DIGITAL COMMUNICATION

Dr. Sanjeev G.
Department of Electronics and Communication Engg

QUANTIZATION

Quantization Error
Signal-To-Quantization Noise Ratio (SQNR)

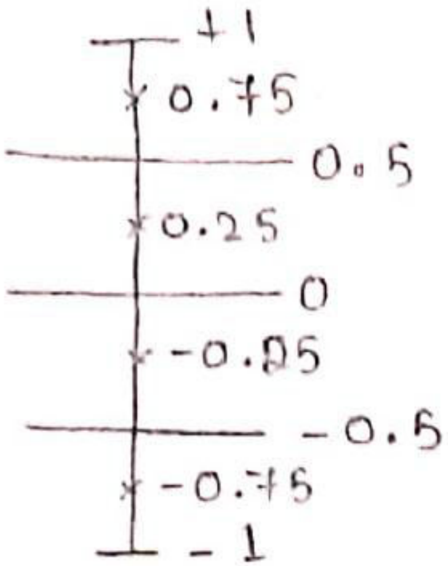
Dr. Sanjeev G.
Department of Electronics and Communication Engineering

QUANTIZATION: AN EXAMPLE

Recall

Let the sequence of samples from the output of a sampler be given as

0.35, 0.51, 0.65, 0.28, -0.06, -0.43, -0.71. Design a uniform quantizer with $L = 4$, in $[-1, 1]$



Quantization:

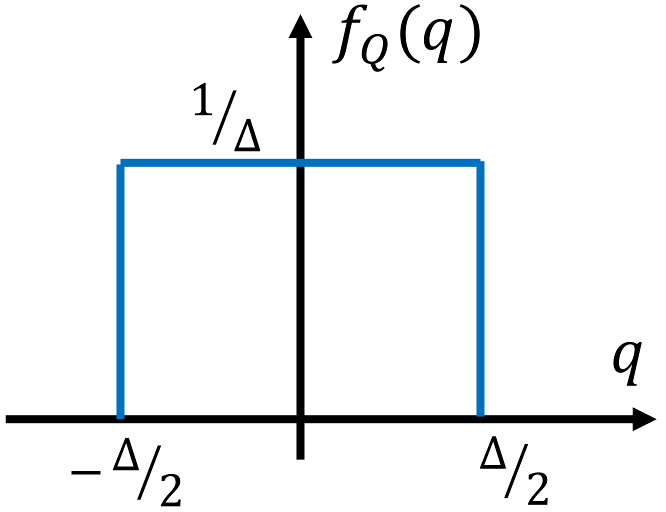
$x(n)$	$y(n)$	Bit Encoding
0.35	0.25	10
0.51	0.75	11
0.65	0.75	11
0.28	0.25	10
-0.06	-0.25	01
-0.43	-0.25	01
-0.71	-0.75	00
⋮	⋮	⋮

$x(n)$	$y(n)$	$q(n) = y(n) - x(n)$
0.38	0.25	-0.13
0.51	0.75	0.24
⋮	⋮	⋮

QUANTIZATION ERROR

Basic Formulation

- The quantization error Is defined as $q(n) = y(n) - x(n)$
- This is the error between the actual value and approximated value
- Quantization error cannot be removed/corrected
- Since $x(n)$ is random, $q(n)$ is also random
- The PDF of $x(n)$, $f_X(x)$ can be general and quantization should be characterized for all $f_X(x)$
- If $L = 2^N$ is large, then $\Delta = {}^{2A}/_{2^N}$ is small, hence we can assume the PDF of $q(n)$ is uniform distribution between $(-{}^{\Delta}/_{2}, {}^{\Delta}/_{2})$, that is $Q \sim U(-{}^{\Delta}/_{2}, {}^{\Delta}/_{2})$
- Recall that if $X \sim U(a, b)$, then
- $\mathbb{E}(X) = \frac{b-a}{2}$, and $var(X) = \frac{(b-a)^2}{12}$
- Therefore, $\mathbb{E}(Q) = 0$, and $var(Q) = \frac{\Delta^2}{12}$



QUANTIZATION ERROR

Derivation of Variance

Show that the variance of the quantization noise is $\frac{(b-a)^2}{12}$

PES UNIVERSITY ONLINE

Sln w.k.t. $\sigma_x^2 = E[X^2] - \mu_X^2$.

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$= \int_a^b x^2 \cdot \frac{1}{(b-a)}\,dx$$

$$= \left.\frac{x^3}{3(b-a)}\right|_a^b$$

$$E[X^2] = \frac{b^3-a^3}{3(b-a)}$$

$$\sigma_x^2 = \frac{b^2+ab+a^2}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{4b^2+4ab+4a^2-3a^2-6ab-3b^2}{12}$$

$$= \frac{b^2-2ab+a^2}{12}$$

$$\therefore \boxed{\sigma_x^2 = \frac{(b-a)^2}{12}}$$

SIGNAL-TO-QUANTIZATION NOISE RATIO

Also Called as Signal-To-Noise Ratio

- A measure of the performance of a quantizer is the signal-to-quantization noise ratio, which is defined as

$$\mathrm{SQNR} = \frac{\text{Average Signal Power}}{\text{Average Quantization Noise Power}}$$

- The signal $x(n)$ is usually assumed to have zero mean and average signal power (or variance) σ_x^2
- Recall that the variance of quantization noise is $\Delta^2/_{12}$. Hence, the SQNR is

$$\mathrm{SQNR} = \frac{12\,\sigma_x^2}{\Delta^2}$$

- Sometimes, SQNR is also called as the signal-to-noise ratio (SNR); however, this should be avoided for reasons that we will see later

THANK YOU

Dr. Sanjeev G.
Department of Electronics and Communication Engineering

sanjeevg@pes.edu

+91 80 2672 1983 Extn 838