

End to End EKF Formulation

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September 24, 2018

Abstract

This is the formulation for the EKF layer. It uses a Error State Kalman Filter. The IMU gets integrated into the nominal states, and the EKF estimates the error states. The error states are used to correct the nominal states.

1 States

1.1 Nominal States

- p global position expressed w.r.t. pose at t_0 , more precisely ${}^I p_{IB}$ where I is the inertial frame and B is the body/vehicle frame
- v global velocity expressed w.r.t. pose at t_0 , more precisely ${}^I v_{IB}$
- q Orientation of the vehicle in quaternion form (w, x, y, z) w.r.t. pose at t_0 , more precisely q_{IB}
- b_a Accelerometer bias in body frame
- b_g Gyroscope bias in body frame

1.2 Error States

- δp global position error
- δv global velocity error
- $\delta \theta$ Orientation error in $\text{so}(3)$
- δb_a Accelerometer bias error
- δb_g Gyroscope bias error

2 Process Model

New IMU measurements are integrated into the nominal state at each time step. The error state process model carries forward the uncertainty of each IMU integration, the errors are estimated at each measurement update. The errors estimate are then injected into the nominal state.

Inputs are assumed constant from time t_k to time t_{k+1} .

- $a_{m_k} = a_k + \eta_a$ Acceleration as measured by the accelerometer. Modeled with white noise.
- $w_{m_k} = w_k + \eta_w$ Angular velocity as measured by the gyroscope. Modeled with white noise.

2.1 Nominal States Discrete Process Model

IMU measurements are integrated to produce predictions of nominal.

$$\Delta t = t_{k+1} - t_k \quad (1)$$

$$\hat{p}_{k+1|k} = p_{k|k} + v_{k|k}\Delta t + \frac{1}{2}(R\{q_{k|k}\}(a_{m_k} - b_{a_{k|k}}) + g)\Delta t^2 \quad (2)$$

$$\hat{v}_{k+1|k} = v_{k|k} + (R\{q_{k|k}\}(a_{m_k} - b_{a_{k|k}}) + g)\Delta t \quad (3)$$

$$\hat{q}_{k+1|k} = q \otimes q\{(w_{m_k} - b_{w_{k|k}})\Delta t\} \quad (4)$$

$$\hat{b}_{a_{k+1|k}} = b_{a_{k|k}} \quad (5)$$

$$\hat{b}_{w_{k+1|k}} = b_{w_{k|k}} \quad (6)$$

2.2 Error States Discrete Process Model

$$\delta \hat{p}_{k+1|k} = \delta p_{k|k} + \delta v_{k|k}\Delta t \quad (7)$$

$$\delta \hat{v}_{k+1|k} = \delta v_{k|k} + (-R\{q_k\}[a_{m_k} - b_{a_{k|k}}]_{\times}\delta\theta - R\{q_{k|k}\}\delta b_{a_{k|k}} + \delta g)\Delta t + v_i \quad (8)$$

$$\delta \hat{\theta}_{k+1|k} = R\{(w_{m_k} - b_{w_{k|k}})\Delta t\}\delta\theta - \delta b_{w_{k|k}}\Delta t + \theta_i \quad (9)$$

$$\delta \hat{b}_{a_{k+1|k}} = \delta b_{a_{k|k}} + a_i \quad (10)$$

$$\delta \hat{b}_{w_{k+1|k}} = \delta b_{w_{k|k}} + w_i \quad (11)$$

where the v_i , θ_i , a_i , and w_i are the random impulses resulted from the integration of noise terms from continuous to discrete. let x be the complete error state, the covariance for the error state process model is given by:

$$Q_k = F_i Q_{i_k} F_i^T$$

$$F_i = \frac{\partial x}{\partial i}$$

$$Q_{i_k} = \begin{bmatrix} \sigma_a^2 \Delta t^2 I_3 & 0 & 0 & 0 \\ 0 & \sigma_w^2 \Delta t^2 I_3 & 0 & 0 \\ 0 & 0 & \sigma_{b_a} \Delta t I_3 & 0 \\ 0 & 0 & 0 & \sigma_{b_w} \Delta t I_3 \end{bmatrix}$$

Note that initializing error states to zero will always produce zero predictions, only the uncertainty is carried forward by the process model.

3 Measurement Model

The neural network is trained to directly output 6 DoF incremental poses in the t_k body frame. let the neural network output at time k be Δx_k . The error state Kalman Filter estimates accumulated error between each measurement update. The error states are injected as follows:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} \oplus \delta \hat{x}_{k+1|k+1}$$

$$z_k = \begin{bmatrix} \hat{p}_{k+1|k+1} \\ \hat{v}_{k+1|k+1} \\ \hat{q}_{k+1|k+1} \\ \hat{b}_{a_{k+1|k+1}} \\ \hat{b}_{w_{k+1|k+1}} \end{bmatrix} = \begin{bmatrix} \hat{p}_{k+1|k} + \delta \hat{p}_{k+1|k+1} \\ \hat{v}_{k+1|k} + \delta \hat{v}_{k+1|k+1} \\ \hat{q}_{k+1|k} \otimes q\{\delta \hat{\theta}_{k+1|k+1}\} \\ \hat{b}_{a_{k+1|k}} + \delta \hat{b}_{a_{k+1|k+1}} \\ \hat{b}_{w_{k+1|k}} + \delta \hat{b}_{w_{k+1|k+1}} \end{bmatrix}$$

Note that ESKF uses global pose and velocity error, and local angle error. Since the network estimates 6 DoF incremental poses, we are interested in using the position error $\hat{p}_{k+1|k+1}$ and angle errors $\delta \hat{\theta}_{k+1|k+1}$ as part of the measurement update. We can measure the error by finding the different between the IMU prediction and network prediction at time k . For $\hat{p}_{k+1|k+1}$, we have the following:

$$\hat{p}_{k+1|k}$$

We can directly measure the error state by taking the difference between the neural network estimation and IMU estimation.

$$z_k = x_k \oplus \delta x_k^{NN} \ominus \hat{x}_{k+1}$$

Since the error state Kalman Filter uses local angle errors and global position errors, let $T_{GI_k} = T\{x_k\}$, the measurement becomes as follows:

$$z_k = \begin{bmatrix} \delta p \\ \delta \theta \end{bmatrix} = \begin{bmatrix} T_{GI_k} p_k^{NN} - \hat{p}_{k+1} \\ \theta_k^{NN} - (w_{m_k} - b_{w_k}) \Delta t \end{bmatrix}$$

The measurement matrix simply become identity with respect to the position and angle errors. The neural network measurement covariances on position is propagated through rotational portion of T_{GI_k} and the covariances on rotation remains unchanged.

4 Injecting Error States into Nominal States

Estimate for the error state are then injected into the nominal states. The error state set to zero while the error state uncertainty propagates.

To inject the error state into nominal state:

$$p_{k+1} = \hat{p}_{k+1} + \delta \hat{p}_{k+1} \quad (12)$$

$$v_{k+1} = \hat{v}_{k+1} + \delta \hat{v}_{k+1} \quad (13)$$

$$q_{k+1} = \hat{q}_{k+1} \otimes q\{\delta \hat{\theta}_{k+1}\} \quad (14)$$

$$b_{a_{k+1}} = \hat{b}_{a_{k+1}} + \delta \hat{b}_{a_{k+1}} \quad (15)$$

$$b_{w_{k+1}} = \hat{b}_{w_{k+1}} + \delta \hat{b}_{w_{k+1}} \quad (16)$$

To propagate error through reset:

$$P_{k+1}^{reset} = G P_{k+1} G^\top$$

$$G = \begin{bmatrix} I_6 & 0 & 0 \\ 0 & I_3 - [\frac{1}{2}\delta\theta]_\times & 0 \\ 0 & 0 & I_6 \end{bmatrix}$$