DIGITAL COMMUNICATION

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QUANTIZATION

Non-Uniform Quantization
Robust Quantization
Companding

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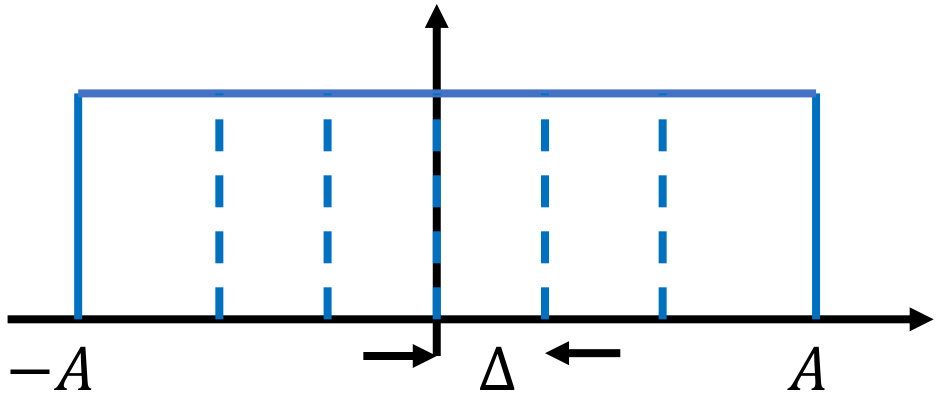
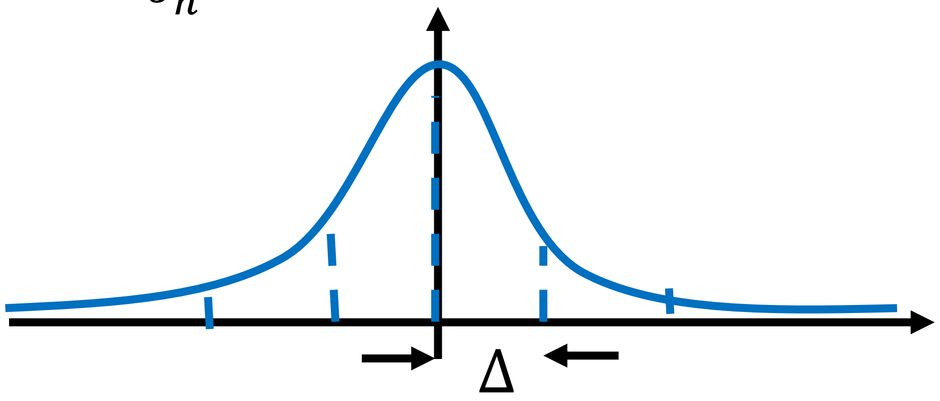
NON-UNIFORM QUANTIZATION

Motivation

- The SNR (SQNR) of a quantizer is defined as $\mathrm{SNR} = \sigma_X^2 / \sigma_n^2$





- In uniform pdf the probability of the input signal being in any level remains the same as the area of each interval remains constant
- In Gaussian pdf (for example), the probability of occurrence of one level is higher than the other levels
- Recall that for Gaussian PDF, $A = 4\sigma$. Therefore, $L/_2$ number of levels lie in $(-2\sigma, 2\sigma)$
- But, the probability of occurrence of samples in $(-2\sigma, 2\sigma)$ is about 0.95

NON-UNIFORM QUANTIZATION

Motivation

- With uniform quantization for input signal with uniform PDF, all the levels are equally likely to occur
- But for the Gaussian input the levels around the mean are much more likely to occur than the levels at the extremes
- Speech signal can be considered to have a Gaussian PDF
- About half of the quantization values are wasted when the input is non-uniform
- This necessitates for the design of a non-uniform quantization
- The number of levels are same $L = 2^N$, but the step sizes are not equal
- Recall that for a discrete random variable Y

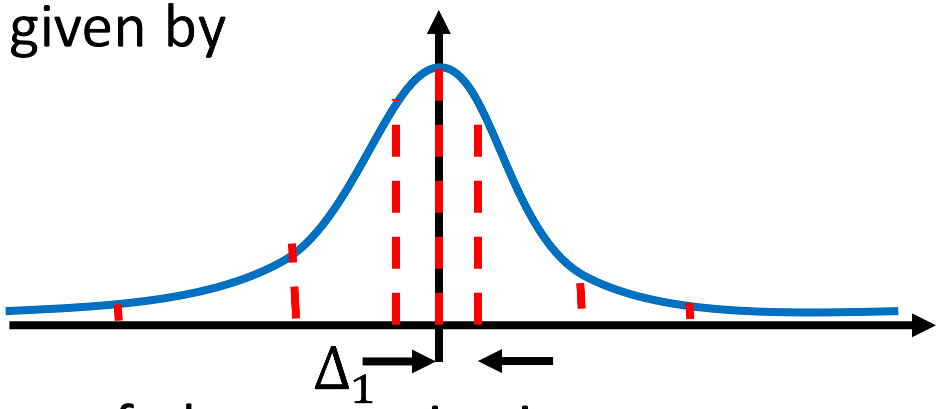
$$\mathbb{E}Y = \sum_{k=0}^{L-1} y\, p_Y(y)\,, \qquad \mathbb{E}\big(g(Y)\big) = \sum_{k=0}^{L-1} g(y) p_Y(y)$$

NON-UNIFORM QUANTIZATION

Motivation

- Let the width of k^{th} level be given by Δ_k
- The probability of input signal being in the k^{th} interval is given by

$$p_k = \int_{b_k}^{b_{k+1}} f_X(x)\ dx$$



- Assuming that N is large, we can consider the variance of the quantization error in the k^{th} interval is given by ${\Delta_k^2}/{12}$
- Therefore, the average quantization error across all levels can be calculated (from the property of expectation) as

$$\sigma_Q^2 = \sum_{k=0}^{L-1} p_k \left(\frac{\Delta_k^2}{12}\right)$$

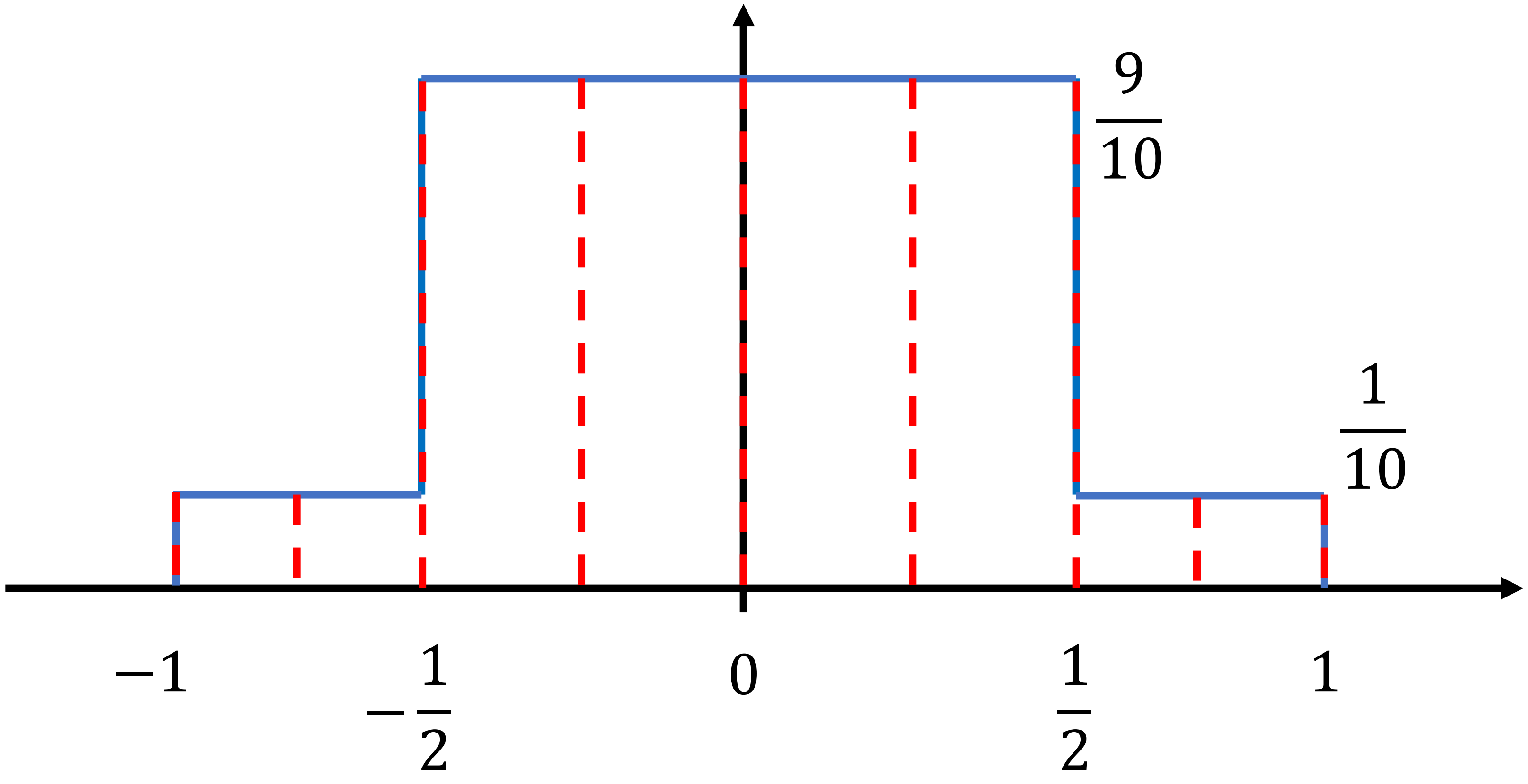
For uniform quantizer, $\Delta_k = \Delta$

$$\sigma_Q^2 = {\Delta^2}/{12}$$

NON-UNIFORM QUANTIZATION

Problem 1

For the given PDF with following details, design a 3-bit uniform quantizer.



NON-UNIFORM QUANTIZATION

Problem 1

For the given PDF with following details, design a 3-bit uniform quantizer.

$$\Delta = \frac{2A}{2^N} = \frac{2}{8} = 0.25$$

$$\sigma_x^2 = \int_{-1/2}^{1/2} x^2\left(\frac{9}{10}\right)dx + 2\int_{1/2}^{1} x^2\frac{1}{10}\,dx$$

$$= 2\cdot\frac{9}{10}\,\frac{x^3}{3}\Bigg|_0^{1/2} + \frac{2}{10}\,\frac{x^3}{3}\Bigg|_{1/2}^{1}$$

$$= \frac{3}{40} + \frac{7}{120}$$

$$\sigma_x^2 = \frac{2}{15}$$

$$SNR = \frac{\sigma_x^2}{\sigma_Q^2}$$

$$= \frac{\frac{2}{15}}{\Delta^2/12} = \frac{2/15}{0.25^2/12}$$

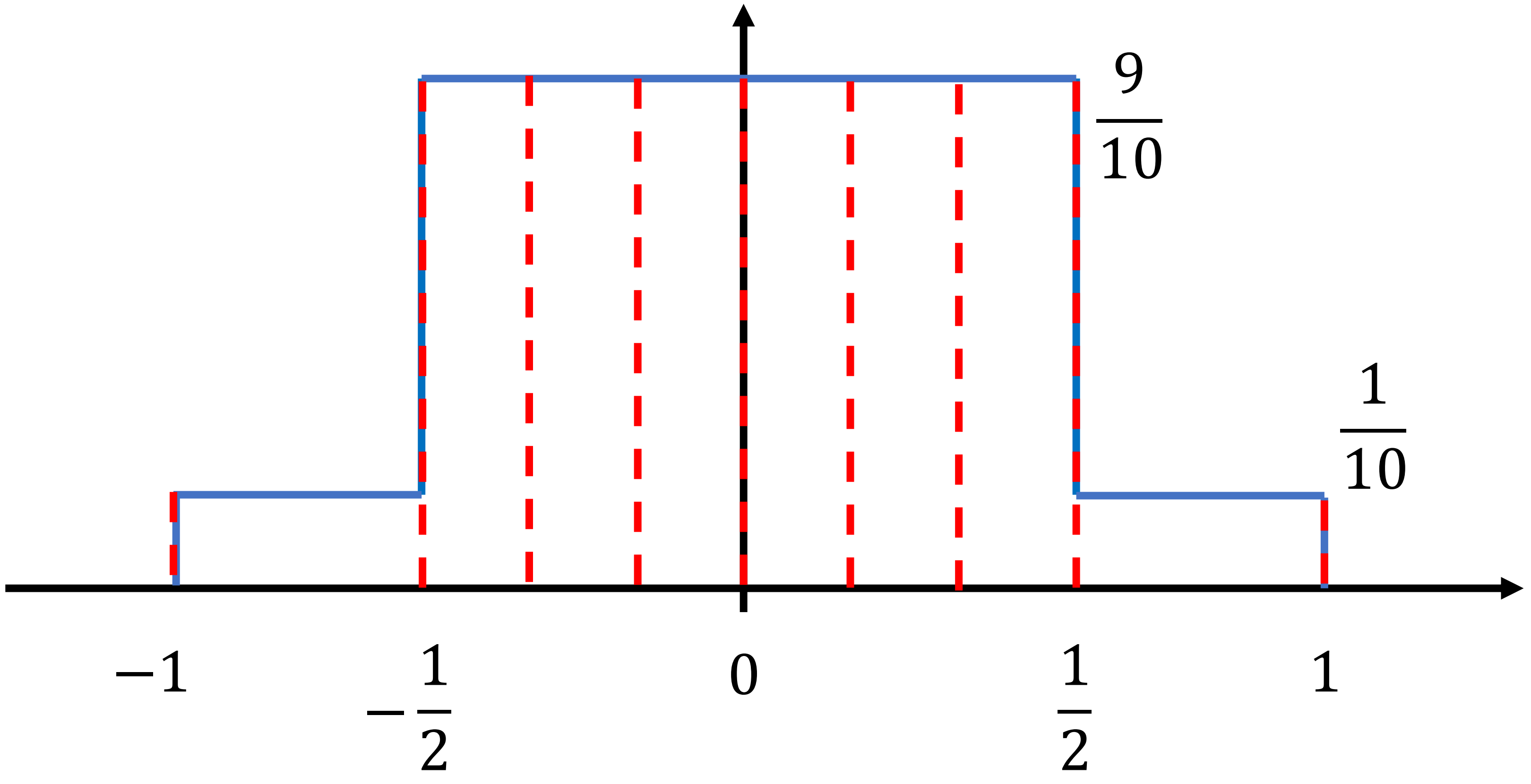
$$\boxed{SNR = 25.6}$$

$$\boxed{SNR_{dB} = 14.08\,dB}$$

NON-UNIFORM QUANTIZATION

Problem 2

For the given PDF with following details, design a 3-bit non-uniform quantizer.



NON-UNIFORM QUANTIZATION

Problem 1

For the given PDF with following details, design a 3-bit uniform quantizer.

$$\sigma_Q^2 = \sum_{k=0}^{7} p_k \frac{\Delta_k^2}{12}$$

for $k = 1$ to 6

$$p_k = \frac{9}{10} \times \frac{1}{6} = \frac{3}{20} \qquad \Delta_k = \frac{1}{6}$$

for $k = 0$ & $k = 7$

$$p_k = \frac{1}{20} \qquad \Delta_k = 0.5$$

$$\sigma_Q^2 = 6 \times \frac{3}{20} \times \frac{1}{36} \times \frac{1}{12} + 2 \times \frac{1}{20} \times 0.25 \times \frac{1}{12}$$

$$\sigma_Q^2 = \frac{1}{240}$$

$$SNR = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{2/15}{1/240}$$

$$\boxed{SNR = 32}$$

$$\boxed{SNR_{dB} = 15.05}$$

ROBUST QUANTIZATION

Definition

- Human perceptual considerations requisites the SNR to be constant across the different input power levels
- This means more levels with smaller step size have to be provided when signal power is low and the fewer levels with larger step size when signal power is high
- Such a quantization scheme in which the SNR is almost same across the different input power levels is called **Robust Quantization** **(a non-uniform quantization)**
- The step sizes are chosen to make SNR almost same across all power levels
- In practice, we first perform a non-linear transformation of the input signal and then apply a uniform quantizer. This transformation is called Compression
- At the receiver we perform the inverse transformation called Expansion
- Together, this process is called companding

THANK YOU

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