

## Hall Effect

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Author: Jacob Cluff

Partners: Jason Pickering, Philip Rybak

### Abstract:

In this experiment, we delve into some of the properties of semiconductors using the Hall effect, positive doped Germanium (p-Ge), negative doped Germanium (n-Ge) and intrinsic Germanium (i-Ge). We find the magnetic leakage to be  $B_{leak} = 0.05 \pm 0.06 \%$ . We estimate the ratio of the magnetic permeability ( $\mu_r$ ) as **find  $\mu_r$** . The magnetic remnance of the iron core is found to be **find remnance**. We determine the band gap of i-Ge to be **band gap of i-Ge**. We determine the resistivity for p-Ge at 300 K to be **p-Ge resistivity**.

# 1 Introduction

The Hall effect was discovered by Edwin Hall in 1879 and published in, "On a New Action of the Magnet on Electric Currents".<sup>1</sup> The effect occurs when an electric field  $\vec{E}$  and a magnetic field  $\vec{B}$  pass through a conductor at  $90^\circ$  to each other. passing electrons are deflected to one edge of the conductor. The excess negative charges on one edge (-) and a lack of negative charges on the opposing edge (+) creates a potential difference between the opposing edges.

# 2 Theory

To find a theoretical value for  $\vec{B}$ , we use Ampere's law to calculate the so called "magneto-motive force", which is the sum of product between the auxiliary (or magnetization) field  $\vec{H}$  and path length for each material. This magneto-motive force is found in Equation 1 and is solved for  $B$ .

$$\begin{aligned}
 NI &= H_{core}L_{core} + H_{gap}L_{gap} \\
 &= B \left( \frac{L_{core}}{\mu} + \frac{L_{gap}}{\mu_0} \right) \\
 \mu \gg \mu_0 \Rightarrow NI &= B \left( \frac{L_{gap}}{\mu_0} \right) \\
 \therefore B &= \frac{N\mu_0}{L_{gap}} I
 \end{aligned} \tag{1}$$

Here,  $N$  is the number of coils in the solenoid,  $I$  is the current,  $H_{core}$  and  $H_{gap}$  are the auxiliary fields through the core and the air in the gap,  $L_{core}$  and  $L_{gap}$  are the path lengths of the magnetic field through the core and the air gap, and  $\mu$  and  $\mu_0$  are the magnetic permeability of the core and air gap. The result in Equation 1 shows that the magnetic field is proportional to  $I$  and can be linearized with  $L_{gap}$  by multiplying  $B$  by  $L_{gap}^2$ , which is shown in Figure 2.

The magnetic leakage can be found by setting the total field ( $B_T$ ) equal to the measured field ( $B_D$ ) plus the leaked field ( $B_L$ ). We then take  $B_T$  as the theoretical field,  $B_D$  as the measured field, and we find  $B_L$  using Equation 2 and Figure 2.

$$\begin{aligned}
 B_T &= B_D + B_L \\
 \Rightarrow B_L &= B_T - B_D \\
 y_l &= y_t - y_d \\
 B_L \cdot gap^2 &= B_T \cdot gap^2 - B_D \cdot gap^2 \\
 B_L &= B_T - B_D
 \end{aligned} \tag{2}$$

$y_l$ ,  $y_t$ , and  $y_d$  represent the plotted lines in Figure 2. Since the y-axis is obtained by multiplying by the gap squared for all fields, the gap squared cancels out and the leakage field can be found by directly subtracting the theory and data arrays. The value for  $B_L$  is then taken as the average plus or minus the standard deviation.

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<sup>1</sup>[https://en.wikipedia.org/wiki/Hall\\_effect](https://en.wikipedia.org/wiki/Hall_effect)

### 3 Results

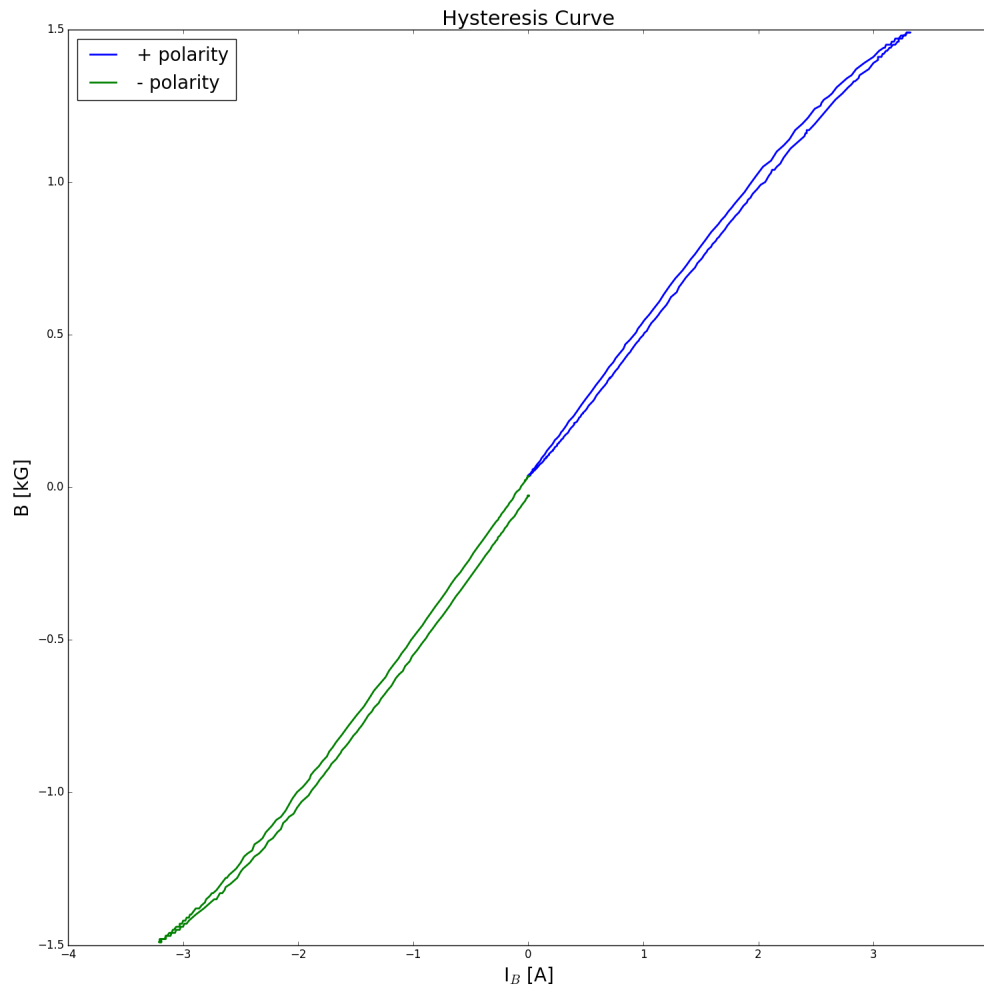


Figure 1: Hysteresis Curve

The iron core that makes up the electromagnet has a property called hysteresis (to lag behind), which means that iron "remembers" its magnetization history

After we take the positive data (shown in blue), we reverse the positive and negative leads, which reverses the polarity, and redo the measurement. The hysteresis is shown by the separation between the data when scrolling in opposite directions. Since the positive measurement was taken first, the negative measurement begins where the last positive data was measured.

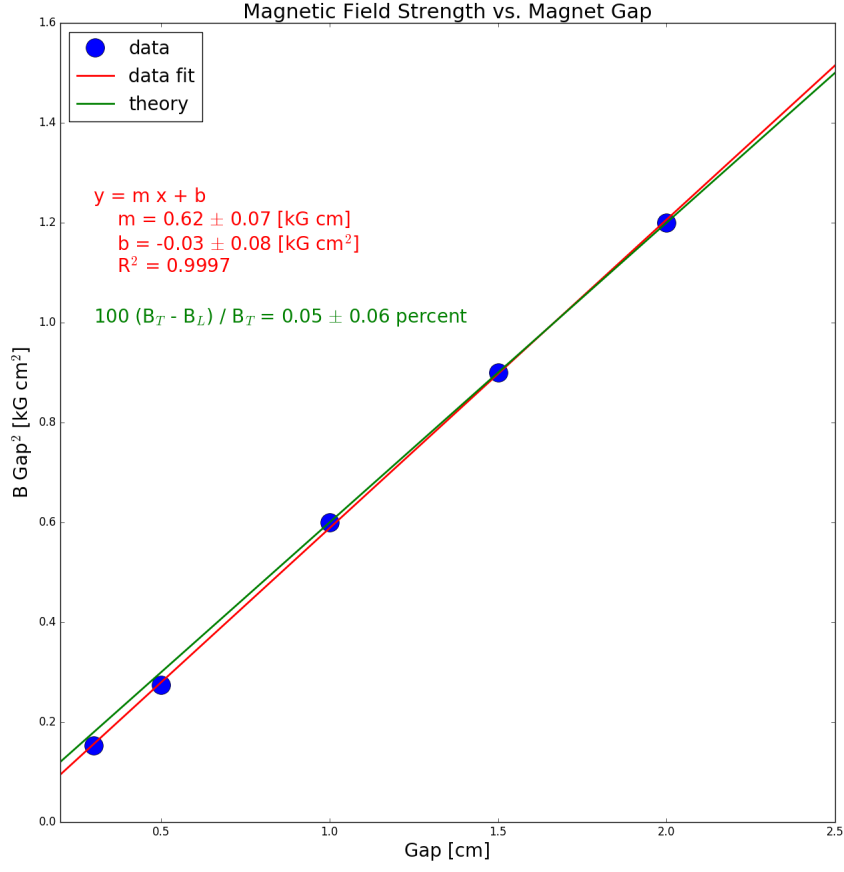


Figure 2: Magnetic Field Strength vs. Magnet Gap

This figure shows the magnetic field strength with respect to the gap distance that has been linearized. We found that if the field strength is multiplied by the square gap distance, the data becomes linearized quite nicely with an  $R^2$  value of 0.9997. This shows that the field strength vs gap distance could be expressed as a power law such as  $B(gap) = -\frac{a_1}{gap} + a_2$ , where  $a_1$  and  $a_2$  positive fitting parameters.

The theoretical curve (shown in green) is calculated using Equation 1 and multiplying gap squared. We determine the magnetic leakage by subtracting the data vales from theory values as seen in Equation 2. The resulting average and standard deviation are  $B_{leak} = 0.05 \pm 0.06 \%$ .

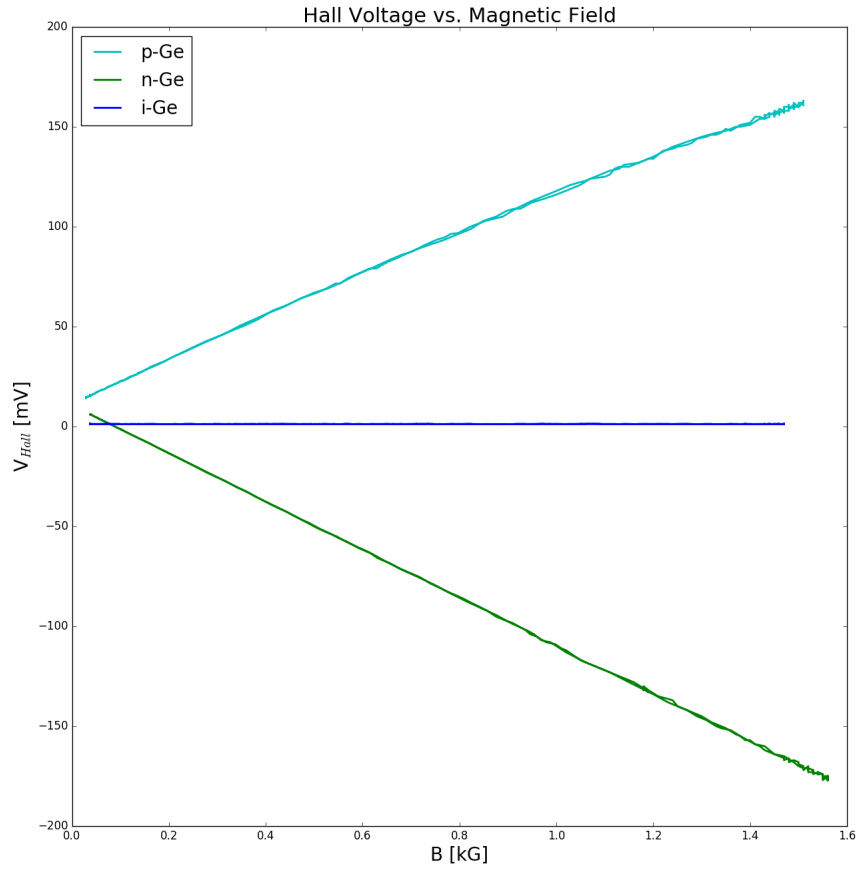


Figure 3: Hall Voltage vs. Magnetic Field

The Hall voltage vs magnetic field for positive doped Germanium (cyan), negative doped Germanium (green), and the intrinsic Germanium (blue) all show straight lines with relatively constant slope, which imply that the hall voltage is proportional to the magnetic field. The offset in the y-intercept of the positive, negative, and intrinsic data could be explained by the hysteresis of the core.

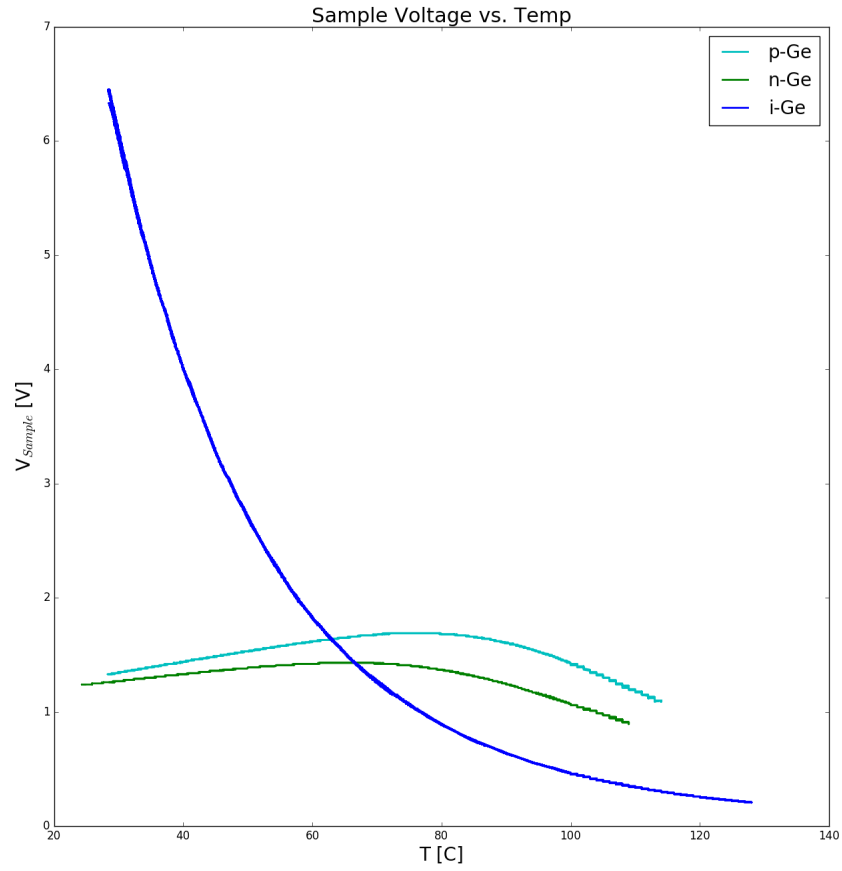


Figure 4: Sample Voltage vs. Time

In this experiment the source current is kept at zero and the temperature is slowly increased over time. Both the positive and negative doped Germanium exhibit metal like behavior before their respective maxima; in their metal-like phase, the increasing temperature causes the electrons to jostle around more violently which increases the resistance in the material which increases the potential. At some point, the additional thermal energy starts pulling electrons off the Germanium atoms and the increase in free electrons causes the potential to start dropping.

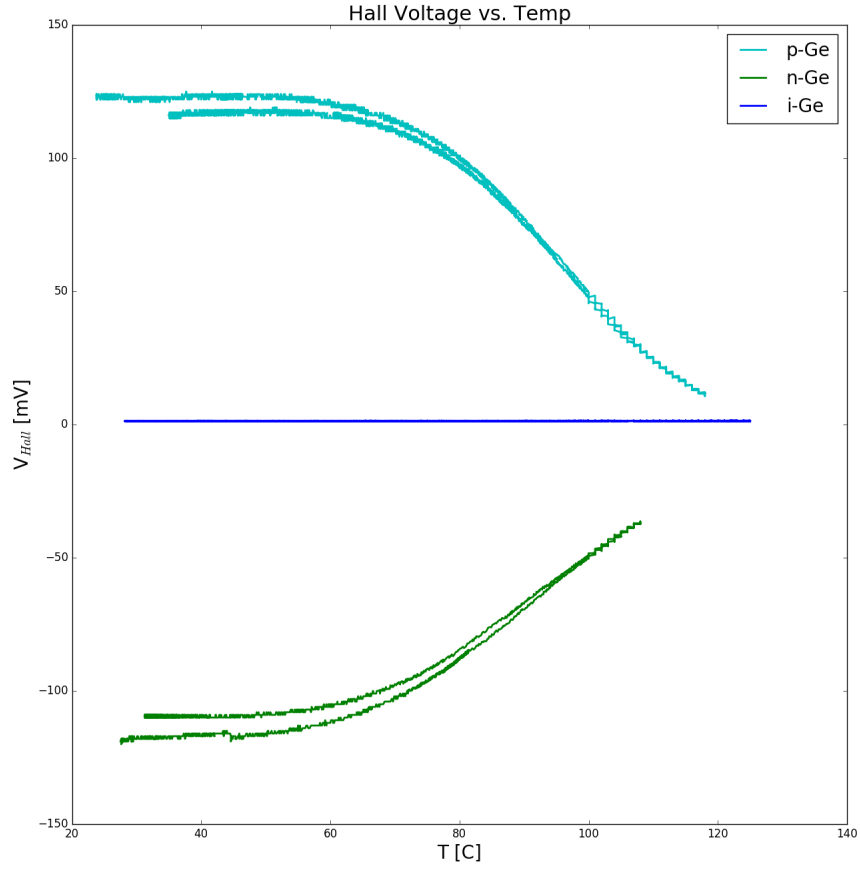


Figure 5: Hall Voltage vs Temperature

The Hall voltage across the positive, negative, and intrinsic germanium show different patterns. The Hall voltage across i-Ge stays relatively steady over the increase in temperature because the fractional change in resistivity depends on the square of the  $\mu_n$  or  $\mu_p$ . The negative charge carriers are drawn easily towards the p-doped (holes) which leads a drop voltage drop; the n-doped (barriers) are seen as potential increase.

## 4 Discussion

The magnetic leakage seen in Figure 2

## 5 Appendix

*Pre-Lab:*

### 1 Characterization of Electromagnet

- A Remove any sample from the console, taking care to not stress the connector wires. Connect the magnet supply to the magnet coils, taking care that the fields "add" not "subtract". Note the 0.1 ohm current-sense resistor near the ground-lead connection. This gives  $I_B = V/R$ .
- B Set the pole pieces for a gap of  $g = 1\text{cm}$  using a spacer-block and tighten the thumb-screws firmly.
- C Measure B vs Current, for  $I_B = 0$  to Max to 0. You may want a short dwell-time, like 100 ms. Use voltage control (current dial at max). This will prevent a nasty spark and possible damage when you unplug the cables. Then measure opposite polarity, by swapping the banana plugs at the supply (with  $V=0!$ ), and run  $I_B = 0$  to Max to 0. Note that  $R_{sense}$  captures the polarity. Swap the wires back when finished.
- D Measure B vs. gap size "g" for a fixed current, say  $I_B = 1$  Amp. A single polarity will suffice. Be sure that  $V = 0$  when you loosen the pole pieces to change the gap (maybe unplug a cable). Use spacer blocks to fix the gap and tighten the pole pieces to prevent accidental clamping shut (damage to sensor and fingers).

## 2 Data for p-Ge

- A Loosen and retract the front pole-piece; the back pole-piece can remain fixed. Insert the sample card, taking care to not bend the contacts or twist the board. The blank side of the sample card should face towards the front. Place the 1/2 cm spacer block just below the circuit board. Close and tighten the pole pieces.
- B Measure hall voltage  $U_h$  vs B, with  $I_S = 30$  mA. Do not exceed 3 Amp for the field current. If you decide to reverse the field, do so only when the current is zero. (However it is recommended that field sweeps be taken only in the positive direction (up and down), because that is how the field was calibrated in table 1). Note that sample voltage is also contained in the same file, giving you  $U_s$  vs B with  $I_S = 30$  mA, showing the "magneto-resistance" effect.
- C Measure sample voltage  $U_h$  vs temperature T (up to 170° C maximum) with  $I_S = 30$  mA,  $B = 0$ . Check that the sample board is not touching the magnet or sensor. Record a full heat/cool cycle, and note the hysteresis due to thermal gradients if T changes too quickly. Suitable data should result using 1000 ms time step during the sweep process, perhaps changing the heater voltage in several steps.
- D Measure Hall voltage  $U_h$  vs T, with  $I_B = 2$  Amp,  $I_S = 30$  mA.

## 3 Data for n-Ge

- A Repeat the above for n-Ge.

## 4 Data for i-Ge

- A Repeat the above for i-Ge. In this case, you may need to lower  $I_s$  to keep the sample voltage below 10 V.

## 5 Data analysis and Questions



- A Calculate the expected magnetic field, knowing  $N = 600$  turns for each coil.  
(Hint: using Ampere's law and assume  $\mu_r$  very large). Estimate the flux leakage (as %) by comparing the calculated and measured magnetic field.
- B Estimate  $\mu_r$  (dimensionless ratio  $\mu/\mu_0$  for the pole pieces, knowing the inductance  $L = 60$  Henry for the two coils in series.
- C Estimate the magnetic remnants of the pole pieces.
- D Plot Hall voltage vs field for all three samples on a single plot.
- E Plot sample voltage vs T for all three samples on a single plot.
- F Make a linearized plot for i-Ge, using the expected functional behavior, and from this find the band gap (in eV) for Ge.
- G Plot hall voltage vs T for all three samples on a single plot. Why does the p-Ge data change sign?
- H (opt) Find the mobilities  $\mu_p$  and  $\mu_n$  from the magneto-resistance data.
- I Find the resistivity (ohm-cm) for p-Ge at  $T = 300$  K from the I-V data and sample geometry. From this, estimate the doping density ( $\#/cm^3$ ).