Cavendish Balance

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Concepts

- Torsion balance; Driven oscillator; Optical lever; “Noise” suppression;

Background Reading and Notes

- TEL-RP2111 Manual for Cavendish Balance
- Mercer document
- Cavendish_3.vi (Labview program)

Precautions

- The tungsten wire for the torsion pendulum is only 25 microns thick and is easily broken and difficult to repair. Do not move or bump the instrument while the boom is suspended.
- Do not allow the laser beam or its reflection into the eye.
- Keep RE magnets far from any computer, monitor, phone, etc.

Background:

You will use a torsion balance to measure the gravitational force between pairs of lead balls, and thereby determine a value for the gravitational constant G. The value of G is the least well-known of the fundamental constants, particularly for small objects and short distances. The gravitational force is extremely weak, even though we feel its presence always, and it holds celestial bodies in their orbits. The recent discovery of Gravity Waves brings renewed interest in fundamental aspects of gravity. The LIGO experiment represents a triumph of exquisite instrumentation that could detect a strain of 10^{-21}. Your task of measuring the force between kg size objects is much simpler than LIGO, but it still presents a significant challenge.

The “torsion balance” is shown in Figure 1. This instrument allows measurement of the extremely small horizontal force between lead balls in the presence of an immensely larger vertical force on the balls by the Earth. This is accomplished using a balanced, symmetrical arrangement, in which the force of the Earth is canceled by symmetry. The outer balls M,M’ are positioned approximately distance w/2 from the small balls. You will make two measurements for G: static and dynamic, as detailed below.

1. ***Background activities while waiting for the boom to settle:***
   a. Estimate the angular range (mRad) for the fine-adjust lever from its dimensions.
   b. Estimate the interaction force and the expected static deflection angle when (M,m) are separated by w/2, based on the elasticity of tungsten, and the known value for G.
   c. Estimate the deflection angle caused by gravity from your body in proximity to the instrument.

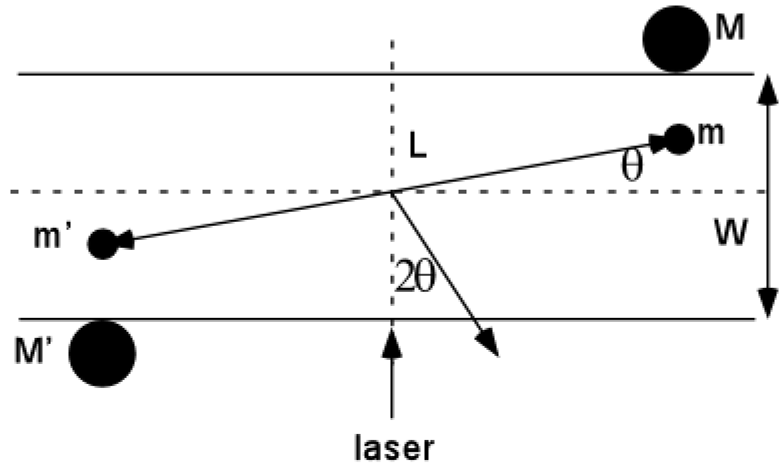

Figure 1. Schematic of torsion balance (top view). A thin tungsten wire supports a boom of length L with attached masses m, m'. The force between M, m causes a small deflection θ, which can be read by capacitive sensor and optical lever. Tungsten support wire: L = 65mm, d=25um. Glass walls with spacing "w" protect the boom from air currents.

Procedure

2. Setup:
   You will first do measurements with no "m" and no "M". This gives the I of the empty boom and also allows a convenient way to calibrate the mRad sensor.
   a. Measure mass and diameter of the four lead balls (M, M', m, m'). Avoid gravity experiments that dent the balls. Measure mass and dimensions of the replica boom, and the outside width "w" of the window-box.
   b. (TA assist!) Begin with no M and no "m". Level the apparatus. Using the patented lifter mechanism, carefully raise the pendulum so it swings freely near the vertical center of the gap. Do not bump or move the instrument while the pendulum is free. Start the Cavendish.vi data acquisition program. At first, the boom may bounce at one or both windows (about 120mR), but it will soon settle down to a nice damped sinewave. Note the physical direction (CW or CCW, top view) for positive mRad reading, since tiny motions will not be visible later. Adjust the rotary position of the vertical shaft (first coarse, then fine) so the center of oscillation is near zero (within ~10 mrad).
3. Angle calibration:
   The mRad scale shown by the VI is only approximate, so you will calibrate the sensor using an "optical lever". Note that a boom deflection of θ gives a beam deflection of 2θ. Set the laser beam to reflect from the boom mirror onto the far wall. An efficient procedure is to mark each turning point during the damped oscillation. You can then plot laser vs VI for a range of values to get an accurate calibration. This also yields the period of oscillation for the boom alone, from which you can get I_{boom} (at least in proportion to the boom loaded with 2m).
4. Background "noise" (M=0, m=0):

It is interesting and useful to observe sensitive to environmental influences. These include vibrations (oddly, very immune b/c of the differential mode); air currents (wind – mostly blocked by the glass windows), electrostatics and gravity from bodies. Your body motions should be small and slow. You might see electrostatics as you stand close, far, allowing time for the boom to move. This is a genuine displacement, not a capacitive-sensor effect.

5. Static measurement of G
   (TA assist!) Lower the boom and carefully add m,m' using tweezers. Next add M,M' to the carriage and set it to center position. Start VI and let the boom to settle. You can hasten to approach to small amplitude using a RE magnet (very powerful!) to provide a gentle push on one end of the boom. This is best done by coaxing the velocity to zero as the amplitude approaches zero – thus simultaneously minimizing both potential and kinetic energy. When the oscillation is below 5mRad, you are ready for data: Rotate M,M' to gently touch the window. Record a few cycles, then swing M,M' to touch the opposite window, and record a few more cycles. The difference between "centers" comprises the static displacement (twice that). Continue, to get several "center values", to establish an uncertainty.
6. Driven (resonant) measurement of G:
   Set M,M' to the center position. Wait for motion < 1mR amplitude. Then swing M,M' in resonance to increase the motion. What is the phase at resonance and how do you move that way? Continue until the amplitude approaches the asymptotic value (6-7 cycles). Then set M,M' to center and leave the recording running to capture the natural decay for another ~ 10 cycles.

Analysis

7. For the static data, a simple eyeball fit will suffice to find the center(s) of the oscillations. For the dynamic data, you should fit the decaying sinewave (Eq 3). The GA program is good for this, provided you help it a bit, as follows:
   a. Chop out "regions of interest (ROI)" and put them into New Cols.
   b. Shift the time-origin to a +turning point (max). This allows to fit cos() with no phase angle.
   c. Oscillating functions have multiple turning points, which can be tricky to handle. Adjust "manual fits" until you are reasonably close before changing to Auto-fit. When fitting the "driven" oscillator, you should in principle add the decaying signal that precedes it (and continues "under" it!). If small, it can be ignored.
8. Analysis and questions:
   a. Find and compare G values for static vs dynamic method. Identify and justify the uncertainties in each experiment. Which uncertainty is dominant, and how might you improve that? For systematic error, propose a source and solution.
   b. Suppose the boom resting angle was 10 mrad (instead of 0), and you have measured offsets from this value. How would that offset change your static G?
   c. (opt) Do the calculation for the torque due to the distant m (-7% in equation 10).
   d. (opt) Find the torque "correction" due to I_{boom} directly from the period of motion with m=0 (boom alone).
   e. (opt) Estimate the force of the RE magnet on m, assuming a field of 100G at a distance of 5cm.

Appendix: Labview program "Cavendish.vi"

The boom position is read by a differential capacitance sensor built into the slot above/below the boom. The capacitance values at 4 points near the ends of the boom provide a "difference signal" proportional to angular displacement (about the suspension wire) while suppressing the "common mode" signal from translational displacements (horiz or vertical). This angular position is read by a LabView program "Cavendish.vi". This VI writes a temp file Cavendish.txt on the desktop when you hit the "STOP" button ("Abort" does not write the file!). You must copy your data and close this temp file at the end of each run, b/c it is overwritten at each new "RUN". An efficient procedure is to open yourfile.xls in a convenient location, and then copy/paste from temp.xls into yourfile.xls, where you can use multiple tabs, make charts and notations, etc. Pin the boom at center position. Then start the program and set sample time = 1 sec. Adjust the capacitive sensor "zero adjust" to give a signal near zero. The "gain sensitivity" should be set at maximum - do not adjust this, as it is funky.

There are two important inputs: "Range" should be at 10 for full scale readings during setup, but set at 1 for high-sensitivity data (which pins at 40mRad). "Nave" is the number of pts averaged at 10pt/sec. You must restart the VI to change these parameters.

Appendix: Driven oscillator

The static response of the torsion pendulum is given by

$$\tau_p = -K\theta \,, \tag{1}$$

where τ_p is the torque about the support, θ is the displacement from equilibrium and K is the restoring spring constant. The equation of motion then is

$$I\ddot{\theta} - b\dot{\theta} + K\theta = \tau \,, \tag{2}$$

where I is the moment of inertia about the support, b is a damping parameter, and τ is an external applied torque. For τ =0, this well-known DEQ has solution

$$\theta(t) = \theta_0 + Ae^{-\gamma t} e^{i(\omega t)} \,, \tag{3}$$

with

$$\omega^2 = \omega_0^2 - \gamma^2 ,\ \omega_0^2 = K / I ,\ \text{and } \gamma = {}^{b}\!/_{2I} \,. \tag{4abc}$$

We have suppressed the phase angle in (wt+Φ) by choice of t_0.

For a static measurement of G, we simply equate the constant gravitational torque to the balance torque, as

$$\tau_g \approx \frac{2MmGL/2}{s_{M,m}^2} = -K\theta_s \,, \tag{5}$$

where θ_s is the static deflection angle, L/2 is the moment arm of the pendulum, and M, m are separated by distance

$$S_{M,m} = W/2 + R_M \,. \tag{6}$$

Here, we ignore the interaction between M and distant mass m', and we assume that M is touching the window and that θ_s is small, so m is on the centerline. For a mass-less boom and point-masses at radius L/2, we have I = $mL^2/2$ and obtain the simple expression for G as

$$G = \frac{s^2 \omega^2 L \theta_s}{2M} \,. \tag{7}$$

For a "resonance" measurement of G, the pendulum is driven at resonance (with phase π/2!) by applied torque

$$\tau_w(t) = \tau_g e^{i(\omega t + \pi/2)}. \quad (8)$$

The response is then given by (See Telatomic manual)

$$\theta_d(t) = \theta_0 + \frac{4Q}{\pi}\theta_s(1 - e^{-\gamma t})e^{i(\omega t)}, \quad (9)$$

where $\theta_s = \tau_g/K$ is the static displacement that would occur for a static driving force of this amplitude, and $Q=\omega/2\gamma$ is the "quality factor" for the resonance. Note that the amplitude of the steady-state (long time) response is larger than the static response θ_s by a factor Q for sine-wave excitation, and another factor $4/\pi$ for square-wave excitation. Higher harmonics are strongly suppressed by the resonance curve.

Corrections to the torque:
The expression for torque (Eq 5) ignores the force exerted by M on the distant mass m', and on the supporting boom. Using values appropriate for this apparatus, the total (corrected) gravitational torque may be written as (See Telatomic manual)

$$\tau_{tot} = \tau_m + \tau_{m'} + \tau_{boom} \approx \tau_m(1 - .07 + .03). \quad (10)$$

The boom correction may be included by calculation of I from dimensions (as here), or better by direct measurement of the period of oscillation with m=0.