LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

Unit 3. Linear Transformations and Orthogonality

Orthogonal Vectors & Subspaces

Definition:

The ***norm or length*** of a n-dimensional vector $x = (x_1, x_2, ..., x_n)$ is written as $\| x \|$ and is defined as

$$\|x\| = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$$

We can also write
$$\|x\|^2 = x^T x$$

Note : Zero is the only vector whose norm is 0.

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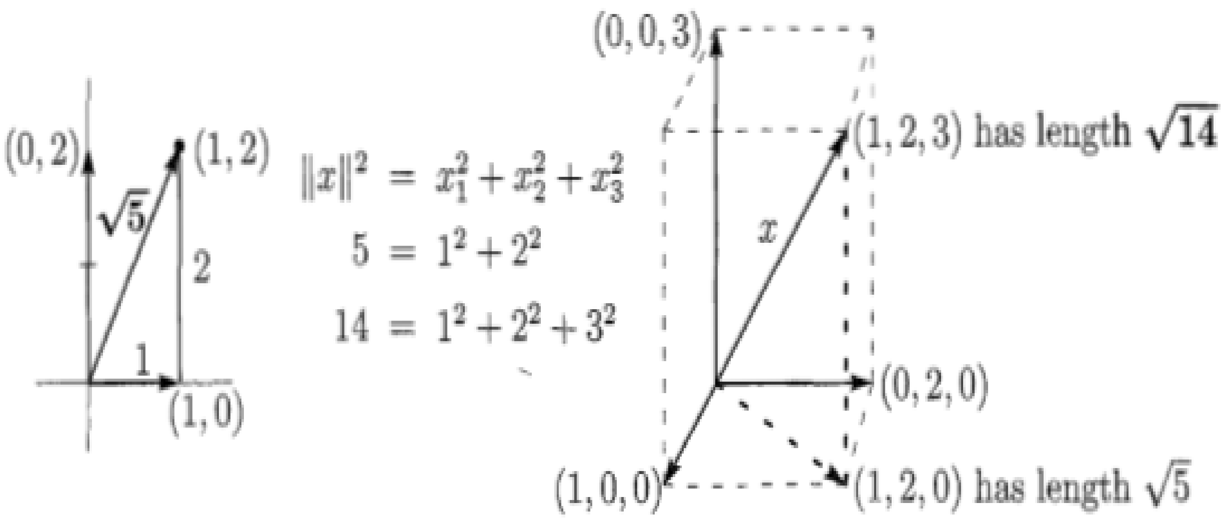


(a) (b)

Orthogonal Vectors & Subspaces

Definition:

The ***inner product*** or dot product or scalar product of two vectors x = (x_1, x_2, …., x_n) and y = (y_1, y_2, …., y_n) is denoted by

$$x^T y \ or \ x \circ y \ or \ \langle x , y \rangle$$

and is defined by

$$x^T y = x_1 y_1 + x_2 y_2 + ... + x_n y_n$$

Note that $$x^T y = y^T x$$

Unit 3. Linear Transformations and Orthogonality

Orthogonal Vectors & Subspaces

Definition :

Two vectors x = (x_1, x_2, …., x_n) and y = (y_1, y_2, …., y_n) are said to be ***orthogonal*** if

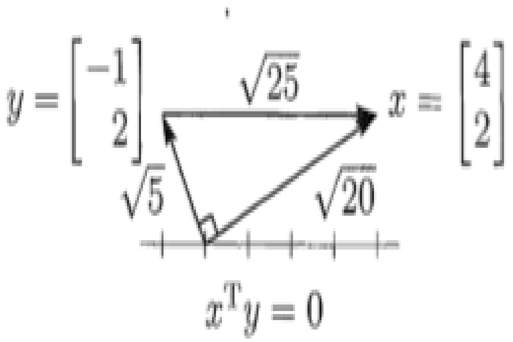
$$x^T y = y^T x = 0$$

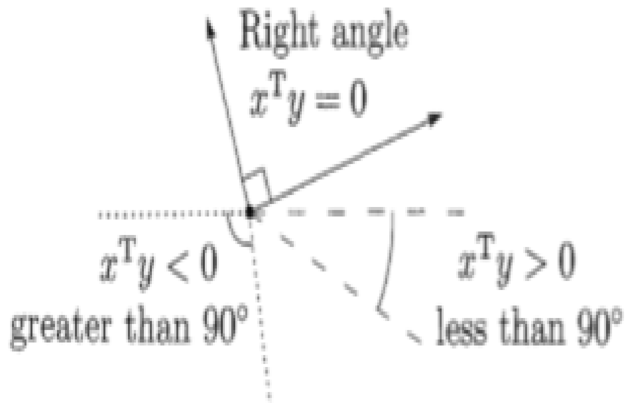
Note :

1. Zero is the only vector that is orthogonal to itself.
2. Zero is the only vector that is orthogonal to every other vector.

Unit 3. Linear Transformations and Orthogonality

Orthogonal Vectors & Subspaces





Orthogonal Vectors & Subspaces

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Examples

1. The coordinate vectors (1, 0, …., 0) , (0, 1, 0,…, 0) , ….., (0, 0, …., 0, 1) are mutually orthogonal in R^n.
2. The vectors (c, s) , (-s, c) are orthogonal in R^2.
3. The vectors (2, 1, 0) , (-1, 2, 0) are orthogonal in R^3.

Orthogonal Vectors & Subspaces

Theorem : If the non-zero vectors $v_1, v_2, \ldots ..v_k$ are mutually orthogonal then these vectors are linearly independent but convex need not be true.

Example : Vectors (2, 1) and (1, 2) are linearly independent but they are not mutually orthogonal

Orthogonal Vectors & Subspaces

Definition :

Two subspaces S and T of a vector space V are ***orthogonal*** if every vector x in S is orthogonal to every vector y in T. Thus,

$$x^T y = 0$$

for all x ϵ S and y ϵ T.

Orthogonal Vectors & Subspaces

Examples

1. $Z = \{0\}$ is orthogonal to all subspaces.

2. In R^2, a line can be orthogonal to another line.

3. In R^3, a line can be orthogonal to another line or a plane. But, a plane cannot be orthogonal to another plane.

Note :

If S and T are orthogonal in V then

$$\dim S + \dim T \leq \dim V$$

Orthogonal Vectors & Subspaces

Fundamental theorem of Orthogonality :

Let A be an m x n matrix then row space of A is orthogonal to its null space in R^n and the column space is orthogonal to left null space in R^m .

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THANK YOU