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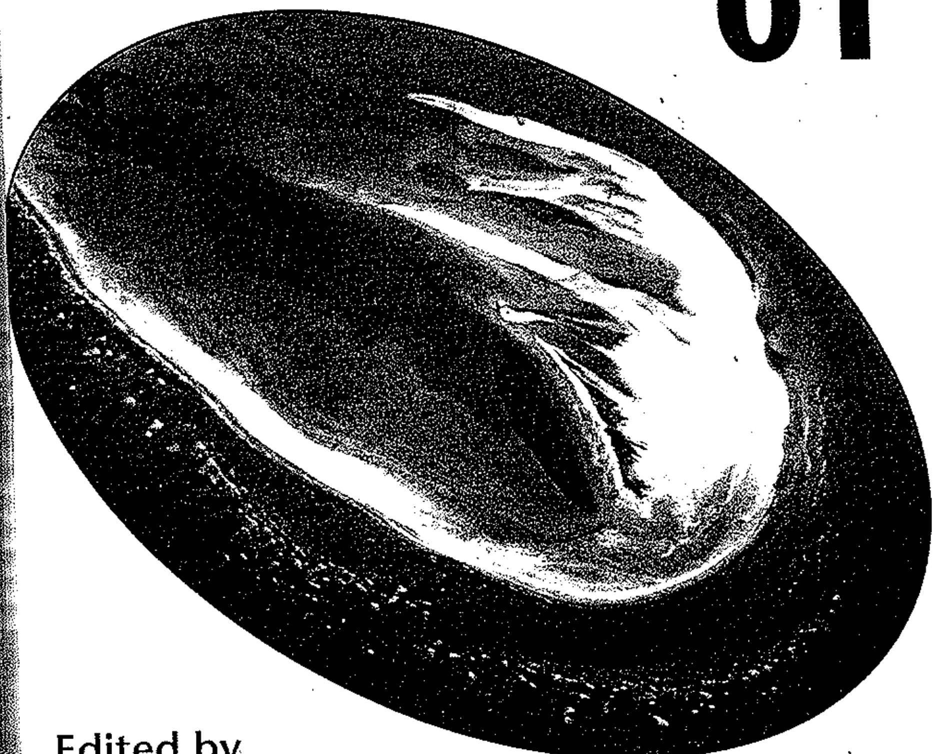

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SPH Modelling of Water Waves

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Abstract: Advances in computational power have permitted the use of Lagrangian particle methods to model fluids, particularly separated flows and free surface flows with splashing. Here we present a smoothed particle hydrodynamics (SPH) approach to a numerical wave tank, capable of studying several types of waves impinging on structures and walls.

INTRODUCTION

Computer modeling of the details of wave breaking and the turbulence in the surf zone can only be done by intricate computational models, including such techniques as Direct Numerical Simulation (DNS) of the Navier–Stokes equations, Large Eddy Simulation (LES), and Volume of Fluid (VOF). All of these methods are Eulerian and have complicated algorithms to determine the instantaneous location of the free surface. The situation becomes more difficult when splashing and air entrainment becomes important.

Recently, increases in computational power have led to the use of Lagrangian methods and, in particular, particle methods, which do not require any special techniques at the free surface or for rotational flows. For these methods, wave nonlinearities and wave breaking are naturally captured.

There are a variety of particle methods that have been developed—all involving applying Newton's Second Law to particles that represent large parcels of water molecules. One such method is Smoothed Particle Hydrodynamics (SPH), developed primarily by Lucy (1977) and Monaghan and colleagues over the last 30 years for astrophysics. More recently Monaghan (1994) has begun applying the technique to 2–D free surface flows, including the dam break problem, a bore, and waves shoaling on a beach. Monaghan and Kos (1999) have examined the run–up of a solitary wave (in

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the context of tsunami inundation) and, in follow–up work, they (Monaghan and Ko[illegible] 2000) have also looked in detail at the flow around the wavemaker in Scott Russell's original experimental apparatus for studying solitary waves in the laboratory.

A similar particle technique has been used by Koshizuka, Tomako and Oka (199[illegible]), Koshizuka, Nobe and Oka (1998), and Gotoh and Sakai (2000) to examine the run[illegible] of waves on a beach and the flow as the wave hits a seawall. Their technique is called the Moving Particle Semi–implicit (MPS) method. The major difference between the method and the SPH is that they solve the Poisson equation for the fluid pressure while in SPH an equation of state is used for the pressure–––that is, the fluid is treated as slightly compressible.

Here we discuss the development of a 2–D numerical wave tank utilizing SPH to study large amplitude waves and wave breaking.

METHODOLOGY

The numerical development of SPH schemes is based on a particle representation of field quantities. Particle representations are generally based on an *approx[illegible]* identity, which is typically expressed as:

$$\zeta(\boldsymbol{r})\approx\int_{r'}\zeta(\boldsymbol{r})\,W_\delta(\boldsymbol{r}-\boldsymbol{r}')\,d\boldsymbol{r}' \tag{1}$$

Here, $\zeta(\boldsymbol{r})$ is a generic field quantity, $\boldsymbol{r}$ is the position vector, δ is a nume[illegible] parameter called the core radius, coomparable to the particle size, while W [illegible] rapidly–decaying, radial smoothing function. The dependence of the smoothing function on the core radius is given by:

$$W_\delta(\boldsymbol{x})=\frac{1}{\delta^n}W\left(\frac{|\boldsymbol{x}|}{\delta}\right) \tag{2}$$

where n is the dimension of the space. Eq. 1 is an approximate identity since W [illegible] converges to the Dirac delta function, $\delta\,(r)$ in the sense of distributions as the [illegible] radius δ decreases to zero. In this limit one recovers the original function ζ and the equality in Eq. 1 is obtained.

The particle representation of $\zeta(\boldsymbol{r})$ is then obtained by approximating the integral in Eq. 1 using numerical quadrature with a cell size $h \lessdot \delta$, resulting in:

$$\zeta(\boldsymbol{r})=\sum_{i=1}^{N}\zeta_i h^n W_\delta(\boldsymbol{r}-\boldsymbol{r}_i) \tag{3}$$

where N is the total number of particles, ζ_i is the value of $\zeta(\boldsymbol{r}_i)$ for the i–th particle and $\boldsymbol{r}_i$ is the position of the i–th particle. In application, h is about 20% larger than the [illegible] of the computational particle radius.

Application of the SPH to free surface flows is based on using this particle

[illegible]tation scheme to represent the density and velocity fields. This is [illegible]plished using a distribution of Lagrangian particles that are described in terms of [illegible] positions $\boldsymbol{r}_i$, densities ρ_i, masses m_i, and velocities $\boldsymbol{v}_i$. The evolution of the [illegible] is then described in terms of a coupled system of ODEs that express the mass [illegible] momentum conservation laws. As discussed in Gingold and Monaghan (1977), Monaghan (1982), and Monaghan (1992), this system can be expressed as:

$$\begin{aligned}
&\frac{dm_i}{dt}=0\\
&\frac{d\boldsymbol{r}_i}{dt}=\boldsymbol{v}_i\\
&\frac{d\rho_i}{dt}=\sum_{j=1}^{n} m_j\left(\boldsymbol{v}_i-\boldsymbol{v}_j\right)\cdot\nabla_i W_\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)\\
&\frac{d\boldsymbol{v}_i}{dt}=-\sum_{j=1}^{N} m_j\left(\frac{p_i}{\rho_i^2}+\frac{p_j}{\rho_j^2}+P_{ij}\right)\nabla_i W_\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)+\boldsymbol{F}_i
\end{aligned} \tag{4}$$

[illegible] t is time, $\boldsymbol{P}_{ij}$ is the viscosity term, and $\boldsymbol{F}_i$ is a body force term (including gravity). [illegible] quantity p_i is the pressure at the i th particle. The last two equations represent the [illegible]vation of mass and the conservation of momentum equations in SPH format. [illegible] equations show the advantage of the kernel smoothing functions W as derivatives [illegible]ntities are found by using derivatives of the kernel function. The viscosity term [illegible]portant for viscous flows (see, e.g., Cleary and Monaghan, 1999); however, in the [illegible]ples below, it has been neglected.

[illegible] nearly incompressible free surface flows, the fluid pressure can be obtained from [illegible] equation of state (Batchelor, 1967; Monaghan, 1994):

$$p=B\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma}-1\right] \tag{5}$$

[illegible] B and γ are constants, and ρ_o is the density at the free surface; here we use [illegible] and ρ_o = 1000 kg/m^3. Monaghan uses $B= 200\ \rho\ g\ H\ /\ \gamma$, where H is a [illegible]resentative depth. This modified equation of state actually gives a sound speed ([illegible]$=dP/d\rho$) that is much slower than the actual speed of sound in the water. This [illegible]portant as the numerical time stepping is determined by the apparent speed of [illegible] in the fluid.

[illegible] kernel function used here is due to Monaghan and Lattanzio (1985):

$$W(r,h)=\frac{10}{7\pi h^2}\begin{pmatrix} 1-\frac{3}{2}(\frac{r}{h})^2+\frac{3}{4}(\frac{r}{h})^3\ ;0\le(\frac{r}{h})\le 1 \\ \frac{1}{4}\left(2-(\frac{r}{h})\right)^3\ ;1\le(\frac{r}{h})\le 2 \\ 0;2<(\frac{r}{h}) \end{pmatrix} \quad (6)$$

This spline kernel has continuous first and second derivatives. Notice that for dist[illegible] greater than 2 h, the kernel is zero. This means that the kernel has compact suppo[illegible] the interactions between particles are only important in the neighborhood of a [illegible] particle. These leads to immense computational time savings in computing the su[illegible] Eqn. 4. Further, Eqn 4 shows that for isolated particles, the equation of mo[illegible] forced simply by gravity.

NUMERICAL SIMULATION

The time stepping of the governing equations is done with a predictor–co[illegible] scheme utilized by Monaghan (1994). For an equation of the form:

$$\frac{dv}{dt}=-\gamma v+F \quad (7)$$

The predictor step for a midpoint calculation is:

$$v^{n+1/2}=\frac{v^n+0.5\,dt\,F^n}{1+0.5\,\gamma\,dt} \quad (8)$$

The corrector step gives

$$v^{n+1/2}=\frac{v^n+0.t\,dt\,F^{n+1/2}}{1+0.5\,\gamma\,dt} \quad (9)$$

and the final calculation for time level n is

$$v^{n+1}=2v^{n+1/2}-v^n \quad (10)$$

Other time–stepping schemes are possible (e.g. Fulk and Quinn, 1995)

Time Step size:

Monaghan (1992) provides several criteria for chosing the step size; one of w[illegible] that the time step is chosen to be less than the travel time of the speed of sound [illegible] medium to traverse the particle size h. Since the speed of sound is much faster th[illegible] particle speeds, this gives small time steps. For most problems, resolution [illegible] requires the use of many particles, this means that the size of h is small, dec[illegible] with the number of particles. So, the higher the resolution required, the small[illegible]

[illegible] step and the longer computation time. In our calculations, time steps are in the [illegible] of 10^{-4} s.

[illegible]ghbors list

[illegible] is crucial that the compact support of the kernel be recognized in the numerical [illegible] and link (or neighbor) lists be used. This is essentially a list maintained during [illegible] computations that keeps track of all the particles that are with 2 h of each other. [illegible] procedure for doing this is to divide the computational domain into square cells of [illegible] 2h. A linear array is maintained that refers to all the particles in a given square. [illegible] each particle, the numerical calculations in Eqn. 4 are carried out only over the [illegible] of particles located in the 9 cells centered near the particle position. This leads to [illegible] saving in computational time.

[illegible] symmetry in the kernels W_{ij} can be further exploited to effectively halve the [illegible] of computations.

[illegible]dary conditions

[illegible]naghan and colleagues have used several boundary conditions either to ensure slip [illegible] boundary or to have a no–slip condition. The first technique was to model the [illegible]rd–Jones forces between stationary boundary particles and the fluid (Monaghan, [illegible] but these radial forces produced a bumpy bottom with fluid particles bouncing [illegible] each boundary particle. Monaghan and Kos (1999) developed a second condition [illegible] smoothed the boundary repulsive force such that it was always acting normal to the [illegible]dary, regardless of the fluid particle position. Here we take a third approach, [illegible] is to establish several rows of boundary particles, which enter into the force [illegible]ctions but are not allowed to move. By experimentation we found that two rows [illegible]tionary particles on all boundaries was sufficient to contain the fluid. This [illegible]nique provides several advantages: there is no additional programming needed to [illegible] the boundaries and it is far easier to specify irregular boundaries by placing [illegible] rows of particles than to specify analytic boundaries as in Monaghan and Kos [illegible]

[illegible] application, this boundary condition provides a check on the time step. Should the [illegible] fail after a number of iterations, it is likely due to particles penetrating the [illegible]dary—implying a time step that is too large.

[illegible]MERICAL WAVETANK (2–D)

[illegible] SPH model that we have developed can be considered a two–dimensional wave [illegible] We have a variable left lateral boundary consisting of either a fixed wall or a [illegible]maker producing a variety of wave forms, including a solitary wave or a periodic [illegible]ain. The periodic wave is created by moving the wall back and forth with a [illegible]soidal velocity in piston or flap motion. The solitary wave is created by Goring's [illegible] (1978). The other end of the tank is an arbitrary slope beach. In between is a [illegible]horizontal bottom. Objects can be placed anywhere within the tank to represent [illegible] and the forces on these objects can be obtained from the fluid pressure.

RESULTS

Prior to the development of the numerical wave tank, a few tests were carried o[illegible] ensure that we could replicate Monaghan's (1994) results. These tests include[illegible] dam break problem (along with a triangular jump on the bed) and wave breaking [illegible] slope. In addition, we performed a variety of test cases, such as dropping colum[illegible] fluid and examining the sloshing of a closed rectangular tank.

For the dropping fluid column, several cases were carried out. First, dis[illegible] columns of liquid were dropped onto the bottom boundary to determine the numb[illegible] fixed rows of particles necessary in the boundary. Interestingly, when the numb[illegible] particles comprising the column is large, the behavior of the falling fluid co[illegible] impinging on the bottom is realistic; however, when the number of particles is sm[illegible] including the case of a single particle, the falling particles bounce elastically from [illegible] bottom boundary! This is the continuum hypothesis at work.

Containers onboard ships can be excited into resonant sloshing modes by w[illegible] action. Faltinsen *et al.* (2000) develop an analytical eigenfunction approac[illegible] nonlinear sloshing of rectangular containers. Their experiments included such [illegible] amplitude sloshing that the fluid impinged on the roof of the container; however [illegible] analytical model did not include the presence of the roof. Figure 1 shows our SP[illegible] approach to the problem. The forcing of the tank is accomplished, not by moving [illegible] tank, but by fixing the coordinate system with the tank and modifying the accelera[illegible] of gravity to either include horizontal accelerations associated with a periodic slidin[illegible] the tank horizontally or by rotating the gravitational vector in a pendulum like mo[illegible] which mimics the rolling of a ship.

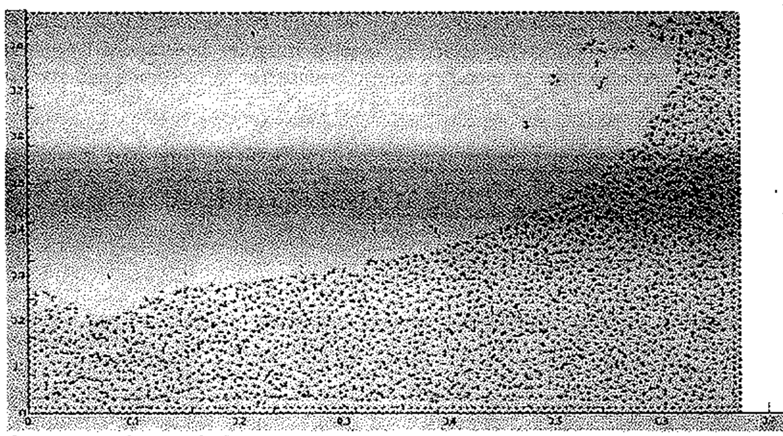
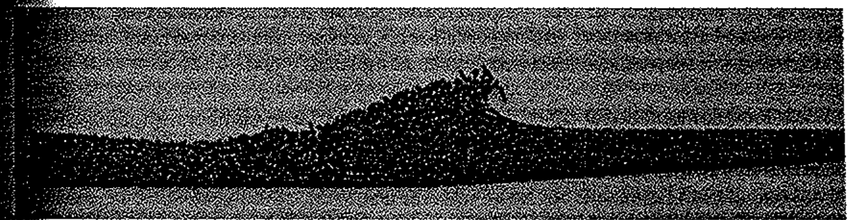


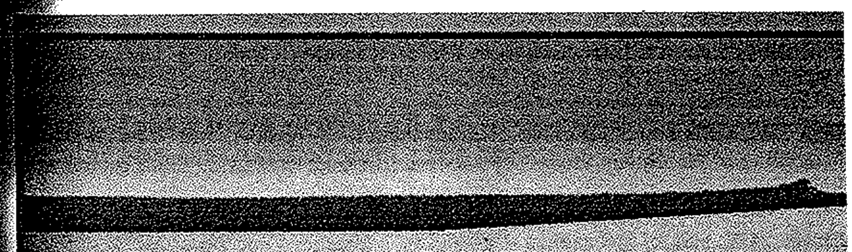
Figure 1Nonlinear sloshing in a tank.

[illegible]e breaking of a solitary wave is shown in Figure 2. This wave was initialized in [illegible] basin using the velocities and displacements of a solitary wave; however, its initial [illegible] height was greater than the stable wave height for that depth. Within a few [illegible]nds, the initially symmetric wave form deformed into the plunging breaker shown [illegible] figure. Further computations, after plunger touch–down shows the development [illegible]urbulent bore.

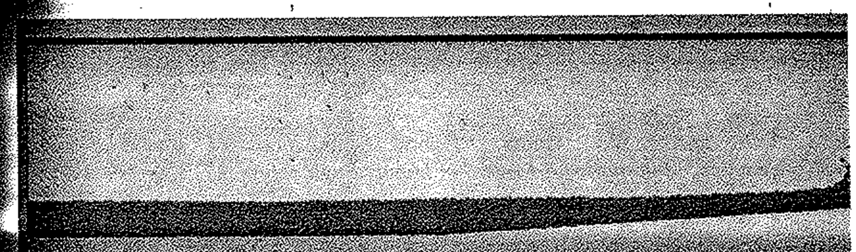


[illegible]e 2Breaking solitary wave

[illegible]e last two figures (3 and 4) show the propagation of a solitary wave generated by [illegible]ng's wavemaker approach. This wave propagates on a flat bottom section prior to [illegible]ing at a sloping beach (1/19.85). At the end of the tank, the wave impacts and [illegible]up on the end wall, with splashing. This models the laboratory experiments [illegible]ed out by Li, Raichlen and Lee (2000), who were examining the splash–up at the [illegible]



[illegible]re 3Solitary wave on beach (right side)



[illegible]re 4Splash-up of solitary wave on a wall (at right)

CONCLUSIONS

Particle methods are useful in predicting wave motions, particularly for the case where conventional models fail, such as when wave splash occurs or within turbulent bores after wave breaking.

There are a variety of issues still to be addressed on the topic of SPH modeling. The appropriate sound speeds, fluid viscosity, and particle number densities are all issues to be addressed by numerical experimentation. Quantititative comparisons with a wide range of laboratory experiments needs to be completed.

The long computational times, associated with the large numbers of particle needed for high resolution, dictate the use of parallel computing and large computing clusters due to the immense amount of computation required. However, it is likely in the future that computation times will not be an impediment.

The numerical wave tank provides a useful tool for testing when two–dimensional studies are sufficient.

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