Lab Report 2

Double Slit

October 10, 2016

Author: Jacob Cluff

Partners: Josef Rinderer, David Turk

Abstract:
In this experiment, the concepts of single and double slit diffraction are explored. The slit width, a, and separation, d, are found experimentally to be $83.4 \pm 11.8 \mu m$ and $414 \pm 8 \mu m$. The counts per second (CPS) frequency are plotted and shown to have Gaussian and Poisson distributions.

Introduction:

The single and double slit experiments explore the wave-particle duality of photons by having a single photon particle create a interference pattern, an inherent property of waves, after passing through two slits side by side. This conflicts with the classical view of two distinct wave fronts causing interference because the photon is a single particle/wave.

Theory:

Equation 1 describes single slit diffraction, where I_0, a, λ, and θ represent the maximum intensity, slit width, wavelength of light, and the angle subtending the normal of the slit plate and the light "ray" coming through the slit; therefore, $\sin\theta$ gives the height on the diffracting screen from where the intensity is maximum.

$$I(\theta) = I_0 \left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi}{\lambda}\sin\theta} \right] \tag{1}$$

Equation 2 makes some simple substitutions where the new variables V_0, and δ represent the maximum voltage and the phase-shift. The phase-shift is useful because maximum found in the data will not correspond to $\theta = 0$, so it serves to shift the theory curves over so that $\theta = 0$ lines up with V_0. Since the angles concerned are so small, $\sin\theta \approx \theta$.

$$\begin{aligned} V(x) &= V_0 \frac{\sin\alpha}{\alpha} \\ \alpha &= \frac{\pi a}{\lambda}\theta \\ \theta &= x - \delta \end{aligned} \tag{2}$$

Equation 3 is the for double slit diffraction. The only new variable introduced here is d, the slit separation.

$$I(\theta) = 4I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) \left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi}{\lambda}\sin\theta} \right]^2 \tag{3}$$

Equation 4 similarly recasts Equation 3 with no new variables introduced.

$$\begin{aligned} V(x) &= 4V_0 \cos^2\beta \left[\frac{\sin\alpha}{\alpha} \right]^2 \\ \beta &= \frac{\pi d}{\lambda}\theta \\ \alpha &= \frac{\pi a}{\lambda}\theta \\ \theta &= x - \delta \end{aligned} \tag{4}$$

Equations 5, 6, and 7 are used for statistical analysis. Equation 5 is used to normalize an array of data so that the total probability of a measurement between the minimum CPS and maximum CPS measured is equal to 1. The CPS densities are compared to the Gaussian and Poisson distributions and found to be similar, but not quite the same. The difference between these two distributions is that the Poisson distribution works over a discrete domain, while the Gaussian distribution works over a continuous domain.

$$1 = \frac{\Psi}{\sqrt{\langle\Psi|\Psi\rangle}} \tag{5}$$

$$P_{Poisson}(x) = \frac{\Lambda^x e^{-\Lambda}}{x!} \tag{6}$$

$$P_{Gaussian}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \tag{7}$$

Procedure:

First, special care must be taken to keep the shutter closed when it will be exposed to laser light, or even the room lighting. Only when the special bulb and the apertures are in place should the shutter be opened. This is because the detector is ultra-sensitive, enough to detect a single photon.

The various lengths between the source, detector, slits, and apertures are measured to help in the conclusion that only one photon is expected to be in the box at any given time.

The aperture is adjusted to find the full range and to see at what settings a single slit diffraction patter (SSDP) and when a double slit diffraction pattern (DSDP) will emerge.

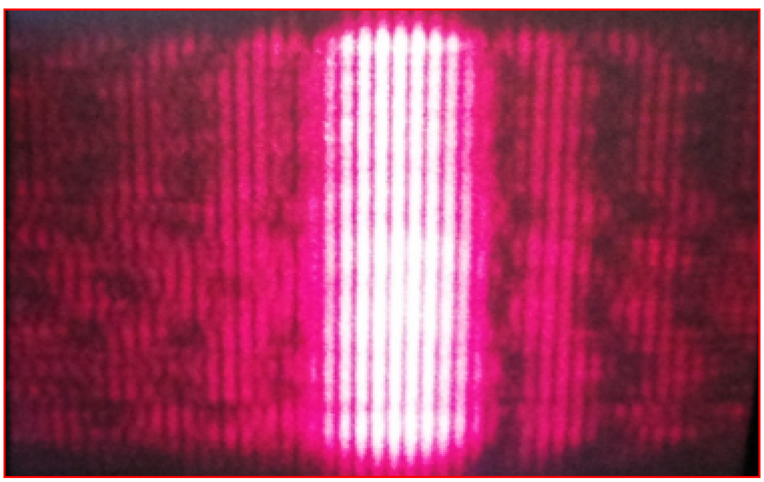

Figure 1: A photo of the double slit diffraction pattern

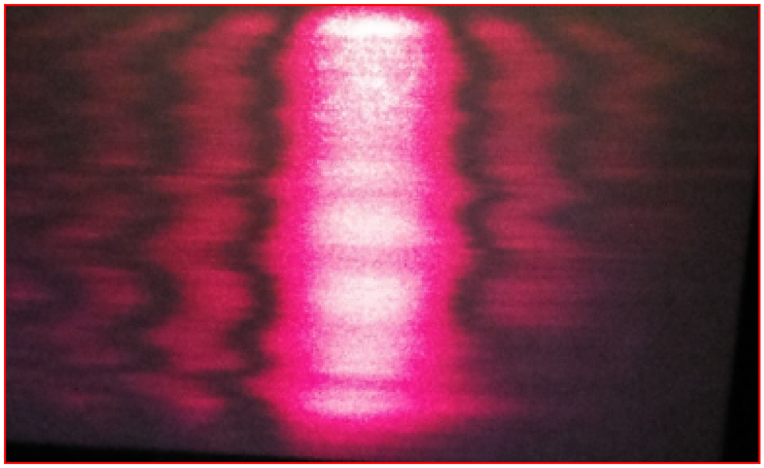

Figure 2: A photo of the single slit diffraction pattern

Two software programs are used. The first is to measure the voltage from the photons striking the detector in a narrow range; in order to get the full range, the micrometer is steadily adjusted through the full range. A small phase shift was noted when scrolling the micrometer forward vs backward. To compensate for this, we started at one end of the range, scrolled all the way to the other end, then back to the beginning. This should allow the data to averaged and a more accurate result produced.

The second piece of software is used to track the CPS over time. Data collected here is used for statistical analysis.

Results:

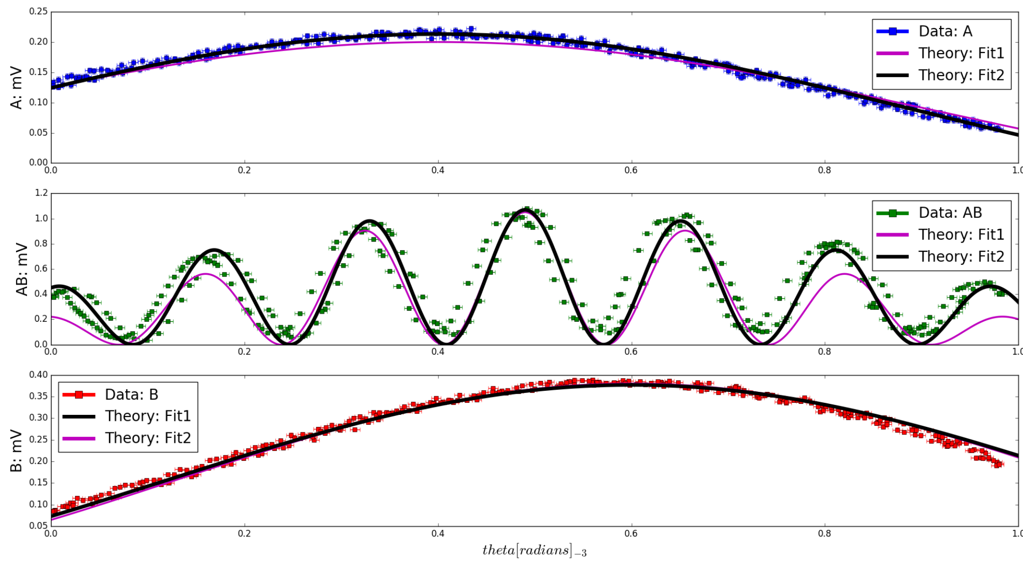

Figure 3: The top, middle and bottom graphs show the SSDP from slit A, DSDP from slits A and B, and the SSDP from slit B. The blue, green and red squares are the data points for each graph. The magenta curve is plotted using 2 for the single slits, while 4 is used for the double slit. V_0, a, λ, and d (for the double slit) are taken to be given values. δ is adjusted manually to fit the data. The black curve in each plot represents the best fit using a least squares approach.

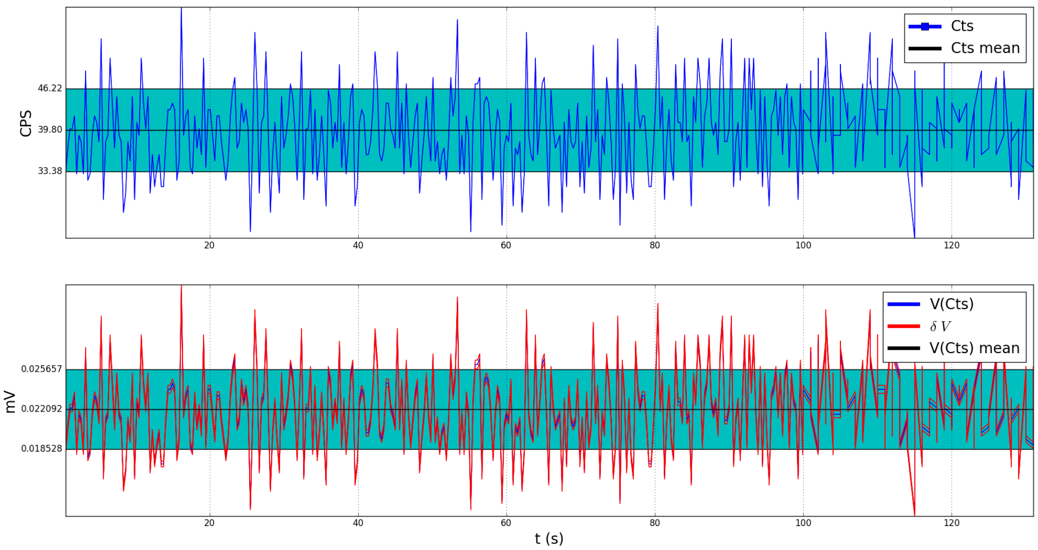

Figure 4: The top plot shows the dark current, or the background noise, CPS vs time; the bottom blue curve represents an attempt to convert the CPS to mV using 8, in the discussion. Although it is not clear how accurate this conversion is, it does seem to give reasonable results. the red curve on the bottom plot shows the error propagation, which mostly overlaps with the blue curve. In both plots, the black line is the average of all values; the cyan box bounds the points within one standard deviation on either side.

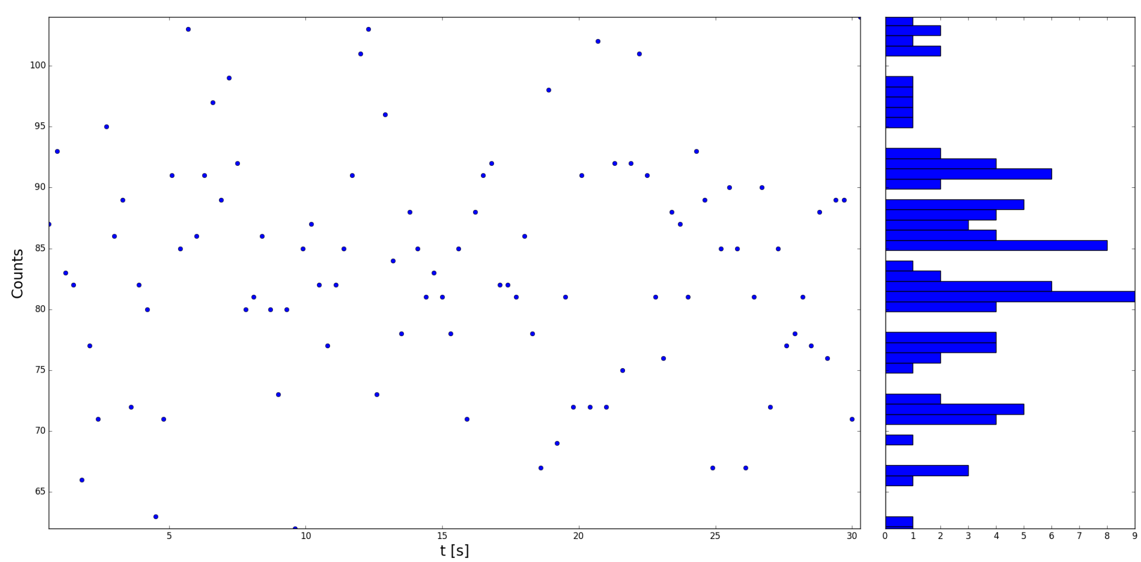

Figure 5: This plot shows the CPS vs time when the micrometer is set around a minimum point. The histogram on the right shows the relative frequency of the CPS; if more points were taken, this histogram would have a Gaussian profile.

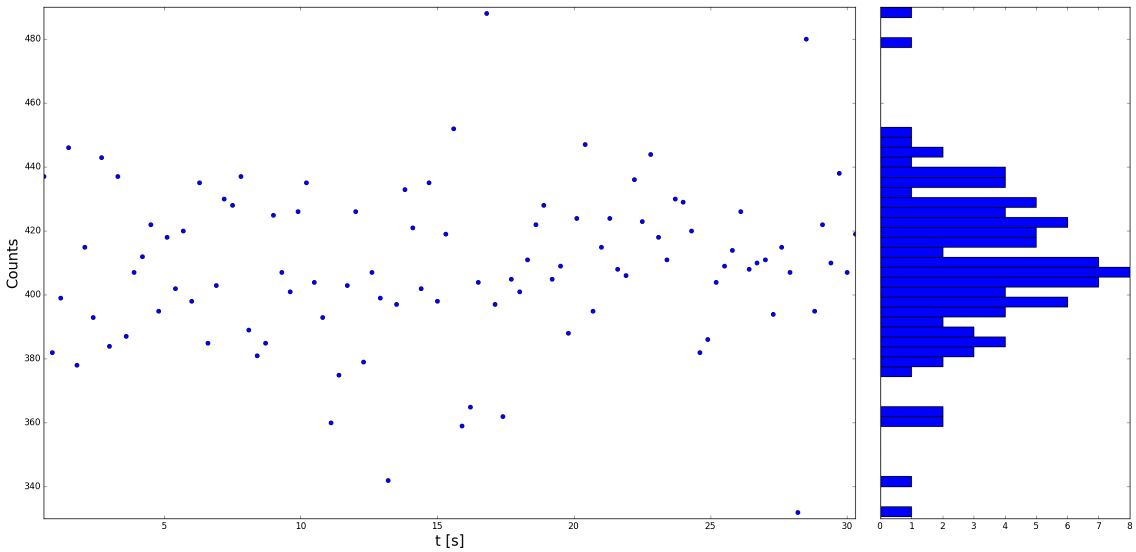

Figure 6: This plot shows CPS vs time when the micrometer is set around a maximum point. The histogram shows the relative CPS frequency.

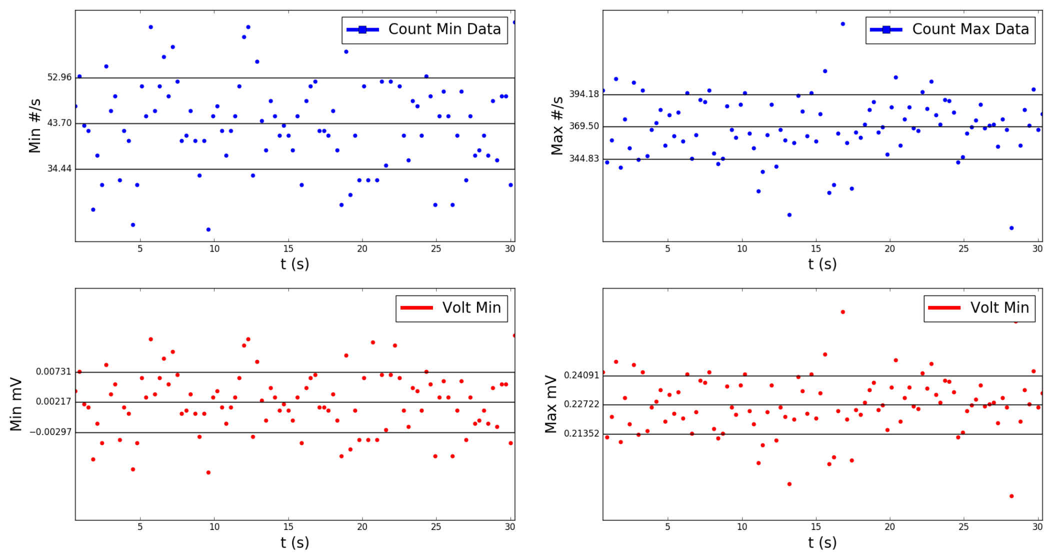

Figure 7: These four plots show the min and max plots of the CPS vs time (on top) and the min and max mV vs time (on bottom). Similar to Figure 4, the black line through the middle is the mean, and the box bounds the data within one σ of the mean.

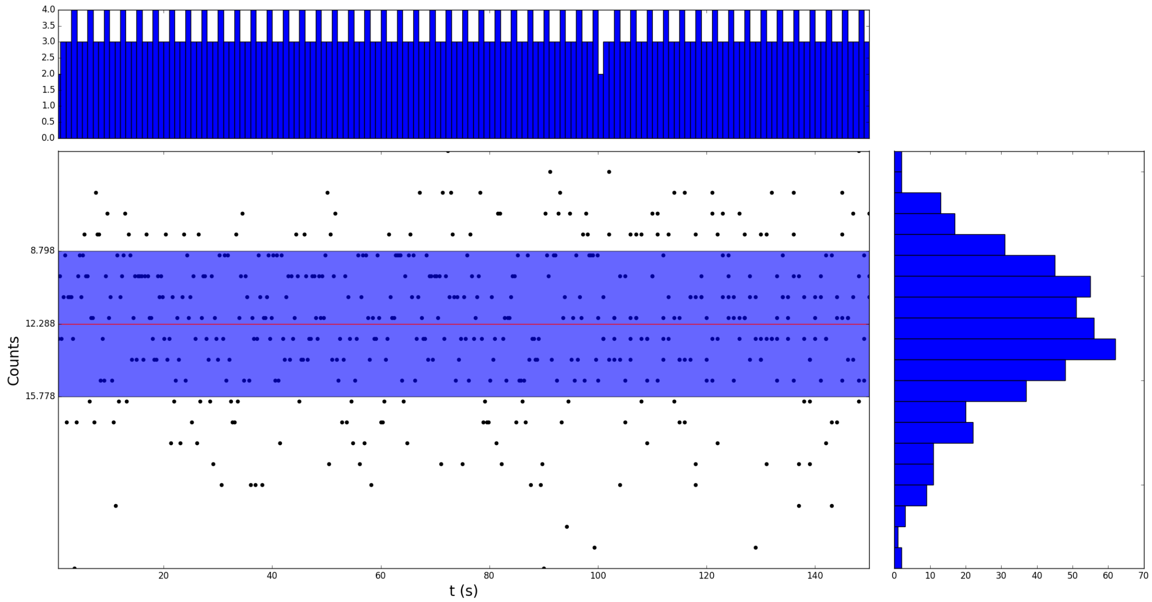

Figure 8: This plot shows a larger data set of around 500, with a CPS average of about 12. The red line shows the mean of the data, the blue box bounds the data within 1 σ of the mean, the histogram on the right shows the relative CPS frequency, and the histogram at the top just says that the data recorded is steady.

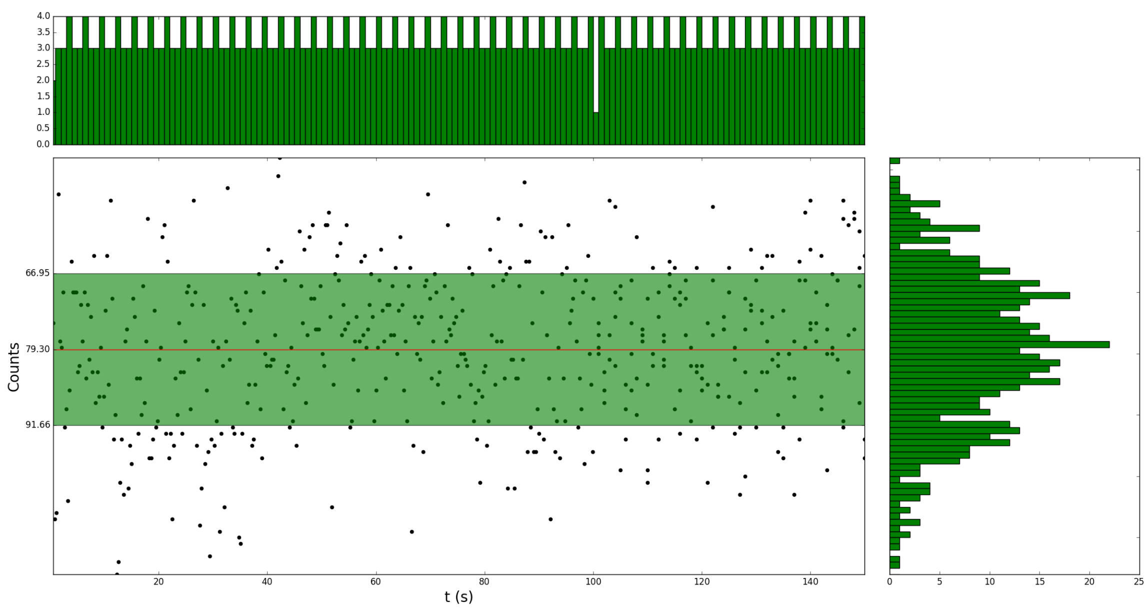

Figure 9: This plot shows a larger data set of around 500, with a CPS average of about 80. The red line shows the mean of the data, the green box bounds the data within 1 σ of the mean, the histogram on the right shows the relative CPS frequency, and the histogram at the top just says that the data recorded is steady.

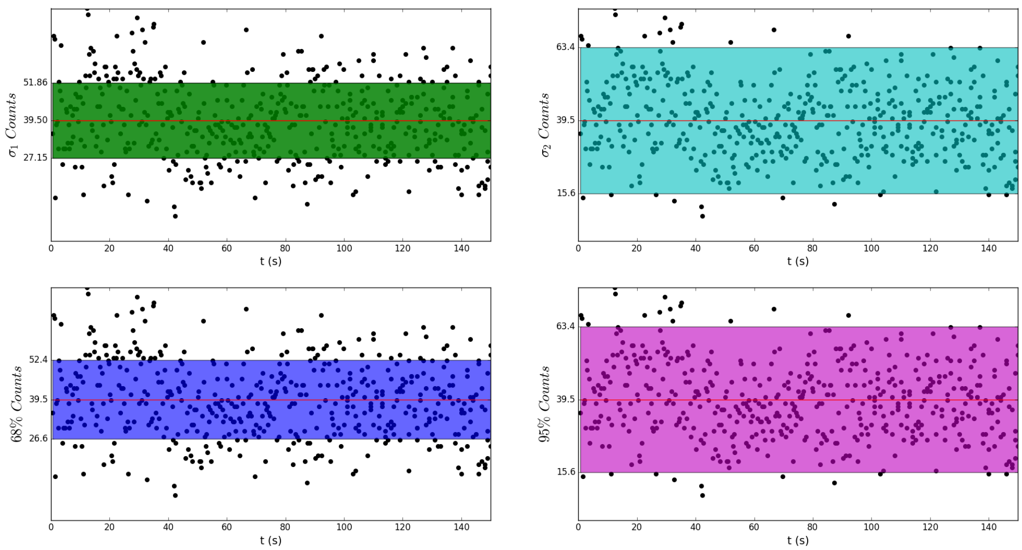

Figure 10: Four plots are shown here that compare σ_1 and σ_2 (see Equation 10) to 68% and 95% data capture. For the σ_1 and σ_2 plots, the mean and standard deviation from the mean are found and represented by the green and cyan bounding boxes. For the 68% and 95% bounding boxes, a computer algorithm is created to count the number of data points with in a incremental distance from the mean and stops when the desired amount is reached. σ_1 and 68% are found to be very close; σ_2 and 95% agree exactly.

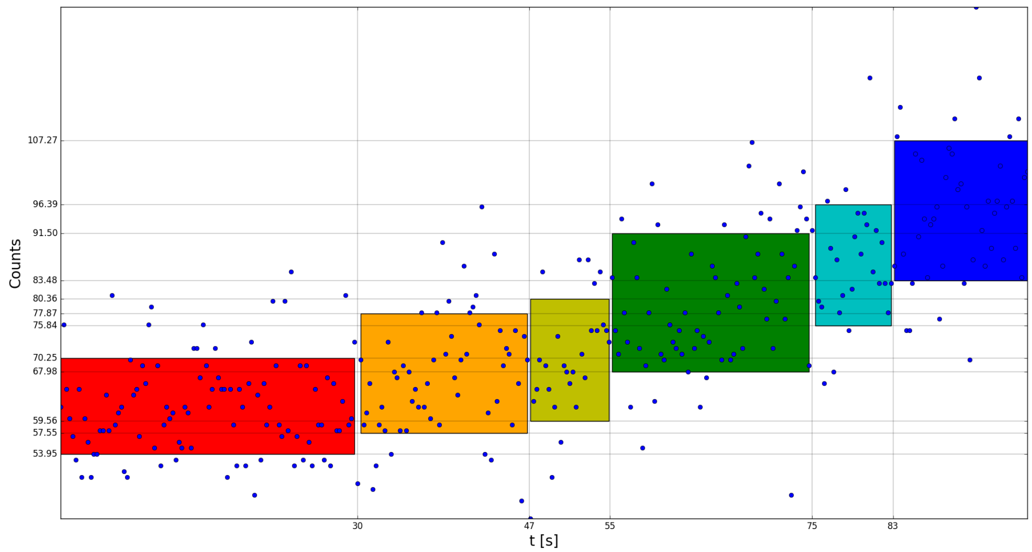

Figure 11: This plot shows the CPS vs time when the bulb intensity is increased incrementally over time. The different colored boxes show standard deviation from the mean (centered around the mean) for each region where the bulb intensity is constant. The red, orange, yellow, green cyan and blue regions correspond to bulb intensities: 5.5, 7, 8, 9, and 10.

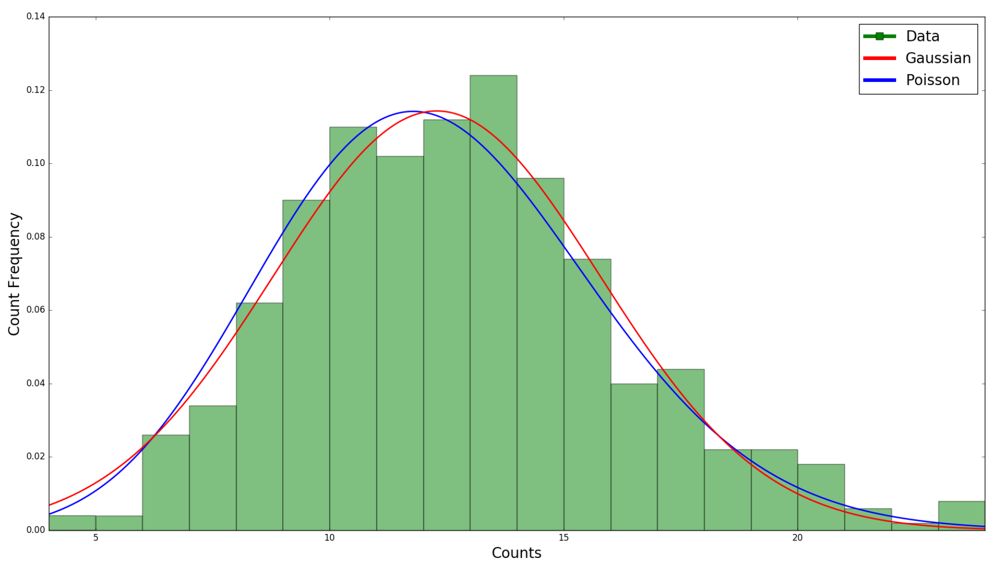

Figure 12: This plot shows the relative CPS frequency of the low count data (see Figure 8) that is normalized using Equation 5. The red curve shows a Gaussian distribution (see Equation 7) while the red curve shows a Poisson distribution (see Equation 6).

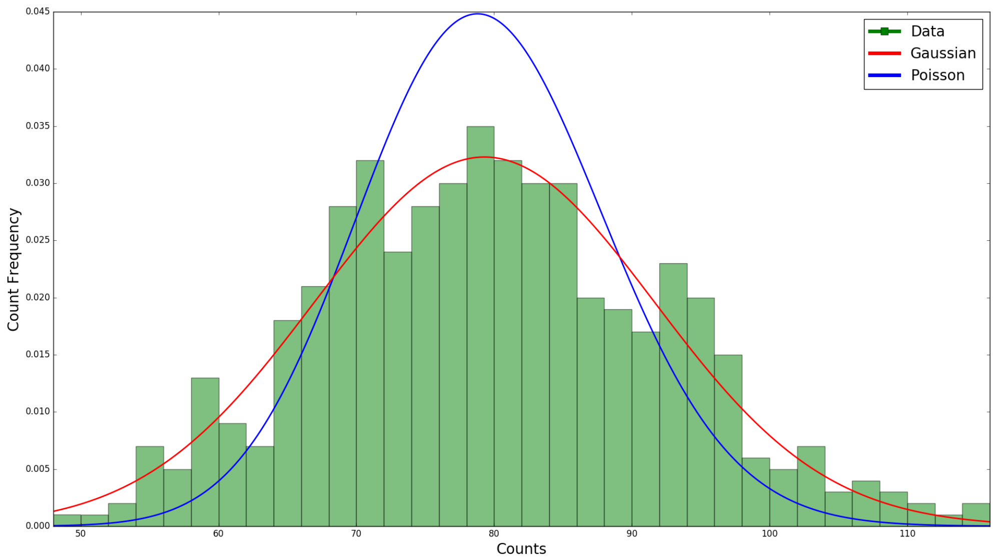

Figure 13: This plot shows the relative CPS frequency of the high count data (see Figure 9) that is normalized using Equation 5. The red curve shows a Gaussian distribution (see Equation 7) while the red curve shows a Poisson distribution (see Equation 6).

Discussion:

Although not necessarily part of the experiment, Equation 8 is used to try and understand how the counts per second and the voltage are related. It describes the the energy, measured in eV is equal to the number of photons per second, divided by the energy per photon and multiplied by the time interval between measurements. The α here is used to describe the gain/loss in the conversion.

Equation 9 is used to find the error propagation. In this experiment, it is used to find the red error curve in Figure 4.

When determining the slit width and separation and the associated error, a different approach is used. The fitting parameters for a and λ were found separately, which led to some difficulty in determining a. To work around this, the ratio is taken for each of the three cases, A, B, and AB, and multiplied by the known wavelength. a is then taken as the average found from the three cases $\pm\sigma$. The slit width found is $83.4 \pm 11.8 \mu m$. More detailed results printed from the Python program created can be found in the appendix.[1]

Since only only the A,B case involved d, the error found is simply the difference between the given value and the value found experimentally; $d = 414 \pm 8 \mu m$. An alternative, and more involved approach, would be to use Equation 9

[1]See Figure 12

When determining the standard deviation higher than σ_1, Equation 10 is used.

$$\begin{aligned} \text{Energy} &= \frac{\text{photons}}{s}\frac{\text{Energy}}{\text{photon}}\Delta t \\ \Rightarrow \alpha eV &= (cnt)\frac{hc}{\lambda}\Delta t \\ \Rightarrow V &\propto \frac{(cnt)\Delta t}{e}\frac{hc}{\lambda} \end{aligned} \tag{8}$$

$$\delta p = \sum_i \frac{\partial p}{\partial q_i}\delta q_i \tag{9}$$

$$\sigma_z = erf\left(\frac{z}{\sqrt{2}}\right) \tag{10}$$

Conclusion:

Appendix:

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Figure 3 shows diffraction data for A, B, AB on red laser and fitted using 'SSFit' and
'DSFit'
Fit A:
a/wavelength = 135.379
slit width 'a' = R1*lambda = 90704.064 nm

Fit B:
a/wavelength = 138.527
slit width 'a' = R1*lambda = 92813.117 nm

Fit AB:
a/wavelength = 66827.262
slit width 'a' = R1 * lambda = 66827.262 nm
d = 414243.655 nm

sig1 statistics: half-width = 6.42058940348 , per = 0.65, delta_per = 0.001
sig1 statistics: half-width = 0.00356428958616 , per = 0.65, delta_per = 0.001
sig1 statistics: half-width = 9.26120942426 , per = 0.65, delta_per = 0.001
sig1 statistics: half-width = 24.6720489623 , per = 0.65, delta_per = 0.001
sig1 statistics: half-width = 0.00514121527352 , per = 0.65, delta_per = 0.001
sig1 statistics: half-width = 0.0136963013299 , per = 0.65, delta_per = 0.001

single photon diffraction table?
get 5 mW, convert to eV, divide by energy/photon, gives photons/sec
photons/sec = (5/1.602E-19) * (1/1000) * (670/1239.8) = 1.6866721083240242e+16 photons/sec
multiply photons/sec by (L/c), where L is length of box and c is speed of light.
this gives number of photons in the box at a time using the red laser.
photons in the box at a time = 27736135.737282988

Statistis of 100 count Data
sig1 statistics: half-width = 12.3527966064 , per = 0.65, delta_per = 0.001
sig2 statistics: half-width = 23.9 , per = 0.954499736104, delta_per = 0.00249973610364
68 statistics: half-width = 12.9 , per = 0.68, delta_per = -0.0119999999999999
95 statistics: half-width = 23.9 , per = 0.95, delta_per = -0.0020000000000000018
sig1 statistics: half-width = 3.49013696006 , per = 0.65, delta_per = 0.001
sig1 statistics: half-width = 12.3527966064 , per = 0.65, delta_per = 0.001
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Figure 14: Printed results from Python code.