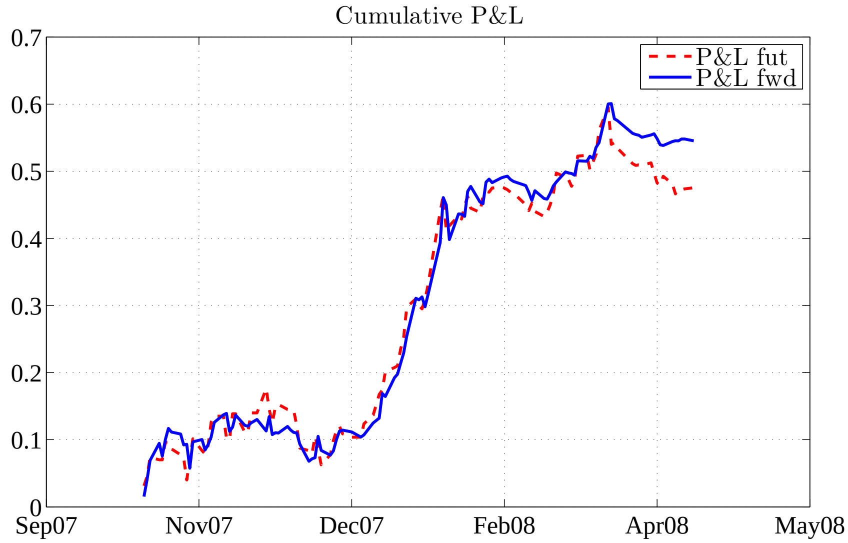

Fig. 3. P&L from the futures and forward contracts

Exercise 4.

Use futures and forward prices available on October 16, 2007:

a.	P^{fut}	95.39
	$f_4^{fut}(t, T_1)$	4.61%
	Min. price to accept:	$\frac{100}{1+f_4^{fut}(t,T_1)/4} = \$98.86$

b.	P^{fwd}	98.78
	$f^{fwd}(t, T_1, T_2)$	4.94%
	Min. price to accept:	$\frac{100}{1+f^{fwd}(t,T_1,T_2)/4} = \$98.78$

Exercise 5.

		Price	P&L	Total	3mo Libor	Total amount available to the firm after 90 days	
a.,c.	Futures	98.86	0.475	\$99.34	2.7088%	$99.34 \times (1 + 2.7088\%/4) =$	\$100.009mn
b.,d	Forward	98.78	0.547	\$99.33	2.7088%	$99.33 \times (1 + 2.7088\%/4) =$	\$100mn

Note: Use the results from Exercise 3 for the P&L.

Exercise 6.

a.i. The empirical probability that the firm has enough cash for the lawsuit is 79% (see Figure 4, a.i).

a.ii The firm would have enough cash in 54% of scenarios (see Figure 4, a.ii).

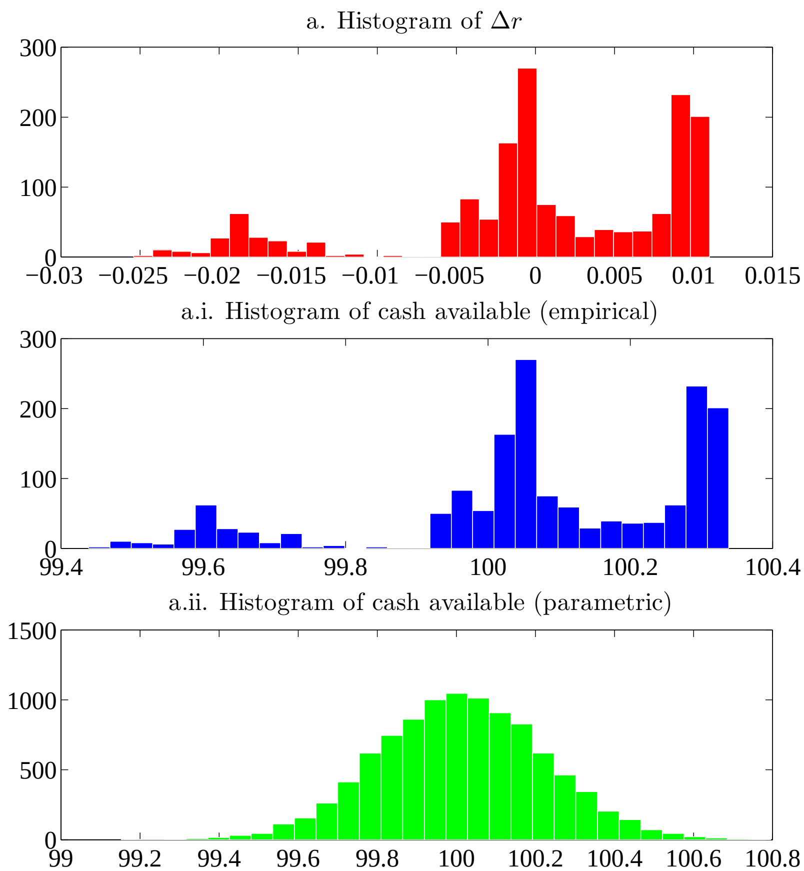

Fig. 4. Distribution of possible cash flows for the firm

α	0.00404
β	0.88152
s.e.	0.00853
$r(0)$	5.2088%

b. Ex post the company will not have enough cash: $98.78 \times (1 + 2.7088\%/4) = 99.4489$.

Exercise 7.

	Version	Price	P&L	Total	3mo Libor	After 90 days	Final ($ mn)
a.	Tbl 6.11	$98.86	0.475	99.336	2.7088%	100.009	0.009
b.	Tbl 6.12	$98.86	-0.475	98.385	6.5112%	99.987	-0.013

Under scenario in Table 6.12, the firm is short on the cash they need, receiving 99.987 out of 100 required.

Exercise 8.

Now, we account for the investing (borrowing) of the futures proceeds (to cover the futures losses). The P&L changes, and so does the amount available to the firm for the lawsuit. The last column compares the current results to the case when futures proceeds are not invested nor losses are covered by borrowing.

	Price	P&L	Total	3mo Libor	After 90 days	Final ($ mn)	Ex(8)-Ex(7)
a., b.	$98.86	0.482	99.342	2.7088%	100.015	0.015	0.00657
c., d.	$98.86	-0.482	98.379	6.5112%	99.980	-0.020	-0.00663

Exercise 9.

	Version	Price	Tailed P&L	Total	3 mo Libor	After 90 days	Untailed Ex(7)	Ex(9)-Ex(7)
a.	Tbl 6.11	$98.86	0.475075	99.335706	2.7088%	100.008	100.009	-0.000226
b.	Tbl 6.12	$98.86	-0.475075	99.335706	6.5112%	99.987	99.987	0.000229

Exercise 10.

a. The securities are not correctly priced, as the put-call parity is violated:

Put	$0.1044
Call	$0.2934
$P^{fwd}(0, 0.5, 0.75)$	98.96
K	99.12
$Z(0, 0.5) \times (P^{fwd} - K)$	-0.1590
Call from P/C parity	($0.0546)

b. Strategy long call, short put, short forward gives a positive cash-flow of $0.3480 at no risk.

Solutions to Chapter 7

Exercise 1.

a.,b. See Figure 5.

c. Sum of squared errors is lower for the second model:

Model 1: $SSR_1 \times 10^4 = 1.8692$

Model 2: $SSR_2 \times 10^4 = 0.8644$.

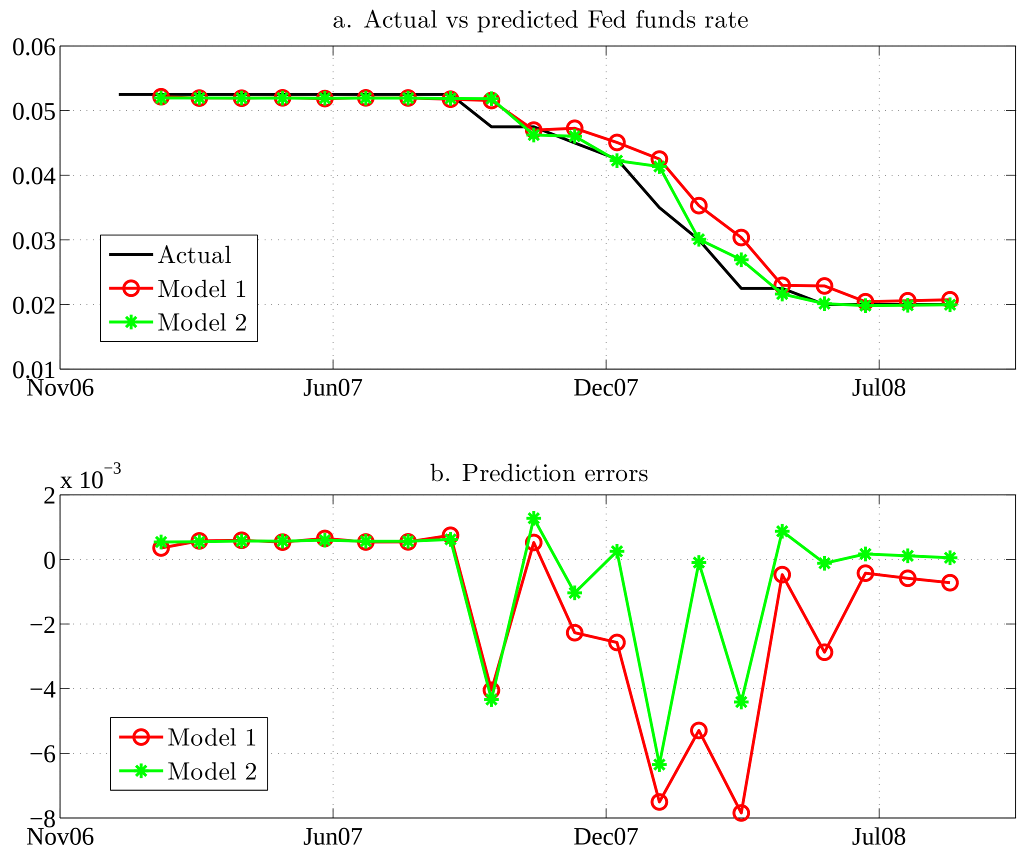

Fig. 5. Prediction of the Fed funds rate from two models

Exercise 2.

a. Using Equation 7.28, minimize squared distance between the observed and the model-based prices (computed from equation 7.27) to obtain the parameters in the table below. Using the parameters compute real yields and the discount factors (see Figure 6).

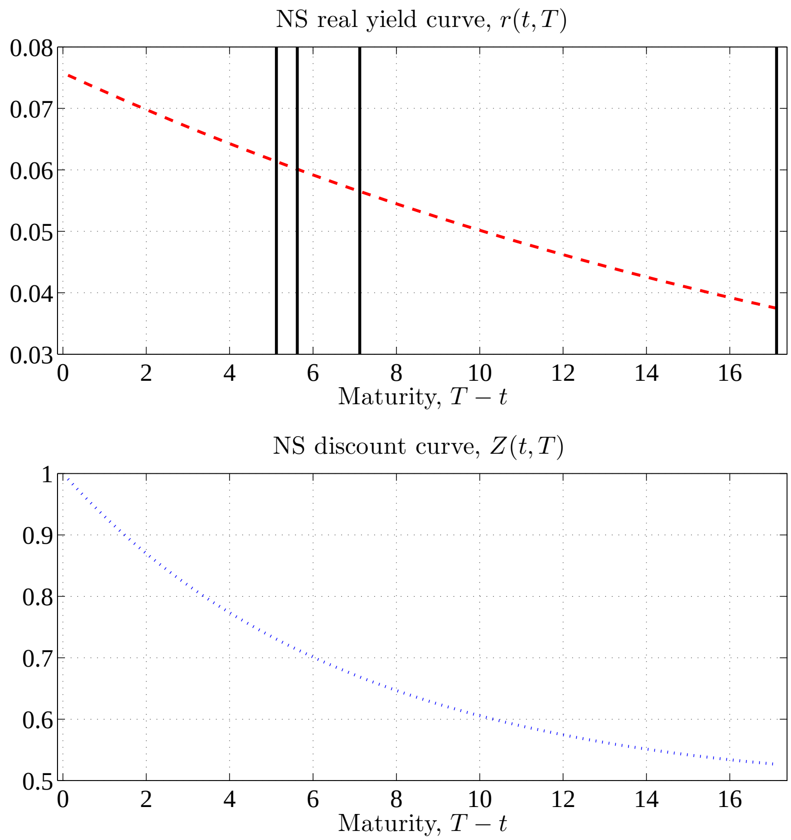

Fig. 6. NS yields and discount factors. Vertical lines indicate the maturities of bonds used in estimation.

Parameter	Value
θ_0	6278.301
θ_1	-6278.23
θ_2	-6289.19
θ_3	-0.18757
κ_1	27056.49
κ_2	32.19053

b. Obtain the analytical expression for the short interest rate by taking the limit of $r(0,T)$ implied by the Nelson-Siegel model:

$$\lim_{T\to 0} r(0,T) = \theta_0 + \theta_1 = 7.58\% \tag{31}$$

Use the fact that:

$$\lim_{T\to 0} \frac{1-e^{-T/\kappa_1}}{T/\kappa_1} = 1 \text{ and } \lim_{T\to 0} = e^{-T/\kappa_1} = 1. \tag{32}$$

c.,d. Using the NS yield curve, the TIPS with maturity 3.372 years (maturing 4/14/2012) is priced at \$86.23. This is \$2.755 lower relative to the traded price.

Exercise 3.

a. In real terms, but not in nominal terms, the value of the total position is zero. In real terms, you essentially go short and long the same portfolio of strips.

b. Make sure you use the dirty prices to compute the real curve. The nominal price of each position is obtained as $\hat{P} \times \text{IndexRatio}$:

	$\hat{P}$	IndexRatio	Price
strip1	4.63	1.18406	5.48
strip3	4.63	1.16067	5.37
			-0.11

You have to pay money.

c. Cash-flow on 01/15/09

	IndexRatio	Coupon 2% semi	Nominal CF
strip1	1.16196	1	1.16
strip3	1.08173	1	1.08
			-0.08

d. It is like holding inflation risk: if inflation increases, you lose on the position.

Exercise 4.

a. The ratio is:

$$\frac{\text{Price strip 1}}{\text{Price strip 3}} = \frac{5.48}{5.37} = 1.0202 \tag{33}$$

You have to purchase 1.0202 of Security 3.

b.,c. The nominal coupon stream from the Security 3 will be:

$$1.0202 \times 1\% \times \text{IndexRatio} = 1.18 \tag{34}$$

Therefore:

	IndexRatio	Coupon 2% semi	Nominal CF	Real stream
strip1	1.18406	1	1.18	1
strip3	1.16067	1	1.18	1.0202
			0.00	**0.0202**

d. In real terms, the price will be $-4.63 + 1.0202 \times 4.63 = 0.09$ units. You cannot back out the nominal price of the position because of different index ratios for both securities.

Exercise 5.

b. If there is no inflation, the security will pay real coupon $\times 1.00031$, as the index ratio will not change. If there is a deflation, and should the index ratio for the security go below 1, it will be automatically reset to 1. In this case, the security will pay an amount equal to the real coupon.

c. The bond is more valuable to investors. Compared to the other securities, the bond offers a deflation floor, and this attractive feature will be priced in.

d. The price of the security using the NS real curve is 89.57. (You have to compute dirty prices of bonds first.)

e. The squared pricing error for this security is

$$(\hat{P}^{NS} - \text{Dirty}\hat{P})^2 = (89.57 - 100.7)^2 = 123.92, \tag{35}$$

and is much larger than on all other securities.

f. The model-implied clean price is: $(89.57 - 0.85) \times 1.00031 = 88.74$. The squared error on this price is: $(88.74 - 99.88)^2 = 123.99$.

g. Yes.

Solutions to Chapter 8

Exercise 2.

Follow the lines of Example 8.2 and Table 6.3. Plug in the NS parameters to compute the discount curve.

a. The price is \$316.39 mn, and is above the \$300 mn par value of the security.

b. Under the (unrealistic) assumption of constant PSA, you can apply the definition of duration in Chapter 3, and compute it in a standard way (see Section 3.2.3). The duration is 6.47.

c.,d. Compute the prices of the pass through under the different scenarios taking into account the parallel shift in the curve and the change in the PSA. Use definitions 8.1 and 8.2 to obtain the effective duration and convexity, respectively.

Prices under the three scenarios:	
P(dr = 0bps, PSA=150%)	316.39
P(dr = +50bps, PSA=120%)	306.68
P(dr = −50bps, PSA=200%)	323.89

Effective Duration	5.44
Effective Convexity	-278.44

The effective duration is lower than the one obtained under the assumption that the change in rates does not affect the PSA. Standard duration overstates the sensitivity of the MBS price to changes in interest rates.

In contrast to the Treasury bonds, the convexity of an MBS is negative (i.e. the value profile is concave with respect to interest rate changes). Therefore, convexity presents a source of risk to investors. This risk is associated with the prepayment option that homeowners have. Effectively, an MBS investor is short an American call option to the homeowners.

Exercise 3.

a. PSA = 150%	Principal	r^{PT}	"Price"	Duration
Tranche A	175	6.50%	181.53	3.88
Tranche B	75	6.50%	80.35	8.60
Tranche C	30	6.50%	32.58	11.23
Tranche D	20	6.50%	21.92	12.96
Total	300	6.50%	316.39	6.47

b.,c.	Tranche A	Tranche B	Tranche C	Tranche D
Effective Duration	3.02	7.23	9.97	12.17
Effective Convexity	-232.50	-426.40	-318.82	-56.55

Prices under the three scenarios:				
P(dr = 0bps, PSA=150%)	181.53	80.35	32.58	21.92
P(dr = +50bps, PSA=120%)	178.26	77.02	30.83	20.57
P(dr = −50bps, PSA=200%)	183.75	82.83	34.08	23.24

d. Buying all tranches is equivalent to buying the MBS (same price and duration). We assume that all tranches have the same quality.

Exercise 4.

a. PSA = 150%	Principal	r^{PT}	"Price"	Duration
Tranche A	175	6.50%	181.21	3.65
Tranche B	75	6.50%	79.97	7.73
Tranche C	30	6.50%	32.31	9.56
Tranche Z	20	6.50%	22.90	20.00
Total	300	6.50%	316.39	6.47

b.,d.	Tranche A	Tranche B	Tranche C	Tranche Z
Effective Duration	2.88	6.57	8.55	17.18
Effective Convexity	-202.65	-352.80	-279.54	-674.87

Prices under the three scenarios:				
P(dr = 0bps, PSA=150%)	181.21	79.97	32.31	22.90
P(dr = +50bps, PSA=120%)	178.14	76.99	30.81	20.74
P(dr = −50bps, PSA=200%)	183.36	82.24	33.58	24.68

Tranche Z has the largest convexity risk that is significantly higher than for tranche D in Exercise 3. Indeed, tranche Z is most sensitive to the prepayment risk.

c. Tranche Z has a higher sensitivity to interest rate changes than tranche D, and thus a higher duration. When supported with tranche Z, the duration of all other tranches A, B, C decreases compared to the case when tranche D is included.

e. Buying all tranches is equivalent to holding the MBS.

Exercise 5.

a.	Principal	r^{PT}	"Price"	Duration
PAC	181.34	6.50%	190.26	5.57
S (support)	118.66	6.50%	126.13	7.81
Total	300.00	6.50%	316.39	6.47

b.,c.	PAC	S
Effective Duration	5.58	5.23
Effective Convexity	48.41	-771.50

Prices under the three scenarios:		
P(dr = 0bps, PSA=150%)	190.26	126.13
P(dr = +50bps, PSA=120%)	185.07	121.61
P(dr = −50bps, PSA=200%)	195.68	128.21

The two tranches have similar durations, but they have very different convexities. The PCA has deterministic future cash-flows, and thus can be priced as any coupon bond. Its value is not affected by the prepayment risk. The support tranche absorbs all the prepayment risk, which is reflected in the strongly negative convexity.

d. Buying all tranches is equivalent to holding the MBS.

Exercise 6.

a.	"Price"	Duration
Interest Only (IO)	127.56	5.54
Principal Only (PO)	188.82	7.09
Total	316.39	6.47

b.,c.	IO	PO
Effective Duration	-16.66	20.37
Effective Convexity	-1678.04	667.08

Prices under the three scenarios		
P(dr = 0bps, PSA=150%)	127.56	188.82
P(dr = +50bps, PSA=120%)	135.51	171.17
P(dr = −50bps, PSA=200%)	114.26	209.63

Value of the IO drops when interest rates decline and the PSA increases. The PO must increase dramatically, to counterbalance this effect and to ensure the higher total value of the pass through. Therefore, IO has a negative duration, while PO has a positive duration. In general, PO is a bullish instrument that benefits from falling interest rates. IO, instead, it a bearish security that benefits from raising interest rates. In the falling interest rate environment, the PO (IO) has a negative (positive) convexity. In the increasing rate environment, these properties reverse. In either case, the convexity, has a dampening effect in that it reduced the profits and limits the losses on each security.

d. Buying all tranches is equivalent to holding the MBS.

Exercise 7.

Tranche	A	B	C	E	G	H
Face	127.50	51.00	25.50	68.00	59.50	93.50
	a. 10/1/1993, PSA = 450%					
Price	119.76	36.06	11.38	57.83	51.09	72.48
Duration	1.99	4.81	7.53	2.82	2.71	2.86
	b. 4/4/1994, PSA = 450%					
Price	108.52	34.45	9.55	53.76	47.80	70.37
Duration	1.67	4.30	6.93	2.43	2.31	2.23
	c. 4/4/1994, PSA = 200%					
Price	108.52	34.45	9.55	53.76	38.55	23.55
Duration	1.67	4.30	6.93	2.43	2.70	7.71

b.i. The interest rates have increased (in a nonparallel fashion).

Behavior of the G+H portfolio				a.	b.ii.	c.iv.
PSA				450%	450%	200%
Date				10/1/1993	4/4/1994	4/4/1994
	Tranche	Investment	Weight	Price	Price	Price
	G	50	0.98	51.09	47.80	38.55
	H	50	0.69	72.48	70.37	23.55
Port.		100		100	95.32	53.98
ΔPort.					-4.68	-46.02

c.i. Yes, the term structure has increased, so the PSA has declined (lower prepayment speed).

c.ii. No. Since all tranches are PO, the portfolio is a bet on decreasing interest rates (it wins when interest rates decline). Between October and April interest rates moved up, therefore the portfolio is losing money. The computed duration confirms this intuition.