Deep Probability Estimation

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Acknowledgements

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Joint work with

- Sheng Liu
- Aakash Kaku
- Weicheng Zhu
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- Haoxiang Huang
- Laure Zanna
- Narges Razavian
- Jonathan Niles-Weed

Probability Estimation

Goal: Estimate probability of uncertain events from high-dimensional input (images, videos)

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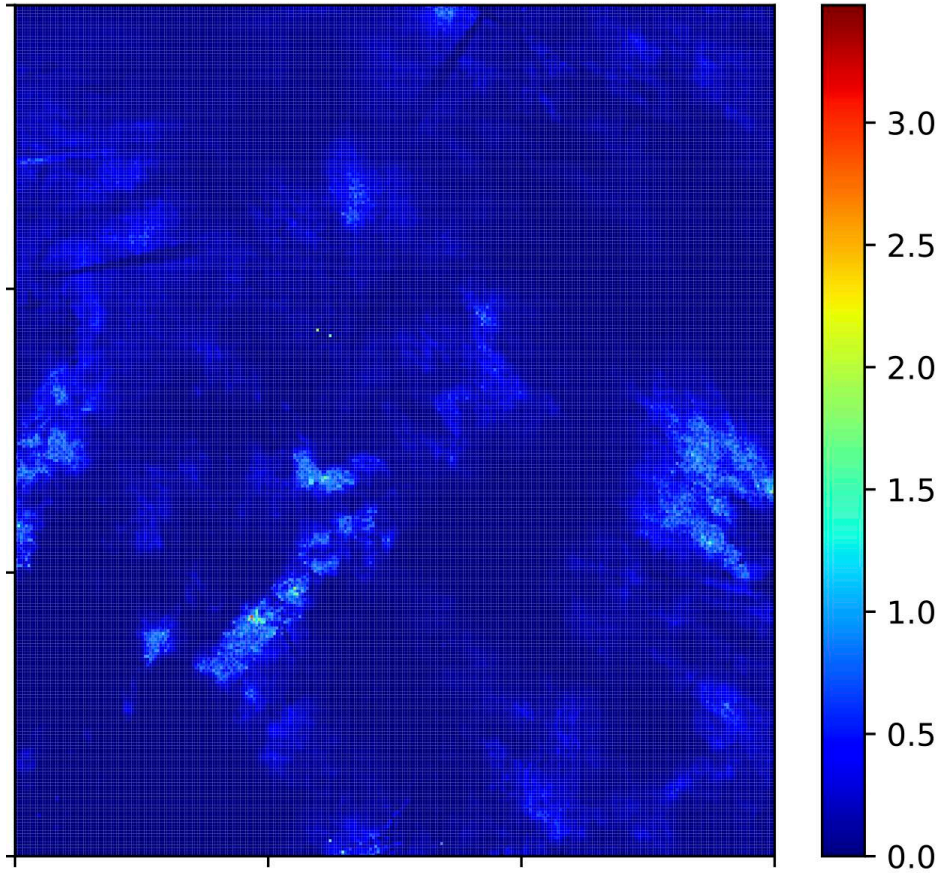
Not equivalent to classification because of inherent uncertainty

Weather Forecasting

Event of interest: Will it rain?

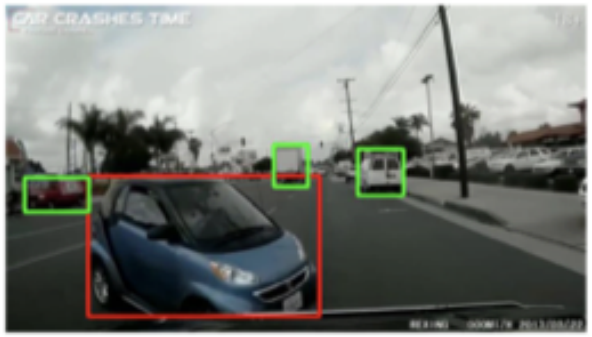
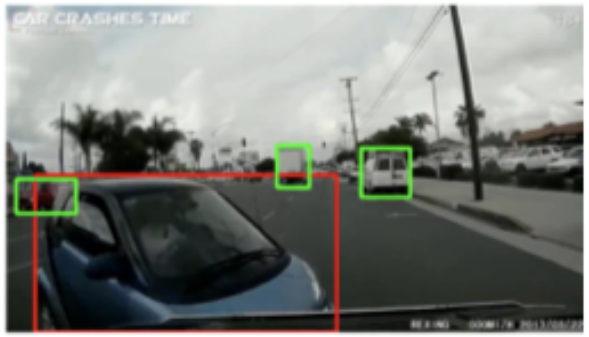
Input: Map of past precipitation



Collision Prediction

Event of interest: Will there be a car crash?

Input: Dashboard video

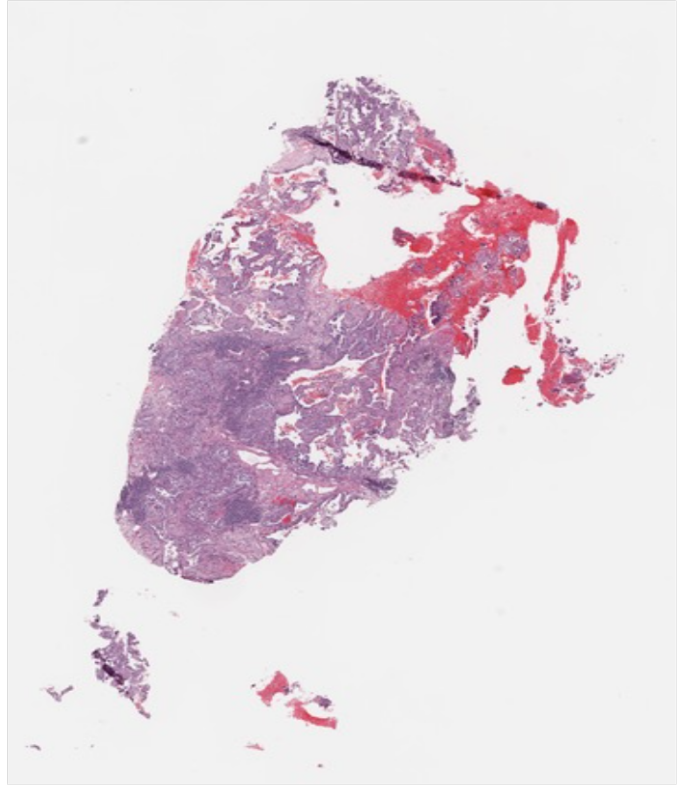




Crash to not crash: Learn to identify dangerous vehicles using a simulator. Kim, H., Lee, K., Hwang, G., and Suh, C. AAAI 2019

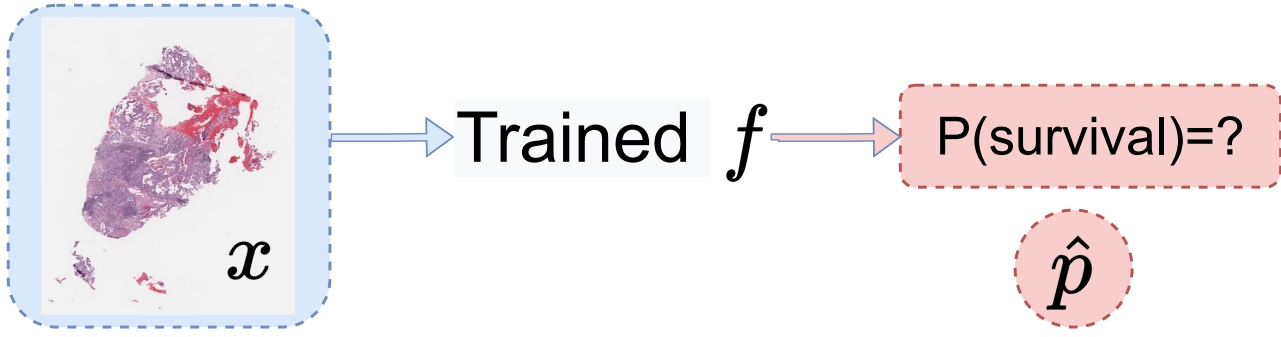
Survival Prediction

Event of interest: Will a cancer patient die in the next 5 years?

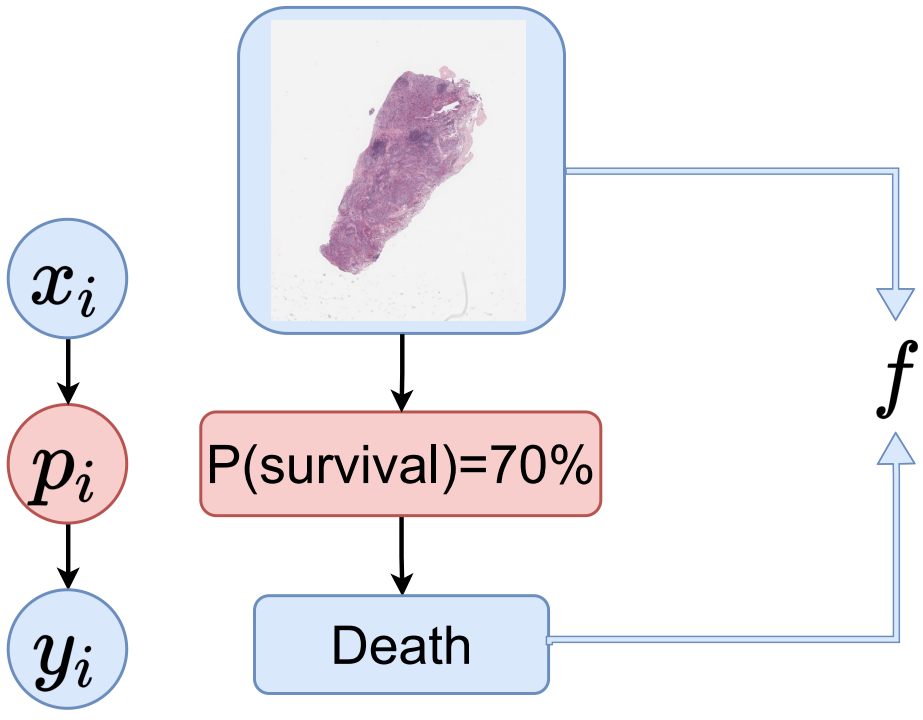
Input: Histopathology slide



Probability estimation via deep learning



Training



Evaluation

How do we evaluate whether estimated probabilities are calibrated?

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Among examples with estimated probability 0.7, are 70% equal to one?

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Bin points according to estimated probability

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Compare estimated probability to empirical probability in each bin

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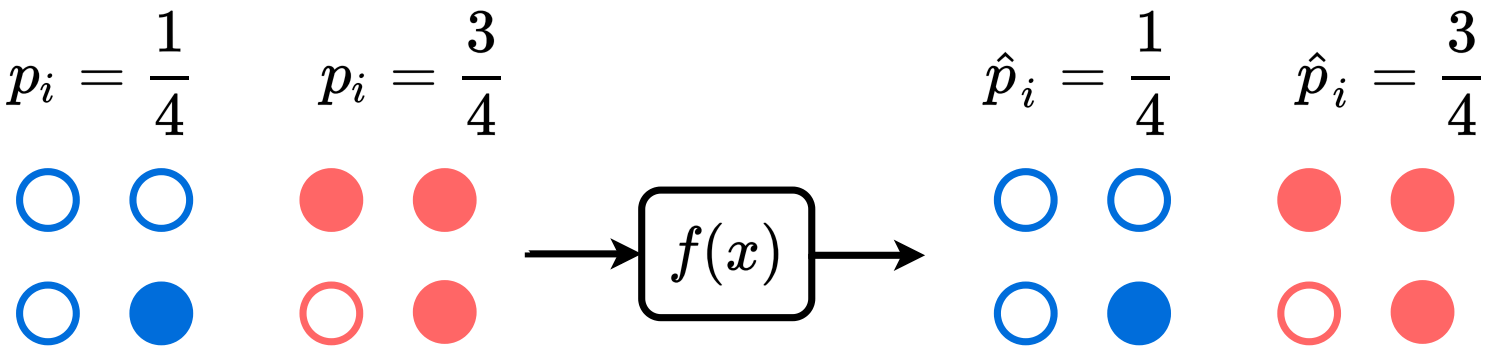
Bin points according to estimated probability

Compare estimated probability to empirical probability in each bin

Is calibration enough?

Example



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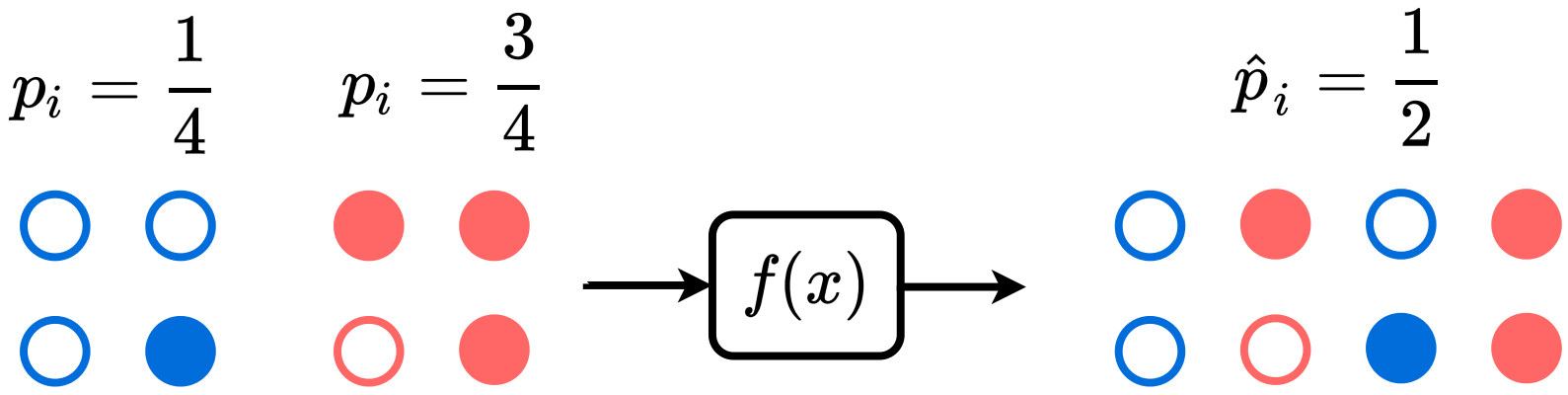

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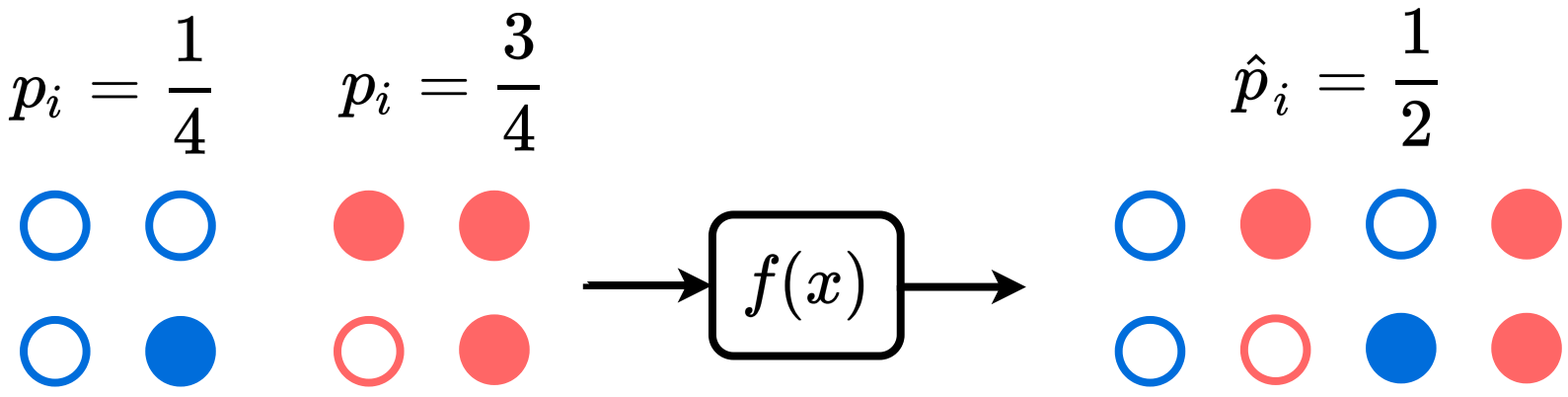

Good calibration does not imply good probability estimation

Synthetic Dataset: Face-based Risk Prediction

x_i

Synthetic Dataset: Face-based Risk Prediction

x_i	Age z_i
	20

Synthetic Dataset: Face-based Risk Prediction

x_i	Age z_i	$p_i = \frac{z_i}{100}$
	20	0.2

Synthetic Dataset: Face-based Risk Prediction

x_i	Age z_i	$p_i = \frac{z_i}{100}$	y_i
	20	0.2	0

Synthetic Dataset: Face-based Risk Prediction

x_i	Age z_i	$p_i = \frac{z_i}{100}$	y_i
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Synthetic Dataset: Face-based Risk Prediction

x_i	Age z_i	$p_i = \frac{z_i}{100}$	y_i
	20	0.2	0
	70		

Synthetic Dataset: Face-based Risk Prediction

x_i	Age z_i	$p_i = \frac{z_i}{100}$	y_i
	20	0.2	0
	70	0.7	

Synthetic Dataset: Face-based Risk Prediction

x_i	Age z_i	$p_i = \frac{z_i}{100}$	y_i
	20	0.2	0
	70	0.7	1

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x_i	Age z_i	$p_i = \frac{z_i}{100}$	y_i
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Face-Age dataset from *Age Progression/Regression by Conditional Adversarial Autoencoder*. Zhang, Z., Song, Y., and Qi, H., CVPR 2017

Cross-entropy

Standard cross-entropy loss is a *proper scoring rule*

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Probabilities estimated by minimizing cross entropy are well calibrated in an infinite data regime

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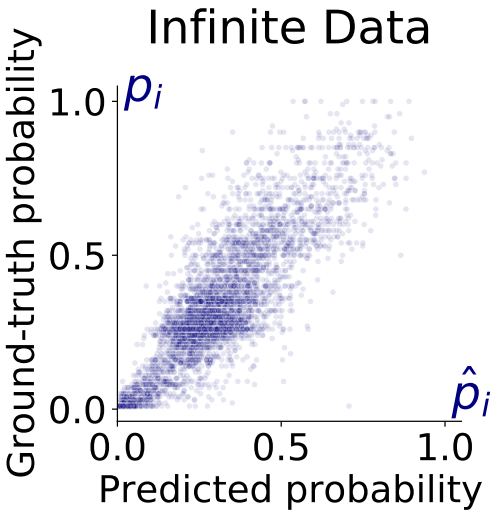
To simulate this, we resample y_i at each epoch based on p_i

Face-based Risk Prediction



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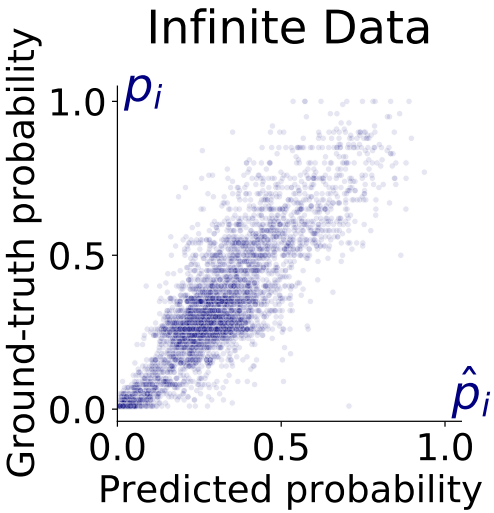
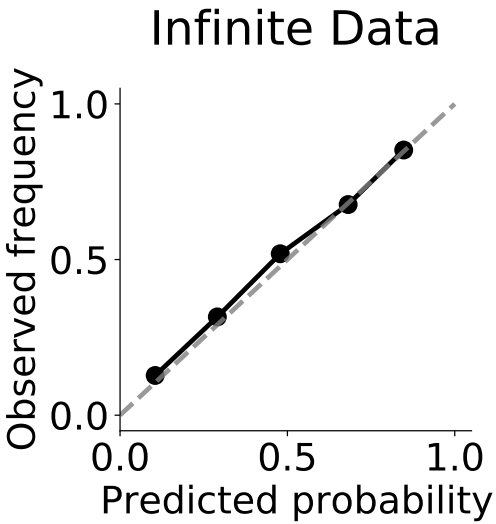
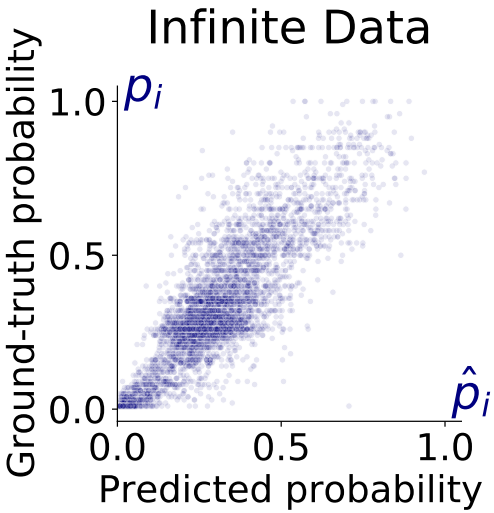
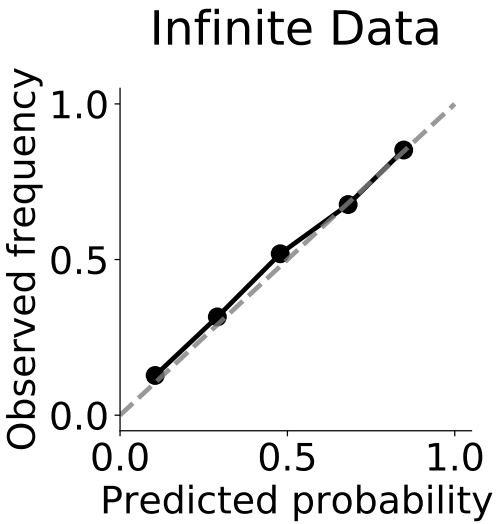




Face-based Risk Prediction

What happens if dataset is finite?





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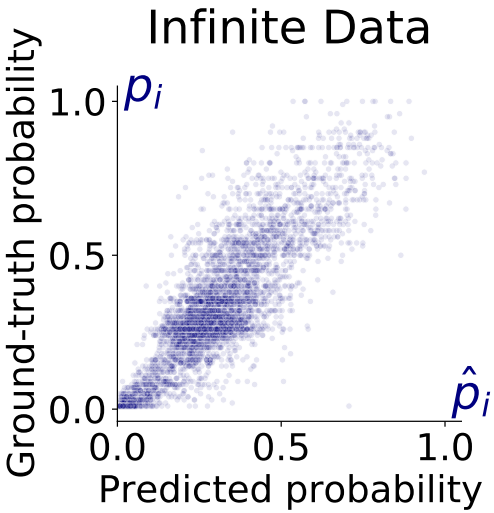

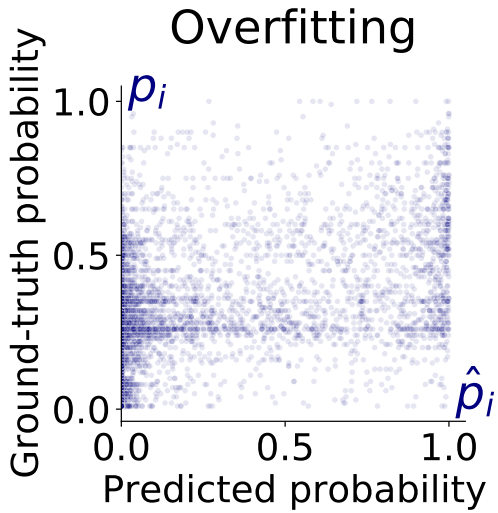

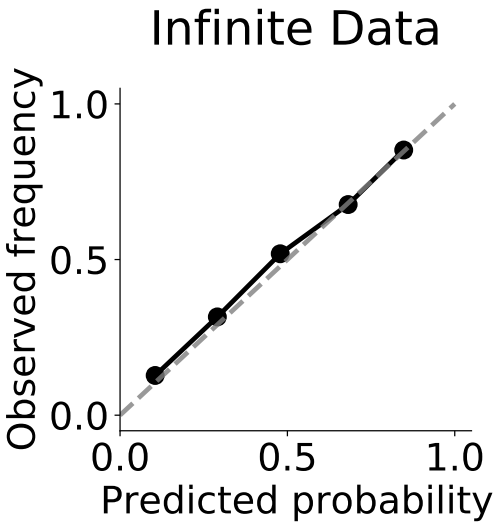

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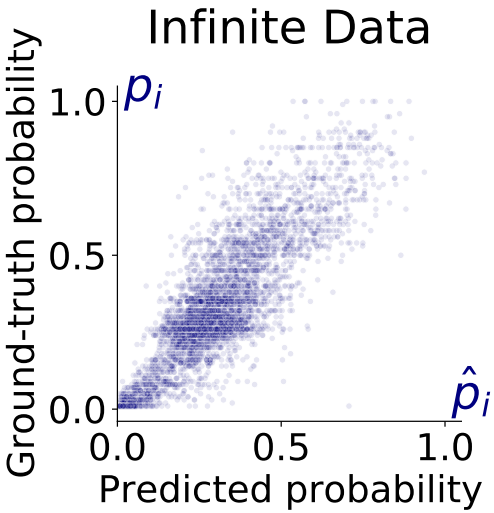

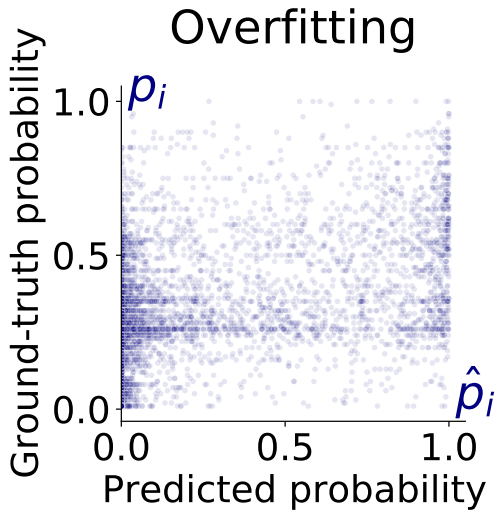

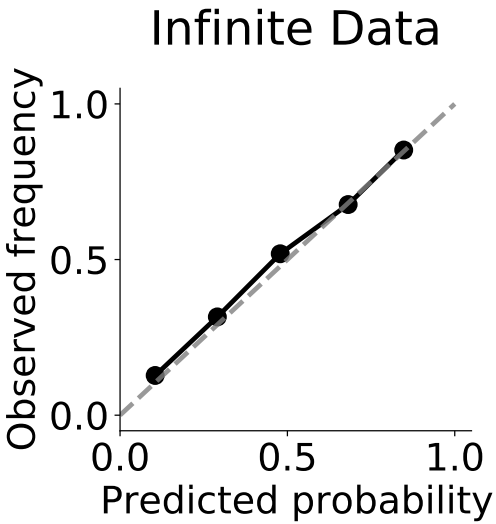

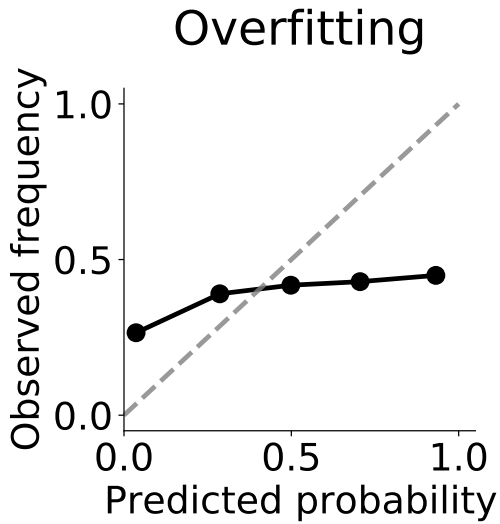

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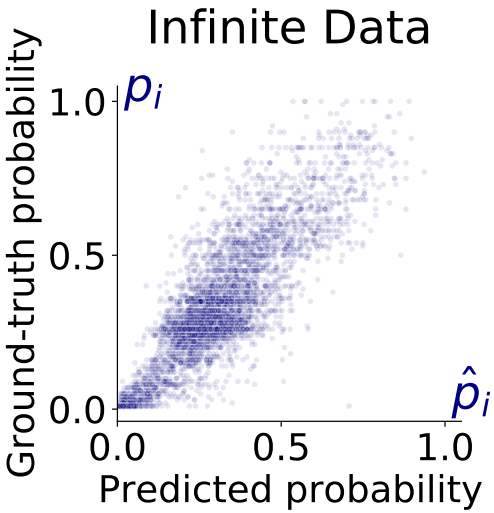

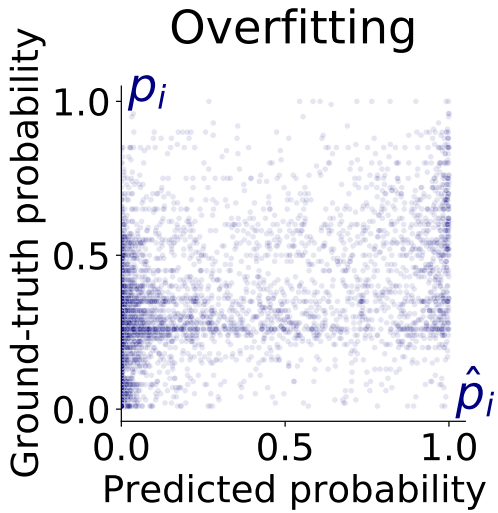

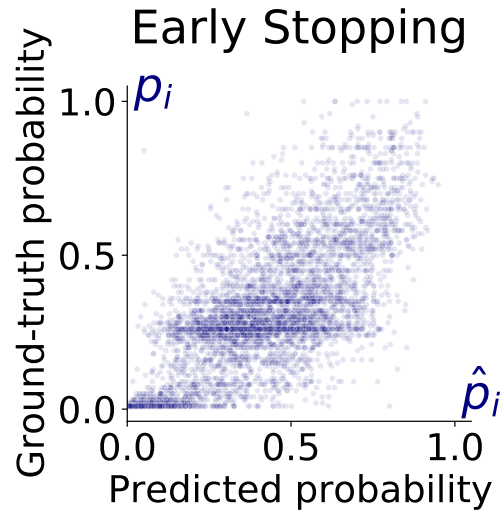

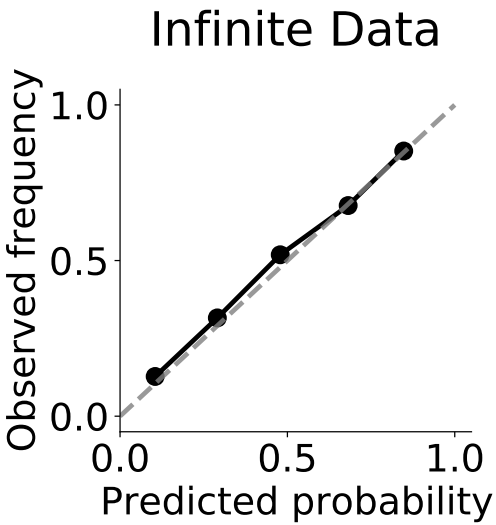

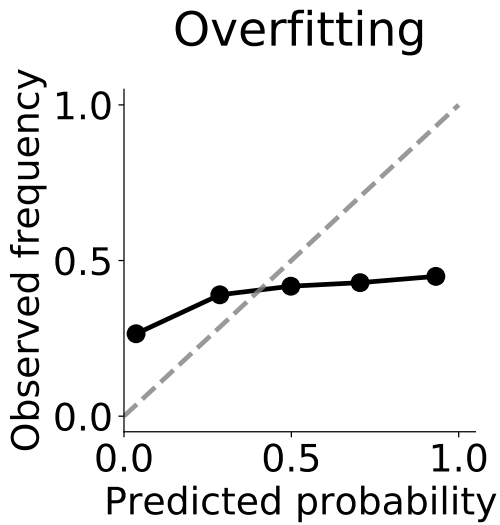

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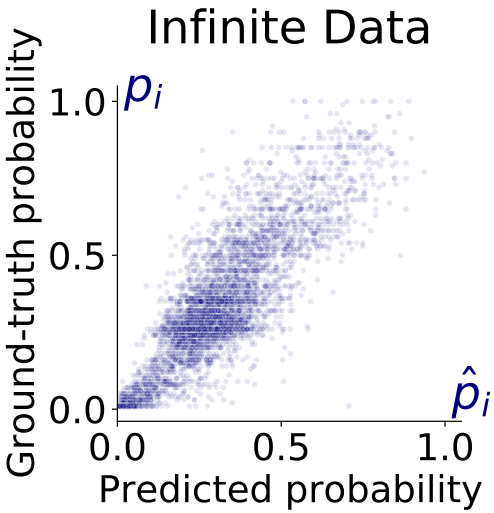

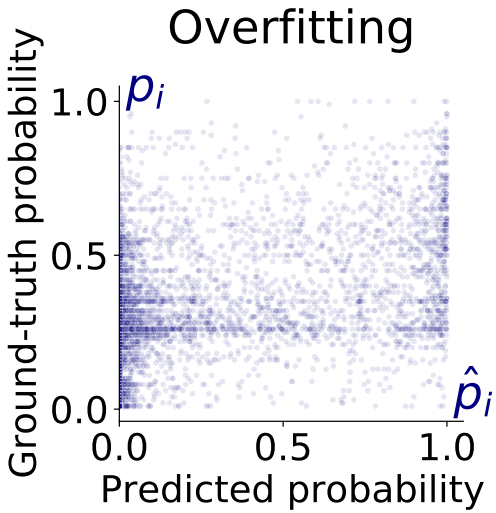

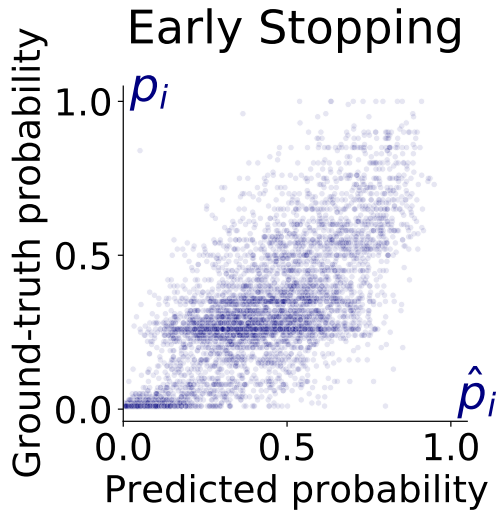

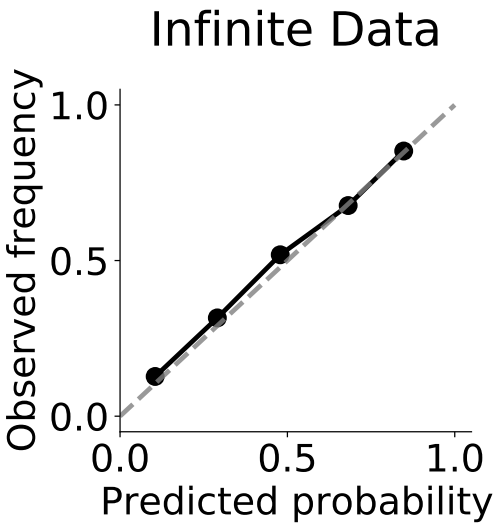

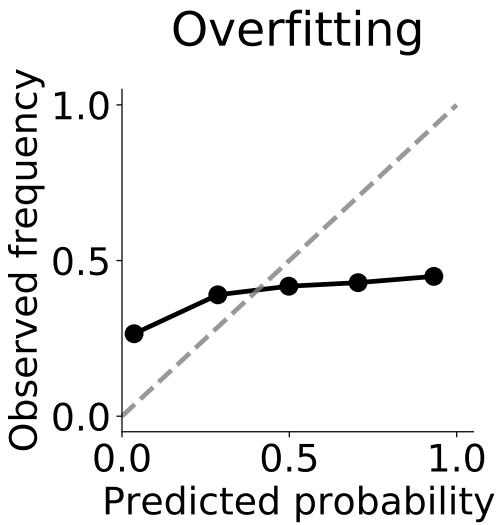

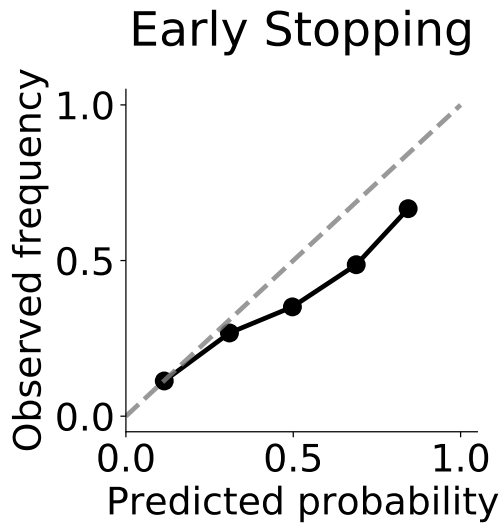

Fundamental phenomenon

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This occurs even for linear models!

Calibrated Probability Estimation (CaPE)

Goal: Improve model *while remaining calibrated*

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1. Minimize cross-entropy loss until memorization begins

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1. Minimize cross-entropy loss until memorization begins
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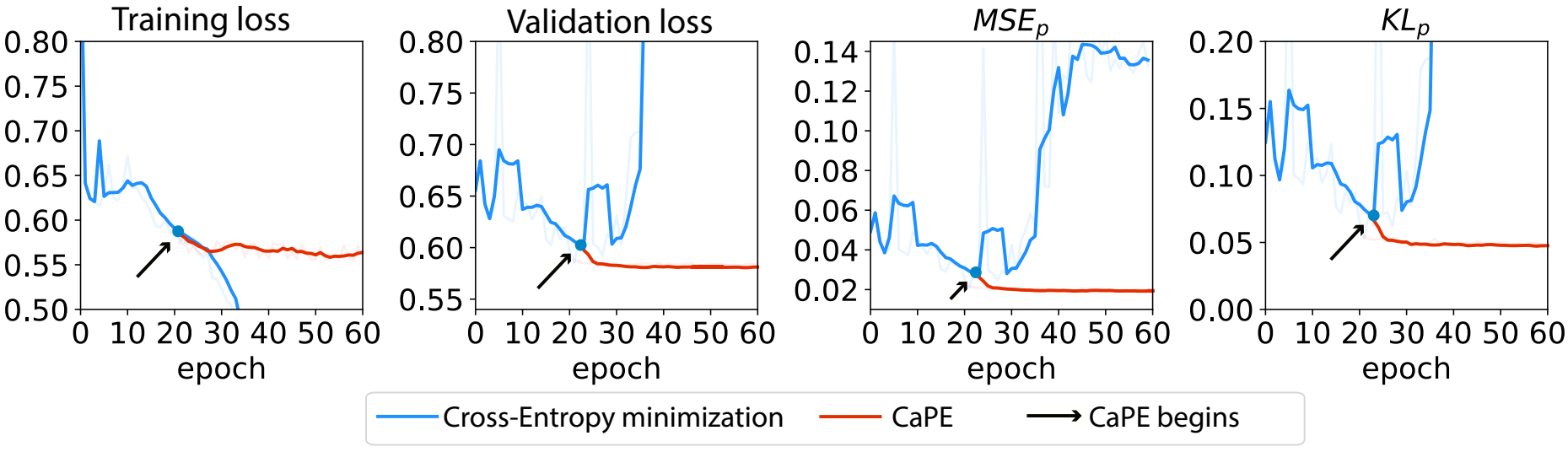
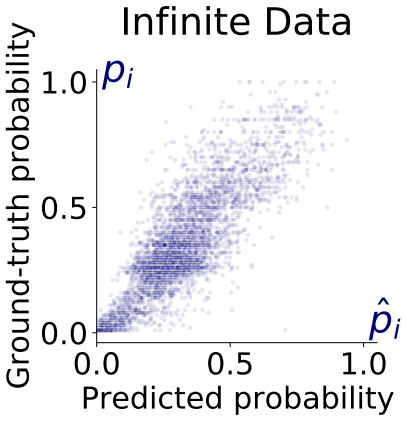
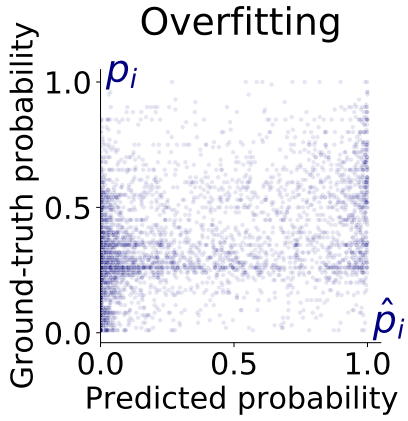
   - Enforcing calibration
   - Minimizing cross entropy

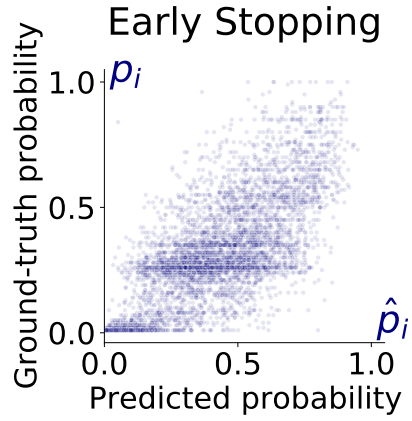
Calibrated Probability Estimation

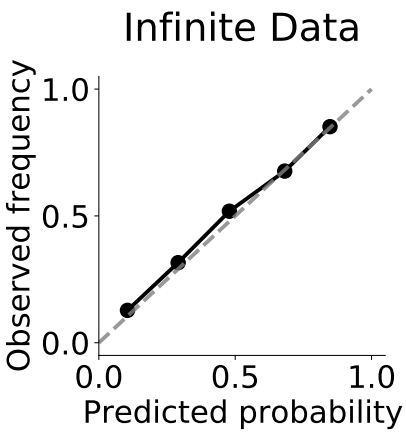


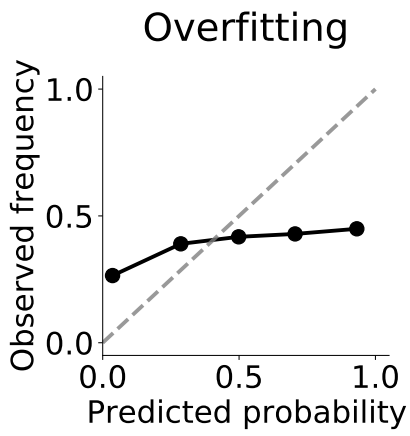
Face-based Risk Prediction

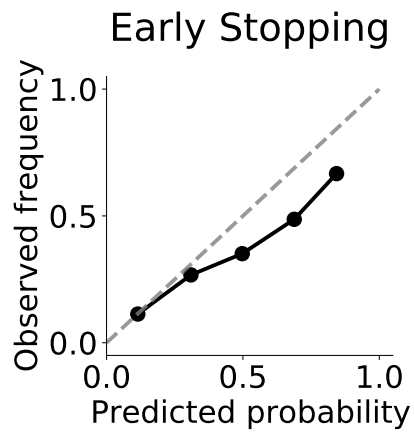












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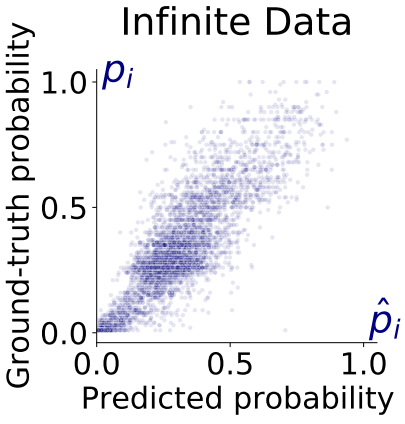
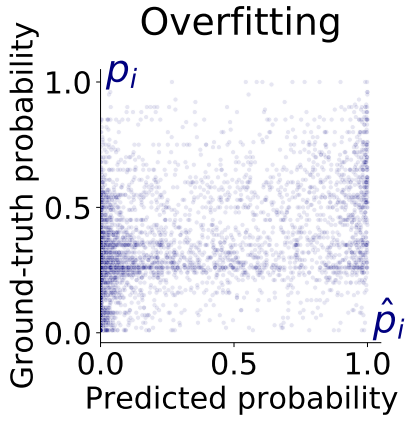
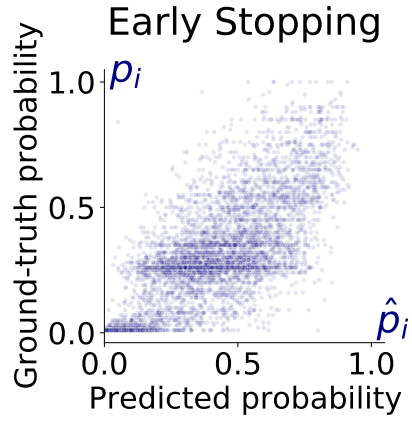
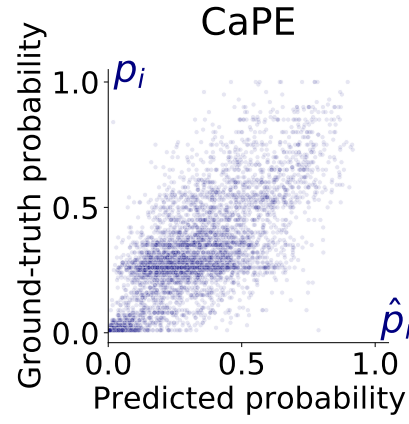
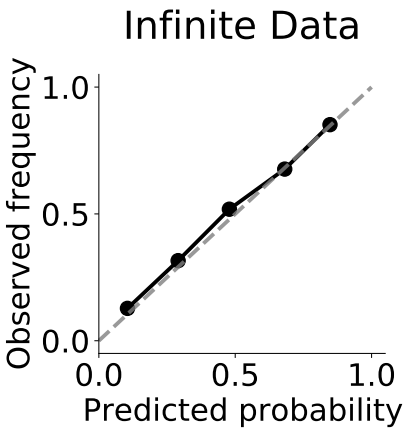
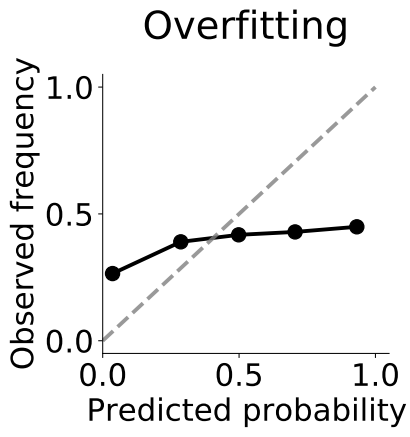


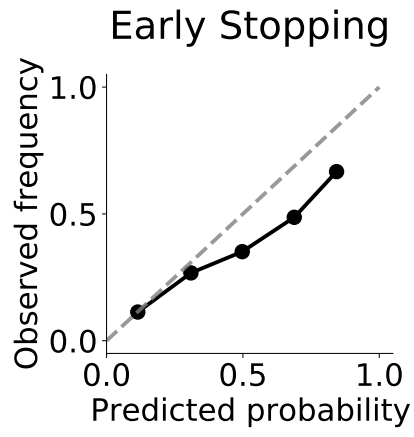




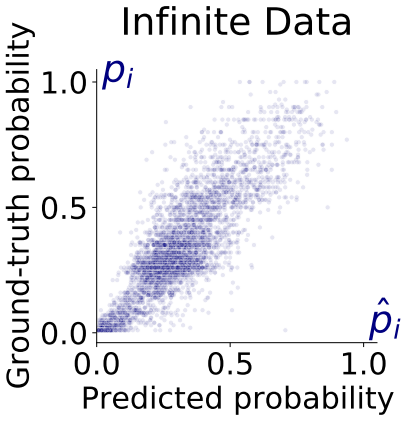


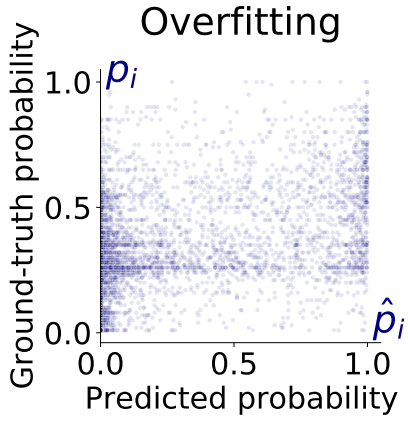


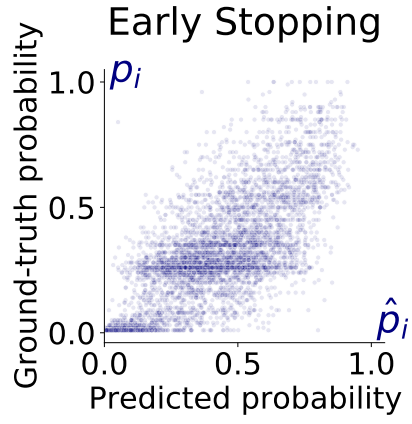


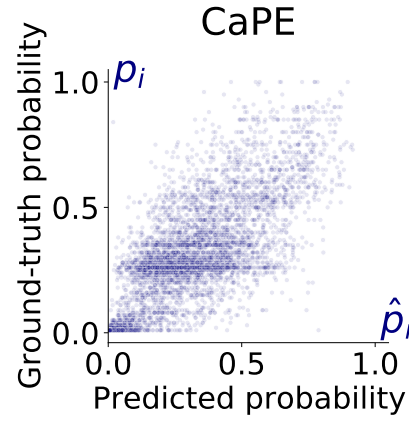


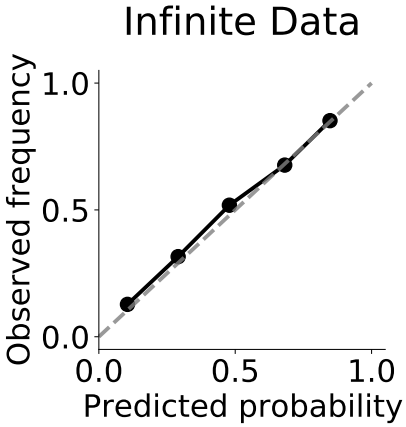
Face-based Risk Prediction

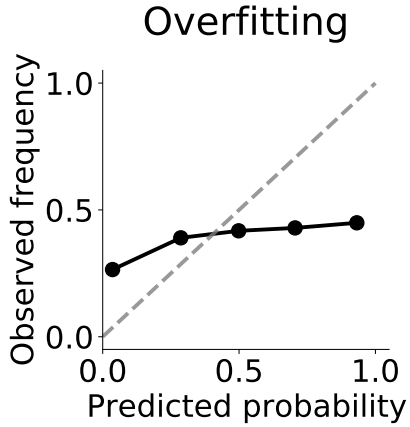


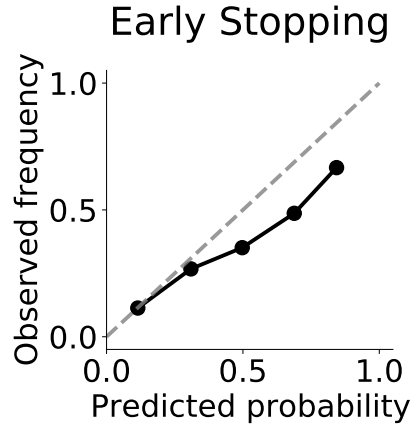


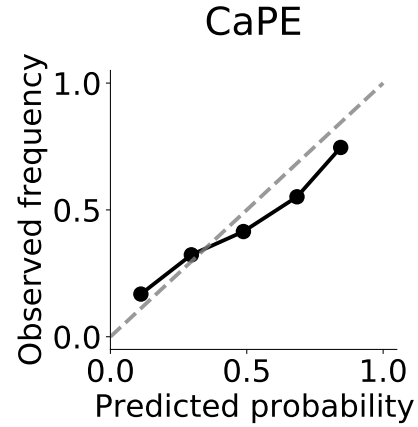












Existing approaches

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- Modified cost function: Focal loss (Mukhoti et al. 2020), entropy-maximizing loss (Pereyra et al. 2017), and kernel mean embeddings (Kumar et al., 2018)

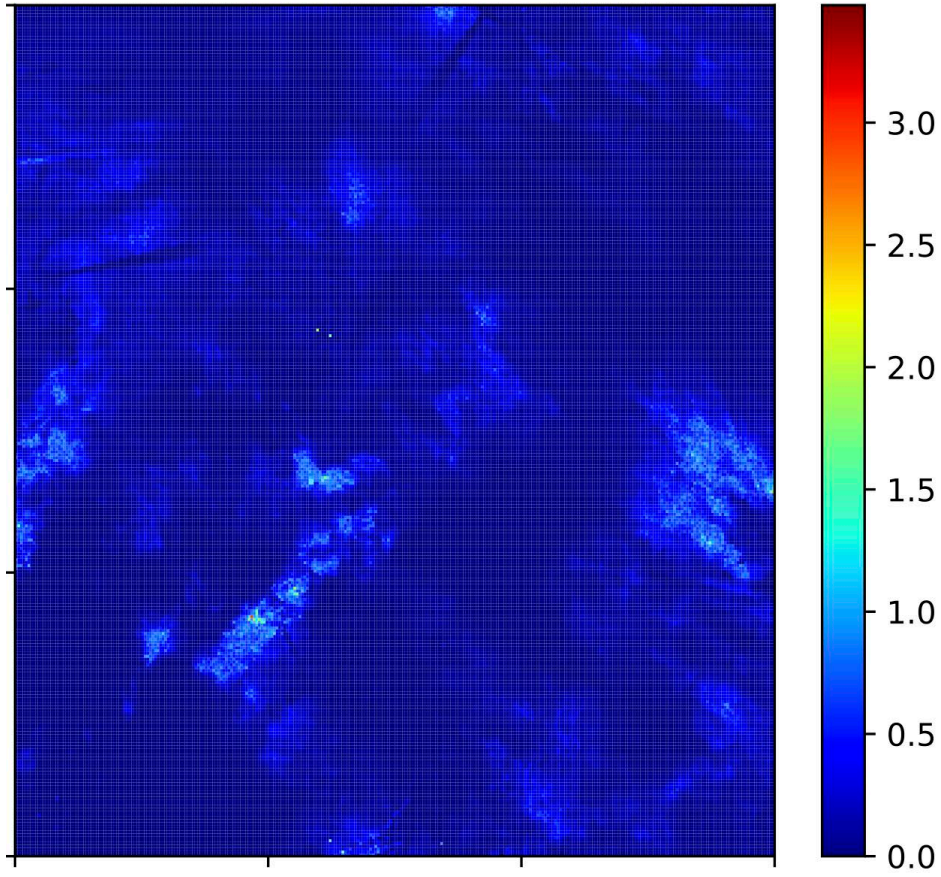
Results

Method ($\times 10^{-2}$)	Cancer Survival AUC	Cancer Survival ECE	Cancer Survival Brier	Weather forecasting AUC	Weather forecasting ECE	Weather forecasting Brier	Collision Prediction AUC	Collision Prediction ECE	Collision Prediction Brier
CE early-stop	58.88	12.25	23.96	77.64	10.91	20.57	85.68	4.36	8.59
Temperature	58.88	12.07	23.73	77.64	8.66	20.21	85.68	4.56	8.51
Platt Scaling	58.91	10.28	23.33	77.65	6.97	19.53	85.76	3.04	8.23
Dirichlet Cal.	49.89	13.83	24.08	77.51	14.29	21.89	83.36	5.78	8.78
Mix-n-match	58.88	12.16	23.67	77.64	8.65	20.21	85.68	4.40	8.52
Focal Loss	55.02	12.15	23.31	76.18	8.32	20.27	82.21	9.07	9.82
Entropy Reg.	56.29	11.73	23.62	79.01	10.53	19.77	83.15	14.54	11.10
MMCE Reg.	48.45	11.84	23.73	76.69	8.46	20.12	85.18	**2.94**	8.48
Deep Ens.	52.46	9.99	23.47	**79.86**	7.41	18.82	85.27	3.15	8.55
CaPE (bin)	**61.44**	12.31	23.20	78.99	5.16	**18.37**	85.70	3.16	8.18
CaPE (kern.)	61.22	**9.48**	**23.18**	79.00	**5.08**	18.39	**85.95**	3.22	**8.13**

Weather Forecasting

Event of interest: Will it rain?

Input: Map of past precipitation



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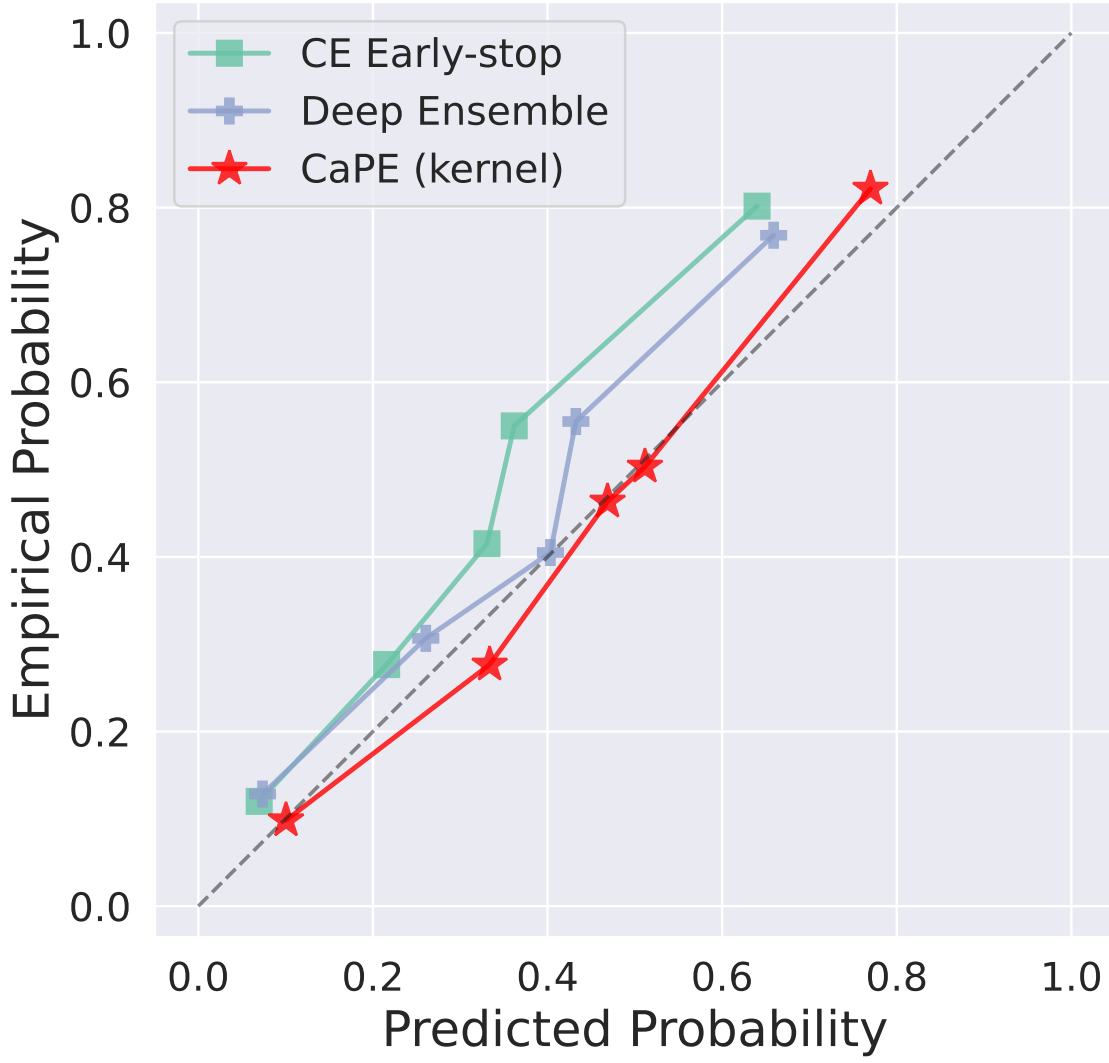
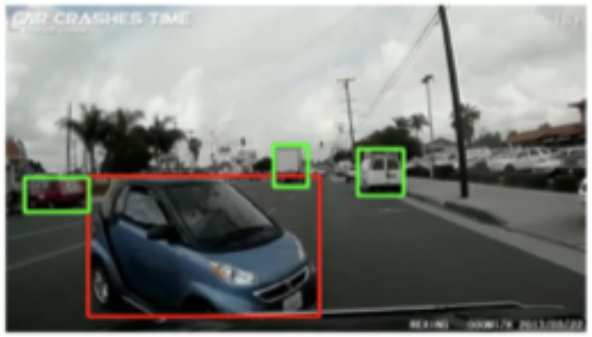
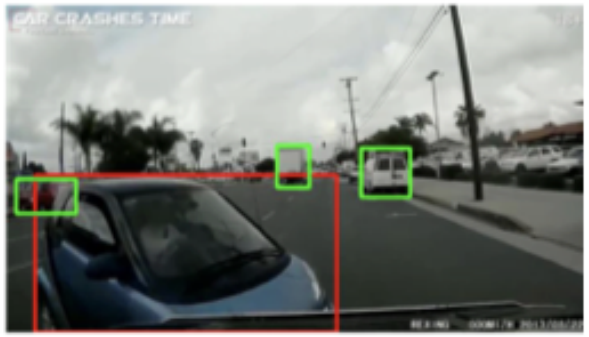


Collision Prediction

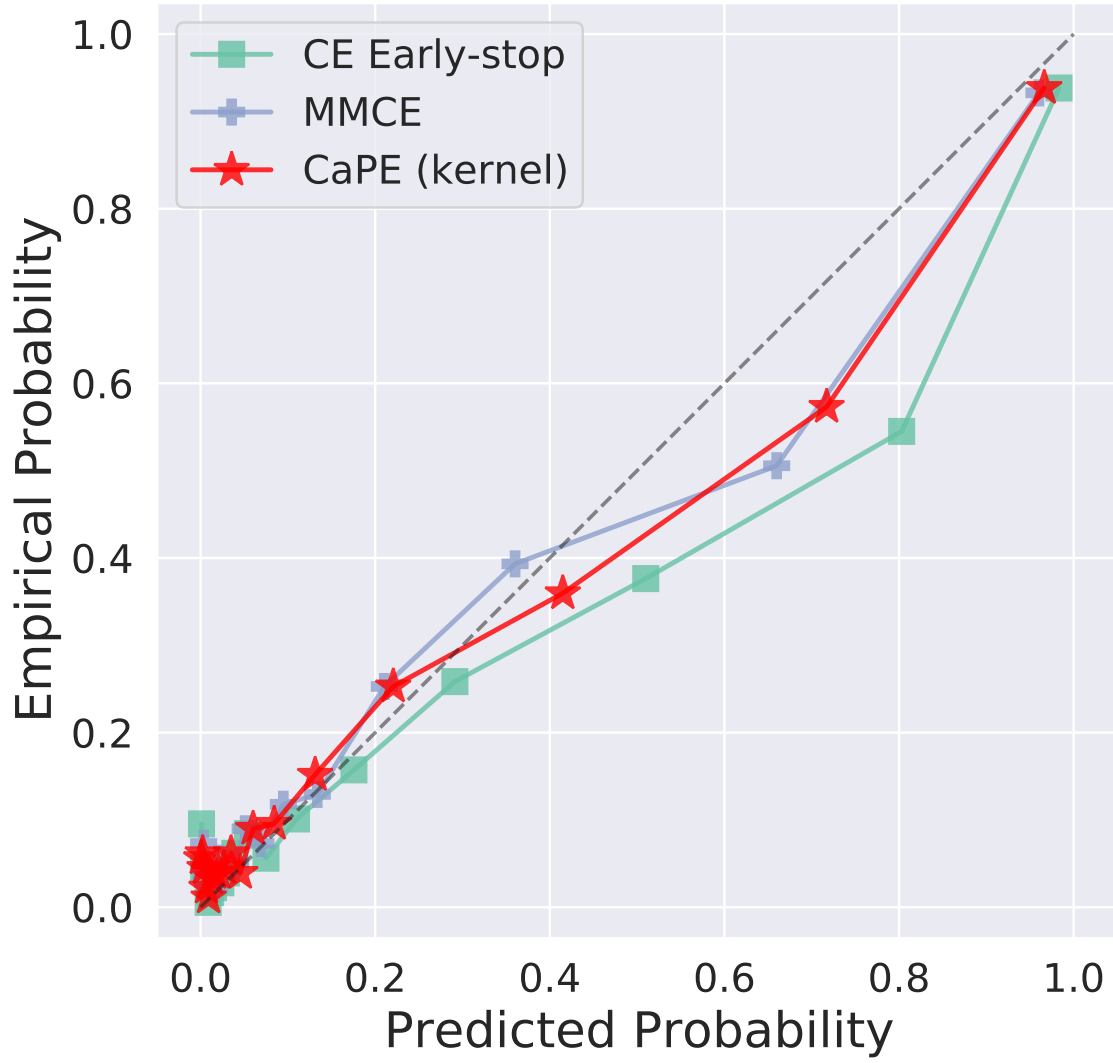
Event of interest: Will there be a collision?

Input: Dashboard video

Crash to not crash: Learn to identify dangerous vehicles using a simulator. Kim, H., Lee, K., Hwang, G., and Suh, C. AAAI 2019

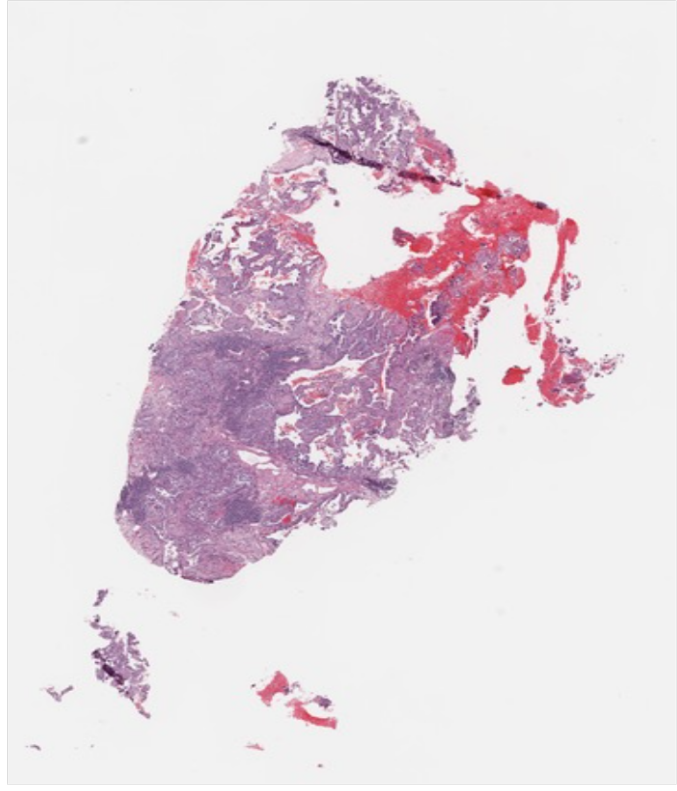
Collision Prediction



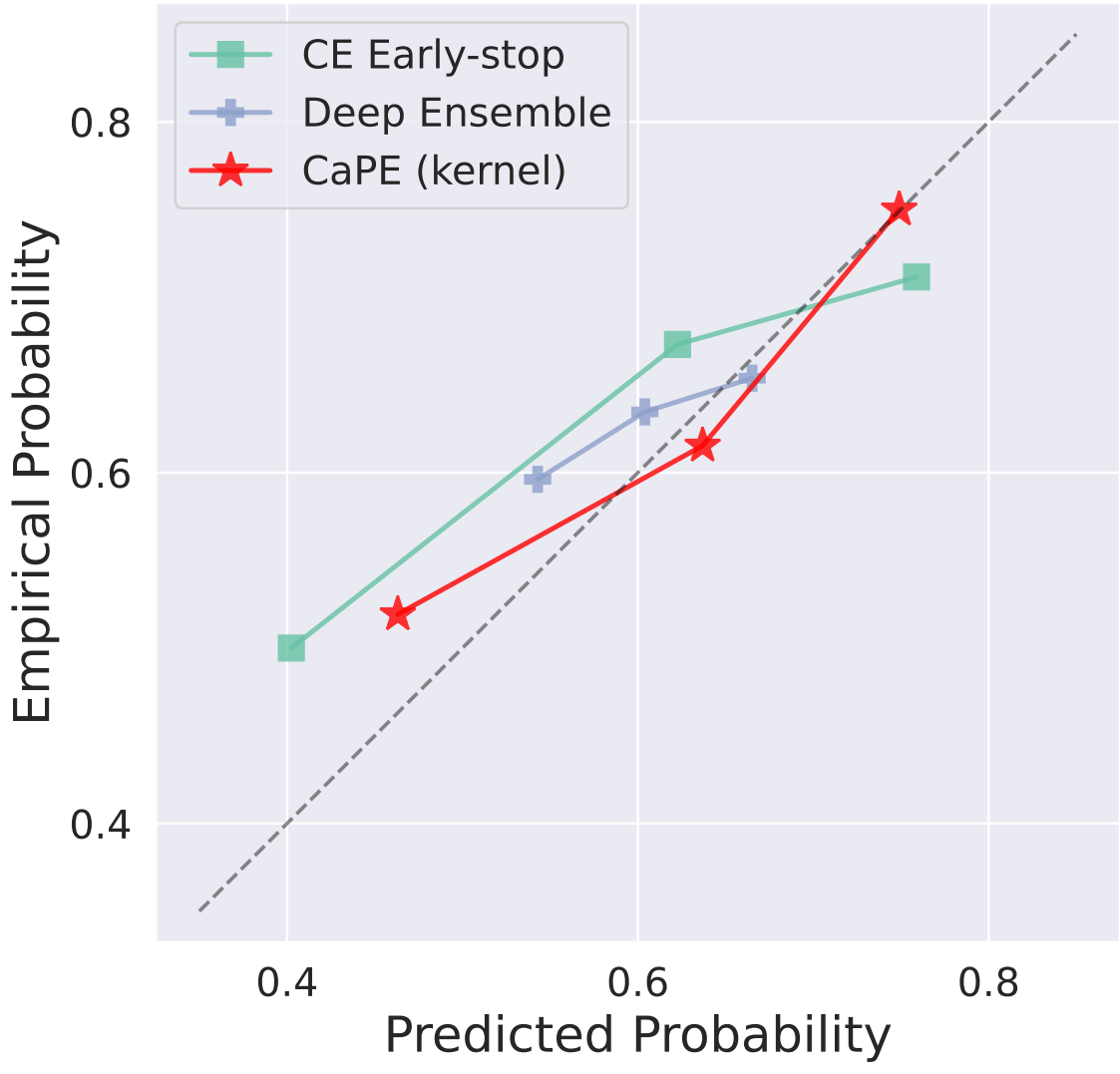
Survival Prediction

Event of interest: Will a cancer patient die in the next 5 years?

Input: Histopathology slide



Survival Forecasting



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- More benchmark datasets are needed!

For more information

Deep Probability Estimation

S. Liu, A. Kaku, W. Zhu, M. Leibovich, S. Mohan, B. Yu, H. Huang, L. Zanna, N. Razavian, J. Niles-Weed, C. Fernandez-Granda