Hall Effect

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Concepts

- Semiconductor, Fermi level, carrier density, conductivity, mobility, magneto-resistance, electromagnet (field calculation, permeability, hysteresis).

Background Reading

- Melissanos ch2.1-2.4
- Sze, Semiconductor Devices

Special Equipment and Skills

- Dual (I, V) regulated supply; Hall Probe;

Precautions

- Do not twist or bend the sample boards ($800) while mounting them.
- "Pinch" the console buttons to avoid twisting the module.
- The sample boards can get hot (170C).
- The magnet coils can be run *briefly* at a max of 3.5A.
- The Bell Hall probe (for magnetic field) is fragile and expensive.

Background: Semiconductors

The invention of the transistor, and by extension, integrated circuits, has revolutionized the world. The field-effect transistor is an electronic valve made from semiconducting materials in which current flow is controlled by applied voltages. Semiconductors, as the name implies, are partially conducting materials, with the critical attribute that conductance can be controlled by externally applied electric fields.

The current and voltage in a linear (passive) circuit obey Ohm's law,

$$V = IR\,. \tag{1.1}$$

This equation implies a geometry for current flow between two contacts. It is an integral form of the constituent equation,

$$j = \sigma \mathrm{E} = E/\rho\,, \tag{1.2}$$

where j=I/A is current density, E is electric field, σ is conductivity and ρ = 1/σ is resistivity. Note that j and E are vector functions of position, and σ may be a tensor. Here we are only concerned with isotropic, scalar and uniform σ, which is a material property, independent of geometry and size. For uniform current flow along length L, with cross section A = W x t, Eq 1.2 integrates simply as

$$I = jA = \frac{(\sigma A)}{L}(EL) = GV\,, \tag{1.3}$$

where G = 1/R is the conductance of the macroscopic circuit, as shown in Fig. 1.

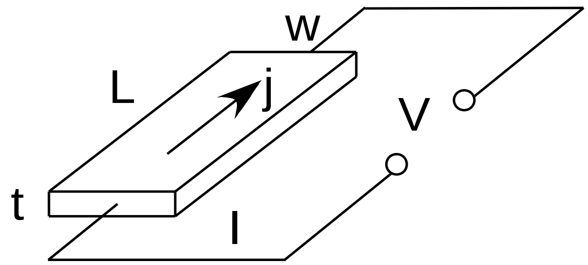

Figure 1. Geometry for uniform current flow through an object. The Ge samples for these experiments have dimensions 17 x 10 x 1.5 mm.

The conductivity of a semiconductor is given by

$$\sigma = e(\mu_n n + \mu_p p) \tag{1.4}$$

where e is the electron charge, μ_n, μ_p are the mobilities and n,p are the density (#/vol) of negative (electrons) or positive (holes) "mobile charge carriers" (see below). Carrier mobility μ defines the drift velocity of carriers in an electric field as

$$\mathrm{v} = \mu E\,. \tag{1.5}$$

A "hole" can be thought of as a bubble in an otherwise filled "sea" (continuous band of quantum-allowed states) of electrons. Note that current flow consists of both positive and negative charges drifting in opposite directions under an applied electric field. Typically, one carrier type will strongly dominate, comprising n-type or p-type behavior (see Eq. 2.4).

Electrons are Fermions, hence they cannot occupy the same quantum state. The crystal bonding in a semiconductor gives rise to a range of energies where no quantum states exist, called the "band gap". The *occupation* of the quantum states is described by the Fermi distribution function, given by

$$F(E) = \frac{1}{\exp[(E - E_F)/kT] + 1} \tag{1.6}$$

where E_F is the "Fermi Energy" and k is the Boltzmann constant. F(E) is essentially a broadened unit step function. Thus, F(E) is ~ 1 for $E<E_F$, and ~0 for $E>E_F$, with a cross-over width $\Delta E \sim kT$. Roughly speaking, E_F separates filled from empty states. Furthermore, due to the Pauli exclusion principle, electrons well below E_F cannot change their motion because there are no unoccupied states at the slightly higher energy implied by motion. On the other hand, electrons above E_F are free to move. These are the "mobile charge carriers" in equation 1.4 above. The carrier densities are given by

$$n = n_i \exp\left[\frac{E_F - E_i}{kT}\right],\ \mathrm{p} = n_i \exp\left[\frac{E_i - E_F}{kT}\right] \qquad (1.7a,b,c)$$
$$np = n_i^2 = N_0^2 \exp\left[E_g / kT\right]$$

where n_i the intrinsic carrier concentration, N_0^2 is independent of temperature and E_i is the intrinsic Fermi energy, which is essentially fixed at mid-gap. Note the mass-action law for the product $np = n_i^2$. For $E_F > E_i$, we have n-type behavior, with n >> p, while for $E_F < E_i$, we have p-type behavior, with p >> n. The temperature dependence for each type is given by

$$n(T) \sim n_i(T) \sim N_0 \exp\left[E_g / 2kT\right] \qquad (1.8a,b)$$
$$p(T) \sim n_i(T) \sim N_0 \exp\left[E_g / 2kT\right]$$

Semiconductors often are "doped" n-type or p-type, which adds a fixed number N_D of mobile charge carriers that are easily excited into the conduction band, even at 300K.

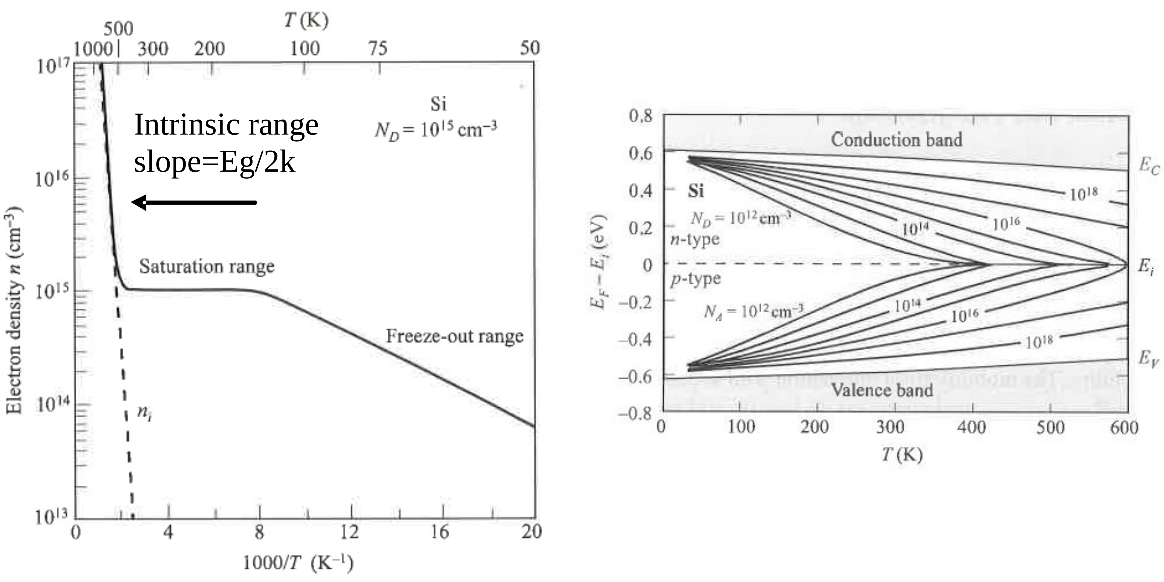

Figure 2. a) Mobile carrier density vs 1/T for n-type Si. b) Fermi level vs T for Si with various doping levels. (after Sze, Physics of Semiconductor Devices). ***Note: E_g^{Si}=1.12eV, E_g^{Ge}=0.66eV at T =300K.***

With the above information, we can explain the variation of conductivity with temperature, which is shown in Fig. 2, for n-type Si ($N_D = 10^{15}$cm^{-3}). For temperatures near 300K ("saturation range"), the carrier density is nearly constant at n ~ N_D. At high temperatures, however, (T > 500K, "intrinsic range") carriers are thermally excited across E_g, and n increases exponentially. This behavior is associated with the change of E_F with temperature, as shown in panel b. Note that at high temperature, E_F approaches E_i (mid-gap), and n~p~n_i.

Background: Hall Effect and Magneto-Resistance

The basic setup for a Hall effect measurement is shown in Fig. 3. Sample voltage V drives a current I along the length of a conductor (x-axis). A magnetic field B_z

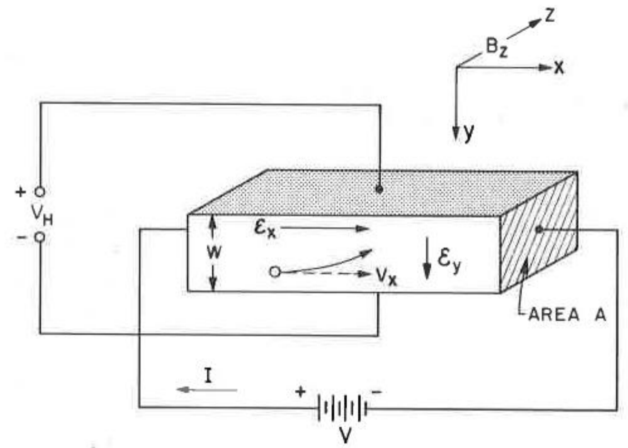

Figure 3. Geometry for Hall effect measurement.

is applied perpendicular to the current flow. Charge carriers are deflected along y by the Lorentz force. In steady state, charges accumulate along the y-edges until no current flows along y. This comprises the Hall voltage/field given as

$$qE_y = qv_x B_z \,. \tag{2.1}$$

The Hall field then can be written as

$$E_y = \frac{1}{e}\left(\frac{J_p}{p} - \frac{J_n}{n}\right) B_z = R_H J_{tot} B_z \tag{2.2}$$

with Hall coefficient

$$R_H = \frac{\left(p\mu_p^2 - n\mu_n^2\right)}{e\left(p\mu_p + n\mu_n\right)^2} \,. \tag{2.3}$$

For a single dominant carrier (n-type with n>>p or p-type with p>>n), the mobility cancels and R_H reduces to

$$R_H \rightarrow \frac{1}{ep} \text{ (holes only), } \rightarrow \frac{-1}{en} \text{(electrons only)} \qquad \text{(2.4a,b)}$$

and the Hall voltage takes a simple form

$$V_H = R_H J_{tot} B_z W. \tag{2.5}$$

This Hall resistance gives a direct value for the type and number of charge carriers, with no adjustable parameters.

The mobility is an important fundamental parameter for semiconductor materials. This can be measured directly via magneto-resistance (see Wikipedia, and Sze), which is the change in electrical resistance with an applied magnetic field. For a dominant carrier type (n or p), the fractional change in resistivity is given approximately by

$$\frac{\Delta\rho}{\rho} \sim \mu_n^2 B^2 \text{ or } \mu_p^2 B^2 . \tag{2.6}$$

Note the SI units of mobility are $m^2V^{-1}s^{-1}$ or Tesla. Although the effect is relatively small (~ 1% in classical systems), it does give a simple and direct way to measure mobility with no adjustable parameters. Point of interest: A related effect, "Giant Magneto-Resistance (GMR)", is the physical basis for all hard-drive computer read heads - a huge industry!

Procedure and Analysis

1. Characterization of electromagnet (no sample)
   a. Remove any sample from the console, taking care to not stress the connector wires. Connect the magnet supply to the magnet coils, taking care that the fields "add" not "subtract". Note the 0.1 ohm current-sense resistor near the ground-lead connection. This gives $I_B = V/R$.

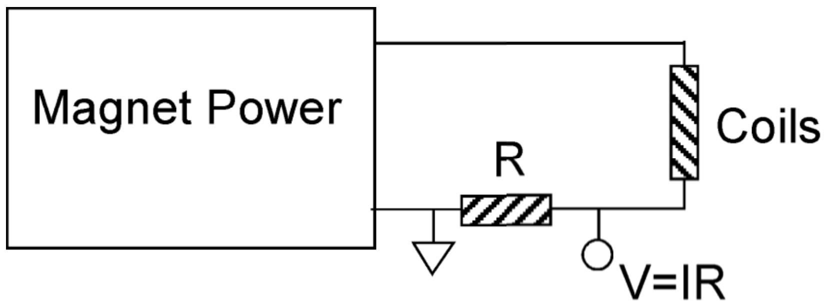

Figure 4. Circuit for magnet current with sense-resistor R=0.1 ohm inserted in the ground-return wire.

   b. Set the pole pieces for a gap of g = 1cm using a spacer-block, and tighten the thumb-screws firmly.
   c. Measure B vs current, for I_B = 0 to Max to 0. You may want a short dwell-time, like 100 ms. Use voltage control (current dial at max). This will prevent a nasty spark and possible damage when you unplug the cables. Then measure opposite polariy, by swapping the banana plugs at the supply (with V=0!), and run I_B = 0 to Max to 0. Note that R_{sense} captures the polarity reversal. Swap the wires back when finished.
   d. Measure B vs gap size "g" for a fixed current, say I_B = 1 Amp. A single polarity will suffice. Be sure that V = 0 when you loosen the pole pieces to change the gap (maybe unplug a cable) Use spacer blocks to fix the gap, and tighten the pole pieces to prevent accidental clamping shut (damage to sensor and fingers).

I (A)	0	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.2
ramp up (G)	60.6	149.2	235.5	341	443	640	847	1055	1265
down (G)	64.0	180.3	295.0	401	507	728	941	1144	1367
I (A)	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
ramp up (G)	1469	1670	1863	2048	2225	2412	2567	2725	2827
down (G)	1572	1786	1993	2168	2363	2530	2680	2755	

Table 1. Typical values for magnetic field (in Gauss) as a function of current (in amps). Note the hysteresis between ramp up and ramp down. Interpolate from this table, or from your own calibrations, for the magnetic field in parts 3-5.

2. LabView interface: Hall_Effect.vi
   a. Gently insert the p-Ge sample into the control console, without stressing the connector wires or the sample board.
   b. Launch Hall_Effect.vi from the desktop. This program is essentially an electronic pencil that will record all parameters for you at a regular interval (dwell time). Input values must be adjusted manually: sample current, heater setting, magnetic current. One chart shows all parameter values vs time. You can highlight those of interest by setting "plot scale" to 0 for the others (can tab through values, while program is running). A second chart shows X vs. Y for any pair of parameters (in drop-down boxes). The entire data file is written to Hall_Effect_Data.xls on the desktop when you hit "STOP". You must copy or rename this file (and close it!), before each "RUN", since it will be over-written. An efficient procedure is to copy from this file (then close it) into your user file, which you can leave open.
   c. Start the VI, and verify that the VI readings match the console readings. You may need to adjust "U_h" to cancel small mV offset, with B=0 and I_S=0. This should be checked at the start of all U_h runs.
3. Data for p-Ge:
   a. Loosen and retract the front pole-piece; the back pole-piece can remain fixed. Insert the sample card, taking care to not bend the contacts or twist the board. The blank side of the sample card should face towards the front. Place the 1/2" spacer block just below the circuit board. Close and tighten the pole pieces,
   b. Measure Hall voltage U_h vs field B, with I_S=30mA. Do not exceed 3 Amps for the field current. If you decide to reverse the field, do so only when the current is zero. (However it is recommended that field sweeps be taken only in the positive direction (up and down), because that is how the field was calibrated in table 1.) Note that sample voltage is also contained in the same file, giving you U_s vs B with I_S=30mA, showing the "magneto-resistance" effect.
   c. Measure sample voltage U_s vs temperature T (up to 170 ^{0}C maximum) with I_S=30mA, B=0. Check that the sample board is not touching the magnet or sensor. Record a full heat/cool cycle, and note the hysteresis due to thermal gradients if the T changes too quickly. Suitable data should result using 1000ms time step during the sweep process, perhaps changing the heater voltage in several steps.
   d. Measure Hall voltage U_h vs T, with B current =2 Amp, and Is = 30mA.
4. Data for n-Ge:
   a. Repeat the above for n-Ge.
5. Data for i-Ge:
   a. Repeat the above for i-Ge. In this case, you may need to lower I_S to keep the sample voltage below 10V.

6. Data analysis and questions
   a. Calculate the expected magnetic field, knowing N = 600 turns for each coil. (Hint: using Ampere's law, and assume μ_r very large). Estimate the flux leakage (as %) by comparing the calculated and measured magnetic field.
   b. Estimate μ_r (dimensionless ratio μ/μ_0 for the pole pieces, knowing the inductance L = 60 Henry for the two coils in series.
   c. Estimate the magnetic remnance of the pole pieces.
   d. Plot Hall voltage vs field for all three samples on a single plot.
   e. Plot sample voltage vs T for all three samples on a single plot.
   f. Make a linearized plot for i-Ge, using the expected functional behavior, and from this find the band gap (in eV) for Ge.
   g. Plot Hall voltage vs T for all three samples on a single plot. Why does the p-Ge data change sign?
   h. (opt) Find the mobilities μ_p and μ_n from the magneto-resistance data.
   i. Find the resistivity (ohm-cm) for p-Ge at T = 300K from the I-V data and the sample geometry. From this, estimate the doping density (#/cm^3).