Thermionic Emission

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Abstract:
In this experiment, thermionic emissions are explored using Stefan-Boltzmann's, Child's, and Richardson's laws. The emissivity and temperature dependence to the power input are experimentally determined using the Stefan-Boltzmann law, $P = \epsilon A \sigma T^n$, where $\epsilon = .57 \pm .30$ and $n = 3.95 \pm .05$. Child's Law, $I_s = l \frac{8\pi\epsilon_0}{9} \sqrt{2\frac{e}{m}} V_0^{3/2} r_a^{-1} \beta^{-2}$, is used to extrapolate values for V_0 and I_0, which are then used in Richardson's equation, $J_0 = A_0 T^2 \exp\left(-e\phi/kT\right)$, which in turn is used to calculate the work function of tungsten. The value for V_0 found is $1.2 \pm 0.10\ V$. On the other hand, an array of values for I_0 are found that correspond with different power inputs; these are: $.135 \pm .01\ mA$, $.82 \pm .1\ mA$, $4 \pm 10\ mA$, $8.9 \pm 1\ mA$, and $12.2 \pm 1\ mA$. The work function for tungsten is then found to be $2.99 \pm 1.0\ V$.

Introduction:

When a metal is heated to high temperatures it starts to glow, a process which is described by the Stefan-Boltzmann Law. When in thermal equilibrium, if an object radiates away all the energy it receives, it is said to be a "blackbody". In real life, such objects don't exist and are better described as "greybodies" that only emit a portion of their incoming energy.

When a strong enough electric potential is applied across the filament, loosely bound valence electrons start to fly off and are directed towards the anode by an electric field, which results in an electric current. However, the higher the emission current, the more electrons are present at any given time outside the filament, which provides an increasing electric force that retards further emission. This process results in an asymptotic limit of the emission current and is described by Child's Law.

The work function of tungsten can be directly calculated from the minimum potential required to produce an emission current. The relationship between the resulting minimum current or, interchangeably, current density and minimum voltage is described by Richardson's equation.

Theory:

The total electric power to the filament is found by

$$P = IV \tag{1}$$

where I is the current and V is voltage. The Stefan-Boltzmann law is given by

$$P = \epsilon A \sigma T^4 \tag{2}$$

where P, ϵ, A, σ, and T are the radiated power, emissivity, surface area of the filament, Stefan-Boltzmann constant ($5.68 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$), and the temperature in Kelvin. The emissivity can be thought of as a likeness factor relating the greybody to an idealized blackbody, where $\epsilon = 1$.

The derivation Stefan-Boltzmann's equation comes by integrating the Planck radiation formula over all wavelengths.[1]

$$\frac{dP}{d\lambda}\frac{1}{A} = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)} \tag{3}$$

The asymptotic emission current, taking into account a "cylindrical" diode, is taken to be [2]

$$I_{asymptotic} = l\frac{8\pi\epsilon_0}{9}\sqrt{2\frac{e}{m}}V_a^{3/2}r_a^{-1}\beta^{-2} \tag{4}$$

where $I_{asymptotic}$ is the maximum the emission current seems to approach, l is the filament length, $\frac{e}{m} = \eta$ is the ratio of the electric charge to mass, V_a is the applied potential, r_a is the anode radius, and β^{-2} is some constant.[3]

[1] This derivation can be found in many physics book or from `<http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/stefan2.html>`

[2] Given as equation 2.4c on page 73 of Melissino's "Experiments in Modern Physics."

[3] See Table 1 in the appendix for these constants' values.

The Richardson equation is taken as[4]

$$J_0 = A_0 T^2 e^{-\frac{e\phi}{kT}} \tag{5}$$

where J_0 is the saturation current density extrapolated to 0, ϕ is the work function of tungsten, and A_0 is a constant equal to

$$A_0 = \frac{4\pi m e k_b^2}{h^3} = 1.2 \times 10^6\ A\ m^{-2} K^{-2} \tag{6}$$

Procedure:

The filament power supply, anode power supply, and other necessary elements are first connected as shown in Figure 8.[5] After the pyrometer is carefully aligned with the small opening in the anode and focused, a few special considerations are made. First, the filament current must not exceed 2.5 Amps. Second, The scaling knob seems unimportant, but it determines which of the three Temperature scales will be used when recording data. If the Temperature readings (in Kelvin) are not around the 1900 - 2400 K, the incorrect scale is most likely being used.

The first set of measurements is used to determine the uncertainty in the brightness temperature readings. An initial reading is taken by recording the filament current and voltage and adjusting the pyrometer until it blends with the filament as close as possible. The temperature is recorded and the pyrometer is adjusted slightly above and below the best reading until one can just see that the filament and pyrometer are not perfectly aligned. These temperatures are also recorded and used to find the uncertainty in the brightness temperature readings.

The second set of measurements is used to experimentally determine the emissivity and temperature dependence of the Stefan-Boltzmann equation. Enough measurements are taken across the full range of filament current allowed (1-2.5 A) to ensure a sufficient statistical fit.

The third set of measurements is used to explore the principles behind Child's law and Richardson's equation (the stopping potential and the asymptotic current extrapolated to zero) and is comprised of subsets of measurements spanning the allowed filament current range. Start by adjusting the filament current around the middle of its range (1.7 A or so) then take several readings of the anode current by adjusting the applied potential across the full range (the brightness temperature need not be recorded). Seven to ten measurements for each subset should be adequate. Several subsets should be taken by adjusting the filament current from 1.7 A to 2.5 A. Special care should be taken to include the 1.7 A and 2.5 A in the subsets for the filament current.

Now that all the data is recorded, it can now be manipulated and explored. First, the electrical power is calculated for the first two sets using Equation 1 and all brightness temperatures are converted to Kelvin (just add 273.15). All of the data sets and subsets are then imported into Python where it is analysed.[6]

Results:

First, the true temperatures are calculated; a linear fit is constructed from known brightness to temperature values[7] and then used to calculate the true temperatures. Figure 1 and Figure 3 show the

[4]Equation 2.2 on page 68 of Melissino's-"Experiments in Modern Physics." The derivation is also available.

[5]See Appendix

[6]All data collected and the Python code written for this project can be requested at the following url, `<https://drive.google.com/drive/folders/0BzjQ_RadlQmdcHFxdVlTTEcwTTg>`

[7]The data for this is taken from the table, "Roeser and Wensel, National Bureau of Standards" in the handout.

results from the linear fit and the true temperature calculations.

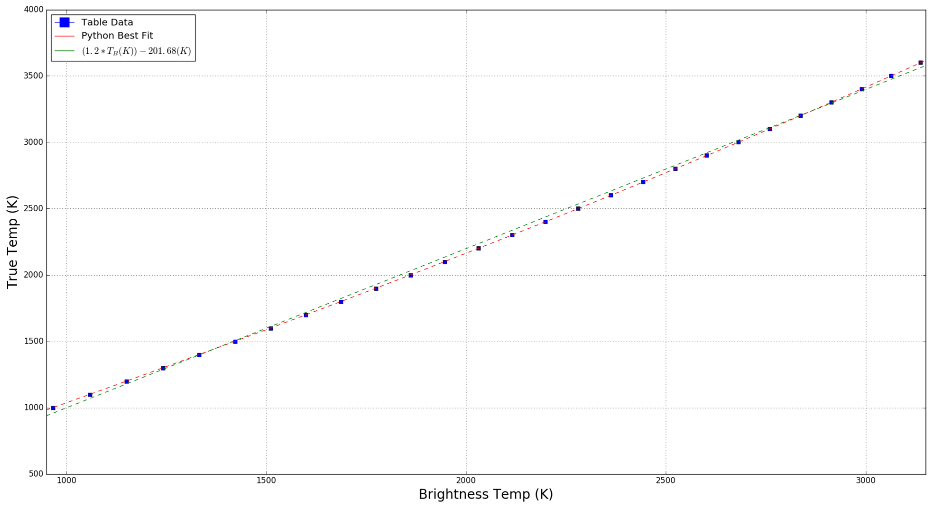

Figure 1: Fitting True Temperature to Brightness Temperature

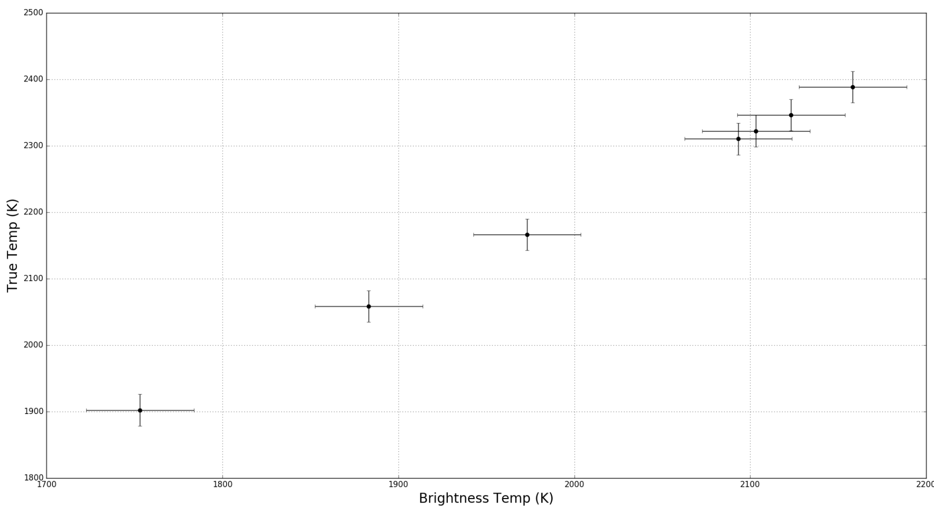

Figure 2: True Temperatures

An ϵ value of $.570 \pm .295$ is then found by fitting it to the power array (y-axis) and the brightness temperature array (x-axis) to Equation 2, using T^4 and $A = 1.30 \times 10^{-5}\ m^2$, which can be seen in Figure 3.

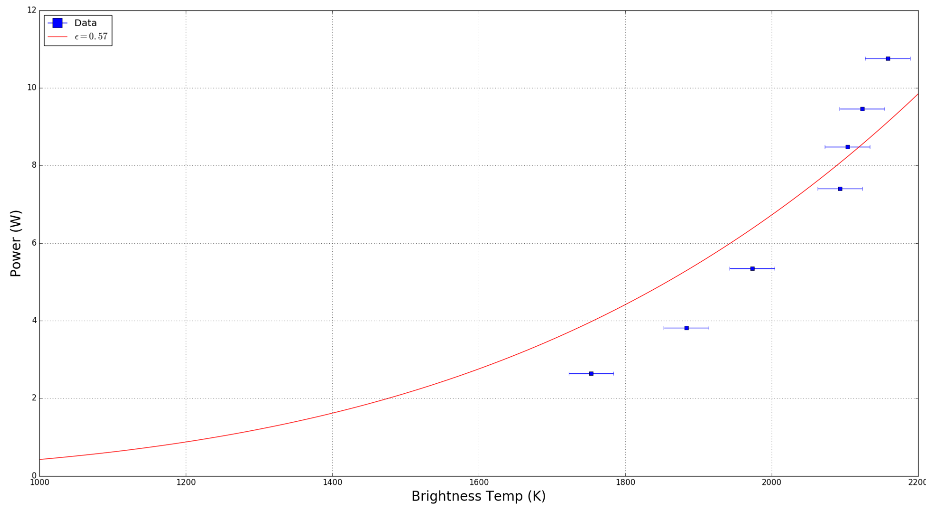

Figure 3: Fitting Epsilon

The temperature dependence $n = 3.95 \pm .05$ is found by fitting the power array (y-axis) and the true temperature array (x-axis) to Equation 2, using previously found ϵ. In Figure 4, the best fit line is also compared with the known dependence of $n = 4$.

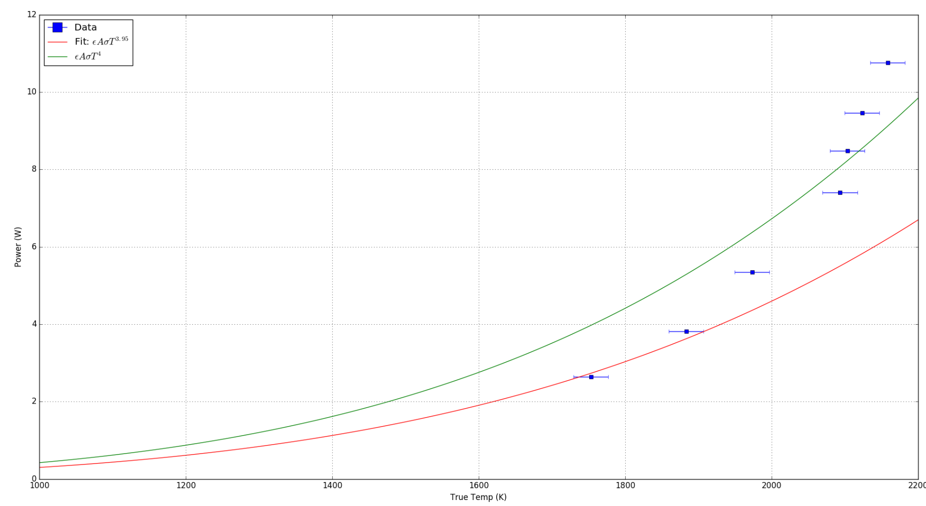

Figure 4: Temperature Dependence

Figure 5 is produced by plotting the subsets of data in the third set of measurements. The subset where $I_{filament} = 2.29\ A$ is used to extrapolate the stopping potential by leaving off the last two data points, fitting Equation 4 to η and V^n and finally extrapolating the curve to 0. V_0 is found to be $1.2 \pm .1\ V$, $\eta = \frac{e}{m}$ is found to be $4.14 \times 10^{11} \pm 2.38 \times 10^{11} C\ kg^{-1}$, and n is found to be $1.30 \pm .198$.

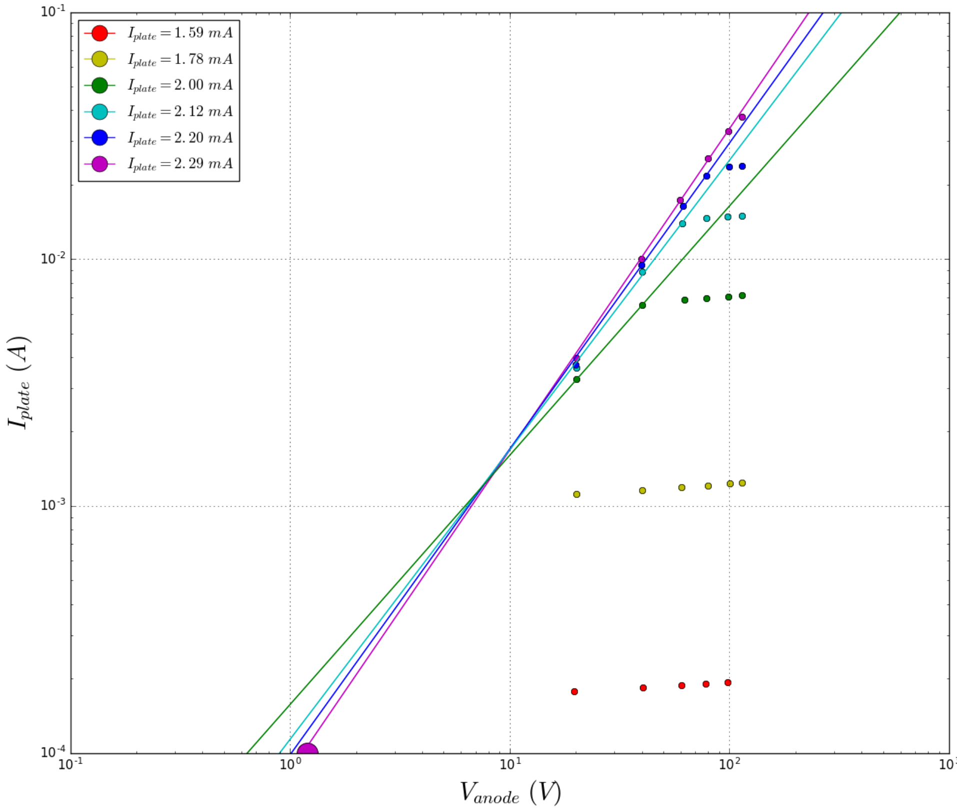

Figure 5: Extrapolating V_0

Figure 6 is produced using the same data as Figure 5, but I_0 is extrapolated to 0 on each curve by leaving off an appropriate amount of data points at the beginning of each array and again fitting to Equation 4. The magenta data set ($I_{filament} = 2.29\ A$) is not used. It is noted that the best data sets to find V_0 are also the hardest to find I_0 values. The I_0 values found are as follows: $.135 \pm .01\ mA$, $.82 \pm .1\ mA$, $4 \pm 10\ mA$, $8.9 \pm 1.0\ mA$, $12.2 \pm 1.0\ mA$.

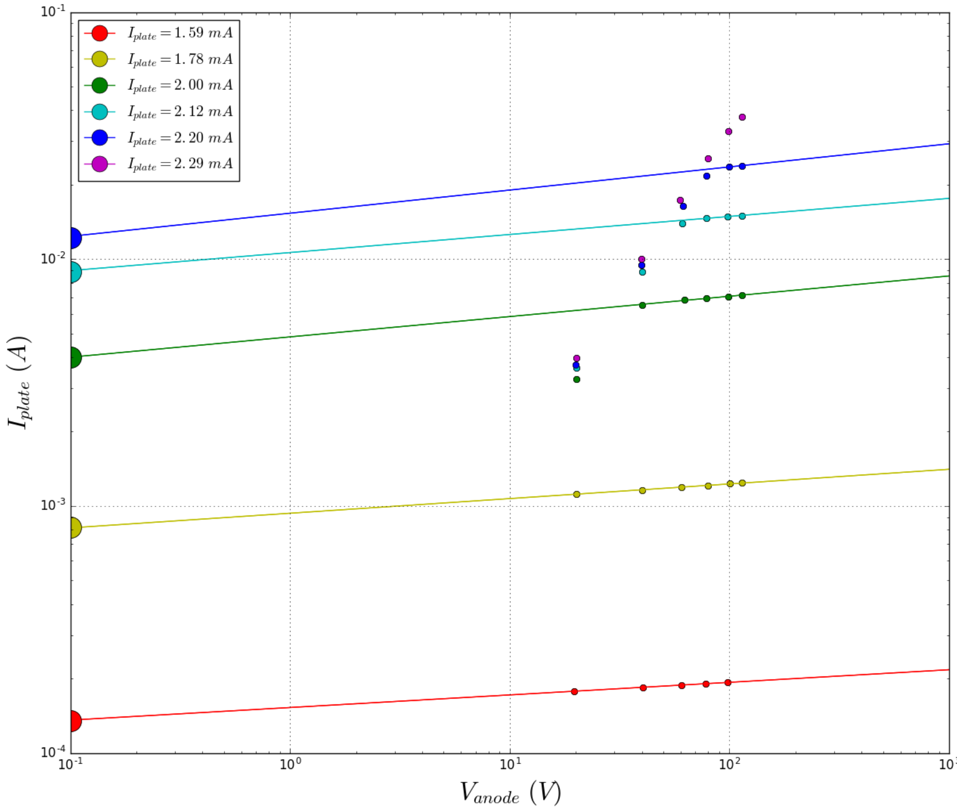

Figure 6: Extrapolating I_0

The I_0 values found are then converted to their respective J_0 values by simply dividing them by $A = 2\pi r_a l$. The J_0 and true temperature arrays are then used to make the semi-log plot shown in Figure 7. The slope of this graph, in theory, is in units of Kelvin and is equal to $\frac{e}{k_b}\phi$. The work function is then calculated as follows:

$$\begin{aligned} m &= (1000) \times \frac{\log Y[4] - \log Y[0]}{X[4] - X[0]} \\ \Rightarrow m &= 34701 \pm 385\ K \\ \phi &= m(K) \times \frac{k(J/K)}{e} \frac{(eV)}{(1.602 \times 10^{-19}\ (J))} \\ \Rightarrow \phi &= 2.99 \pm 1.0\ V \end{aligned} \tag{7}$$

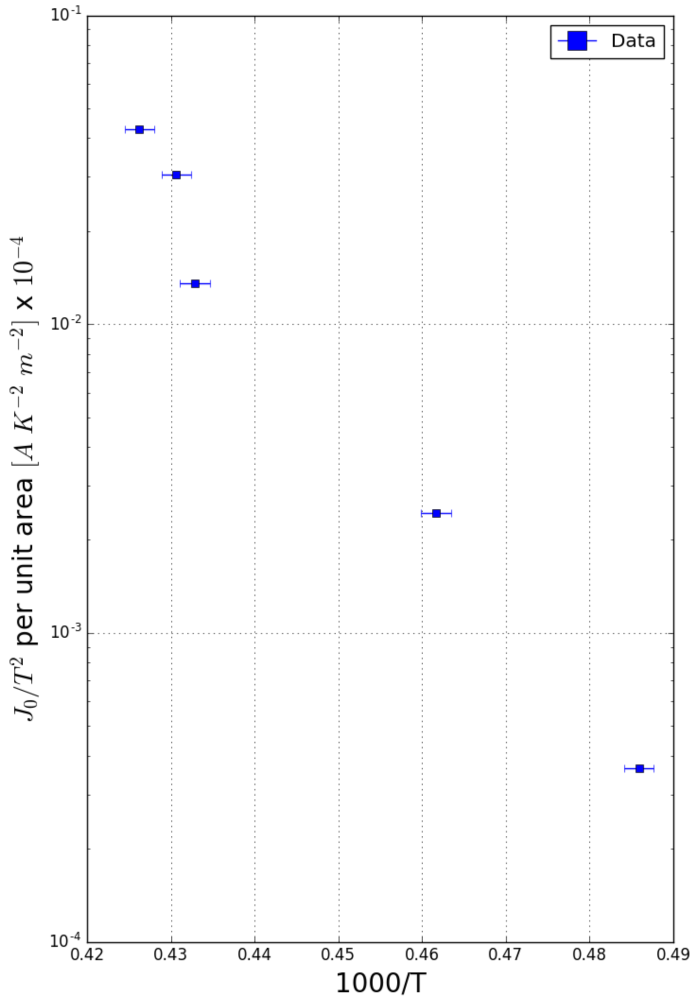

Figure 7: Finding Work Function

Discussion:

The uncertainty in all current and voltage measurements is taken as $\Delta.005$ due to the precision of the ammeter and voltmeters used. The resulting err in the power calculations is found by

$$P = (I[i] \times \Delta V) + (\Delta I \times V[i]) + (\Delta A \times \Delta V) \tag{8}$$

where i is the i^{th} index in the associated array. $\Delta P[i]$ is plotted in Figures 3 and 4, but they are tiny compared to the scale of the graph. The uncertainty in the brightness temperature is the standard deviation of the three measurements in the first set and found to be $\Delta B\ Temp = 30.6\ K$. The uncertainty in the true temperature is found by

$$\begin{aligned} \Delta T\ Temp &= std(PythonFit[i] - LinearFit[i]) \\ &= 30.7\ K \end{aligned} \tag{9}$$

The error found $\Delta\epsilon$ was found by subtracting the experimental value from the known value as was the n for the temperature dependence.

The uncertainty in extrapolating V_0 and I_0 is found experimentally in Python by making minute changes to the large markers in Figures 5 and 6 and centering them by eye.

The uncertainty obtained in calculating the work function is a little more tricky and is started by finding the uncertainty in the y-axis, since its slope is taken.

$$\begin{aligned} T &= \sqrt{T[\,]\cdot T[\,]} \\ I &= \sqrt{I[\,]\cdot I[\,]} \\ \Delta Y[i] &= \frac{I[i] + \Delta I}{T[i]^2 + 2T[i]\Delta T + \Delta T^2}\frac{T^2}{I} \end{aligned} \tag{10}$$

where T is the magnitude of the true temperature array $T[\,]$, I is the magnitude of the I_0 array $I_0[\,]$, ΔT and ΔI are the respective uncertainties previously obtained, and $\Delta Y[i]$ is the uncertainty associated with the y-axis in Figure 7, which is needed to find the slope. The uncertainty in the x-axis is simpler at $\Delta X = \frac{1}{\Delta T^2}$. The uncertainty in the slope m is then

$$\begin{aligned} \Delta m &= std(\frac{\log \Delta Y[i]}{\Delta X}) \\ &= 385\ K \end{aligned} \tag{11}$$

There are many feasible ways of determine the uncertainty in the work function, but the simplest one is found to be

$$\begin{aligned} \Delta\phi &= std\left(\frac{J_0[i]}{T[i]}\right) \\ &= 1.0\ V \end{aligned} \tag{12}$$

Conclusion:

The main results and associated errors are listed for convenience in the Appendix. All in all the results and associated error largely agree with expected values. One of the largest errors seen is $\Delta\eta = -2.4E11\ C/kg$, which is 135 percent over the expected value. Although large, it is still acceptable.

Appendix:

Table 1: Constants for Child's Law

l	filament length	3.17×10^{-2} (m)
ϵ_0	permittivity of free space	8.854×10^{-12} (F/m)
r_a	anode radius	7.90×10^{-3} (m)
β^{-2}	correction coefficient	0.93

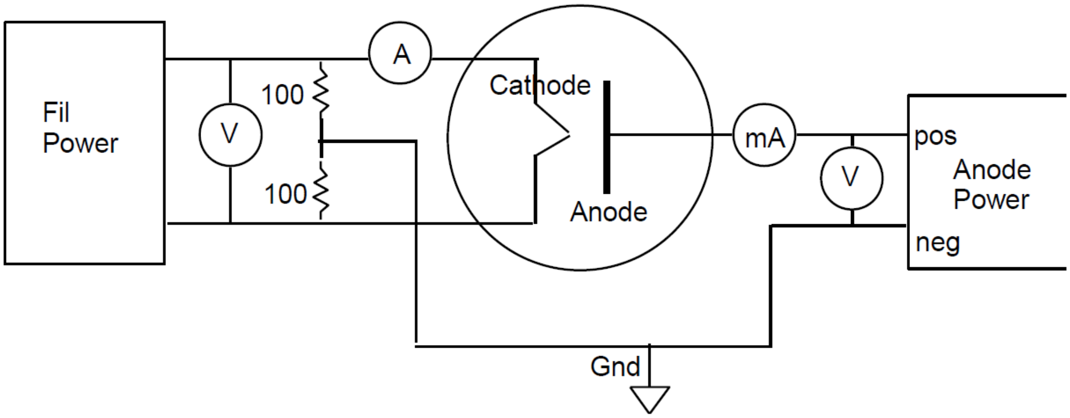

Figure 8: Circuit Diagram

Error in Brightness Temp = 30.5941170816 K
Error in True Temp = 23.7325259224 K

Error in Fig 2:
X err = 30.5941170816 K
Y err = 23.7325259224 K

T Temp = [1902.1 2058.1 2166.1 2310.1 2322.1 2346.1 2388.1] K

Results for Fig 3:
Data Fit ep = 0.569148051006
Known Average ep for range = 0.273666666667
epsilon error = -0.29548138434

Brightness Temp Error = 30.5941170816 K
Power Error = [0.016475 0.019975 0.023925 0.028525 0.030625 0.032525 0.034975] W

Results for Fig 4:
Experimental n = 3.94992620512
Error in n = 0.0500737948829
True Temp err = 23.7325259224 K
Power err = [0.016475 0.019975 0.023925 0.028525 0.030625 0.032525 0.034975] W

Results from Fig 5:
Extrapolated value for $V_0 = 1.2V$
$V_0 err = 0.1V$
error not shown on graph because it looks sloppy

e/m and n error:

em = 4.14047462534 E11 C/kg
Error in e/m = -2.38022849854 E11 C/kg
Error in e/m = -135.2213456 percent
n = 1.30153960091
Error in n = 0.198460399095

Results from Fig 6:
$I_0 = [0.1350.824.8.912.2] mA$
$I_0 uncertainty = [0.010.110.1.1.] mA$
error not shown on graph because it looks sloppy

Error in Fig 7:
Y err not shown on graph; looks clunky in semi-log plot
Error in Temp = 23.7325259224 K
Error in $J_0/T^2 = [0.05010278, 0.27507073, 1.1857542, 2.60160752, 3.49421427] AK^-2m^-2$
Slope = 34701.3189604 K
Slope error = 385.356534718 K
Work Function for Tungsten = 2.99076848244 V
Error in Work Function = 1.0 V