



LINEAR ALGEBRA AND ITS APPLICATIONS

UE19MA251

Unit 3. Linear Transformations and Orthogonality

Projection Matrices



The matrix P that projects onto $C(A)$ is given by

$$\text{Projection matrix} \quad P = A(A^T A)^{-1} A^T.$$

Also , if P and Q are the matrices that project onto orthogonal subspaces then it is always true that $PQ = 0$ and $P + Q = I$

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Least Squares Fitting Of Data



Suppose we do a series of experiments and expect the output b to be a linear function of the input t . We look for a straight line

$$b = C + Dt$$

If there is no experimental error then two measurements of b will determine the line. But, if there is error, we minimize it by the method of least squares and find the optimal straight line.

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Least Squares Fitting Of Data



Consider the following system of equations:

$$C + Dt_1 = b_1$$

$$C + Dt_2 = b_2 \dots$$

$$C + Dt_m = b_m$$

In matrix form,

$$\begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_m \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \text{ or } Ax = b$$

The best solution \hat{x} can be obtained by solving the normal equations.

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Problems on Least Squares Fitting Of Data



Use the method of least square to fit the best line to the data $b = (4, 3, 1, 0)$ at $t = -2, -1, 0, 2$ respectively. Find the projection of $b = (4, 3, 1, 0)$ onto the column space of A . Calculate the error vector ' e ' and check that ' e ' is orthogonal to the columns of A .

Solution: Let $C + Dt = b$ be the best fit straight line for the given data.

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Given that $b = 4, 3, 1, 0$ at $t = -2, -1, 0, 2$

$$\Rightarrow \left. \begin{array}{l} C + D(-2) = 4 \\ C + D(-1) = 3 \\ C + D(0) = 1 \\ C + D(2) = 0 \end{array} \right\} \Rightarrow \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow Ax = b$$

The system is inconsistent. To find least square solution \hat{x} , we have to solve normal equation

$$\text{i.e } A^T A \hat{x} = A^T b$$

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$$\Rightarrow \begin{bmatrix} 4 & -1 \\ -1 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 61/35 \\ -36/35 \end{pmatrix}$$

\therefore The best straight line fit for the given data is .

$$b = \frac{61}{35} - \frac{36}{35} t$$

Let p be the projection of $b = (4, 3, 1, 0)$ onto $C(A)$

$$\therefore p = A \begin{bmatrix} 1 \\ x \end{bmatrix} = \frac{1}{35} \begin{pmatrix} 133 \\ 97 \\ 61 \\ -11 \end{pmatrix}$$

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Problems on Least Squares Fitting Of Data



The error vector

$$e = b - \hat{b} = \begin{pmatrix} 1/5 \\ 8/35 \\ -26/35 \\ 11/35 \end{pmatrix}.$$

The error vector e is orthogonal to both the columns of A

i.e $e^T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$ and $e^T \begin{pmatrix} -2 \\ -1 \\ 0 \\ 2 \end{pmatrix} = 0$

Therefore the vector ' e ' is orthogonal to column space of A .



THANK YOU
