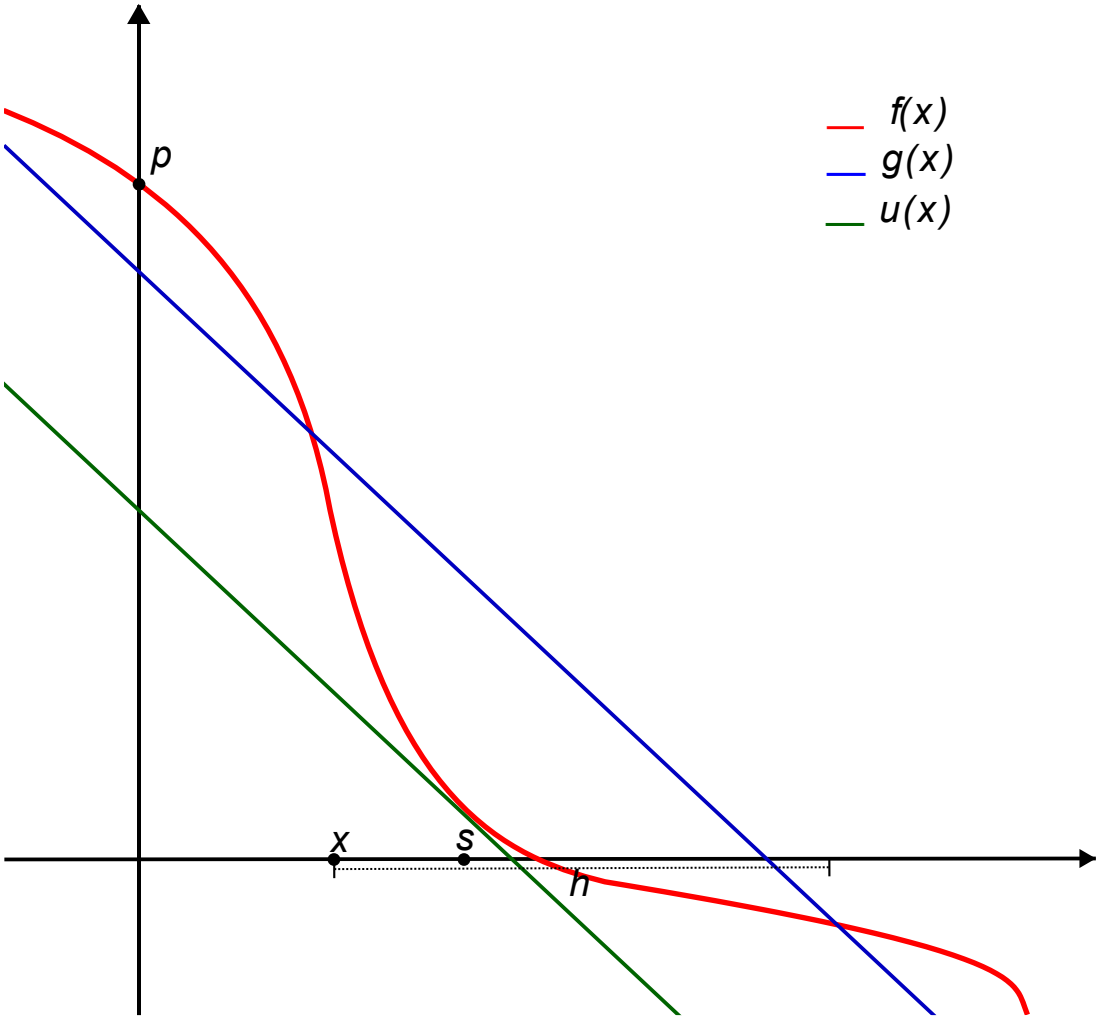
Data Science Prerequisite Mathematics Exam

For questions 1 through 6, consider figure 1 below.



1. Which point marked on the figure is equal to $f(0)$?
2. Consider the following expression: $\frac{f(x+h)-f(x)}{h}$. What is this equivalent to, in terms of figure 1?
3. The function $g(x)$ is a line, so it is of the form $mx + b$. What information about $f(x)$ can we take from this?
4. The functions $g(x)$ and $u(x)$ are both lines, so let $g(x) = mx + b$ and $u(x) = nx + c$. What is $f'(s)$?
5. Which point marked on the figure represents a potential optimum?
6. Is $f''(s)$ equal to 0, greater than 0, or less than 0?

For questions 7 through 24, consider the following matrices and vectors.

$$A = \begin{bmatrix} 6 & 8 & 6 & 7 \\ 5 & 3 & 0 & 9 \\ 7 & 7 & 1 & 4 \\ 2 & 5 & 5 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 2 & 0 & 9 & 9 \\ 1 & 9 & 6 & 1 & 1 \\ 5 & 2 & 0 & 2 & 5 \\ 0 & 1 & 7 & 4 & 7 \end{bmatrix} \qquad u = \begin{bmatrix} 3 \\ 7 \\ -6 \\ 9 \end{bmatrix} \qquad v = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 4 \end{bmatrix}$$

7. Evaluate the following expression: $\sum_{i=1}^{n} v_i$.
8. Evaluate the following expression: $\sum_{i=1}^{n} v_i^2$.
9. What are the dimensions of the matrix product AB?
10. What are the dimensions of the matrix product BA?
11. What are the dimensions of the product Av?
12. What are the dimensions of the product $u^T v$?
13. What are the dimensions of the product uv^T?
14. Evaluate the following expression: $\sum_{j=1}^{4} B_{1j}$.
15. Evaluate the following expression: $\sum_{i=1}^{m} A_{i2}$.
16. What is the value of $(AB)_{2,1}$?
17. What are the dimensions of B^T?
18. The L^2 norm of a vector x is defined as follows: $\|x\| = \sqrt{x^T x}$. Calculate $\|v\|$.
19. The L^1 norm of a vector x is defined as follows: $|x| = \sum_{i=1}^{n} |x_i|$. Calculate $|u|$.
20. Consider the columns of the matrix B to be a set of vectors in R^4. Is this set linearly independent, or linearly dependent, and why?
21. Say we removed the fifth column of the matrix B. Is it possible for the set of the remaining columns to span the vector space R^4? Why or why not?
22. What does the matrix I look like in the expression IA?
23. What does the matrix I look like in the expression BI?
24. Say there exists some matrix C such that $AC = CA = I$. What is C?
25. Let M and N be matrices such that MN is defined. Is the product $N^T M^T$ defined?

26. Tickets at Empire Cinema cost £4.00 for children and £6.00 for adults. On a given day, 3,700 tickets were sold. Total ticket revenues were £18,000. Say we wanted to determine the number of children and the number of adults that attended the cinema that day. Give the linear system we would use to find this. Next, give the equivalent coefficient matrix (A) and constant vector (b).
27. In a particular clinic, 10% of patients are prescribed narcotic pain killers. Overall, 5% of the clinic's patients are addicted to narcotics. Out of all the people who are prescribed pain pills, 8% are addicts. If a patient is addicted to narcotics, what is the probability that they will be prescribed pain pills?
28. A guitar manufacturer has factories in Fresno, Nashville, and Memphis. The Fresno factory produces 32% of all the guitars, the Nashville factory produces 43%, and the

Memphis factory produces 25%. 16% of all the guitars produced at the Fresno factory are acoustic, the rest are electric. At the Nashville factory 88% of the guitars produced are electric. At the Memphis factory, 73% of the guitars produced are acoustic.

a. What is the probability that a guitar produced by this company is electric?
b. We select a guitar at random that was produced by this company. The guitar is an acoustic guitar. What is the probability it was manufactured in Memphis?

29. Let X and Y be independent random variables.
    a. Say the probability of some observation $x \in X$ is $P(x) = 0.81$. Now say we observe some $y \in Y$. What is $P(x|y)$?
    b. Say the distribution of X is given by $X \sim p(x\ |\mu, \sigma^2)$. Now, say we take a sample of size N from X. Our sample is $\{x_1, x_2, \dots, x_N\}$. What is the likelihood function of our sample?
30. Starting with the definition for the conditional probability of an event, given the observation of another event $P(A|B)$, derive the formula from Baye's Theorem.
31. A ship is transporting livestock across the sea. The ship contains 38 sheep, 12 cows, and 22 pigs.
    a. What it the probability that an animal on the ship is a sheep?
    b. What is the probability that an animal on the ship is a pig, given that it's not a sheep?
    c. What is the age of the ship's captain?