Solution Manual

to accompany the textbook

Fixed Income Securities: Valuation, Risk, and Risk Management by Pietro Veronesi

Chapters 9 - 13

Preliminary Version

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Solutions to Chapter 9

Exercise 1.

a. The expected return is equal to 2.5%.

b. The forward rate (continously compounded) is equal to 3.0954%. This is lower than the expected rate computed in Part (a). If we observe high forward rates it may be because of two posibilities: either market participants expect higher future interest rates; or they are strongly averse to risk, and thus the price of long term bonds is low today.

c. The market price of risk equals: λ = -0.1980. The high (negative) market price of risk, means that market participants have high risk aversion, which may explain the price of long term bonds today.

d. The risk neutral probability equals: $p* = 0.7000$. The interpretation is the same as in Part (c).

Exercise 2.

a. All three methodologies give the same answer for the option: $V_0 = 1.3861$.

b. The price of the bond option is: $V_0 = 0.3328$.

Exercise 3.

a. The value of the zero coupon bond maturing at $i = 1$ is 98.0199. The value of the zero coupon bond maturing at $i = 2$ is 95.8417.

b. The contiously compounded yield (y_i) for each bond is: $y_1 = 0.0400$ and $y_2 = 0.0425$.

c. The value of the option is $V_0 = 0.9802$.

d. The replicating portfolio holds (N_1) 1.3434 of the bond maturing at $i = 1$, and (N_2) -1.3637 of the bond maturing at $i = 2$. The following table summarizes this:

	Price	Position	Total
P_z(0,1)	98.0199	1.3434	131.6758
P_z(0,2)	95.8417	-1.3637	-130.6956
		Portfolio	0.9802

Of course the value of the replicating portfolio is the same as the option. When interest rates go up from 4% to 6% we have that:

	Price	Position	Total
P_z(1,1)	100.00	1.3434	134.34
P_z(1,2)	97.0446	-1.3637	-132.34
		Portfolio	2.00

When interest rates go down from 4% to 3% we have that:

	Price	Position	Total
P_z(1,1)	100.00	1.3434	134.34
P_z(1,2)	98.5112	-1.3637	-134.34
		Portfolio	0.00

Where both scenarios replicate the payoffs of the option.

e. Using the short-term bond and the option we have:

	Price	Position	Total
P_z(0,1)	98.0199	0.9851	95.5605
P_z(0,2)	0.9802	-0.7333	-0.7188
		Portfolio	95.8417

Where 95.8417 is, as seen before, the price of the bond.

Exercise 4.

You don't have enought information to compute the market price of risk, since everything is done in the risk neutral world. Specifically, note that if we have equation (9.21):

$$\lambda_0 = \frac{e^{-r_0 \times \Delta} E[P_1(2)] - P_0(2)}{P_{1,u}(2) - P_{1,d}(2)} \tag{1}$$

To compute this we need $E[P_1(2)]$, which includes the risk natural probabilities which we don't have. In order to compute λ we need these.

Exercise 5.

a. The value of r_0 is: 5%

b. The following three pairs of values for ($r_{1,u}$, $r_{1,d}$) are consistent with the two bond prices: (7.010, 3.005), (7.000, 3.015) and (6.000, 4.000). In fact the relationship between both interest rate scenarios is linear and can be summarized by the following equation:

$$r_{1,d} = -0.9852 \times r_{1,u} + 9.9115 \tag{2}$$

Note that this is consistent with the idea of risk neutrality. In order to compensate a decrease (increase) in the up state, the down side must increase (decrease); given that the risk neutral probabilities are fixed.

c. The option gives an information on actual prices, so we can infer the market price of risk and therefore can pin down the actual values for ($r_{1,u}$, $r_{1,d}$). These values should be consistent in order to value the option at the given price. The values for ($r_{1,u}$, $r_{1,d}$) are: (7.000, 3.015).

d. If you didn't know $p*$ you would also need the value of the interest rate in the up and down states or the prices of the bonds in these states.

Solutions to Chapter 10

Exercise 1.

a. The bond evolves in the following way:

90.0000	93.2394	100.000
	97.0446	100.000
		100.000

b. The market price of risk is: λ = -0.3709

c. The anwers are the following:

   i. The market price of risk is: λ = -0.3709. Which is the same as the one computed from the bond, this is because the market price of risk is the same across securities since it measures the willingness of agents to hold risky assets. In other words it is a measure on the economic agents and not on the instruments themselves.

   ii. The price of the option is 0.2238.

   iii. The result holds the same.

d. The price of the option is 0.0245.

e. The option has the following structure over time:

0.0245	0.0000	0.0000
	0.2234	0.0000
		2.0199

The positions on the short-term bond (N_1) and on the three-period bond (N_2) vary over time, in order to replicate the option structure, in the following way:

N_1		N_2	
-0.0244	0.0000	0.0287	0.0000
	-0.6632		0.6972

Exercise 2.

a. Using the information provided the risk neutral probability of moving up the tree is 0.7, as pointed out in Case 1. The tree for the 3-year zero coupon bond is the following:

85.8139	87.3940	91.3931	100.00
	93.8042	96.0789	100.00
		98.0199	100.00
			100.00

b. The expected return on holding the 2-year bond for a year is: 5.34%, and the expected return on holding the 3-year for a year is: 7.07%.

c. The answers are the following:

   i. The value of the range bond is: 101.92.

   ii. The 9% VaR of the bond is -10.53, which is -10.33% of the original investment on the bond.

   iii. You should use p to compute the VaR, since VaR is an estimate of loss in the future, the *actual* future. In order to make this estimate we need the risk *natural* probabilites in order to be able to compute the loss we could face. Using the wrong probabilies might underestimate the loss. In this specific case it does, if we use risk *neutral* probabilities we get -3.90 as the 9% VaR.

   iv. If we compared it with a regular coupon bond, we would see that it is cheaper. If there is a high expectation, as it occured around 1993, that interest rates will be within the range for a long time they are very attractive.

Exercise 3.

The following results vary greatly with the dataset used. The results presented use values from September 1, 2009. Which are the following:

LIBOR 3-month

I	1999	5.009%	I	2003	1.288%	I	2007	5.348%
II	1999	5.355%	II	2003	1.116%	II	2007	5.359%
III	1999	6.083%	III	2003	1.160%	III	2007	5.621%
IV	1999	6.005%	IV	2003	1.157%	IV	2007	5.131%
I	2000	6.289%	I	2004	1.111%	I	2008	3.058%
II	2000	6.778%	II	2004	1.604%	II	2008	2.681%
III	2000	6.816%	III	2004	2.005%	III	2008	2.811%
IV	2000	6.403%	IV	2004	2.558%	IV	2008	2.217%
I	2001	4.877%	I	2005	3.100%	I	2009	1.264%
II	2001	3.791%	II	2005	3.505%	II	2009	0.656%
III	2001	2.597%	III	2005	4.006%	III	2009	0.348%
IV	2001	1.883%	IV	2005	4.530%			
I	2002	2.031%	I	2006	4.990%			
II	2002	1.860%	II	2006	5.509%			
III	2002	1.806%	III	2006	5.373%			
IV	2002	1.383%	IV	2006	5.360%			

Swaps and Eurodollar Futures

Maturity	Swap rate	Maturity	Eurodollar Futures
1 year	0.61%	3 months	99.535
2 year	1.29%	6 months	99.305
3 year	1.91%	9 months	98.945
4 year	2.37%	12 months	98.555
5 year	2.72%		
7 year	3.20%		

a. The current 3-month interest rate is 0.3475%, when converted to continous compounding we get: 0.3473%. The regression gives the following parameters (t-stats in parenthesis): $\alpha =$ -0.0003 (-0.1497) and $\beta = 0.97763$ (19.7565). The predicted value for next quarter is $m_{t+i} = 0.3396\%$.

b. We estimate $\sigma = 0.006055$. The tree for the first 3 ($i = 3$) periods looks like this:

$i = 0$	2	3	4
0.35%	0.74%	1.12%	1.50%
	0.13%	0.52%	0.90%
		-0.09%	0.29%
			-0.31%

c. The zero coupon yield curve can be summarized with the following discounts:

$Z(0, 0.25)$	99.9132
$Z(0, 0.50)$	99.7533
$Z(0, 0.75)$	99.5200
$Z(0, 1.00)$	99.3922
$Z(0, 1.25)$	99.0297
$Z(0, 1.50)$	99.0640
$Z(0, 1.75)$	98.0851
$Z(0, 2.00)$	97.4451
$Z(0, 2.25)$	96.7885
$Z(0, 2.50)$	97.2324
$Z(0, 2.75)$	94.8043
$Z(0, 3.00)$	94.3874

These values were computed with the Eurodollar futures and swap data.

d. With the emprical σ you get probabilites outside of the [0,1] range. If you increase σ enough you can go into the range. In this specific case, volatility had to be increased by 10 percentage points, to get the first eight periods to be within the range.

e. When the probabilities are working as they should, we get that the expected risk neutral rate is higher than the one predicted by the regression. This makes sense in order to make economic agents risk neutral, they must be compensated with enough upside. Recall the discussion at the end of chapter 9 (section 9.4.4).

Solutions to Chapter 11

Exercise 1.

a. The zero coupon spot curve for all possible maturities is:

$Z(0,1)$	0.9608
$Z(0,2)$	0.9081
$Z(0,3)$	0.8462

b. The price of the security is: 45.4073.

c. The price of the floor is: 1.1809.

d. The expected change in the spread is zero.

e. The price of the spread call option is: 1.1152.

Exercise 2.

a. The zero coupon spot curve for all possible maturities is:

$Z(0,1)$	0.9802
$Z(0,2)$	0.9584
$Z(0,3)$	0.9377

b. The price of the fixed coupon bond is: 102.25.

c. The price of the floating rate bond is: 100, which is consistent with has been explained in previous chapters about floating rate bonds.

d. The value of the fixed rate payer swap is: -2.2523.

e. The value of the 3-year cap is: 0.2682.

Exercise 3.

a. The price of the zero coupon bonds is:

$Z(0,1)$	0.9608
$Z(0,2)$	0.9141
$Z(0,3)$	0.8659

b. The swap rate $c(3)$ is: 4.89%.

c. For the option described in the exercise:

   i. The value of the option is: 0.1532.

   ii. In order to hedge this security:

1. Choose two securities to hedge this one, for example the 1-year zero coupon bond and the 2-year zero coupon bond.
2. Compute the values for N_1 and N_2 which give the amount of each security that we need to buy. In this case the values are N_1 = 8.1358 and N_2 = -8.3835.
3. Verify that this strategy actually gives the value of the bond:

$$[N_1 \times Z(0,1)] + [[N_2 \times Z(0,2)] = 0.1532$$

You can also verify that this replicates the up and down states, by using the inputs for $i = 1$:

* up. When interest rates go from 4% to 7% we have that:

$$[N_1 \times Z_{up}(1,1)] + [[N_2 \times Z_{up}(1,2)] = 0.3190$$

* dn. When interest rates go from 4% to 3% we have that:

$$[N_1 \times Z_{dn}(1,1)] + [[N_2 \times Z_{dn}(1,2)] = 0.0000$$

Which matches exactly the payoffs of the option.

d. For the Procter & Gamble leveraged swap we have:

i. The value of the security is: -27.5495.

ii. Intuitively the value of $\bar{r}$ that makes the swap value zero should be higher than $c(3)$. P&G is paying more now, through the states in which it has to pay the spread, which means that in order to be compensated it should expect a higher rate from the fixed rate that Bankers Trust pays.

iii. Using MS Excel Solver we find the value of this rate to be: 14.94%.

Exercise 4.

a. The term structure of interest rates is summarized in the following zero coupons:

$Z(0,1)$	0.9608
$Z(0,2)$	0.9186
$Z(0,3)$	0.8742

b. The swap rate $c(3) = 4.57\%$.

c. For the index amortizing swap we have:

i. Intuitively the fixed payer is better off because when rates go down, which means that the fixed payer has to pay, also this notional goes down. This results in lower cash outflows for the fixed rate payer.

ii. The value of the swap assuming $c(3)$ is: 0.8388.

iii. The swap rate is 4.96%. This rate is higher than the $c(3)$ for the same reason pointed i. Since the fixed rate payer is better off, it should be expected that - in order to bring the value of the swap back to zero - he should pay a higher rate to the floating rate payer.

iv. The hedging values using the plain vanilla swap (maturing in $i = 3$) and the short term zero coupon bond are: $N_1 = 0.5857$ and $N_2 = 0.6115$. Yet this doesn't hedge the security perfectly.

Exercise 5.

a. The zeros from the LIBOR curve are the following:

t	$P_z(0,t)$	t	$P_z(0,t)$	t	$P_z(0,t)$
0.25	99.2904	2.75	92.0454	5.25	81.4258
0.50	98.6053	3.00	90.9898	5.50	80.3754
0.75	98.1544	3.25	89.9522	5.75	79.3266
1.00	97.6061	3.50	88.8979	6.00	78.2843
1.25	96.9954	3.75	87.8326	6.25	77.2529
1.50	96.3399	4.00	86.7628	6.50	76.2365
1.75	95.6373	4.25	85.6875	6.75	75.2401
2.00	94.8950	4.50	84.6072	7.00	74.2689
2.25	94.0613	4.75	83.5326	7.25	73.3133
2.50	93.0947	5.00	82.4744	7.50	72.3608

b. The Ho-Lee model in the first 2 years ($i = 8$):

$i = 0$	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
2.85%	2.82%	2.96%	3.42%	4.08%	4.67%	5.28%	5.87%	6.67%
	2.04%	2.17%	2.63%	3.30%	3.89%	4.50%	5.08%	5.89%
		1.39%	1.85%	2.51%	3.11%	3.72%	4.30%	5.11%
			1.07%	1.73%	2.33%	2.93%	3.52%	4.32%
				0.95%	1.54%	2.15%	2.73%	3.54%
					0.76%	1.37%	1.95%	2.76%
						0.59%	1.17%	1.98%
							0.39%	1.19%
								0.41%

c. The following table compares the risk neutral expected future interest rate with the forward rate.

	$\sigma = 0.0078$		$\sigma = 0.05$		$\sigma = 0.001$	
t	$E^*[r_t]$	$f(t\text{-}1,t)$	$E^*[r_t]$	$f(t\text{-}1,t)$	$E^*[r_t]$	$f(t\text{-}1,t)$
0	2.849%	2.849%	2.849%	2.849%	2.849%	2.849%
0.25	2.430%	2.430%	2.438%	2.430%	2.430%	2.430%
0.5	2.174%	2.173%	2.204%	2.173%	2.173%	2.173%
0.75	2.243%	2.241%	2.311%	2.241%	2.241%	2.241%
1	2.514%	2.511%	2.636%	2.511%	2.511%	2.511%
1.25	2.717%	2.712%	2.908%	2.712%	2.712%	2.712%
1.5	2.935%	2.928%	3.209%	2.928%	2.928%	2.928%
1.75	3.126%	3.117%	3.499%	3.117%	3.117%	3.117%
2	3.542%	3.530%	4.029%	3.530%	3.530%	3.530%
2.25	4.147%	4.132%	4.764%	4.132%	4.132%	4.132%
2.5	4.553%	4.534%	5.315%	4.534%	4.534%	4.534%
2.75	4.637%	4.614%	5.558%	4.614%	4.614%	4.614%

Note that as volatility increases, so does the difference between the rates.

d. The value of the 1-year cap is 0.2374, the value of the 2-year cap is 0.6003, and the value of the 3-year cap is 1.3805. In general the model underestimates the price of the securities.

e. The value of the swap is zero, as expected.

f. The value of the swaption is: 1.9956.

Exercise 6.

a. The value of the corridor note is 100.092.

b. The coupon for a fixed coupon bond that generates a similar price to the corridor note is 3.83%. This makes sense since the corridor note only pays coupons in some states; so in order to be equally priced the fixed coupon bond, which pays in all states, must pay a lower coupon.

c. Spot rate duration of the corridor note is 7.77, while the spot rate duration on the fixed coupon bond is 4.35. The duration on the corridor note is higher because the variation of rates affects the coupon payment, making it even more sensitive to changes in interest rates.

d. See Figure 1. Convexity on the coupon bond is 18.02, whereas the convexity on the corridor note is -193.40. The corridor note appears to have negative convexity because when rates increase the price of the bond falls even more so, since coupon payments stop. Also when interest rates decrease coupon payments stop, reducing the usual upside for bonds in this scenario.

e. The price of the corridor note according to the simple BDT model is 80.218. This is consistent with previous findings in which Ho-Lee tends to overestimate (at least in the short run), while BDT underestimates.

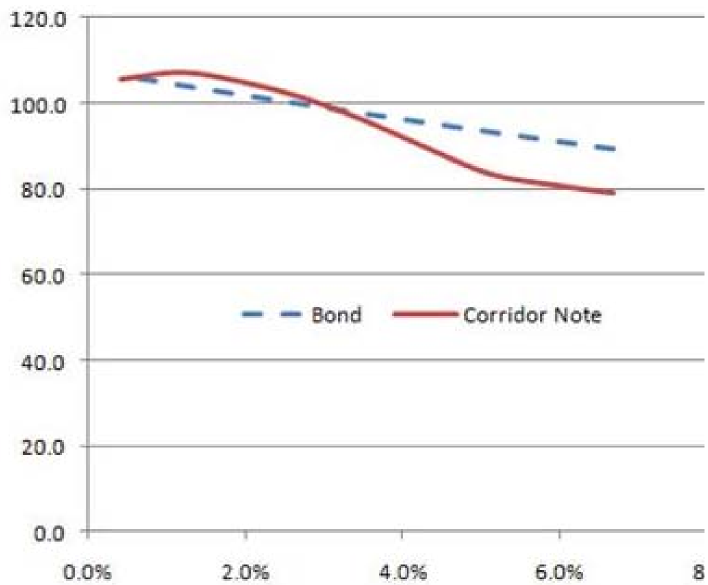

Figure 1: Corridor note and fixed coupon bond

Exercise 7.

a. The following table reports the first values of the LIBOR curve, in discount factors:

t	$P_z(0,t)$
0.50	99.6053
0.75	99.4966
1.00	99.3555
1.25	99.1319
1.50	98.8526
1.75	98.5171
2.00	98.1323
2.25	97.6366
2.50	96.9852

b. The implied volatilities are:

t	$\sigma_t^{implied}$
0.50	0.1775
0.75	0.2288
1.00	0.3460
1.25	0.4298
1.50	0.4616
1.75	0.4804
2.00	0.4801
2.25	0.4788
2.50	0.4752

c. The forward volatilities are:

t	$\sigma_t^{forward}$
0.50	0.2925
0.75	0.2975
1.00	0.3054
1.25	0.2597
1.50	0.2054
1.75	0.1926
2.00	0.1813
2.25	0.1826
2.50	0.1659

d. The price of the corridor note is 80.218.

Solutions to Chapter 12

Exercise 1.

a. The value of the non-callable bond is 100.13 and the value of the callable bond is 99.74. The non-callable bond is more expensive because it doesn't have the prepayment risk that the callable bond has.

b. For the 2-year mortgage we have:

   i. The annual payment is 5,375.87.

   ii. See the following (the value of the mortgage is without prepayment:

$i =$	0	1	2
Interest paid	0.00	497.14	254.60
Principal paid	0.00	4,878.73	5,121.27
Principal outstanding	10,000.00	5,121.27	0.00
Mortgage without prepayment	9,973.14	5,088.18 5,129.05	0.00 0.00 0.00

   iii. The option value is:

$i =$	0	1	2
Value of option	3.70	0.00 7.78	0.00 0.00 retired

   It is optimal to exercise in node (1,1), since the exercise value, 7.78, is higher than the wait value, 0.00.

   iv. The value of the mortgage with prepayment is:

$i =$	0	1	2
Mortgage with prepayment	9,969.44	5,088.18 5,121.27	0.00 0.00 retired

   The homeowner is compensating the lender for the time value of money and the prepayment risk.

c. Duration for the mortgage is calculated by the following formula:

$$D \approx -\frac{1}{P_0} \times \frac{P_{1,u} - P_{1,d}}{r_{1,u} - r_{1,d}} = -\frac{1}{9,969.44} \times \frac{5,088.18 - 5,121.27}{5.5\% - 4.7\%} = 0.4149$$

Exercise 2.

a. The value of the American swaption is 0.2101.

b. The value of the callable bond is 99.74.

c. In order to *only* hedge the prepayment risk you hedge the underlying option, intead of the callable bond itself. The hedging strategy is the following:

$i =$	0	1
Short-term bond (N1)	0.0077	0.0012
American swaption (N2)	-1.6144	-0.1252

Note that there is no hedging strategy for node (1,1) since the option is retired at that point.

d. To hedge the interest rate risk (which includes prepayment risk) of the callable bond the investor can use the following strategy:

$i =$	0	1
Short-term bond (N1)	1.0525	1.0525
American swaption (N2)	-1.8073	-0.9418

As in the previous part, there is no hedging strategy for node (1,1).

Exercise 3.

a. The results on the mortgage are as follows:

i. The value of the coupon is $C = 11,132.65$. The stream of scheduled interest payments, principal payments, and the remaining principal is:

i	t	Interest paid	Principal payments	Remaining principal
0	0.0	0.0	0.0	100,000.00
1	0.5	2,000.00	9,132.65	90,867.35
2	1.0	1,817.35	9,315.31	81,552.04
3	1.5	1,631.04	9,501.61	72,050.43
4	2.0	1,441.01	9,691.64	62,358.79
5	2.5	1,247.18	9,885.48	52,473.31
6	3.0	1,049.47	10,083.19	42,390.12
7	3.5	847.80	10,284.85	32,105.27
8	4.0	642.11	10,490.55	21,614.72
9	4.5	432.29	10,700.36	10,914.37
10	5.0	218.29	10,914.37	0.00

ii. The interest rate tree at semi-annual frequency, for the first eight periods, is:

$i = 0$	1	2	3	4	5	6	7	8	9
1.74%	2.87%	4.69%	6.39%	8.72%	10.67%	12.17%	13.72%	16.19%	19.89%
	2.17%	3.53%	4.82%	6.58%	8.04%	9.17%	10.34%	12.20%	14.99%
		2.66%	3.63%	4.96%	6.06%	6.91%	7.79%	9.19%	11.30%
			2.73%	3.73%	4.57%	5.21%	5.87%	6.93%	8.51%
				2.81%	3.44%	3.92%	4.43%	5.22%	6.42%
					2.60%	2.96%	3.34%	3.93%	4.84%
						2.23%	2.51%	2.97%	3.64%
							1.90%	2.23%	2.75%
								1.68%	2.07%
									1.56%

– iii. The value of the mortgage without the prepayment option is:

$i = 0$	1	2	3	4	5	6	7	8	9
100,359	88,791	77,636	67,110	57,045	47,527	38,283	29,049	19,678	10,079
	91,416	80,252	69,578	59,266	49,384	39,716	30,054	20,282	10,329
		82,305	71,519	61,015	50,845	40,840	30,839	20,752	10,521
			73,031	62,377	51,981	41,713	31,447	21,113	10,669
				63,430	52,859	42,387	31,915	21,391	10,781
					53,533	42,903	32,273	21,603	10,867
						43,297	32,546	21,764	10,932
							32,753	21,886	10,981
								21,979	11,018
									11,046

iv. The value of the American option implicit in the mortgage is:

$i = 0$	1	2	3	4	5	6	7	8	9
288	33	5	0	0	0	0	0	0	0
	548	61	10	0	0	0	0	0	0
		753	114	21	1	0	0	0	0
			981	210	42	2	0	0	0
				1071	386	84	4	0	0
					1060	513	168	8	0
						907	441	149	17
							648	271	66
								364	104
									132

v. The option-adjusted value of the mortgage is: 100,071.

A. The prepayment option is going to be exercised when the value of the mortgage exceeds the outstanding principal. This occurs in the following nodes:

$i =$	0	1	2	3	4	5	6	7	8	9
0	H	H	H	H	H	H	H	H	H	H
1		X	H	H	H	H	H	H	H	H
2			R	H	H	H	H	H	H	H
3				R	H	H	H	H	H	H
4					R	X	H	H	H	H
5						R	R	X	H	H
6							R	R	R	X
7								R	R	R
8									R	R
9										R

where H stands for Hold, X for Exercise, and R for Retired.

B. Yes. Any path leading to nodes (7,5) or (9,6).

C. It is fairly priced in the sense that it takes into account the prepayment option, assuming that agents act optimally. But this may not be the case, people may forget to exercise at the right time or they may be other factors affecting their decision that are not taken into account in the model.

D. If the homeowner doesn't refinance then the value of the option is higher than before. Refinancing is done for the homeowner's benefit, if he 'forgets' to do so, it is the holder of the security that gets an extra amount of cash.

b. The results for the mortgage backed securities are as follows:

i. The value of the pass-through is 99,334. The spot rate duration is: 4.4084.

ii. The value of the PO strip is 93,989 and the value of the IO strip is 5,345.

A. The sum of the IO and PO strips equals the value of the pass-trough security.

Exercise 4.

a. The discount factors are the following:

t	$P_z(0,t)$
0.5	97.3802
1.0	94.8396
1.5	92.5016
2.0	90.3145
2.5	88.1076
3.0	85.9810
3.5	83.8522
4.0	81.7794
4.5	79.7269
5.0	77.7215
5.5	75.7400
6.0	73.8007
6.5	71.9030
7.0	70.0459
7.5	68.2339
8.0	66.4614
8.5	64.7279
9.0	63.0324
9.5	61.3743
10.0	59.7528

b. The simple BDT model for the first 5 periods is:

$t =$	0	0.5	1	1.5	2	2.5
$i =$	0	1	2	3	4	5
0	5.31%	6.08%	6.61%	7.30%	8.70%	9.91%
1		4.49%	4.89%	5.39%	6.43%	7.32%
2			3.61%	3.98%	4.75%	5.41%
3				2.94%	3.51%	3.99%
4					2.59%	2.95%
5						2.18%

c. The value of the callable bond is: 94.45. If we have a portfolio with face value 500 million, we have the value of the portfolio is 472.25 million.

d. Duration of the callable bond is 5.4552 and convexity of the callable bond is -0.8583.

e. See Figure 2.

f. Convexity of the callable bond is negative, which means that in situations of low interest rates it doesn't gain as much value as non-callable bonds. Instead in converges into the face value of the bond as the probability of prepayment increases.

g. The following answers address the hedging strategy using an American swaption.

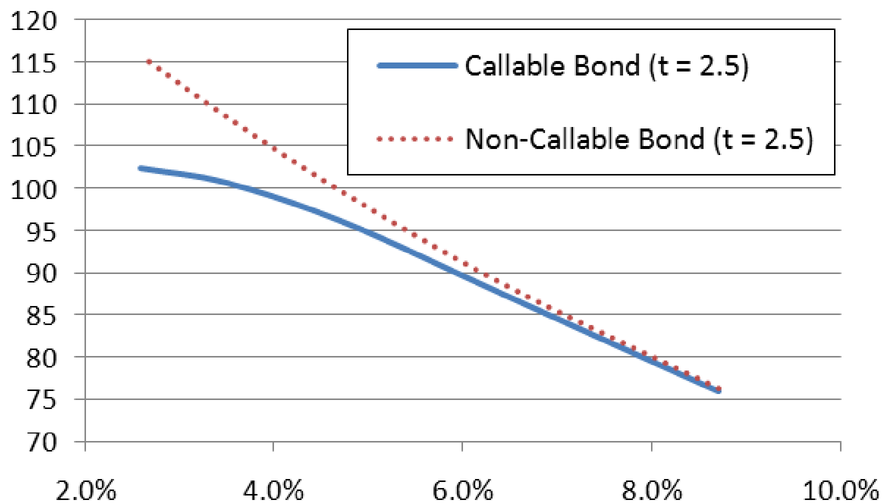

Figure 2: Callable bond and non-callable bond at $t = 2.5$

i. The price of the American swaption, with no lock-out period, is 4.38. With the lockout period is 3.99.

ii. Figure 3 plots the swaption's value. This addressed the investors concerns because it exhibits positive convexity. The opposite to what the callable bond has.

iii. Figure 4 present the results for the portfolio for various scenarios of future interest rates. Note that the total position (callable bond plus Bermudan swaption) takes the investor back to the non-callable bond.

Exercise 5.

a. The following table reports all zero coupon bonds using the bootstrap method. See Figure 5

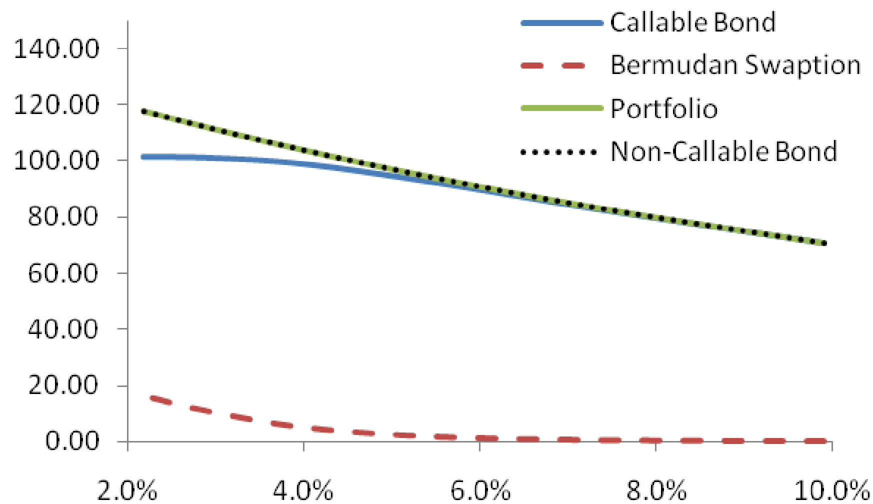

Figure 3: Swaption value at *Call time* -1

T	$P_z(0,T)$
0.5	97.58
1	95.10
1.5	92.76
2	90.49
2.5	88.26
3	86.08
3.5	84.01
4	81.96
4.5	79.89
5	77.97
5.5	76.01
6	74.07
6.5	72.11
7	70.17
7.5	68.32
8	66.60
8.5	64.95
9	63.22
9.5	61.59
10	59.77

b. The following shows the values of the BDT tree for the first 6 periods, using $\sigma = 0.2018$.

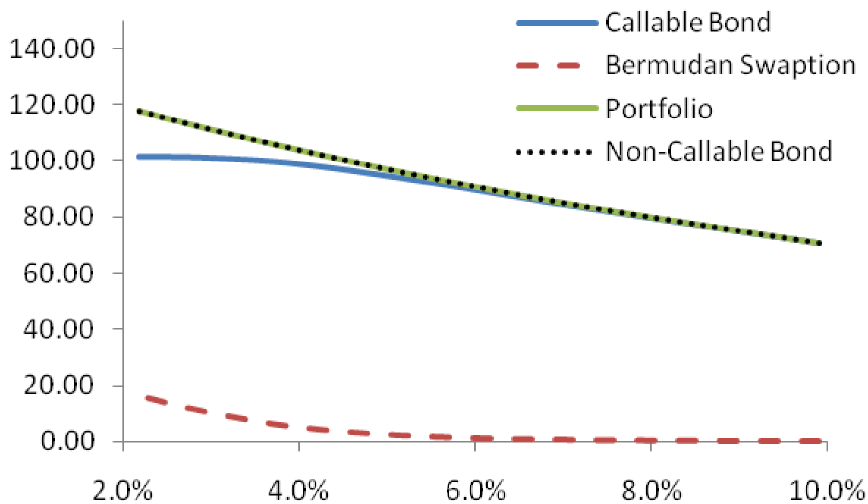

Figure 4: Scenarios for investor's portfolio

i=	0	1	2	3	4	5	6
0	4.91%	5.87%	6.51%	7.40%	8.52%	9.74%	10.92%
1		4.41%	4.89%	5.56%	6.40%	7.32%	8.21%
2			3.68%	4.18%	4.81%	5.50%	6.17%
3				3.14%	3.62%	4.14%	4.64%
4					2.72%	3.11%	3.49%
5						2.34%	2.62%
6							1.97%

c. On the Option Adjusted Spread (OAS):

  i. The price of the callable note is 98.818.

  ii. The OAS is: 26 bps.

  iii. In order to make the OAS equal to zero, $\sigma = 0.15$.

d. The price of the callable note is 42.312. The main reason is that volatilty has a bigger impact in the Ho-Lee model, which in turn raises the value of the option and reduces the value of the callable security.

e. The duration of the security is: 3.9973.

f. Convexity:

  i. The dollar spot rate duration at the required nodes is: $D^{\$}_{i+1,j} =$ 443.2313 and $D^{\$}_{i+1,j+1} = 261.7521$.

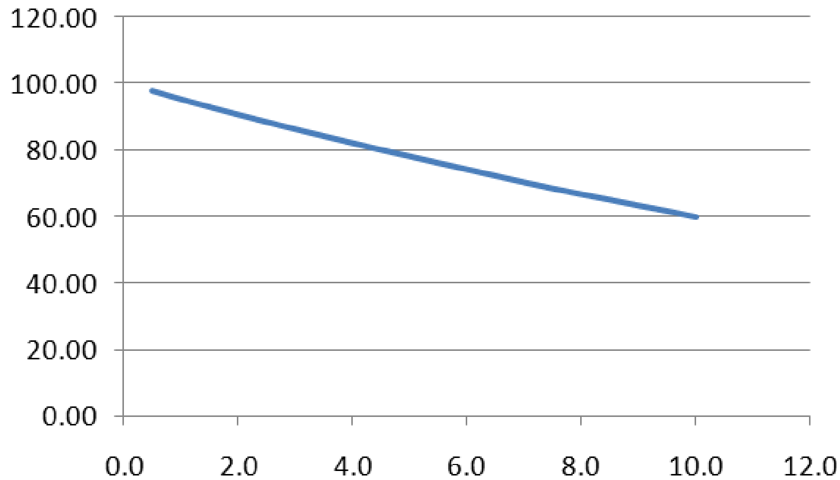

Figure 5: Price of zeros (discounts) for different maturities

ii. Spot rate convexity is then: -126.025.

j. If instead of June, 2009 the lockout period finishes June, 2008 (i.e. instead of lasting two years it lasts one year), the convexity goes to -222.62.

Exercise 6.

a. The discount curve at semiannual intervals is:

T	$P_z(0,T)$	T	$P_z(0,T)$
0.5	97.3977	11.0	56.4337
1.0	94.9996	11.5	54.9050
1.5	92.7237	12.0	53.4164
2.0	90.5208	12.5	51.9670
2.5	88.3629	13.0	50.5561
3.0	86.2352	13.5	49.1828
3.5	84.1315	14.0	47.8464
4.0	82.0500	14.5	46.5459
4.5	79.9921	15.0	45.2804
5.0	77.9603	15.5	44.0491
5.5	75.9576	16.0	42.8511
6.0	73.9871	16.5	41.6856
6.5	72.0515	17.0	40.5517
7.0	70.1531	17.5	39.4485
7.5	68.2936	18.0	38.3753
8.0	66.4744	18.5	37.3312
8.5	64.6964	19.0	36.3155
9.0	62.9602	19.5	35.3274
9.5	61.2660	20.0	34.3662
10.0	59.6137	20.5	33.4311
10.5	58.0031	21.0	32.5215

b. The BDT model is the same as in Exercise 5.

c. The price of the Bermudan zero coupon bond is 24.304. Which is different from the issue price: 26.77.

d. The spot rate duration is: 9.5067.

e. Previously we found that the model priced the 6% callable bond at 98.818, while the quoted price is 100. For the Bermudan zero coupon we found that the model price is 24.304, while the quoted price is 26.77. In both these cases the empirical volatility was used: $\sigma = 0.2018$. If instead we use $\sigma = 0.15$, which we now prices the 6% callable bond at it's quoted price, we get that the price of the Bermudan zero coupon is: 25.464. This means that there is an arbitrage oportunity.

Exercise 7.

a. The following table summarizes the discount factors:

t	$P_z(0,t)$	t	$P_z(0,t)$	t	$P_z(0,t)$	t	$P_z(0,t)$
0.25	98.35	7.00	69.56	13.75	48.08	20.50	33.14
0.50	96.87	7.25	68.63	14.00	47.42	20.75	32.69
0.75	95.50	7.50	67.71	14.25	46.77	21.00	32.24
1.00	94.23	7.75	66.80	14.50	46.13	21.25	31.80
1.25	93.02	8.00	65.90	14.75	45.50	21.50	31.36
1.50	91.87	8.25	65.01	15.00	44.88	21.75	30.93
1.75	90.75	8.50	64.13	15.25	44.26	22.00	30.51
2.00	89.66	8.75	63.27	15.50	43.66	22.25	30.09
2.25	88.59	9.00	62.41	15.75	43.06	22.50	29.68
2.50	87.53	9.25	61.56	16.00	42.47	22.75	29.27
2.75	86.48	9.50	60.73	16.25	41.89	23.00	28.87
3.00	85.44	9.75	59.90	16.50	41.32	23.25	28.48
3.25	84.41	10.00	59.09	16.75	40.75	23.50	28.09
3.50	83.38	10.25	58.28	17.00	40.19	23.75	27.70
3.75	82.35	10.50	57.49	17.25	39.64	24.00	27.32
4.00	81.33	10.75	56.71	17.50	39.10	24.25	26.95
4.25	80.32	11.00	55.93	17.75	38.56	24.50	26.58
4.50	79.30	11.25	55.17	18.00	38.04	24.75	26.22
4.75	78.30	11.50	54.42	18.25	37.52	25.00	25.86
5.00	77.30	11.75	53.67	18.50	37.00	25.25	25.50
5.25	76.30	12.00	52.94	18.75	36.50	25.50	25.16
5.50	75.32	12.25	52.22	19.00	36.00	25.75	24.81
5.75	74.34	12.50	51.50	19.25	35.50	26.00	24.47
6.00	73.36	12.75	50.80	19.50	35.02	26.25	24.14
6.25	72.40	13.00	50.10	19.75	34.54	26.50	23.81
6.50	71.44	13.25	49.42	20.00	34.07	26.75	23.48
6.75	70.49	13.50	48.74	20.25	33.60	27.00	23.16

b. The simple BDT model in the first few nodes looks like the following (using $\sigma = 0.15$):

i⟶	0	1	2	3	4	5
0	6.64%	6.54%	6.56%	6.67%	6.88%	7.18%
1		5.63%	5.64%	5.74%	5.92%	6.18%
2			4.86%	4.94%	5.10%	5.32%
3				4.25%	4.39%	4.58%
4					3.78%	3.94%
5						3.39%

c. The price of the pass-through security is: 97.49.

d. The price of the pass-through is outside the bid-ask spread. Specifically it is underpriced. This might be due to the model, as seen before the BDT model tends to underprice securities. We may need to incorporate a variance that changes over time. The OAS is: -70 bps.

Solutions to Chapter 13

Exercise 1.

a. The zero coupon spot curve for all possible maturities, through 1,000 Monte Carlo simulations, is:

t	$Z(0,t)$	standard error	conf. interval upper bound	conf. interval lower bound
1	0.9608	0.0000	0.9608	0.9608
2	0.9086	0.0009	0.9104	0.9069
3	0.8465	0.0017	0.8500	0.8430

b. The price of the security is: 45.5469 with standard error 1.3393. The confidence interval is [39.8683 45.2254].

c. The price of the 3-year floor is: 1.1847 with standard error 0.0452. The confidence interval is [1.0944 1.2751].

d. The expected change in the spread is zero with standard error 0.0010. The confidence interval is [-0.0020 0.0020].

e. The price of the spread call option is: 1.1666 with standard error 0.0376. The conficence interval is [1.0914 1.2418].

Exercise 2.

a. The monthly Ho-Lee tree looks like the following:

$t=0$	1	2	3	4	5	6	7	8	9	10	11
5.00%	5.29%	5.58%	5.87%	6.16%	6.44%	6.73%	7.02%	7.31%	7.60%	7.89%	8.18%
	4.71%	5.00%	5.29%	5.58%	5.87%	6.16%	6.45%	6.73%	7.02%	7.31%	7.60%
		4.42%	4.71%	5.00%	5.29%	5.58%	5.87%	6.16%	6.45%	6.74%	7.02%
			4.13%	4.42%	4.71%	5.00%	5.29%	5.58%	5.87%	6.16%	6.45%
				3.85%	4.13%	4.42%	4.71%	5.00%	5.29%	5.58%	5.87%
					3.56%	3.85%	4.14%	4.42%	4.71%	5.00%	5.29%
						3.27%	3.56%	3.85%	4.14%	4.43%	4.72%
							2.98%	3.27%	3.56%	3.85%	4.14%
								2.69%	2.98%	3.27%	3.56%
									2.40%	2.69%	2.98%
										2.12%	2.41%
											1.83%

b. The price of the interest rate barrier option would be 0.1316. The problem with the backward methodology is that we cannot distinguish the states that go down and out, since this is path dependant.

c. Through the BDT model we get a price of 0.0131, which is significantly lower than the previous value. This is because the BDT tends to be more concentrated around the center than the Ho-Lee model. This means that positive results are less extreme than in the Ho-Lee model. Thus the lower value.

Exercise 3.

a. The trigger rates are:

i	r_{-i}
0.5	2.17%
1.0	2.66%
1.5	2.73%
2.0	2.81%
2.5	3.44%
3.0	2.96%
3.5	3.34%
4.0	2.97%
4.5	3.64%

b. The value of the mortgage is: 100,477, which is still far from the value of the security obtained in the previous section: 100,071 (still outside of the confidence intervals). This may be because only 1,000 simulations were used.

c. The value of the 3.5% passthrough security is: 99,154.

d. Adding some probability to the model we get:

   i. Adding a 50 PSA we get that the price of the mortgage is now: 99,301.

   ii. If 80% of homeowners forget to exercise at the optimal time we get that the price of the passthrough is now: 99,294.

e. This is good news for the bank issuing the mortgage since now it gets more money. As seen in the previous question, when homeowners don't exercise optimally it means that mortgages increase in value.

f. The price of the IO strips is 9,259 with spot rate duration -3.81. The price of the PO strips is 89,895 with spot rate duration -5.95. Note that here only optimal exercise is considered.

Exercise 4.

a. The results are the same as in chapter 11, exercise 5.

b. See chapter 11, exercise 5.

c. After around 600 simulations the average error over 40 periods goes below 10 cents on the value of a zero coupon bond with face value 100.

d. The price from the tree model for a 1-year cap is 0.2401. Through the simulation approach (1000 simulations) we get 0.2381, with a a standard error of 0.0051. The price from the tree is well within the confidence intervals. Sigma used to price a 1-year cap is 0.00795.

e. The price of the Asian cap is 0.2249 with a standard error of 0.0119. Sigma for pricing a 2 year bond is 0.012354.

f. Spot rate duration for the option is -250.557.

Exercise 5.

a. In order to compute the monthly discounts from the data given we need to:

1. Bootstrap the data, in order to obtain the quarterly discounts, up to year 10. The following are the discounts:

t	$Z(0,t)$	t	$Z(0,t)$
0.25	0.9948	5.25	0.8229
0.50	0.9890	5.50	0.8131
0.75	0.9826	5.75	0.8032
1.00	0.9758	6.00	0.7932
1.25	0.9686	6.25	0.7831
1.50	0.9610	6.50	0.7729
1.75	0.9530	6.75	0.7627
2.00	0.9446	7.00	0.7524
2.25	0.9363	7.25	0.7429
2.50	0.9276	7.50	0.7334
2.75	0.9187	7.75	0.7239
3.00	0.9095	8.00	0.7144
3.25	0.9003	8.25	0.7049
3.50	0.8910	8.50	0.6954
3.75	0.8814	8.75	0.6858
4.00	0.8717	9.00	0.6762
4.25	0.8621	9.25	0.6667
4.50	0.8524	9.50	0.6571
4.75	0.8426	9.75	0.6475
5.00	0.8326	10.00	0.6380

2. Use the Extended Nelson Siegel model to compute the yield curve and discounts at a monthly basis. The parameters are the following: θ_0 = 6278.3013, θ_1 = -6278.2814, θ_2 = -6289.1879, θ_3 = 0.3114642, λ_0 = 27056.491, λ_1 = 32.191094.

Figure 6 summarizes the results.

b. Using the Ho-Lee model might produce some inconsistencies, since the negative interest rates would be outside of the ranges of the corridor bond. Additionally, using the 3-month LIBOR for the coupon payment far ahead would also bring negative cash flows. For this I use the simple BDT model. The first 6 steps of the simple BDT tree are:

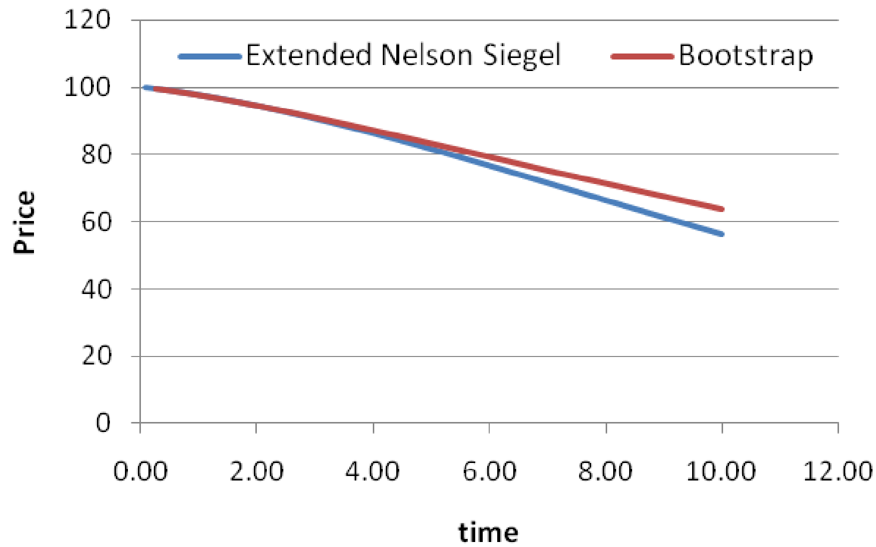

Figure 6: Discount factors from Extended Nelson Siegel model and Bootstrap method

0	0.0833	0.1666	0.25	0.3333	0.4166	0.5
0	1	2	3	4	5	6
2.02%	2.23%	2.46%	2.70%	2.96%	3.25%	3.55%
	1.97%	2.17%	2.38%	2.62%	2.87%	3.14%
		1.92%	2.11%	2.31%	2.53%	2.77%
			1.86%	2.04%	2.24%	2.45%
				1.81%	1.98%	2.17%
					1.75%	1.91%
						1.69%

c. The price of the security is: 82.71.

d. These bonds are cheaper than regular bonds. Additionally these bonds can offer value to an investor who has a view on interest rates such that they will not go above the upper barrier. Also if he thinks that the 6 month LIBOR will be lower than the 3 month LIBOR, allowing him to have a higher cash flow without going over the upper bound.

e. The spot rate duration of these notes is 13.6768. Figure 7 shows the value for different starting short rates.

Exercise 6.

a. The value of the GNSF 6 is 106.71. Which is different from the previous price of 98.76.

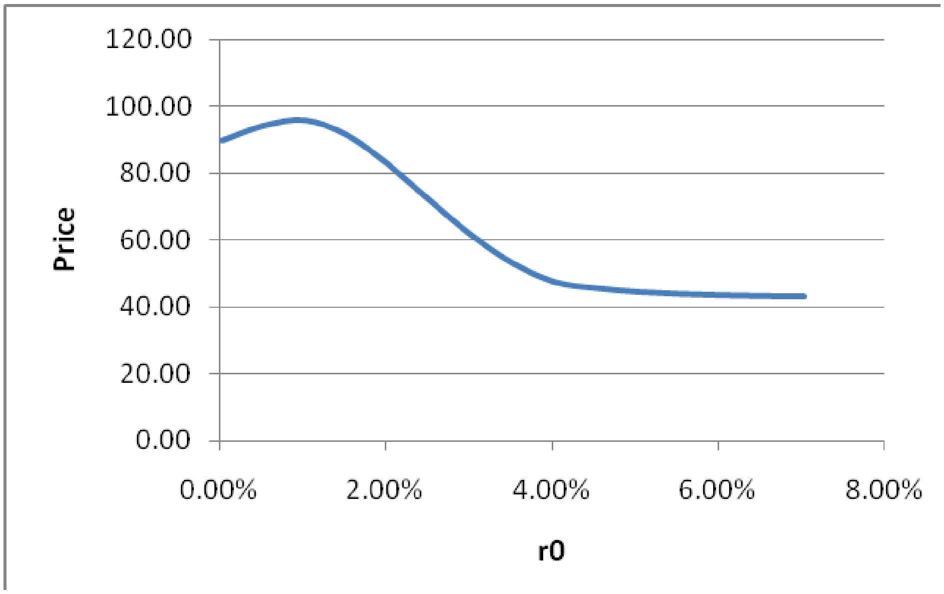

Figure 7: Discount factors from Extended Nelson Siegel model and Bootstrap method

b. Including suboptimal prepayment we have:

   i. Including a PSA type prepayment (65% PSA) we have that the passthrough is: 103.56.

   ii. Including a prepayment that allows forgeting to exercise the option (20%) when optimal gives a value of: 97.42. A probability of 14% for forgeting to exercise the option at the optimal time makes the model price equal to the quoted price (99.39).

c. The spot rate duration is -7.93, which is different from 2.39 from the theoretical model.

d. The price of the principal only (PO) is 43.62, with spot rate duration: -10.49. The price of the interest only (IO) is 630.97, with spot rate duration: -6.15.