Principles of Digital Signal Processing

Dr. B. Niranjana Krupa

Department of Electronics and Communication.

DSP

Discrete Fourier Transform

Dr. B. Niranjana Krupa

Department of Electronics and Communication.

PES UNIVERSITY ONLINE

Recovering x(n):

When $x(n)$ has infinite duration, the equally spaced frequency samples do not represent $x(n)$.

They (frequency samples) only correspond to a periodic sequence $x_p(n)$, the aliased version of $x(n)$, as in

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$$

DFT

When $x(n)$ has finite duration of length $L \leq N$, then

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$$

$$x_p(n) = \begin{cases} x(n), & 0 \leq n \leq L-1 \\ 0, & L \leq n \leq N-1 \end{cases}$$

Frequency domain sampling

DFT

It is important to note that zero padding doesnt provide any additional information about the spectrum of the sequence. However padding zeros and computing N point DFT only results in better display.

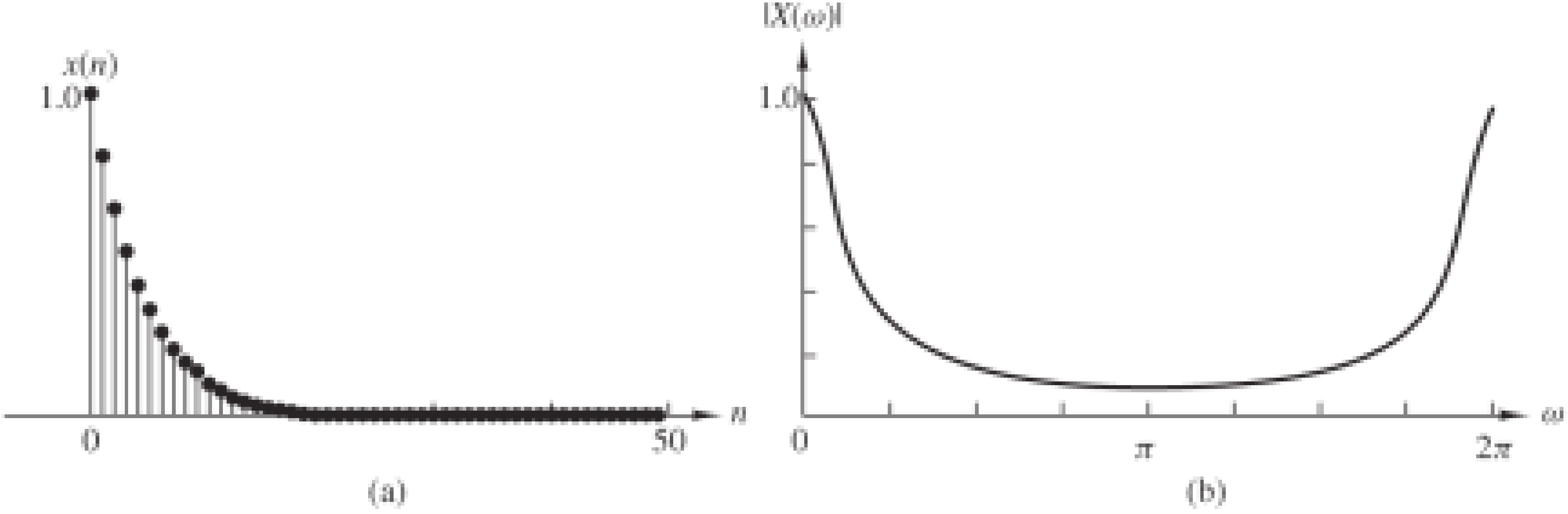

Figure 7.1.4 (a) Plot of sequence $x(n) = (0.8)^n u(n)$; (b) its Fourier transform

Frequency domain sampling

DFT

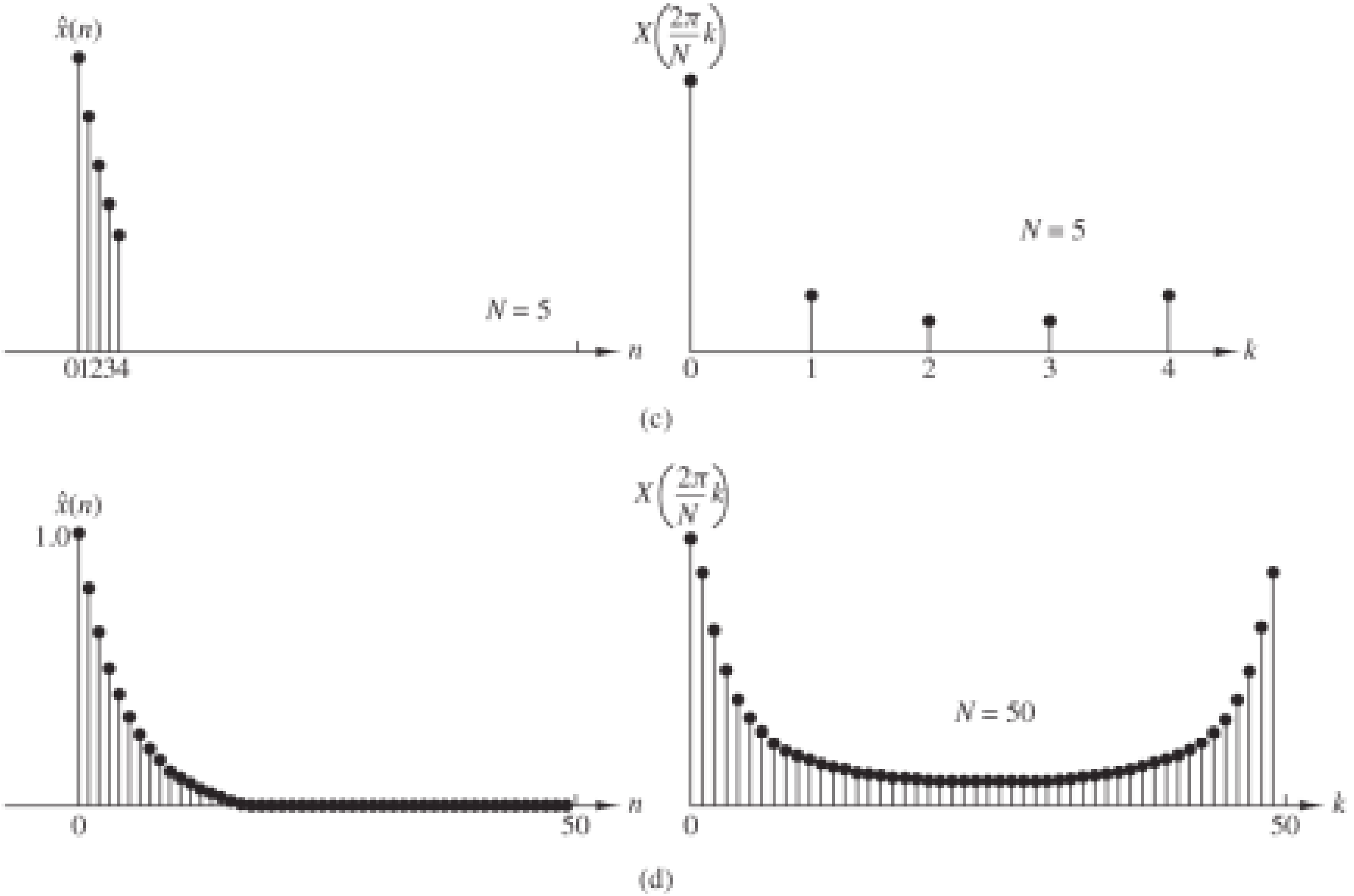

Figure 7.1.4 (a) Plot of sequence $x(n) = (0.8)^n u(n)$; (b) its Fourier transform (magnitude only); (c) effect of aliasing with $N = 5$; (d) reduced effect of aliasing with $N = 50$.

Frequency domain sampling

DFT

In summary, a finite-duration sequence $x(n)$ of length L [i.e., $x(n) = 0$ for $n < 0$ and $n \geq L$] has a Fourier transform

$$X(\omega) = \sum_{n=0}^{L-1} x(n)e^{-j\omega n} \qquad 0 \leq \omega \leq 2\pi$$

where the upper and lower indices in the summation reflect the fact that $x(n) = 0$ outside the range $0 \leq n \leq L-1$. When we sample $X(\omega)$ at equally spaced frequencies $\omega_k = 2\pi k/N$, $k = 0, 1, 2, \ldots, N-1$, where $N \geq L$, the resultant samples are

$$X(k) \equiv X\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{L-1} x(n)e^{-j2\pi kn/N}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \qquad k = 0, 1, 2, \ldots, N-1$$

Frequency domain sampling

DFT

To summrise DFT and IDFT

$$\text{DFT:}\quad X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \qquad k = 0, 1, 2, \ldots, N-1$$

$$\text{IDFT:}\quad x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}, \qquad n = 0, 1, 2, \ldots, N-1$$

Frequency domain sampling

DFT as linear transform

DFT and IDFT expressed as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad k = 0, 1, \ldots, N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \qquad n = 0, 1, \ldots, N-1$$

Where, $W_N = e^{-j2\pi/N}$

The *Nth* root of unity

Frequency domain sampling

DFT as linear transform

- Computational complexity of DFT
  - Each point: N complex multiplications and $(N-1)$ complex additions
  - Hence, N-point DFT:
    - $N*N$ complex multiplications and
    - $N*(N-1)$ complex additions
- DFT and IDFT as linear transformations on $x(n)$ and $X(k)$
  - Consider N-*point* vectors x_N of $x(n)$ and X_N of $X(k)$ and an NXN matrix W_N as:

Frequency domain sampling

DFT as linear transform

$$\mathbf{x}_N = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}, \qquad \mathbf{X}_N = \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}$$

Frequency domain sampling

DFT as linear transform

$$
\mathbf{W}_N = \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\
 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\
\vdots & \vdots & \vdots & \vdots\vdots\vdots & \vdots \\
1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)}
\end{bmatrix}
$$

DFT as linear transform

N-point DFT expressed in matrix form as:

$$\mathbf{X}_N = \mathbf{W}_N \mathbf{x}_N$$

W_N is the matrix of linear transformation and is a s
ymmetric matrix

If we assume that the inverse of W_N exists then,

IDFT $\rightarrow$ $$\mathbf{x}_N = \mathbf{W}_N^{-1} \mathbf{X}_N \qquad \mathbf{x}_N = \frac{1}{N} \mathbf{W}_N^{*} \mathbf{X}_N$$

Frequency domain sampling

DFT as linear transform

Comparing

$$\mathbf{W}_N^{-1} = \frac{1}{N}\mathbf{W}_N^*$$

$$\mathbf{W}_N\mathbf{W}_N^* = N\mathbf{I}_N$$

W_N is an orthogonal (unitary) matrix

And its inverse exists, given as shown above

THANK YOU

Dr. B. Niranjana Krupa

Department of Electronics and Communication

bnkrupa@pes.edu

+91 80 6666 3333 Ext 777