measurements, using helium as the target gas. In addition to the elastically scattered electrons, there are other groups of electrons of lower energy, corresponding to scattering processes in which the bombarding electrons lose kinetic energy and the electrons of the target atoms are excited to higher energy levels. Since kinetic energy is not conserved in such interactions, we call them *inelastic collisions*.

1-10 X-RAY SPECTRA

Some features of x-ray spectra provide striking confirmations and extensions of the Bohr theory. We mentioned earlier the continuous spectrum of x rays that is produced when ener-
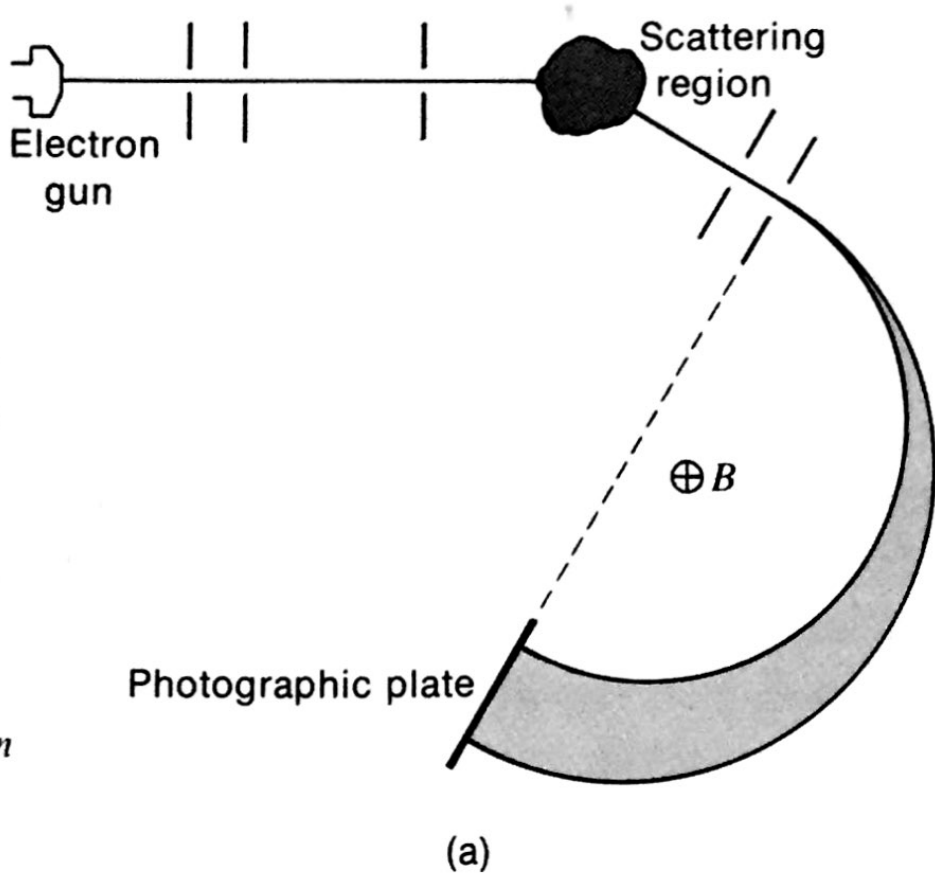
Fig. 1-13 (a) Experimental arrangement for electron scattering experiments. (b) Results on elastic and inelastic scattering processes, showing the production of characteristic excited states of sharply defined energy in atoms of the target helium gas. The scattering peaks associated with the production of excited helium states are marked He*, with the excitation energy in parentheses. [After L. C. Van Atta, Phys. Rev. 38, 876 (1931).]
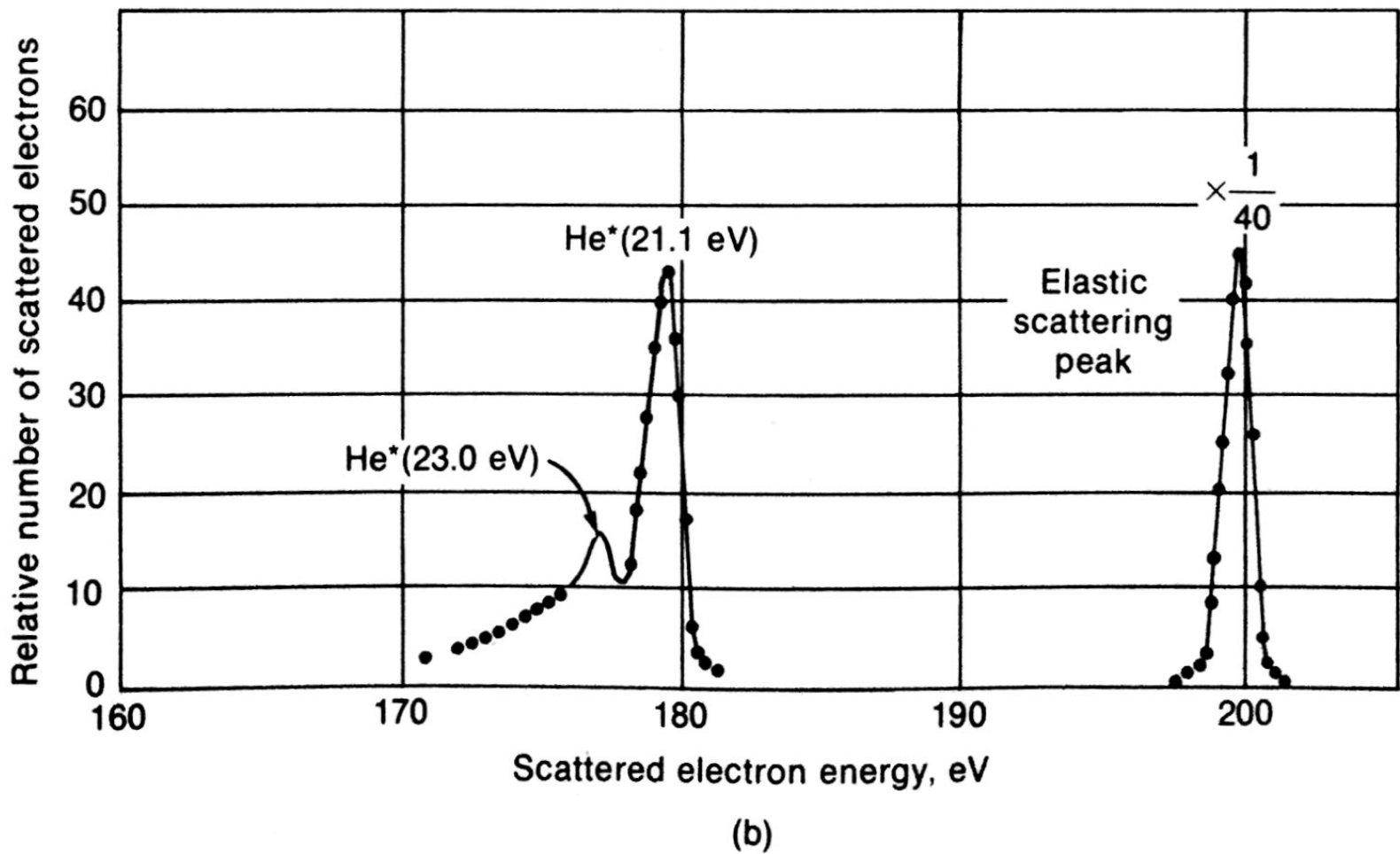
(a)

(b)

Fig. 1-14 X-ray spectra produced by 35-keV electrons striking tungsten, molybdenum, and chromium targets. The three spectra exhibit the same short-wavelength limit, as demanded by the Duane-Hunt law. Contrast this feature with the two very sharp and intense lines which appear only in the spectrum obtained with the molybdenum target. [After C. T. Ulrey, Phys. Rev. 11, 405 (1918).]
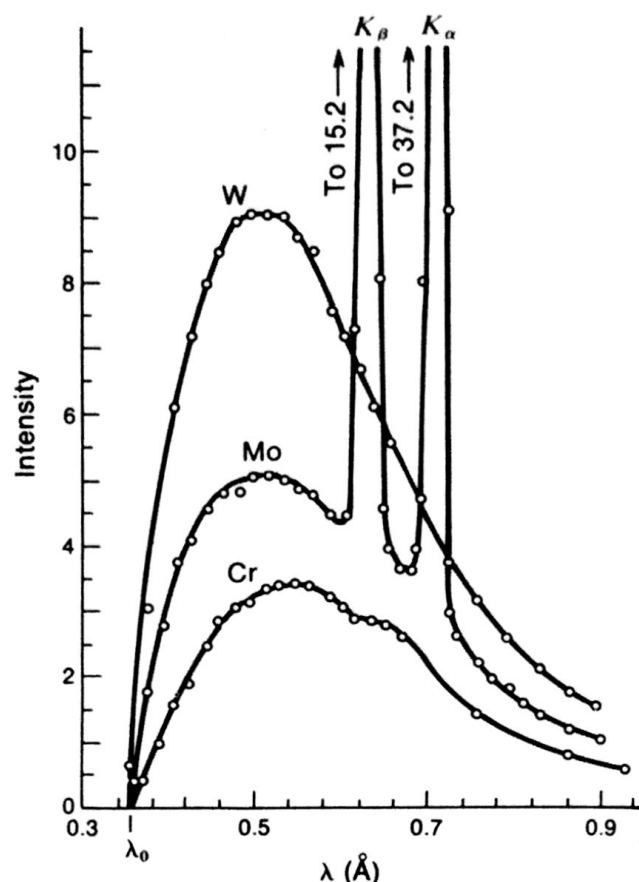
getic electrons (10–100 keV) strike a target (see Figure 1-9a). However, in addition to this continuous (bremsstrahlung) radiation, the x-ray spectrum may also contain a few sharp lines, as illustrated in Figure 1-14. (For a brief description of how x-ray spectra are analyzed, see the next section.) The wavelengths of these x-ray lines are characteristic of the bombarded target atoms, but (in contrast to the great diversity of optical spectra) there is a remarkable family likeness between the x-ray line spectra of all the different elements. Also, the x-ray spectra are far simpler than most optical spectra. The systematics of these x-ray spectra were explored by the British physicist H. G. J. Moseley in a brilliant and intensive period of research in 1913–1914 (just before he went off to World War I and his own death).[29] Figure 1-15 shows a sample of Moseley's results for a number of different atomic species. And what Moseley also did was to show how these results were to be understood in terms of an extension of the Bohr theory.

[29]H. G. J. Moseley, Phil. Mag. 26, 1024 (1913); 27, 703 (1914). The story of Moseley and his work is well told in Bernard Jaffé, *Moseley and the Numbering of the Elements*, Doubleday Anchor, New York, 1971, and in J. L. Heilbron, *H. G. J. Moseley*, University of California Press, Berkeley, 1974.

Simple models of the atom

Fig. 1-15 Spectra showing the characteristic x-ray lines for several elements. In this composite photograph, the abscissa is wavelength, which increases toward the right. The two strongest lines in each spectrum were designated K_α and K_β, and the spectra were arranged so that these line wavelengths increase from bottom to top. [Figure reproduced from H. G. J. Moseley, Phil. Mag. 26, 1024 (1913). Reprinted with the permission of the Philosophical Magazine.]
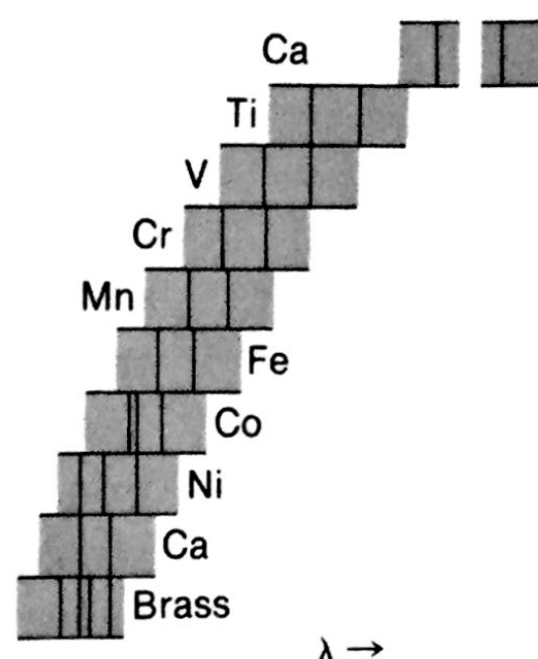
The basis of the theoretical analysis is a simple model of a many-electron atom. We begin with a central nuclear charge, and consider the kind of Bohr orbits that one would have for a single electron. But now we introduce the idea that each orbit can accommodate only a certain number of electrons. This leads to a picture of an atom as built up of successive "shells" of electrons, as indicated in Figure 1-16. By long-established tradition these shells are labeled by the successive letters K, L, M, ... In fact the periodic similarity of chemical properties, summarized in the periodic table of the elements, is associated with the recurrence of similar electron configurations in the outermost shells as one goes from lighter to heavier elements. For example, beryllium, which has its two outermost electrons in the L shell, labeled by the Bohr quantum number $n = 2$, is chemically similar to magnesium, which has its two outermost electrons in the M shell, belonging to $n = 3$.

Suppose now that a many-electron atom is hit so hard (by

Fig. 1-16
(a) Shell model of many-electron atom, showing the four innermost shells. Each shell is taken to have a limited capacity for electrons. (b) Standard notation for x-ray spectral lines. [Note: This figure is purely schematic—the radii used in the figure were arbitrarily chosen.]

1-10   X-ray spectra
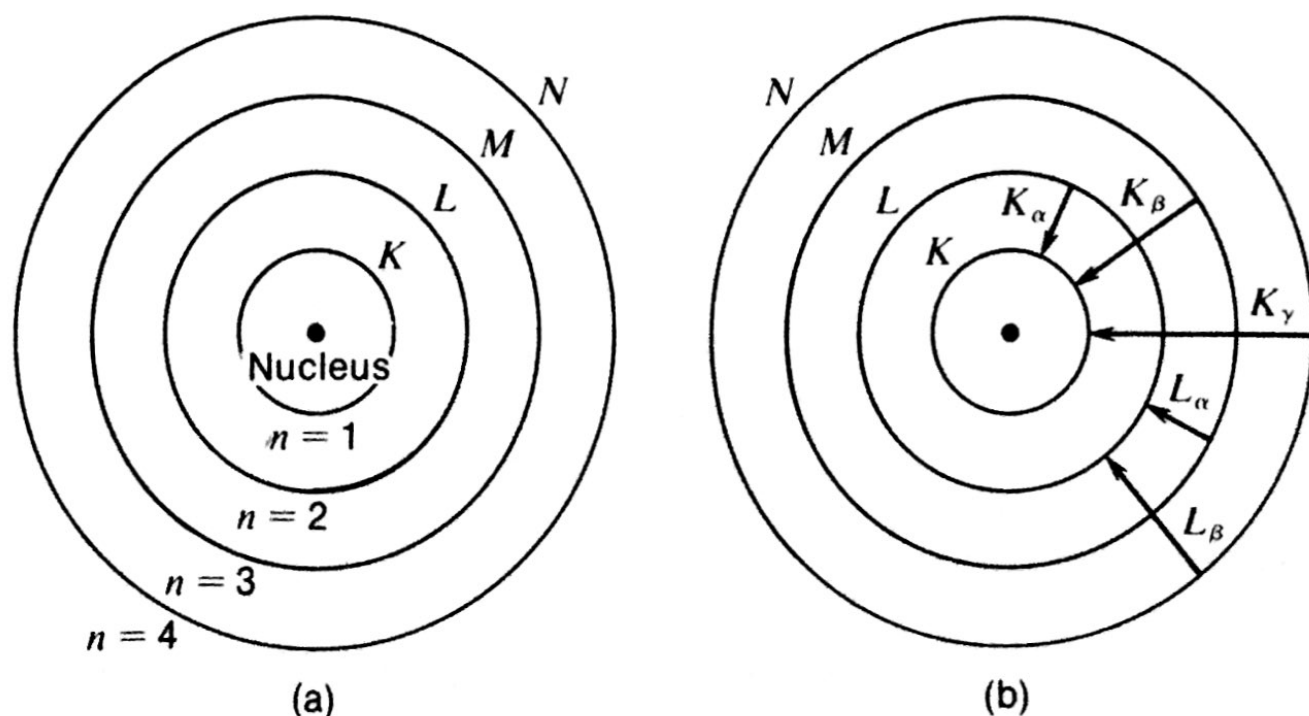
an energetic electron from outside, for example) that one of the electrons in the filled innermost orbit ($n = 1$) is knocked out of the atom completely. The ejected electron leaves a vacancy into which an electron from one of the larger orbits (for example, $n = 2$ or $n = 3$) can fall, emitting a photon in the process. The energy of the emitted photon will be the energy difference between the initial and final states of this many-electron atom. In principle, this energy difference must take into account the interactions among *all* of the electrons in the atom. However, an easily visualized approximate model which makes use of the Bohr theory ignores these multiple interactions and analyzes the downward transition as that of an electron moving under the influence of a single effective central charge. By Gauss's law, the effective central charge is that of the nucleus diminished by the charge of any electrons remaining in orbits which lie inside that of the radiating electron. (This reduction of effective charge is called *screening*.) Call the effective central charge Q. Then from the Bohr formula, Eq. 1-25, with $n_f = 1$, who have

$$\nu = \frac{c}{\lambda} = \frac{2\pi^2 m e^2 Q^2}{h^3}\left(1 - \frac{1}{n_i^2}\right) \tag{1-26}$$

If we take $n_i = 2$ in Eq. 1-26, we define what Moseley recognized as the line known as K_α in the characteristic x-ray spectra. It is the exact counterpart of the first (longest-wavelength) line in the Lyman series of hydrogen. (The x-ray lines are coded according to the shell on which the quantum jump terminates. A companion line, called K_β, corresponds to $n_i = 3$ and has a shorter wavelength.) Thus we have

$$\nu(K_\alpha) = \frac{3}{4} \cdot \frac{2\pi^2 m e^2 Q^2}{h^3} \tag{1-27}$$

Then if one measures the frequency of the K_α transition for all the different elements, the value of $\sqrt{\nu}$ should be a measure of the effective central charge Q in each case. And what Moseley found was that the graph of $\sqrt{\nu}$ against the chemical atomic number Z was a straight line (Figure 1-17a). In Moseley's own words: "We have here a proof that there is in the atom a fundamental quantity, which increases by regular steps as we pass
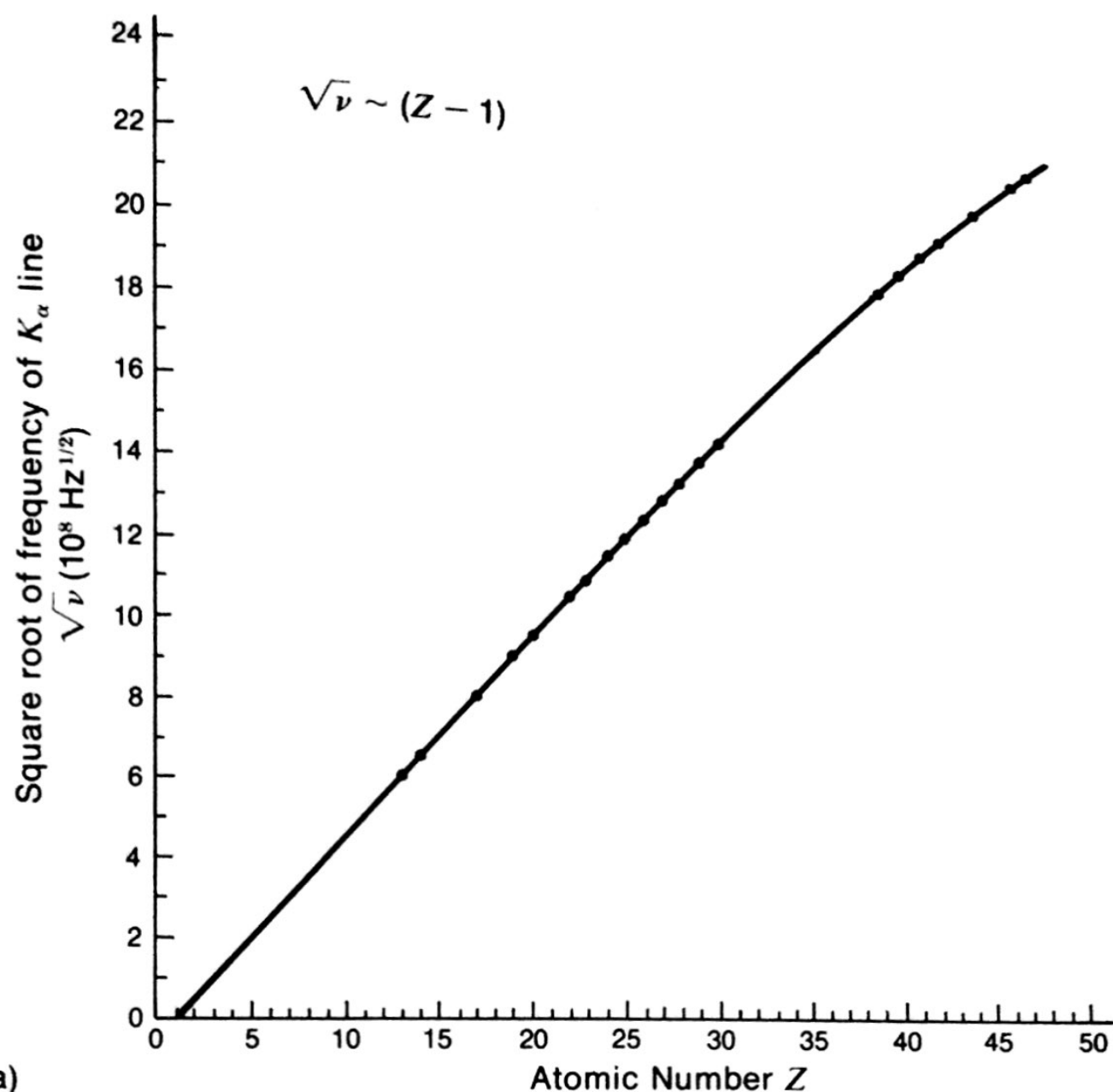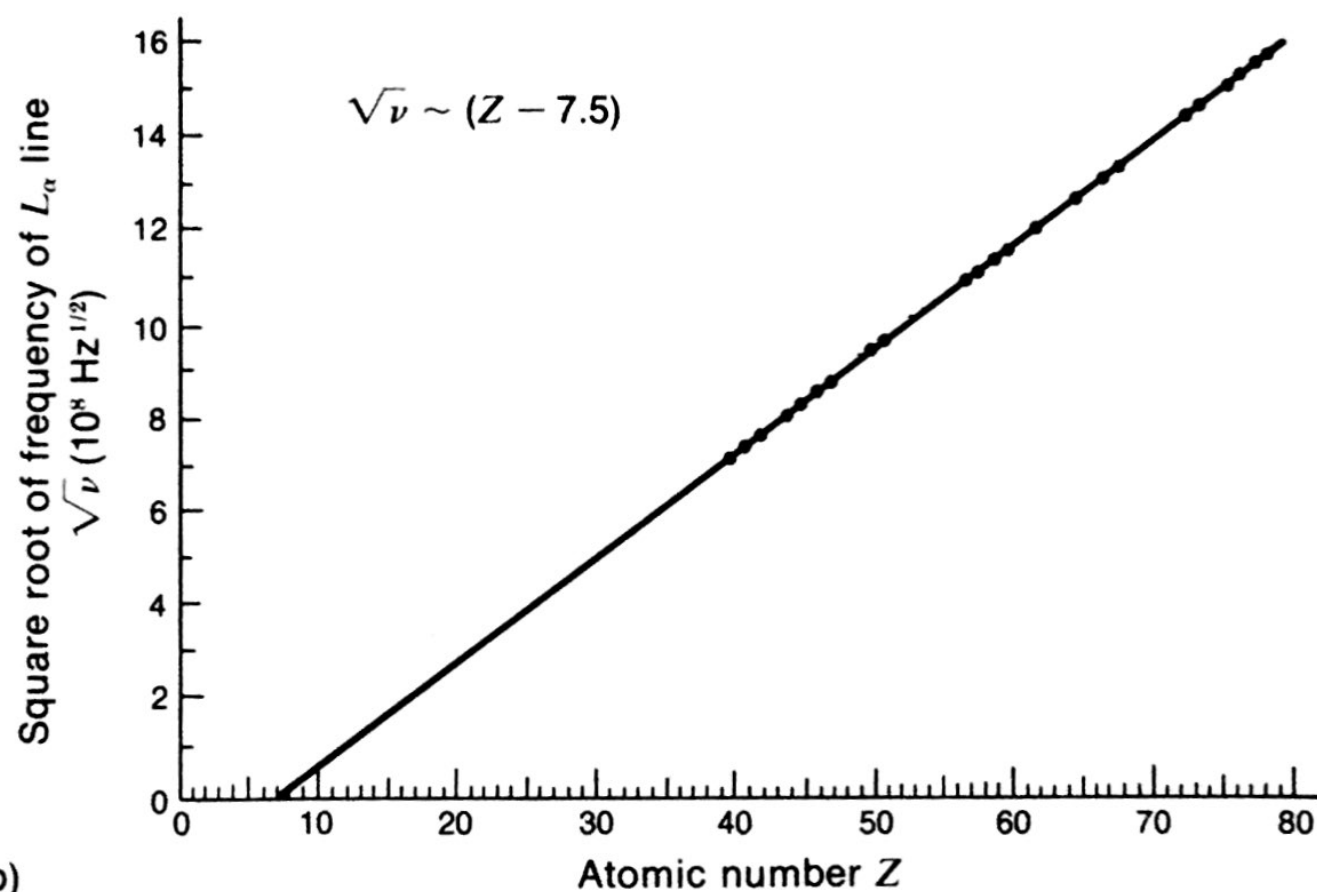
Simple models of the atom

$\sqrt{\nu} \sim (Z - 1)$

(a)

$\sqrt{\nu} \sim (Z - 7.5)$

(b)

Fig. 1-17 The physical basis for chemical atomic number. (a) Square root of K_α transition frequency as a function of chemical atomic number Z. (The line is adjusted to intercept the horizontal axis at Z = 1.) (b) Square root of L_α transition frequency as a function of Z. [Graphs based on data from H. G. J. Moseley, Phil. Mag. 27, 703 (1914).]

from one atom to the next. This quantity can only be the charge on the atomic nucleus."[30]

The graph of $\sqrt{\nu}$ against Z actually has an intercept at about 1 on the Z axis. This implies that the effective central

[30] H. G. J. Moseley, Phil. Mag. 26, 1031 (1913).

1-10   X-ray spectra

charge for the K_α transition is equal to $(Z-1)e$, and hence that one electron was left in the $n=1$ orbit when its companion was originally removed. The implication is that the inner orbit normally contains two electrons.

In a similar way it was possible to analyze the so-called L transitions—those that end on the $n=2$ orbit. (These correspond to lines in the Balmer series of hydrogen.) For these L transitions the effective central charge was found to be equal to about $(Z-7.5)e$, indicating that the shielding of the nucleus represented by the electrons remaining in the $n=1$ and $n=2$ shells together was the equivalent of between 7 and 8 electrons (Figure 1-17b). This suggests that 8 or 9 electrons populate the inner two orbits when filled. We shall see later that a more realistic value is 10 electrons. Evidently we need a better theory to account accurately for all the details.

Let us illustrate the above discussion with one typical example.

Molybdenum $(Z=42)$ has a characteristic K_α line of about 0.71 Å. The corresponding photon energy is given by

$$E_{\text{photon}} = \frac{1.24 \times 10^4}{\lambda(\text{Å})} \text{ eV} \approx 17.5 \text{ keV}$$

Theoretically, by Eq. 1-27, the photon energy should be given by

$$h\nu = \frac{3}{4}(41)^2\left(\frac{2\pi^2 me^4}{h^2}\right) \approx 1260\left(\frac{2\pi^2 me^4}{h^2}\right)$$

But the quantity $2\pi^2 me^4/h^2$ is the ionizaton energy of the hydrogen atom, that is, 13.6 eV, so we would expect the energy of the molybdenum K_α radiation to be about 1260×13.6 eV, or 17.1 keV, which checks pretty well.

We can recognize that the way in which these x-ray spectra are generated is very similar to the stimulation of optical emission lines following electron bombardment in the Franck-Hertz experiment. The only basic difference is that in the Franck-Hertz experiment the radiation comes from electrons

Simple models of the atom

Fig. 1-18 Log-log plot showing the frequency dependence of x-ray absorption in lead and in iron. For increasing frequency, the absorption coefficient shows a general decline but exhibits an abrupt increase when the x-ray photon energy becomes large enough to remove an electron from an inner shell. [From data of S. J. M. Allen, Phys. Rev. 27, 266 and 28, 907 (1926).]
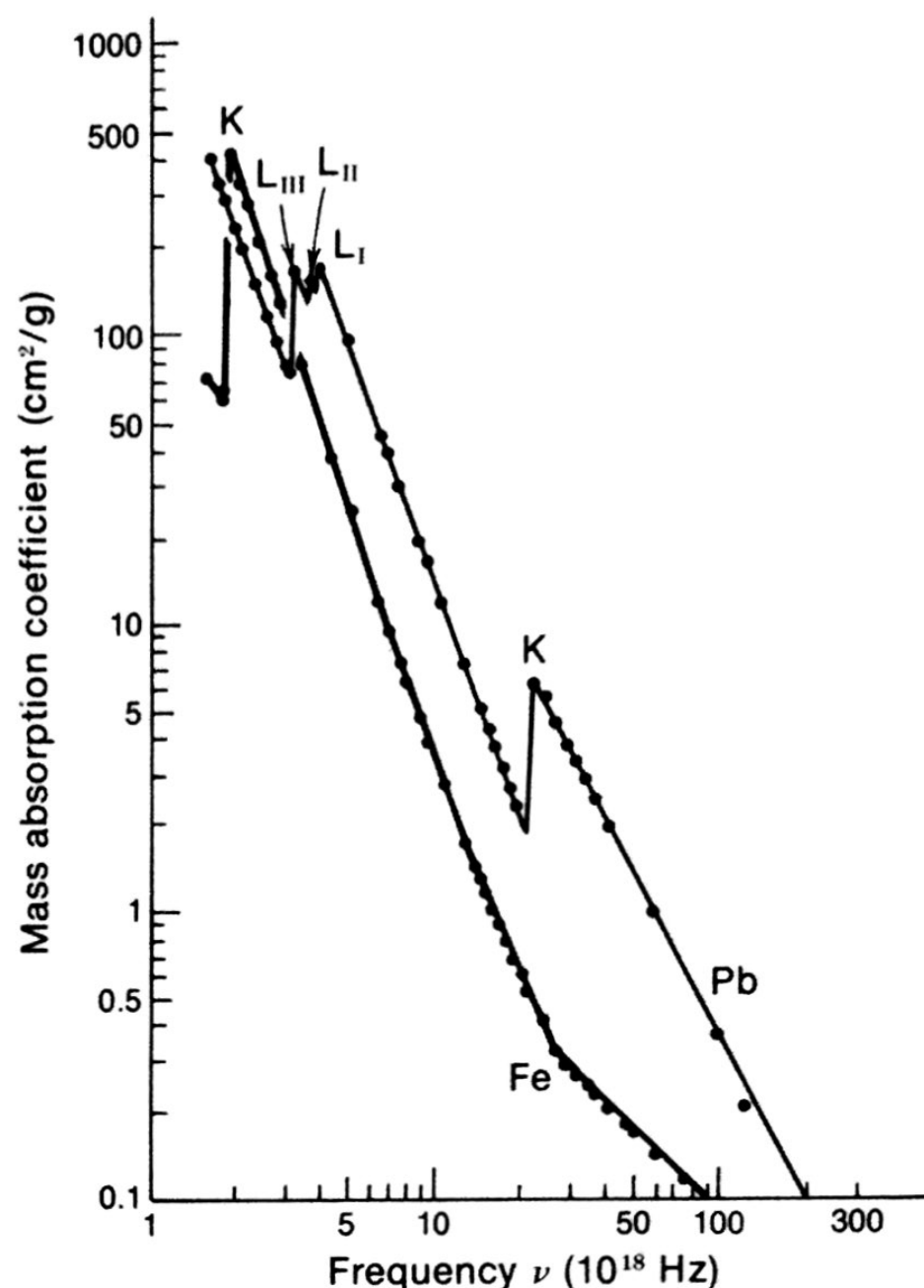
falling back into their original state after being temporarily dislodged, whereas the x-ray emission comes from an outer electron falling into the vacancy created by the complete ejection of an inner electron from the atom. In connection with this last point, it is clear that the initial process of removing an electron from the K or L shell requires a definite minimum energy equivalent to the ionization energy of that shell. This is manifested in the "absorption edges" that appear when a continuous x-ray spectrum is passed through a material—the absorption as a function of photon frequency ν shows a sharp increase at each value of ν corresponding to removal of an electron from a particular shell. (See Figure 1-18.)

1-11 A NOTE ON X-RAY SPECTROSCOPY

The early study of x rays was one of the most exciting and important episodes in the history of physics. It was in 1895 that W. K. Roentgen first observed the production of penetrating radiation when cathode rays (themselves not yet fully understood) were stopped in the walls of a discharge tube.[31] Sev-

[31]W. K. Roentgen, Sitzungsber, Wurzburger Phys.-Med. Ges., December 28, 1895; translated, Nature 53, 274 (1896).

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