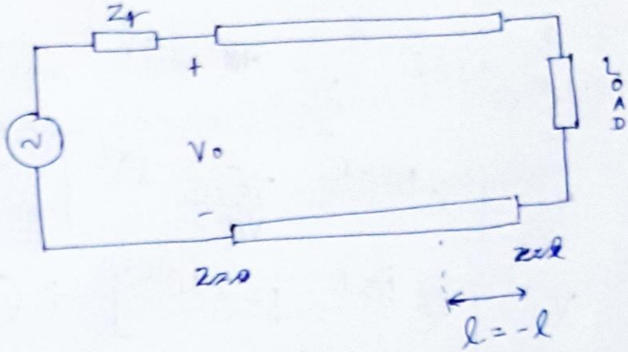
POWER

The transmission line is used in transferring power from source to the load

Consider the transmission line



In general

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

For a lossless line $\alpha = 0$

$$V_s(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$

At $z = -l$ [Because it is measured from load]

$$V_s(l) = V_o^+ e^{-j\beta(-l)} + V_o^- e^{+j\beta(-l)}$$

$$V_s(l) = V_o^+ . e^{j\beta l} + V_o^- e^{-j\beta l}$$

$$V_s(l) = V_o^+ \left[e^{j\beta l} + \frac{V_o^-}{V_o^+} e^{-j\beta l} \right]$$

$$V_s(l) = V_o^+ \left[e^{j\beta l} + \Gamma e^{-j\beta l} \right] \rightarrow (1)$$

Γ = Reflection coefficient

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{+\gamma z}$$

$\alpha = 0$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{+j\beta z}$$

$z = -l$

$$I_s(l) = \frac{V_o^+}{Z_o} e^{+j\beta l} - \frac{V_o^-}{Z_o} e^{-j\beta l}$$

$$I_s(l) = \frac{V_o^+}{Z_o} \left[e^{+j\beta l} - \frac{V_o^-}{V_o^+} e^{-j\beta l} \right]$$

$$I_s(l) = \frac{V_o^+}{Z_o} \left[e^{j\beta l} - \Gamma e^{-j\beta l} \right] \rightarrow 2$$

$$P_{ave} = \frac{1}{2} Re \left[V_s(l) \, I_s^*(l) \right]$$

$$P_{ave} = \frac{1}{2} Re \left[V_o^+ \left[e^{j\beta l} + \Gamma e^{-j\beta l} \right] \times \frac{(V_o^+)^*}{Z_o} \left[e^{-j\beta l} - \Gamma^* e^{+j\beta l} \right] \right]$$

$$= \frac{1}{2} Re \left[\frac{|V_o^+|^2}{Z_o} \left[1 - |\Gamma|^2 + \Gamma e^{-j2\beta l} - \Gamma^* e^{j2\beta l} \right] \right]$$

since Last two terms are imaginary

$$P_{ave} = \frac{1}{2 Z_o} \left[|V_o^+|^2 \left(1 - |\Gamma|^2 \right) \right]$$

$$P_{ave} = \frac{|V_o^+|^2}{2 Z_o} \left(1 - (|\Gamma|^2) \right)$$

$$\boxed{P_{ave} = \frac{|V_o^+|^2}{2 Z_o} \left[1 - |\Gamma|^2 \right]} \rightarrow 3$$

We know $P_t = P_i - P_r$

P_t = transmitted power
P_i = Incident Power
P_r = Reflected power

From Equation 3 it is evident that the power is Constant and it does not depend on "l"

when $\Gamma = 0$, Maximum power is transmitted

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{when } Z_L = Z_0 \quad \Gamma_L = 0$$

Reflection coefficient for different cases of lossless line

There are three different cases they are

A) Shorted Line ($Z_L = 0$)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0 - Z_0}{0 + Z_0} = -1$$

$$S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1+1}{1-1} = \infty$$

where $\alpha = 0$

$$Z_{sc} = Z_{in}\Big|_{Z_L=0} = Z_0 \left[\frac{\cancel{Z_L}^{0} + jZ_0 \tan \beta l}{Z_0 + j\cancel{Z_L}^{0} \tan \beta l} \right]$$

$$Z_{sc} = Z_{in}\Big|_{Z_L=0} = jZ_0 \tan \beta l$$

B) Open circuited Line $(Z_L = \infty)$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\cancel{Z_L}\left[1 - \cancel{\frac{Z_0}{Z_L}}^{\to 0}\right]}{\cancel{Z_L}\left[1 + \cancel{\frac{Z_0}{Z_L}}^{\to 0}\right]} = 1$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1+1}{1-1} = \infty$$

where $\alpha = 0$

$$Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = \lim_{Z_L \to \infty} Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= \lim_{Z_L \to \infty} Z_{in} = Z_0 \times \frac{\cancel{Z_L}}{\cancel{Z_L}} \left[\frac{1 + j\cancel{\frac{Z_0}{Z_L}}^{\to 0} \tan\beta l}{\cancel{\frac{Z_0}{Z_L}}^{\to 0} + j\tan\beta l}\right]$$

$$= \lim_{Z_L \to \infty} Z_{in} = Z_0 \left[\frac{1}{j\tan\beta l}\right]$$

$$= -jZ_0 \cot\beta l$$

c) Matched Line $(Z_L = Z_0)$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L - Z_L}{Z_L + Z_L} = 0$$

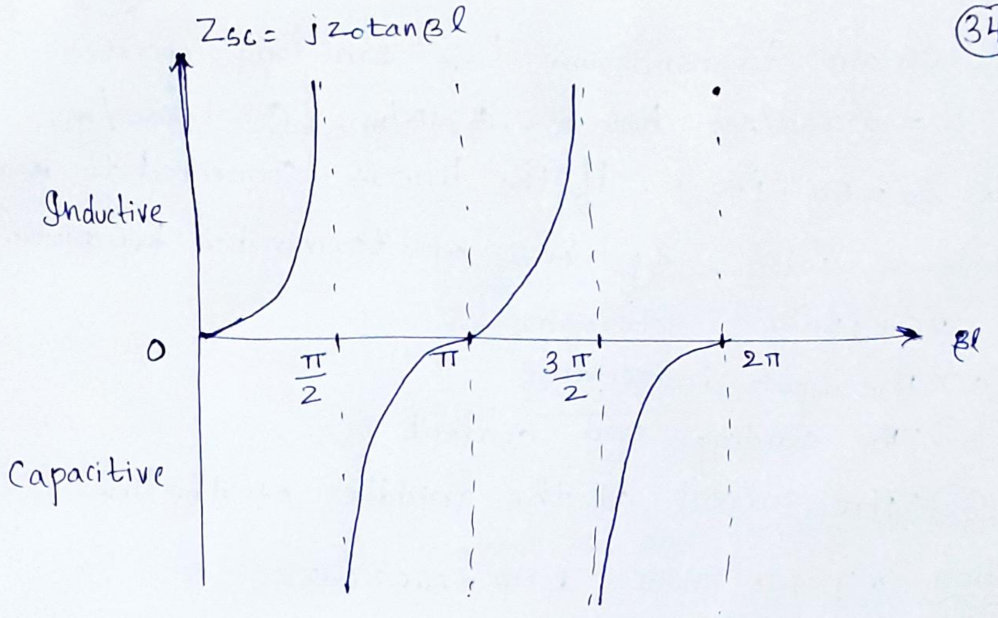
where $\alpha = 0$

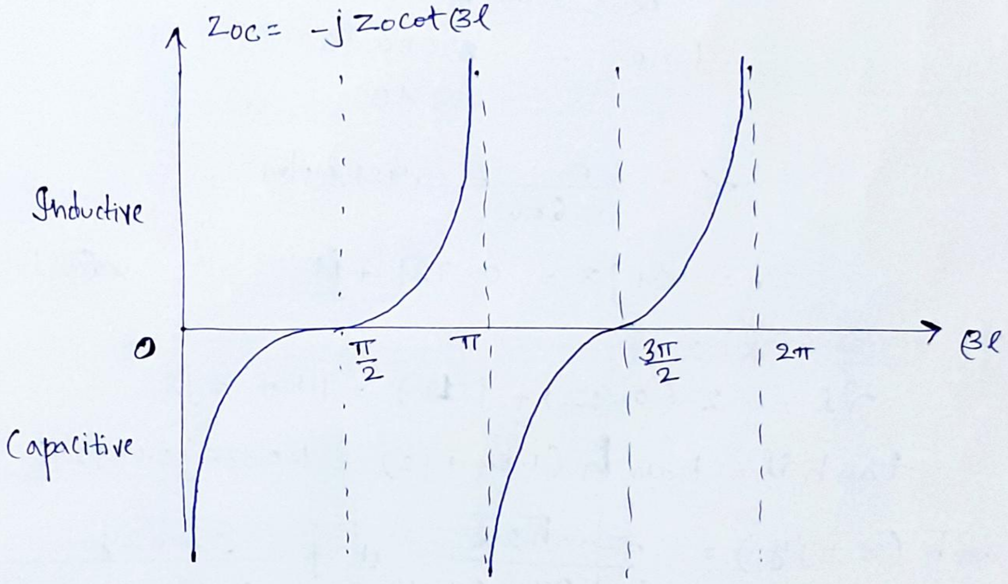
$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$Z_{in} = Z_0 \left[\frac{Z_0 + jZ_0 \tan\beta l}{Z_0 + jZ_0 \tan\beta l}\right] = Z_0$$

$$Z_{in} = Z_0$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1$$


Zsc= jZotanβl
Inductive
0
π/2
π
3π/2
2π
βl
Capacitive


Zoc= -jZocotβl
Inductive
0
π/2
π
3π/2
2π
βl
Capacitive

1) A certain transmission line 2m long operates at $\omega = 10^6$ rad/sec has $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_0 = 60 + j40\ \Omega$. If the line is connected to a Source of $10\angle 0^\circ$, $Z_g = 40\Omega$ and terminated by a load of $20 + j50\ \Omega$, determine

(a) The input impedance
(b) The sending end current
(c) The current at the middle of the line

Solution: (a) We know $1\ Np = 8.686\,dB$

$$\alpha = 8\,dB/m$$

$$1\ NP \quad - \quad 8.686\ dB$$

$$? \quad - \quad 8\ dB$$

$$\alpha = \frac{8}{8.686} = 0.921\ Np/m$$

$$\gamma = \alpha + j\beta = 0.921 + j1 \qquad \beta = 1$$

$$l = 2$$

$$\gamma l = 2(0.921 + j(1)) = 1.84 + j2$$

$$\tanh \gamma l = \tanh(1.84 + j2) = 1.033 - j0.03929$$

$$\tanh(x \pm jy) = \frac{\sinh 2x}{\cosh 2x + \cos 2y} \pm j\frac{\sin 2y}{\cosh 2x + \cos 2y}$$

$$Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right]$$

$$= (60 + j40)\left[\frac{(20 + j50) + (60 + j40)(1.033 - j0.03929)}{(60 + j40) + (20 + j50)(1.033 - j0.03929)}\right]$$

$$Z_{in} = 60.25 + j38.79\ \Omega = 71.66\angle 32.77^\circ$$

b) The sending end current

$$I(z=0) = I_0$$

$$I(z=0) = \frac{V_g}{Z_{in}+Z_g} = \frac{10}{60.25+j38.79+40}$$

$$I(z=0) = 93.03 \angle -21.15^\circ \times 10^{-3} \text{ A}$$

c) To find the current at any point, we need V_0^+ and V_0^-

$$I_0 = I(z=0) = 93.03 \angle -21.15^\circ \times 10^{-3}$$

$$V_0 = Z_{in} I_0 = (71.66 \angle 32.77^\circ)(93.03 \times 10^{-3} \angle -21.15^\circ)$$

$$V_0 = 6.667 \angle 11.62^\circ$$

We know $V_0^+ = \frac{1}{2}\left[V_0 + I_0 Z_0\right]$

$$= \frac{1}{2}\left[6.667 \angle 11.62^\circ + \left[93.03 \times 10^{-3} \angle -21.15^\circ\right]\left[60+j40\right]\right]$$

$$= 6.687 \angle 12.08^\circ$$

$$V_0^- = \frac{1}{2}\left[V_0 - I_0 Z_0\right]$$

$$= 0.0518 \angle 260^\circ$$

At the middle of the line, $z = l/2$,

$$\gamma z = 0.921 + j1$$

$$I_s(z=l/2) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$I_s(z=l/2) = 35.10 \angle 281^\circ \times 10^{-3} \text{ A}$$

Note: Velocity is generally termed as phase velocity

(1) A transmission line operating at 500 MHz has $Z_0 = 80\,\Omega$, $\alpha = 0.04$ NP/m, $\beta = 1.5$ rad/m. Find the line parameters R, L, G and C

Solution: Since $\alpha = 0.04$ NP/m $\neq 0$ and $Z_0 = 80\,\Omega$,

~~which is real~~ $Z_0 =$ Real, the given transmission line is a distortionless transmission line.

$$\therefore \quad RC = GL \rightarrow (1)$$

$$\alpha = \sqrt{RG} \rightarrow (2)$$

$$\beta = \omega\sqrt{LC} \rightarrow (3)$$

But $C = \frac{GL}{R} \rightarrow (4)$

using (4) in (3) $\beta = \omega\sqrt{\frac{L \times GL}{R}}$

$$\beta = \omega L\sqrt{\frac{G}{R}} \rightarrow (5)$$

But we know for a distortion less line $Z_0 = \sqrt{\frac{L}{C}} \rightarrow (6)$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}} = 80 \rightarrow (7)$$

$$\beta = \omega L \times \frac{1}{\sqrt{\frac{R}{G}}} \Rightarrow \frac{\omega L}{Z_0} = \beta \rightarrow (8)$$

consider $\alpha = \sqrt{RG} \rightarrow (2)$ $\quad Z_0 = \sqrt{\frac{R}{G}} \rightarrow (7)$

$$\alpha Z_0 = \sqrt{RG} \times \sqrt{\frac{R}{G}} = R$$

$$0.04 \times 80 = R \Rightarrow \boxed{R = 3.2\,\Omega/m}$$

We know $Z_o = \sqrt{\frac{R}{G}} \Rightarrow Z_o^2 = \frac{R}{G}$

$$G = \frac{R}{Z_o^2} = \frac{3.2}{80 \times 80} = 5 \times 10^{-4} \text{ S/m}$$

$$\therefore \boxed{G = 5 \times 10^{-4} \text{ S/m}}$$

From (8) $\frac{\omega L}{Z_o} = \beta \Rightarrow L = \frac{Z_o \beta}{\omega}$

$$L = \frac{80 \times 1.5}{2\pi \times f} = \frac{80 \times 1.5}{2 \times \pi \times 500 \times 10^6} = 38.2 \times 10^{-9} \text{ H}$$

$$\boxed{L = 38.2 \times 10^{-9} \text{ H/m}}$$

$$RC = GL \Rightarrow C = \frac{GL}{R}$$

$$C = \frac{5 \times 10^{-4} \times 38.2 \times 10^{-9}}{3.2} = 5.968 \times 10^{-12} \text{ F/m}$$

$$\boxed{C = 5.968 \times 10^{-12} \text{ F/m}}$$

(2) A telephone line has $R = 30\,\Omega/km$, $L = 100\,mH/km$, $G = 0$ and $C = 20\,\mu F/km$. At $f = 1\,kHz$ obtain

(a) The characteristic impedance of the line

(b) The propagation constant

(c) The phase velocity

Solution: Given $R = 30\times10^{-3}\,\Omega/m$ $L = 100\times10^{-3}\times10^{-3}\,H/m$,

$G = 0\,℧/m$ $C = 20\times10^{-6}\times10^{-3}\,F/m$

(a)
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{30\times10^{-3} + j2\times\pi\times1\times10^{3}\times100\times10^{-6}}{0 + j2\times\pi\times1\times10^{3}\times20\times10^{-6}\times10^{-3}}}$$

$$= 70.73 - j1.688 = 70.75\angle{-1.367^\circ}\ \Omega$$

(b)
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(30\times10^{-3} + j2\pi\times10^{3}\times100\times10^{-6})(0 + j2\pi\times10^{3}\times20\times10^{-9})}$$

$$\gamma = 2.121\times10^{-4} + j8.888\times10^{-3}\ /m$$

(c) The phase velocity $u = \frac{\omega}{\beta} = \frac{2\pi\times10^{3}}{8.888\times10^{-3}}$

$$u = 7.069\times10^{5}\ m/s$$

Note: (i) Velocity is generally termed as phase Velocity

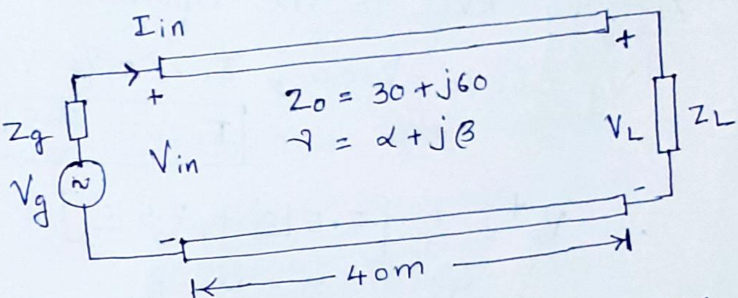
3) The transmission line shown in figure is 40m long and has $V_g = 15\angle 0^\circ\ V_{rms}$, $Z_0 = 30 + j60\ \Omega$ and $V_L = 5\angle -48^\circ\ V_{rms}$. If the line is matched to the load, Calculate

(a) The input impedance Z_{in}

(b) The sending-end current I_{in} and Voltage V_{in}

(c) The propagation constant.

Solution :

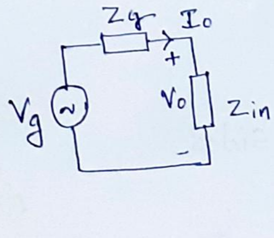


(a) Given that transmission line is matched

$\therefore Z_0 = Z_L = 30 + j60\ \Omega$

$\therefore \boxed{Z_{in} = Z_0 = 30 + j60\ \Omega}$

(b) $V_{in} = V_0 = \dfrac{Z_{in}}{Z_{in} + Z_g} V_g = \dfrac{Z_0 V_g}{Z_0 + Z_0}$ Because $Z_{in} = Z_0$, $Z_g = Z_0$



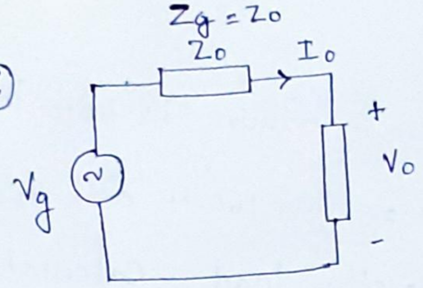
$V_{in} = \dfrac{V_g}{2} = \dfrac{15\angle 0^\circ}{2} = 7.5\angle 0^\circ\ V_{rms}$

$I_{in} = I_0 = \dfrac{V_g}{Z_g + Z_{in}}$ Since the line is matched $Z_g = Z_0$ $Z_{in} = Z_0$

$I_{in} = I_0 = \dfrac{V_g}{Z_0 + Z_0} = \dfrac{15\angle 0^\circ}{2(Z_0)} = \dfrac{15\angle 0^\circ}{2(30 + j60)}$

$= 0.1118\angle -63.43^\circ$

(c)



Consider the equivalent circuit as shown in figure

Since line is matched $Z_0 = Z_L \Rightarrow \Gamma = 0 \Rightarrow V_0^- = 0$

We know $V_0^+ = \frac{1}{2}\left[V_0 + I_0 Z_0\right]$

$V_s(z)\big|_{z=0} = V_0 = V_0^+ + V_0^-$

$I_s(z)\big|_{z=0} = I_0 = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$

$V_0 = 7.5 \angle 0^\circ$

Applying KVL to the equivalent circuit

$V_g - I_0 Z_0 - V_0 = 0$, $I_0 Z_0 = V_g - V_0 = 15\angle 0^\circ - 7.5\angle 0^\circ$

$\boxed{I_0 Z_0 = 7.5 \angle 0^\circ}$

$\therefore V_0^+ = \frac{1}{2}\left[7.5\angle 0^\circ + 7.5\angle 0^\circ\right] = 7.5\angle 0^\circ$

$V_L = V_0^+ e^{-\gamma l} + \cancel{V_0^-}^{\,0} e^{+\gamma l}$ [Because $\Gamma = 0$]

$\frac{V_L}{V_0^+} = e^{-\gamma l} \Rightarrow e^{+\gamma l} = \frac{V_0^+}{V_L} = \frac{7.5\angle 0^\circ}{5\angle -48^\circ}$

$\therefore e^{+\gamma l} = 1.5\angle 48^\circ \Rightarrow e^{(\alpha + j\beta)l} = 1.5\angle 48^\circ$

$e^{\alpha l} \cdot e^{j\beta l} = 1.5 e^{j48^\circ} \rightarrow (1)$

Equating real components

$e^{\alpha l} = 1.5$

Taking Natural log on both sides

$\alpha l = \ln(1.5)$

$\alpha = \frac{\ln(1.5)}{l}$, where $l = 40$

$\alpha = \frac{\ln(1.5)}{40} = 0.0101$ NP/m

Equating imaginary components

$$e^{j\beta l} = e^{j48^\circ}$$

Converting degrees into radians

$$\pi - 180^\circ$$

$$\left(\frac{\pi \times 48^\circ}{180}\right)? - 48^\circ$$

$$e^{j\beta l} = e^{j\frac{48^\circ \times \pi}{180^\circ}}$$

$$\beta l = \frac{48^\circ \times \pi}{180^\circ}$$

$$\beta = \frac{1}{l} \times \frac{48^\circ \times \pi}{180^\circ} = \frac{1 \times 48^\circ \times \pi}{40 \times 180^\circ}$$

$$\beta = 0.02094 \text{ rad/m}$$

length of the tr line $[\because l = 40m]$

$$\gamma = \alpha + j\beta$$

$$\gamma = 0.0101 + j0.02094 \,/m$$

4) a) show that at high frequencies ($R \ll \omega L$, $G \ll \omega C$) 52

$$\gamma = \left(\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

b) obtain a similar formula for Z_0

Solution :

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = j\omega\sqrt{LC}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$\gamma = j\omega\sqrt{LC}\sqrt{1 - \frac{RG}{\omega^2 LC} + \frac{R}{j\omega L} + \frac{G}{j\omega C}}$$

$\frac{RG}{\omega^2 LC}$ is very small and it can be neglected

Because $R \ll \omega L$ and $G \ll \omega C$

$$\gamma \cong j\omega\sqrt{LC}\sqrt{1 + \frac{R}{j\omega L} + \frac{G}{j\omega C}}$$

$$= j\omega\sqrt{LC}\sqrt{1 + \frac{1}{j}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)}$$

$$\gamma = j\omega\sqrt{LC}\left(1 + \frac{1}{j}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right)^{1/2}$$

Applying Binomial theorem to the term $\left[1 + \frac{1}{j}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right]^{1/2}$ we get

$$\gamma = j\omega\sqrt{LC}\left[1 + \frac{1}{2j}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) + \cdots\right]$$

Higher order terms of Binomial theorem expansion are neglected.

$$\gamma = j\omega\sqrt{LC}\left[1 + \frac{R}{2j\omega L} + \frac{G}{2j\omega C}\right]$$

$$\gamma = \cancel{j\omega}\sqrt{LC} \times \frac{R}{2\cancel{j\omega}L} + \cancel{j\omega}\sqrt{LC}\cdot\frac{G}{2\cancel{j\omega}C} + j\omega\sqrt{LC}$$

$$\boxed{\gamma = \left(\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}}$$

(b)

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}\sqrt{\frac{\left(1 + \frac{R}{j\omega L}\right) \times \cancel{j\omega}}{\left(1 + \frac{G}{j\omega C}\right) \times \cancel{j\omega}}}$$

$$Z_0 = \sqrt{\frac{L}{C}}\left(1 + \frac{R}{j\omega L}\right)^{1/2}\left(1 + \frac{G}{j\omega C}\right)^{-1/2}$$

Applying Binomial theorem

$$Z_0 = \sqrt{\frac{L}{C}}\left[1 + \frac{R}{2j\omega L}\right]\left[1 - \frac{G}{2j\omega C}\right]$$

Higher order terms of Binomial theorem are neglected.

$$Z_0 = \sqrt{\frac{L}{C}}\left[1 + \frac{R}{j2\omega L} - \frac{G}{2j\omega C} - \cancelto{0}{\frac{RG}{4j^2\omega^2 LC}}\right]$$

$$Z_0 \simeq \sqrt{\frac{L}{C}}\left[1 - \frac{jR}{2\omega L} + \frac{jG}{2\omega C}\right]$$

$$\boxed{Z_0 \simeq \sqrt{\frac{L}{C}}\left[1 + j\left(\frac{G}{2\omega C} - \frac{R}{2\omega L}\right)\right]}$$

1) On a distortionlessline, the voltage wave is given by

$$V(l') = 120 e^{0.0025 l'} \cos(10^8 t + 2l') + 60 e^{-0.0025 l'} \cos(10^8 t - 2l')$$

where l' is the distance from the load.

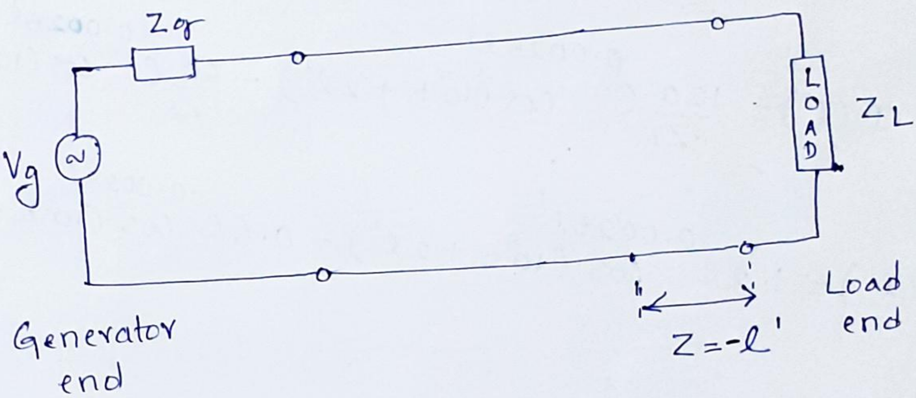
If $Z_L = 300 \Omega$, find (a) α, β and u (b) Z_0 and $I(l')$

Solution: We know that

(a) $V(z,t) = Re\left[V_s(z) e^{j\omega t}\right]$

$$V(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{+\alpha z} \cos(\omega t + \beta z) \quad (1)$$

where z = distance measured from the generator



In the given problem l' is the distance from the load. There fore equation (1) becomes (2) by substituting $z = -l'$

$$\therefore V(l') = V_0^+ e^{-\alpha(-l')} \cos(\omega t - \beta(-l')) + V_0^- e^{+\alpha(-l')} \cos(\omega t + \beta(-l'))$$

$$\therefore V(l') = V_0^+ e^{+\alpha l'} \cos(\omega t + \beta l') + V_0^- e^{-\alpha l'} \cos(\omega t - \beta l') \rightarrow (2)$$

$$\therefore V(l') = 120 e^{0.0025 l'} \cos(10^8 t + 2l') + 60 e^{-0.0025 l'} \cos(10^8 t - 2l') \rightarrow (3)$$

comparing (2) and (3) we get

$V_0^+ = 120$, $V_0^- = 60$, $\alpha = 0.0025$, $\beta = 2$,

$\omega = 10^8$ rad/sec

$\therefore$ $\alpha = 0.0025$ Np/m

$\beta = 2$ rad/m

$$u = \frac{\omega}{\beta} = \frac{10^8}{2} = 0.5 \times 10^8 \text{ m/s}$$

(b) $$\Gamma = \frac{V_0^-}{V_0^+} = \frac{60}{120} = 0.5$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow 0.5 = \frac{300 - Z_0}{300 + Z_0}$$

$$\therefore \quad Z_0 = 100\,\Omega$$

$$I(l') = \frac{120}{Z_0} e^{0.0025l'} \cos(10^8 t + 2l') - \frac{60}{Z_0} e^{-0.0025l'} \cos(10^8 t - 2l')$$

$$I(l') = 1.2 e^{0.0025l'} \cos(10^8 t + 2l') - 0.6 e^{-0.0025l'} \cos(10^8 t - 2l') \text{ A}$$

2) A distortionless cable is 4m long and has a characteristic impedance of 60 Ω. An attenuation of 0.24 dB is observed at the receiving end. Also, a signal applied to the cable is delayed by 80 μs before is measured at the receiving end. Find R, G, L and C for the cable

Solution: For a distortionless cable

$$RC = LG \rightarrow (1)$$

$$Z_0 = \sqrt{\frac{L}{C}} = 60 \rightarrow (2)$$

phase velocity $u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{\text{length}}{\text{time}} = \frac{4}{80 \times 10^{-6}}$

$$\frac{1}{\sqrt{LC}} = \frac{4}{80 \times 10^{-6}} \rightarrow (3)$$

$$(2) \times (3) \quad \sqrt{\frac{L}{C}} \times \frac{1}{\sqrt{LC}} = \frac{60 \times 4}{80 \times 10^{-6}}$$

$$\frac{1}{C} = \frac{60 \times 4}{80 \times 10^{-6}}$$

$$\boxed{C = 333.33 \; nF/m}$$

$$Z_0 = \sqrt{\frac{L}{C}} = 60$$

$$\frac{L}{C} = 60 \times 60$$

$$L = 60 \times 60 \times 333.33 \times 10^{-9}$$

$$\boxed{L = 1.20 \times 10^{-3} \; H/m}$$

$\alpha l = 0.24\ dB$

$\alpha l = \frac{0.24}{8.686}$ ~~dB~~ $\quad$ 1 NP - 8.686 dB

0.24 dB

$\alpha l = 0.0276\ NP$

$\alpha = \frac{0.0276}{l} = \frac{0.0276}{4} = 6.9\times10^{-3}$

$\alpha = \sqrt{RG} \qquad \alpha^2 = RG$

$RG = (6.9\times10^{-3})^2 \rightarrow (4)$

$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}} = 60$ $\quad \frac{R}{L} = \frac{G}{C} \quad \frac{R}{G} = \frac{L}{C}$

$\frac{R}{G} = 60\times60 \rightarrow (5)$

(4) × (5) $\quad R^2 = (6.9\times10^{-3})^2 \times 60\times60$

$\boxed{R = 0.414\ \Omega/m}$

$\alpha = \sqrt{RG}$

$\alpha^2 = RG$

$\boxed{G = \frac{\alpha^2}{R} = \frac{(6.9\times10^{-3})^2}{0.414} = 115\times10^{-6}\ S/m}$

3) A coaxial line 5.6m long has distributed parameter $R = 6.5\,\Omega/m$, $L = 3.4\,\mu H/m$, $G = 8.4\,mS/m$ and $C = 21.5\,pF/m$. If the line operates at 2 MHz, calculate the characteristic impedance and the end to end propagation time delay.

Solution :

$$R + j\omega L = 6.5 + j2\pi \times 2\times10^{6} \times 3.4\times10^{-6}$$

$$= 6.5 + j42.73 = 43.22 \angle 81.35^\circ$$

$$G + j\omega C = 8.4\times10^{-3} + j2\pi \times 2\times10^{6} \times 21.5\times10^{-12}$$

$$= (8.4 + j0.2701)\times10^{-3} = 8.404\times10^{-3} \angle 1.84^\circ$$

$$Z_0 = \sqrt{\frac{43.22 \angle 81.35}{8.404\times10^{-3} \angle 1.84}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = 71.71 \angle 39.75^\circ$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{43.22 \angle 81.35^\circ \times 8.404\times10^{-3} \angle 1.84}$$

$$\gamma = 0.6026 \angle 41.595$$

$$\gamma = 0.4506 + j0.3999 /m$$

$$\alpha = 0.4506 \quad \beta = 0.3999$$

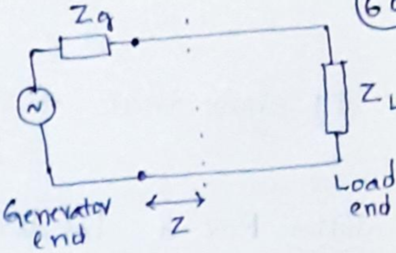
$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{\text{velocity}} = \frac{5.6}{u}$$

$$\text{time} = \frac{5.6}{\frac{\omega}{\beta}} = \frac{\beta \times 5.6}{2\pi\times2\times10^{6}} = \frac{0.3999\times5.6}{2\times\pi\times2\times10^{6}}$$

$$= 0.1782\,\mu sec$$

Transmission Line Equations - (Additional)



Consider $\frac{d^2 V_s}{d z^2} - \gamma^2 V_s = 0$

The solution to above linear homogeneous differential equation is

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

where V_o^+ and V_o^- are wave amplitudes

The instantaneous expression for voltage is

$$V(z,t) = Re\left[V_s(z) e^{j\omega t}\right]$$

$$V(z,t) = V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{+\alpha z} \cos(\omega t + \beta z)$$

III'ly $\frac{d^2 I_s}{d z^2} - \gamma^2 I_s = 0$

The solution to above linear homogeneous differ-ential equation is

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$$

where I_o^+ and I_o^- are wave amplitudes

The instantaneous expression for current is

$$I(z,t) = Re\left[I_s(z) e^{j\omega t}\right]$$

$$I(z,t) = I_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + I_o^- e^{+\alpha z} \cos(\omega t + \beta z)$$

$$I(z,t) = \frac{V_o^+}{Z_o} e^{-\alpha z} \cos(\omega t - \beta z) - \frac{V_o^-}{Z_o} e^{+\alpha z} \cos(\omega t + \beta z)$$

The second term in the above equation is negative because $Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-}$

In all the above expressions "z" is measured from generator end as shown in figure above.

① show that for a lossless line

$$Z_{sc} Z_{oc} = Z_0^2$$

(61)

Solution: For a lossless line when $Z_L = 0$

$$Z_{sc} = Z_{in}\Big|_{Z_L=0} = jZ_0 \tan\beta l$$

For a lossless line when $Z_L = \infty$

$$Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = -jZ_0 \cot\beta l$$

$$Z_{sc} \times Z_{oc} = jZ_0 \tan\beta l \times -jZ_0 \cot\beta l$$

$$= -j^2 Z_0^2 \,(1)$$

$$Z_{sc} \times Z_{oc} = Z_0^2$$

2) Show that a transmission coefficient may be defined as $\tau_L = \frac{V_L}{V_0^+} = 1 + \Gamma_L = \frac{2Z_L}{Z_L + Z_0}$

where τ_L = Transmission Coefficient

Γ_L = Reflection Coefficient.

Solution:

$$\tau_L = \frac{V_L}{V_0^+} = \frac{I_L Z_L}{\frac{1}{2}[V_L + Z_0 I_L]}$$

$$= \frac{I_L Z_L}{\frac{1}{2}[I_L Z_L + Z_0 I_L]}$$

$$= \frac{2 I_L Z_L}{I_L Z_L + I_L Z_0}$$

$$= \frac{2Z_L}{Z_L + Z_0} \rightarrow (1)$$

$$1 + \Gamma_L = 1 + \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{2Z_L}{Z_L + Z_0} \rightarrow (2)$$

$$(1) = (2)$$

$$\boxed{\tau_L = 1 + \Gamma_L = \frac{2Z_L}{Z_L + Z_0}}$$

1. Two voltage waves having equal ~~amplitudes~~ frequencies and amplitudes propagate in opposite directions on a lossless transmission line. Determine the total voltage as a function of time and position

Solution: Since the waves have the same frequency we can write their combination using their phasor forms: Assuming phase constant β and real amplitude V_0, the two wave voltages combine in this way

$$V_{ST}(z) = V_0 e^{-j\beta z} + V_0 e^{+j\beta z}$$

$$= 2V_0 \cos(\beta z)$$

In the real instantaneous form, this becomes

$$V(z,t) = Re\{2V_0 \cos(\beta z) e^{j\omega t}\}$$

$$= 2V_0 \cos \beta z \cos \omega t$$

This represents a standing wave, in which the amplitude varies as $\cos(\beta z)$ and oscillates in time as $\cos(\omega t)$

2) A lossless transmission line is 80cm long and operates at a frequency of 600 MHz. The line parameters are $L = 0.25\,\mu H/m$, $C = 100\,pF/m$. Find the characteristic impedance, the phase constant and the velocity.

<u>Solution:</u> Since the line is lossless

$R = G = 0$

The characteristic Impedance

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.25\times10^{-6}}{100\times10^{-12}}} = 50\,\Omega$$

Since $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\beta = \omega\sqrt{LC}$$

Phase constant $\beta = 2\pi\,(600\times10^{6})\sqrt{0.25\times10^{-6}\times100\times10^{-12}}$

$$\beta = 18.85\ \text{rad/m}$$

Phase velocity u or $v_p = \frac{\omega}{\beta} = \frac{2\pi\times600\times10^{6}}{18.85}$

$$= 2\times10^{8}\ \text{m/s}$$

3) A distortionless transmission line operating at 250 MHz has $R = 30\ \Omega/m$, $L = 200\,nH/m$ and $C = 80\,pF/m$.

(a) Determine the characteristic Impedance Z_0, the propagation constant γ and the velocity of propagation along the line.

(b) After how many meters travelling along the line will the voltage wave get reduced to 30% of its initial value?

(c) How far should the voltage wave travel along the line in order to undergo a phase shift of 20°.

Solution :: (a) For a distortionless line $\frac{R}{L} = \frac{G}{C}$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{200 \times 10^{-9}}{80 \times 10^{-12}}} = 50\ \Omega$$

propagation constant γ given by

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

But $\frac{R}{L} = \frac{G}{C} \Rightarrow G = \frac{R}{L}C$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)\left(\frac{R}{L}C + j\omega C\right)}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)\left(\frac{C}{L}\right)(R + j\omega L)}$$

$$\gamma = \alpha + j\beta = (R + j\omega L)\sqrt{\frac{C}{L}}$$

$$\gamma = \alpha + j\beta = R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

$$\gamma = 30\sqrt{\frac{80 \times 10^{-12}}{200 \times 10^{-9}}} + j\, 2\pi \times 250 \times 10^{6} \sqrt{200 \times 10^{-9} \times 80 \times 10^{-12}}$$

$$\gamma = 0.6 + j\,6.2832\ /m$$

(P4)

$\gamma = 0.6 + j\,6.2832\ /m$

$\gamma = 0.6 + j\,6.2832/m$

$\gamma = \alpha + j\beta$

Attenuation constant $\alpha = 0.6\ Np/m$ $\quad\beta$ = phase constant $= 6.2832\ rad/m$

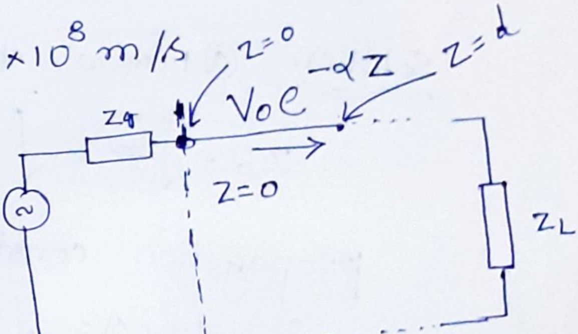
The velocity of propagation or phase velocity is given by

$$u \text{ or } V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{200\times10^{-9} \times 80\times10^{-12}}}$$

$$= 2.5\times10^{8}\ m/s$$

(b)



Let us assume that the wave is travelling along +ve z-direction and the magnitude of voltage wave be V_0 at $z=0$.

The magnitude of this voltage wave at any arbitrary position along the line would then given by $V_0 e^{-\alpha z}$. The reflected wave is not considered.

Assume at $z=d$, the magnitude of this wave get reduced to 30% of V_0.

$$\therefore V_0 e^{-\alpha d} = 0.3V_0$$

$$\Rightarrow e^{-\alpha d} = 0.3$$

$$\ln e^{-\alpha d} = \ln 0.3$$ (P5)

$$\cancel{-}\alpha d = \cancel{-}1.2039$$

$$d = \frac{1.2039}{\alpha} = \frac{1.2039}{0.6} = 2.006\,m$$

(c) As the wave travels along the line, it would continuously undergo a phase shift. If it is assumed that the wave is travelling along the +Z direction and at the initial position this wave is expressed as $V_0 e^{-j(0)}$, then this wave can be expressed at any arbitrary position along the line by $V_0 e^{-j\beta z}$.

At $Z = l$ $\quad \beta l = 20^\circ \times \frac{\pi}{180}$ radians

i.e After travelling 'l' m phase shift will be 20° or $20^\circ \times \frac{\pi}{180}$ radians

$$\beta l = 20 \times \frac{\pi}{180}$$

$$6.2832\, l = 20 \times \frac{\pi}{180}$$

$$l = \frac{20 \times \pi}{6.2832 \times 180} = 5.56 \times 10^{-2}\,m$$

$$= 55.56 \times 10^{-3}\,m$$

4) A 50Ω lossless transmission line is terminated by a load impedance $Z_L = 50 - j75\Omega$. If the incident power is 100mW, find the power delivered by the load.

Solution: Reflection coefficient $= \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{(50-j75) - (50)}{(50 - j75) + 50}$$

$$= 0.36 - j0.48$$

$$= 0.6\, e^{-j0.93}$$

$$P_t = (1 - |\Gamma|^2)\, P_i$$

$$P_t = (1 - |0.6|^2) \times 100 \times 10^{-3}$$

$$P_t = 64\ mW$$

5) A transmission line operating at 500 M rad/sec has $L = 0.5\,\mu H/m$, $C = 32\,pF/m$, $G = 100\,\mu \mho/m$ and $R = 25\,\Omega/m$

(a) Calculate values of $\gamma, \alpha, \beta, u, \lambda$ and Z_0

(b) What distance down the line can a voltage wave travel before it is reduced to 10% of its initial value of amplitude?

(c) What distance must it travel to undergo a 90° phase shift?

Solution: (a) $L = 0.5\,\mu H/m$ $C = 32\,pF/m$, $G = 100\,\mu \mho/m$

$R = 25\,\Omega/m$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(25 + j \times 500 \times 10^6 \times 0.5 \times 10^{-6})(100 \times 10^{-6} + j\, 500 \times 10^6 \times 32 \times 10^{-12})}$$

$$\gamma = 0.1061 + j\,2.002\,/m$$

$$\gamma = \alpha + j\beta$$

$$\alpha = 0.1061 \qquad \beta = 2.002$$

Wave length $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.002} = 3.14\,m$

Phase velocity u or $V_p = \frac{\omega}{\beta} = \frac{500 \times 10^6}{2.002}$

$= 2.5 \times 10^8\,m/s$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{25 + j\,500 \times 10^6 \times 0.5 \times 10^{-6}}{100 \times 10^{-6} + j\,500 \times 10^6 \times 32 \times 10^{-12}}}$$

$$= 125.3 \angle -2.68^\circ\ \Omega$$