06: Multivariate Linear Regression Questions

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1

Let A an invertable symmetric matrix. Show that A^{-1} is symmetric.

$$\begin{aligned} A &= A^T \\ (A)^{-1} &= (A^T)^{-1} \\ &= (A^{-1})^T \end{aligned}$$

We start with the fact that A is symmetric. Then, since A is invertable, we take the inverse of both sides. Then we use the rule that the transpose of an inverse matrix is equal to the inverse of the transpose matrix. In the end, we have $A^{-1} = (A^{-1})^T$, which means that the inverse of the symmetric matrix we started with is equal to its own inverse, which means A^{-1} is also symmetric.

2

Let A, B, and C be matrices such that CA = I and AB = I. Show that C = B.

$$\begin{aligned} CA = 1 &\Rightarrow C = A^{-1} \\ AB = 1 &\Rightarrow B = A^{-1} \\ C &= A^{-1} = B \\ \therefore C &= B \end{aligned}$$

Since the product CA is the identity, A is invertable and C must be its inverse. Likewise for the product AB.

3

Say we are considering the likelihood function of a multivariate linear regression. Let Σ be the covariance matrix for the N multivariate Gaussians which are multiplied together in the likelihood function. How would we interpret the value Σ_{kk}?

$$\Sigma_{kk} = \sigma_{\phi_k}\sigma_{\phi_k} = \sigma^2_{\phi_k} = tr[\Sigma]$$

These are all different ways of writing the same thing. Essentially, Σ_{kk} are all the elements of Σ where the row index and the column index are equal – in other words, it is is the trace of Σ, which represents the variance of X along each index of k.

4

Say we have a model $\hat{Y} = \Phi W$. Let $W = [w_1, w_2, ...w_K]$. What is the result of Φw_k for $1 \leq k \leq K$?

$$\hat{y}_k = \Phi w_k, \ \forall k \in \{1 \leq k \leq K\}$$

$\hat{Y}$ is an N x K matrix modeling K targets with N observations. $\hat{y}_k$ is then the N x 1 matrix of the target model for the kth set of targets.