Digital Signal Processing

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Properties of DFT

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Properties of DFT
Linear Convolution

$$y(n_0) = \sum_{k=-\infty}^{\infty} x(k)h(n_0 - k)$$

1. *Folding*. Fold $h(k)$ about $k = 0$ to obtain $h(-k)$.
2. *Shifting*. Shift $h(-k)$ by n_0 to the right (left) if n_0 is positive (negative), to obtain $h(n_0 - k)$.
3. *Multiplication*. Multiply $x(k)$ by $h(n_0 - k)$ to obtain the product sequence $v_{n_0}(k) \equiv x(k)h(n_0 - k)$.
4. *Summation*. Sum all the values of the product sequence $v_{n_0}(k)$ to obtain the value of the output at time $n = n_0$.

Properties of DFT
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Example :

The impulse response of a linear time-invariant system is

$$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$$

Determine the response of the system to the input signal

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

Solution:

$$y(n) = \{\ldots, 0, 0, 1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1, 0, 0, \ldots\}$$

Properties of DFT
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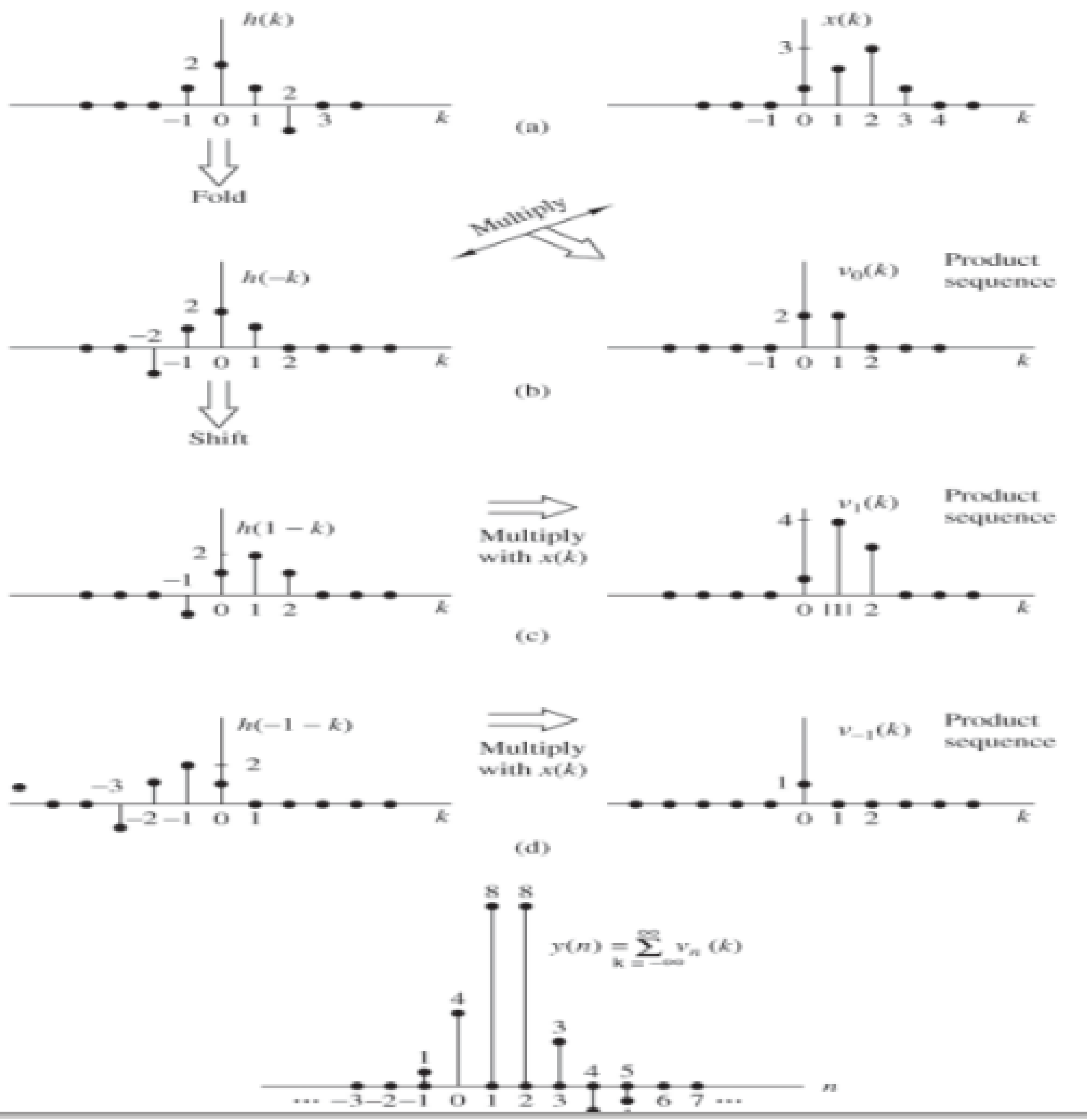


Properties of DFT

Multiplication of two DFTs and Circular convolution

Two finite duration sequences and their respective N -point DFTs

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi nk/N} \qquad k = 0, 1, \ldots, N-1$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi nk/N} \qquad k = 0, 1, \ldots, N-1$$

$$X_3(k) = X_1(k) X_2(k) \qquad k = 0, 1, \ldots, N-1$$

Properties of DFT

Multiplication of two DFTs and Circular convolution

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k)e^{j2\pi km/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X_1(k)X_2(k)e^{j2\pi km/N}$$

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{N-1} x_1(n)e^{-j2\pi kn/N}\right]\left[\sum_{l=0}^{N-1} x_2(l)e^{-j2\pi kl/N}\right]e^{j2\pi km/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x_1(n)\sum_{l=0}^{N-1} x_2(l)\left[\sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N}\right]$$

Properties of DFT
Multiplication of two DFTs and Circular convolution

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & a = 1 \\ \dfrac{1-a^N}{1-a}, & a \neq 1 \end{cases}$$

$$a = e^{j2\pi(m-n-l)/N}$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & l = m-n+pN = ((m-n))_N, \quad p \text{ an integer} \\ 0, & \text{otherwise} \end{cases}$$

Properties of DFT

Multiplication of two DFTs and Circular convolution

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)x_2((m-n))_N \qquad m = 0, 1, \ldots, N-1$$

Hence, we conclude that multiplication of the DFTs of two sequences is equivalent to the circular convolution of the two sequences in time domain

Properties of DFT

Multiplication of two DFTs and Circular convolution

Example
:

Perform the circular convolution of the following two sequences:

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

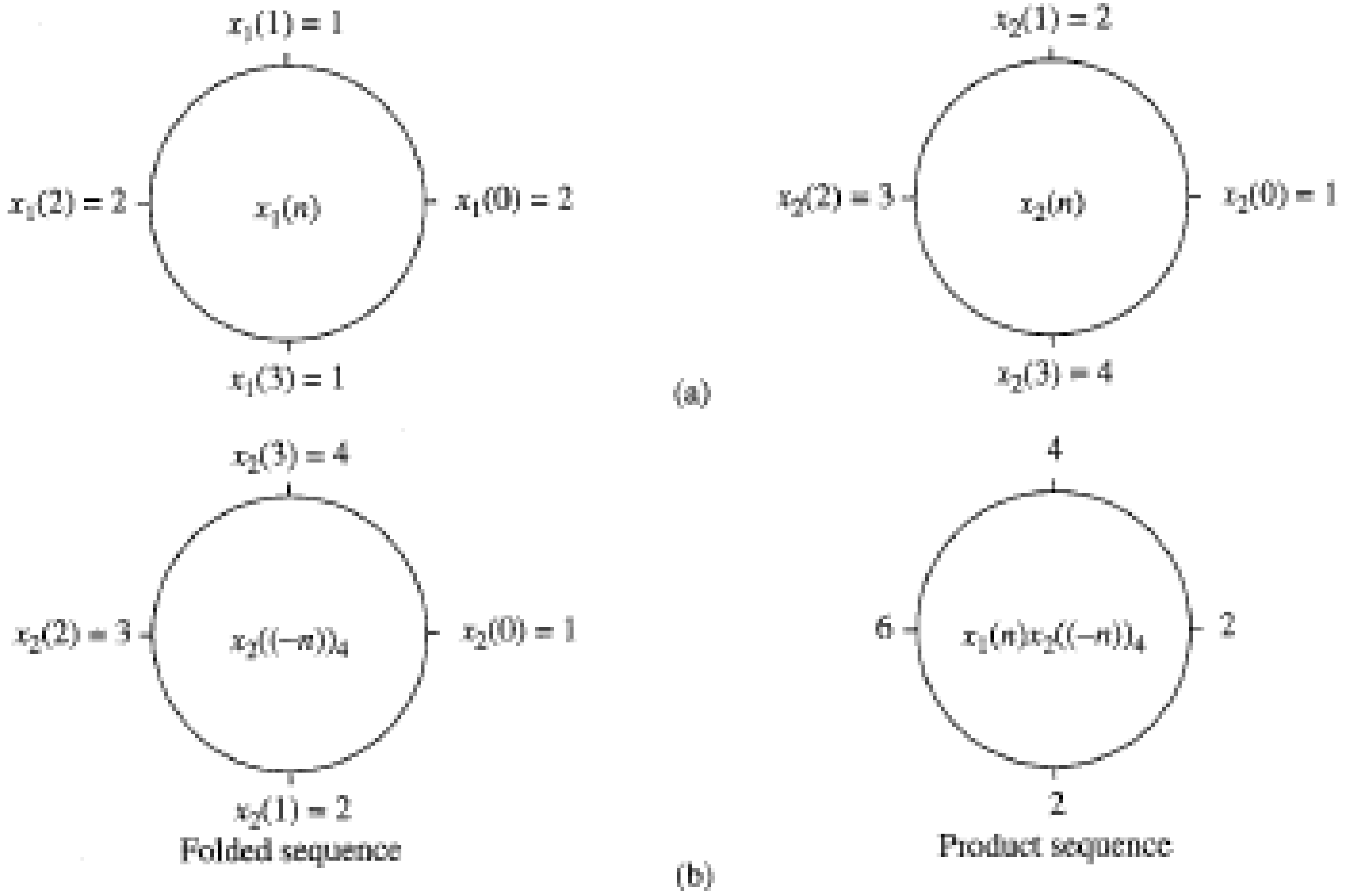
$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Solution

$$x_3(n) = \{14, 16, 14, 16\}$$

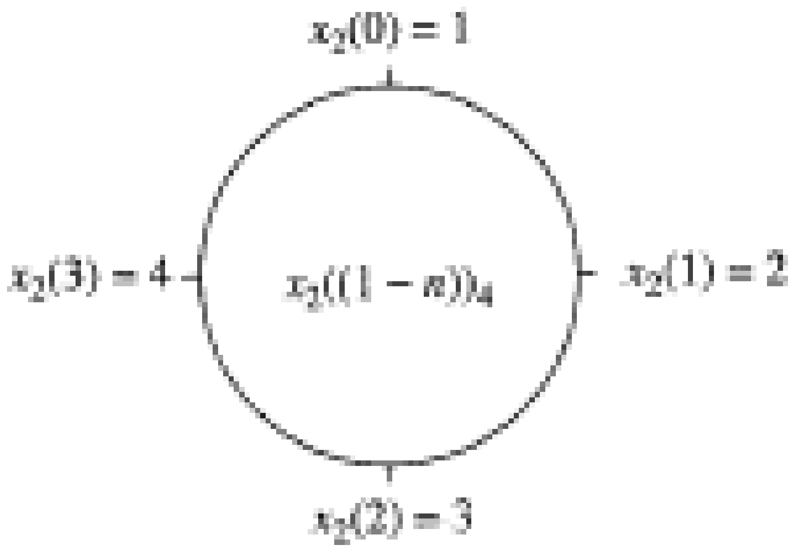
Properties of DFT

Multiplication of two DFTs and Circular convolution

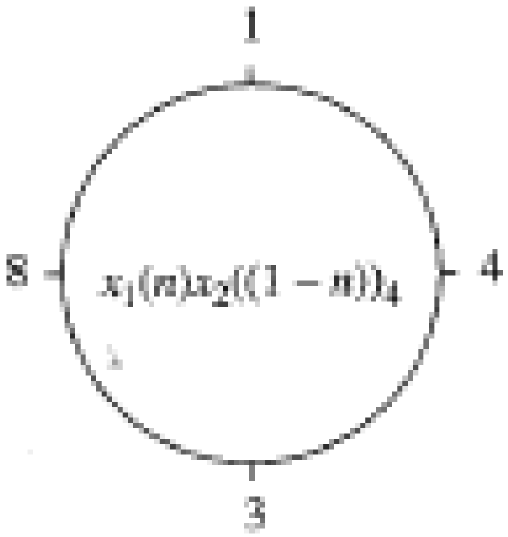


Properties of DFT

Multiplication of two DFTs and Circular convolution



Folded sequence rotated by one unit in time



Product sequence

(c)

Properties of DFT

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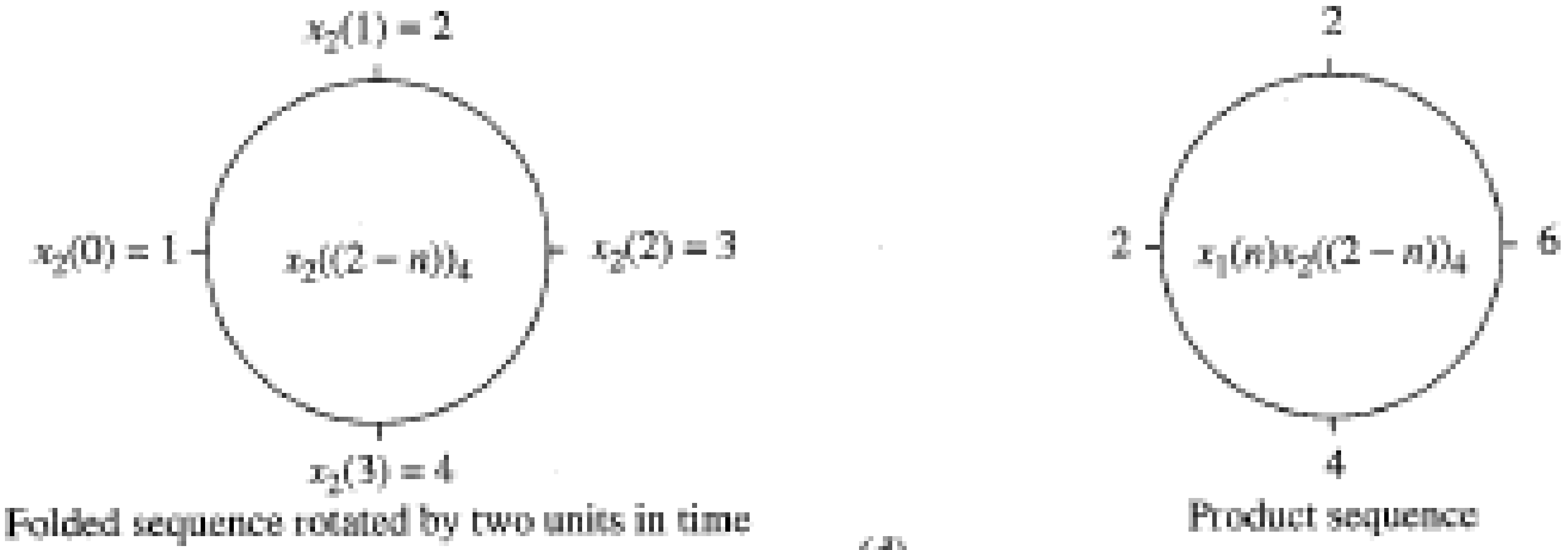
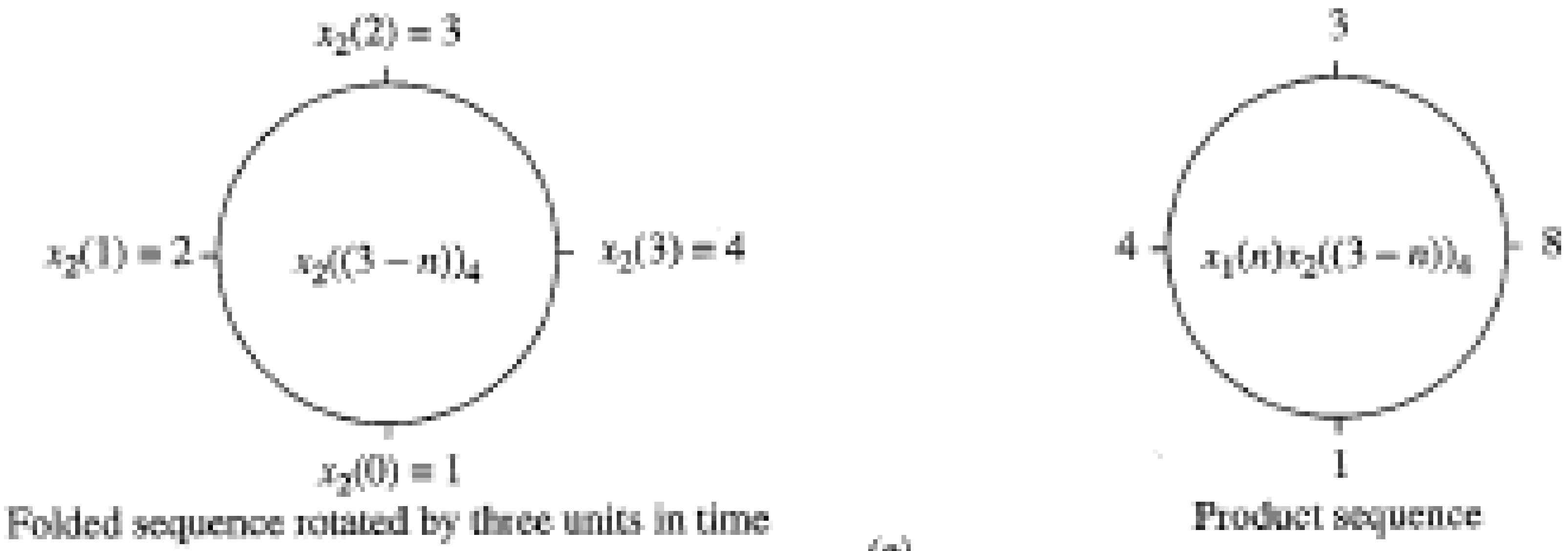


(d)

Properties of DFT

Multiplication of two DFTs and Circular convolution



(e)

Properties of DFT

Multiplication of two DFTs and Circular convolution

By means of DFT and IDFT , determine the sequence x 3 (n) corresponding to the circular convolution of the sequences in the previous example:

$$X_1(k) = \sum_{n=0}^{3} x_1(n)e^{-j2\pi nk/4} \qquad k = 0, 1, 2, 3$$

$$= 2 + e^{-j\pi k/2} + 2e^{-j\pi k} + e^{-j3\pi k/2}$$

$$X_1(0) = 6 \qquad X_1(1) = 0 \qquad X_1(2) = 2 \qquad X_1(3) = 0$$

$$X_2(k) = \sum_{n=0}^{3} x_2(n)e^{-j2\pi nk/4} \qquad k = 0, 1, 2, 3$$

$$= 1 + 2e^{-j\pi k/2} + 3e^{-j\pi k} + 4e^{-j3\pi k/2}$$

$$X_2(0) = 10 \qquad X_2(1) = -2 + j2 \qquad X_2(2) = -2 \qquad X_2(3) = -2 - j2$$

Properties of DFT

Multiplication of two DFTs and Circular convolution

$$X_3(k) = X_1(k)X_2(k)$$

$$X_3(0) = 60 \qquad X_3(1) = 0 \qquad X_3(2) = -4 \qquad X_3(3) = 0$$

$$x_3(n) = \sum_{k=0}^{3} X_3(k)e^{j2\pi nk/4} \qquad n = 0, 1, 2, 3$$

$$= \frac{1}{4}(60 - 4e^{j\pi n})$$

$$x_3(0) = 14 \qquad x_3(1) = 16 \qquad x_3(2) = 14 \qquad x_3(3) = 16$$

Properties of DFT

Multiplication of two DFTs and Circular convolution

Circular Convolution

$$x_1(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X_1(k)$$

$$x_2(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X_2(k)$$

$$x_1(n) \textcircled{N} x_2(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X_1(k)X_2(k)$$

THANK YOU

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