UE20MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

<u>Unit-2-Vector Spaces:</u>

Vector Spaces and Subspaces (definitions only) , Linear Independence, Basis and Dimensions, The Four Fundamental Subspaces.
Self Learning Component: Examples of Vector Spaces and Subspaces.

Class No.	Portions to be covered
15-16	Vector Spaces and Subspaces (Definition & Examples)
17	Echelon Form, Row Reduced Form, Pivot Variables , Free variables
18-19	Linear Dependence, Independence, Basis and Dimensions
20	**Matlab Class Number 3 – LU Decomposition**
21-22	The Four Fundamental Subspaces-Column Space and Row Space
23	Null Space
24	Left Null Space
25-26	Problems on Four Fundamental Subspaces
27	**Matlab Class Number 4 -Inverse of a Matrix by Gauss Jordan Method**
28	**Applications**

Classwork problems:

1.	Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors: (a) $\{(1,3,1,-2),(2,5,-1,2),(1,3,7,-2)\}$ (b) $\{(1,1,2),(1,2,5),(5,3,4)\}$ (c) $\{t^2-t+5,\ 2t^2-3t,\ -t^2+2t+5\}$ (d) $\left\{\begin{pmatrix}1 & -5\\ -4 & 2\end{pmatrix},\begin{pmatrix}1 & 1\\ -1 & 5\end{pmatrix},\begin{pmatrix}-2 & 3\\ 5 & 2\end{pmatrix},\begin{pmatrix}1 & -2\\ -5 & 3\end{pmatrix}\right\}$ in $M_{2x2}(R)$ **Answer:** (a)independent (b) dependent $c_1=-7c_3$, $c_2=2c_3$(c)dependent, $p_1(t)-p_2(t)=p_3(t)$. (d) independent.
2.	(a)Determine whether these vectors $\{(1,1,1,1),(1,2,3,2),(2,5,6,4),(2,6,8,5)\}$ form a basis of R^4. If not, find the dimension of the subspace S they span. (b) If S is a subspace of R^4 , extend the basis of S to a basis of R^4. **Answer:** They do not form a basis of R^4. They span a subspace S of dimension 3.

3.	Reduce the following matrices to Row Reduced Echelon form and determine their ranks $\begin{pmatrix} 2 & -4 & 4 & -2 \\ 4 & -9 & 7 & -3 \\ 1 & -4 & 8 & 0 \end{pmatrix}$. $\begin{pmatrix} 0 & 3 & 1 & 4 \\ 1 & 1 & 2 & 1 \\ 3 & 4 & 5 & 2 \\ 4 & 8 & 8 & 7 \end{pmatrix}$ Identify the pivot variables and free variables. Find the special solutions to Ax=0. **Answer:** (-1,-7/8,1/8,1); (13/4,-3/4,-7/4,1)
4.	Find the Column space and Null space for the following matrices: $\begin{pmatrix} 2 & 4 & -2 & 2 \\ -2 & 5 & 7 & 3 \\ -3 & 6 & -8 & 6 \end{pmatrix}$ $\begin{pmatrix} -2 & 1 & 2 & 1 \\ 0 & 3 & -1 & -2 \\ 2 & -3 & 0 & 4 \\ 1 & -1 & 2 & -2 \end{pmatrix}$ **Answer:** C(A) is a 3-d plane in $\mathbb{R}^3$ and N(A) is a line in $\mathbb{R}^4$ C(A) is a 4-d plane in $\mathbb{R}^4$ and N(A) is origin in $\mathbb{R}^4$
5.	For which vector b=(a,b,c) does the following system Ax=b have a solution? 2x+2y+3z=a; 3x-y+5z=b; x-3y+2z=c **Answer:** c-b+a=0
6.	Which combination of vectors (a,b,c) are in the column space of A? $\begin{pmatrix} 1 & 0 & 5 & 3 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & -4 & 1 \end{pmatrix}$ **Answer:** The column space contains all vectors with a-b+c=0.
7.	Find a basis for the set of vectors in $\mathbb{R}^3$ in the plane 2x-3y+4z=0. **Answer:** {(3,2,0), (-2,0,1)}
8.	For which vector (b_1,b_2,b_3, b_4) is this system solvable? $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & 9 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ **Answer:**. $b_2=2b_1$ and $3b_1-3b_3+b_4=0$
9.	If the set of vectors {u,v,w} are linearly independent vectors, then show that the set $\{u+v,\ u-v,\ u-2v+w\}$ is linearly independent.

10.	Find a basis and dimension of the subspace W of $V=M_{2x2}$ spanned by $A=\begin{pmatrix}1 & -5\\ -4 & 2\end{pmatrix}, B=\begin{pmatrix}1 & 1\\ -1 & 5\end{pmatrix}, C=\begin{pmatrix}2 & -4\\ -5 & 7\end{pmatrix} D=\begin{pmatrix}1 & -7\\ -5 & 1\end{pmatrix}.$ **Answer:** Basis of W = $\left\{\begin{pmatrix}1 & -5\\ -4 & 2\end{pmatrix}, \begin{pmatrix}1 & 1\\ -1 & 5\end{pmatrix}\right\}$ Dim=2
11.	Find a basis and the dimension of the subspaces of $V=\{(a,b,c,d)/a-2b=4c, 2a=c+3d\}$ in $\mathbb{R}^4$ **Answer:** Basis of V = {(1,1/2,0,2/3), (0,-2,1,-1/3)} Dim=2
12.	Find conditions on a,b,c so that v=(a,b,c) in $\mathbb{R}^3$ belongs to W =span(u_1, u_2, u_3) where u_1=(1,2,0), u_2=(-1,1,2), U_3=(3,0,-4) (i) Do u_1, u_2, u_3 span $\mathbb{R}^3$? (ii) Is W a subspace of $\mathbb{R}^3$? (iii) Find a basis and the dimension of W. **Answer:** 3c+2b-4a=0 (i)No, u_1, u_2, u_3 do not span $\mathbb{R}^3$ (ii)Yes (iii) Basis={(u_1,u_2)}
13.	If the column space of A is spanned by the vectors (1,0,-1), (2,1,3), (4,2,6), (3,1,2), find all those vectors that span the null space of A. Determine whether or not the vector b=(-2,-2,0,2) is in that subspace What are the bases and dimensions of $C(A^T)$ and $N(A^T)$. **Answer:** (0,-2,1,0),(-1,-1,0,1) $b\in N(A)$,Dim of $C(A^T)$=2, Dim of $N(A^T)$=1
14.	Obtain the four fundamental subspaces , their basis and dimension given $\begin{pmatrix}-3 & 6 & -1 & 1 & -7\\ 1 & -2 & 2 & 3 & -1\\ 2 & -4 & 5 & 8 & -4\\ 4 & -2 & 1 & -5 & -7\end{pmatrix}$. Also describe the four fundamental subspaces.
15.	Find left / right inverse (whichever possible) for the following matrices (i) $\begin{pmatrix}3 & 0 & 1\\ 1 & 0 & -2\end{pmatrix}$ (ii) $\begin{pmatrix}3 & 1\\ -1 & 1\\ 1 & -1\end{pmatrix}$