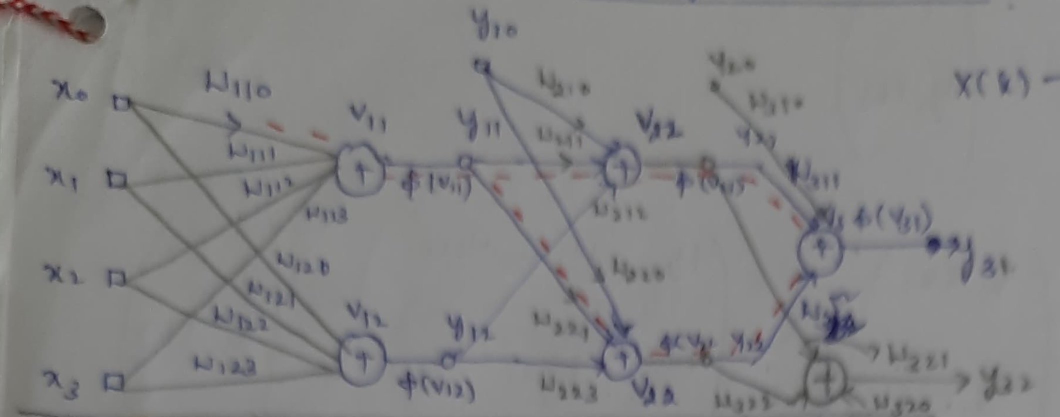
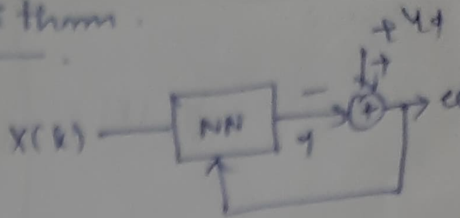
Back propogation Algorithm





Layer 0	1	2	3
index j	i	s	q
Nodes no	m_1	m_2	m_3

Feedforward pass:

Layer 0

input: $X(k) = \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix}$ $\quad$ o/p: $y_0(k) = \begin{pmatrix} x_0(k) \\ x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix} = \begin{pmatrix} 1 \\ X(k) \end{pmatrix}.$

Layer 1:

input: $y_0(k) = \begin{pmatrix} 1 \\ X(k) \end{pmatrix}.$ $\quad$ output: $\bar{y}_1(k) = \begin{pmatrix} y_{11}(k) \\ y_{12}(k) \end{pmatrix}.$

Where $\boxed{\bar{y}_{10}(k) = \begin{pmatrix} \phi_1(v_{11}(k)) \\ \phi_1(v_{12}(k)) \end{pmatrix} = \phi_1(v_1(k))}$ $\quad$ $y_1(k) = \begin{pmatrix} 1 \\ \bar{y}_1(k) \end{pmatrix}.$

$$v_{11}(k) = w_{110}x_0(k) + w_{111}x_1(k) + w_{112}x_2(k) + w_{113}x_3(k).$$

$$v_{12}(k) = w_{120}x_0(k) + w_{121}x_1(k) + w_{122}x_2(k) + w_{123}x_3(k).$$

Let $V_1(k) = \begin{pmatrix} v_{11}(k) \\ v_{12}(k) \end{pmatrix} = \begin{pmatrix} w_{110} & w_{111} & w_{112} & w_{113} \\ w_{120} & w_{121} & w_{122} & w_{123} \end{pmatrix} \begin{pmatrix} 1 \\ x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix}$

2×4

$m_1 \times (m_0 + 1)$

$$V_1(k) = W_1 Y_0(k)$$

Layer 2:

input $y_1(k) = \begin{pmatrix} 1 \\ \bar{y}_1(k) \end{pmatrix}$.

output:

$$V_2(k) = \begin{pmatrix} V_{21}(k) \\ V_{22}(k) \end{pmatrix} = \underbrace{\begin{pmatrix} W_{210} & W_{211} & W_{212} \\ W_{220} & W_{221} & W_{222} \end{pmatrix}}_{\substack{2\times 3 \\ m_2 \quad m_1+1}} \begin{pmatrix} 1 \\ \bar{y}_1(k) \end{pmatrix}$$

$$V_2(k) = W_2\, Y_1(k)$$

$$\phi_2(V_2(k)) = \begin{pmatrix} \phi_2(V_{21}(k)) \\ \phi_2(V_{22}(k)) \end{pmatrix}.$$

$$\bar{y}_2(k) = \begin{pmatrix} y_{21}(k) \\ y_{22}(k) \end{pmatrix} = \begin{pmatrix} \phi_2(V_{21}(k)) \\ \phi_2(V_{22}(k)) \end{pmatrix} = \phi_2(V_2(k))$$

Layer 3:

input $y_2(k) = \begin{pmatrix} 1 \\ \bar{y}_2(k) \end{pmatrix}$.

o/p:

$$V_3(k) = \underbrace{\begin{pmatrix} W_{310} & W_{311} & W_{312} \\ W_{320} & W_{321} & W_{322} \end{pmatrix}}_{m_3 \times (m_2+1)} \begin{pmatrix} 1 \\ \bar{y}_2(k) \end{pmatrix} = W_3\, Y_2(k).$$

1×3.

$$\phi_3(V_3(k)) = \begin{pmatrix} \phi_3(V_{31}(k)) \\ \phi_3(V_{32}(k)) \end{pmatrix} = y_3(k) = \begin{pmatrix} y_{31}(k) \\ y_{32}(k) \end{pmatrix}.$$

ckward pass:

$\cancel{e(k) = (e_1(k))}$

$\cancel{e_1(k) = y_{d_1}(k) - y_{31}(k)}$.

$\xi(k) = \frac{1}{2} \sum_{q=1}^{m_3} e_q^2(k) = \frac{1}{2} e_1^2(k) = e_1(k)$.

WKT error is a fun of W. (indirect relation).

~~Now~~ consider error at the 1st neuron in the o/p layer,

ayer 3:

$$e_1(k) = y_{d_1}(k) - y_{31}(k).$$

Now differentiate $\xi(k)$ with respect to W_3.

But W_3 has 3 components i.e. W_{310} & W_{311} & W_{312}.

$$\frac{\partial \xi(k)}{\partial W_{310}(k)} = \underset{①}{\frac{\partial \xi(k)}{\partial e_1(k)}} \cdot \underset{②}{\frac{\partial e_1(k)}{\partial y_{31}(k)}} \cdot \underset{③}{\frac{y_{31}(k)}{\partial V_{31}(k)}} \cdot \underset{④}{\frac{\partial V_{31}(k)}{\partial W_{310}(k)}} \quad \text{——— ①}$$

① $= e_1(k)$ ② $= -1$ ③ $= \phi_3'(V_{31}(k))$.

④ $= y_{20}(k)$.

Eq ① becomes.

$$\frac{\partial \xi(k)}{\partial W_{310}(k)} = -e_1(k)\, \phi_3'(V_{31}(k))\, y_{20}(k). \quad \text{——— Ⓐ}$$

~~similarly~~: $\cancel{\frac{\partial \xi(k)}{\partial W_{311}(k)} = \frac{\partial \xi(k)}{\partial e_1(k)} \cdot \frac{\partial e_1(k)}{\partial y_{31}(k)} \cdot \frac{\partial y_{31}(k)}{}}$

$$\frac{\partial \xi(k)}{\partial W_{311}(k)} = \frac{\partial \xi(k)}{\partial e_1(k)} \cdot \frac{\partial e_1(k)}{\partial y_{31}(k)} \cdot \frac{\partial y_{31}(k)}{\partial V_{31}(k)} \cdot \frac{\partial V_{31}(k)}{\partial W_{311}} = -e_1(k)\, \phi_{31}'\, y_{21}(k)$$

$$\frac{\partial \xi(k)}{\partial w_{312}(k)} = -e_1(k)\, \phi_3'(v_{31}(k))\, y_{22}(k).$$

similarly consider the error at neuron 2 in the o/p layer.

$$e_2(k) = y_{d_2} - y_{32}(k).$$

$$\frac{\partial \xi(k)}{\partial w_{320}(k)} = \frac{\partial \xi(k)}{\partial e_2(k)} \cdot \frac{\partial e_2(k)}{\partial y_{32}(k)} \cdot \frac{\partial y_{32}(k)}{\partial v_{32}(k)} \cdot \frac{\partial v_{32}(k)}{\partial w_{320}(k)}$$

$$= -e_2(k)\, \phi_3'(v_{32}(k)) \cdot y_{20}(k). \quad \text{— (B)}$$

From Eq^n (A) & (B). we can write general exp^n for layer 3 as follows.

$$\frac{\partial \xi(k)}{\partial w_{3qs}(k)} = -e_q(k) \cdot \phi_3'(v_{3q}(k))\, y_{2s}(k)$$

Now define the local gradient.

at 1st neuron in the 3rd layer.

$$\delta_{31}(k) = -\frac{\partial \xi}{\partial v_{31}} = e_1(k)\, \phi_3'(v_{31}(k)).$$

$$\delta_{32}(k) = \frac{-\partial \xi}{\partial v_{32}(k)} = e_2(k)\, \phi_3'(v_{32}(k)).$$

in general.

$$\delta_{3q}(k) = -\frac{\partial \xi}{\partial v_{3q}} =$$

$$\therefore \delta_3(k) = \begin{pmatrix} \delta_{31}(k) \\ \delta_{32}(k) \end{pmatrix} = \begin{pmatrix} e_1(k) \\ e_2(k) \end{pmatrix} \odot \begin{pmatrix} \phi_3'(v_{31}(k)) \\ \phi_3'(v_{32}(k)) \end{pmatrix}$$

$$\delta_3(k) = e_3(k) \odot \phi_3'(v_3(k)).$$

ight update:

$$W(k+1) = W(k) - \eta_3 \frac{\delta \xi(k)}{\partial W_{3qs}(k)}$$

for ex:

$$\Delta W_{310}(k) = -\eta_3 \frac{\partial \xi(k)}{\partial W_{310}(k)}$$

$$= \eta_3 \, e_1(k) \, \phi_3'(V_{31}(k)) y_{20}(k).$$

$$\Delta W_3(k) = \begin{pmatrix} \Delta W_{310}(k) & \Delta W_{311}(k) & \Delta W_{312}(k) \\ \Delta W_{320}(k) & \Delta W_{321}(k) & \Delta W_{322}(k) \end{pmatrix}.$$

$$= \eta \begin{pmatrix} \delta_{31} y_{20} & \delta_{31} y_{21} & \delta_{31} y_{22} & \delta_{31} y_{23} \\ \delta_{32} y_{20} & \delta_{32} y_{21} & \delta_{32} y_{22} & \delta_{32} y_{23} \end{pmatrix}.$$

$$= \eta \begin{pmatrix} \delta_{31} & \delta_{32} \end{pmatrix}^T \odot \begin{pmatrix} y_{20} & y_{21} & y_{22} & y_{23} \end{pmatrix}.$$

$$\Delta W_3(k) = \eta \; \delta_3(k) \, y_2^T(k).$$

2nd Hidden layer:

$$\frac{\partial \xi(k)}{\partial W_{210}} = \frac{\partial \xi}{\partial e_1} \cdot \frac{\partial e_1}{\partial y_{31}} \cdot \frac{\partial y_{31}}{\partial V_{31}} \cdot \frac{\partial V_{31}}{\partial y_{21}} \cdot \frac{\partial y_{21}}{\partial V_{21}} \cdot \frac{\partial V_{21}}{\partial W_{210}} +$$

$$\frac{\partial \xi}{\partial e_2} \cdot \frac{\partial e_2}{\partial y_{32}} \cdot \frac{\partial y_{32}}{\partial V_{32}} \cdot \frac{\partial V_{32}}{\partial y_{21}} \cdot \frac{\partial y_{21}}{\partial V_{21}} \cdot \frac{\partial V_{21}}{\partial W_{210}}.$$

$$= -e_1 \, \phi_3'(V_{31}(k)) \cdot W_{311} \, \phi_2'(V_{21}(k)) \, y_{10} \; -$$

$$e_2 \, \phi_3'(V_{32}(k)) \, W_{321} \, \phi_2'(V_{21}(k)) \, y_{10}(k)$$

$\leftarrow \delta_{3q}(k)$

$$= -\sum^{2} \left(e_q \, \phi_q'(V_{3q}(k)) \right) W_{3q1} \, \phi_2'(V_{21}(k)) \, y_{10}(k)$$

$$\frac{\xi(k)}{\partial w_{210}} = -\sum_{q=1}^{2} \delta_{3q}(k)\, w_{3q1}\, \phi_2'(v_{21}(k))\, y_{10}(k).$$

Similarly :

$$\frac{\partial \xi(k)}{\partial w_{220}} = \frac{\partial \xi}{\partial e_1} \cdot \frac{\partial e_1}{\partial y_{31}} \cdot \frac{\partial y_{31}}{\partial v_{31}} \cdot \frac{\partial v_{31}}{\partial y_{22}} \cdot \frac{\partial y_{22}}{\partial v_{22}} \cdot \frac{\partial v_{22}}{\partial w_{220}} +$$

$$\frac{\partial \xi}{\partial e_2} \cdot \frac{\partial e_2}{\partial y_{32}} \cdot \frac{\partial y_{32}}{\partial v_{32}} \cdot \frac{\partial v_{32}}{\partial y_{22}} \cdot \frac{\partial y_{22}}{\partial v_{22}} \cdot \frac{\partial v_{22}}{\partial w_{220}}.$$

$$= -e_1 \phi_3'(v_{31}(k)) \cdot w_{312}\, \phi_2'(v_{22}(k)) \cdot y_{20} - e_2 \phi_3'(v_{32}(k))\, w_{322}\, \phi_2'(v_{22}(k)) \cdot y_{20}.$$

$$= -\sum_{q=1}^{2} \overbrace{e_q\, \phi_3'(v_{3q}(k))}^{\delta_{3q}}\, w_{3q2}\, \phi_2'(v_{22}(k)) \cdot y_{20}.$$

In general.

$$\frac{\partial \xi(k)}{\partial w_{2si}^{(k)}} = -\sum_{q=1}^{2} \delta_{3q}\, w_{3qs} \cdot \phi_2'(v_{2s}(k)) \cdot y_{1i}.$$

Now define the local gradient for layer 2 as follows.

$$\delta_{2s} = +\sum_{q=1}^{2} \delta_{3q}\, w_{3qs} \cdot \phi_2'(v_{2s}(k))$$

$$= \mp \frac{\partial \xi(k)}{\partial v_{2s}} \qquad 1 \le s \le 2.$$

$$\therefore \frac{\partial \xi(k)}{\partial w_{2si}} = \mp \cdot \delta_{2s} \cdot y_{1i}(k).$$

$$\delta_2(k) = \begin{pmatrix} \delta_{21}(k) \\ \delta_{22}(k) \end{pmatrix} = \begin{pmatrix} \delta_{31} w_{311} + \delta_{32} w_{321} \\ \delta_{31} w_{312} + \delta_{32} w_{322} \end{pmatrix} \odot \begin{pmatrix} \phi(v_{21}) \\ \phi(v_{22}) \end{pmatrix}$$

$$= \bar{W}_3^T(k)\, \delta_3(k) \odot \phi_2'(v_2(k)).$$

$$\cancel{\delta_2(k) = \begin{pmatrix} \delta_{21}(k) \\ \delta_{22}(k) \end{pmatrix}}$$

$$\bar{W}_3 = \begin{pmatrix} W_{310} & W_{311} & W_{312} \\ W_{320} & W_{321} & W_{222} \end{pmatrix}.$$

$$\Delta W_2 = \begin{pmatrix} \Delta W_{210} & \Delta W_{211} & \Delta W_{212} \\ \Delta W_{220} & \Delta W_{221} & \Delta W_{222} \end{pmatrix}.$$

$$= \eta_2 \begin{pmatrix} \delta_{21} \\ \delta_{22} \end{pmatrix} \odot \begin{pmatrix} y_{10} & y_{11} & y_{12} \end{pmatrix}.$$

$$= \eta_2 \, \delta_2(k) \, y_1^T(k).$$

$$\boxed{\Delta W_{2si} = -\eta_2 \frac{\partial \xi(k)}{\partial W_{2si}(k)} = \eta_2 \, \delta_{2s} \, y_{1i}(k).}$$

First Hidden layer:

$$\frac{\partial \xi(k)}{\partial w_{110}(k)} = \frac{\partial \xi}{\partial e_1(k)} \cdot \frac{e_1(k)}{\partial y_{31}(k)} \cdot \frac{\partial y_{31}(k)}{\partial v_{31}(k)} \left[\frac{\partial v_{31}(k)}{\partial y_{21}} \cdot \frac{\partial y_{21}}{\partial v_{21}} \cdot \frac{\partial v_{21}}{\partial y_{11}} \cdot \frac{\partial y_{11}}{\partial v_{11}} \cdot \frac{\partial v_{11}}{\partial w_{110}} + \frac{\partial v_{31}(k)}{\partial y_{22}} \cdot \frac{\partial y_{22}}{\partial v_{22}} \cdot \frac{\partial v_{22}}{\partial y_{11}} \cdot \frac{\partial y_{11}}{\partial v_{11}} \cdot \frac{\partial v_{11}}{\partial w_{110}} \right]$$

$$+ \frac{\partial \xi}{\partial e_2(k)} \cdot \frac{\partial e_2}{\partial y_{32}} \cdot \frac{\partial y_{32}}{\partial v_{32}(k)} \cdot \left[\frac{\partial v_{32}}{\partial y_{21}} \cdot \frac{\partial y_{21}}{\partial v_{21}} \cdot \frac{\partial v_{21}}{\partial y_{11}} \cdot \frac{\partial y_{11}}{\partial v_{11}} \cdot \frac{\partial v_{11}}{\partial w_{110}} + \frac{\partial v_{32}}{\partial y_{22}} \cdot \frac{\partial y_{22}}{\partial v_{22}} \cdot \frac{\partial v_{22}}{\partial y_{11}} \cdot \frac{\partial y_{11}}{\partial v_{11}} \cdot \frac{\partial v_{11}}{\partial w_{110}} \right]$$

$$= -e_1 \phi_3'(v_{31}(k)) \left[w_{311} \cdot \phi_2'(v_{21})) w_{211} + w_{312} \phi_2'(v_{22}(k)) w_{221} \right] \phi_1'(v_{11}) y_{00} - e_2 \phi_3'(v_{32}(k)) \left[w_{312} * \phi_2'(v_{21}(k)) w_{211} + w_{322} \cdot \phi'(v_{22}(k)) w_{221} \right] \phi_1'(v_{11}) \cdot y_{00}.$$

$$\underset{1ij}{\frac{\partial \xi(k)}{\partial w_{110}(k)}} = \left[-\sum_{q=1}^{2} \underbrace{e_q(k) \phi_3'(v_{3q}(k))}_{\delta_{3q}} \sum_{s=1}^{2} w_{3qs} \phi_2'(v_{2s}(k)) w_{2s1} \right] \phi_1'(v_{11}) y_{00}$$

$$= -\sum_{q=1}^{2} \sum_{q=1}^{2} \underbrace{\delta_{3q}(k)\, w_{3qs} \phi_2'(v_{22}) w_{2s1}}_{\delta_{2s}} \phi_1'(v_{11}) y_{00}.$$

$$= -\underbrace{\sum_{s=1}^{2} \delta_{2s} w_{2s1} \phi_1'(v_{11})}\, y_{00}.$$

$$\boxed{\frac{\partial \xi}{\partial w_{1ij}} = -\delta_{1i}\, y_{0i}(k).}$$

where $\delta_{1i} = \sum_{s=1}^{2} \delta_{2s} w_{2si} \phi_1'(v_{1i})$

(6)

$$\delta_1 = \begin{pmatrix} \delta_{11} \\ \delta_{12} \end{pmatrix} = \begin{pmatrix} \delta_{21} W_{211} + \delta_{22} W_{221} \\ \delta_{22} W_{212} + \delta_{22} W_{222} \end{pmatrix} \odot \begin{pmatrix} \phi_1'(v_{11}) \\ \phi_1'(v_{12}) \end{pmatrix}$$

$$= \begin{pmatrix} W_{211} & W_{222} \\ W_{212} & W_{222} \end{pmatrix} \begin{pmatrix} \delta_{21} \\ \delta_{22} \end{pmatrix} \odot \begin{pmatrix} \phi_1'(v_{11}) \\ \phi_1'(v_{12}) \end{pmatrix}.$$

$$\delta_1 = \bar{W}_2^T \delta_2(k) \odot \phi_1'(v_1).$$

$$W(k+1) = W(k) + \Delta W(k)$$

$$\Delta W = \begin{pmatrix} \Delta W_{110} & \Delta W_{111} & \Delta W_{112} & \Delta W_{113} \\ \Delta_{120} & \Delta W_{121} & \Delta W_{122} & \Delta W_{123} \end{pmatrix}.$$

$$\Delta W = \eta_1 \begin{pmatrix} \delta_{11} \\ \delta_{22} \end{pmatrix} \odot (1 \; y_{01} \; y_{02} \; y_{03})$$

Summary:

Backward pass:

output layer: $\delta_3 = \begin{pmatrix} \delta_{31} \\ \delta_{32} \end{pmatrix} = e \odot \phi_3'(V_3(k))$

$$= \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \odot \begin{pmatrix} \phi_3'(V_{31}(k)) \\ \phi_3'(V_{32}(k)) \end{pmatrix}$$

$$\Delta W_3(k) = \eta_3 \delta_3(k) y_2^T(k).$$

2nd Hidden layer:

$$\delta_2 = \begin{pmatrix} \delta_{21} \\ \delta_{22} \end{pmatrix} = \bar{W}_3^T \delta_3 \odot \phi_2'(V_2(k))$$

$$=$$

$$\Delta W_2(k) = \eta_2 \delta_2 y_1^T(k).$$

1st hidden layer: $\delta_1 = \begin{pmatrix} \delta_{11} \\ \delta_{12} \\ \delta_{13} \end{pmatrix} = \bar{W}_2^T \delta_2 \odot \phi_1'(V_1(k))$

$$\Delta W_1(k) = \eta_1 \delta_1 \odot y_0^T(k).$$

(10)

Consider 1:4:1 N/W.

$$W_1 = \begin{pmatrix} 0.2 & -0.4 \\ 0.25 & -0.4 \\ 0.4 & -0.1 \\ -0.1 & 0.4 \end{pmatrix} \qquad \eta_1 = \eta_2 = 0.01$$

$\phi_1(v) = \tanh(v)$ $\quad \phi_2(v) = v.$

$W_2 = (-0.25 \quad 0.25 \quad 0.15 \quad -0.5 \quad 0.3)$.

Data: $\{(\frac{x}{1/2}, \frac{y_d}{1})\}$ Use BPA and find the Next set of wts.

Sol^n:

$v_1 = W^T X$ where $X = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$.

$$= \begin{pmatrix} 0.2 & -0.4 \\ 0.25 & -0.4 \\ 0.4 & -0.1 \\ -0.1 & 0.4 \end{pmatrix} \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 0.2 - 0.2 \\ 0.25 - 0.2 \\ 0.4 - 0.05 \\ -0.1 + 0.2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.05 \\ 0.35 \\ 0.1 \end{pmatrix}$$

$4 \times 2 \quad 2 \times 1$

$$\bar{y}_1 = \phi_1(v_1) = \begin{pmatrix} \tanh(0) \\ \tanh(0.05) \\ \tanh(0.35) \\ \tanh(0.1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0.499 \\ 0.3363 \\ 0.0996 \end{pmatrix}.$$

0.461

$$y_1 = \begin{pmatrix} 1 \\ 0.0 \\ 0.499 \\ 0.3363 \\ 0.0996 \end{pmatrix}.$$

$$V_2 = W_2 Y_2 = (-0.25 \quad 0.25 \quad 0.15 \quad -0.5 \quad 0.3) \begin{pmatrix} 1 \\ 0 \\ 0.499 \\ 0.3363 \\ 0.0996 \end{pmatrix}.$$

$= -0.380785.$

$y_2 = -0.380785.$

$e = y_d - y_2 = \underline{1.3808}$

1:4:1

$$\delta_2 = e_1 \odot \phi_2'(U_2(k)).$$

$$= e \odot 1$$

$$= 1.3808$$

$$\Delta W_2 = \eta_2 \delta_2 y_1^T = 0.01 \times 1.3808 \times (1 \quad 0 \quad 0.05 \quad 0.35 \quad 0.1)$$

$$\Delta W_2 = [0.0138 \quad 0 \quad 0.00069 \quad 0.0048 \quad 0.00128].$$

$$\delta_1 = (\bar{W}_2^T \delta_2) \odot \phi_1'(V_1(k)).$$

$$= \begin{pmatrix} 0.25 \\ 0.15 \\ -0.5 \\ 0.3 \end{pmatrix} \times 1.3808 \odot \begin{pmatrix} 1-0 \\ 1-0.05^2 \\ 1-0.35^2 \\ 1-0.1^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3452 \\ 0.2071 \\ -0.6904 \\ 0.4142 \end{pmatrix} \odot \begin{pmatrix} 1 \\ 0.9975 \\ 0.8869 \\ 0.9901 \end{pmatrix}$$

$$\delta_1 = \begin{pmatrix} 0.3452 \\ 0.2066 \\ -0.6123 \\ 0.4101 \end{pmatrix}.$$

$$\Delta W_1 = \eta \delta_1 y_0^T$$

$$= 0.01 \times \begin{pmatrix} 0.3452 \\ 0.2066 \\ -0.6123 \\ 0.4101 \end{pmatrix} (1 \quad 0.5).$$

$$= \begin{pmatrix} 0.0034 & 0.0017 \\ 0.0020 & 0.0010 \\ -0.0061 & -0.0031 \\ 0.0041 & 0.0021 \end{pmatrix}.$$

$$(\tanh x)' = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

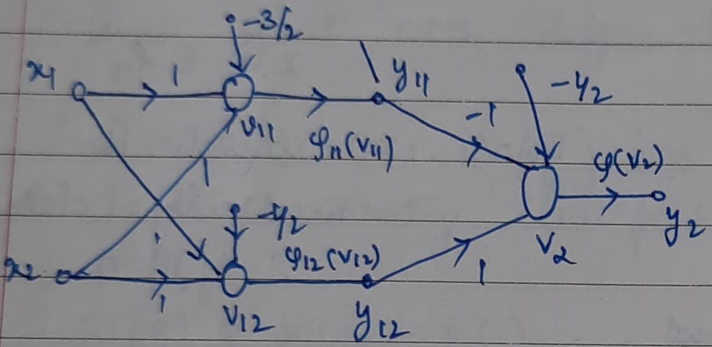
..._PG_PO8	... reports and design documents by adhering to the appropriate standards, make effective presentations, give and receive clear instructions
UE17ECE_PG_PO9	Engage in lifelong learning with persistent scientific temper for professional advancement and effective communication of the technical information.
UE17ECE_PG_PO10	Become a complete professional with high integrity and ethics, with excellent professional conduct and with empathy towards the environmental and contribute to the community for sustainable development of society
UE17ECE_PG_PO11	Critically evaluate the outcomes of one's actions and apply self corrective measures to improve the performance.

Example :

① X-OR Gate 3 neurons . (total).

$$y_{11} = \varphi(-3/2 + x_1 + x_2)$$
$$y_{12} = \varphi(x_1 + x_2 - 1/2)$$
$$y_2 = \varphi(-y_{11} + y_{12} - 1/2)$$

K	0	1	2	3
$x_1(k)$	0	0	1	1
$x_2(k)$	0	1	0	1
$d(k)$	0	1	1	0
	l_2	l_1	l_1	l_2



$$\varphi(v) = \begin{cases} 1 & v \geq 0 \\ 0 & v < 0 . \end{cases}$$

K	$x_1(k)$	$x_2(k)$	v_{11}	y_{11}	v_{12}	y_{12}	v_2	y_2
1	0	0	$-3/2$	0	$-1/2$	0	$-1/2$	0
2	0	1	$-1/2$	0	$1/2$	1	$1/2$	1
3	1	0	$-1/2$	0	$1/2$	1	$1/2$	1
4	1	1	$1/2$	1	$1/2$	1	$-1/2$	0