DIGITAL IMAGE PROCESSING-1

Unit 4: Lecture 46

Dr. Shikha Tripathi
Department of Electronics &
Communication Engineering

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Unit 4: Image Filtering and Restoration

Dr. Shikha Tripathi
Department of Electronics & Communication Engineering

Last Session

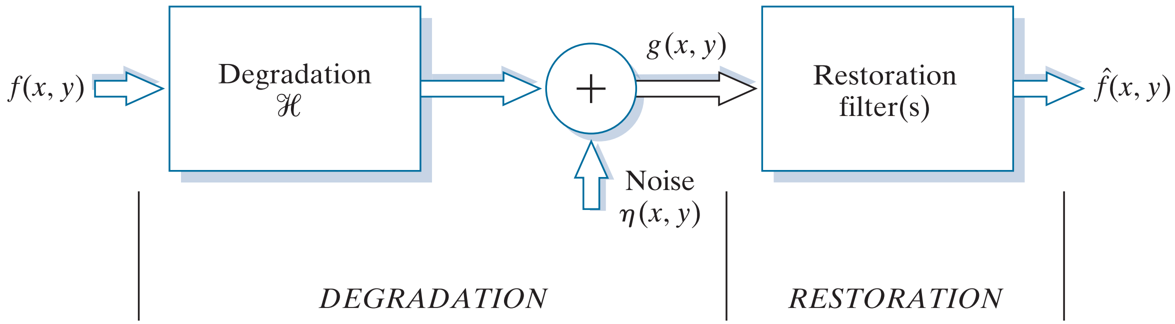
- Estimation of degradation function
- Image Restoration in presence of degradation only
    - Restoration using Inverse filtering
- Image Restoration in presence of degradation and noise

This Session

- Image Restoration in presence of degradation and noise
  - Restoration using Weiner Filters
  - Constrained least square filters

Restoration in the Presence of Degradation Only

Degradation / Restoration Model



- Spatial Domain: $g(x,y) = h(x,y)*f(x,y) + \eta(x,y)$
- Frequency Domain: $G(u,v) = H(u,v).F(u,v) + N(u,v)$

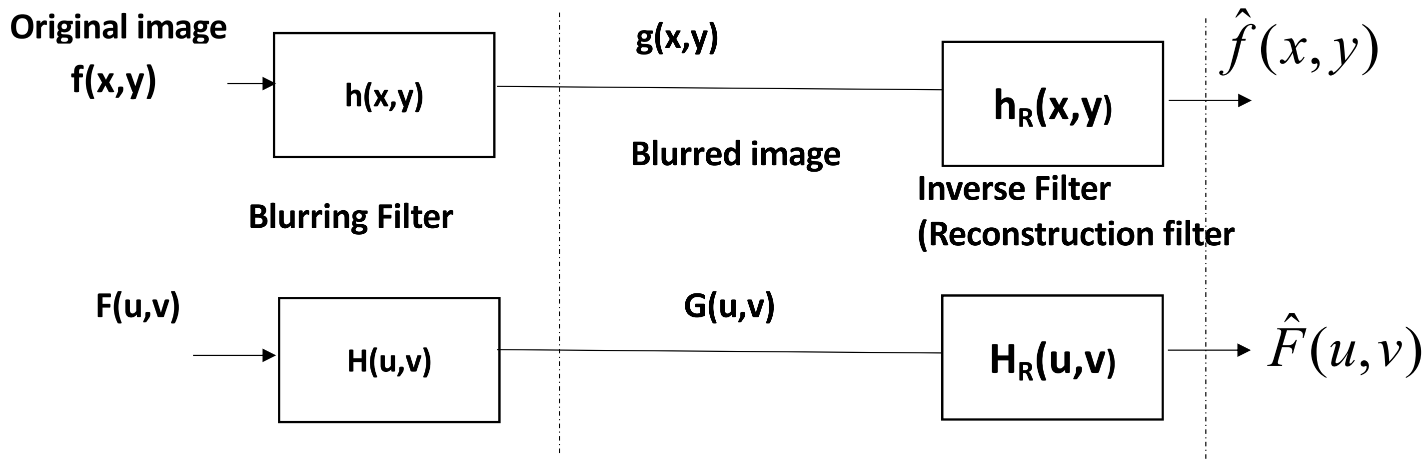
- **Restoration**

✓ **by Inverse Filtering**

2. Minimum Mean Square Error Filtering (Wiener Filtering)

3. Constrained Least Square Filtering

Inverse Filter in Time and Frequency Domain



$$G(u,v) = H(u,v).F(u,v)$$

And $$\hat{F}(u,v) = H_R(u,v).G(u,v)$$

$$\therefore \hat{F}(u,v) = \underbrace{H_R(u,v).H(u,v)}_{1}.F(u,v)$$

Inverse Filter in Time and Frequency Domain

- Reconstructed image can be obtained by inverse filter

$$H_R(u,v) = \frac{1}{H(u,v)}$$

- Consider restoration of images degraded by degradation filter H (which is given or obtained by known methods)
- Simplest approach is direct inverse filtering

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Image with Degradation and Noise

- Consider image with degradation and Noise

$$G(u,v) = F(u,v).H(u,v) + N(u,v)$$

$$Or\ \hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

- We can’t recover f(x,y) exactly even if we know H(u,v) as N(u,v) is not known
- Also if H(u,v) has zero or small values, the ratio could dominate F(u,v), making it more noisy

Image with Degradation and Noise

- Problems:
  - We don't know $N(u,v)$
  - $H(u,v)$ often has zero values or small values.

If $H(u,v) = 0, \quad N(u,v)/H(u,v) \rightarrow \infty$

If $H(u,v) \approx 0, \quad N(u,v)/H(u,v) \rightarrow max$

Thus noise is amplified & dominates output.

Solution:

- One way to resolve small value problem is to limit the filter frequencies to values near origin. We know that H(0,0) is usually the highest value of H(u,v) spectrum. **(Zero Filtering)**
- Hence by limiting analysis near origin we avoid small values

Limitations of Inverse Filtering

- It is an unstable filter
- It is sensitive to noise

- **In practice inverse filter is not popularly used**

✓By Inverse Filtering

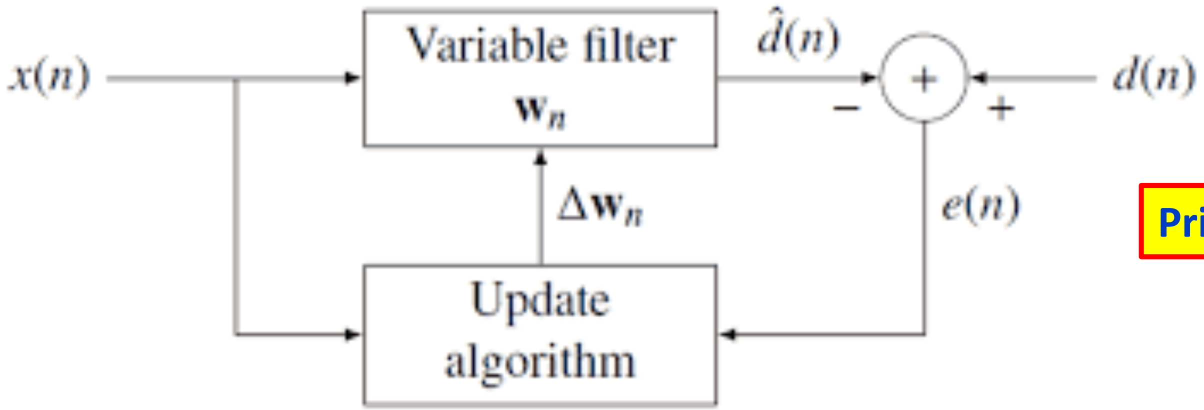
1. **Minimum Mean Square Error Filtering (Wiener Filtering)**
2. Constrained Least Square Filtering

Minimum Mean Square Error (Wiener) Filtering

- Inverse filtering approach makes no explicit provision for handling noise
- Wiener filtering approach incorporates both degradation function & statistical characteristics of noise for restoration
- Here image and noise are considered as random variables
- Assumption: Image and noise have zero mean

Wiener Filter

- **Wiener filter** is a filter used to produce an estimate of a desired or target random process by linear time-invariant (LTI) filtering of an observed noisy process, assuming known stationary signal and noise spectra, and additive noise.



Principle: To minimize mean square error

Minimum Mean Square Error (Wiener) Filtering

- Wiener filter restores the image in presence of blur (degradation) as well as noise
- **The Objective of Wiener filter is to minimize mean square error**

$$e^2 = E[(f - \hat{f})^2]$$

***Conditions:**

1. Noise intensities & image intensities are uncorrelated.
2. Intensity levels in the estimate($\hat{f}$) are linear function of levels in the degraded image ‘g’

Minimum Mean Square Error (Wiener) Filtering

- Based on these assumptions the estimate of restored image in frequency domain is given as

$$\hat{F}(u,v) = \left[\frac{H^*(u,v) S_f(u,v)}{S_f(u,v) |H(u,v)|^2 + S_\eta(u,v)} \right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

Where

$\hat{F}(u,v) =$ *Fourier transform of the estimate of the undegraded image*

$G(u,v) =$ *Fourier transform of the degraded image*

$H(u,v) =$ *Degradation transfer function*

$H^*(u,v) =$ *Complex conjugate of* $H(u,v)$

$|H(u,v)|^2 = H^*(u,v) H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2 =$ *Power spectrum of noise*

$S_f(u,v) = |F(u,v)|^2 =$ *Power spectrum of undegraded image*

Minimum Mean Square Error (Wiener) Filtering

- The restored image in spatial domain is given by the Inverse Fourier transform of the frequency domain estimate of $\hat{F}(u,v)$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

- If noise is zero then noise power spectrum vanishes and Wiener filter reduces to inverse filter

Minimum Mean Square Error (Wiener) Filtering

- When dealing with white noise, spectrum is constant thus simplifying the estimate

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

- However power spectrum of undegraded image and noise is seldom known

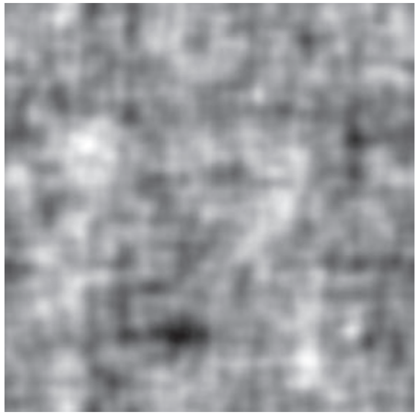
- An approach frequently used is to approximate the expression to

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + k} \right] G(u,v)$$

  - Where k is a specified constant added to all terms of $|H(u,v)|^2$

Example 1

Severe turbulence,
$k = 0.0025$



Result of full inverse filtering

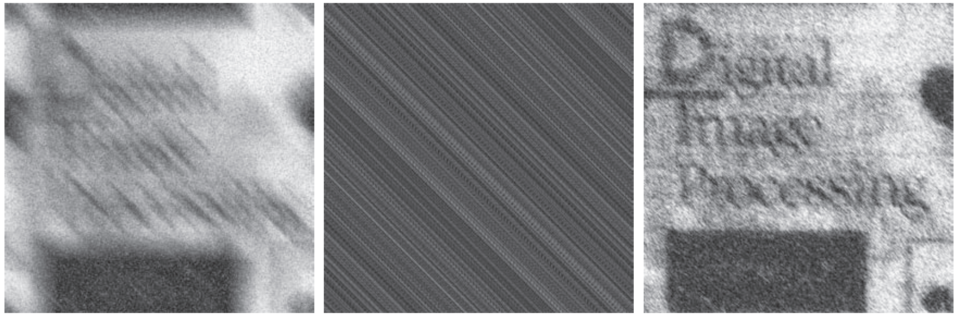
Radially limited inverse filter result

Wiener filter result.

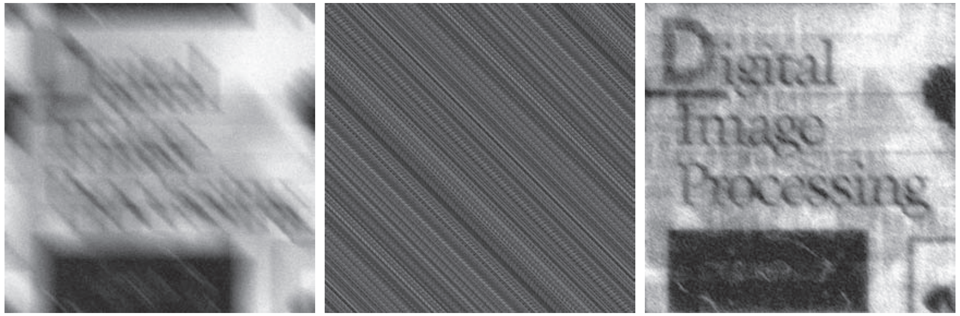
Comparison of inverse and Wiener filtering.

- Here k is chosen iteratively to give best visual result

Example 2



8-bit image corrupted by motion blur and additive noise.



Same sequence, but with noise variance one order of magnitude less.

Result of inverse filtering and Result of Wiener filtering.

Same sequence, but noise variance reduced by five orders

Note in middle figure how the deblurred image is quite visible through a "curtain" of noise.

Several Cases to test Wiener Filter

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

- Case 1: When noise is zero,

  Wiener filter = Inverse filter

- Case 2: If

$$\frac{S_\eta(u,v)}{S_f(u,v)} << 1 \quad i.e,\ k << 1$$

  - Wiener filter acts like a passband filter and allows the signal to pass through without any attenuation

Several Cases to test Wiener Filter

- Case 3: If

$$\frac{S_\eta(u,v)}{S_f(u,v)} >> 1 \quad i.e,\ k >> 1$$

- $H_R(u,v) \approx 0$ and Wiener filter acts like a stopband filter for signal and doesn't allow signal to pass, thus attenuating noise
- If noise frequency is high, Wiener filter will not consider that filter coefficient

Summary: Minimum Mean Square Error (Wiener) Filtering

- The estimate of restored image in frequency domain using minimum mean square error is given as

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

Minimum Mean Square Error (Wiener) Filtering

- Drawback: Power spectrum of undegraded image and noise is seldom known
- An approach frequently used is to approximate the expression to

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + k} \right] G(u,v)$$

Limitations of these Methods

- The problem of having to know something about the degradation function *H* is common to all methods discussed so far
- However, the Wiener filter presents an additional difficulty
  - **The power spectra of the undegraded image and noise must also be known**
  - **In some cases it is possible to achieve acceptable results using the approximation but a constant value for the ratio of the power spectra is not always a suitable solution**

1. By Inverse Filtering
2. Minimum Mean Square Error Filtering (Wiener Filtering)
3. **Constrained Least Square Filtering**

Constrained Least Square Filter

- Drawback of Wiener filter: performance depends upon the correct estimation of the value of k
- Constrained Least Square Filtering approach doesn't make any assumption about the original undegraded image.
- **It only requires knowledge of the mean and variance of the noise which can be calculated from the degraded image**

Constrained Least Square Filter

- Degradation process can be expressed in matrix form as $\mathbf{g = Hf + \eta}$

  Where g,f and η are vectors of size: MNx1

  Matrix H has dimension MN x MN

- **H is sensitive to noise → Optimality of restoration is based on the measure of smoothness (i.e, Minimize Laplacian operator)**

Constrained Least Square Filter

- Second order derivative operator or Laplacian operator tries to enhance the irregularities or discontinuities in the image.

- So if we can minimize the Laplacian of reconstructed image, that will ensure that the reconstructed image will be smooth

  → Optimization problem

Technique

- Find the minimum of criterion function, C

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x,y) \right]^2$$

- Subject to the constraint $\left\| \mathbf{g} - \mathbf{H}\hat{\mathbf{f}} \right\|^2 = \left\| \boldsymbol{\eta} \right\|^2$

(Since $\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\eta}$)

where $\left\| \mathbf{a} \right\|^2 \triangleq \mathbf{a}^T \mathbf{a}$ is the Euclidean norm

Technique

- The frequency domain solution to this optimization problem is given by

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

- Where γ has to be adjusted to satisfy the constraint $\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\boldsymbol{\eta}\|^2$
- The function reduces to inverse filtering if γ=0

Technique

- And P(u,v) is the Fourier transform of padded version of Laplacian p(x,y) given by

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

- Image is of size (MxN), so before we compute Fourier transform of p(x,y) which is (3x3), we have to pad appropriate number of zeros so that p(x,y) is also of (MxN) size, then compute P(u,v)

Technique

- γ can be adjusted iteratively until acceptable results are achieved
- For optimality this parameter has to be adjusted to satisfy the constraint
- Procedure for computing γ by iteration:
- Define a residual vector **r** as

$$\mathbf{r} = \mathbf{g} - \mathbf{H}\hat{\mathbf{f}}$$ **Since η = g - Hf**

- $\hat{F}(u,v)$ (and also $\hat{f}$) is a function of γ.
- Hence **r** is also a function of γ

Technique

- It has been shown (Hunt [1973], Gonzalez and Woods [1992]) that

$$\phi(\gamma) = \mathbf{r}^T\mathbf{r}$$
$$= \|\mathbf{r}\|^2$$

is a monotonically increasing function of γ

- We need to adjust γ so that

$$\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2 \pm \alpha$$

where α is an accuracy factor

Technique

- If $\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2$ the constraint will be strictly satisfied
- Because $\phi(\gamma)$ is monotonic, finding desired value of γ is not difficult.
- Iterative procedure is as follows:
- Step 1: Specify an initial value of γ
- Step 2: Compute $\|\mathbf{r}\|^2$

$$r = g - H\hat{f}$$

$$\|r\|^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} r^2(x, y)$$

Technique

- Step 3: Stop if. $\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2 \pm \alpha$ condition is satisfied otherwise return to Step 2 and increase or decrease γ suitably

else

if $\|r\|^2 \leq \|n\|^2 - a$, then increase γ and go to 2

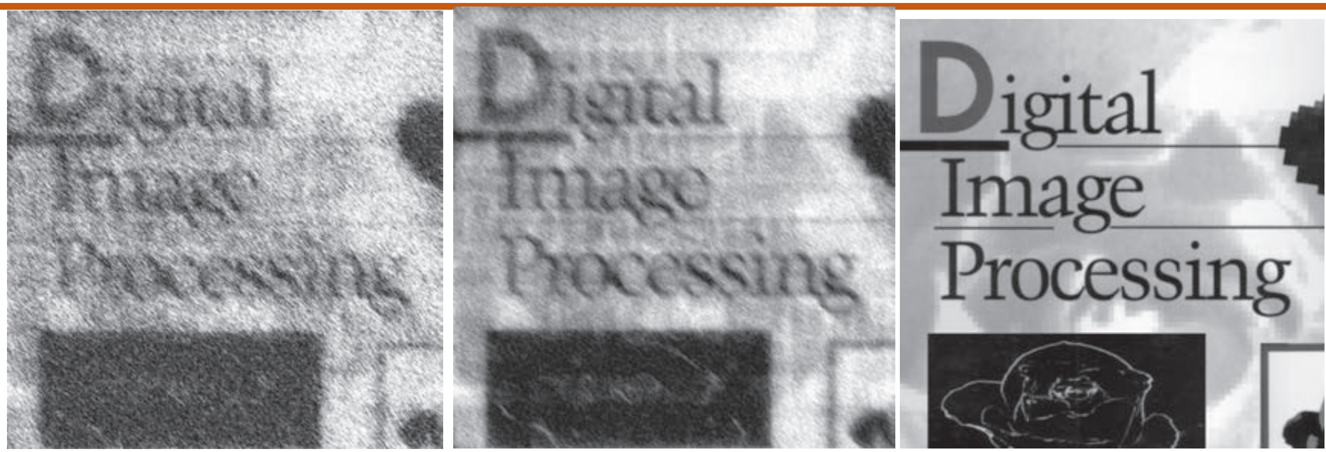
if $\|r\|^2 \geq \|n\|^2 + a$, then decrease γ and go to 2.

Use new value of γ to recompute

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma\,|P(u,v)|^2}\right] G(u,v)$$

$$\hat{f} = \mathrm{IDFT}\left[\hat{F}(u,v)\right]$$

Example



Result of Wiener filtering with different noise variance

Results of constrained least squares filtering

Example

Severe turbulence,
$k = 0.0025$

Iteratively determined constrained least squares restoration using correct noise parameters.

Result obtained with wrong noise parameters.

Next Session

Unit 5: Color Image Processing

THANK YOU

Dr. Shikha Tripathi
Department of Electronics & Communication Engineering

shikha@pes.edu
+91 9482219115