Hall Effect

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Author: Jacob Cluff

Partners: Jason Pickering, Philip Rybak

Abstract:
In this experiment, we delve into some of the properties of semiconductors using the Hall effect, positive doped Germanium (p-Ge), negative doped Germanium (n-Ge) and intrinsic Germanium (i-Ge). We find the magnetic leakage to be $B_{leak} = 0.05 \pm 0.06$ %. We estimate the ratio of the magnetic permeability (μ_r) as $\mu_r \approx 208,400 \pm 40 H/m$). The magnetic remnance of the iron core is found to be $B_r = 0.064$ kGs. We determine the band gap of i-Ge to be $E_g = .7259 \pm .0004$ eV. We determine the resistivity for p-Ge at 300 K to be $\rho = 900 \pm 50\ \Omega$ cm.

1 Introduction

The Hall effect was discovered by Edwin Hall in 1879 and published in, "On a New Action of the Magnet on Electric Currents".[1] The effect occurs when an electric field $\vec{E}$ and a magnetic field $\vec{B}$ pass through a conductor at 90^o to each other. passing electrons are deflected to one edge of the conductor. The excess negative charges on one edge (-) and a lack of negative charges on the opposing edge (+) creates a potential difference between the opposing edges.

2 Theory

To find a theoretical value for $\vec{B}$, we use Ampere's law to calculate the so called "*magneto-motive force*", which is the sum of product between the auxiliary (or magnetization) field $\vec{H}$ and path length for each material. This magneto-motive force is found in Equation 1 and is solved for B.

$$\begin{aligned} NI &= H_{core}L_{core} + H_{gap}L_{gap} \\ &= B\left(\frac{L_{core}}{\mu} + \frac{L_{gap}}{\mu_0}\right) \\ \mu \gg \mu_0 \Rightarrow NI &= B\left(\frac{L_{gap}}{\mu_0}\right) \\ \therefore B &= \frac{N\mu_0}{L_{gap}}I \end{aligned} \tag{1}$$

Here, N is the number of coils in the solenoid, I is the current, H_{core} and H_{gap} are the auxiliary fields through the core and the air in the gap, L_{core} and L_{gap} are the path lengths of the magnetic field through the core and the air gap, and μ and μ_0 are the magnetic permeability of the core and air gap. The result in Equation 1 shows that the magnetic field is proportional to I and can be linearized with L_{gap} by multiplying B by L_{gap}^2, which is shown in Figure 2.

The magnetic leakage can be found by setting the total field (B_T) equal to the measured field (B_D) plus the leaked field (B_L). We then take B_T as the theoretical field, B_D as the measured field, and we find B_L using Equation 2 and Figure 2.

$$\begin{aligned} B_T &= B_D + B_L \\ \Rightarrow B_L &= B_T - B_D \\ y_l &= y_t - y_d \\ B_L \cdot gap^2 &= B_T \cdot gap^2 - B_D \cdot gap^2 \\ B_L &= B_T - B_D \end{aligned} \tag{2}$$

y_l, y_t, and y_d represent the plotted lines in Figure 2. Since the y-axis is obtained by multiplying by the gap squared for all fields, the gap squared cancels out and the leakage field can be found by directly subtracting the theory and data arrays. The value for B_L is then taken as the average plus or minus the standard deviation.

To calculate the ratio of magnetic permeability μ_r, we assume a known value of inductance ($L = 60$ H) and a magnetic field B of a long solenoid. We end up approximating the area as $A = \pi(.04m)^2$ and arranging a

[1] `https://en.wikipedia.org/wiki/Hall_effect`

solution where the uncertainty from the A won't play a major role.

$$\begin{aligned} L &= \frac{N\Phi}{I} \\ &= \frac{N}{I}BA \\ &= \left(\frac{\mu_0\mu_r NI}{l}\right)\frac{NA}{I} \\ &= \left(\frac{\mu_0 NI}{l}\right)\frac{\mu_r A}{I} \\ &= \frac{B\mu_r A}{I} \\ \Rightarrow \frac{IL}{A} &= B\mu_r \end{aligned} \tag{3}$$

We then plot suitable data to model the results of Equation 3, were $\frac{IL}{A}$ is the y-axis, B is the x-axis, and μ_r is the slope m. Based on Equation 3, we determine the uncertainty in μ_r based off the slope and its associated uncertainty alone. This is shown in Equation 4 and in Figure 7.

$$\Rightarrow \mu_r = m \pm \Delta m \tag{4}$$

The next topic we explore is resistance (ρ). The resistivity of a material is proportional to the Resistance R and cross sectional area A_{sample} of the material and is inversely proportional to the length of the material l_{sample}. When we plot the data, the natural log of ρ is put on the y-axis and $1000/T$ is put onto the x-axis. Equation 5 shows the analysis of ρ.

$$\begin{aligned} \rho &= \frac{A_{sample}}{l_{sample}}R \\ &= \frac{A_{sample}}{l_{sample}}\frac{V}{I} \\ \Rightarrow ln(\rho) &= mx + b \\ \Rightarrow \rho &= e^{mx+b} \\ \Rightarrow \Delta\rho &= \left[\left(\frac{\partial\rho}{\partial m}\Delta m\right)^2 + \left(\frac{\partial\rho}{\partial b}\Delta b\right)^2\right]^{1/2} \\ &= \rho[(x\Delta m)^2 + (\Delta b)^2]^{1/2} \end{aligned} \tag{5}$$

3 Results

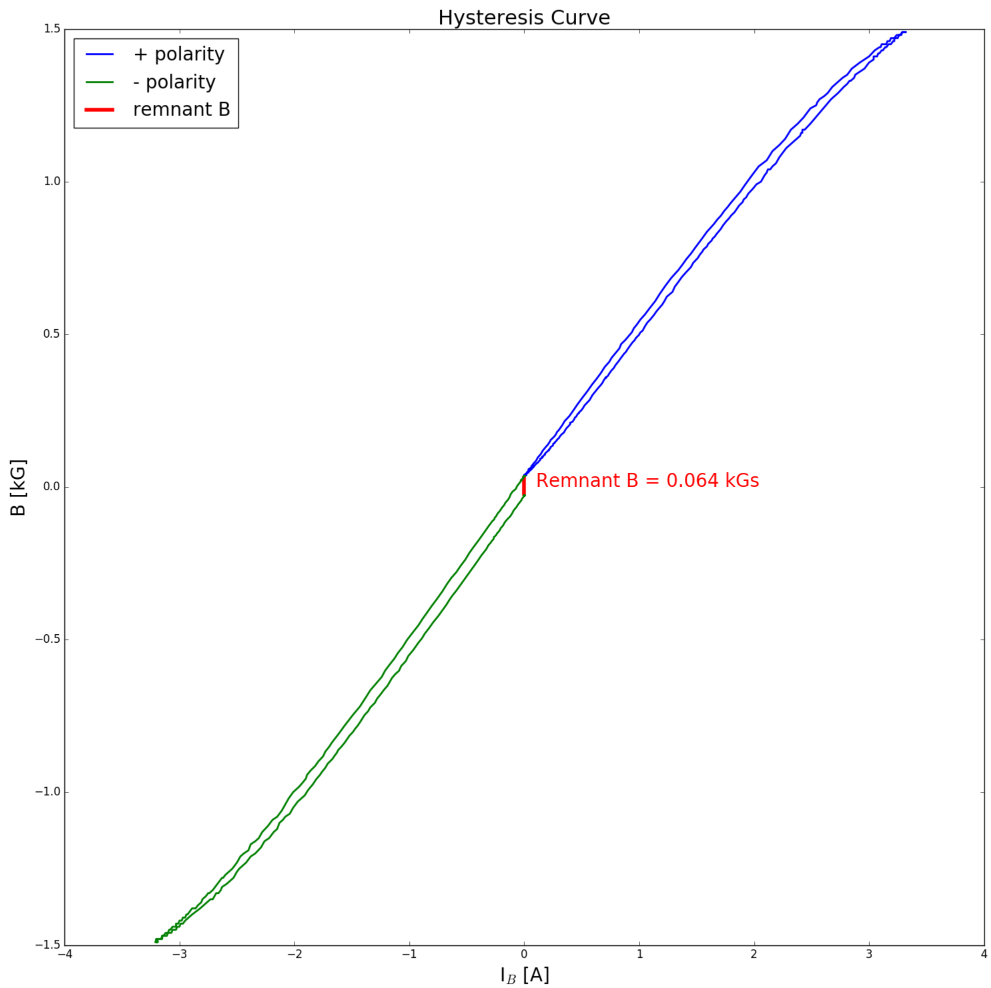

Figure 1: Hysteresis Curve

The iron core that makes up the electromagnet has a property called hysteresis (to lag behind), which means that iron "remembers" its magnetization history

After we take the positive data (shown in blue), we reverse the positive and negative leads, which reverses the polarity, and redo the measurement. The hysteresis is shown by the separation between the data when scrolling in opposite directions. Since the positive measurement was taken first, the negative measurement begins where the last positive data was measured.

We approximate the remnant magnetic field by finding the difference between where the Hysteresis curve ends and where it begins. This is done because it begins and ends at $I_B = 0$ and it has a remnant field. The value we find is $B_r = 0.064$ kGs.

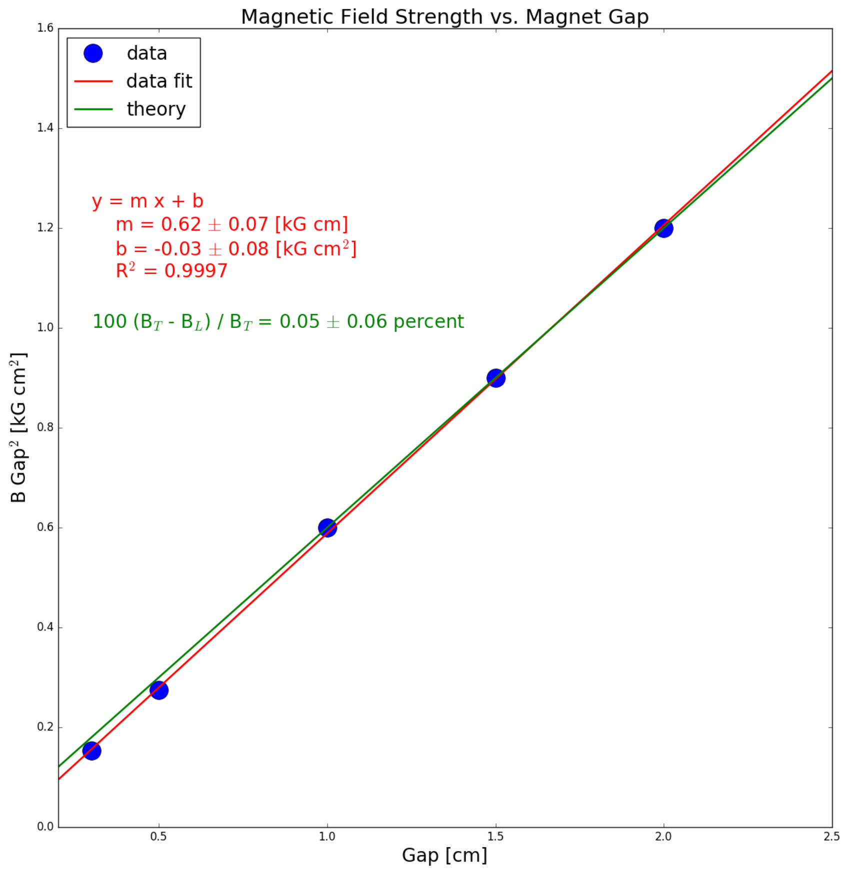

Figure 2: Magnetic Field Strength vs. Magnet Gap

This figure shows the magnetic field strength with respect to the gap distance that has been linearized. We found that if the field strength is multiplied by the square gap distance, the data becomes linearized quite nicely with an R^2 value of 0.9997. This shows that the field strength vs gap distance could be expressed as a power law such as $B(gap) = -\frac{a_1}{gap} + a_2$, where a_1 and a_2 positive fitting parameters.

The theoretical curve (shown in green) is calculated using Equation 1 and multiplying gap squared. We determine the magnetic leakage by subtracting the data vales from theory values as seen in Equation 2. The resulting average and standard deviation are $B_{leak} = 0.05 \pm 0.06$ %.

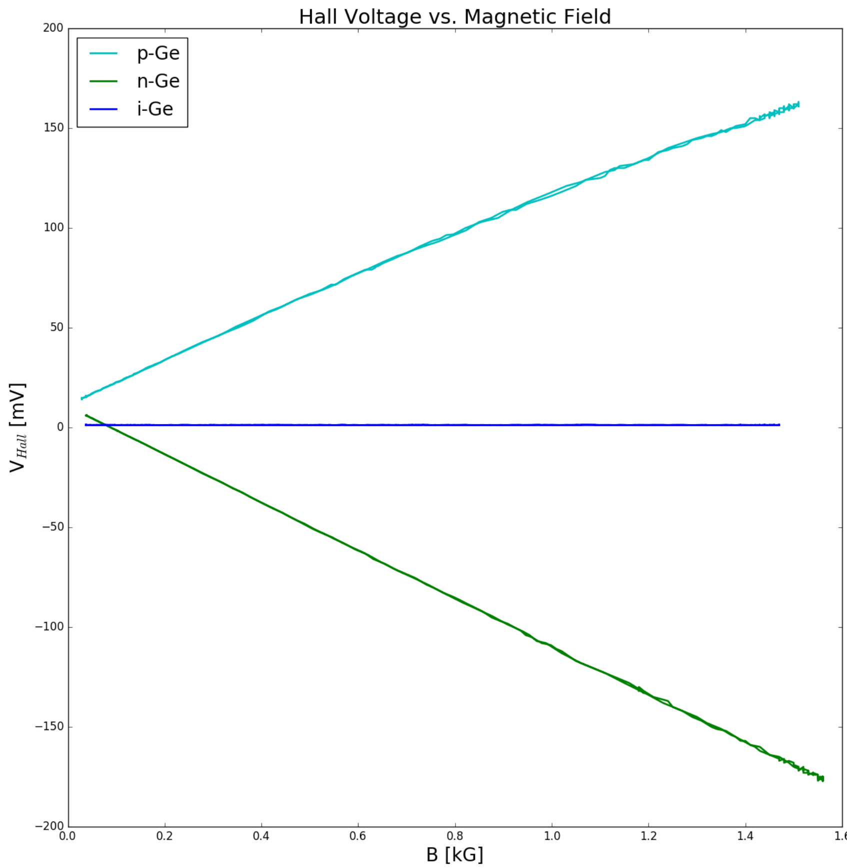

Figure 3: Hall Voltage vs. Magnetic Field

The Hall voltage vs magnetic field for positive doped Germanium (cyan), negative doped Germanium (green), and the intrinsic Germanium (blue) all show straight lines with relatively constant slope, which imply that the hall voltage is proportional to the magnetic field. The offset in the y-intercept of the positive, negative, and intrinsic data could be explained by the hysteresis of the core.

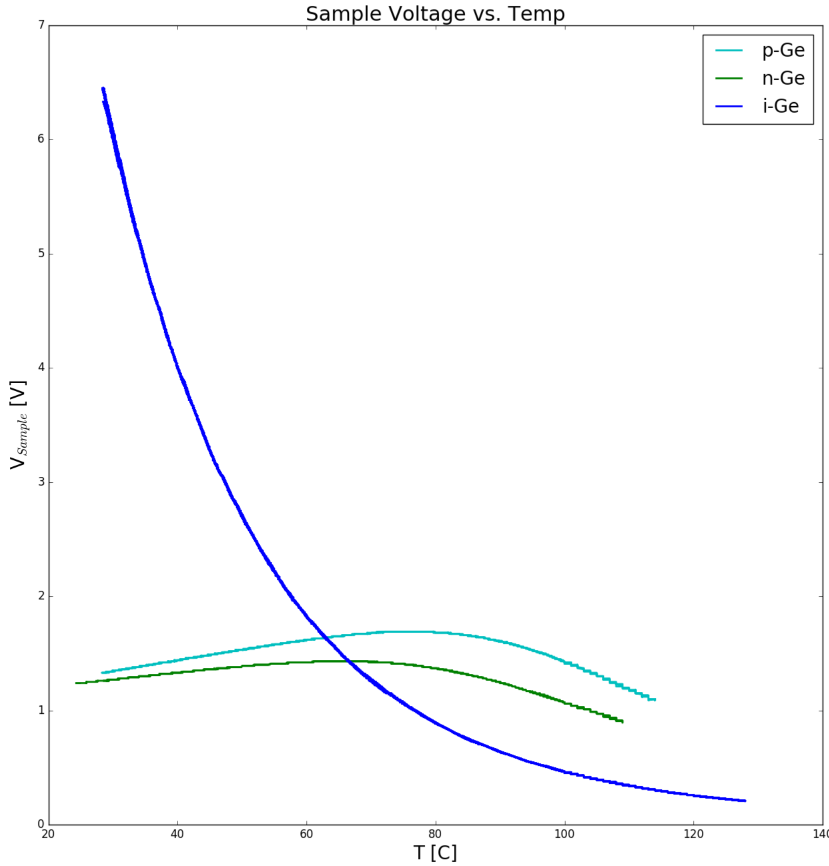

Figure 4: Sample Voltage vs. Time

In this experiment the source current is kept at zero and the temperature is slowly increased over time. Both the positive and negative doped Germanium exhibit metal like behavior before their respective maxima; in their metal-like phase, the increasing temperature causes the electrons to jostle around more violently which increases the resistance in the material which increases the potential. At some point, the additional thermal energy starts pulling electrons off the Germanium atoms and the increase in free electrons causes the potential to start dropping.

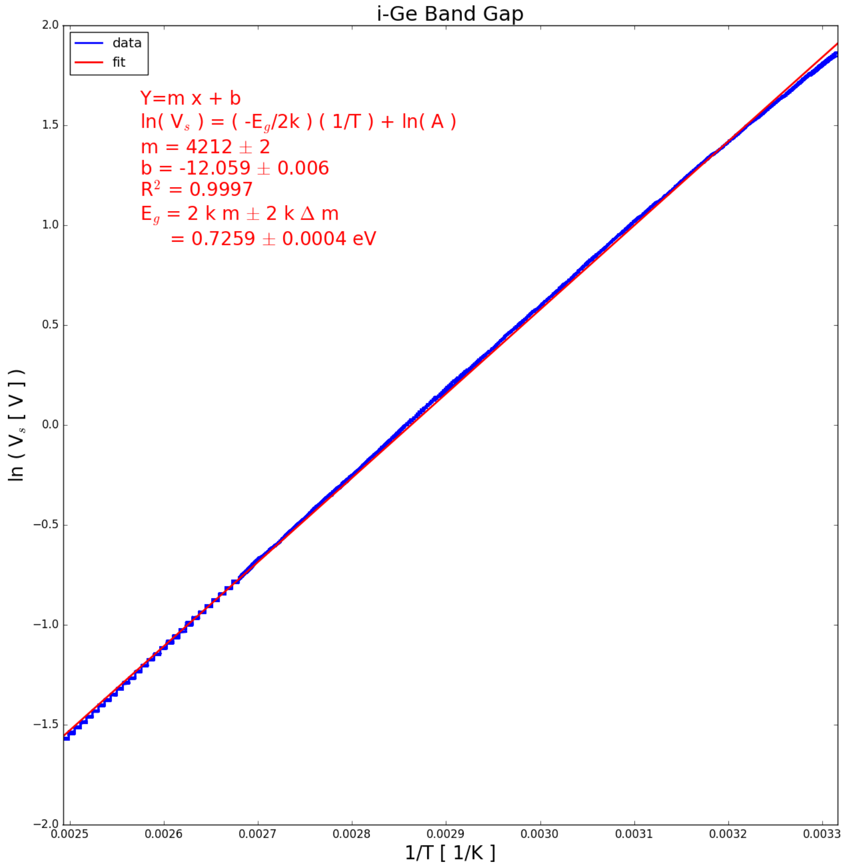

Figure 5: Band Gap of i-Ge

First, we take note that V_s for i-Ge, as seen in Figure 4, has an exponential curve. We assume it has the form $V_s = Ae^{-E_g/2kT}$ and linearize it by taking the natural log of both sides, then plot $ln(V_s)$ vs $1/T$, making sure to convert T to Kelvin. We then find that the slope of the line is equal to $-E_g/2kT$ and we use the value and uncertainty in the slope to calculate E_g. Doing this, we find E_g to be $0.7259 \pm .0004$ eV.

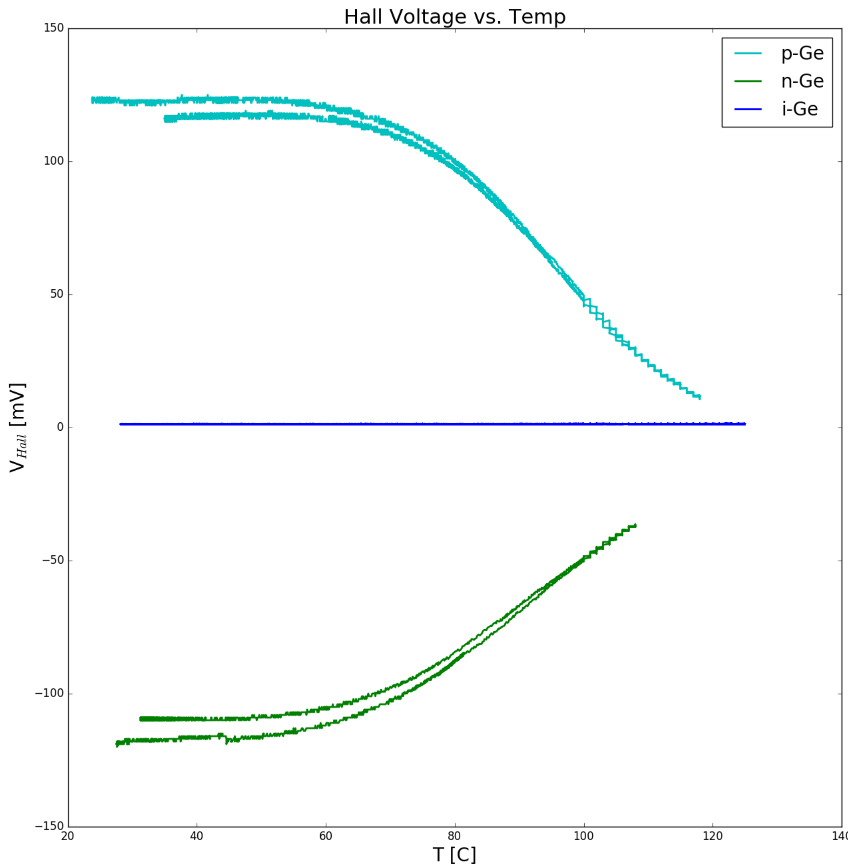

Figure 6: Hall Voltage vs Temperature

The Hall voltage across the positive, negative, and intrinsic germanium show different patterns. The Hall voltage across i-Ge stays relatively steady over the increase in temperature because the fractional change in resistivity depends on the square of the μ_n or μ_p. The negative charge carries are drawn easily towards the p-doped (holes) which leads a drop voltage drop; the n-doped (barriers) are seen as potential increase.

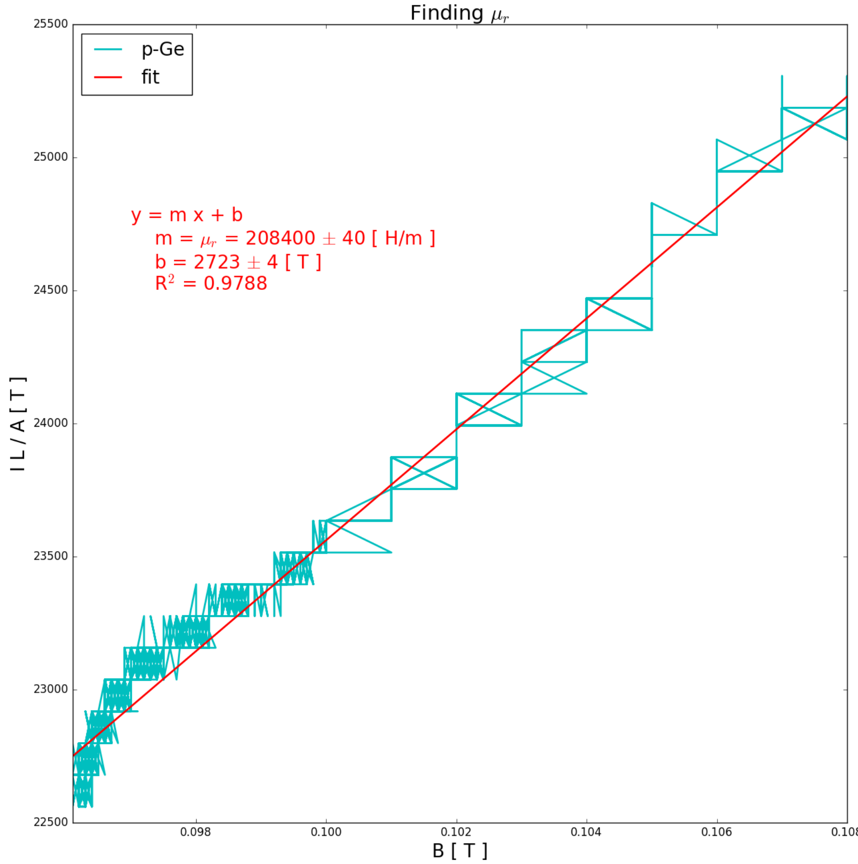

Figure 7: Finding μ_r

The patchwork compilation of the y-axis, which includes a complete guess at the area of the solenoid still gives a rough representation of the expected behavior. This method shows that $\mu_r \approx 208,400 \pm 40 H/m$.

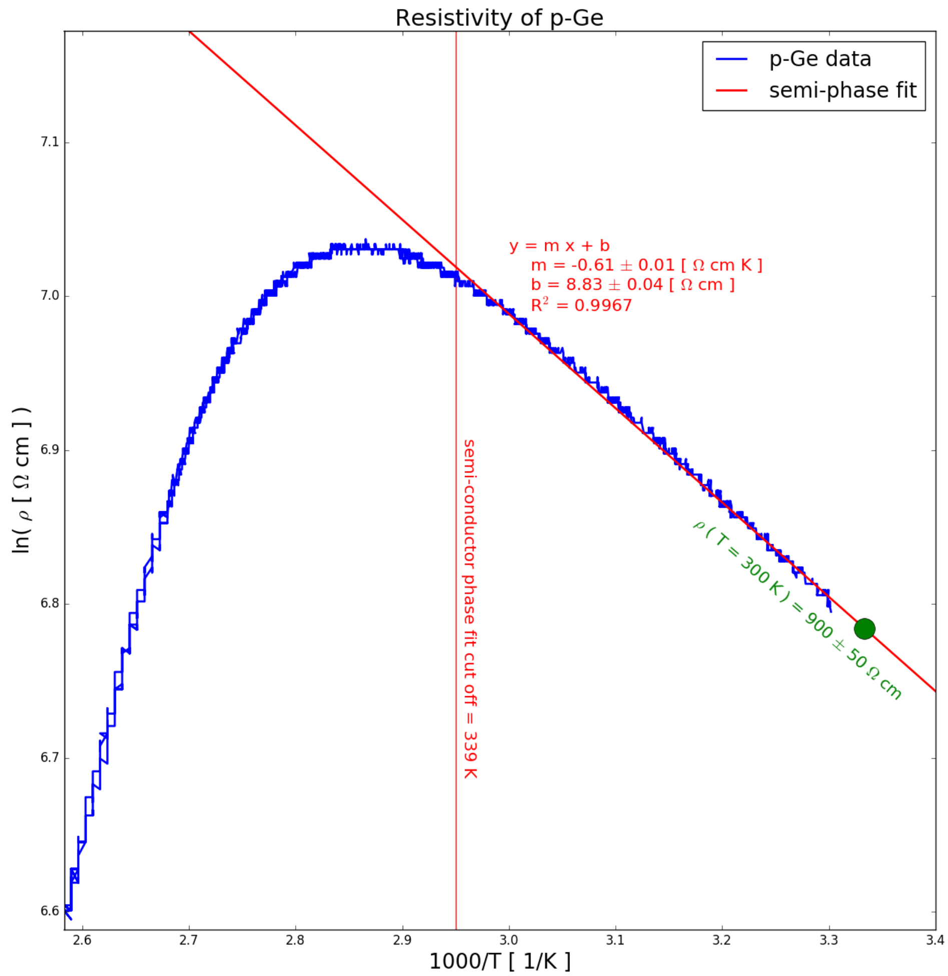

Figure 8: Finding Resistivity of p-Ge

First, we put Y-axis together using the data from part 2C in the Procedure (Appendix) and using Equation 5. We then draw a vertical line at $T = 399$ K to mark the beginning of the semi-conductor phase of p-Ge (higher temperature means lower resistance). Next, we fit the appropriate section of the data to a straight line and find the correlating resistivity for $T = 300$ K. Lastly we use equation 5 again to calculate the uncertainty in ρ. The resistivity we find at $T = 300$ K is $\rho = 900 \pm 50\ \Omega$ cm.

4 Discussion

The magnetic leakage seen in Figure 2 is found to be $B_{leak} = 0.05 \pm 0.06$ %, which seems to imply that solenoid does indeed tend to keep most of its field contained inside. This value seems reasonable.

For the magnetic permeability ratio, we found $\mu_r \approx 208,400 \pm 40H/m$, which seems in excellent agreement with other highly reputable sources.[2]

For the remnant magnetic field, we found $B_r 0.064$ kGs. We didn't find any sources to cross check to see if our result was reasonable, but its seems to fit within expectation.

We found the energy band gap to be $E_g = .7259 \pm .0004$ eV, which seems reasonable because E_g was given as 0.699 eV for a nearby temperature.

We found the resistivity of p-Ge at 300 K to be $\rho = 900 \pm 50\ \Omega$ cm.

5 Appendix

Pre-Lab:

1. **Characterization of Electromagnet**

   A Remove any sample from the console, taking care to not stress the connector wires. Connect the magnet suppy to the magnet coils, taking care that the fields "add" not "subtract". Not the 0.1 ohm current-sense resistor near the ground-lead connection. This gives $I_B = V/R$.

   B Set the pole pieces for a gap of g = 1cm using a spacer-block and tighten the thumb-screws firmly.

   C Measure B vs Current, for $I_B = 0$ to Max to 0. You may want a short dwell-time, like 100 ms. Use voltage control (current dial at max). This will prevent a nasty spark and possible damage when you unplug the cables. Then measure opposite polarity, by swapping the banana plugs at the supply (with V=0!), and run $I_B = 0$ to Max to 0. Note that R_{sense} captures the polarity. Swap the wires back when finished.

   D Measure B vs. gap size "g" for a fixed current, say $I_B = 1$ Amp. A single polarity will suffice. Be sure that V = 0 when you loosen the pole pieces to change the gap (maybe unplug a cable). Use spacer blocks to fix the gap and tighten the pole pieces to prevent accidental clamping shut (damage to sensor and fingers).

2. **Data for p-Ge**

   A Loosen and retract the front pole-piece; the back pole-piece can remain fixed. Insert he sample card, taking care to not bend the contacts or twist the board. The blank side of the sample card should face towards the front. Place the 1/2 cm spacer block just below the circuit board. Close and tighten the pole pieces.

   B Measure hall voltage U_h vs B, with $I_S = 30$ mA. Do not exceed 3 Amp for the field current. If oyu decide to reverse the field, do so only when the current is zero. (However it is recommended that field sweeps be taken only in the positive direction (up and down), because that is how the field was calibrated in table 1). Note that sample voltage is also contained in the same file, giving you U_s vs B with $I_S = 30$ mA, showing the "magneto-resistance" effect.

[2] just joking...`https://en.wikipedia.org/wiki/Permeability_(electromagnetism)`

C Measure sample voltage U_h vs temperature T (up to 170^o C maximum) with $I_S =$ 30 mA, B = 0. Check that the sample board is not touching the magnet or sensor. Record a full heat/cool cycle, and note the hysteresis due to thermal gradients if T changes too quickly. suitable data should result using 1000 ms time step during the sweep process, perhaps changing the heater voltage in several steps.

D Measure Hall voltage U_h vs T, with $I_B = 2$ Amp, $I_S = 30$ mA.

3 Data for n-Ge

A Repeat the above for n-Ge.

4 Data for i-Ge

A Repeat the above for i-Ge. In this case, you may need to lower I_s to keep the sample voltage below 10 V.

5 Data analysis and Questions

A Calculate the expected magnetic field, knowing N = 600 turns for each coil. (Hint: using Ampere's law and assume μ_r very large). Estimate the flux leakage (as %) by comparing the calculated and measured magnetic field.

B Estimate μ_r (dimensionless ratio μ/μ_0 for the pole pieces, knowing the inductance L = 60 Henry for the two coils in series.

C Estimate the magnetic remnants of the pole pieces.

D Plot Hall voltage vs field for all three samples on a single plot.

E Plot sample voltage vs T for all three samples on a single plot.

F Make a linearized plot for i-Ge, using the expected functional behavior, and from this find the band gap (in eV) for Ge.

G Plot hall voltage vs T for all three samples on a single plot. Why does the p-Ge data change sign?

H (opt) Find the mobilities μ_p and μ_n from the magneto-resistance data.

I Find the resistivity (ohm-cm) for p-Ge at T = 300 K from the I-V data and sample geometry. From this, estimate the doping density (#/cm^3).