श्री:

Communication Engineering

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UNIT - I

Amplitude Modulation.

Introduction to Communication Engineering

* Idea of communication : Transfer of information

* Electrical communication : how it developed ?
  → Telegraph, Telephone, triode, TV, shannon's idea, transistor . . .

Basic block diagram of a comm. system

Source — Transmitter — channel — Receiver — Destⁿ.

Source : Can be speech, audio, image, video — .

Transmitter : can include
i) conversion to electrical form (transducer)
ii) compression
iii) modulation
iv) conversion from electrical to electromagnetic/optical etc.

channel : provides the physical medium for communication. can be wired, wireless, optical, storage etc.

Receiver : processes the received signal to provide an estimate of the source signal.

Why study analog Comm^n ?

* Many analog communication systems are still in use.
  ex: Broadcast systems
* Helps in understanding digital communication
* Real time operations
* At very high frequencies, ADC/DAC operation becomes difficult. Analog communication is useful in such situations.
  Ex: 5G systems, where hybrid analog / digital techniques are used.

Review of Fourier Transform :

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

some properties :

Time shift : $x(t-t_0) \xleftrightarrow{F} X(f) e^{-j2\pi f t_0}$

freq. shift : $e^{j2\pi f_0 t} x(t) \xleftrightarrow{F} X(f-f_0)$

reversal : $x(-t) \xleftrightarrow{F} X(-f)$

conjugation : $x^*(t) \xleftrightarrow{F} X^*(-f)$

differentiation : $\frac{d\, x(t)}{dt} \xleftrightarrow{F} j2\pi f\, X(f)$

integration : $\int_{-\infty}^{t} x(\tau)\, d\tau \xleftrightarrow{F} \frac{X(f)}{j2\pi f} + \frac{1}{2} X(0) \delta(f)$

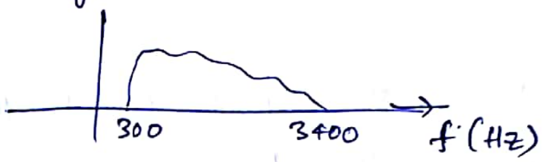
duality : if $x(t) \xleftrightarrow{F} X(f)$, then $X(t) \xleftrightarrow{F} x(-f)$

parseval's theorem : $\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$

Modulation

The information bearing signal is called the message signal / modulating signal / baseband signal.
The term "baseband" refers to the band of frequencies occupied by the message signal.

Ex: Speech signal



Modulation is the process by which some characteristic of a carrier wave is varied in accordance with the message signal.

The carrier is typically a sinusoid. The result of modulation is called the "modulated signal".
Modulation results in a shift of the freq. band of the modulating signal.

Depending on the parameter of the carrier wave that is varied, we have
i) Amplitude modulation
ii) Frequency modulation } Angle modulation.
iii) phase modulation

Need for modulation

1. Antenna height

An antenna can be considered to be an opened out transmission line that converts electrical energy to electromagnetic energy. It requires standing wave pattern for its operation.
$\therefore$ The height of the antenna should be at least $\lambda/4$, where λ is the wavelength of the message signal.

Ex: Find the required antenna height if $f_0 = 2\,kHz$.

Ans: $\lambda = \frac{c}{f_0} = \frac{3 \times 10^8}{2 \times 10^3} = 150\ km$

$\therefore \frac{\lambda}{4} = 37.5\ km \Rightarrow$ impractical.

Ex: Consider the speech signal that occupies the band from 300 Hz to 3 kHz.

$f_0 = 300\ Hz \Rightarrow \frac{\lambda}{4} = \frac{3 \times 10^8}{4 \times 3 \times 10^2} = 250\ km$

$f_0 = 3000\ Hz \Rightarrow \frac{\lambda}{4} = \frac{3 \times 10^8}{4 \times 3 \times 10^3} = 25\ km$

$\Rightarrow$ single antenna won't be able to handle the signal.

By shifting the signal to a high frequency band, we can reduce the required antenna height.

$\therefore$ we go for radio frequency transmission.

[Radio frequency : Frequency that can be efficiently transmitted using a practicable antenna]

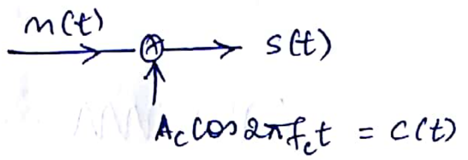
2. To avoid interference from other baseband sources. Since "separation in time" is not practical, we go for "separation in frequency".

3. Efficient utilization of ~~th~~ the available spectrum

4. To avoid man-made noise in baseband frequencies.

DSBSC

This technique is the direct application of the frequency shift property of fourier Transform.



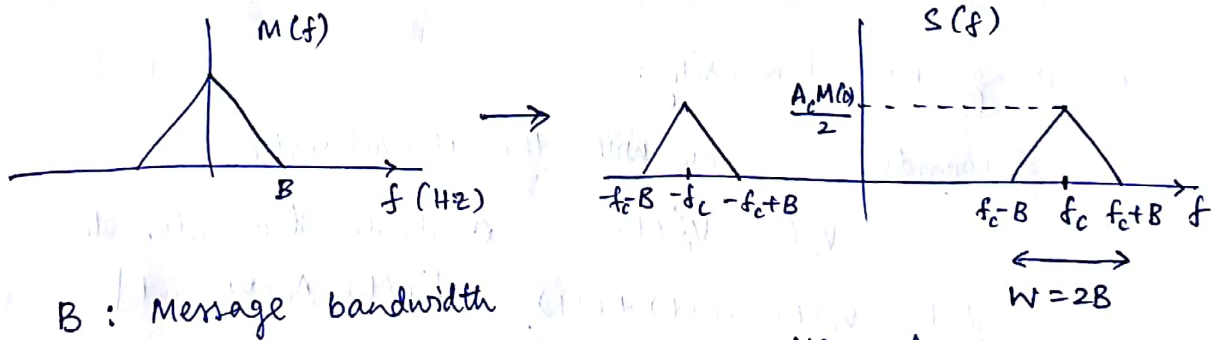
The message signal m(t) is multiplied with the carrier $A_c \cos 2\pi f_c t$ to obtain the modulated waveform s(t).

$$s(t) = A_c\, m(t) \cos 2\pi f_c t$$

$$= A_c\, m(t) \left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right]$$

By the freq. shift property of FT

$$S(f) = \frac{A_c}{2} \left[M(f - f_c) + M(f + f_c) \right]$$



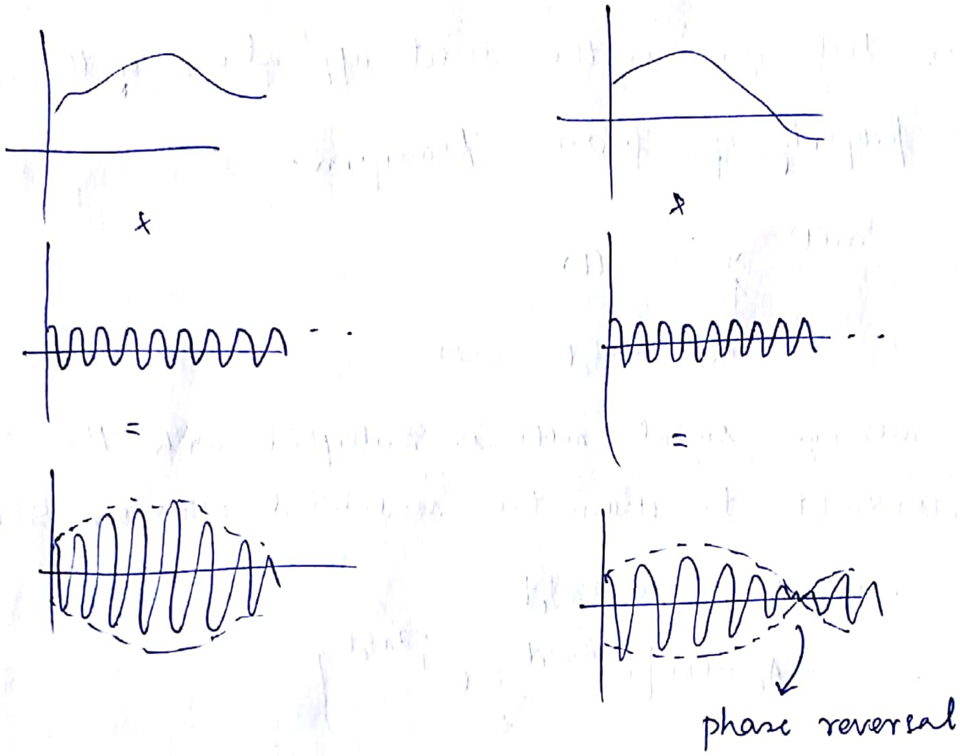
B : Message bandwidth

W = 2B : Transmission bandwidth

Narrowband commn ⇒ $10W \leq f_c \leq 100W$

[NOTE : Deterministic signals do not contain information; only random signals do. Hence we actually need to consider random processes and their power spectra. Here, for the purpose of illustration, we consider deterministic signals & their spectra]

Time domain illustration



DSBSC causes envelope distortion.

Generation of DSBSC

1. using non-linearity:

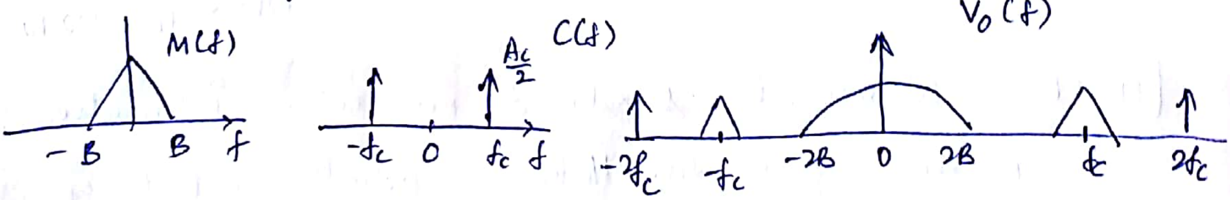
Consider a device with the characteristic

$$V_o(t) = V_i^2(t) \qquad \text{ex: diode, transistor etc.}$$

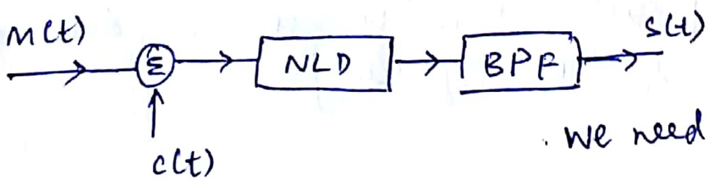
$$[c(t) = A_c \cos 2\pi f_c t]$$

Let $v_i(t) = m(t) + c(t)$

$\therefore V_o(t) = m^2(t) + c^2(t) + 2m(t)c(t)$



A BPF can be used to recover $m(t)c(t)$



We need $\boxed{f_c \geq 3B}$

Suppose $V_o(t) = a_1 v_i(t) + a_2 v_i^2(t)$

Let $v_i(t) = m(t) + c(t)$

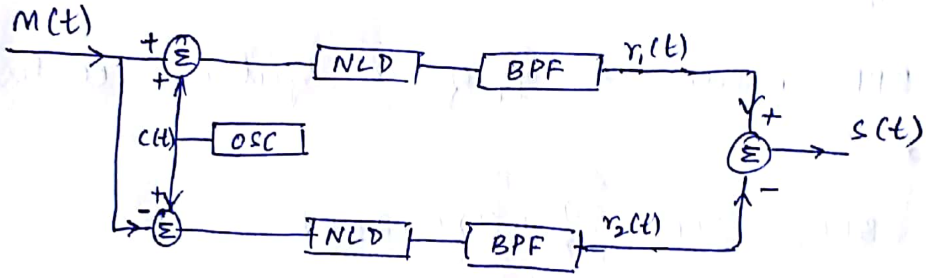
Then $V_o(t) = a_1 [m(t) + c(t)] + a_2 [m^2(t) + c^2(t) + 2m(t)c(t)]$

After BPF, we get $r_1(t) = a_1 c(t) + 2a_2 m(t) c(t)$

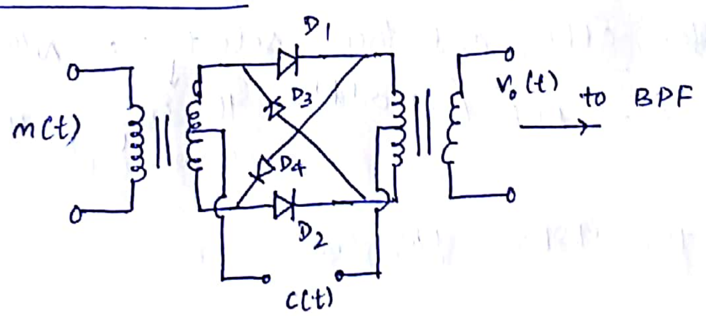
$|||^{ly}$, if $v_i(t) = c(t) - m(t)$, after BPF, we have

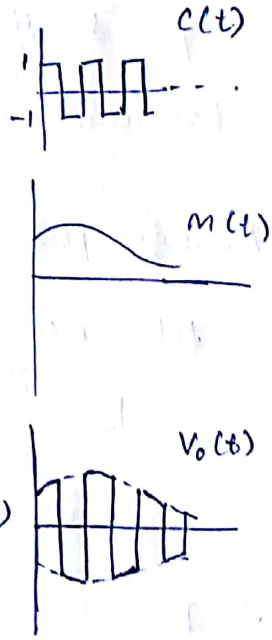
$r_2(t) = a_1 c(t) - 2a_2 m(t) c(t)$

$\therefore$ we have the following structure



Ring modulator :





The transformers are center-tapped.

when $c(t)$ is '+'ve, D_1 & D_2 conduct, & ~~s(t)~~ $\Rightarrow$

$V_o(t) = m(t)$

when $c(t)$ is '−'ve, D_3 & D_4 conduct &

$V_o(t) = -m(t)$

Recall the Fourier series of $c(t)$

$$c(t) = \frac{4}{\pi} \sum_{K=1}^{\infty} \frac{(-1)^{K-1}}{2K-1} \cos[2\pi f_c t (2K-1)]$$

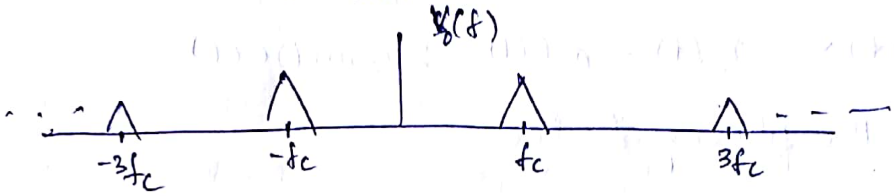
$\therefore\ v_o(t) = c(t)\,m(t)$

$$= \frac{4}{\pi}\sum_{K=1}^{\infty}\frac{(-1)^{K-1}}{2K-1}\cos\left[2\pi f_c t(2K-1)\right] m(t)$$

$$\therefore\ V_o(f) = \left[\frac{4}{\pi}\sum_{K=1}^{\infty}\frac{(-1)^{K-1}}{2K-1}\left\{\delta\left(f-\{2K-1\}f_c\right) + \delta\left(f+\{2K-1\}f_c\right)\right\}\right] * M(f)$$

[multiplication property of FT]

$$= \frac{4}{\pi}\sum_{K=1}^{\infty}\frac{(-1)^{K-1}}{2K-1}\left[M\left(f-\{2K-1\}f_c\right) + M\left(f+\{2K-1\}f_c\right)\right]$$



The BPF recovers the copy of the spectrum centered around f_c.

After BPF, $s(t) = \frac{4}{\pi}\,m(t)\cos 2\pi f_c t$.

* In fact, we can use any periodic waveform with period $\frac{N}{f_c}$ for $c(t)$, and then select the Nth harmonic using the BPF. (N odd) the copy of $M(f)$ around the

Average power of the DSBSC waveform

Let $m(t)$ have an average power of P_m W.

$$P_m = \int_{-\infty}^{\infty}|m(t)|^2\,dt = \int_{-\infty}^{\infty}|M(f)|^2\,df$$

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

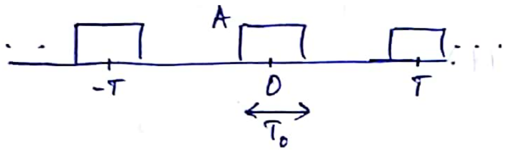
$$\therefore\ P_s = \int_{-\infty}^{\infty}|S(f)|^2\,df = \frac{A_c^2}{4}\left[\int_{-\infty}^{\infty}|M(f-f_c)|^2\,df + \int_{-\infty}^{\infty}|M(f+f_c)|^2\,df\right]$$

$$= \frac{A_c^2}{4}\cdot\left[P_m + P_m\right] = \frac{A_c^2 P_m}{2}$$

F.S for the square wave

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi f_c t}$$

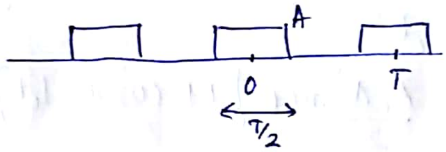
$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)e^{-jk2\pi f_c t}\,dt$$



$\xleftrightarrow{F}$ $a_k = Ad \text{ sinc } kd$

where $d = \frac{T_0}{T}$ = duty cycle.

$$\therefore \quad a_k = A.d \frac{\sin k\pi d}{k\pi d} \qquad a_0 = Ad.$$

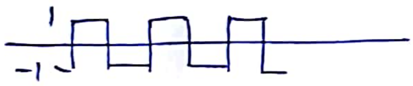


$d = 1/2 \Rightarrow a_k = \frac{A}{\cancel{2}} \frac{\sin k\pi/2}{k\pi/\cancel{2}}$

$a_0 = A/2$

$\Rightarrow a_k = A \frac{\sin k\pi/2}{k\pi}, \quad a_0 = 0$

$\therefore$



$\Rightarrow a_k = \frac{2}{k\pi}.\sin k\pi/2$

$$\therefore \quad a_1 = a_{-1} = 2/\pi$$
$$a_2 = a_{-2} = 0$$
$$a_3 = a_{-3} = \frac{-2}{3\pi}$$
$$a_4 = a_{-4} = 0$$
$$a_5 = a_{-5} = \frac{2}{5\pi}$$
$$\vdots$$

all even harmonics are zero

$$\therefore x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk.2\pi f_c t}$$

$$= \frac{2}{\pi}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right] - \frac{2}{3\pi}\left[e^{j2\pi.3f_c t} + e^{-j2\pi.3f_c t}\right]$$
$$+ \frac{2}{5\pi}\left[e^{j2\pi 5f_c t} + e^{-j2\pi 5f_c t}\right] - \frac{2}{7\pi}\left[e^{j2\pi 7f_c t} + e^{-j2\pi 7f_c t}\right] + \ldots$$

$$\therefore x(t) = \frac{4}{\pi}\cos 2\pi f_c t - \frac{4}{3\pi}\cos 2\pi 3f_c t + \frac{4}{5\pi}\cos 2\pi 5 f_c t - \ldots$$

$$= \frac{4}{\pi}\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1}\cos\left[2\pi f_c t(2k-1)\right]$$

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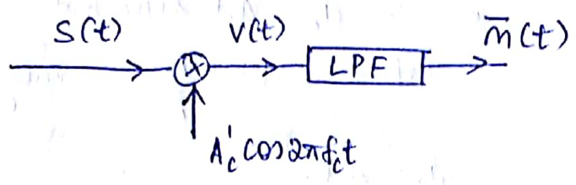
Demodulation

For DSBSC, we use "coherent" demodulation.

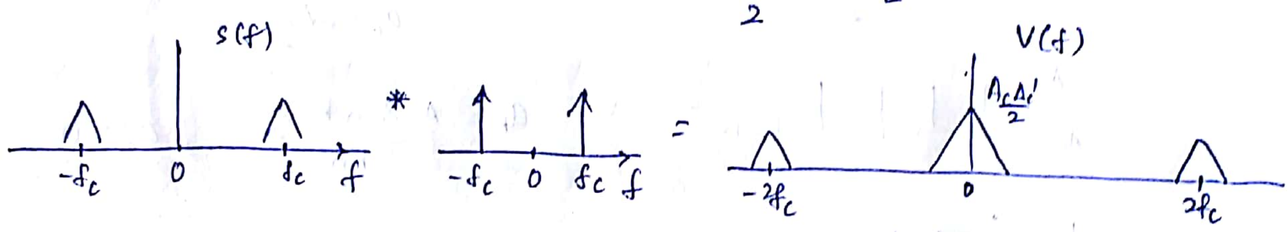
coherent = A local copy of the carrier (with the same phase) is used at the receiver.



$$v(t) = s(t) \cdot A_c' \cos 2\pi f_c t$$

$$= A_c m(t) \cos 2\pi f_c t \cdot A_c' \cos 2\pi f_c t$$

$$= A_c A_c' m(t) \cdot \cos^2 2\pi f_c t = \frac{A_c A_c' m(t)}{2}\left[1 + \cos 4\pi f_c t\right]$$



After LPF,

$$\bar{m}(t) = \frac{A_c A_c'}{2} m(t).$$

Frequency error

Suppose the local oscillator generates $A_c' \cos 2\pi (f_c + \Delta f) t$

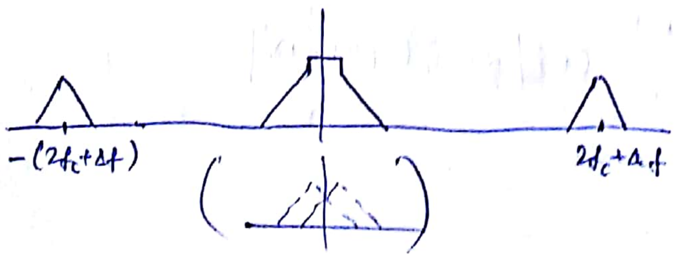
Then $v(t) = A_c A_c' m(t) \cos 2\pi f_c t \cdot \cos 2\pi (f_c + \Delta f) t$

$$= \frac{A_c A_c'}{2} m(t) \left[\cos 2\pi (2f_c + \Delta f) t + \cos 2\pi \cdot \Delta f t\right]$$

After LPF,

$$\bar{m}(t) = \frac{A_c A_c'}{2} m(t) \cdot \cos 2\pi \cdot \Delta f t$$

⇒ There is a residual DSBSC effect. This results in distortion. This is known as "beat effect".



phase error

Let $v(t) = A_c m(t) \cdot \cos 2\pi f_c t \cdot A_c' \cos(2\pi f_c t + \phi)$

$$= \frac{A_c A_c'}{2} m(t) \left[\cos(2\pi \cdot 2f_c t + \phi) + \cos\phi \right]$$

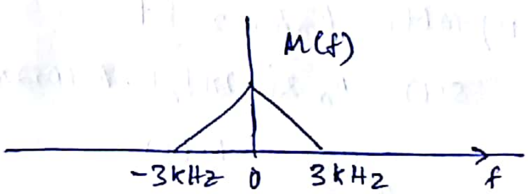
After LPF, $\bar{m}(t) = \frac{A_c A_c'}{2} m(t) \cdot \cos\phi$.

* phase error results only in scaling by $\cos\phi$, but no distortion.

* If $\phi = \pm\frac{\pi}{2}$, then $\bar{m}(t) = 0$. This is known as quadrature null effect ..

Problems :

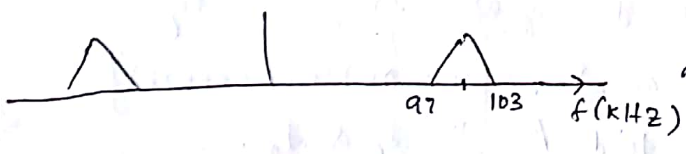
1. consider the signal $m(t)$ with the spectrum shown below. Find the required antenna height at the cutoff frequencies for
i) no modulation
ii) DSBSC with $f_c = 100$ kHz
iii) DSBSC with $f_c = 1000$ kHz



Ans: i) no modulation

$$\frac{\lambda}{4} = \frac{3\times10^8}{4\times3\times10^3} = \frac{10^5}{4} = 25 \text{ km}$$

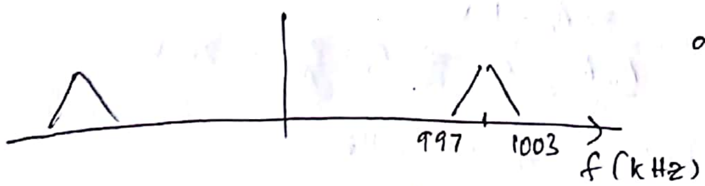
ii) $f_c = 100$ kHz



at 97 kHz, $\frac{\lambda}{4} = \frac{3\times10^8}{4\times97\times10^3} = 773$ m

at 103 kHz, $\frac{\lambda}{4} = \frac{3\times10^8}{4\times103\times10^3} = 728$ m

iii) $f_c = 1000$ kHz



at 997 kHz, $\frac{\lambda}{4} = \frac{3\times10^8}{4\times997\times10^3}$

$= 75.2$ m

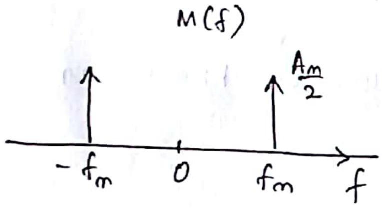
at 1003 kHz, $\frac{\lambda}{4} = \frac{3\times10^8}{4\times1003\times10^3} = 74.7$ m

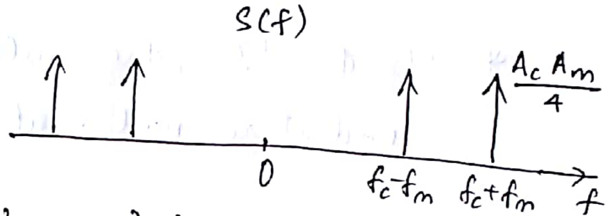
2. Draw the spectrum of the modulated wave for
i) $m(t) = A_m \cos 2\pi f_m t$
ii) $m(t) = A_m \sin 2\pi f_m t$
with ~~f_c as the carrier frequency~~ $c(t) = A_c \cos 2\pi f_c t$.

Also find the power of $s(t)$

i) $m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t = \frac{A_m A_c}{2}\left[\cos 2\pi (f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$$



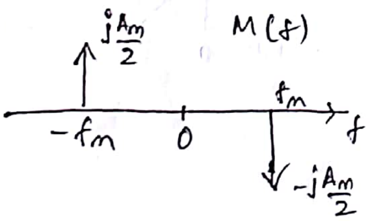


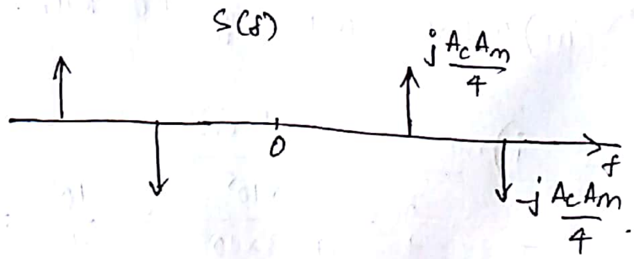
Avg. power : $P_s = \frac{A_c^2}{2} \cdot P_m = \frac{A_c^2}{2} \cdot \frac{A_m^2}{2} = \frac{A_c^2 A_m^2}{4}$

or, from $S(f)$: $P_s = \left(\frac{A_c A_m}{4}\right)^2 \times 4 = \frac{A_c^2 A_m^2}{4}$

ii) $m(t) = A_m \sin 2\pi f_m t$

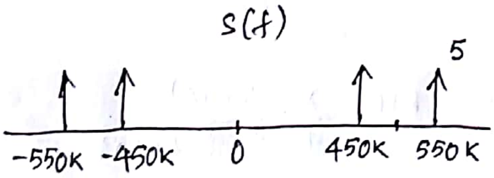
$$s(t) = A_m \sin 2\pi f_m t \cdot A_c \cos 2\pi f_c t = \frac{A_c A_m}{2}\left[\sin 2\pi(f_c+f_m)t - \sin 2\pi(f_c-f_m)t\right]$$





$P_s = \frac{A_c^2 A_m^2}{4}$.

3. The spectrum of the modulated signal is given by
$$S(f) = 5\left[\delta(f-550K) + \delta(f-450K) + \delta(f+450K) + \delta(f+550K)\right]$$
If $m(t)$ has power $P_m = 0.5$ W, find A_c, P_s, f_c & f_m.



$m(t) = A_m \cos 2\pi f_m t \quad P_m = \frac{A_m^2}{2} = 1/2$

$\Rightarrow A_m = 1$ V

$\frac{A_m A_c}{4} = 5 \Rightarrow A_c = \frac{20}{A_m} = 20$ V.

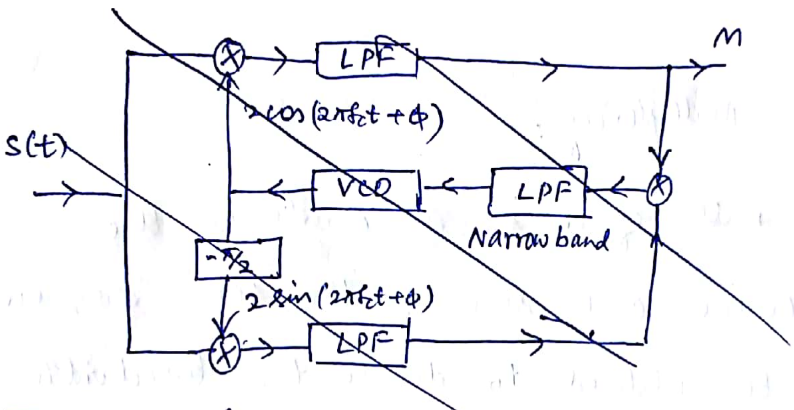
$P_s = \frac{A_c^2}{2} \cdot P_m = 100$W

$f_c = 500$ KHZ

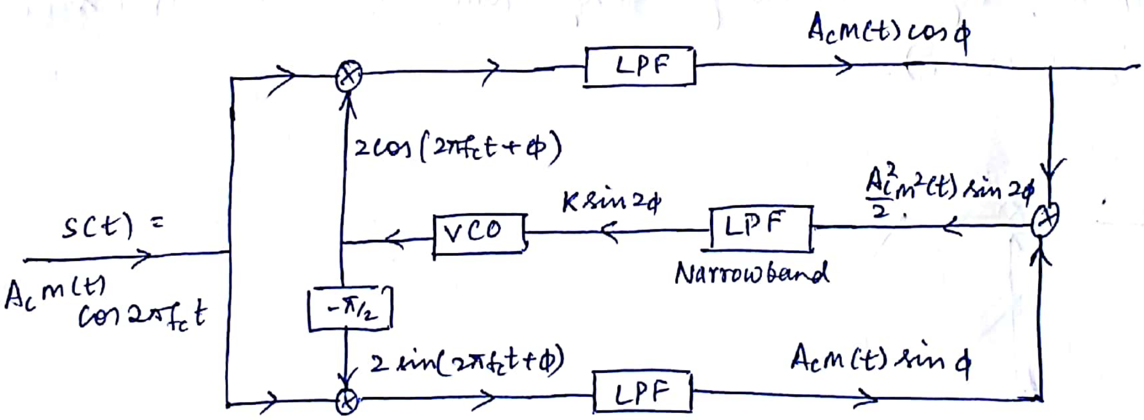
$f_m = 50$ KHZ.

Costas receiver

Costas receiver recovers the carrier phase from the modulated signal s(t)



The VCO (voltage controlled oscillator) is an oscillator whose frequency & phase of oscillation can be controlled by a voltage input. The costas receiver is a negative feedback system.



To start with, the VCO produces a cosine wave of ~~the~~ frequency f_c Hz, but with a phase shift ϕ. It is multiplied with s(t) in the upper branch, followed by LPF, resulting in the signal $A_c m(t) \cos\phi$. In the lower branch, s(t) is multiplied with $2\sin(2\pi f_c t+\phi)$, followed by LPF, resulting in $A_c m(t) \sin\phi$. The two signals are then input to the phase discriminator (multiplier followed by narrowband LPF).

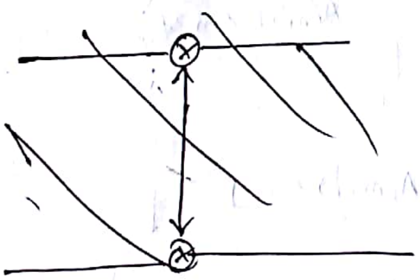
We have $(A_c m(t) \cdot \cos\phi)(A_c m(t) \sin\phi) = A_c^2 m^2(t) \sin 2\phi$.

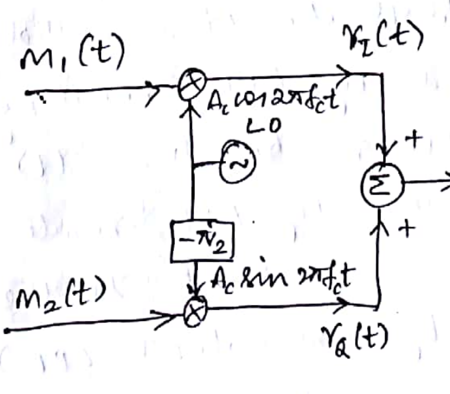
Since the LPF is narrowband, it captures the ~~dc~~ mainly the zero frequency content, which is proportional to $\sin 2\phi$.

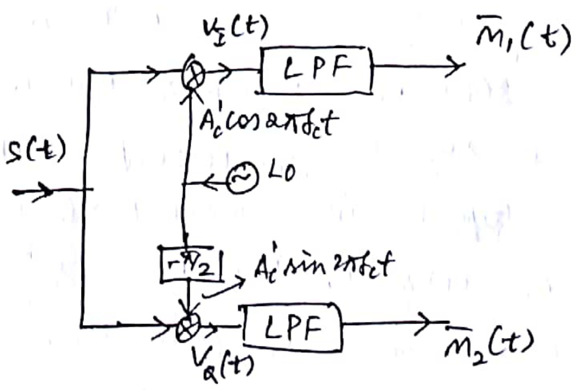
since ~~sinφ~~ $\sin\theta \approx \theta$ for small θ, the output of the phase discriminator is proportional to 2ϕ, which is given to the VCO, in order to correct the phase error.

Quadrature Carrier Multiplexing:

The quadrature null effect is a result of the orthogonality of cosine and sine of the same frequency. This property can be utilized to double the bandwidth efficiency of DSBSC. i.e, we can send two message signals of bandwidth B Hz in a transmission bandwidth of 2B Hz, by modulating ~~using~~ with two carriers (sine and cosine).







In the transmitter, the signal $m_1(t)$ is multiplied with the "in phase" carrier $A_c \cos 2\pi f_c t$, and $m_2(t)$ is multiplied with the "quadrature" carrier $A_c \sin 2\pi f_c t$. The two modulated signals are added, resulting in

$$s(t) = A_c \left[m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t \right]$$

At the receiver, $s(t)$ is fed to both the I & Q branches.

In the I branch,

$$v_I(t) = s(t)\, A_c' \cos 2\pi f_c t$$

$$= A_c A_c' \cdot \left[m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t \right] \cdot \cos 2\pi f_c t$$

$$= \frac{A_c A_c'}{2} \left[m_1(t) + m_1(t) \cos 2\pi (2f_c) t + m_2(t) \sin 2\pi (2f_c) t \right]$$

The LPF will block the high frequency components, and hence we have

$$\bar{m}_1(t) = \frac{A_c A_c'}{2} m_1(t)$$

In the Q branch,

$$v_Q(t) = s(t)\, A_c' \sin 2\pi f_c t$$

$$= A_c A_c' \left[m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t \right] \sin 2\pi f_c t$$

$$= \frac{A_c A_c'}{2} \left[m_1(t) \sin 2\pi (2f_c) t + m_2(t) - m_2(t) \cos 2\pi (2f_c) t \right]$$

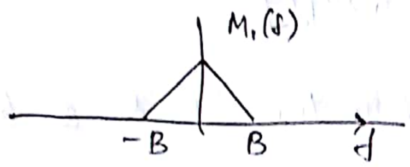
After LPF

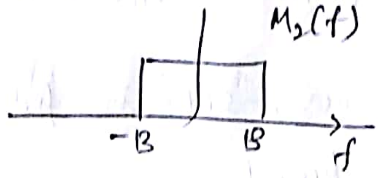
$$\bar{m}_2(t) = \frac{A_c A_c'}{2} m_2(t)$$

Thus both $m_1(t)$ & $m_2(t)$ can be recovered from $s(t)$

Exercise:

1. consider $m_1(t)$ & $m_2(t)$ with the spectra shown below.
   plot the spectra of $r_I(t)$, $r_Q(t)$, $s(t)$, $v_I(t)$, $v_Q(t)$, & $\tilde{m}_1(t)$ & $\tilde{m}_2(t)$.





Application

* DSBSC is used in stereo FM transmission
* QCM is used in TV transmission (for chrominance/luminance signals).

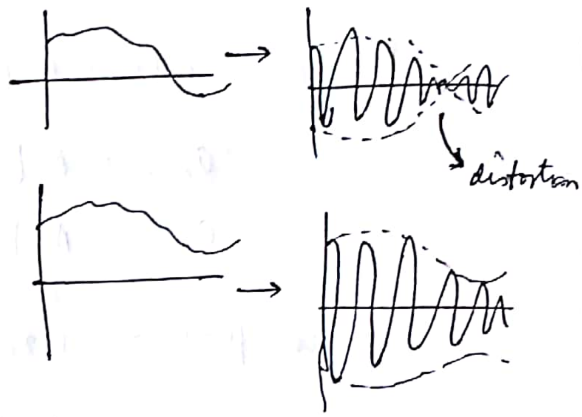
Disadvantages of DSBSC

1. Receiver is complex
2. Transmission BW is double the message BW.

The DSBSC receiver is complex because of the need for phase lock circuitry. In a broadcast scenario with one transmitter and possibly millions of receivers, it makes sense to have as simple a receiver structure as possible, while accommodating all the complexity at the transmitter. This is achieved in "standard AM".

Standard AM

A simpler alternative to coherent demodulator is the envelope detector. But it cannot be used in DSBSC since the envelope in DSBSC is distorted. In DSBSC, if the signal m(t) has no zero crossings, then the envelope will be an undistorted version of m(t). This can be achieved by adding a d.c value to the scaled version of m(t).



$\therefore \quad s(t) = [1 + K_a m(t)] A_c \cos 2\pi f_c t$

K_a: amplitude sensitivity (volt^{-1})

It is chosen in such a way that

$|K_a m(t)| \leq 1 \quad \forall t$, to ensure that envelope distortion is avoided.

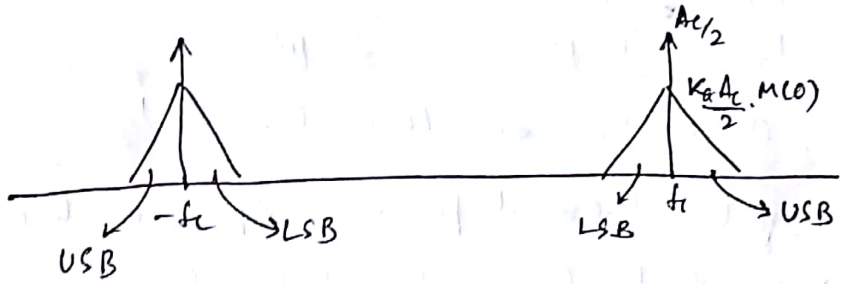
$|K_a m(t)|_{max}$ is called the "modulation index" 'μ'.

[or $|K_a m(t)|_{max} \times 100$ is percentage modulation]

We need $\mu \leq 1$ to avoid envelope distortion.

$$s(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier component}} + \underbrace{A_c K_a m(t) \cos 2\pi f_c t}_{\text{sidebands}}.$$

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{K_a A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$



Single tone modulation

Let $m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c\left[1 + K_a A_m \cos 2\pi f_m t\right]\cos 2\pi f_c t$$

$$= A_c\left[1 + \mu \cos 2\pi f_m t\right]\cos 2\pi f_c t \qquad \left[\because m(t)_{max} = A_m\right]$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\cos 2\pi (f_c + f_m)t + \frac{A_c \mu}{2}\cos 2\pi (f_c - f_m)t]$$

Envelope: $a(t) = A_c\left[1 + \mu \cos 2\pi f_m t\right]$

$$a_{max} = A_c[1+\mu] \qquad \text{or} \qquad \mu = \frac{a_{max} - a_{min}}{a_{max} + a_{min}}.$$
$$a_{min} = A_c[1-\mu]$$

when $\mu = 1 \Rightarrow 100\%$ modulation, $a_{min} = 0$.

carrier power: $P_c = \frac{A_c^2}{2}$

Sideband power: $P_{SB} = \frac{A_c^2 \mu^2}{4}$.

$\therefore$ total power: $P_T = P_c + P_{SB} = \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$

Modulation/power efficiency

$$\eta = \frac{P_{SB}}{P_T} = \frac{\mu^2/2}{1 + \mu^2/2} = \frac{\mu^2}{2+\mu^2}$$

for $0 \leq \mu \leq 1$, we have $0 \leq \eta \leq 1/3$.

$\Rightarrow$ most of the transmit power is spent on the carrier, that carries no information.

For a general signal $m(t)$,

$$s(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t.$$

$$P_c = \frac{A_c^2}{2}. \qquad P_{SB} = \frac{A_c^2 k_a^2}{2} P_m$$

$$\therefore \eta = \frac{P_{SB}}{P_c + P_{SB}} = \frac{k_a^2 P_m}{1 + k_a^2 P_m}.$$

For $0 \le \mu \le 1$, it can be shown that $0 \le \eta \le 0.5$.

[exercise].

AM generation

1. Squaring modulator

Consider the NLD with $V_o(t) = a_1 V_i(t) + a_2 V_i^2(t)$.

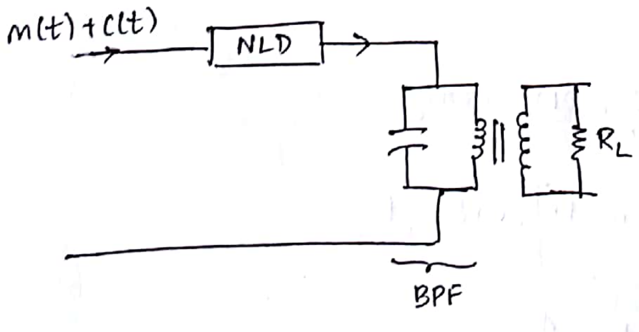
Let $V_i(t) = m(t) + c(t)$, then after BPF, we have

$$s(t) = a_1 c(t) + 2a_2 c(t) m(t)$$

$$= a_1 \cdot A_c \cos 2\pi f_c t + 2a_2 A_c m(t) \cos 2\pi f_c t$$

$$= A_c \cdot a_1 \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t$$

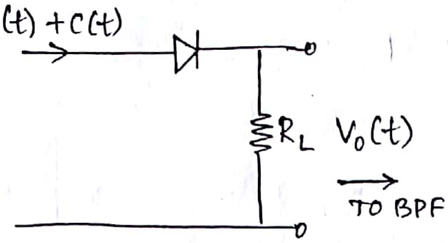
$$\Rightarrow k_a = \frac{2a_2}{a_1}$$



We need $f_c \geq 3B$

Switching modulator

$V_i(t) = m(t) + c(t)$

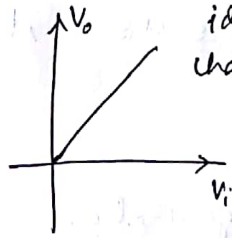


It is assumed that

i) the diode acts like an ideal switch

ii) $A_c >> |m(t)|$

$V_i(t) = m(t) + A_c \cos 2\pi f_c t$



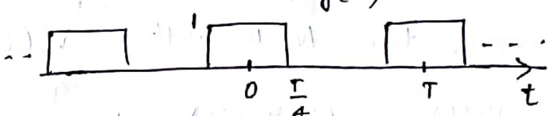
ideal diode characteristic

Due to the ideal nature of the diode, we have

$$V_o(t) = \begin{cases} V_i(t) & c(t) > 0 \\ 0 & c(t) < 0 \end{cases}$$

or $\quad V_o(t) = V_i(t) \cdot g(t)$

$g(t)$



WKT $\quad g(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1} \cos 2\pi (2k-1) f_c t \qquad T = \frac{1}{f_c}$

$$\therefore V_o(t) = \left[m(t) + A_c \cos 2\pi f_c t\right]\left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t - \frac{2}{3\pi} \cos 2\pi (3f_c) t + \cdots\right]$$

$$= \left[\frac{A_c}{2} \cos 2\pi f_c t + \frac{2}{\pi} m(t) \cos 2\pi f_c t\right] + \underbrace{\left[\frac{m(t)}{2} + \frac{2A_c}{\pi} \cos^2 2\pi f_c t + \cdots\right]}_{\text{blocked by BPF}}$$

After BPF,

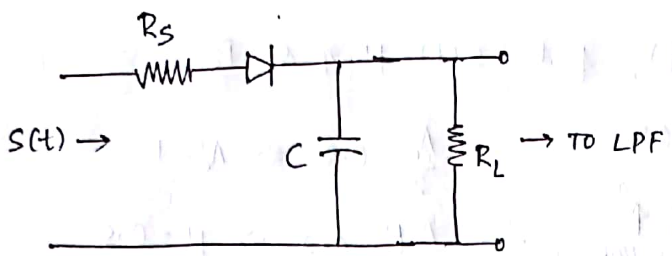
$$s(t) = \frac{A_c}{2} \cos 2\pi f_c t + \frac{2}{\pi} m(t) \cos 2\pi f_c t$$

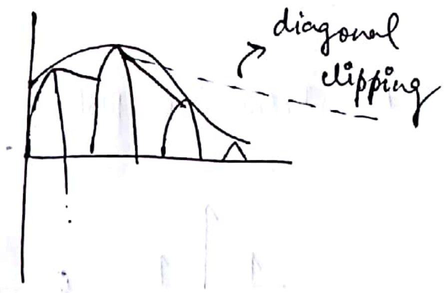
$$= \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos 2\pi f_c t$$

$$k_a = \frac{4}{\pi A_c}$$

* To increase k_a, we need to decrease A_c (but should be large enough w.r.t $|m(t)|$, so that diode acts like an ideal switch.

Envelope detector





Envelope detector can be used to demodulate the AM wave only if $\mu \leq 1$. If $\mu > 1$, coherent demodulation needs to be used.

We assume ideal diode behaviour.

* It is assumed that the source resistance R_s is very small, & hence $R_s C << \frac{1}{f_c}$. Hence, during the positive half cycle of $S(t)$, the capacitor charges up almost instantaneously to the peak voltage. (the diode is forward biased &)

* As the input signal decreases from the peak, the diode becomes reverse biased, and the capacitor discharges through R_L. R_L is chosen such that $R_L C >> \frac{1}{f_c}$, hence the capacitor discharges slowly till the next half cycle. Once $S(t)$ exceeds the capacitor voltage, the diode again becomes forward biased, and the capacitor charges up to the new peak voltage. And the process repeats.

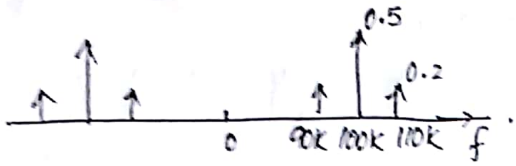
* If the envelope varies rapidly w.r.t the time constant $R_L C$, then the capacitor discharge will be too slow, and many peaks may be missed. This is known as diagonal clipping. To avoid this, we need to ensure $R_L C << \frac{1}{B}$.

$\therefore$ overall, we require $\frac{1}{f_c} << R_L C << \frac{1}{B}$.

* The remaining ripple is removed by using a LPF.

problems :

1. From the plot of $s(f)$, find $s(t)$, μ, η & P_s.



$\frac{A_c^2}{2} = 0.5 \Rightarrow A_c = 1$

$\frac{A_c \mu}{4} = 0.2 \Rightarrow \mu = 0.8$

$\eta = \frac{0.8^2}{2+0.8^2} = \frac{0.64}{2.64} = 0.24$

$s(t) = [1 + 0.8 \cos 2\pi 10^4 t] \cos 2\pi . 10^5 t$

$P_s = \frac{A_c^2}{2}\left[1+\frac{\mu^2}{2}\right] = 0.66\,W$

2. If P_c is 90% of P_T, find μ.

$\eta = \frac{P_T - P_c}{P_T} = 0.1 = \frac{\mu^2}{2+\mu^2} \Rightarrow \mu = \frac{\sqrt{2}}{3}$.

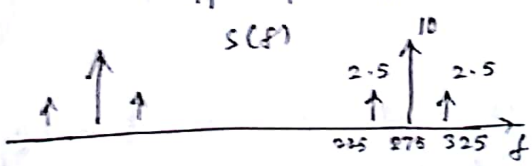
3. Let $s(t) = 5 \cos 450\pi t + 20 \cos 550\pi t + 5 \cos 650\pi t$

Find f_c, f_m, A_c, μ, P_T & η. Also draw the power spectrum.

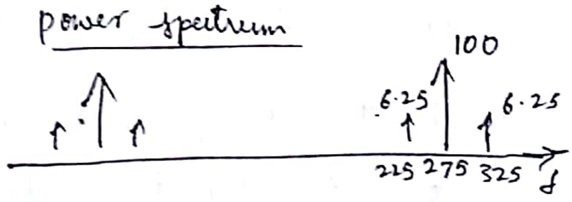
$s(t) = 20\,[1 + 0.5 \cos 100\pi t] \cos 550\pi t$

$\therefore f_c = 275\,Hz$, $f_m = 50\,Hz$, $A_c = 20V$, $\mu = 0.5$, ~~$A_c = 10V$~~

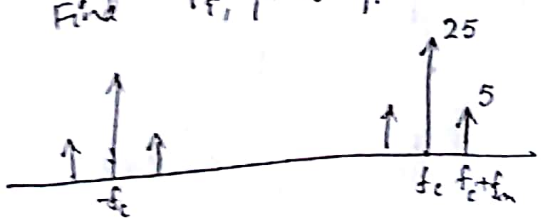
$\eta = 1/9$ $\quad P_T = 225W$.



power spectrum



4. The power spectrum of an AM wave is shown below.
Find P_T, μ & η.



Ans: From the figure, $P_c = 50W$ & $P_{SB} = 20W$

$\therefore P_T = 70W$, $\eta = \frac{P_{SB}}{P_T} = \frac{20}{70} = \frac{2}{7}$

$\frac{2}{7} = \frac{\mu^2}{2+\mu^2} \Rightarrow 4 + 2\mu^2 = 7\mu^2$ or $5\mu^2 = 4$, $\mu^2 = \frac{4}{5}$

$\mu = \frac{2}{\sqrt{5}}$.

5. The tuned circuit of the oscillator in an AM transmission system employs a 40 μH coil & a 12nF capacitor. If the message signal is a sinusoid of frequency 5 kHz, find the lower & upper sideband frequencies & the transmission bandwidth.

Ans: $f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40\times10^{-6}\times12\times10^{-9}}} \simeq 230$ kHz

$\therefore f_{USB} = 230+5 = 235$ kHz

$f_{LSB} = 230-5 = 225$ kHz

BW = 10 kHz.

6. A 400 W carrier is modulated to a depth of 80% by a sinusoid. Find P_T.

Ans: $P_T = \frac{A_c^2}{2}\left[1+\frac{\mu^2}{2}\right] = P_c\left[1+\frac{\mu^2}{2}\right] = 400\left[1+\frac{0.8^2}{2}\right] = 528$ W

7. The total current in an AM transmitter is 5A. If the modulation index is 0.6 (single tone modulation), find the antenna current when only the carrier is sent.

Ans: $I_T = I_c\sqrt{1+\frac{\mu^2}{2}}$ $\therefore I_c = \frac{I_T}{\sqrt{1+\mu^2/2}} = \frac{5}{\sqrt{1+\frac{0.6^2}{2}}} = 4.6$ A.

8. The carrier $10\cos(50\times10^5\pi t)$ is modulated by the signal $m(t) = 6\cos 500\pi t - 3\sin 1000\pi t + 4\cos 1500\pi t$. If $k_a = 0.1$, find P_T, μ_{eff} & η.

Ans: $\mu_1 = 6\times0.1 = 0.6$, $\mu_2 = 0.1\times3 = 0.3$ & $\mu_3 = 0.1\times4 = 0.4$

$\therefore \mu_{eff} = \sqrt{\mu_1^2+\mu_2^2+\mu_3^2} = \sqrt{0.6^2+0.3^2+0.4^2} = 0.78$

$\therefore P_T = \frac{A_c^2}{2}\left[1+\frac{\mu_{eff}^2}{2}\right] = \frac{100}{2}\left[1+\frac{0.78^2}{2}\right] =$

$\cancel{\eta = \frac{\mu^2}{2+\mu^2}}$

9. The o/p voltage of an AM transmitter is given by $s(t) = 400\left[1+0.4\cos 2000\pi t\right]\cos(\pi\times10^7 t)$. This voltage is fed to a load of 600Ω.

Find i) f_c ii) f_m iii) carrier power iv) total power.

Ans: $f_c = 5\text{ MHz}$ & $f_m = 1000\text{ Hz}$.

$$P_c = \frac{A_c^2}{2R} = \frac{400^2}{2\times600} = 133.33\,W$$

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right] = 133.33\left[1+\frac{0.4^2}{2}\right] = 144\,W.$$

10. Consider the AM signal $s(t) = \left[1+\alpha\cos\omega_m t + \alpha\cos 2\omega_m t\right]\cos\omega_c t$, $\alpha > 0$.

show that, to avoid envelope distortion, $\alpha \le \frac{8}{9}$.

Ans: To avoid distortion, we need the envelope $A(t) \geq 0 \quad \forall t$

or, $\min\limits_t \{A(t)\} \geq 0 \Rightarrow \frac{dA(t)}{dt} = 0$.

$$A(t) = \left[1+\alpha\cos\omega_m t + \alpha\cos 2\omega_m t\right]$$

$$\therefore \frac{dA(t)}{dt} = -\alpha\omega_m\sin\omega_m t - 2\alpha\omega_m\sin 2\omega_m t = 0$$

$$\Rightarrow -\alpha\omega_m\sin\omega_m t\left[1+4\cos\omega_m t\right] = 0 \quad \left[\text{use } \sin 2\theta = 2\sin\theta\cos\theta\right]$$

$\therefore$ the solutions are : $\sin\omega_m t = 0 \Rightarrow \omega_m t = 0, \pi, 2\pi, \ldots$

$1+4\cos\omega_m t = 0 \Rightarrow \cos\omega_m t = -1/4$.

The corresponding values of $A(t)$ are

$A(t) = 1+2\alpha$ for $\omega_m t = 0, 2\pi, 4\pi, \ldots$

$= 1$ for $\omega_m t = \pi, 3\pi, \ldots$

since $\alpha > 0$, these two sol^m do not give the minimum.

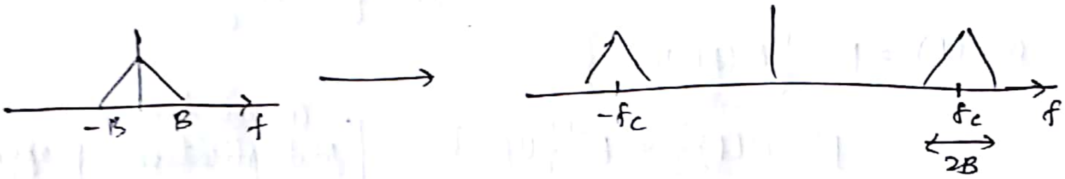
For $\cos\omega_m t = -1/4$, $\cos 2\omega_m t = 2\cos^2\omega_m t - 1 = 2\left(-\frac{1}{4}\right)^2 - 1 = -\frac{7}{8}$,

hence $A(t) = 1 - \frac{\alpha}{4} - \alpha\cdot\frac{7}{8} = 1 - \frac{9\alpha}{8}$.

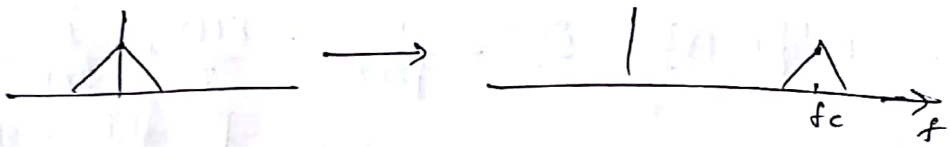
$$\therefore \left(1-\frac{9\alpha}{8}\right) \geq 0 \Rightarrow \alpha \le \frac{8}{9}$$

Single Sideband Modulation

Both DSBSC and standard AM (which is DSB-FC, i.e, DSB with full carrier) are wasteful of bandwidth since the transmission BW is twice the message BW.



It is clear that only one copy of the ~~spectrum~~ message signal's spectrum is enough to recover the signal at the receiver. So can we just transmit one half of the spectrum, as follows?

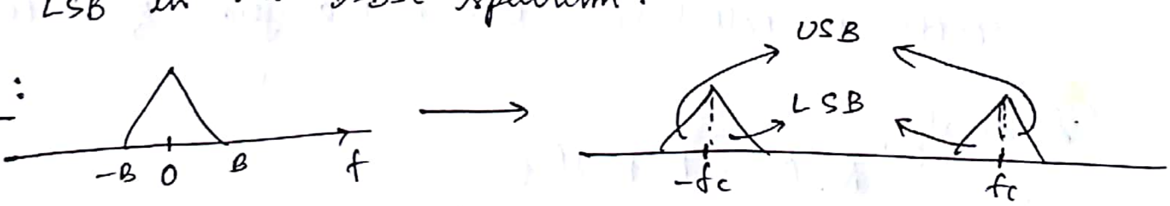


[Ans: This won't improve BW efficiency. why?, or ~~as~~ as an exercise, show that this scheme won't improve BW efficiency]

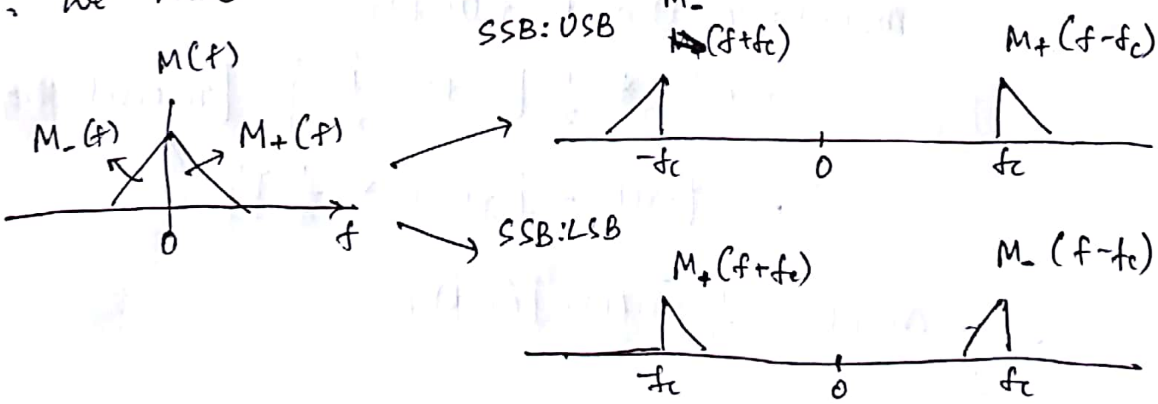
[Hint : is s(t) real?].

A better approach is to consider either just the USB or the LSB in the DSBSC spectrum.

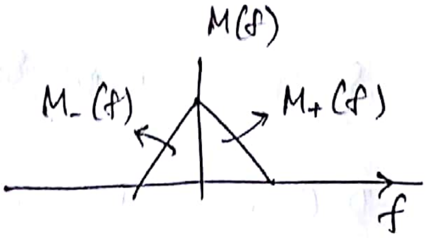
DSBSC:



∴ we have



Expression for $s(t)$



we have

$M_+(f) = M(f) \cdot U(f)$, where $U(f)$ is the unit step fn in frequency.

$$\therefore m_+(t) = F^{-1}\{M(f) \cdot U(f)\}$$

$$= F^{-1}\{M(f)\} * F^{-1}\{U(f)\} \quad [\text{convolution } \cancel{\text{Multiplication}} \text{ property}]$$

$$= m(t) * F^{-1}\{U(f)\}.$$

Recall : $F\{u(t)\} = \frac{\delta(f)}{2} + \frac{1}{j2\pi f}$

$$\therefore F^{-1}\{U(f)\} = \frac{\delta(t)}{2} - \frac{1}{j2\pi t} = \frac{\delta(t)}{2} + \frac{j}{2\pi t}$$

[duality property]

$$\therefore m_+(t) = m(t) * \frac{1}{2}\left[\delta(t) + \frac{j}{\pi t}\right]$$

$$= \frac{1}{2}\left[m(t) + j\left\{m(t) * \frac{1}{\pi t}\right\}\right]$$

Let $\hat{m}(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau}\, d\tau$

$$\therefore m_+(t) = \frac{1}{2}\left[m(t) + j\hat{m}(t)\right] \longrightarrow (1)$$

$\hat{m}(t)$ is called the "Hilbert transform" of $m(t)$.

Similarly, $M_-(f) = M(f)U(-f)$

$$\therefore m_-(t) = m(t) * F^{-1}\{U(-f)\}$$

$$= m(t) * \frac{1}{2}\left[\delta(t) - \frac{j}{\pi t}\right] \quad [\text{reversal property}]$$

$$= \frac{1}{2}\left[m(t) - j\left\{m(t) * \frac{1}{\pi t}\right\}\right]$$

$$\therefore m_-(t) = \frac{1}{2}\left[m(t) - j\hat{m}(t)\right]$$

For USB

$$S(f) = M_+(f-f_c) + M_-(f+f_c)$$

$$\therefore \quad s(t) = m_+(t)\, e^{j2\pi f_c t} + m_-(t)\, e^{-j2\pi f_c t} \qquad \text{[freq. shift property]}$$

$$= \frac{1}{2}\left[m(t) + j\hat{m}(t)\right]\left[\cos 2\pi f_c t + j \sin 2\pi f_c t\right]$$
$$+ \frac{1}{2}\left[m(t) - j\hat{m}(t)\right]\left[\cos 2\pi f_c t - j \sin 2\pi f_c t\right]$$

simplifying, we get

$$s(t) = m(t)\cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t$$

For LSB

$$S(f) = M_+(f+f_c) + M_-(f-f_c)$$

$$\therefore \quad s(t) = m_+(t)\, e^{-j2\pi f_c t} + m_-(t)\, e^{j2\pi f_c t}$$

$$= \frac{1}{2}\left[m(t) + j\hat{m}(t)\right]\left[\cos 2\pi f_c t - j \sin 2\pi f_c t\right]$$
$$+ \frac{1}{2}\left[m(t) - j\hat{m}(t)\right]\left[\cos 2\pi f_c t + j \sin 2\pi f_c t\right]$$

$$\therefore \quad s(t) = m(t)\cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t.$$

The Hilbert transform

It is not a "transform" as such, but just an LTI system with impulse response $\frac{1}{\pi t}$.

$$m(t) \longrightarrow \boxed{h(t)} \longrightarrow \hat{m}(t) \qquad h(t) = \frac{1}{\pi t}.$$

$$\hat{m}(t) = m(t) * \frac{1}{\pi t}$$

$$\therefore \quad \hat{M}(f) = M(f) \cdot F\left\{\frac{1}{\pi t}\right\}$$

WKT, $\mathrm{sgn}(t) \xleftrightarrow{F} \frac{1}{j\pi f}$

or $j\,\mathrm{sgn}(t) \xleftrightarrow{F} \frac{1}{\pi f}$

$\therefore$ By duality

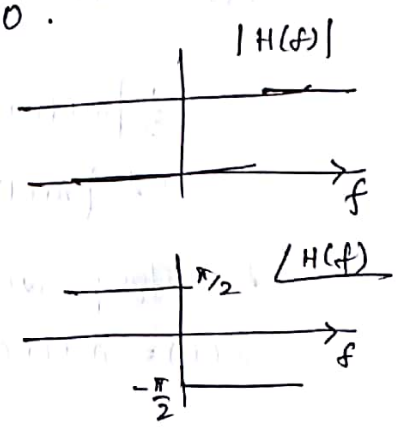
$$\frac{1}{\pi t} \xleftrightarrow{F} -j\,\mathrm{sgn}(f)$$

$$\therefore \quad \hat{M}(f) = M(f)\{-j\,\mathrm{sgn}(f)\}$$

or
$$\hat{M}(f) = \begin{cases} -j\,M(f) & f > 0 \\ j\,M(f) & f < 0 \end{cases}$$

or $\hat{M}(f) = \begin{cases} M(f)\, e^{-j\pi/2} & f>0 \\ M(f)\, e^{j\pi/2} & f<0 \end{cases}$

$= \begin{cases} |M(f)|\, e^{j[\theta(f) - \pi/2]} & f>0 \\ |M(f)|\, e^{j[\theta(f) + \pi/2]} & f<0. \end{cases}$

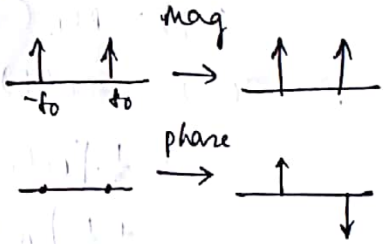


* Magnitude spectrum remains unchanged *
* Only the phase spectrum is modified.
* Hilbert transform is a "wideband $-\frac{\pi}{2}$ phase shifter".

Examples:

1. $m(t) = A\cos 2\pi f_0 t$

   $\hat{m}(t) = A\cos[2\pi f_0 t - \frac{\pi}{2}] = A\sin 2\pi f_0 t$



2. $m(t) = A\sin 2\pi f_0 t$

   $\hat{m}(t) = A\sin[2\pi f_0 t - \pi/2] = -A\cos 2\pi f_0 t$

properties

1. $\hat{m}(t)$ has the same magnitude spectrum as $m(t)$ [Except at $f=0$, since $\hat{M}(f)=0$].

2. The phase spectrum is modified. $\hat{M}(f)$ has a phase shift of $\pm\pi/2$ w.r.t $M(f)$.

3. $H\{\hat{m}(t)\} = -m(t)$

   proof: $\hat{M}(f) = -j\,\mathrm{sgn}(f)\, M(f)$

   $\hat{\hat{M}}(f) = [-j\,\mathrm{sgn}(f)]^2 M(f)$

   $= -M(f)$

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4. $m(t)$ and $\hat{m}(t)$ are orthogonal to each other.

<u>proof</u>: $\int_{-\infty}^{\infty} m(t)\, \hat{m}^*(t).dt = \int_{-\infty}^{\infty} M(f)\, \hat{M}^*(f).df$

[By parseval's theorem]

$$= \int_{-\infty}^{\infty} M(f)\,[-j\, sgn(f).M(f)]^*.df$$

$$= j \int_{-\infty}^{\infty} \underbrace{|M(f)|^2}_{\text{even}}.\underbrace{sgn(f)}_{\text{odd}}.\,df = 0.$$

[$x(t)$ & $y(t)$ are said to be orthogonal to each other if their inner product $\langle x(t), y(t)\rangle$ is zero. Here,

$$\langle x(t), y(t)\rangle = \int_{-\infty}^{\infty} x(t)\, y^*(t).dt \;]$$

<u>Exercise</u>: show that the Hilbert transform of $x(t) = \frac{1}{1+t^2}$ is $\hat{x}(t) = \frac{t}{1+t^2}$

[Hint: use $e^{-a|t|} \xleftrightarrow{F} \frac{2a}{a^2+4\pi^2 f^2}$ to find the time signal whose F.T is $\frac{1}{1+f^2}$, and then use duality].

<u>Ans</u>: $e^{-a|t|} \xleftrightarrow{F} \frac{2a}{a^2+4\pi^2 f^2}$

$$\therefore e^{-2\pi|t|} \xleftrightarrow{F} \frac{4\pi}{4\pi^2+4\pi^2 f^2} = \frac{1}{\pi}.\frac{1}{1+f^2}$$

By duality,

$$\frac{1}{1+t^2} \xrightarrow{F} \pi . e^{-2\pi|f|}$$

$$\therefore \hat{M}(f) = -j\, sgn(f)\pi e^{-2\pi|f|}$$

WKT,

$$e^{-a|t|}\, sgn(t) \xleftrightarrow{F} \frac{-j4\pi f}{a^2+4\pi^2 f^2}$$

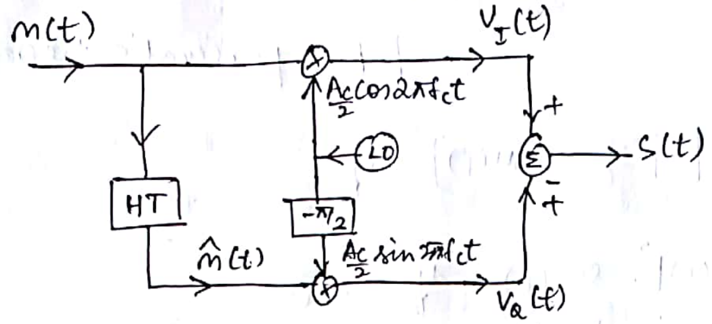
$$\therefore -j\pi\, sgn(t)\, e^{-2\pi|t|} \xleftrightarrow{F} \frac{-4\pi^2 f}{4\pi^2+4\pi^2 f^2} = \frac{-f}{1+f^2}$$

By duality,

$$\hat{m}(t) = \frac{t}{1+t^2}$$

SSB generation

1. phase shift method

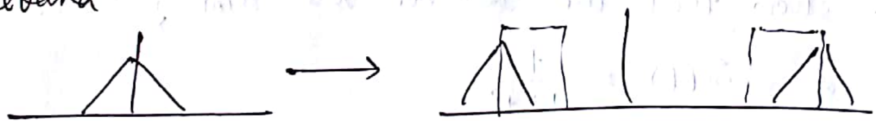


$$s(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \mp \hat{m}(t)\sin 2\pi f_c t\right]$$

* In practice, it is difficult to implement wideband 90° phase shifter.
* Exercise: plot spectra of $v_I(t)$, $v_Q(t)$ & $s(t)$

2. Frequency discrimination method

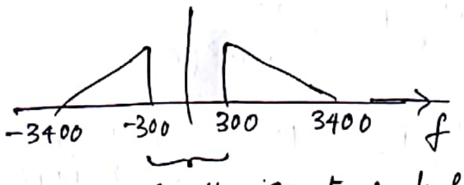
we can first perform DSBSC & then filter the required sideband



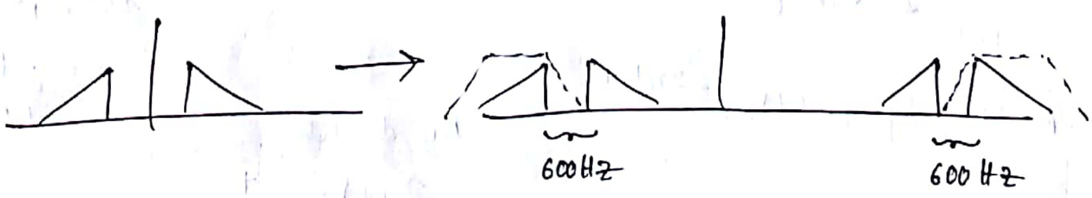
But this requires ideal filter, hence not practical.

But this becomes feasible if m(t) has a spectral hole near zero frequency.

Ex: speech signal

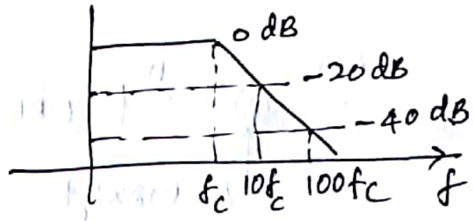


∴ we can have



But the problem here is the required filter order

1st order filter ⇒ 20 dB/decade



with 1st order filter, for 40 dB attenuation, transition band ≈ $100 f_c$

" 2nd " " " " ≈ $10 f_c$

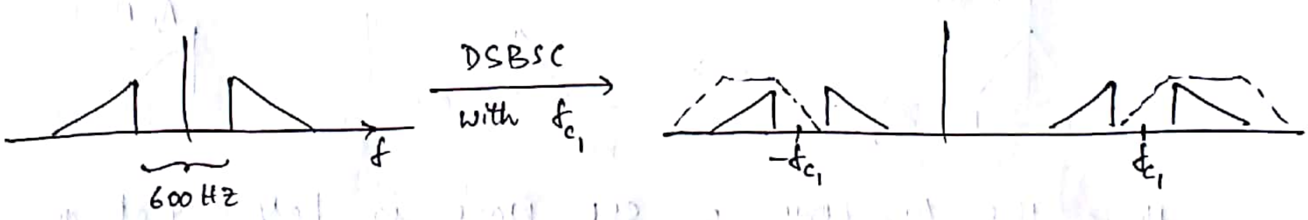
" 3rd " " " " ≈ $5 f_c$

⋮

10th " " " " ≈ $0.2 f_c$

But f_c typically is in 100s of kHz. So the transition band cannot be accommodated in the 600 Hz gap, for practical filter orders.

Therefore, we go for multistage implementation.



After BPF,