UE20EC303 EMF THEORY
UNIT -3

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EMF THEORY

TEXT BOOK AND REFERENCES

Textbook:

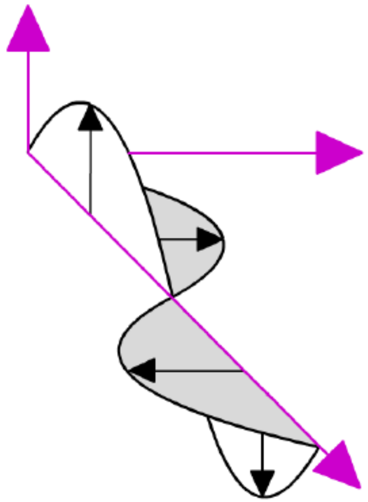
1. "Principles of Electromagnetics", Matthew N. O. Sadiku, 4th / 6th Edition, Oxford University Press, 2007 / 2018.

Reference Books:

1. "Electromagnetic Field Theory Fundamentals", Bagh Singh Guru, Huseyin R Hiziroglu, Cambridge University Press, 2nd Edition, 2002.
2. "Engineering Electromagnetics", William H Hayt Jr, J.A. Buck, 7th Edition, Tata McGraw Hill, 2007.
3. "Microwave Devices and Circuits", Samuel. Y. Liao, Third Edition, Pearson, 2006.
4. "Engineering Electromagnetics Essentials", B. N. Basu, Universities Press (India), 2015.

UNIT - 3

Maxwell's Equations for Time – Varying Fields



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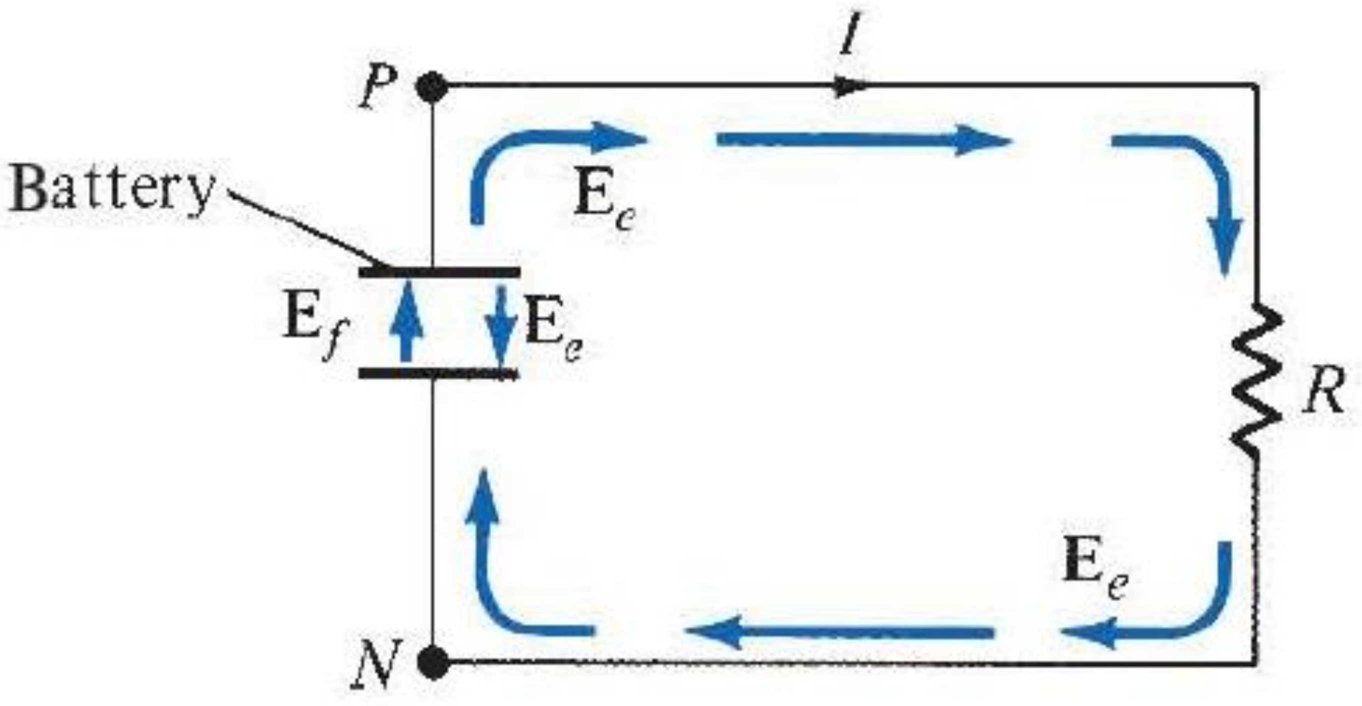
EMF Theory – Unit – 3 {Text Book Sec. 9.1 - 9.5, 9.7, 5.9, 8.7, 10.1 – 10.7}

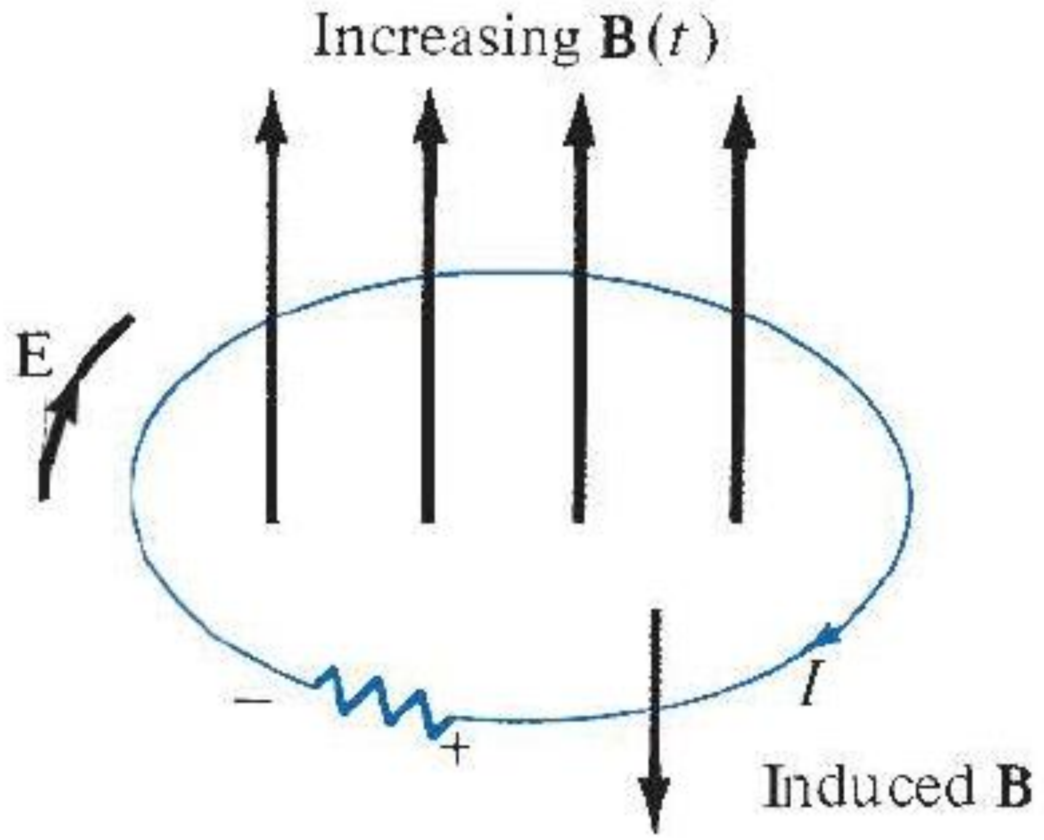
CONTENTS

- Faraday's Law.
- Transformer & Motional Electromotive Forces.
- Displacement Current.
- Maxwell's Equations in their final forms.
- Time Harmonic Fields.
- Electric & Magnetic Boundary conditions.
- Waves, propagation of waves in different media.
- Wave polarisation.

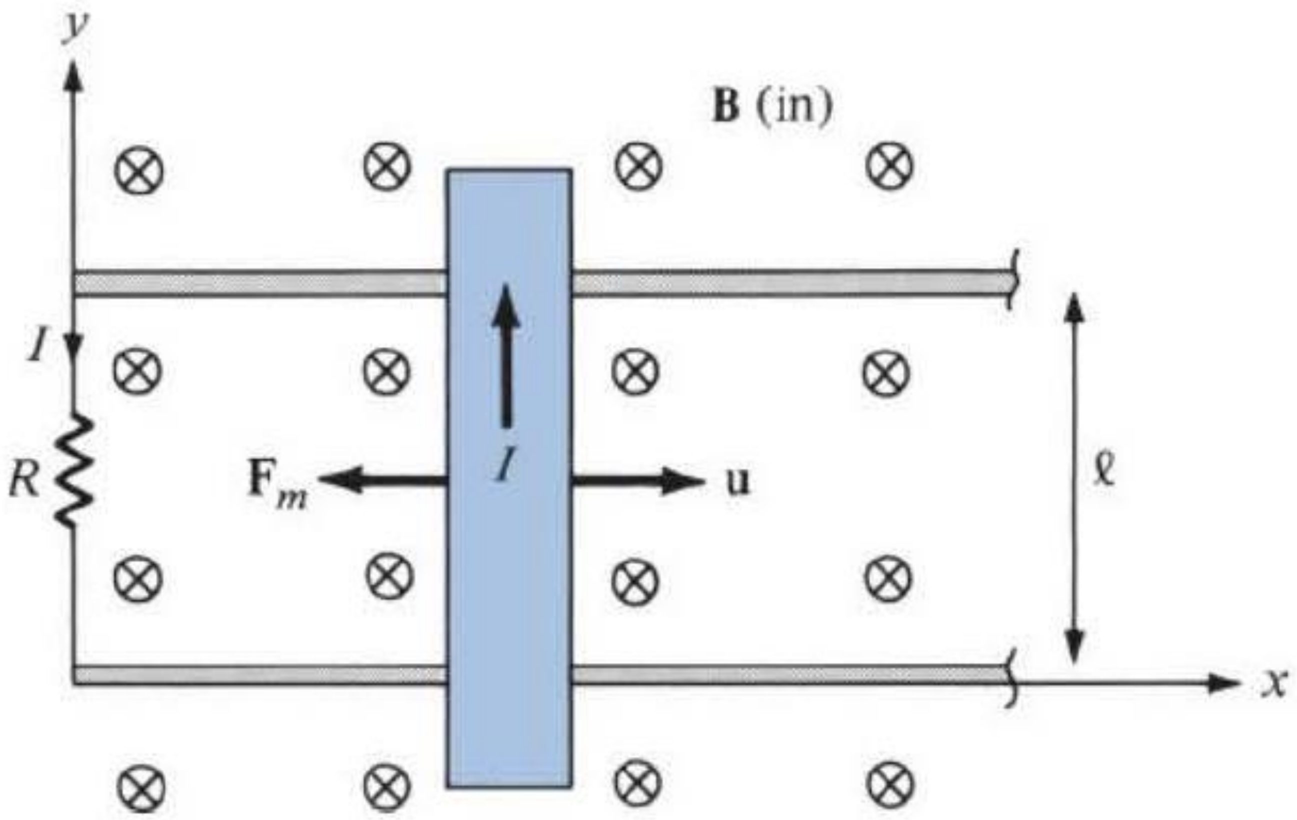
FARADAY'S LAW

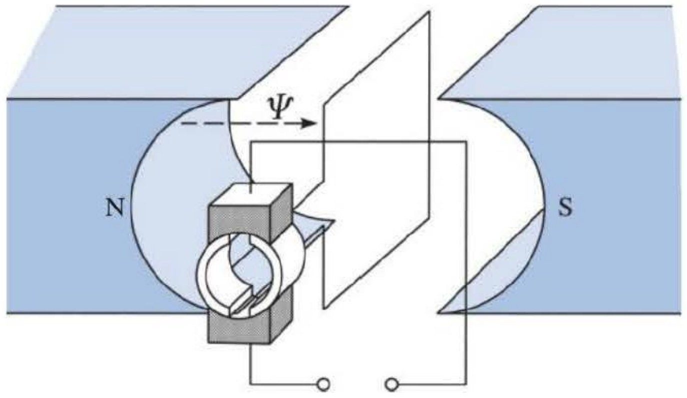


TRANSFORMER EMF

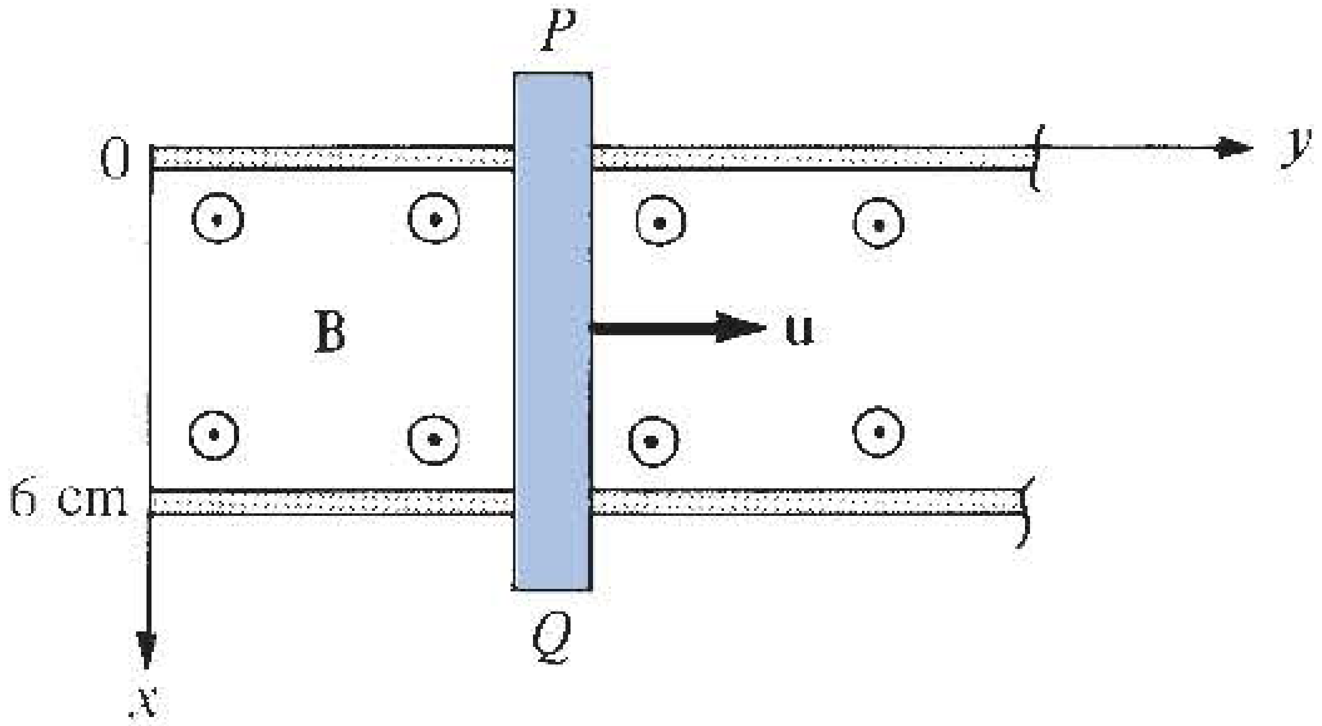


MOTIONAL EMF





NUMERICALS



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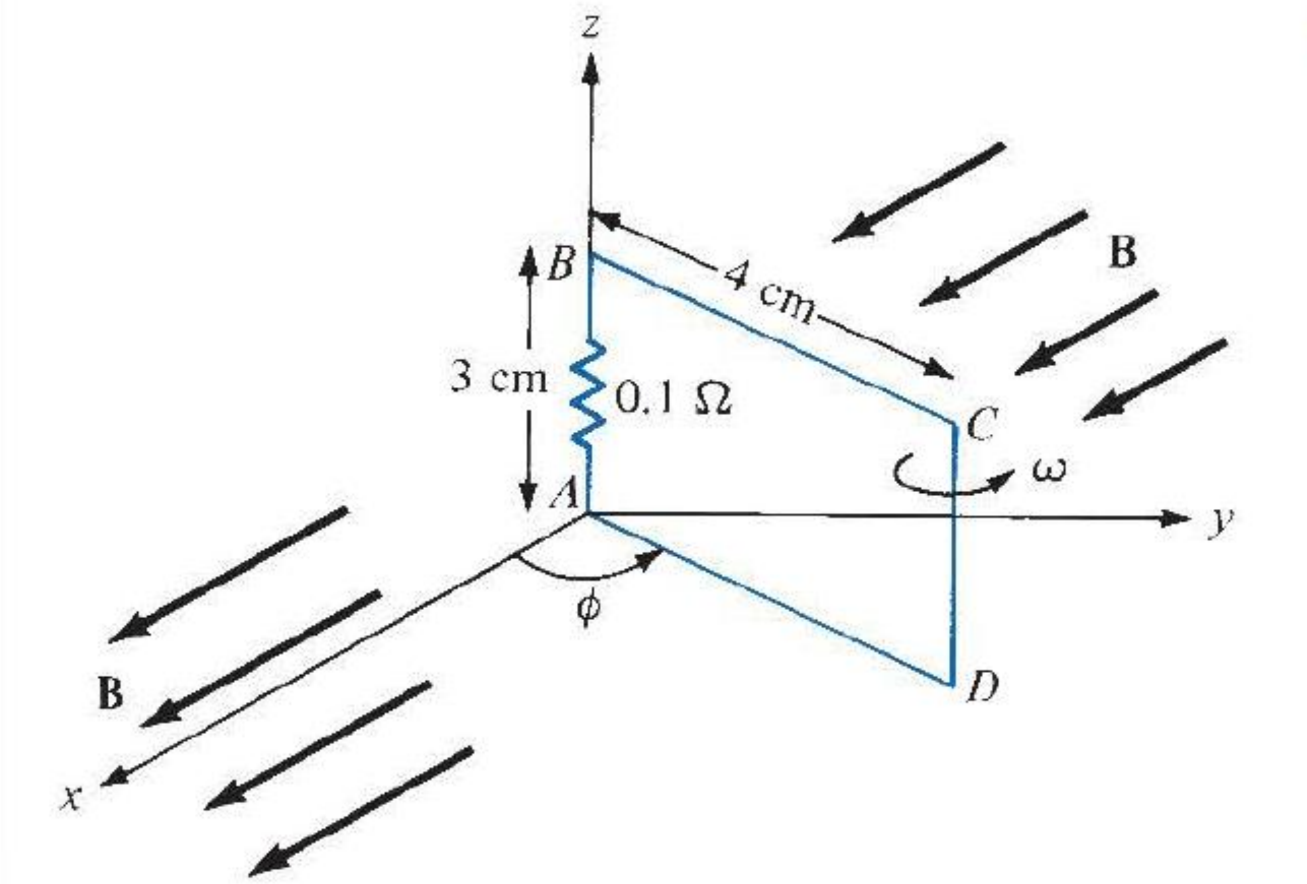
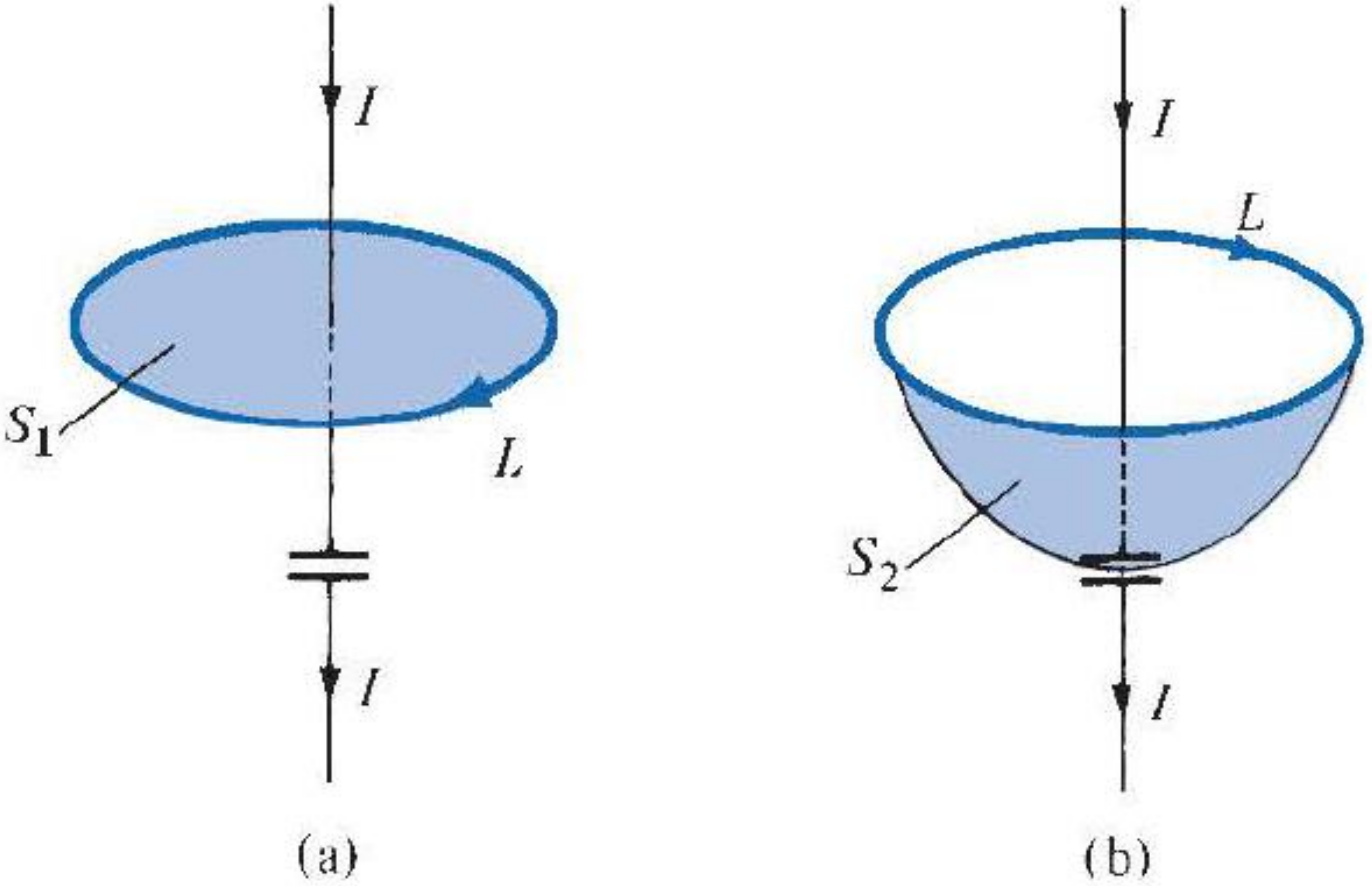


DISPLACEMENT CURRENT



Self Study from Text Book

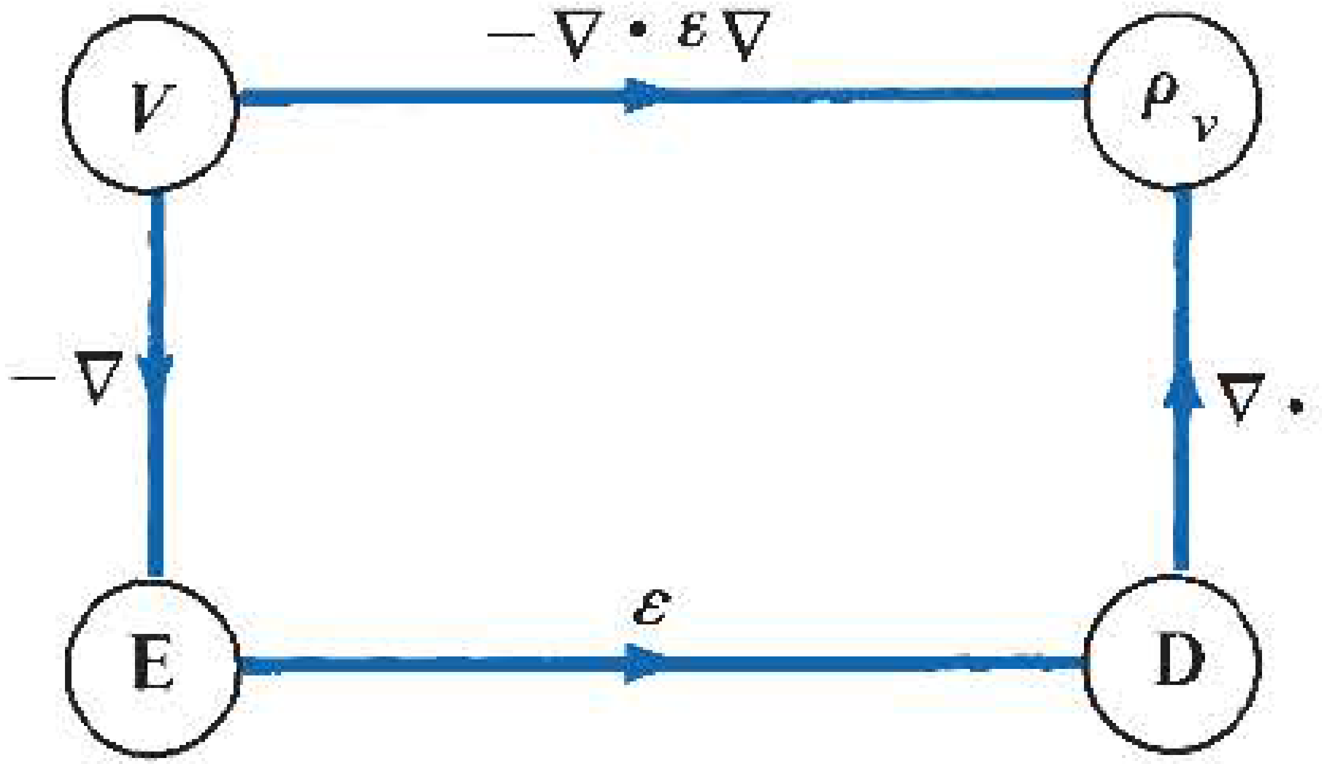
GENERALISED MAXWELL'S EQUATIONS

TABLE 9.1 Generalized Forms of Maxwell's Equations

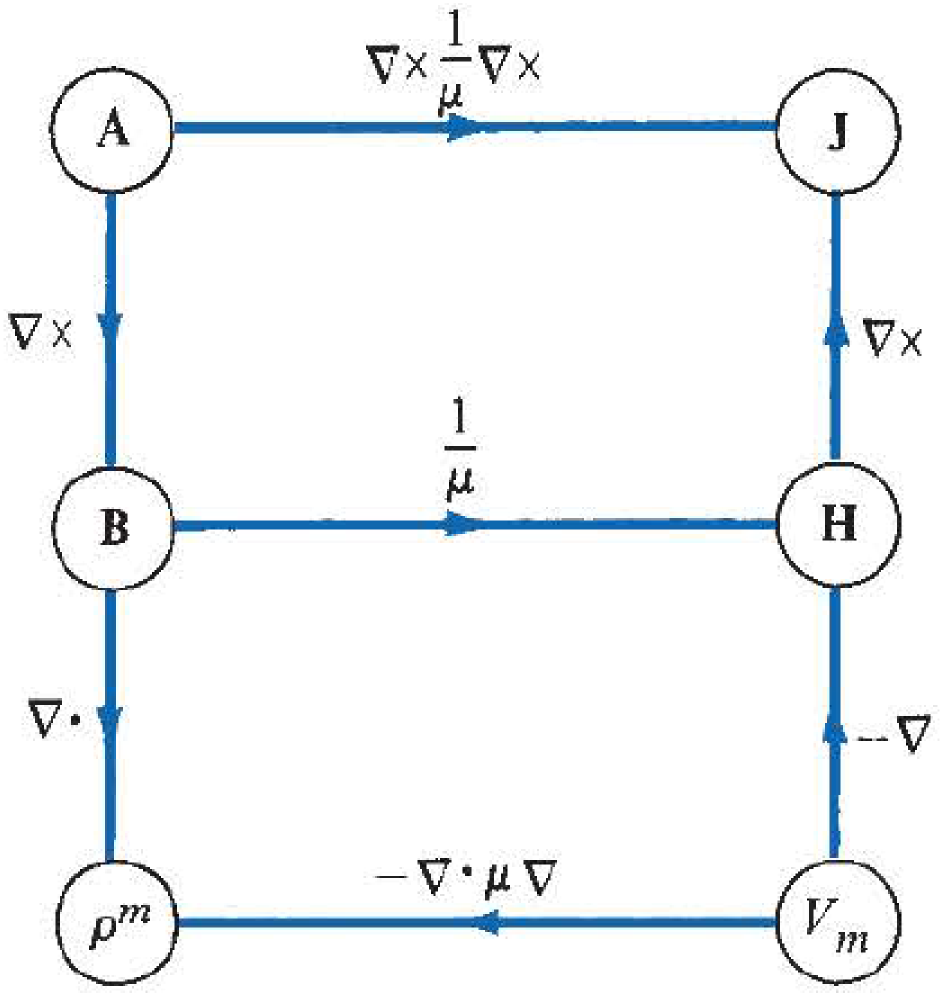
Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isolated magnetic charge*
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\dfrac{\partial}{\partial t}\int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{S}$	Ampère's circuit law

*This is also referred to as Gauss's law for magnetic fields.

FLOW DIAGRAM – ELECTROSTATIC SYSTEM



FLOW DIAGRAM – MAGNETOSTATIC SYSTEM



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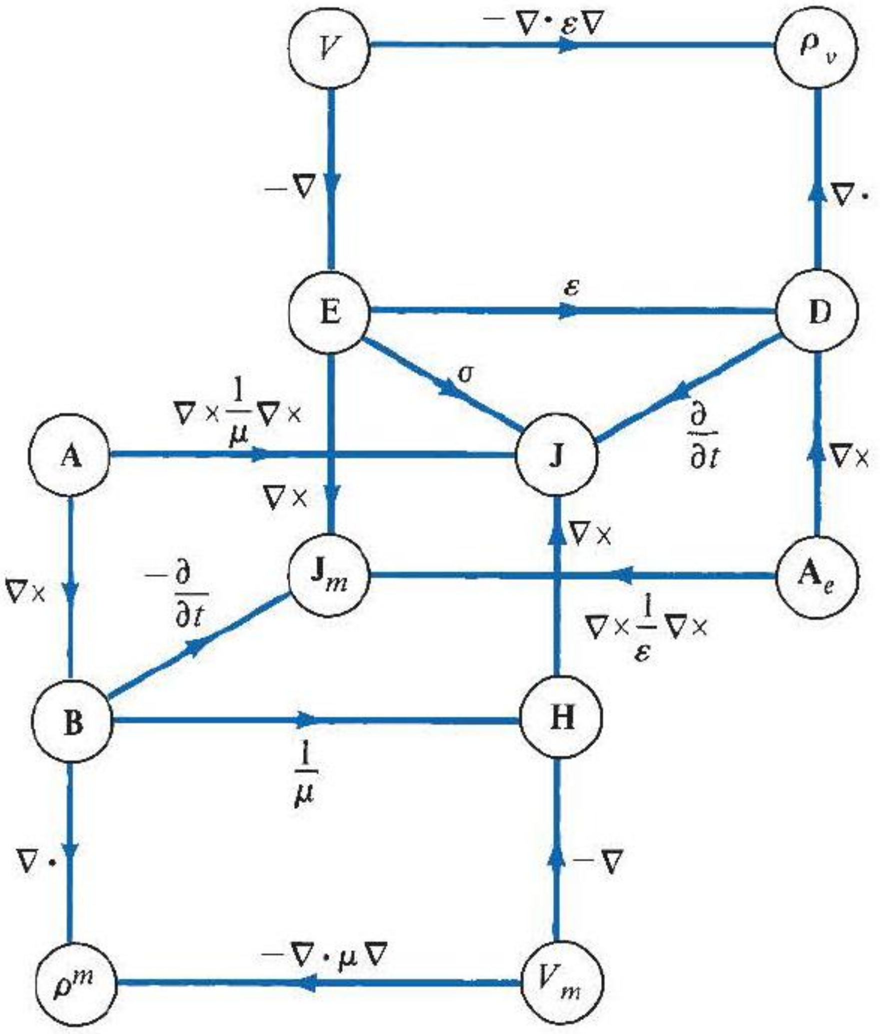
FLOW DIAGRAM – ELECTROMAGNETIC SYSTEM



TIME-HARMONIC MAXWELL'S EQUATIONS – PHASOR FORM

Point Form	Integral Form
$\nabla \cdot \mathbf{D}_s = \rho_{vs}$	$\oint \mathbf{D}_s \cdot d\mathbf{S} = \int \rho_{vs}\, dv$
$\nabla \cdot \mathbf{B}_s = 0$	$\oint \mathbf{B}_s \cdot d\mathbf{S} = 0$
$\nabla \times \mathbf{E}_s = -j\omega \mathbf{B}_s$	$\oint \mathbf{E}_s \cdot d\mathbf{l} = -j\omega \int \mathbf{B}_s \cdot d\mathbf{S}$
$\nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega \mathbf{D}_s$	$\oint \mathbf{H}_s \cdot d\mathbf{l} = \int (\mathbf{J}_s + j\omega \mathbf{D}_s) \cdot d\mathbf{S}$

Boundary Conditions.

→ Till now, the discussion considered the existence of electric field in a homogeneous medium.

→ If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called Boundary Conditions.

→ These conditions are helpful in determining the field on one side of the boundary if the field on the other side is known.

→ Subsequent discussions would be for the following interfaces separating:

(a) Dielectric (ϵ_{r_1}) & Dielectric (ϵ_{r_2})

(b) Conductor & dielectric

(c) Conductor & free space.

→ To determine the boundary conditions, we use the following Maxwell's equations:

$$\oint_S \bar{D} \cdot d\bar{S} = Q_{enc} \quad \text{- First eqn.}$$

$$\oint_L \bar{E} \cdot d\bar{l} = 0 \quad \text{- Second Eqn.}$$

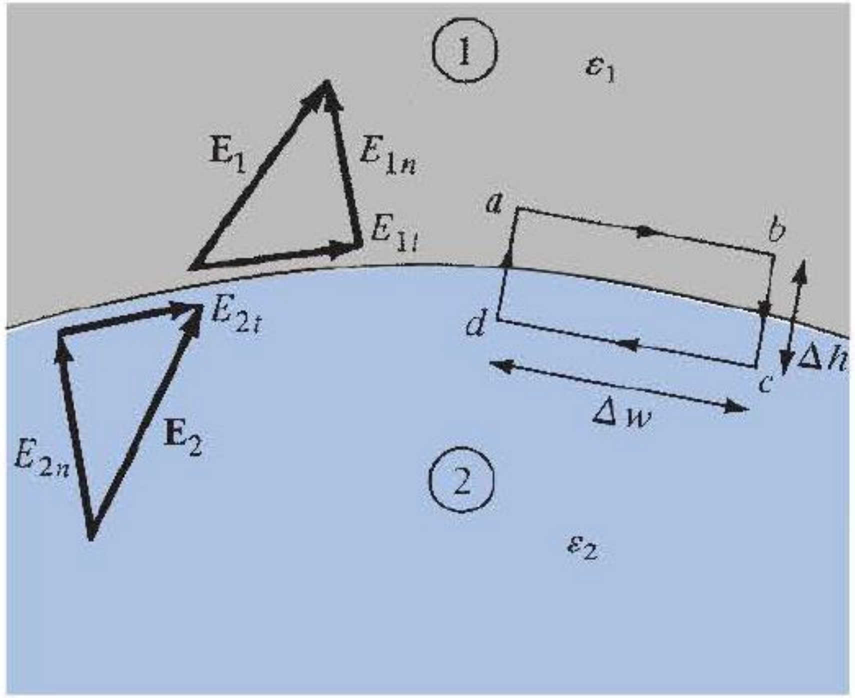
→ The electric field intensity $\bar{E}$ is decomposed into two orthogonal components

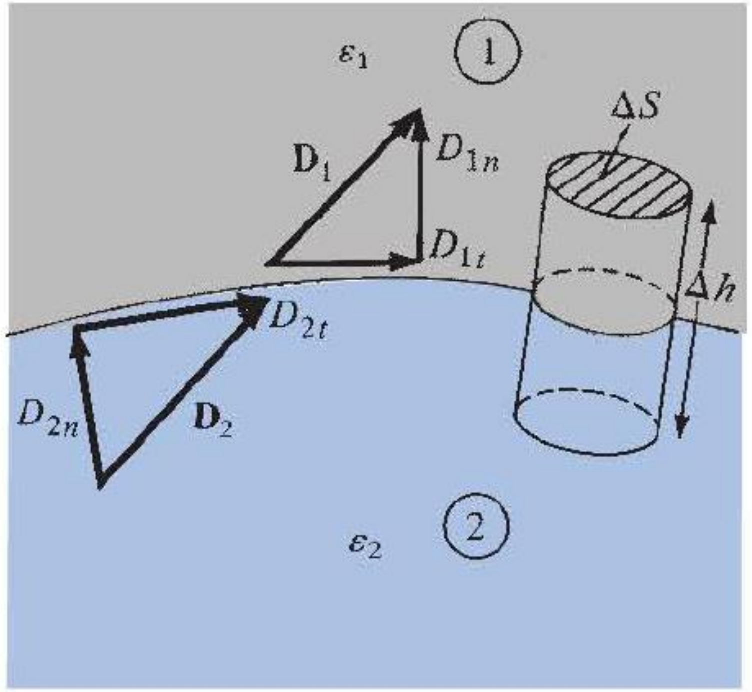
$\bar{E} = \bar{E}_t + \bar{E}_n$; $\bar{E}_t$ - Tangential component

$\bar{E}_n$ - Normal component.

→ A similar decomposition can be done for the Electric flux density $\bar{D}$.

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS





Dielectric – Dielectric Boundary Conditions

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

Dielectric - Dielectric Boundary Conditions.

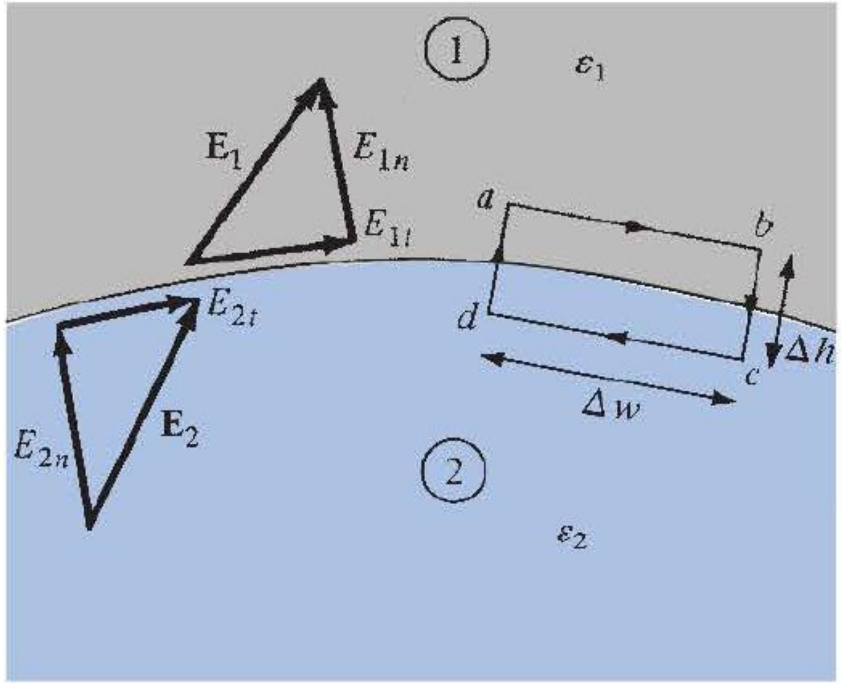
→ Consider the $\bar{E}$ field existing in a region that consists of a different dielectrics characterised by $\epsilon_1 = \epsilon_0 \epsilon_{r1}$ & $\epsilon_2 = \epsilon_0 \epsilon_{r2}$.

→ The fields $\bar{E}_1$ & $\bar{E}_2$ in media 1 & 2, respectively can be decomposed into.

$$\bar{E}_1 = \bar{E}_{1t} + \bar{E}_{1n} \quad \& \quad \bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n}$$

→ while applying the eqn. $\oint \bar{E} \cdot d\bar{l} = 0$ to the closed path abcda as shown, assuming that the ~~path~~ path is very small w.r.t spatial variations, we have

$$0 = \underbrace{E_{1t}\Delta w}_{ab} \overbrace{- E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2}}^{bc} \overbrace{- E_{2t}\Delta w}^{cd} \underbrace{+ E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2}}_{da}$$



(a)

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

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(*) The signs are obtained referring the direction of travel along abcda & the direction of the component vectors.

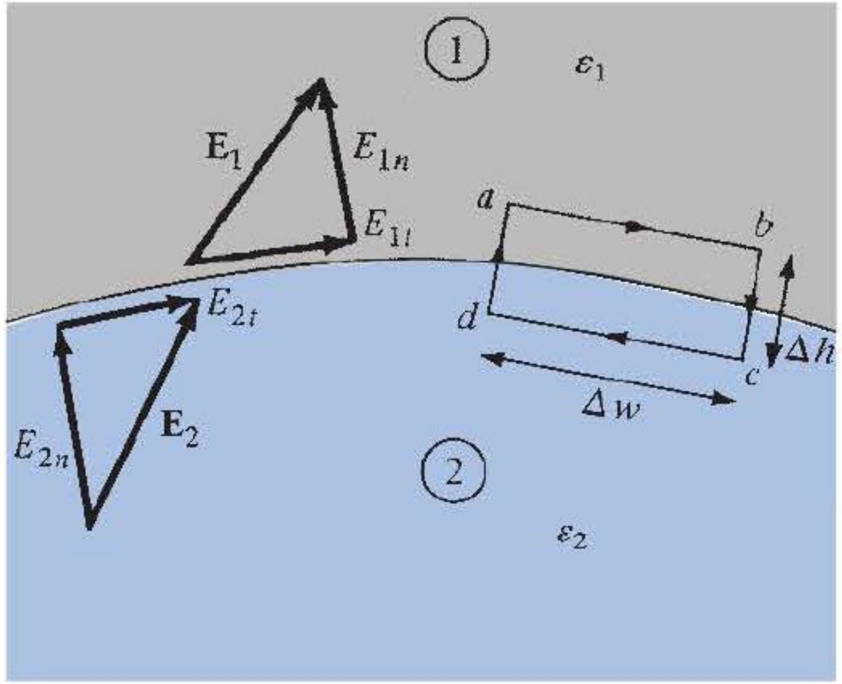
→ Also $E_t = |\bar{E}_t|$ & $E_n = |\bar{E}_n|$. The $\frac{\Delta h}{2}$ terms cancel resulting in

$$0 = E_{1t}\Delta w + E_{2t}\Delta w = (E_{1t} - E_{2t})\Delta w$$

or $$\boxed{E_{1t} = E_{2t}}$$

→ (*) Thus the tangential components of $\bar{E}$ are same on the two sides of the boundary.

→ (*) In other words, $\bar{E}_t$ undergoes no change on the boundary & it is said to be continuous across the boundary.



(a)

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

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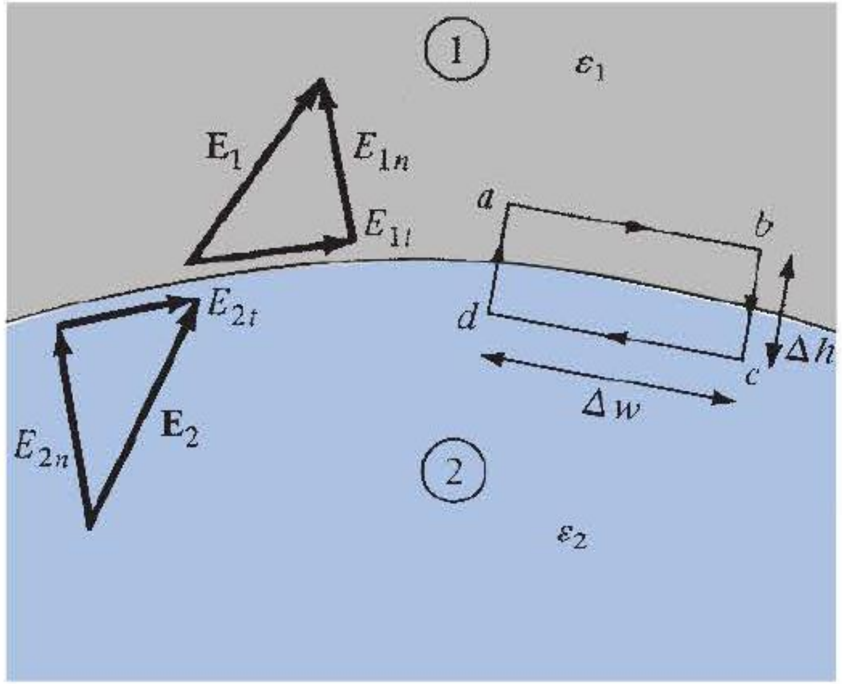
→ Since $\bar{D} = \varepsilon \bar{E} = \bar{D}_t + \bar{D}_n$, the equation $\bar{E}_{1t} = E_{2t}$ can be expressed as

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

or

$$\boxed{\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}}$$

→ $\bar{D}_t$ undergoes some change across the interface. Hence $\bar{D}_t$ is said to be "discontinuous" across the interface.



(a)

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

→ Similarly, when apply the Equation $\oint_S \bar{D}\cdot d\bar{S} = Q_{enc}$ to the cylindrical Gaussian Surface in the other figure,

→ The contribution due to the sides vanishes. Allowing $\Delta h \to 0$ gives

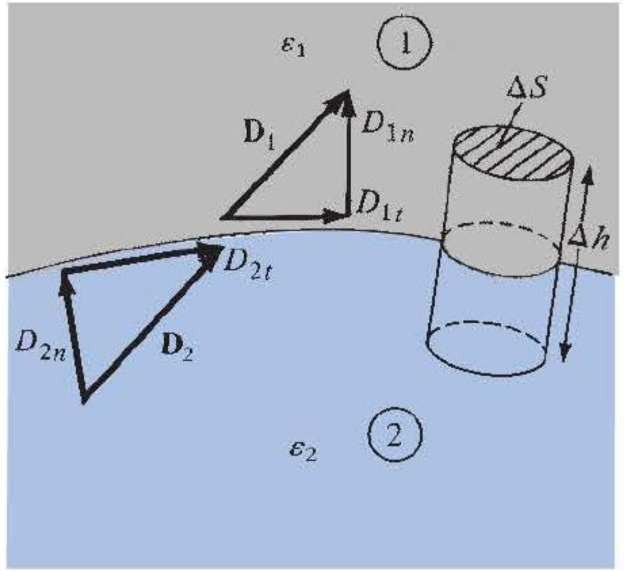
$$\Delta Q = \rho_s \Delta S = D_{1n}\Delta S - D_{2n}\Delta S$$

[–ve sign due to opposite direction at the bottom]

or $\boxed{D_{1n} - D_{2n} = \rho_s}$

where ρ_s is the free charge density placed deliberately at the boundary.

→ It is to be noted that the eqn. $D_{1n} - D_{2n} = \rho_s$ is based on the assumption that $\bar{D}$ is directed from Region 2 to Region 1 & it should be applied accordingly.



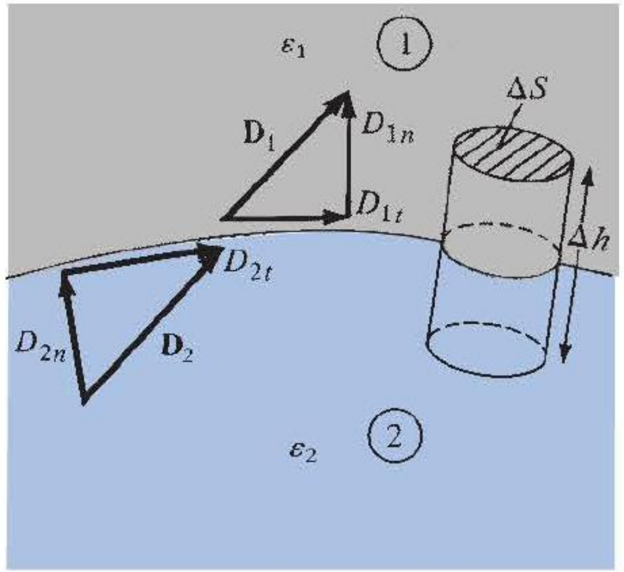
(b)

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

→ If no free charges exist at the interface (i.e charges are not deliberately placed), $\rho_s = 0$, then

$$\boxed{D_{1n} = D_{2n}} \quad \text{with } \rho_s = 0$$

→ Thus the normal component of $\bar{D}$ is continuous across the interface (or) D_n undergoes no change at the boundary.



(b)

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

(20)

→ Since $D = \varepsilon E$, we have

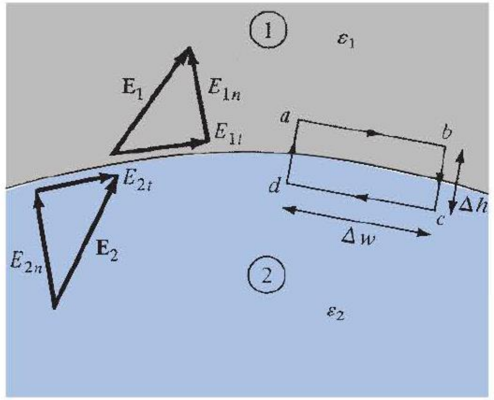
$$\boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}}$$

meaning the normal component of E is discontinuous at the boundary.

→ Summarising the Boundary conditions that need to be satisfied by an electric field at the boundary separating two different dielectrics,

$$\boxed{E_{1t} = E_{2t}} \quad ; \quad \boxed{\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}}$$

$$\boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}} \quad ; \quad \boxed{D_{1n} = D_{2n}} \text{ with } \rho_s = 0$$



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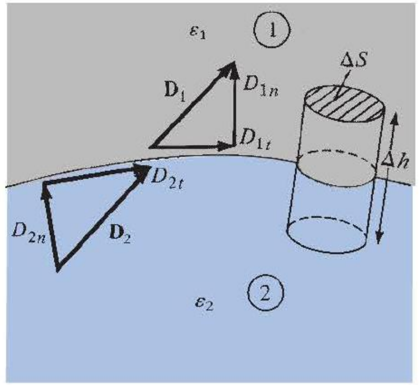

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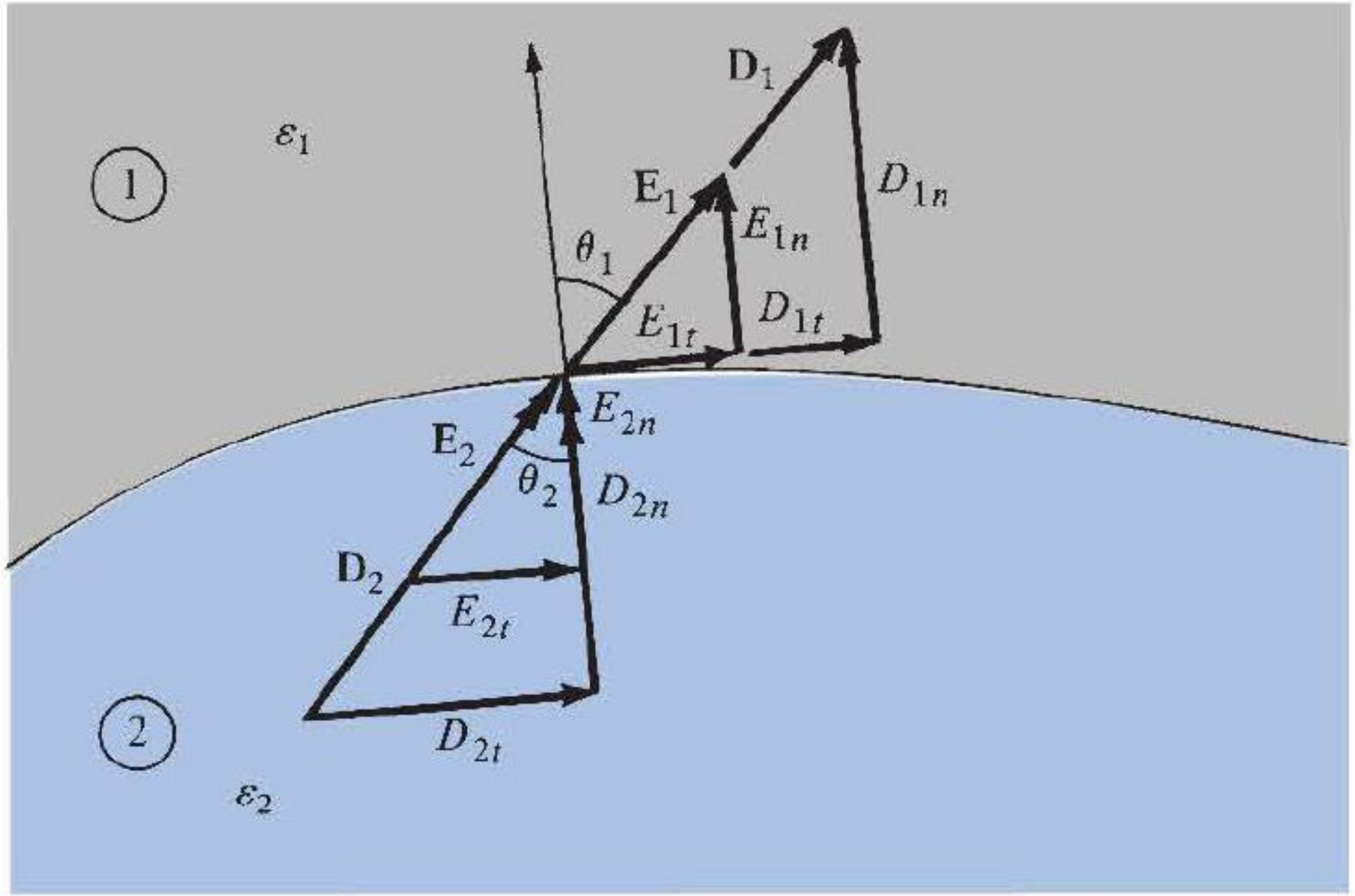
ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

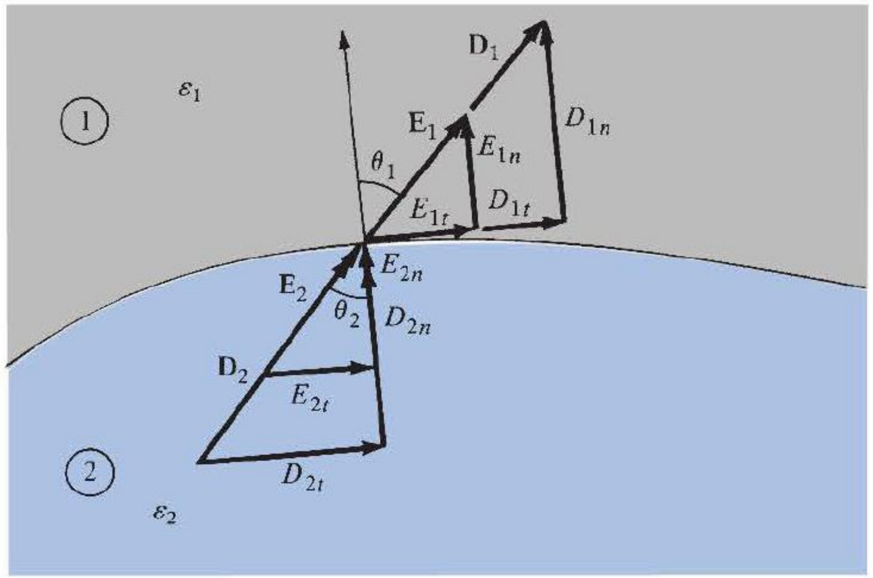


Refraction in Dielectric – Dielectric Boundary

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

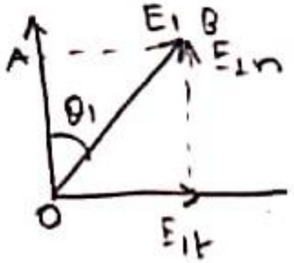
Refraction of Electric field using Boundary Conditions.

→ Refer the figure. Consider $\bar{D_1}$ or $\bar{E_1}$ & $\bar{D_2}$ or $\bar{E_2}$ making angles θ_1 & θ_2 with the normal to the interface as shown.



→ Using the Eqn. $E_{1t} = E_{2t}$, we have.

$$E_1 \sin\theta_1 = E_{1t} = E_{2t} = E_2 \sin\theta_2 \quad \text{(in magnitude form)}$$



E_{1n} $\quad \sin\theta_1 = \dfrac{AB}{OB} = \dfrac{E_{1t}}{E_1} \Rightarrow E_{1t} = E_1 \sin\theta_1$

or

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad \text{— (1)}$$

→ Similarly, by applying $\qquad \cos\theta_1 = \dfrac{OA}{AB} = \dfrac{E_{1n}}{E_1}$

$$\varepsilon_1 E_1 \cos\theta_1 = D_{1n} = D_{2n} = \varepsilon_2 E_2 \cos\theta_2$$

(or)

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2 \quad \text{— (2)}$$

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

Using ① + ② we have

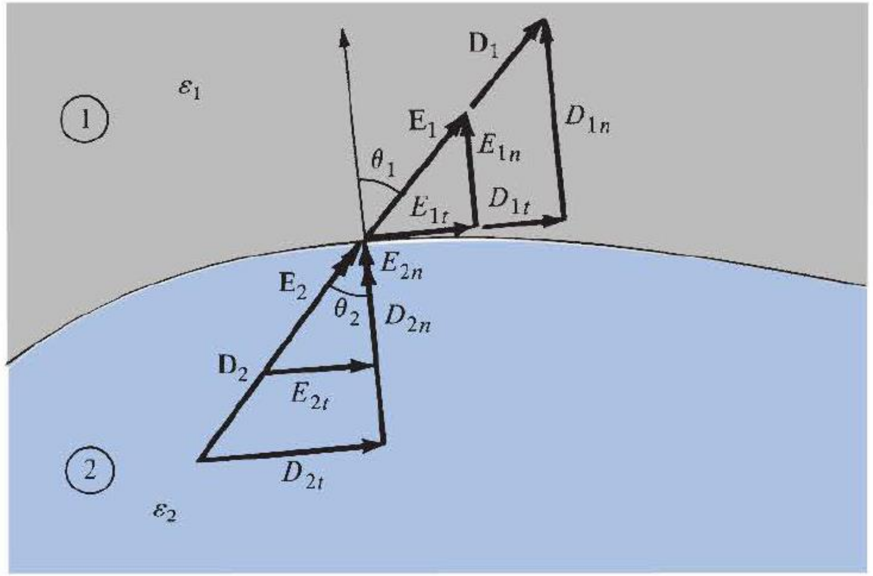
$$\boxed{\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}}$$

→ Since $\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$ & $\varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$, we have

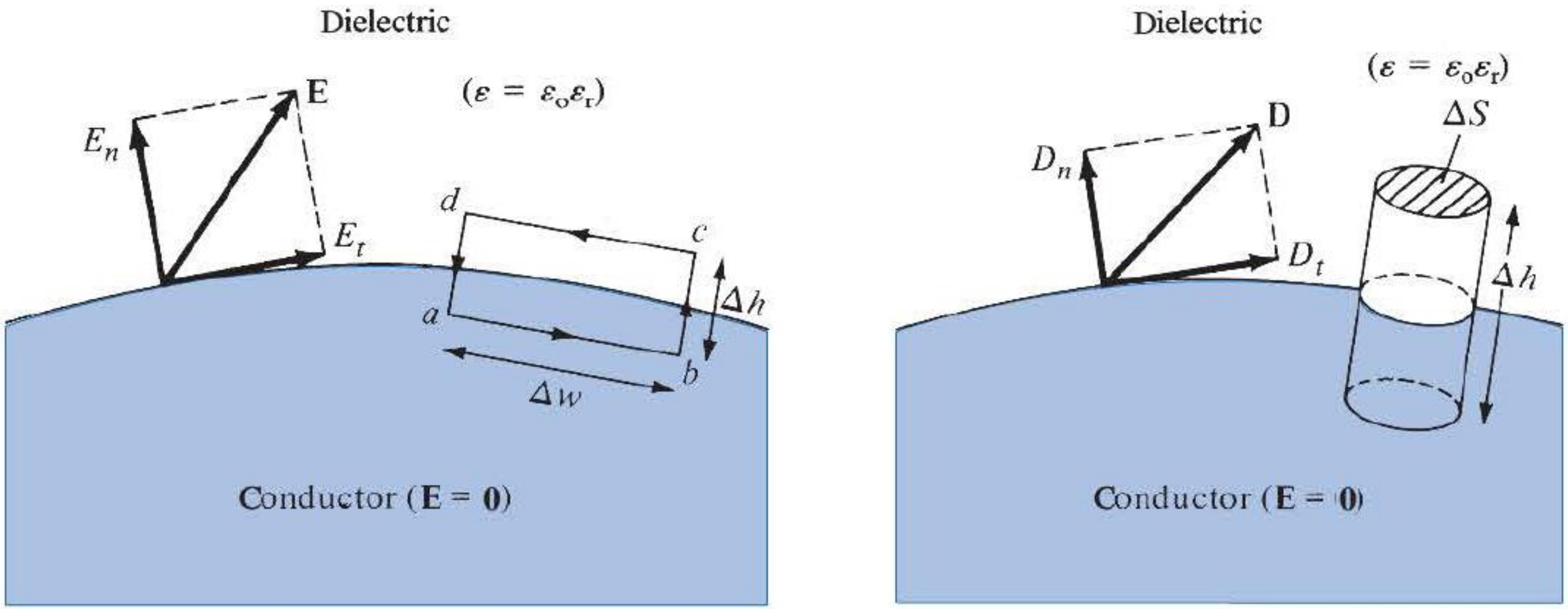
$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

→ This is "law of refraction" of electric field at the boundary, free of charge ($\rho_s = 0$ assumed)

→ In general, an interface between 2 dielectrics produce a bending of flux lines.



ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS



Conductor – Dielectric Boundary Conditions

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

Conductor - Dielectric Boundary conditions.

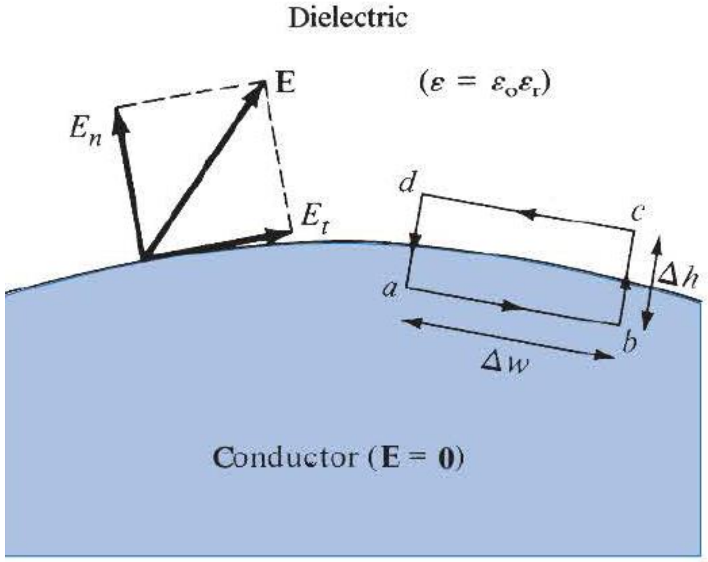
→ The conductor is assumed to be perfect ($\sigma \to \infty$).

→ To determine the boundary conditions, we follow the same procedure, except that $E = 0$ inside the conductor.

→ Using the equation $\oint \vec{E} \cdot d\vec{l} = 0$ to the closed path 'abcda' as in the figure, gives

$$0 = \cancel{0 \cdot \Delta w} + \cancel{0 \cdot \frac{\Delta h}{2}} + E_n \frac{\Delta h}{2} - E_t \cdot \Delta w - \cancel{E_n \frac{\Delta h}{2}} + \cancel{0 \cdot \frac{\Delta h}{2}}$$

→ As $\Delta h \to 0$ $\boxed{E_t = 0}$



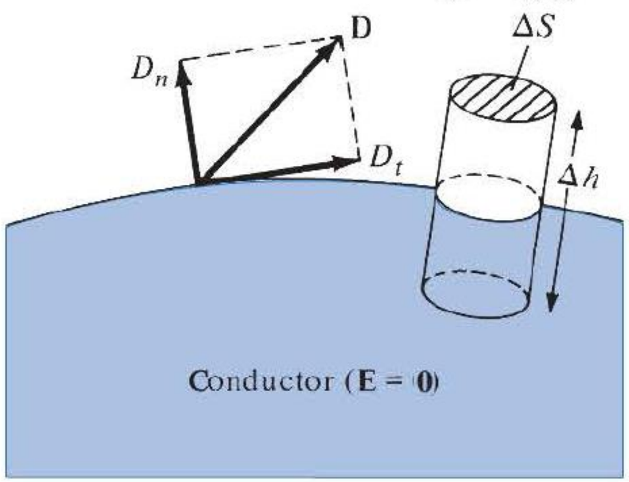
(a)

→ Similarly, using $\oint_S \bar{D}\cdot d\bar{S} = Q_{enc}$ to the cylindrical Gaussian Surface & letting $\Delta h \to 0$, we have

$$\Delta Q = D_n \cdot \Delta S - \cancel{0 \cdot \Delta S} = \rho_s \Delta S$$

→ Since $\bar{D} = \varepsilon \bar{E} = 0$ inside the conductor, we have

$$\boxed{D_n = \rho_s}$$



(b)

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

(22)

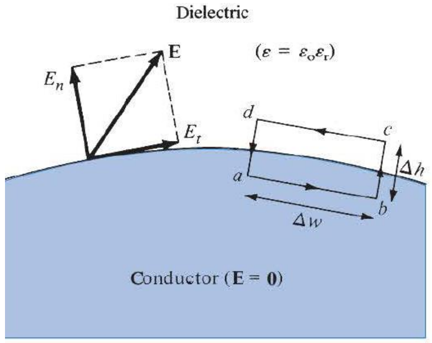
Conclusions: under static conditions, following can be inferred / concluded for a perfect conductor:

→ No electric field may exist within the conductor. $\boxed{\rho_v = 0 \; ; \; \bar{E} = 0}$

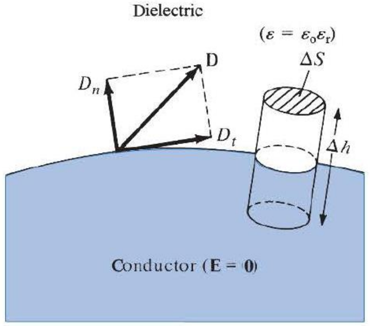
→ Since $\bar{E} = \nabla V = 0$, there can be no potential difference between any 2 points in the conductor i.e a conductor is an equipotential body.

→ An electric field $\bar{E}$ must be external to the conductor & must be normal to its surface.

$$\boxed{D_t = \varepsilon_0 \varepsilon_r E_t = 0 \;\&\; D_n = \varepsilon_0 \varepsilon_r E_n = \rho_s}$$



(a)



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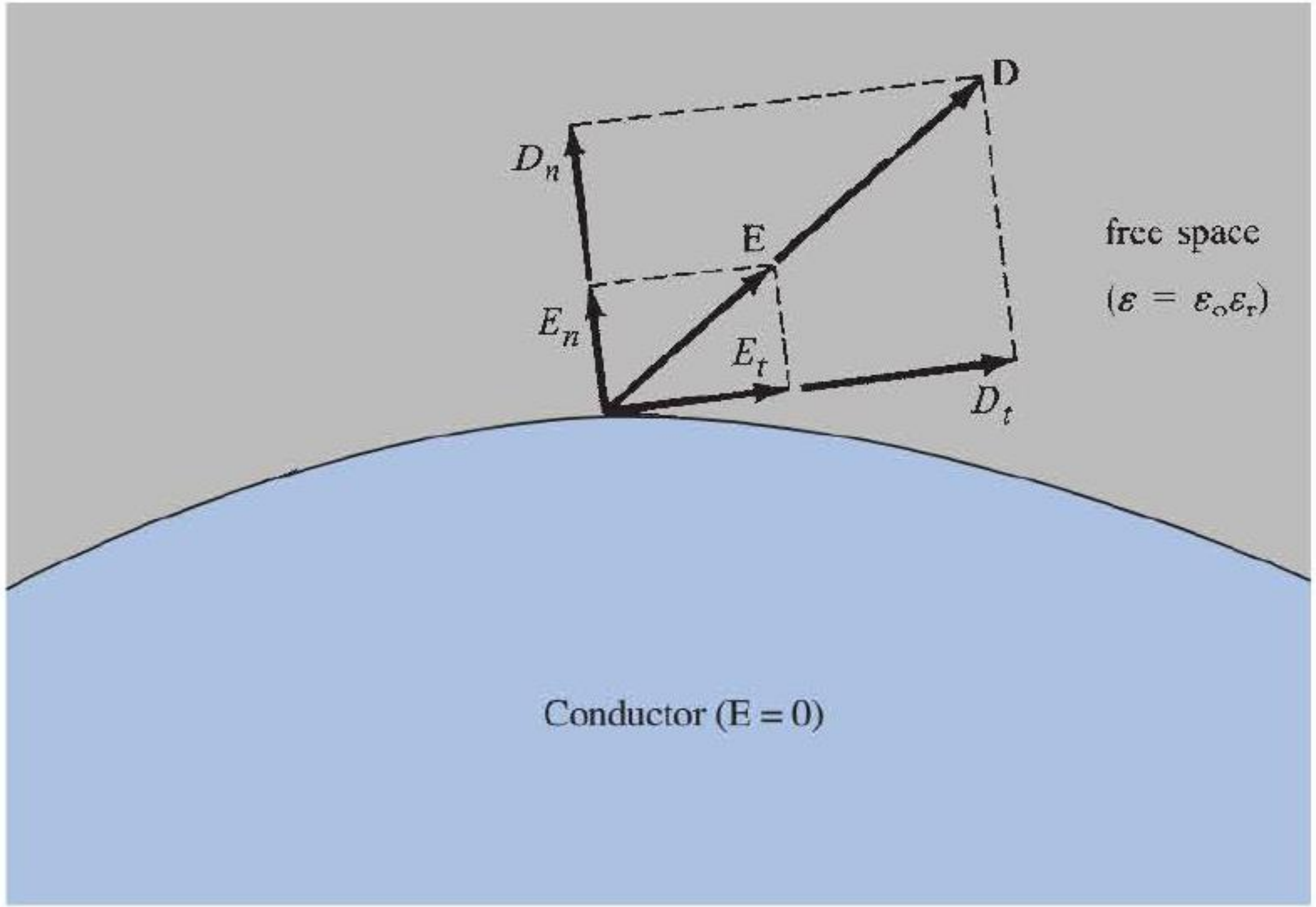
ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS



Conductor – Free Space Boundary Conditions

ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

Conductor - Free Space Boundary Conditions.

→ Refer the figure.

→ Boundary conditions can be obtained from

$$D_t = \varepsilon_o \varepsilon_r E_t = 0 \;\; \& \;\; D_n = \varepsilon_o \varepsilon_r E_n = \rho_s$$

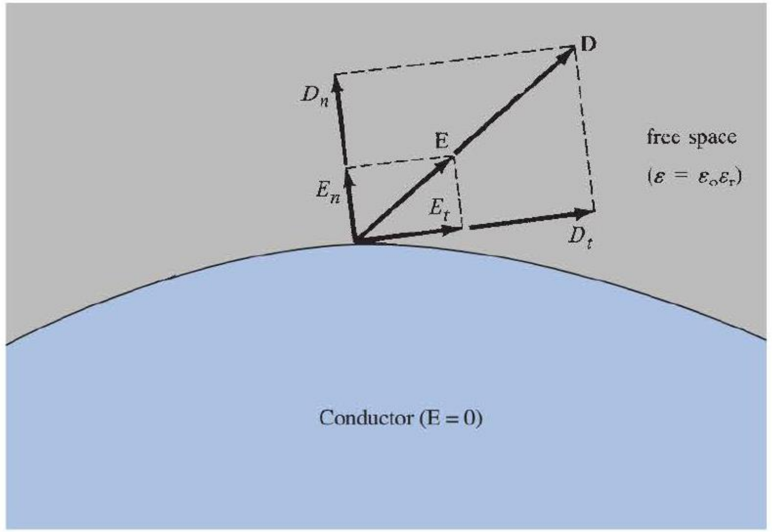
by replacing ε_r by 1 (∵ free space may be regarded as a special dielectric for which $\varepsilon_r = 1$).

→ The electric field $\bar{E}$ must be external to the conductor & normal to its surface.

→ Then $\boxed{D_t = \varepsilon_o E_t = 0 \;\; \& \;\; D_n = \varepsilon_o E_n = \rho_s}$

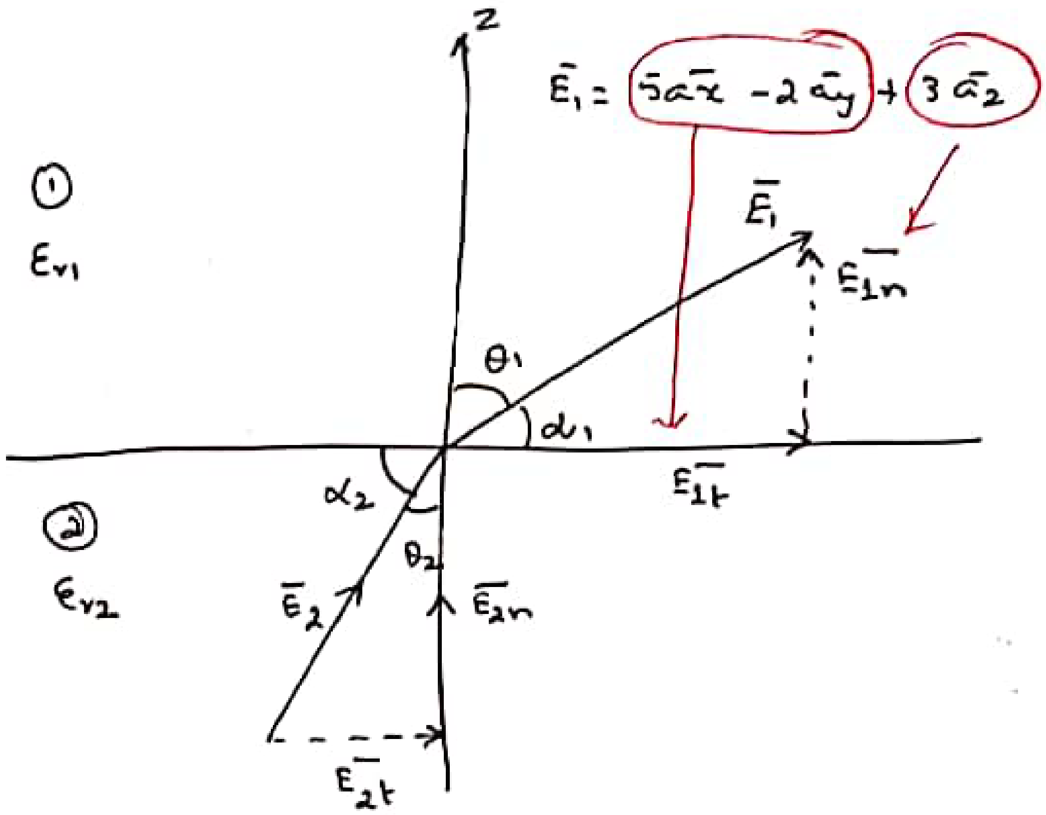
→ The above relation implies that the $\bar{E}$ field must approach the conducting surface normally.

→ $\boxed{\text{Ex-8}}$



(23)

Two extensive homogeneous, isotropic dielectrics meet on plane $z=0$. for $z>0$; $\varepsilon_{r1}=4$, & for $z<0$ $\varepsilon_{r2}=3$. A uniform electric field $\bar{E}_1 = 5\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z$ kV/m exists for $z \geq 0$.



ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

(a) Find $\bar{E}_2$ for $z \leq 0$

Since $\bar{a}_z$ (unit vector in z-direction) is normal to the boundary plane, we can obtain the normal components as

$$E_{1n} = \bar{E}_1 \cdot \bar{a}_n = (5\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z) \cdot (\bar{a}_z) = 3.$$

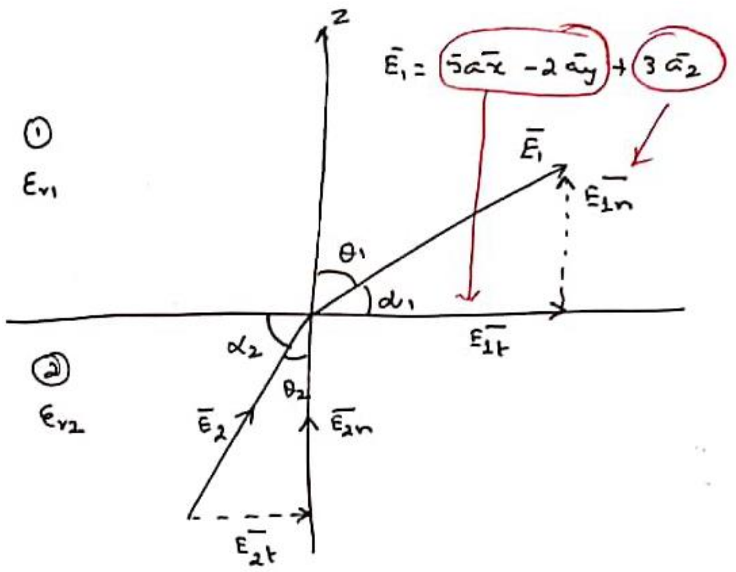
$\therefore \bar{E}_{1n} = 3\bar{a}_z$ (in the +ve z direction)

Similarly $\boxed{\bar{E}_{2n} = (\bar{E}_2 \cdot \bar{a}_z)\bar{a}_z}$

Now $\bar{E} = \bar{E}_t + \bar{E}_n \Rightarrow \bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n} = 5\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z - 3\bar{a}_z = 5\bar{a}_x - 2\bar{a}_y$

Now using the boundary conditions of D-D, we have

$\bar{E}_{1t} = \bar{E}_{2t}$. Hence $\bar{E}_{1t} = \bar{E}_{2t} = 5\bar{a}_x - 2\bar{a}_y$



ELECTRIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS

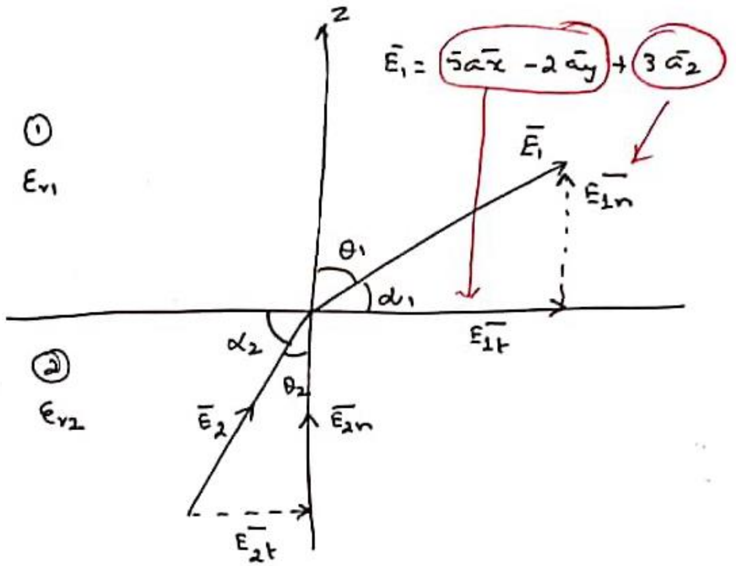
(24)

Similarly $\quad \bar{D}_{1n} = \bar{D}_{2n}$ or $\epsilon_{r1} \bar{E}_{1n} = \epsilon_{r2} \bar{E}_{2n}$

or. $\quad \bar{E}_{2n} = \dfrac{\epsilon_{r1}}{\epsilon_{r2}} \bar{E}_{1n} = \dfrac{4}{3}(3\bar{a}_z) = 4\bar{a}_z$

Thus $\quad \bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n} = 5\bar{a}_x - 2\bar{a}_y + 4\bar{a}_z$ kV/m

x ———— x ———— x ———— x ————x



(b) The angles $\bar{E}_1$ & $\bar{E}_2$ make with the interface.
from the diagram $\alpha_1 = 90° - \theta_1$ & $\alpha_2 = 90° - \theta_2$

method-1 Now $|\bar{E}_{1n}|$ or $E_{1n} = 3$; $|\bar{E}_{1t}| = E_{1t} = \sqrt{5^2 + (-2)^2}$
$= \sqrt{29}$

Hence $\tan\theta_1 = \dfrac{|\bar{E}_{1t}|}{|\bar{E}_{1n}|} = \dfrac{E_{1t}}{E_{1n}} = \dfrac{\sqrt{29}}{3}$

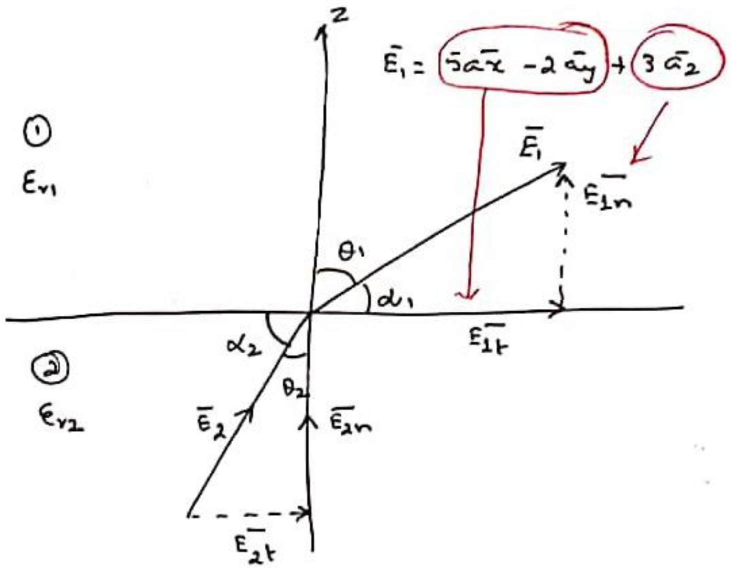
or $\theta_1 = \tan^{-1}\left(\dfrac{\sqrt{29}}{3}\right) = 60.8784°$

$\therefore \alpha_1 = 90° - 60.8784 = 29.1216°$

method-2 $\bar{E}_1 \cdot \bar{a}_n = |\bar{E}_1| \cdot |\bar{a}_n| \cos\theta_1$

or $\cos\theta_1 = \dfrac{\bar{E}_1 \cdot \bar{a}_n}{|\bar{E}_1|(1)}$ since $\bar{a}_n = \bar{a}_z$ $= \dfrac{3}{\sqrt{5^2 + (-2)^2 + 3^2}}$

$\cos\theta_1 = \dfrac{3}{\sqrt{38}}$ $= 60.8784°$ $\Rightarrow \alpha_1 = 90° - 60.8784 = 29.1216°$



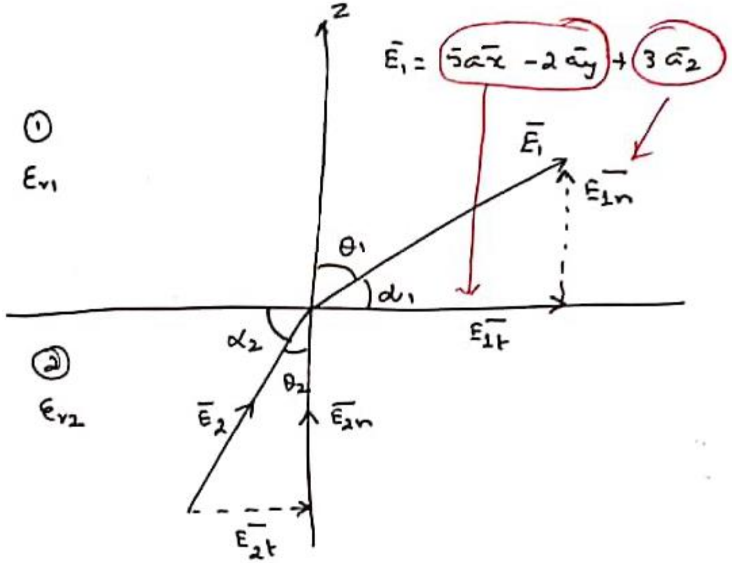
Similarly $|\bar{E}_{2n}| = |4\bar{a}_z| = 4$; $E_{2t} = E_{1t} = \sqrt{29}$

$\therefore \tan\theta_2 = \dfrac{E_{2t}}{E_{2n}} = \dfrac{\sqrt{29}}{4} = 1.346 \Rightarrow \theta_2 = \tan^{-1}(1.346)$

$= 53.3957^\circ$

Hence $\alpha_2 = 90^\circ - \theta_2 = 36.6043^\circ$

Check $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\epsilon_{r1}}{\epsilon_{r2}} \Rightarrow 1.333$



(c) The energy densities in both dielectrics. (25)

Recall $w_E = \frac{1}{2} \bar{D} \cdot \bar{E} = \frac{1}{2} \varepsilon E^2$ $E = |\bar{E}|$; $\varepsilon = \varepsilon_0 \varepsilon_r$

Then

$$w_{E1} = \frac{1}{2} \varepsilon_1 |\bar{E}_1|^2 = \frac{1}{2} \times \underbrace{4 \times \frac{10^{-9}}{36\pi}}_{\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}} \times \underbrace{(25+4+9) \times 10^6}_{|\bar{E}_1|^2}$$

$$= 672 \ \mu J/m^3$$

Similarly

$$w_{E2} = \frac{1}{2} \varepsilon_2 |\bar{E}_2|^2 = \frac{1}{2} \times 3 \times \frac{10^{-9}}{36\pi} \times (25+4+16) \times 10^6$$

$$= 597 \ \mu J/m^3$$

(d) The energy within a cube of side 2m centred at (3,4,-5).

At the centre (3,4,-5), of the cube of side 2m, $z = -5 < 0$

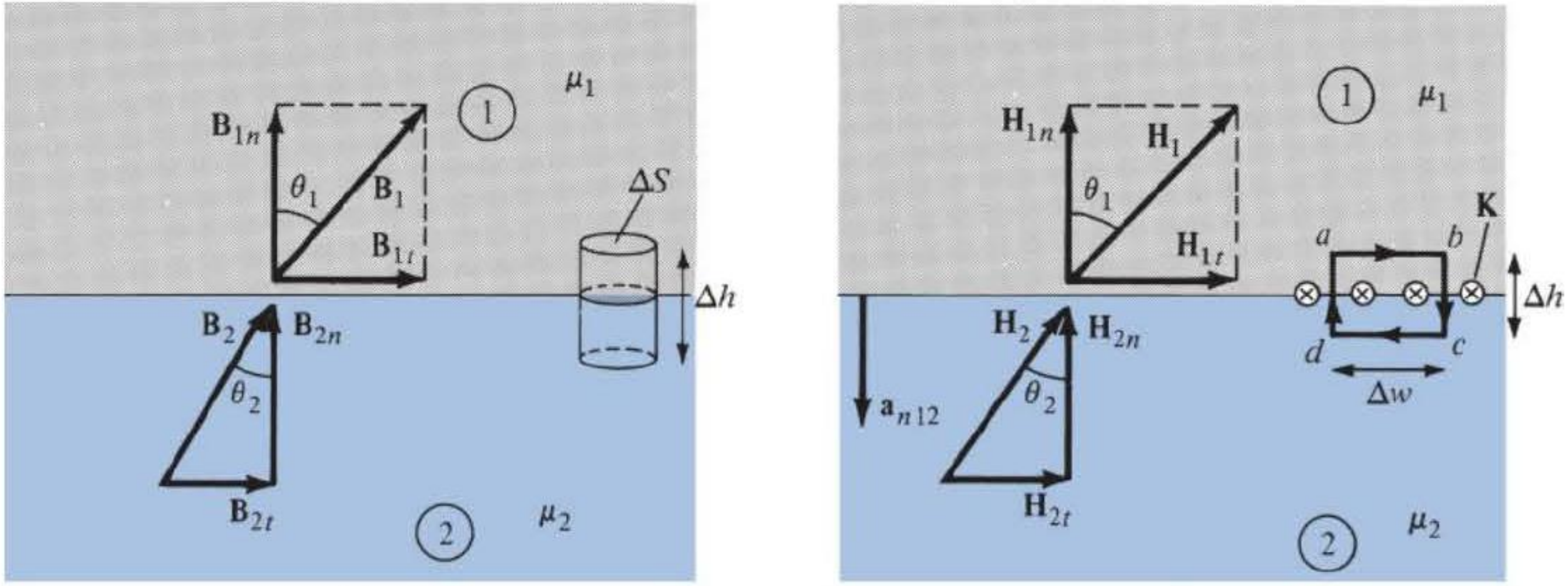
i.e the cube is in Region ② with $2 \le x \le 4$, $3 \le y \le 5$

& $-6 \le z \le -4$. Therefore

$$W_E = \int w_{E2}\, dv = \int_{x=2}^{4} \int_{y=3}^{5} \int_{z=-6}^{-4} w_{E2}\, dx\, dy\, dz.$$

$$= w_{E2}\,(2)(2)(2) = 597 \times 8 = \underline{\underline{4.776\ \text{mJ}}}$$

MAGNETIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS



(a) (b)

Summary:

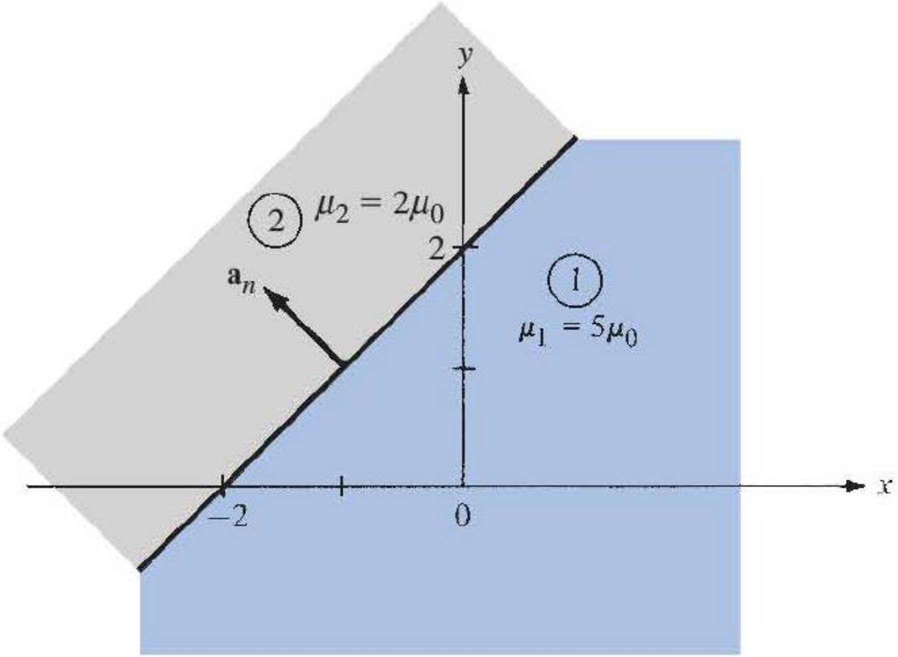
$\boldsymbol{B_{1n} = B_{2n}}$ or $\mu_1 \boldsymbol{H_{1n}} = \mu_2 \boldsymbol{H_{2n}}$

$(\boldsymbol{H_{1n}} - \boldsymbol{H_{2n}}) \times \boldsymbol{a_{n12}} = \boldsymbol{K}$

$\boldsymbol{H_{1t} = H_{2t}}$ or $\dfrac{\boldsymbol{B_{1t}}}{\mu_1} = \dfrac{\boldsymbol{B_{2t}}}{\mu_2}$ **with the boundary free of current.**

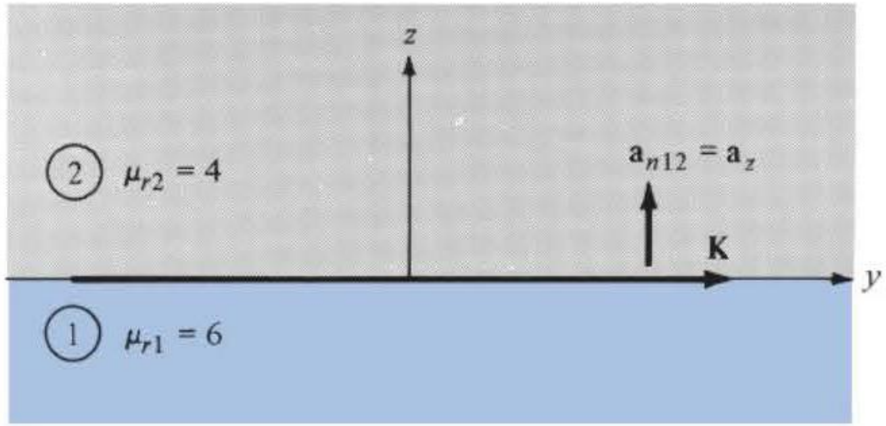
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$

MAGNETIC FIELDS IN MATERIAL SPACE – BOUNDARY CONDITIONS



Ex. 1

Ex. 2



THANK YOU

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