CONTROL SYSTEMS

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UNIT 3: THE STABILITY OF LINEAR FEEDBACK SYSTEMS

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Concept of Stability

- Stability is important in the design and control of feedback control systems
- Closed loop system (CLS) is either stable or unstable is referred as **absolute stability**
- Given the CLS is stable system, we can further characterize the degree of stability is referred as **relative stability**.
- Ex, aircraft design

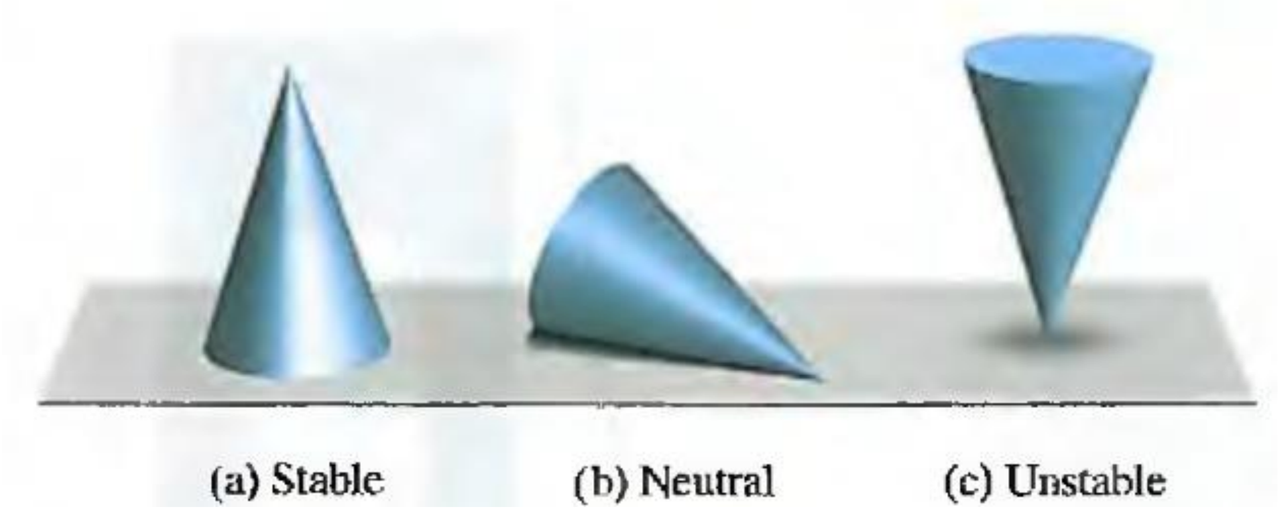
Concept of Stability

- **Stability**

- A stable system is defined as a system with a bounded (limited) system response. That is, if the system is subjected to a bounded input or disturbance and the response is bounded in magnitude, the system is said to be stable.

- A stable system is a dynamic system with a bounded response to a bounded input.

- Illustrated as shown in figure.



THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Concept of Stability

➔ Determining stability

- The stability of a dynamic system is defined in a similar manner. The response to a displacement, or initial condition, will result in either a decreasing, neutral, or increasing response.
- Specifically, it follows from the definition of stability that a linear system is stable if and only if the absolute value of its impulse response g(t), integrated over an infinite range, is finite.
- i.e., $\int_0^\infty |g(t)|dt < \infty$

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Determining stability

- The location of the poles in the s-plane of a system indicates the resulting transient response.
- There are 4 categories based on the pole location and their corresponding responses

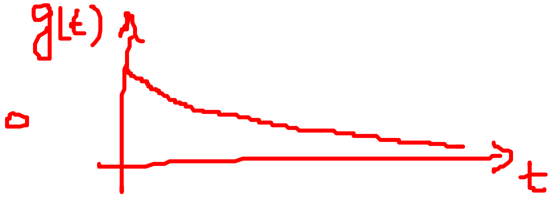
THE STABILITY OF LINEAR FEEDBACK SYSTEMS

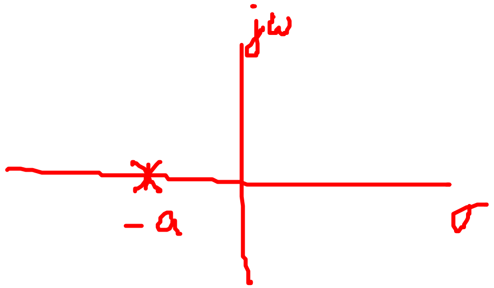
Determining stability

- The poles in the left-hand portion of the s-plane result in a decreasing response for disturbance inputs.

ex, $G(s) = \frac{1}{s+a} \Rightarrow$ poles $s = -a$

$g(t) = e^{-at}, \; t \geq 0$

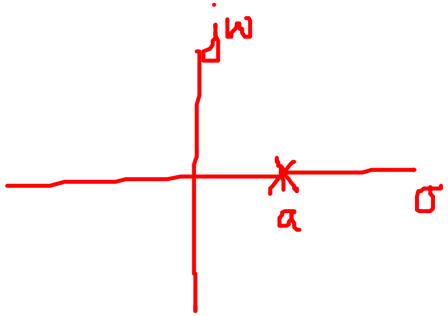




- The poles on the right-hand plane result in a increasing response for a disturbance inputs.

ex, $G(s) = \frac{1}{s-a}$

$g(t) = e^{at}, \; t \geq 0$



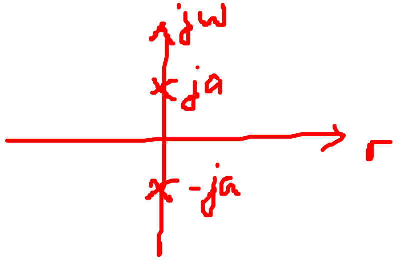
THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Determining stability

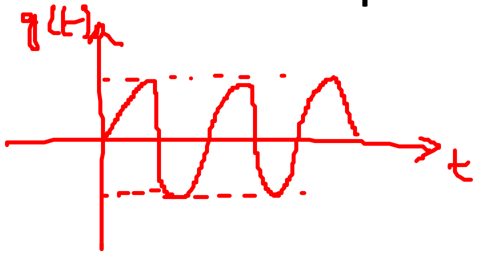
- The simple poles on the $j\omega$-axis result in a neutral response for a disturbance input.

Ex, $G(s) = \dfrac{a}{s^2+a^2}$,

poles, $s = \pm ja$



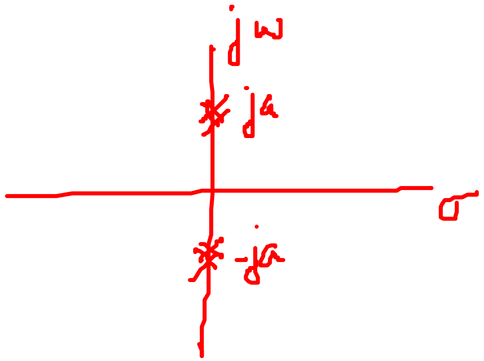
$g(t) = \sin at,$
$t \geq 0$



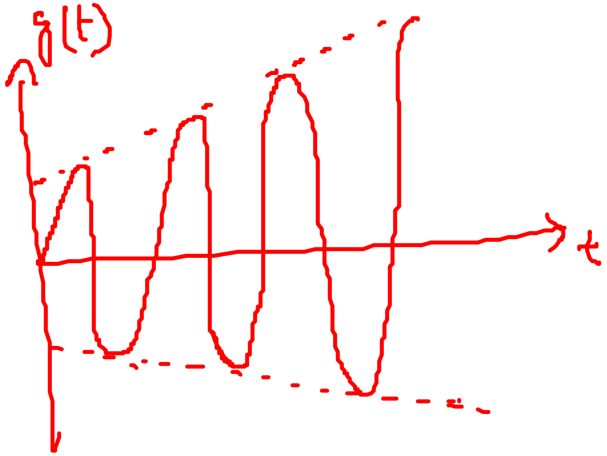
- Similarly, multiple poles on the $j\omega$-axis result in a increasing response for a disturbance input.

Ex, $G(s) = \dfrac{2as}{(s^2+a^2)^2}$

poles $s = \pm ja, \pm ja$



$g(t) = t\sin at,$
$t \geq 0$



- Therefore, the poles of desirable dynamic systems **must lie in the left-hand portion** of the s-plane for the **system to be stable.**

Conditions for stability

- **A necessary and sufficient condition for a feedback system to be stable** is that all the poles of the system transfer function have negative real part.
- A system is **stable,** if all the poles of the transfer function are in the left-hand side of s-plane.
- A system is **not stable,** if not all the roots are in the left-hand side of s-plane.

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Conditions for stability

- If the characteristic equation has **simple roots on the imaginary axis** ($j\omega$-axis) with all other roots in the left half-plane, then the steady-state output will be sustained oscillations for a bounded input. The system is called **marginally stable.**
- If there are **multiple poles on jw-axis**, with all other roots in the left half-plane, then the steady-state output will be rising with oscillations for a bounded input. For this case, the output becomes unbounded. The system is called **unstable**.

Methods to Determine Stability

- Routh-Hurwitz
- Root Locus

Time domain

- Bode Plot
- Nyquist Criterion

frequency domain

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Routh Hurwitz Stability Criterion

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Routh Hurwitz Criterion (RH Criterion)

- In 1800, A. Hurwitz and E. J. Routh introduced the method of determining stability of linear systems.
- Consider that the characteristic equation of LTI SISO system

$$\Delta(s) = q(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0.$$

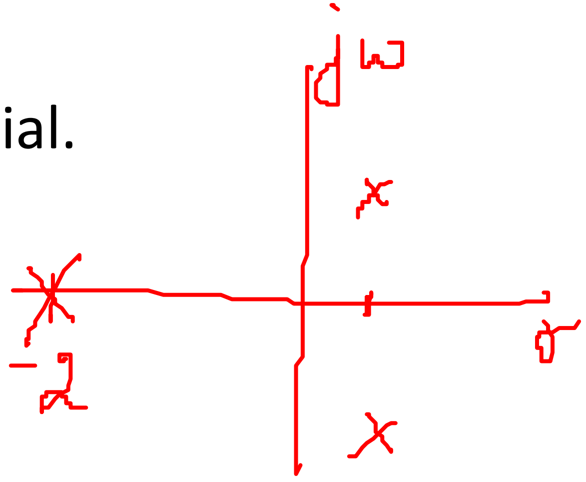
- Where all the coefficients are real

R H Criterion

Necessary condition: The coefficients of the characteristic polynomial should be positive and non-zero. This implies that all roots of the characteristic equation should have negative real parts.

$$T(s) = \frac{p(s)}{q(s)} = \frac{K\prod_{i=1}^{M}(s+z_i)}{s^N\prod_{k=1}^{Q}(s+\sigma_k)\prod_{m=1}^{R}[s^2+2\alpha_m s+(\alpha_m^2+\omega_m^2)]},$$

The denominator polynomial D(s) represents the characteristic polynomial.



Ex. $s^3+s^2+2s+8=0$ $\Rightarrow$ $s = -2,\ 0.5 \pm j\,1.92$

Sufficient Condition: All the elements of the first column of the Routh-Hurwitz Array should have same sign.

Characteristic equation (C.E) in factored form

$$a_n(s - r_1)(s - r_2)\cdots(s - r_n) = 0,$$

where r_i is the i^{th} root of the C.E

R H Criterion

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- After multiplying the factors

$$\begin{aligned} q(s) = a_n s^n &- a_n(r_1 + r_2 + \cdots + r_n)s^{n-1} \\ &+ a_n(r_1r_2 + r_2r_3 + r_1r_3 + \cdots)s^{n-2} \\ &- a_n(r_1r_2r_3 + r_1r_2r_4 \cdots)s^{n-3} + \cdots \\ &+ a_n(-1)^n r_1r_2r_3 \cdots r_n = 0. \end{aligned}$$

- In other words,

$$\begin{aligned} q(s) = a_n s^n &- a_n \text{ (sum of all the roots) } s^{n-1} \\ &+ a_n \text{ (sum of the products of the roots taken 2 at a time) } s^{n-2} \\ &- a_n \text{ (sum of the products of the roots taken 3 at a time) } s^{n-3} \\ &+ \cdots + a_n(-1)^n \text{ (product of all } n \text{ roots)} = 0. \end{aligned}$$

- Ex. Find the polynomial of the following roots $-2, -1-j, -1+j$

R H Criterion

$$q(s) = a_n s^n - a_n\,(\text{sum of all the roots})\, s^{n-1}$$
$$+ a_n\,(\text{sum of the products of the roots taken 2 at a time})\, s^{n-2}$$
$$- a_n\,(\text{sum of the products of the roots taken 3 at a time})\, s^{n-3}$$
$$+ \cdots + a_n(-1)^n\,(\text{product of all } n \text{ roots}) = 0.$$

- Ex. Find the polynomial of the following roots $\overset{r_1}{-2}, \overset{r_2}{-1-j}, \overset{r_3}{-1+j}$, $a_n = 1$, $n = 3$

$$q(s) = s^3 - (-2-1-j-1+j)\,s^2 + [-2(-1-j) + (-1-j)(-1+j) + (-2)(-1+j)]\,s + (-1)^3[-2(-1-j)(-1+j)] = 0$$

$$= s^3 - (-4)s^2 + [2 + \cancel{2j} + (1+1) + 2 - \cancel{2j}]\,s + (-1)(-2(2))$$

$$= s^3 + 4s^2 + 6s + 4$$

R H Criterion

Procedure for solving by Routh Hurwitz :

1. Construct Routh table
2. Interpret the Routh table as follows
3. If the sign of the entries in the first columns are same then the system is stable otherwise the number of poles on the right side depends on the number of sign changes in the first column.

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

R H Criterion

Procedure for solving by Routh Hurwitz :

Routh Table:

$$\Delta(s) = q(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0.$$

$$\begin{array}{c|ccc}
s^n & a_n & a_{n-2} & a_{n-4} \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} \\
s^{n-2} & b_{n-1} & b_{n-3} & b_{n-5} \\
s^{n-3} & c_{n-1} & c_{n-3} & n_{n-5} \\
\vdots & \vdots & \vdots & \vdots \\
s^0 & h_{n-1} & &
\end{array}$$

where

$$b_{n-1} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix},$$

$$b_{n-3} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix},$$

$$c_{n-1} = \frac{-1}{b_{n-1}} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix},$$

R H Criterion

Consider the 4th order system, $G(s) = \dfrac{N(s)}{(a_4s^4+a_3s^3+a_2s^2+a_1s^1+a_0)} = \dfrac{N(s)}{D(s)}$

Procedure for solving by Routh Hurwitz :

Write the coefficients in 2 rows

- First row starts with a_n
- Second row starts with a_{n-1}
- Other coefficients alternate between rows
- Both rows should be same length
  - Continue until no coefficients are left
  - Add zero as last coefficient if necessary

s^4	a_4	a_2	a_0
s^3	a_3	a_1	a_0
s^2	b_1	a_0	
s^1	c_1		
s^0	a_0		

TABLE 6.1 Initial layout for Routh table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$b_1 = \dfrac{a_4 a_1 - a_3 a_2}{a_3}$	$b_2 = \dfrac{a_3 a_0 - a_4 0}{a_3} = a_0$	
s^1	c_1		
s^0	$d_1 = a_0$		

$c_1 = \dfrac{b_1 a_1 - a_3 a_0}{b_1}$

$d_1 = \dfrac{c_1 a_0 - c_1 0}{c_1}$

$= a_0$

Procedure for solving by Routh Hurwitz :

Complete the third row.

- Call the new entries $b_1, \cdots, b_k$
  - The third row will be the same length as the first two

$$b_1 = -\frac{\det\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} \qquad b_2 = -\frac{\det\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} \qquad b_3 = -\frac{\det\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3}$$

- The denominator is the first entry from the previous row.
- The numerator is the determinant of the entries from the previous two rows:
  - The first column
  - The next column following the coefficient

$$b_k = -\frac{\det\begin{vmatrix} a_n & a_{n-2k} \\ a_{n-1} & a_{n-2k-1} \end{vmatrix}}{a_{n-1}}$$

  - If a coefficient doesn't exist, substitute 0.

R H Criterion

TABLE 6.2 Completed Routh table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

If the sign of the entries in the first columns are same then the system is stable otherwise the number of poles on the right side depends on the number of sign changes in the first column.

Exceptions

Case1: No element in the first column is zero

Solve the given polynomial by using RH table directly and check requirement for a stable second-order system which is simply that all the coefficients be positive or all the coefficients be negative else not stable.

Ex. $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Exceptions

Case1: No element in the first column is zero

s^4	2	3	10	$+$
s^3	1	5	0	$+$
s^2	$\frac{3-10}{1} = -7$	10		$-$
s^1	$\frac{-35-10}{-7} = \frac{+45}{7}$			$+$
s^0	10			$+$

$s|m$ is unstable,

$\because$ 2 sign changes

$\Rightarrow$ 2 poles on RHS

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Exceptions

Ex: Consider the overall transfer function $T(s)=1000/(s^3+10s^2+31s^1+1030)$.

Determine whether the system is stable or not using RH criteria.

s^3	1	31	$+$
s^2	10	1030	$+$
s^1	$\dfrac{10 \times 31 - 1030}{10} = -72$		$-$
s^0	$\dfrac{-72 \times 1030 - 0 \times 10}{-72} = 1030$		$+$

2 sign changes $\Rightarrow$ 2 poles on RHS s-plane

$\Rightarrow$ s/m is unstable

The sign changes are 2 implies that there are 2 poles lie on the right side of the s-plane.

Hence, the system is unstable.

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

R H Criterion

- **Examples,** $(s-1+j\sqrt{7})(s-1-j\sqrt{7})(s+3)=0$
- $s^3+s^2+2s+24=0$ $s^3-s^2+2s+24=0$
- The polynomial satisfies all the necessary conditions because all the coefficients exist and are positive.

s^3	1	2
s^2	1	24
s^1	-22	0
s^0	24	0

- Because two changes in sign appear in the first column, we find that the two roots of q(s) lie in the right-hand plane.

Exceptions

Case 2: zero only in first column

- Replace zero by ε when ε is some small positive constant greater than 0.

Then let ε tend to 0 from left to right.

OR

- Replace $s = \frac{1}{z}$ resulting polynomial will have roots which are reciprocal of the roots of the original polynomial. Hence they will have the same sign. Resulting polynomial can be written by a polynomial with coefficient in reverse order.

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

Exceptions

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Case 2: zero only in first column, Ex. $s^4 + s^3 + 2s^2 + 2s + 3 = 0$

method 1:

s^4	1	2	3	+
s^3	1	2		+
s^2	$0 \; \epsilon$	3		+
s^1	$\frac{2\epsilon - 3}{\epsilon}$			−
s^0	3			+

$$\lim_{\epsilon \to 0} \frac{2\epsilon - 3}{\epsilon} = 2 - \lim_{\epsilon \to 0} \frac{3}{\epsilon}$$

-ve

2 sign changes

⇒ 2 poles on RHS

∴ s/m is unstable

method 2: substitute $s = 1/z$ in C·E

$$3z^4 + 2z^3 + 2z^2 + z + 1 = 0$$

z^4	3	2	1
z^3	2	1	
z^2	1/2	1	
z^1	-3		
z^0	1		

Examples

- $q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 - 11s + 10$, check the no of poles on RHS of s-plane
- The Routh array is then

s^5	1	2	-11
s^4	2	4	10
s^3	0 ϵ	-16	
s^2	$\frac{4\epsilon+32}{\epsilon}$	10	
s^1	d_1		
s^0	10		

$$\lim_{\epsilon \to 0} 4 + \frac{32}{\epsilon} = +ve$$

$$d_1 = \frac{\frac{-16(4\epsilon+32)}{\epsilon} - 10\epsilon}{\frac{4\epsilon+32}{\epsilon}} = -ve$$

$\because$ 2 sign changes

$\Rightarrow$ 2 poles are on RHS

Examples

- $q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 - 11s + 10$, check the no of poles on RHS of s-plane
- The Routh array is then

s^5	1	2	-11
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s^2	$\frac{4\epsilon+32}{\epsilon}$	10	
s^1	d_1		
s^0	10		

$$\lim_{\epsilon \to 0} 4 + \frac{32}{\epsilon} = +ve$$

$$d_1 = \frac{\frac{-16(4\epsilon+32)}{\epsilon} - 10\epsilon}{\frac{4\epsilon+32}{\epsilon}} = -ve$$

$\because$ 2 sign changes

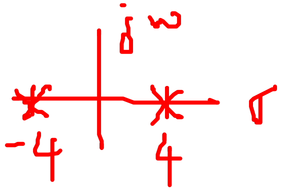
$\Rightarrow$ 2 poles are on RHS

Case 3: Entire row is 0

When all the elements in one row are zeros before the tabulation is terminated, it indicates that one or more of the following conditions may exists.

1. The equation has at least one pair of real roots with equal magnitude but opposite signs.

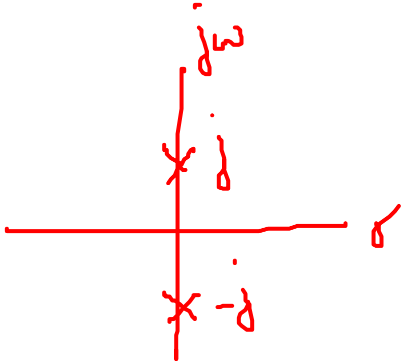
$3s^2 - 48 = 0 \Rightarrow s = \pm 4$



2. The equation has one or more pairs of imaginary roots.

$s^2 + 1 = 0$

$s = \pm j$



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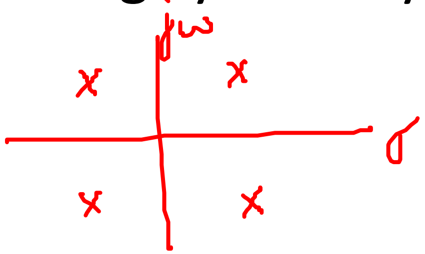
R H Criterion - Exceptions

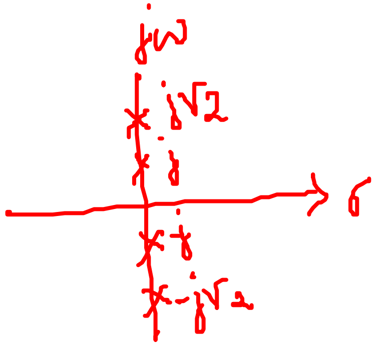
1. The equation has pairs of complex-conjugate roots forming symmetry about the origin of the s-plane

   ex, $s = -1 \pm j \,,\; 1 \pm j$



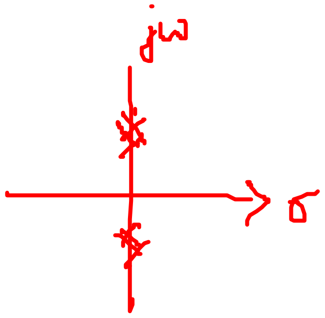
2. The equation has non repeated pairs of roots located on jw axis

   ex, $s^4 + 3s^2 + 2 = 0 \,,\quad s = \pm j \,,\; \pm j\sqrt{2}$



3. The equation has repeated pairs of roots located on jw axis

   ex, $s^4 + 2s^2 + 1 = 0 \,,\quad s = \pm j \,,\; \pm j$



R H Criterion - Exceptions

Steps to complete the Routh table:

- Form the auxiliary equation from the row above the zero row.
- Differentiate the auxiliary polynomial with respect to s and replace the row of zeros by its coefficient .
- Continue with the construction of RH table and the stability from the number of sign changes in the first column.
- An entire row of 0 will appear in the RH table when purely even or purely odd polynomial is a factor of original polynomial.

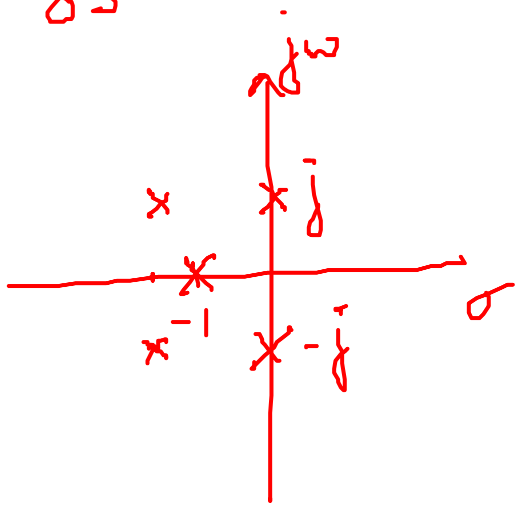
R H Criterion - Exceptions

Case 3: Entire row is 0 Ex, $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

s^5	1	8	7
s^4	4	8	4
s^3	6	6	
s^2	4	4	
s^1	0 8		
s^0	4		

$A(s) = 4s^2 + 4 = 0 \Rightarrow s = \pm j$

$\frac{dA}{ds} = 8s$



⇒ no sign changes
∴ no poles on RHS
⇒ ∴ system is marginally stable

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

R H Criterion

$$\frac{1}{4}s^3 + s^2 + \frac{7}{4}s + 1$$

$$4s^2+4 \,\big)\, s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4$$

$$\frac{s^3}{4} + s^2 + \frac{7}{4}s + 1 = 0$$

$$s^3 + 4s^2 + 7s + 4 = 0$$

$$s = -1,\ -1.5 \pm j1.323$$

R H Criterion

- q(s) = s^3 + 2s^2 + 4s + K. Find the value of K for which the system is stable and also for the system becoming marginally stable.

$$
\begin{array}{c|cc}
s^3 & 1 & 4 \\
s^2 & 2 & K \\
s^1 & \dfrac{8-K}{2} & 0 \\
s^0 & K & 0
\end{array}
$$

For a stable system, we require that

$$0 < K < 8.$$

- Considering K=8 to get case 3
- U(s) = 2s^2 + Ks^0 = 2s^2 + 8 = 2(s^2 + 4) = 2(s + j2)(s - j2).

R H Criterion

- q(s) = s^3 + 2s^2 + 4s + K. Find the value of K for which the system is stable and also for the system becoming marginally stable.

s^3	1	4
s^2	2	K
s^1	$\frac{8-K}{2}$	0
s^0	K	0

zero

For a stable system, we require that

$$0 < K < 8.$$

when $K = 8$

$A(s) = 2s^2 + 8 = 0$

$\frac{dA}{ds} = 4s$

$s = \pm 2j$

$s = -2$

- Considering K=8 to get case 3
- U(s) = 2s^2 + Ks^0 = 2s^2 + 8 = 2(s^2 + 4) = 2(s + j2)(s - j2).

j2

-2

-j2

R H Criterion

Ex, $s^3 + 3Ks^2 + (K+2)s + 4 = 0$

Determine the value of K for which the system to be stable.

s^3	1	$K+2$
s^2	$3K$	4
s^1	$\dfrac{3K(K+2)-4}{3K}$	
s^0	4	

For the system to be stable

$3K > 0$, $\dfrac{3K(K+2)-4}{3K} > 0$

$(K - 0.527)(K + 2.527) > 0$

$K > 0.527$

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

R H Criterion

Ex. $G(s) = \dfrac{K(s+2)(s+1)}{(s+0.1)(s-1)}$

$T(s) = \dfrac{G(s)}{1+G(s)}$

$C.E = 1 + G(s) = 0$

$1 + \dfrac{K(s+2)(s+1)}{(s+0.1)(s-1)} = 0 \Rightarrow (s+0.1)(s-1) + K(s+2)(s+1) = 0$

$(1+K)s^2 + (3K-0.9)s + (2K-0.1) = 0$

s^2	$1+K$	$2K-0.1$
s^1	$3K-0.9$	
s^0	$2K-0.1$	

i) No poles on RHS

$K+1 > 0 \Rightarrow K > -1$

$3K-0.9 > 0 \Rightarrow K > 0.3$

$2K-0.1 > 0 \Rightarrow K > 0.05$

$\therefore K > 0.3 \Rightarrow$ stable

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

R H Criterion

ii) 1 pole on RHS $\Rightarrow$ one sign change

$1+K>0$ & $2K-0.1<0$ & $3K-0.9<0$

$K>-1$ $\quad K<0.05$ $\quad K<0.3$

$-1<K<0.05$

iii) 2 poles on RHS $\Rightarrow$ 2 sign changes

$3K-0.9<0$ & $2K-0.1>0$

$K<0.3$ $\quad K>0.05$

$0.05<K<0.3$

R H Criterion

Ex, C.E $\ddot{x} - (K+2)\dot{x} + (2K+5)x = 0$

a) Find the value of K for which the system is
i) stable, ii) limitedly stable, iii) unstable

b) For the stable case, for what values of K is the system
i) underdamped (ii) overdamped

C.E, $s^2 - (K+2)s + (2K+5) = 0$

$$\begin{array}{c|cc} s^2 & 1 & 2K+5 \\ s^1 & -(K+2) & \\ s^0 & 2K+5 & \end{array}$$

a) i) stable

$-(K+2) > 0$ and $2K+5 > 0$

$K < -2$ $\quad$ $K > -2.5$

$-2.5 < K < -2$

R H Criterion

ii) limitedly stable

$K = -2$ or $K = -2.5$

iii) unstable

$K > -2$ or $K < -2.5$

b) $s_1, s_2 = \frac{1}{2}\left\{(K+2) \pm \sqrt{(K+2)^2 - 4(2K+5)}\right\}$

For critically damped, $(K+2)^2 - 4(2K+5) = 0$

for stable $-2.5 < K < -2$ $\qquad K = 0.47, -2.47$

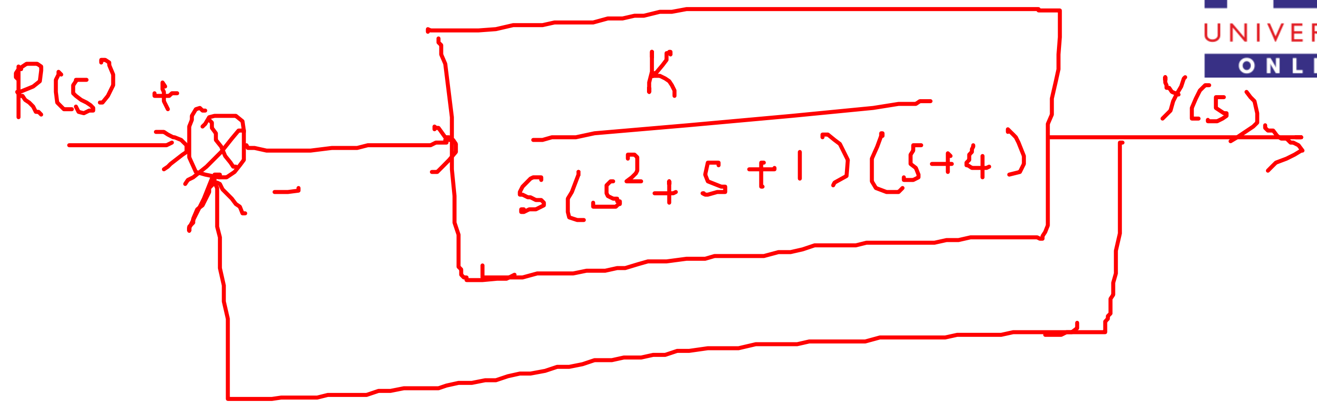
for underdamped, $-2 > K > -2.47$

for overdamped, $-2.47 > K > -2.5$

R H Criterion

Ex,

$$\frac{Y(s)}{R(s)} = \frac{K}{s(s^2+s+1)(s+4)+K}$$



C.E, $s^4 + 5s^3 + 5s^2 + 4s + K = 0$

s^4	1	5	K
s^3	5	4	
s^2	$\frac{21}{5}$	K	
s^1	$\frac{\frac{84}{5} - 5K}{21/5}$		
s^0	K		

For the s/m to be stable

$K > 0 \leftarrow \frac{84}{5} - 5K > 0$

$K < \frac{84}{25}$

$K < 3.36$

$0 < K < 3.36$

R H Criterion

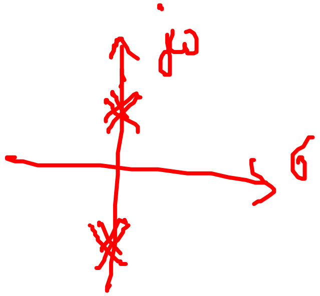
When $K = \frac{84}{25}$

$$A(s) = \frac{21}{5} s^2 + \frac{84}{25} = 0$$

$$s^2 = -\frac{4}{5}$$

$$s = \pm j\sqrt{4/5} \rightarrow \omega$$

frequency of sustained oscillation $\omega = \sqrt{\frac{4}{5}}$ rad/sec



s/m is unstable but from RH => s/m is marginally stable which is not true.

- Case 4: Repeated roots of the characteristic equation on jw axis
  - This is a DRAWBACK of RH criteria.
  - We cannot get the result by using Routh table as if repetitive poles lie on the jw axis which means the system is unstable but by using RH table we get the system is marginally stable.
  - Ex, $s^5 + s^4 + 2s^3 + 2s^2 + s + 1 = 0$

$A_1(s) = s^4 + 2s^2 + 1$

$\frac{dA_1}{ds} = 4s^3 + 4s$

$A_2(s) = s^2 + 1 \Rightarrow \frac{dA_2}{ds} = 2s$

s^5	1	2	1
s^4	1	2	1
s^3	0 → 4	0 → 4	
s^2	1	1	
s^1	0 → 2		
s^0	1		

$A_2 = s^2 + 1 = 0$

$s = \pm j$

$A_1 = s^4 + 2s^2 + 1 = 0$

$s = \pm j, \pm j$

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

R H Criterion

- $q(s) = (s + 1)(s + j)(s - j)(s + j)(s - j)$ Solve using Routh Hurwitz.

The Routh array is

->

$$
\begin{array}{c|ccc}
s^5 & 1 & 2 & 1 \\
s^4 & 1 & 2 & 1 \\
s^3 & \epsilon & \epsilon & 0 \\
s^2 & 1 & 1 & \\
s^1 & \epsilon & 0 & \\
s^0 & 1 & &
\end{array}
$$

- where $\epsilon \longrightarrow 0$. Note the absence of sign changes, a condition that falsely indicates that the system is marginally stable. The impulse response of the system increases with time as $t \sin(t + \phi)$.

- The auxiliary polynomial at the s^2 line is s^2 + 1, and the auxiliary polynomial at the s^4 line is s^4 + 2s^2 + 1 = (s^2 + 1)^2, indicating the repeated roots on the jw-axis.

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

R H Criterion

- q(s) = s^5 + s^4 + 4s^3 + 24s^2 + 3s + 63

The Routh array is

$$\begin{array}{c|ccc} s^5 & 1 & 4 & 3 \\ s^4 & 1 & 24 & 63 \\ s^3 & -20 & -60 & 0. \\ s^2 & 21 & 63 & 0 \\ s^1 & 0 & 0 & 0 \end{array}$$

Therefore, the auxiliary polynomial is

$$U(s) = 21s^2 + 63 = 21(s^2 + 3) = 21(s + j\sqrt{3})(s - j\sqrt{3}). \qquad (6.16)$$

which indicates that two roots are on the imaginary axis. To examine the remaining roots, we divide by the auxiliary polynomial to obtain

$$\frac{q(s)}{s^2 + 3} = s^3 + s^2 + s + 21.$$

Establishing a Routh array for this equation, we have

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 1 & 21 \\ s^1 & -20 & 0 \\ s^0 & 21 & 0 \end{array}$$

- The two changes in sign in the first column indicate the presence of two roots in the right-hand plane, and the system is unstable. The roots in the right-hand plane are s = +1 ± j√6

UNIT 3: THE STABILITY OF LINEAR FEEDBACK SYSTEMS

The Relative Stability of Feedback Control Systems

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The Relative Stability of Feedback Control Systems

- Given the CLS is stable system, we can further characterize the degree of stability is referred as **relative stability**. Ex, Aircraft design
- Relative stability can be determined by finding the settling time
- The settling time being inversely proportional to the real part of the dominant roots, the relative stability can be specified by requiring that all the roots of the characteristic equation be more negative than a certain value.
- i.e., all the roots must lie to the left of the line $s = -\sigma_1 (\sigma_1 > 0)$

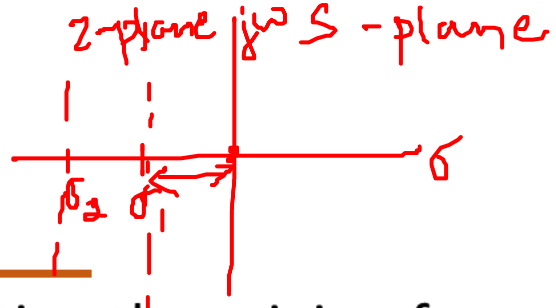
real part

$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

$t_s = \frac{4}{\zeta\omega_n}$

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

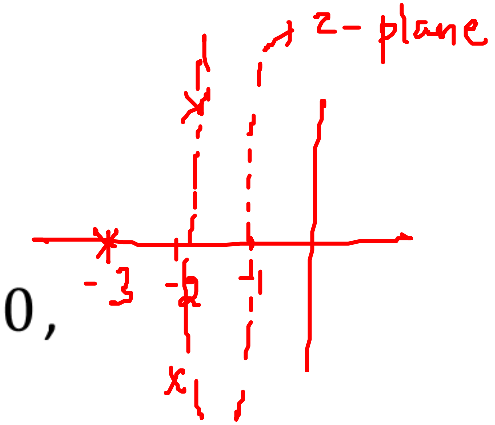
The Relative Stability of Feedback Control Systems



- The C.E of the system under study is then modified by shifting the origin of the s- plane to $s = -\sigma_1$ i.e. by substitution $s = z - \sigma_1$

- If the new C.E in z satisfies the RH criterion are more negative than $-\sigma_1$

- Ex, Consider a 3rd order system with C.E $s^3 + 7s^2 + 25s + 39 = 0$, Determine the relative stability



$s = -3, -2 \pm j3$

let $\sigma_1 = 1$, $s = z - 1$

$\therefore C.E = (z-1)^3 + 7(z-1)^2 + 25(z-1) + 39 = 0$

$z^3 + 4z^2 + 14z + 20 = 0$

z^3	1	14
z^2	4	20
z^1	9	
z^0	20	

$\Rightarrow$ s/m is stable

let $\sigma_1 = 2$, $s = (z-2)$, $C.E =$

THE STABILITY OF LINEAR FEEDBACK SYSTEMS

The Relative Stability of Feedback Control Systems

Ex, $q(s) = s^3 + 4s^2 + 6s + 4$

THANK YOU

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