Noise Fundamentals

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Abstract:
In this experiment we analyze Johnson-Niquist-white noise to obtain an experimental value for Boltzmann's constant at room temperate and at 77 Kelvin. We find Boltzmann's constant at room temperature to be $k = 1.2 \times 10^{-23} \pm 0.2 \times 10^{-23}$ J/K; we find Boltzmann's constant at 77 Kelvin to be $k = 1.2 \times 10^{-23} \pm 0.5 \times 10^{-23}$ J/K. These values agree very well with the known value of 1.38×10^{-23} J/K. The error of this measurement is 1.8×10^{-23} J/K (13%)

Introduction:

Behind every new discovery there are scientific measurements; every measurement is taken with an instrument; each instrument has an imperfect accounting of reality. In electronic instruments, this imperfect accounting is due, largely, to the random motions of conducting electrons in the circuit, which produce an unwanted signal called noise.

In the scientific sense, noise is an unwanted signal that puts a lower limit on the accuracy an instrument can possibly have; when the measurement and noise are comparable, the measurement is no longer reliable. By understanding noise sources and how to account for them, it becomes possible take measurements at the fundamental noise level of the sensitive instruments, allowing us to probe deeper into the nature of reality.

In this series of experiments, we test the nature of small voltage fluctuations caused by random thermal motion. Since resistors dissipate energy as heat, we expect that an analytical accounting of this noise source will be temperature and resistance dependant.

Thermal noise was first measured by Johnson[1] and was first explained by Nyquist[2]. For this reason, thermal noise is called either Johnson or Nyquist noise. Because it distributed over the entire spectrum, it is also classified as "white" noise. Since thermal noise is spread across the spectrum and instruments aren't currently able to measure the entire spectrum, its expected that the noise calculation will also depend on the frequency bandwidth the instrument is sensitive to.

Theory:

Nyquist showed that the average squared value of the voltage fluctuations is given by

$$< V_J^2 >= 4kTR\Delta f, \tag{1}$$

where k is Boltzmann's constant, T is the temperature, R is the resistance, and Δf is the frequency bandwidth over which the noise is measured. As expected, it is dependant on temperature, resistance, and frequency bandwidth. The Boltzmann constant has units of energy per temperature degree and often shows up in temperature calculations and distributions (such as blackbody radiation). As such, it is not surprising that it shows up here as well.

In the experiments, a digital voltmeter is attached the the circuit and is used to measure the voltage fluctuations. Before reaching the voltmeter, the measurement passes through a multiplier which decreases the signal by a factor of ten and squares it - to make it more compatible with the V^2 dependence in Equation 1. Before reaching the multiplier, the signal is also amplified by the "low-level" and the "high level" electronics. The total effect is shown by Equation 2,

$$\begin{aligned} < V_J^2 > &= V_{out}/G \\ G &= G_1 G_2 G_3 \\ G_1 &= 600 \\ G_3 &= 1/\sqrt{10}, \end{aligned} \tag{2}$$

where V_{out} is the reading from the digital voltmeter, G is the total gain in the signal, G_1 is the low-level gain, G_2 is the high-level gain, and G_3 is the multiplier gain. In these experiments, both G_1 and G_3 are kept constant and G_2 is adjusted to keep $V_{out} \approx 1$ (in order to avoid saturation).

Procedure:

After completing the setup (Part I) as described in the pre-lab (Appendix), we start Part II (Full Bandwidth Noise). The full band width is found by switching the oscilloscope to "store" mode and setting mode to "average". This average is then slowly adjusted to trigger off of the maximum voltage and allowed to run until the signal is almost a smooth distribution (perhaps Gaussian?) We make a crude approximation of the distribution's Full Width at Half Max (FWHM) and record it in convenient units of time; the full bandwidth is taken as the inverse of the FWHM. Once the full bandwidth has been approximated, Equations 1 and 2 are compared using a set resistance, room temperature ($\approx 298K$), the full bandwidth, the known value of k, V_{out}, and an appropriate value for G_2.

In Part III, we explore Johnson noise as a function of bandwidth and find an experimental value for k.

First, we make a similar comparison to the one made in Part II between V_{out} and $< V_J^2 >$. A low pass filter ($f_1 = 100\ Hz$) and a high pass filter ($f_2 = 100\ kHz$) is used; the resulting bandwidth ($\Delta_f = 110{,}961\ Hz$) is found by using Table 1.5 in the Teachspin Noise Fundamentals manual. The low end gain is adjusted as needed and the other parameters - R and T - are the same as in Part II.

[1] Phys. Rev 32, 97 (1928)
[2] Phys. Rev 32, 110 (1928)

Next, a series of measurements are taken varying R and adjusting G_2 as needed. By fitting this data along a line, the underlying amplifier noise will be seen as the y-intercept (when $R = 0$). A better value of the Johnson noise ($< V_J^2 >$) is found by subtracting the amplifier noise.

Last, an experimental value for k is found by finding the slope of a linearized plot of the spectral noise as a function of bandwidth.

In Part IV) we repeat the first experiment of Part III using two resistors that are attached to a temperature probe. This probe will later be immersed in a liquid nitrogen bath so that Boltzmann's constant can be calculated at 77 K. Before applying liquid nitrogen, we test the external resistors A_{ext}, B_{ext}, and C_{ext} by repeating the exercise mentioned and checking that the preamp noise agrees with the previous result.

During this exercise, we found that C_{ext} in our probe was lower much lower than expected and was unreliable; we decided to not include any measurements from it.

In Part V) we immerse the probe in liquid nitrogen and allow a sufficient length of time for the probe to approach thermal equilibrium with the liquid nitrogen. A total of three measurements should be taken (one for each resistor). Boltzmann's constant is then found by finding the slope of the linearized fit to three points (two in our case).

In Part VI) we use the software "Labview" to record the spectral density of the Johnson noise. Several spectra are recorded using various low and high frequency filters (f_1 and f_2) to create an effective bandwidth, Δf. The spectra are then compared with the values found on Table 1.5 in the Teachspin Noise Fundamentals manual to make sure that the table values used in the experiments are reasonable.

All of the linear fitting parameters and their uncertainties are found using the "fits" python module.[3]

Results:

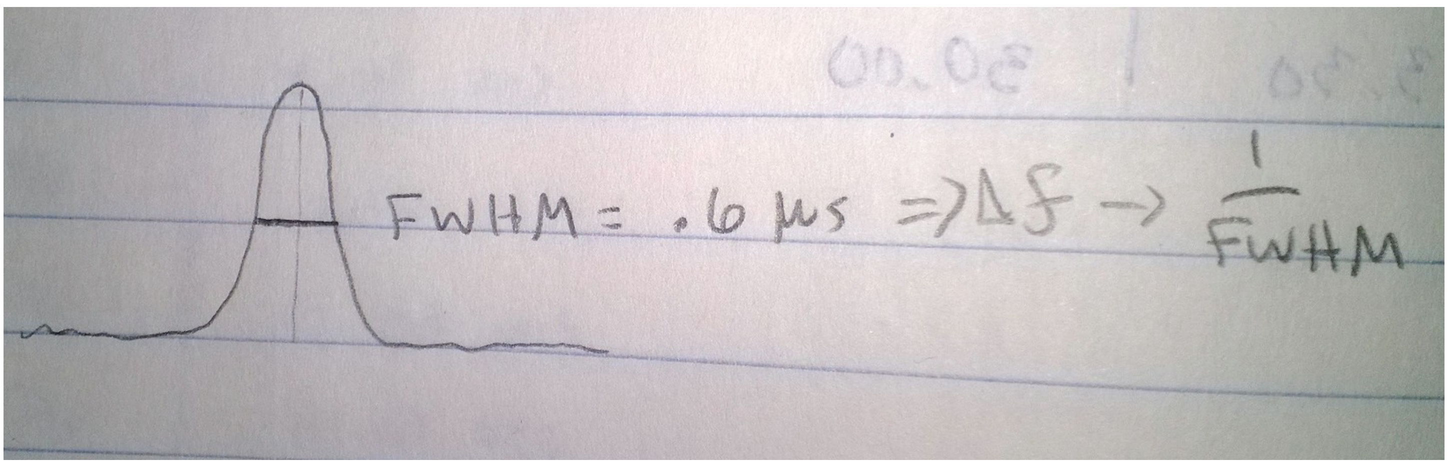

Figure 1: Part II) Full Bandwidth Noise.

This is a sketch of the oscilloscope reading when switched to "store" mode with mode set to "average". The trigger level is adjusted to only trigger from the highest voltage. We find the Full Width at Half Max (FWHM) from this distribution to be .6 μs which implies that Δf is approximately 1,666,666 Hz, or 1.7 MHz. This represents the largest frequency bandwidth the instruments will be able to detect.

The following parameters are used to calculate $< V_J^2 >$: $\Delta f = 1.7\ MHz$, high level gain $G2 = 400$, $k = 1.38 \times 10^{-23}\ J/K)$, $T = 298\ K$, and $R = 10\ k\Omega$. The error between $G < V_J^2 > = 1.27\ V^2$ and $V_{out} = 1.37\ V^2$ is therefore equal to $0.1V$ (9%).

In a similar calculation, V_{out} is converted to V_{rms} by squaring Equation 2 and is compared with the square of Equation 1. The results of these calculations give $V_{rms,conv} = 15\ \mu V$ and $V_{rms,calc} = 17\ \mu V$ with an an error of 1 μV (7%).

[3] `https://github.com/jacluff1/djak/blob/master/fits.py`

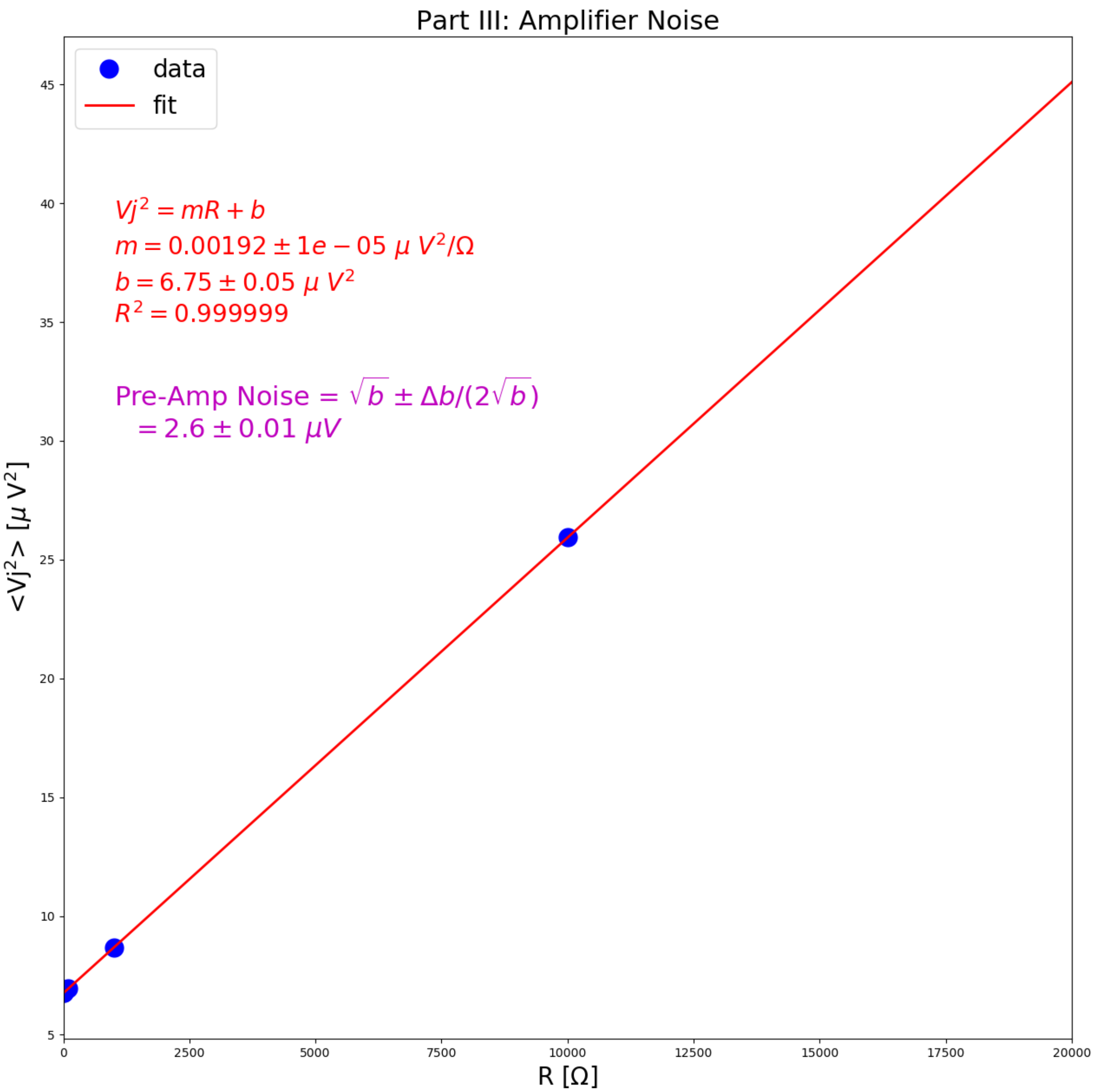

Figure 2: Part III) Pre-Amp Noise at Room Temperature.

The data is taken by recording V_{out} for several values of resistance and corresponding appropriate values of G_2. We convert V_{out} to $< V_J^2 >$ using Equation 2 and fit the data to a line $y = mR + b$.

The preamp noise can be taken as either the $b \pm \Delta b$ in μV^2 (shown in red), or by taking the square root of $\sqrt{b} \pm \Delta b/(2\sqrt{b})$ in μV (shown in magenta).The latter is found using $\Delta b \partial_b(\sqrt{b})$.

We found the best fit by neglecting the two largest resistance values; at higher values of R, the linear behavior is non linear.

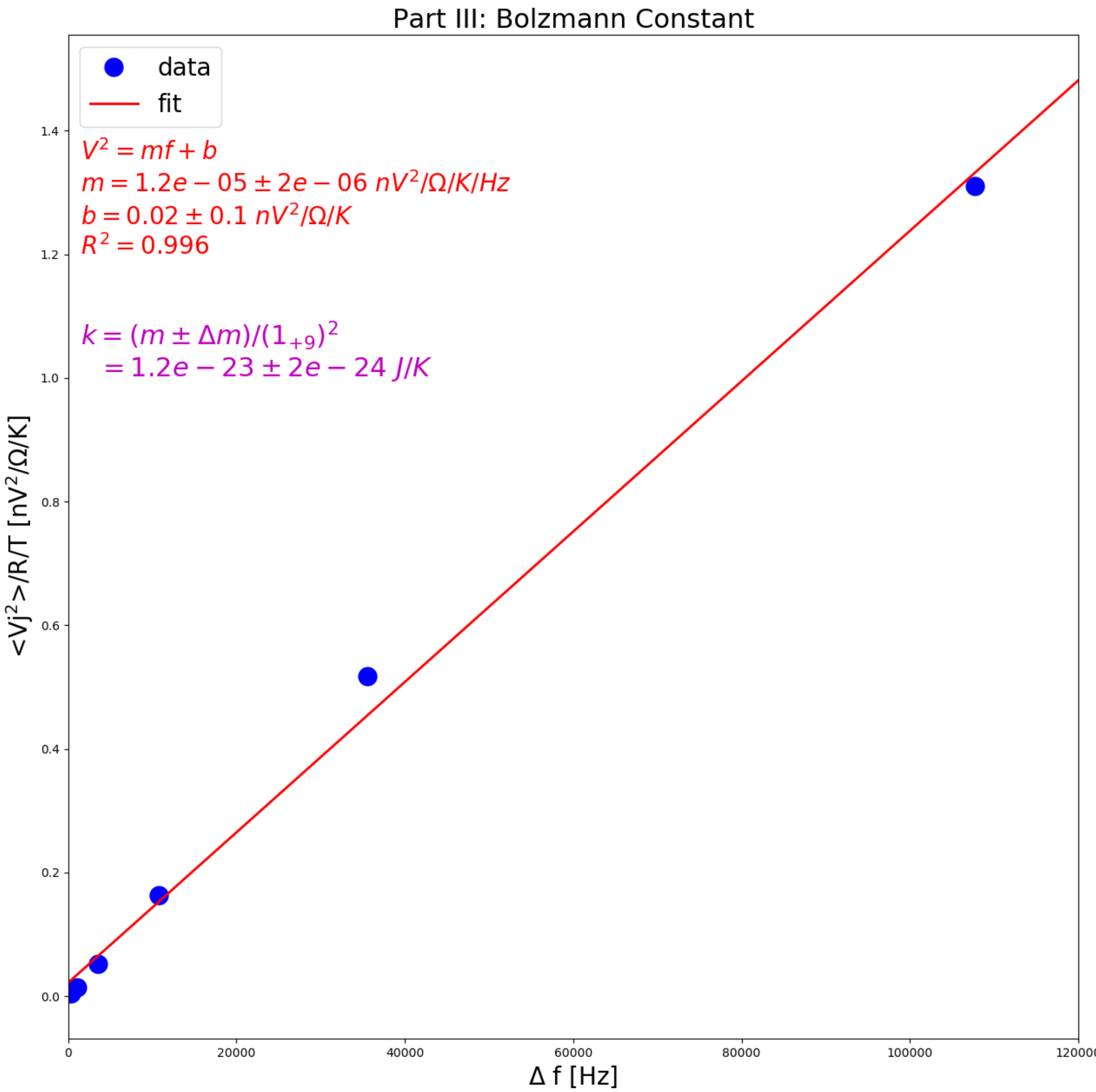

Figure 3: Part III) Experimental Value of Boltzmann's Constant at Room Temperature.

We take the data by recording V_{out} over several sets of f_1 and f_2. The resulting Δf is taken from Table 1.5 in the Teachspin Manual.

Again, V_{out} is converted using Equation 2 and appropriate values of G_2. We then divide $< V_J^2 >$ by the resistance and Temperature so that the fitted slope will only be in terms of Boltzmann's constant.

The result, $k = 1.2 \times 10^{-23} \pm 0.2 \times 10^{-23}$ J/K agrees very well with the known value of 1.38×10^{-23} J/K. The error of this measurement is 1.8×10^{-23} J/K (13%).

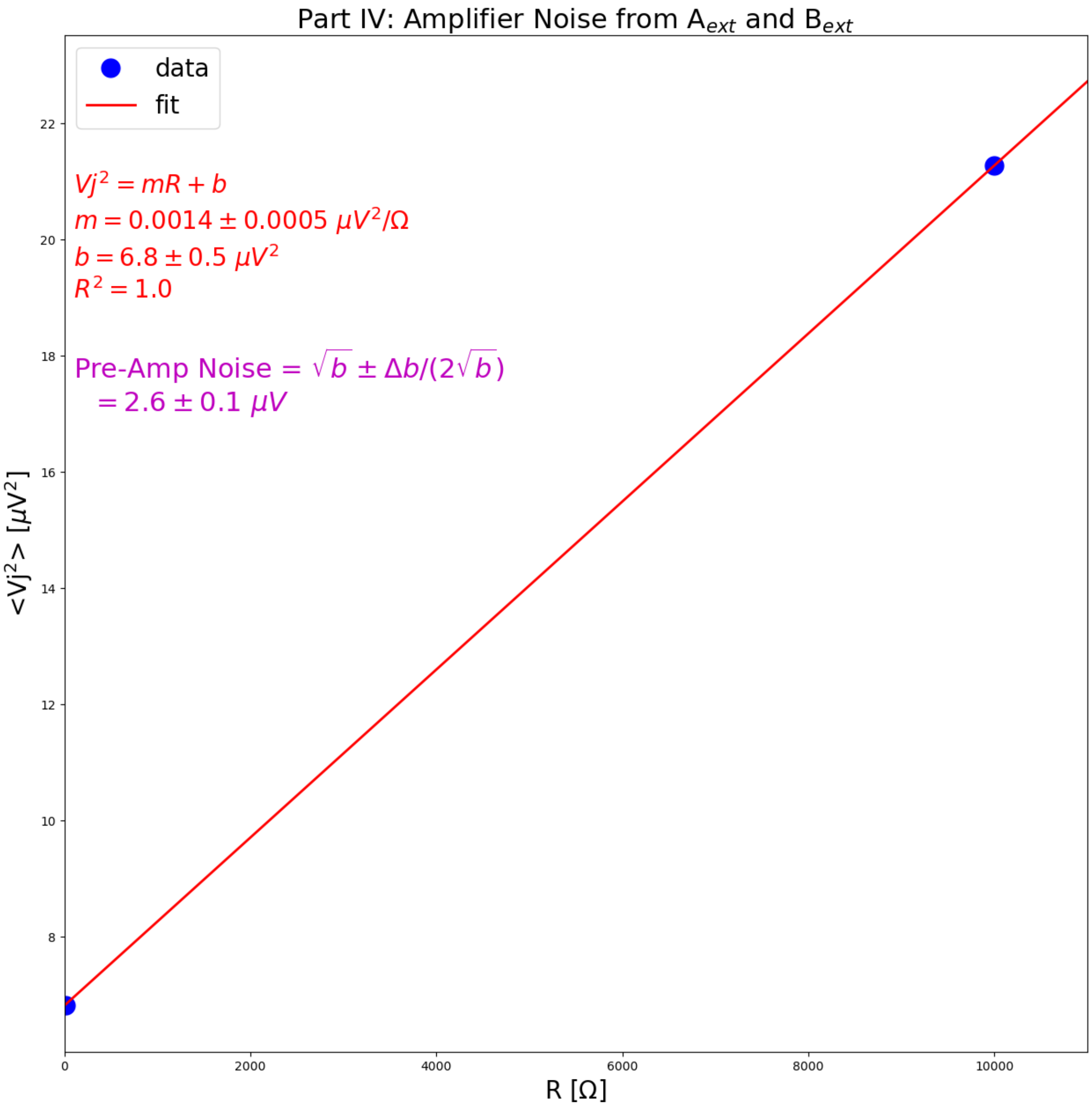

Figure 4: Part IV) Pre-Amp Noise with "External" Resistors.

We take the data by recording V_{out} for resistance A_{ext} and B_{ext} and use corresponding appropriate values of G_2. We convert V_{out} to $< V_J^2 >$ using Equation 2 and fit the data to a line $y = mR + b$.

The preamp noise can be taken as either the $b \pm \Delta b$ in μV^2 (shown in red), or by taking the square root of $\sqrt{b} \pm \Delta b/(2\sqrt{b})$ in μV (shown in magenta).The latter is found using $\Delta b \partial_b(\sqrt{b})$.

Since we only have two data points available, the fitted values are assumed to be reliable to two significant figures. The preamp noise seen here and in Figure 2 are in perfect agreement; within the number of mutually reliable significant figures, they are the save value.

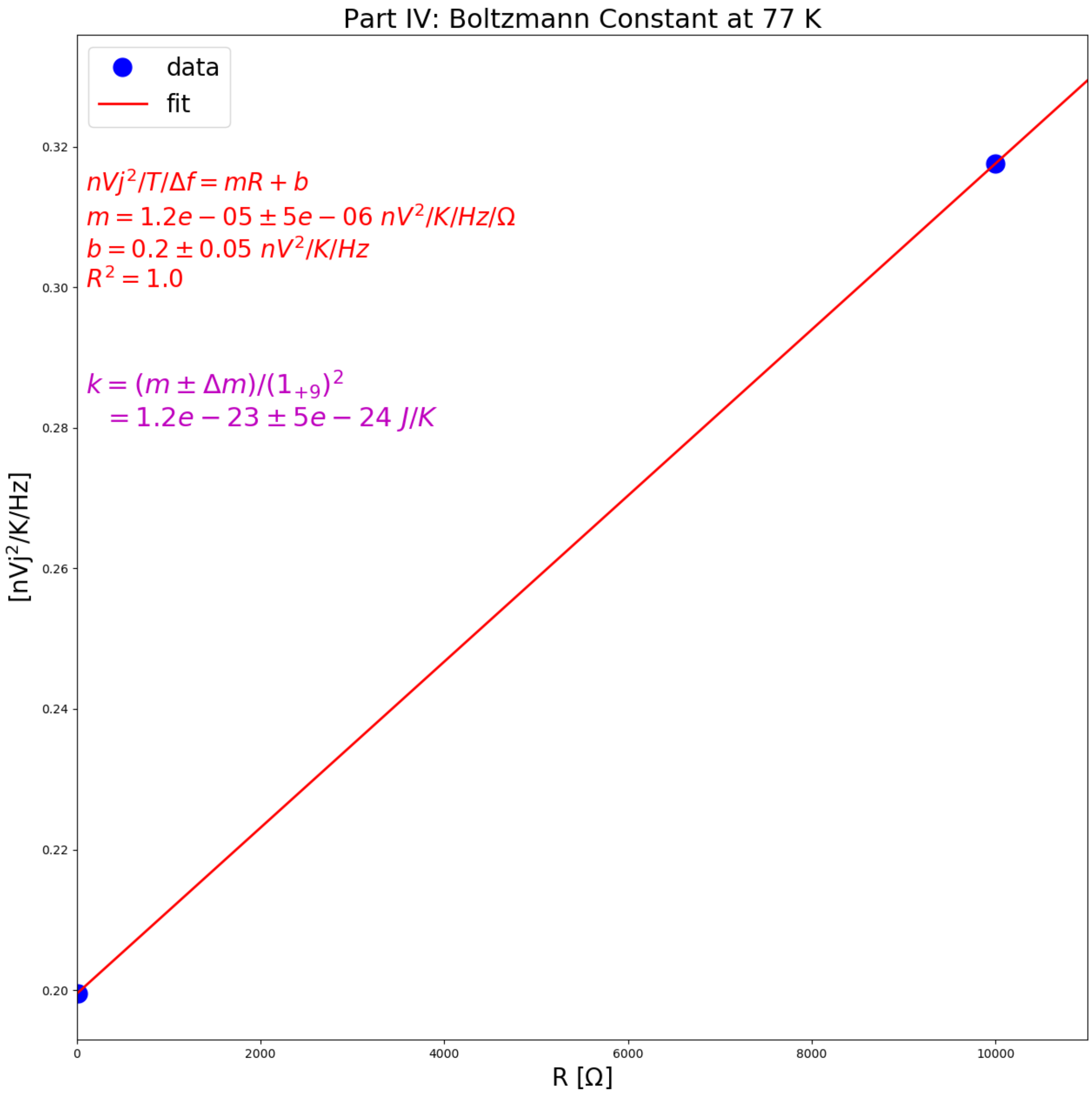

Figure 5: Part V) Experimental Value of Boltzmann's Constant

We take the data by recording V_{out} for resistance A_{ext} and B_{ext} and use corresponding appropriate values of G_2. We convert V_{out} to $< V_J^2 >$ using Equation 2. This result is then divided by the temperature and the bandwidth to get the slope only in terms of Boltzmann's constant and is then fit to a line $y = kR + b$.

The result, $k = 1.2 \times 10^{-23} \pm 0.5 \times 10^{-23}$ J/K agrees very well with the known value of 1.38×10^{-23} J/K. The error of this measurement is 1.8×10^{-23} J/K (13%).

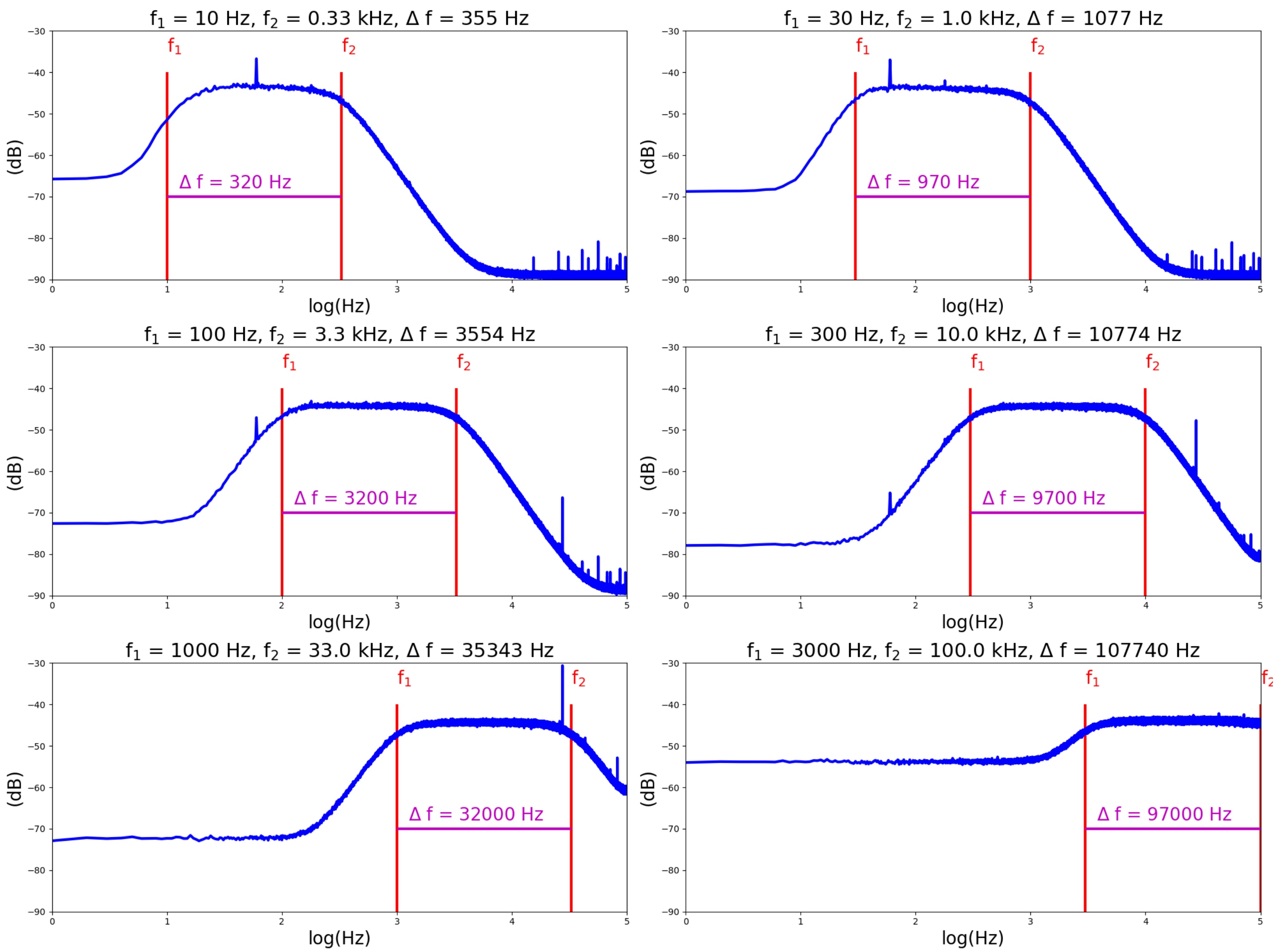

Figure 6: Part VI) Bandwidth from f_1 and f_2.

We recorded the data by setting the low and high band filters appropriately for each measurement and letting "Labview" average each spectra for 250 seconds. The spectra is then exported as a csv vile and imported into Python for plotting.

Each spectra has two vertical lines that correspond with the $log(f_1)$ and $log(f_2)$, respectively. the difference between the f_2 and f_1 is also noted above the horizontal line spanning f_1 and f_2. These lines are visual aid in evaluating how the bandwidth is related to the low and high pass frequency filters.

Comparing the spectra and filters, the bandwidth clearly falls withing expectation. Also, the filter height seems to even out at higher frequencies. We expect noise signal would become indistinguishable around the maximum bandwidth allowed, which seems to agree with the general trend shown in Figure 6.

Since white noise is "visible" at every frequency, removing the low and high band filters would allow all frequencies to be read (theoretically). since the noise is proportional to bandwidth, we would expect that the integrated noise would increase sharply.

Appendix:

Pre-Lab:

1 Observe Low-Frequency Noise (Teach Spin, section 7)

a Verify that the "Low-level Electronics" (LLE, metal case) is attached to the "High-level Electronics" (HLE,wood case) via the "Preamp Power Output."

b Ensure that the low-temperature probe is snugly attached to the "Probe Cable" connector on the front panel of the "Temperature Module" of the Low-level Electronics.

c the "Preamplifier" settings are: input resistor (R_{in}) at B_{ext} (which should connect to the 10 kΩ resistor in the low-temperature probe); feedback resistor (R_f) at 1 kΩ; and feedback capacitor (C_f) at min (not used).

d Connect the "Output" of the Preamplifier to the "IN" of the "Gain" amplifier on the High-level electronics. Typical Gain settings, clockwise from IN are: AC, x1, x10, and 50 on the "Fine adjust."

e Connect the "OUT" of the Gain amplifier to "IN A" of the "Multiplier," and to the input of a digitizing oscilloscope. Typical Multiplier settings are: AC and AxA.

f Connect the "Output" "OUT" to a digital voltmeter. Typical Output settings are 1 s "Time Constant" and 0-2 V "Meter Scale."

g Energize the electronics. Adjust the Gain Fine Adjust to obtain about 1 volt on the Output meter, and on the digital voltmeter.

2 **Full Bandwidth Noise**

a Attacht the "Gain OUT" to channel 1 of the oscilloscope. Change time scales and note that the noise on the oscilloscope covers essentially all frequencies (white noise).

b Measure the AxA output using the digital voltmeter. Take the square root to obtain $\delta\ V_{rms}$. Calculate the absolute rms voltage across the resistor using the Gain values. Be sure to include (1200/200)x100 from the preamp, and /10 from the Output of the multiplier: $V_{out} = [V_{in}(t)]^2/10$.

c Attach the "monitor" output from the Multiplier to CH. 2 on the oscilloscope. Switch to the x-y mode of the oscilloscope, record the signal, and switch back to only Ch. 1 (Gain OUT).

d Switch the oscilloscope to "Store" mode, and the "Mode" to "Average." Adjust the trigger level on the oscilloscope until it triggers only from the highest voltages; the voltage should drop off on either side of the highest voltage, representing a loss in correlation from maximum voltage. Record the time constant for the loss in correlation. Use this time constant to estimate the bandwidth (Δ f) of the amplifiers, which limit the short-time response that you measure.

e Use this Δ f in Eq(4) to calculate the expected V_{rms} from the Output, and compare to your measurements.

3 **Johnson Noise as a Function of Bandwidth (Teach Spin, Chapter 1)**

a Start with the 10 kΩ resistor (B_{ext}).

b Connect the "output" of the Preamplifier to the "In" of the first (left-most) filter on the High-level electronics. Then connect the "High-pass" output to IN of the second filter. Finally connect the "Low-pass" output from this filter to the Gain IN.

c Start with a broad band pass filter: 0.1 kHz on the High-pass and 100 kHz on the Low-pass.

d To avoid saturation, always adjust the Gain (G_2) to keep the output around 1 V.

  i. As in section I, use Eq(4) with Δf (from the filters) to calculate the expected V_{rms} from the Output, and compare to your measurements.
  ii. Repeat a) for several input resistors. Obtain an improved value for the Johnson noise from the resistors by removing amplifier noise, using the formula
  $< V^2(t) >= (G_1 G_2)^2/10\ V \ \ < V_J^2 + V_N^2 >$
  iii. Obtain an experimental value for the Boltzmann's constant at room temperature.
  iv. Use several (5-10) value of the High-pass and Low-pass frequencies, with Δf from Table 1.5, to study how the spectral noise density depends on bandwidth.

4 **Johnson Noise at Two Temperatures (Teach Spin, Chapter 4)**

a Choose an optimal bandwidth. Repeat III b) using 10 Ω (A_{ext}), 10 kΩ (B_{ext}), and 100 kΩ (C_{ext}) for the Low-temperature probe immersed in liquid nitrogen.

b Switch R_{in} to directly compare the noise of the resistors at 77 K to those at room temperature.

c Obtain an experimental value for the Boltzmann's constant at 77 K.

5 **Spectral Density of Johnson Noise (using LabVIEW)**

a Return to the settings from section III a).

b Connect Gain OUT to MyDAQ AI0, and Multiplier Monitor to MyDAQ AI1.

c Start the computer. Load LabVIEW program: White Noise. Start data acquisition. Average the signal for many (¿¿10) sweeps. Press and hold "Restart Averaging" if necessary. Press "stop" to record each spectrum. Be sure to rename files to ovoid overwrite.

i. Record spectra for several different filter ranges. Analyze the spectra in terms of the filters used. Specifically, do the 3 dB points match what you expect? How does the maximum noise amplitude depend on the bandwidth? What is the minimum noise amplitude? What are the maximum and minimum frequencies of white noise? What if you remove the filters: now what limits the frequency range of the white noise?

6 **(Optional) 1/f Noise**

a How can you extend the frequency range found in V a) to lower frequencies?

b Using LabVIEW program 1_f noise; obtain several spectra over the maximum frequency range, and over the frequency range reduced by an order of magnitude. What do you see?

c Using LabVIEW program 1_f noise long; obtain several spectra over the maximum frequency range, and over the frequency range reduced by three orders of magnitude. Be patient, each spectrum must average over 100 seconds before displaying. What do you see?

d Obtain the slope (α) assuming that the noise spectral density varies as $S(f) \propto 1/f^{\alpha}$. Over what frequency range does the noise exhibit 1/f - like behavior.

e Repeat c) and d) for all three resistors in the low-temperature probe, at room temperature and at liquid nitrogen temperature.