Principles of Digital Signal Processing

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DSP

RELATIONSHIP OF DFT WITH OTHER TRANSFORM

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Frequency domain sampling

DFT

To summrise DFT and IDFT

$$\text{DFT:}\quad X(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \qquad k=0,1,2,\ldots,N-1$$

$$\text{IDFT:}\quad x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \qquad n=0,1,2,\ldots,N-1$$

Relationship of DFT with other transform

Relationship to the Fourier series coefficients of a periodic sequence.

Relationship to the Fourier transform of an aperiodic sequence.

Relationship to the z-transform.

Relationship to the Fourier series coefficients of a continuous-time signal.

Relationship of DFT with other transform

Relationship to the Fourier series coefficients of a periodic sequence.

$$x_p(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi nk/N} \qquad -\infty < n < \infty$$

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi nk/N} \qquad k = 0, 1, \ldots, N-1$$

DFT of $x(n) = x_p(n),\ 0 \le n \le N-1$

$$X(k) = N c_k$$

Relationship of DFT with other transform

Relationship to the Fourier transform of an aperiodic sequence.

$$X(k) = X(\omega)|_{\omega=2\pi k/N} = \sum_{n=-\infty}^{\infty} x(n)e^{-j2\pi nk/N} \qquad k = 0, 1, \ldots N\text{-}1$$

DFT coefficients
of periodic seq
of period N

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$$

Frequency domain sampling

Relationship of DFT with other transform

$$\hat{x}(n) = \begin{cases} x_p(n), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = \hat{x}(n) \qquad 0 \le n \le N-1$$

Only when x(n) is of finite duration

Frequency domain sampling

Relationship of DFT with other transform

Relationship to the z-transform.

x(n) having z– transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

X(z) sampled at N equally spaced points on the unit circle

$z_k = e^{j2\pi k/N}$

$$X(k) = X(z)|_{z=e^{j2\pi nk/N}}, \qquad k = 0, 1, \ldots, N-1$$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi nk/N}$$

Frequency domain sampling

Relationship of DFT with other transform

- If x(n) has a finite duration of length N or less the sequence can be recovered from its N–point DFT.
- Hence, its z–transform is uniquely determined by it s N–point DFT.

Frequency domain sampling

Relationship of DFT with other transform

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$$

$$X(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \right] z^{-n}$$

Relationship of DFT with other transform

$$X(z) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} \left(e^{j2\pi k/N} z^{-1}\right)^n$$

$$X(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1 - e^{j2\pi k/N} z^{-1}}$$

Polynomial interpolation formula for X(ω)

$$X(\omega) = \frac{1 - e^{-j\omega N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1 - e^{-j(\omega - 2\pi k/N)}}$$

Relationship of DFT with other transform

Relationship to the Fourier series coefficients of a continuous-time signal.

Continuous time periodic signal with fundamental period

$T_p = 1/F_0$

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0}$$

Fourier series

Sample $x_a(t)$ at a uniform rate $Fs = N/Tp = 1/T$

Frequency domain sampling

Relationship of DFT with other transform

Continuous time periodic signal

The discrete time sequence:

$$x(n) \equiv x_a(nT) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 n T} = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kn/N}$$

$$= \sum_{k=0}^{N-1} \left[\sum_{l=-\infty}^{\infty} c_{k-lN} \right] e^{j2\pi kn/N}$$

DFT formula

Frequency domain sampling

Relationship of DFT with other transform

$$X(k) = N \sum_{l=-\infty}^{\infty} c_{k-lN} \equiv N\tilde{c}_k$$

$$\tilde{c}_k = \sum_{l=-\infty}^{\infty} c_{k-lN}$$

Aliased version of the sequence $\{c_k\}$

THANK YOU

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