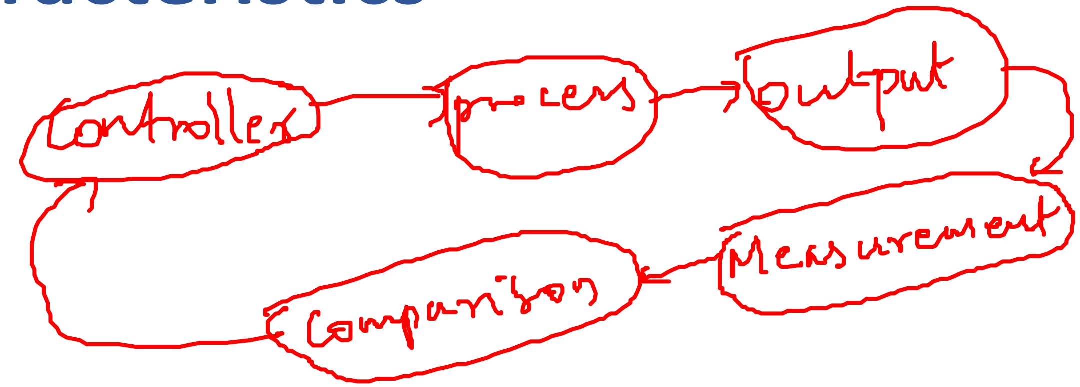
CONTROL SYSTEMS

Karpagavalli S.
Department of Electronics and Communication Engineering

CONTROL SYSTEMS

Feedback Control System Characteristics



Karpagavalli S.

Department of Electronics and Communication Engineering

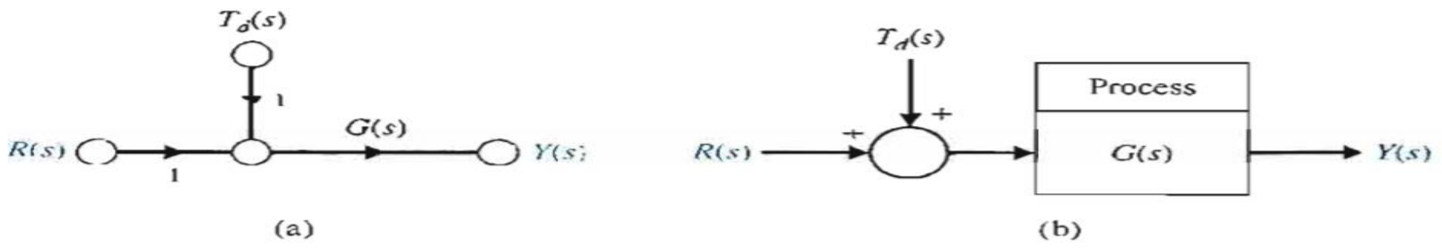
Feedback - Introduction

- **Feedback** as a means of automatic regulation and control is inherent in nature.
- It can be seen in many physical, biological and soft systems.
- Ex, body temperature of any living being is automatically regulated through a process
- There are 2 types of systems
  - Open loop systems
  - Closed loop (feedback) systems

Types of control systems

- Open loop systems(non-feedback)

These are non-feedback control systems. In this type of system, sensing of the actual output and comparing of this output (through feedback) with the desired input does not take place.

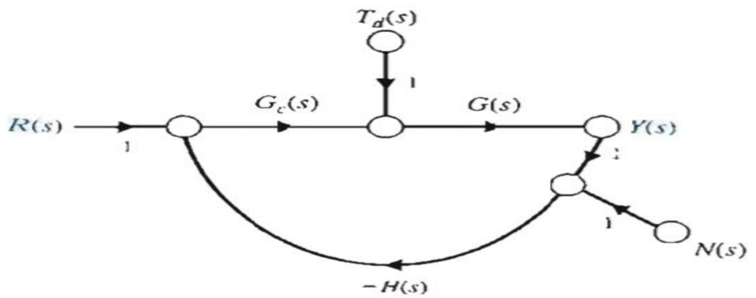


Td(s) is the disturbance signal.

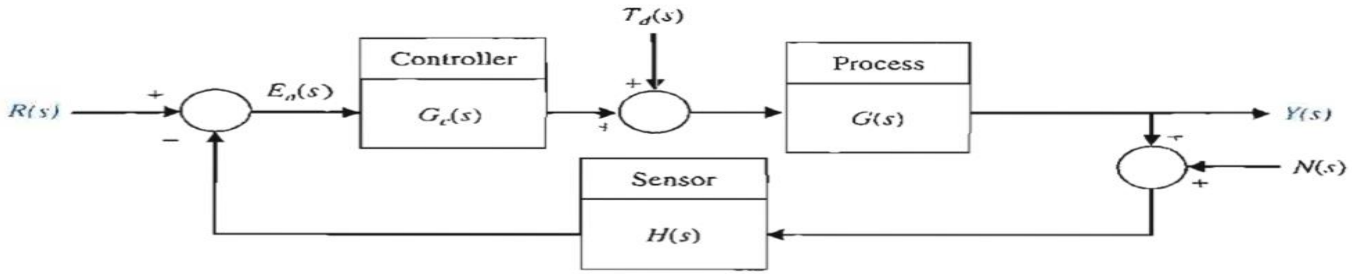
FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Types of control systems

- Closed Loop systems : Systems with feedback



(a)



(b)

N(s) is the unwanted noise signal.

Types of control systems

Advantages of Closed Loop:

- Decreased sensitivity of the system to parameter variations
- Improved rejection of the disturbances
- Improved measurement noise attenuation
- Improved reduction of the steady state error of the system
- Easy control and adjustment of the transient response of the system

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

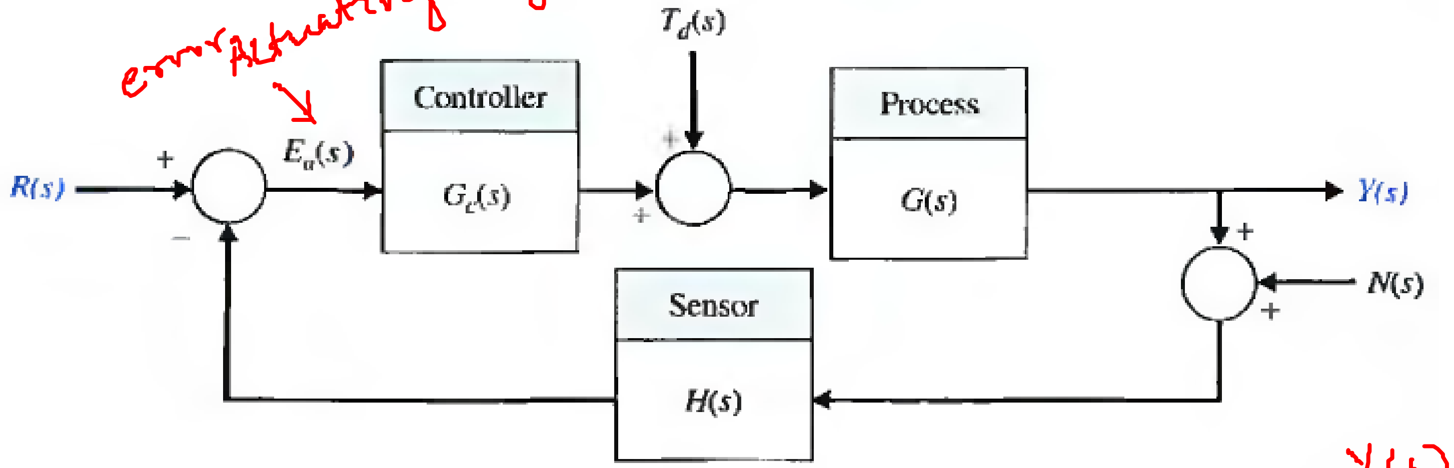
Closed Loop System: Error Signal Analysis

- Error Signal Analysis: The closed-loop feedback control system shown has three inputs— R(s), Td(s), and N(s)—and one output, Y(s). The signals Td(s) and N(s) are the disturbance and measurement noise signals, respectively.
- tracking error as

$$E(s) = R(s) - Y(s)$$

- For ease of discussion, consider a unity feedback system, that is, H(s) = 1 .

error Actuating signal



A unity feedback systems

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} \cdot R(s)$$

$$Y(s) = \frac{G(s)}{1 + G(s)G_c(s)} \cdot T_d(s)$$

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Types of control systems

By using principle of superposition ,we can calculate the output due to each input separately and add them to give the equation for total output.

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s).$$

Since, $E(s) = Y(s) - R(s)$

$$E(s) = \frac{1}{1 + G_c(s)G(s)}R(s) - \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) + \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s).$$

Let $L(s) = G_c(s)G(s).$ as Loop gain

S(s) C(s)

$$E(s) = \frac{1}{1 + L(s)}R(s) - \frac{G(s)}{1 + L(s)}T_d(s) + \frac{L(s)}{1 + L(s)}N(s).$$

Error Signal Analysis

- Define sensitivity function S(s) and complementary sensitivity function C(s) as

  S(s)= 1/(1+L(s)) and C(s)=L(s)/(1+L(s))

$$, L(s) = G_c(s)G(s)$$

$$S(s) = \frac{1}{1+L(s)}$$

$$C(s) = \frac{L(s)}{1+L(s)}$$

- Then the error function in terms of S(s) and C(s) is,

  E(s) = S(s)R(s) - S(s)G(s)Td(s) + C(s)N(s)

- The relationship between S(s) and C(s) is

  S(s) + C(s)=1

- For a given G(s), if we want to minimize the tracking error, both S(s) and C(s) to be small. Remember that S(s) and C(s) are both functions of the controller, $G_c(s)$ which the design engineer must select. We cannot simultaneously make S(s) and C(s) small. Obviously, design compromises must be made.

Error Signal Analysis

- To reduce $T_d(s)$ effect on E(s), L(s) has to be made large over the range of frequencies that characterize the disturbances.
- To attenuate N(s), L(s) has to be made small over the range of frequencies.
- But, we cannot simultaneously make S(s) and C(s) small. Obviously, design compromises must be made.
- Solution: During design phase, the loop gain L(s) made large at low frequencies (associated with range of frequency of disturbance) and L(s) made small at high frequencies (associated with measurement noise).

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Sensitivity of Control System to Parameter Variations

Karpagavalli S.

Department of Electronics and Communication Engineering

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Sensitivity of Control System to Parameter Variations

- The sensitivity of a control system to parameter variations is of prime importance.
- A primary advantage of a closed-loop feedback control system is its ability to reduce the system's sensitivity.
- For the closed-loop case, if $G_c(s)G(s) \gg 1$ for all complex frequencies of interest Then ,

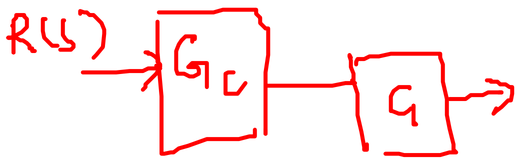
$$Y(s) = R(s)$$

- The output is approximately equal to the input.
- However, the condition $G_c(s)G(s) >> 1$ may cause the system response to be highly oscillatory and even unstable. But the fact that **increasing the magnitude of the loop gain reduces the effect of G(s) on the output ,is an important result.**

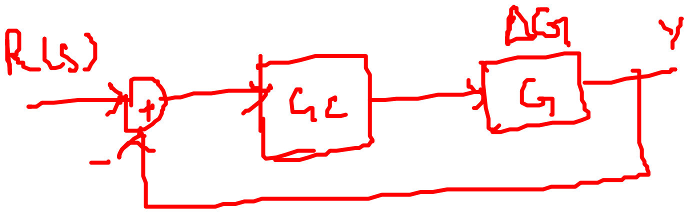
FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Sensitivity of Control System to Parameter Variations

- To illustrate the effect of parameter variations,
- Suppose the process G(s) undergoes a change such that the model is G(s) + ΔG(s). The change is due to changes in the external environment or natural aging or it may represent the uncertainty in certain plant parameters.
- For the open loop case, the change in the transfer function of the output



$$Y(s) = [G(s) + \Delta G(s)]\, G_c(s) \cdot R(s)$$



$$Y(s) = \frac{G_c(s)\,(G(s) + \Delta G(s))}{1 + G_c(s)\,(G(s) + \Delta G(s))} \cdot R(s)$$

Sensitivity of Control System to Parameter Variations

- Suppose the process G(s) undergoes a change such that the model is G(s) + ΔG(s). Then the error signal becomes,

$$E(s) + \Delta E(s) = \frac{1}{1 + G_c(s)(G(s) + \Delta G(s))} R(s).$$

Then the tracking error

$$\Delta E(s) = \frac{-G_c(s)\,\Delta G(s)}{(1 + G_c(s)G(s) + G_c(s)\,\Delta G(s))(1 + G_c(s)G(s))} R(s).$$

Since we usually find that $G_c(s)G(s) \gg G_c(s)\,\Delta G(s)$, we have

$$\Delta E(s) \approx \frac{-G_c(s)\,\Delta G(s)}{(1 + L(s))^2} R(s).$$

the change in the tracking error is reduced by the factor 1 +L(s)

Sensitivity of Control System to Parameter Variations

- For large L(s), we have 1 + L(s) ~ L(s), and we can approximate the change in the tracking error by

$$\Delta E(s) \approx -\frac{1}{L(s)} \frac{\Delta G(s)}{G(s)} R(s).$$

- Larger magnitude L(s) translates into smaller changes in the tracking error (that is, reduced sensitivity to changes in ΔG(s) in the process). Also, larger L(s) implies smaller sensitivity, S(s) $= \frac{1}{1+L(s)}$

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Sensitivity of Control System to Parameter Variations

- **System sensitivity** is defined as the ratio of the percentage change in the system transfer function to the percentage change in process transfer function.

G(s)

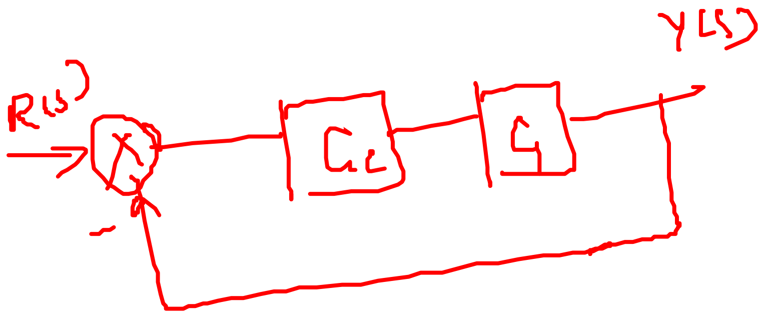
T(s)

- The system transfer function is
- $$T(s)= Y(s)/R(s)$$
- therefore the sensitivity is defined as

$$S= (\Delta T(s)/T(s))/(\Delta G(s)/G(s))$$

- In the limit, for small incremental changes,

$$\boxed{S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}.}$$



- For a closed loop system T(s),

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}. = \frac{Y(s)}{R(s)}$$

Sensitivity of Control System to Parameter Variations

Therefore, the sensitivity of the feedback system is

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{G_c}{(1 + G_cG)^2} \cdot \frac{G}{GG_c/(1 + G_cG)}$$

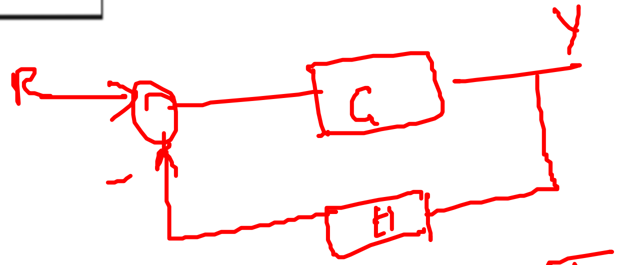
or

$$\boxed{S_G^T = \frac{1}{1 + G_c(s)G(s)}.}$$

$$T = \frac{G_cG}{1 + G_cG} = \frac{u}{v}$$

$$\frac{\partial T}{\partial G} = \frac{(1+G_cG)G_c - G_cG_cG}{(1+G_cG)^2}$$

$$= \frac{G_c + \cancel{G_c^2G} - \cancel{G_c^2G}}{(1+G_cG)^2}$$



$$T = \frac{G}{1+GH}$$

$$S_H^T = \frac{\partial T}{\partial H} \cdot \frac{H}{T}$$

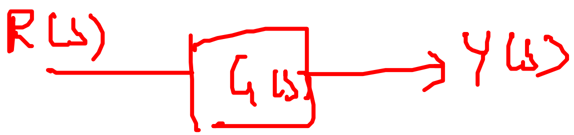
$$\frac{\partial T}{\partial H} \cdot \frac{H}{T} = \cancel{G} \cdot \frac{-G}{(1+GH)^2} \cdot \frac{H}{\frac{G}{1+GH}} = \frac{-GH}{1+GH}$$

Sensitivity of Control System to Parameter Variations

Sensitivity of Open Loop Control System,

T(s) = G(s)

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T} = \frac{G}{T} = \frac{G}{G} = 1$$



$$T(s) = \frac{Y(s)}{R(s)} = G(s)$$

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Sensitivity of Control System to Parameter Variations

- A closed loop system is used to track the sun to obtain maximum power from a photovoltaic array. The tracking system may be represented by

$G(s) = \frac{100}{\tau s+1}$ where $\tau = 3\ seconds$ nominally , $H(s) = 1$

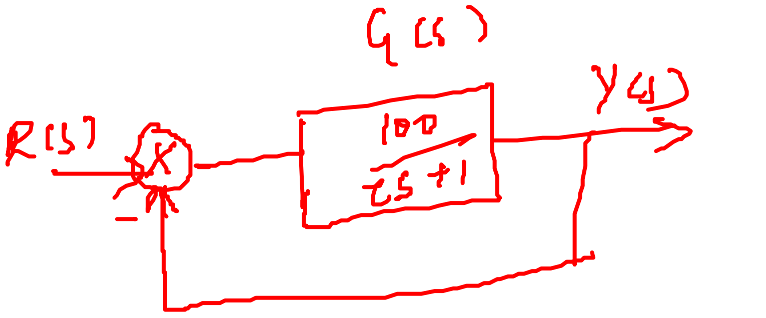
- a) Calculate the sensitivity of this system for a small change in τ
- b) Calculate the time constant of the closed loop system response

Sol: a) $S^T_\tau = S^T_G \, S^G_\tau$

$$\frac{\partial T}{\partial \tau} \cdot \frac{\tau}{T} = \frac{\partial T}{\partial G} \cdot \frac{G}{T} \cdot \frac{\partial G}{\partial \tau} \cdot \frac{\tau}{G}$$

$$S^T_G = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{1}{1+GH} = \frac{1}{1+\frac{100}{\tau s+1}}$$

$$= \frac{\tau s+1}{\tau s+101}$$



$$T(s) = \frac{Y(s)}{R(s)} = \frac{G}{1+GH} = \frac{100/\tau s+1}{1+100/\tau s+1}$$

$$T(s) = \frac{100}{\tau s+101}$$

Sensitivity of Control System to Parameter Variations

$$S_{\tau}^{G} = \frac{\partial G}{\partial \tau} \cdot \frac{\tau}{G} = \frac{-\tau s}{\tau s + 1}$$

$$S_{\tau}^{T} = \frac{3s+1}{3s+101} \cdot \frac{-3s}{3s+1} = \frac{-3s}{3s+101}$$

b) $$T(s) = \frac{100}{\tau s + 101} = \frac{100/101}{\frac{\tau}{101} s + 1}$$ ← Time constant form

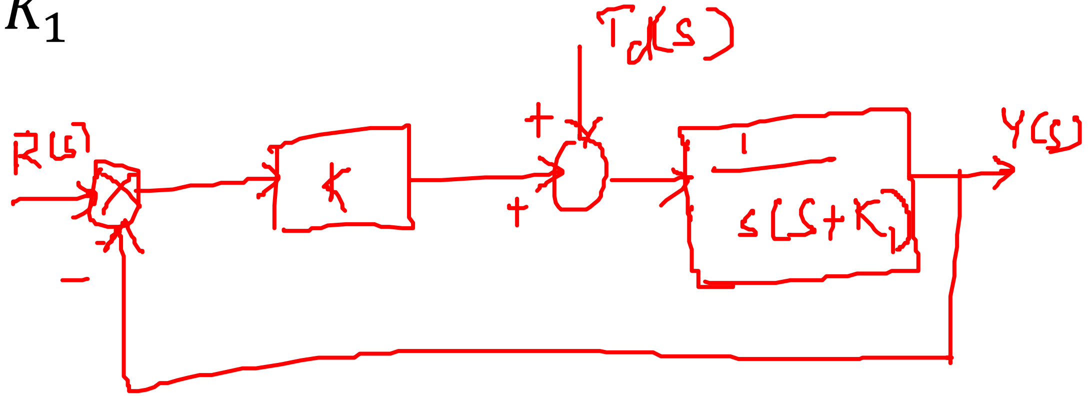
$$\tau_c = \frac{\tau}{101} = \frac{3}{101} = 0.0293 \text{ sec.}$$

time constant of closed loop system

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Sensitivity of Control System to Parameter Variations

- Consider the unity feedback system shown. The system has two parameters, the controller gain K and the constant K_1 in the process.
- A) calculate the sensitivity of CLTF to changes in K_1



Sol: To find CLTF, $T_d(s) = 0$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{K}{s(s+K_1)}}{1 + \frac{K}{s(s+K_1)}}$$

$$= \frac{K}{s^2 + K_1 s + K}$$

$$S^T_{K_1} = \frac{\partial T}{\partial K_1} \cdot \frac{K_1}{T} = \frac{-\cancel{K} \cdot s}{(s^2+K_1 s+K)^{\cancel{2}}} \cdot \frac{K_1}{\frac{\cancel{K}}{\cancel{s^2+K_1 s+K}}} = \frac{-s K_1}{s^2 + K_1 s + K}$$

make K as large as possible, to lose $S^T_{K_1}$

Sensitivity of Control System to Parameter Variations

- Consider the unity feedback system shown. The system has two parameters, the controller gain K = 120 and the constant $K_1 = 10$ in the process.
- A) calculate the steady state error of the closed loop system due to a unit step input with $T_d(s) = 0$
- A) calculate the steady state error of the closed loop system due to a unit step input with $T_d(s) = 1/s$

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Disturbance Signals in a Feedback Control System

Karpagavalli S.

Department of Electronics and Communication Engineering

Disturbance Signals in a Feedback Control System

- A disturbance signal is an unwanted input signal that affects the output signal.
- Example for disturbance and noise
  - Electronic amplifiers have inherent noise generated within the integrated circuits or transistors
  - radar antennas are subjected to wind gusts, and many systems generate unwanted distortion signals due to nonlinear elements.
- The benefit of feedback systems is that the effect of distortion, noise, and unwanted disturbances can be effectively reduced.

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

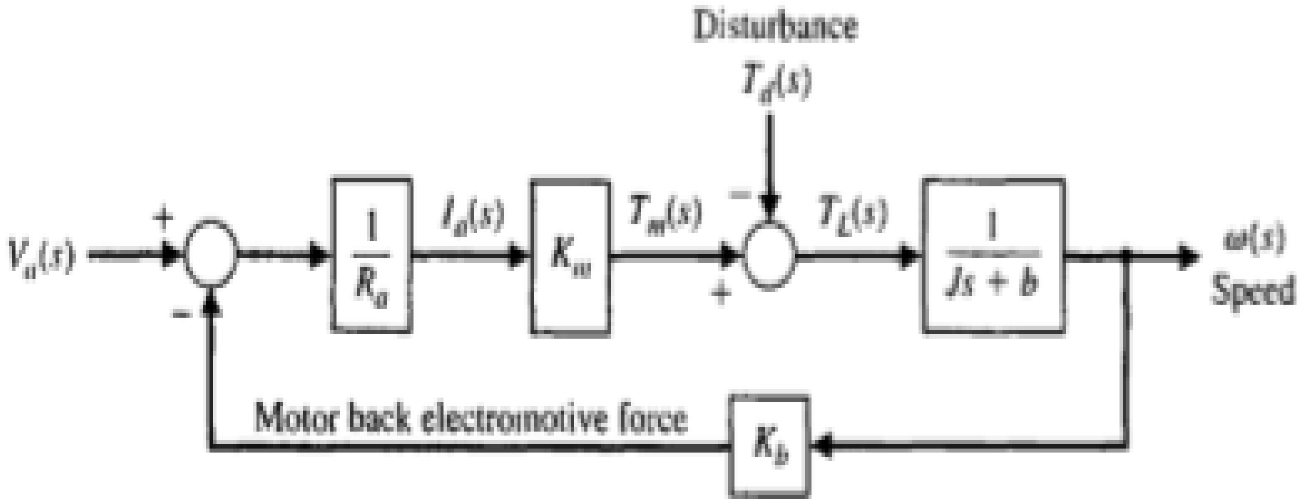
Disturbance Signals in a Feedback Control System

- For Ex, Steel rolling mill speed control system, Disturbance Rejection: when R(s) = 0 and N(s) = 0
- Open loop system

$$E(s) = R(s) - \omega(s)$$

$$= -\omega(s)$$

$$= \frac{1}{(Js+b) + \frac{K_m K_b}{R_a}} \cdot T_d(s)$$



$$\frac{\omega(s)}{T_d(s)} = -\frac{1}{(Js+b) + \frac{K_m K_L}{R_a}}$$

Disturbance Signals in a Feedback Control System

- Disturbance Rejection: when R(s) = 0 and N(s) = 0

Steady state error: $T_D(s) = D/s$

according to final value Theorem

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{(Js+B) + \frac{K_m K_b}{R_a}} \cdot \frac{D}{\cancel{s}}$$

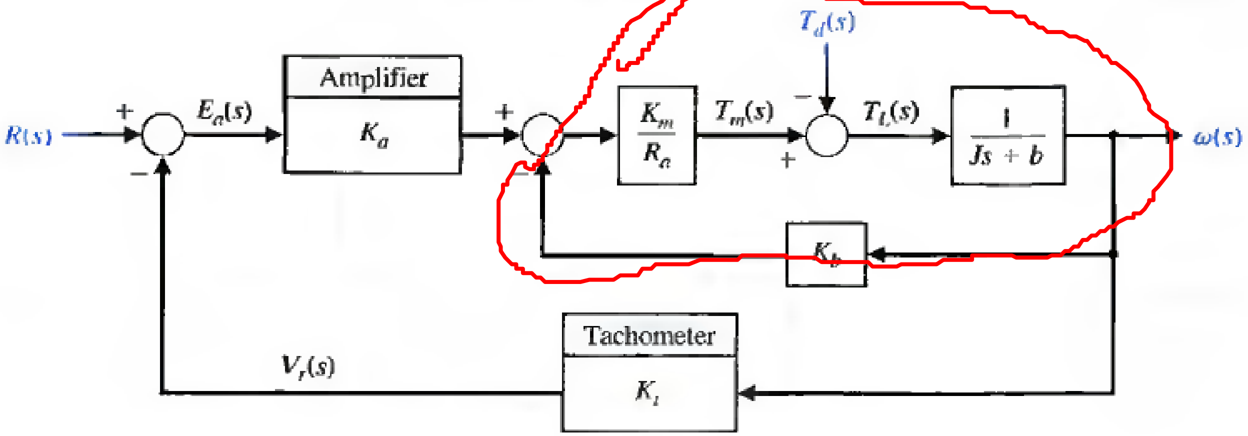
$$e_{ss} = \frac{D}{B + \frac{K_m K_b}{R_a}} = -\omega_0(\infty)$$

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Disturbance Signals in a Feedback Control System

- Disturbance Rejection: For Closed loop system ,when R(s) = 0 and N(s) = 0

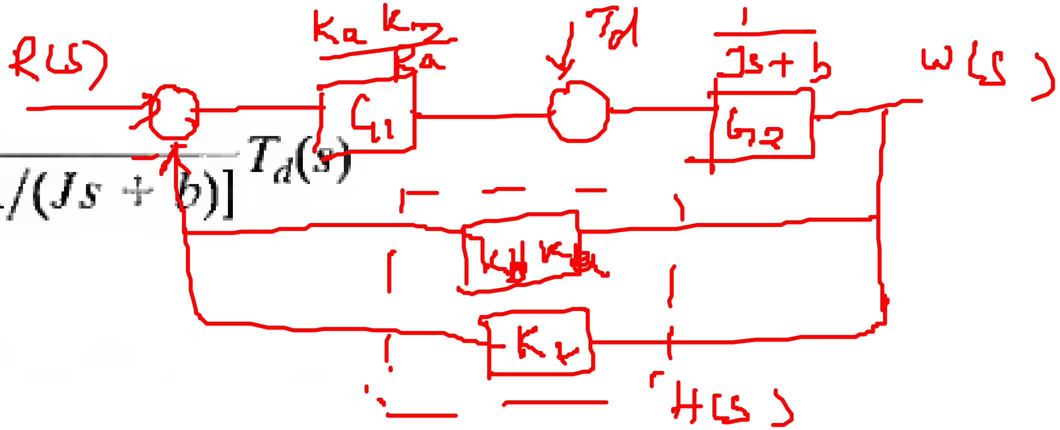
$E(s) = R(s) - \omega(s) = -\omega(s)$



$$G_1(s)H(s) = \frac{K_aK_m}{R_a}\left(K_t + \frac{K_b}{K_a}\right) \approx \frac{K_aK_mK_t}{R_a},$$

$$\omega(s) = \frac{-1/(Js+b)}{1 + (K_tK_aK_m/R_a)[1/(Js+b)] + (K_mK_b/R_a)[1/(Js+b)]}T_d(s)$$

$$= \frac{-1}{Js + b + (K_m/R_a)(K_tK_a + K_b)}T_d(s).$$



Disturbance Signals in a Feedback Control System

$$\lim_{t\to\infty} \omega(t) = \lim_{s\to 0}(s\omega(s)) = \frac{-1}{b + (K_m/R_a)(K_t K_a + K_b)} D;$$

when the amplifier gain K_a is sufficiently high, we have

$$\omega(\infty) \approx \frac{-R_a}{K_a K_m K_t} D = \omega_c(\infty).$$

$$\frac{\omega_c(\infty)}{\omega_0(\infty)} = \frac{R_a b + K_m K_b}{K_a K_m K_t}$$ < 0.02

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Control of the transient response of control systems

Karpagavalli S.

Department of Electronics and Communication Engineering

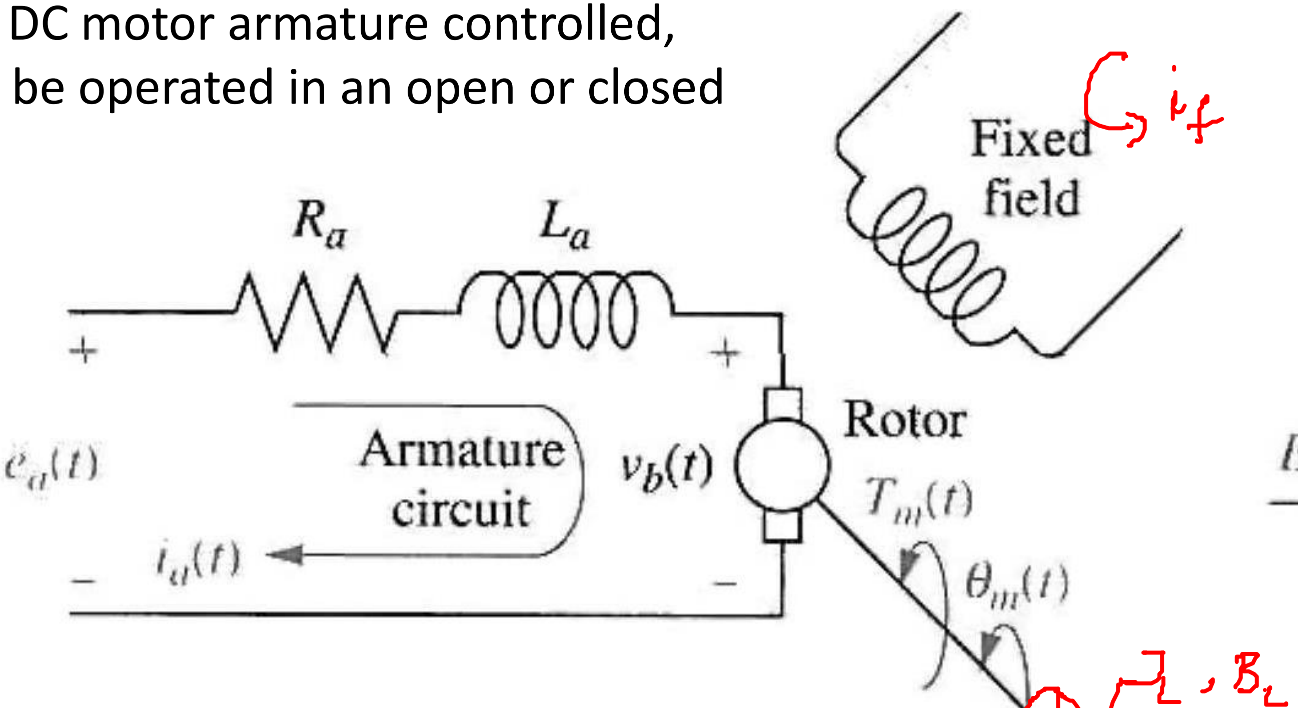
FEEDBACK CONTROL SYSTEM CHARACTERISTICS

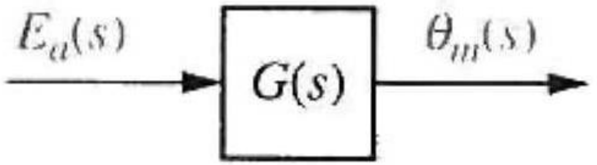
Control of the Transient Response of Control Systems

- Purpose of control system is to provide desired response, the transient response of control system often must be adjusted until it is satisfactory
- Open loop systems doesn't provide a satisfactory response
- It is often possible to alter the response of open loop system by adding cascade controller $G_c(s)$, preceding the process G(s)
- $G_c(s)$G(s) has to be designed so that the resulting transfer function provides the desired transient response.
- A closed loop system can often be adjusted to yield the desired response by adjusting the feedback loop parameters.

Control of the Transient Response of Control Systems

- Consider a DC motor armature controlled, which may be operated in an open or closed loop





$T(t) \propto i_a i_f$ → nonlinear eq.

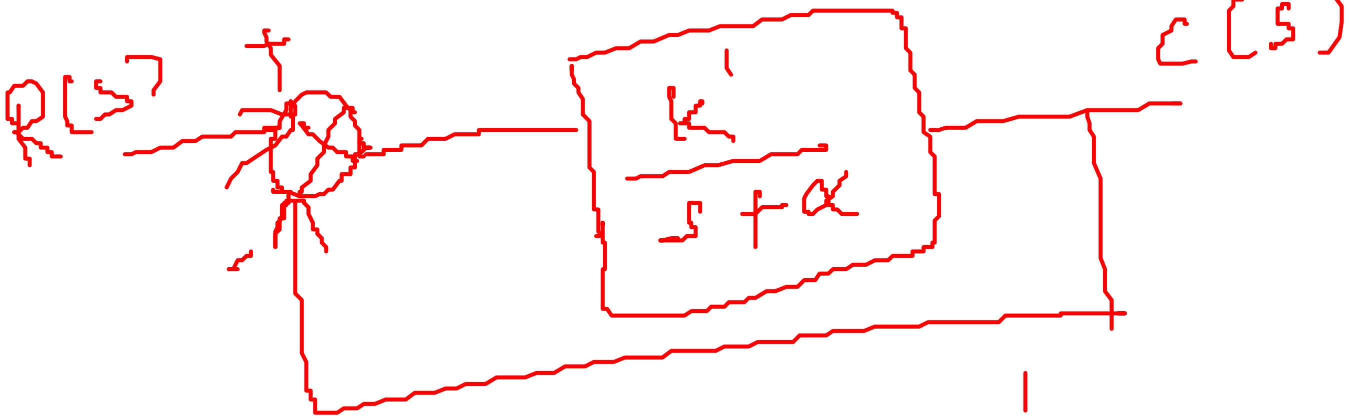
→ fix i_a, $i_f \uparrow\downarrow$ → field controlled

→ fix i_f, $i_a \uparrow\downarrow$ → armature controlled

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Control of the Transient Response of Control Systems

$$\frac{\theta(s)}{E_a(s)} = \frac{K_1}{s\left\{(L_a s + R_a)\left[(J_m+J_L)s+(B_m+B_L)\right]+K_1K_2\right\}}$$



$$\frac{C(s)}{R(s)} = \frac{K'}{s+\alpha+K'}$$

$$G(s) = \frac{K'}{s+\alpha} = \frac{K'}{\alpha\left(\frac{s}{\alpha}+1\right)} = \frac{K'/\alpha}{\boxed{\frac{s}{\alpha}}+1} \Rightarrow \frac{\frac{K'}{\alpha}}{\tau s+1}$$

$$\frac{1}{\alpha} = \tau$$

FEEDBACK CONTROL SYSTEM CHARACTERISTICS

Cost of Feedback

Despite of the immense uses of having feedback in a control systems , there are also certain disadvantages

- Cost of the system

The cost of the system increases as there are more number of components and feedback systems need a sensor ,which is the most expensive component .

- Complexity of the system

Since we have more number of components , the complexity of the system increases.

Cost of Feedback

- Loss of gain

The open-loop gain is Gc(s)G(s) and is reduced to Gc(s)G(s)/(l + Gc(s)G(s)) in a unity negative feedback system. The closed-loop gain is smaller by a factor of 1/(1 + Gc(s)G(s)),

- Possibility of instability

Whereas the open-loop system is stable, the closed-loop system may not be always stable.

The Performance of Feedback Control Systems:

Steady-State Error

error $e(t)$,

$$e_{ss} = \lim_{t \to \infty} e(t) \quad \text{or} \quad \lim_{s \to 0} sE(s)$$

Karpagavalli S.

Department of Electronics and Communication Engineering

STEADY – STATE ERROR

Introduction

- **If the output of a control system at steady state does not exactly match with the input, the system is said to have steady state error**

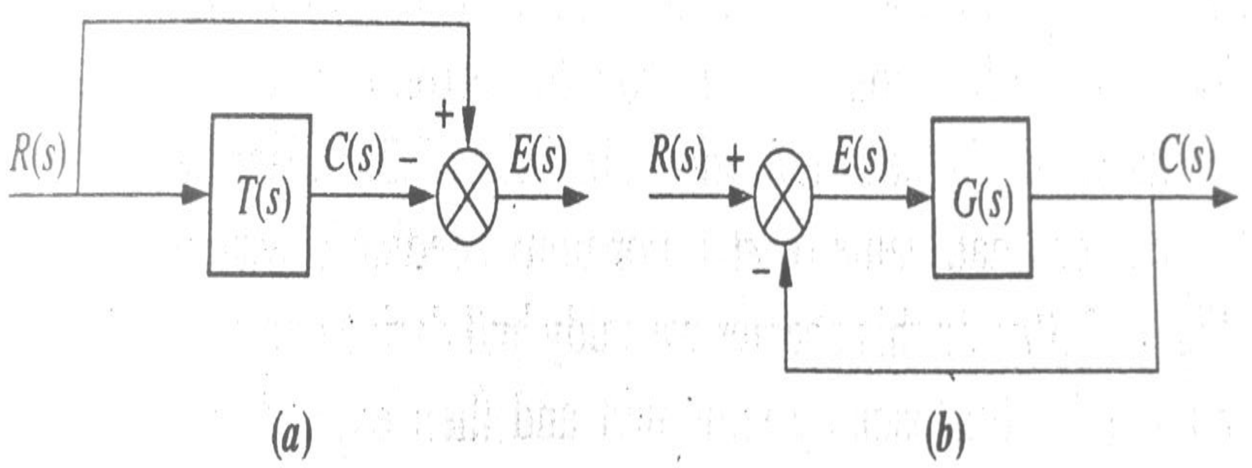
- **Any physical control system inherently suffers steady-state error in response to certain types of inputs.**

- **<u>A system may have no steady-state error to a step input, but the same system may exhibit nonzero steady-state error to a ramp input</u>.**

STEADY – STATE ERROR

Introduction

- **Non linear sources-**
  - **backlash in Gears**
  - **Motor that will not move unless the input voltage exceeds threshold**



a) Closed loop control system error

b) Representation for UFB

STEADY – STATE ERROR

Classification of Control Systems

- **Control systems may be classified according to their ability to follow step inputs, ramp inputs, parabolic inputs, and so on.**

- **The magnitudes of the steady-state errors due to these individual inputs are indicative of the goodness of the system.**

Classification of Control Systems

- Consider the unity-feedback control system with the following open-loop transfer function

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

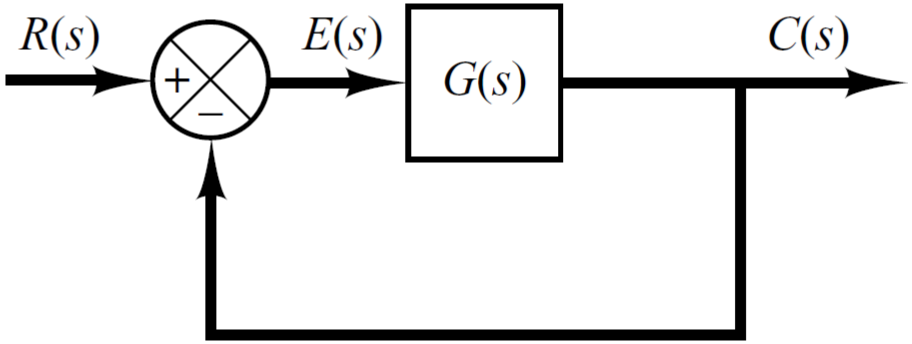
- It involves the term s^N in the denominator, representing N poles at the origin.

- A system is called type 0, type 1, type 2, ... , if N=0, N=1, N=2, ... , respectively.

STEADY – STATE ERROR

Unity Feedback Systems

- **Consider the system shown in following figure.**



- **The closed-loop transfer function is**

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} \qquad G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

STEADY – STATE ERROR

Unity Feedback Systems

- Steady state error is defined as the error between the input signal and the output signal when $t \to \infty$.

- **The transfer function between the error signal E(s) and the input signal R(s) is**

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} \qquad E(s) = \frac{1}{1+G(s)} R(s)$$

- **By Final value theorem , The steady state error is**

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

STEADY STATE ERROR

Static Error Constants

Karpagavalli S.
Department of Electronics and Communication Engineering

STEADY – STATE ERROR

Static Error Constants

- The static error constants are figures of merit of control systems. The higher the constants, the smaller the steady-state error.
- In a given system, the output may be the position, velocity, pressure, temperature, or the like.
- Therefore, in what follows, we shall call the output “position,” the rate of change of the output “velocity,” and so on.
- This means that in a temperature control system “position” represents the output temperature, “velocity” represents the rate of change of the output temperature, and so on.

STEADY – STATE ERROR

Static Error Constants (K_p)

- The steady-state error of the system for a unit-step input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s}$$

$$= \frac{1}{1 + G(0)}$$

- The static position error constant K_p is defined by

$$K_p = \lim_{s \to 0} G(s) = G(0)$$

- Thus, the steady-state error in terms of the static position error constant K_p is given by

$$e_{ss} = \frac{1}{1 + K_p}$$

STEADY – STATE ERROR

Static Error Constants (K_p)

$K_p = \lim_{s \to 0} G(s)$

For a Type 0 system

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

For Type 1 or higher order systems

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 1$$

For a unit step input the steady state error e_{ss} is

$$e_{ss} = \frac{1}{1 + K}, \qquad \text{for type 0 systems}$$

$$e_{ss} = 0, \qquad \text{for type 1 or higher systems}$$

STEADY – STATE ERROR

Static Error Constants (K_v)

- The steady-state error of the system for a unit-ramp input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{sG(s)}$$

- The static velocity error constant K_v is defined by

$$K_v = \lim_{s \to 0} sG(s)$$

- Thus, the steady-state error in terms of the static velocity error constant K_v is given by

$$e_{ss} = \frac{1}{K_v}$$

STEADY – STATE ERROR

Static Error Constants (K_v)

- **For a ramp input the steady state error e_{ss} is**

$$e_{ss} = \frac{1}{K_v} = \infty, \qquad \text{for type 0 systems}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}, \qquad \text{for type 1 systems}$$

$$e_{ss} = \frac{1}{K_v} = 0, \qquad \text{for type 2 or higher systems}$$

Static Acceleration Error Constants (K_a)

- The steady-state error of the system for parabolic input is

$$e_{ss} = \lim_{s\to 0} \frac{s}{1 + G(s)} \frac{1}{s^3}$$

$$= \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

- The static acceleration error constant K_a is defined by

$$K_a = \lim_{s\to 0} s^2 G(s)$$

- Thus, the steady-state error in terms of the static acceleration error constant K_a is given by

$$e_{ss} = \frac{1}{K_a}$$

STEADY – STATE ERROR

Static Acceleration Error Constants (K_a)

- **For a Type 0 system**

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- **For Type 1 systems**

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{s(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- **For Type 2 systems**

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{s^2(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- **For Type 3 or higher order systems**

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 3$$

STEADY – STATE ERROR

Static Acceleration Error Constants (K_a)

- **For a parabolic input the steady state error e_{ss} is**

$$e_{ss} = \infty, \quad \text{for type 0 and type 1 systems}$$

$$e_{ss} = \frac{1}{K}, \quad \text{for type 2 systems}$$

$$e_{ss} = 0, \quad \text{for type 3 or higher systems}$$

STEADY – STATE ERROR

Summary

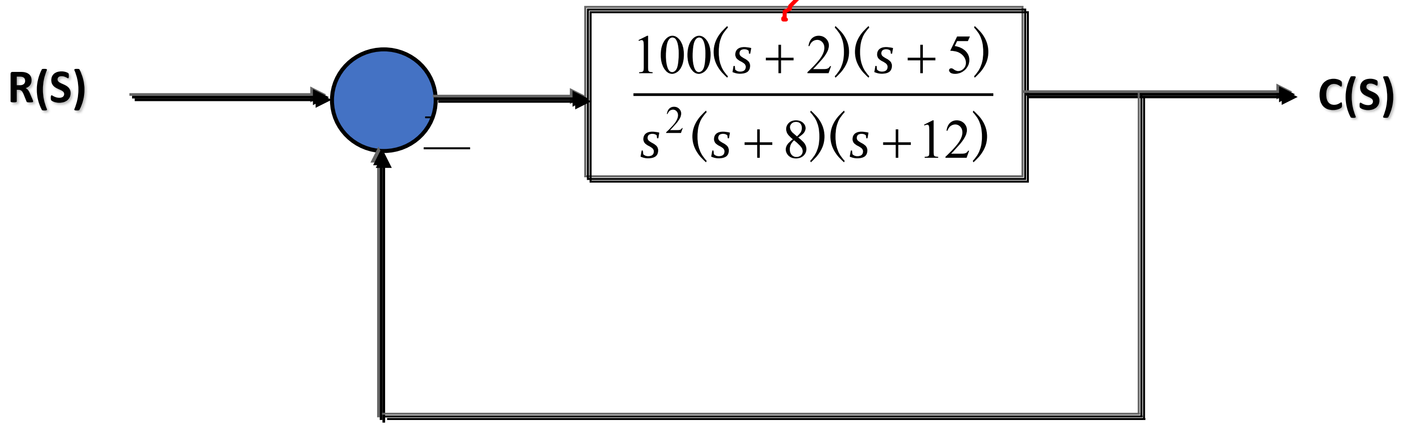
	Step Input $r(t) = 1$	Ramp Input $r(t) = t$	Acceleration Input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1+K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$

STEADY – STATE ERROR

Example

- **For the system shown in figure below evaluate the static error constants and find the expected steady state errors for the standard step, ramp and parabolic inputs.**

$G(s)$ - OLTF



$K_p = \lim_{s \to 0} G(s)$

$K_v = \lim_{s \to 0} sG(s)$

$K_a = \lim_{s \to 0} s^2 G(s)$

STEADY – STATE ERROR

Example

$$G(s) = \frac{100(s+2)(s+5)}{s^2(s+8)(s+12)}$$

$$K_p = \lim_{s\to 0} G(s)$$

$$K_p = \lim_{s\to 0}\left(\frac{100(s+2)(s+5)}{s^2(s+8)(s+12)}\right)$$

$$K_p = \infty$$

$$K_v = \lim_{s\to 0} sG(s)$$

$$K_v = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s)$$

$$K_a = \lim_{s\to 0}\left(\frac{100s^2(s+2)(s+5)}{s^2(s+8)(s+12)}\right)$$

$$K_a = \left(\frac{100(0+2)(0+5)}{(0+8)(0+12)}\right) = 10.4$$

Step

$$e_{ss} = \frac{1}{1+K_p}$$

$$= 0$$

Ramp

$$e_{ss} = \frac{1}{K_v} = 0$$

parabolic

$$e_{ss} = \frac{1}{K_a} = 0.09$$

STEADY – STATE ERROR

Example

$K_p = \infty$ $K_v = \infty$ $K_a = 10.4$

$$e_{ss} = \frac{1}{1 + K_p} = 0$$

$$e_{ss} = \frac{1}{K_v} = 0$$

$$e_{ss} = \frac{1}{K_a} = 0.09$$

STEADY – STATE ERROR

Example

Determine the static error constants of the system represented by the OLTF with unity feedback

$$G(s) = \frac{K(s+2)}{s(s^3+7s^2+12s)} = \frac{K(s+2)}{s^2(s^2+7s+12)}$$

Also determine the type & order of the system. Find the e_{ss} for a unit parabolic input.

Sol: type = 2 , order = 4

$$K_p = \lim_{s \to 0} G(s) = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s) = \infty$$

$$K_a = \lim_{s \to 0} s^2 G(s) = \frac{2K}{12} = K/6$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K/6} = \frac{6}{K}$$

STEADY – STATE ERROR

Example

Find the static error constants for a system

$$G(s) = \frac{s+10}{s(s^3+5s^2+15s)} = \frac{s+10}{s^2(s^2+5s+15)}$$

$$L[t^n] = \frac{n!}{s^{n+1}}$$

assuming UFB, also calculate e_{ss} when $r(t) = 2t\,u(t)$ & $r(t) = 4t^2 u(t)$

$R(s) = 2/s^2$ $\qquad$ $R(s) = 8/s^3$

Sol: type = 2 , order = 4

$$K_p = \lim_{s \to 0} G(s) = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s) = \infty$$

$$K_a = \lim_{s \to 0} s^2 G(s) = \frac{10}{15} = 2/3$$

$$e_{ss} = \frac{2}{K_v} = \frac{2}{\infty} = 0 \quad , \quad e_{ss} = \frac{8}{K_a} = \frac{8}{2/3} = \underline{\underline{12}}$$

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)}$$

THANK YOU

Karpagavalli S.
Department of Electronics and Communication Engineering

karpagavallip@pes.edu

+91 80 2672 1983 Extn 753