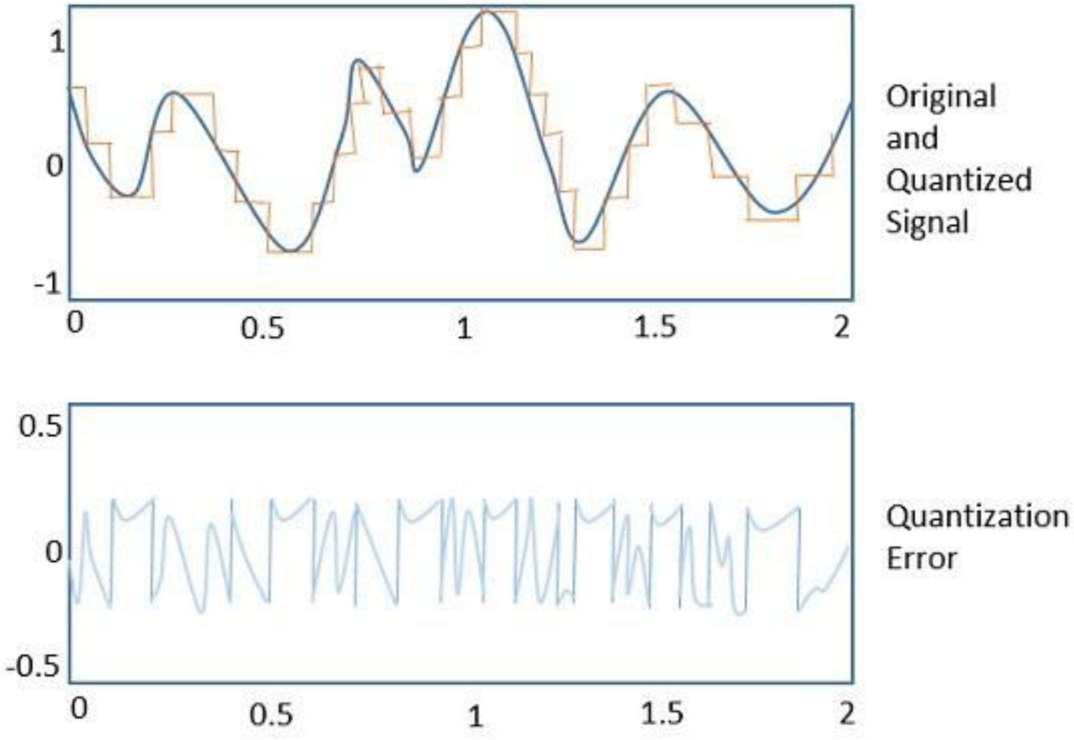
PES UNIVERSITY ONLINE

DIGITAL COMMUNICATION

Dr. Sanjeev G.
Department of Electronics and Communication Engg

QUANTIZATION

Problems



Dr. Sanjeev G.

Department of Electronics and Communication Engineering

QUANTIZATION

Problem 1

Let X be uniform over the range -10 to 10. If it is required that $\sigma_Q^2 < 0.2$. What Is the minimum N required?

Solution: By default, we consider mid-rise quantizer

Given :- $\sigma_Q^2 < 0.2$

$$\frac{\Delta^2}{12} < 0.2$$

$$\Delta < \sqrt{2.4}$$

$$\Delta < 1.549.$$

$$\Delta = \frac{2A}{2^N} < 1.549.$$

$$\frac{2 \times (10)}{2^N} < 1.549$$

$$2^N > \frac{20}{1.549}.$$

$$N > \log_2\left(\frac{20}{1.549}\right)$$

$$N > 3.69$$

$$\boxed{N \geq 4.}$$

Problem 2

Let X be uniform over between $[-A, A]$. Find the SNR for N-bit quantization, assuming that N is large.

Solution:

$$SNR = \frac{\frac{(2A)^2}{12}}{\Delta^2/12}$$

$$SNR = \frac{4A^2}{\Delta^2}$$

w.k.t. $\Delta = \frac{2A}{2^N}$

$$\therefore SNR = \frac{4A^2}{4A^2/2^{2N}}$$

$$\boxed{SNR = 2^{2N}}$$

$$SNR_{dB} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_Q^2}\right).$$

$$= 10\log_{10}(2^{2N}).$$

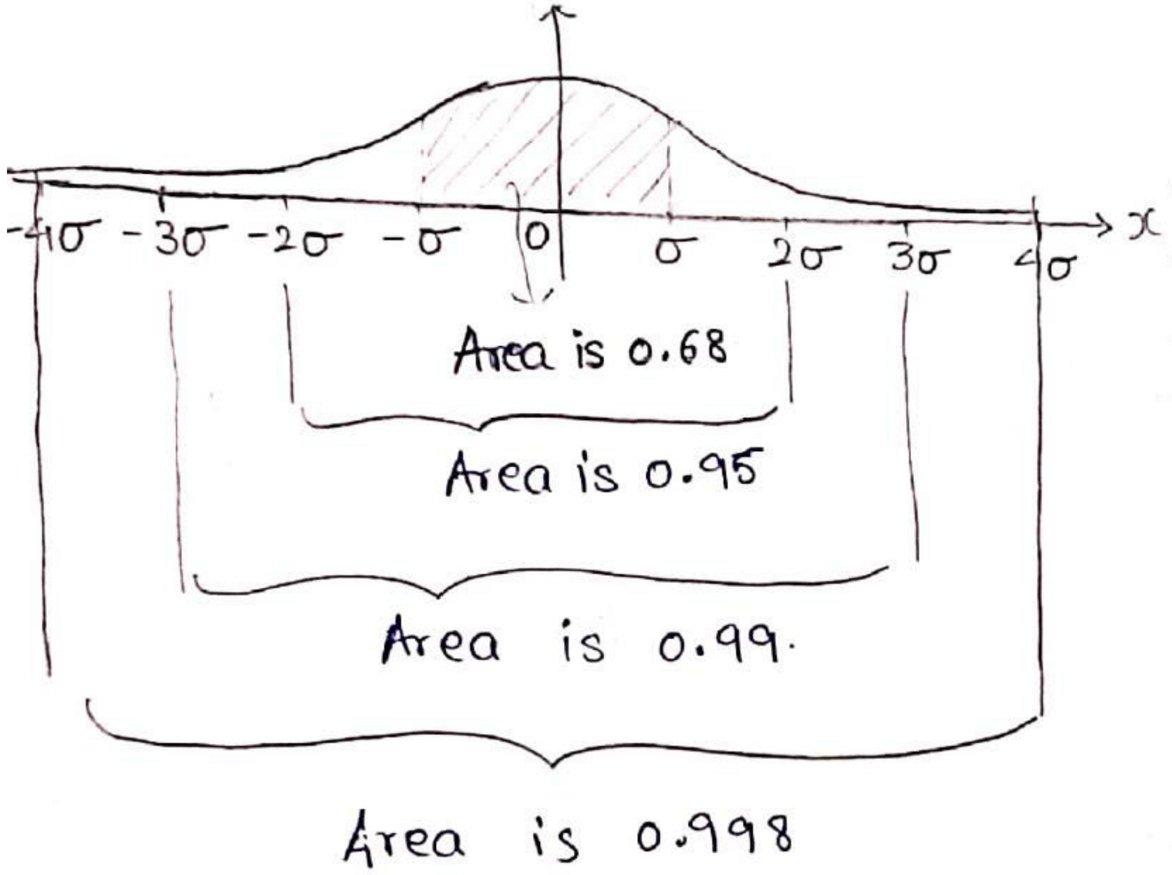
$$SNR_{dB} = 20N\log_{10}(2).$$

$$\boxed{SNR_{dB} = 6.02N \approx 6N}$$

Problem 3

Let $X \sim \mathcal{N}(0, \sigma_X^2)$. Find the SNR for N-bit quantization.

Solution:



* Hence for all practi-cal purposes the value of $x(n)$ can be take between -4σ & 4σ.

QUANTIZATION

Problem 3

Let $X \sim \mathcal{N}(0, \sigma_X^2)$. Find the SNR for N-bit quantization.

Solution:

w.r.t. for Gaussian distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

here $\mu = 0$.

$$\therefore f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\sigma_Q^2 = \frac{\Delta^2}{12} = \frac{(2 \times 4\sigma_X)^2}{12 \times 2^{2N}}$$

$$= \frac{64\sigma_X^2}{12 \times 2^{2N}} = \frac{16}{3}\frac{\sigma_X^2}{2^{2N}}$$

$$SNR = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{\sigma_X^2}{\frac{16}{3}\frac{\sigma_X^2}{2^{2N}}}$$

$$\boxed{SNR = \frac{3}{16} 2^{2N}}$$

$$\boxed{SNR_{dB} = 6N - 7.269}$$

Problem 4

Let $x(n) = A\cos(2\pi f_0 n)$. Find the SNR for N-bit quantization.

Solution: Note that the signal $x(n)$ is deterministic

$$\boxed{SNR = \frac{\text{Avg. Power of Input signal}}{\sigma_Q^2} = \frac{P_x}{\sigma_Q^2}}$$

$$P_x = \frac{A^2}{2}.$$

$$SNR = \frac{A^2}{2} \Big/ \left(\Delta^2/12\right)$$

$$\boxed{SNR = \frac{3}{2}\, 2^N}$$

$$\boxed{SNR_{dB} = 6N + 1.76}$$

QUANTIZATION

Observations

From the above problems, we can conclude the following

- The SQNR (SNR) depends on the PDF of input signal $x(n)$
- Typically, $\mathrm{SNR_{dB}} = 6N + c$, which is an incrementally linear function of N with a slope of 6 dB/bit
- This means that for every additional bit added to represent the quantized signal, we get an improvement of 6 dB in SNR
- When an additional bit is added, the number of levels doubles, and the width size reduces by half, resulting in a smaller quantization error
- In other words, for every additional bit added, σ_Q^2 decreases by a factor of 4 (Note that $10 \times \log_{10} 4 \approx 6$ dB)

THANK YOU

Dr. Sanjeev G.
Department of Electronics and Communication Engineering

sanjeevg@pes.edu

+91 80 2672 1983 Extn 838