LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

Linear Transformations

Definition :

A transformation T is said to be linear if it satisfy the rule of linearity.

i.e., $A(cx+dy) = c\,A(x) + d\,A(y)$ for any scalar c,d are real constants.

Linear Transformations

Example: I n linear system of equations Ax =b , Matrix A is a transformation from R^n to R^m .

Note: Consider a transformation $T : A \to B$

where A and B are subspaces .

1. A is the domain of the transformation.

2. B is the co domain of the transformation.

3. For any x in A , there exist Tx in B, here Tx is the image of T and x is the pre image of Tx

Linear Transformations

4. The set of all images is the subset of B is called Range of the transformation.

5. For all x in A such that Tx =o is called the Kernel of the transformation.

6. Dimension of the range is called rank and dimension of Kernel is called nullity.

Linear Transformations

Definition :

The space of all polynomials in t of degree n is
a vector space called the ***polynomial space***
denoted by P_n.
P_n= { Its basis is 1 , t, t^2, …. , t^n and dimension is n+1 .

Linear Transformations

Example 1 : The operation of differentiation

is linear. It takes P_{n+1} to P_n. The column space is the whole of P_n and the null space is P_0, the 1-dimensional space of all constants.

Example 2: The operation of integration

is linear. It takes P_n to P_{n+1}. The column space is a subspace of P_{n+1} and the null space is just the zero vector.

Linear Transformations

Example 3 :

Multiplication by a fixed polynomial , say $3 + 4t$ is also a linear transformation.

Let $p(t) = a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n$ then

$A\,p(t) = (3+4t)\,p(t) = 3a_0 + \dots + 4a_n t^{n+1}$.

This A sends P_n to P_{n+1}.

PES UNIVERSITY ONLINE

THANK YOU