

# **Principles of Digital Signal Processing**

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# **DSP**



### **RELATIONSHIP OF DFT WITH OTHER TRANSFORM**

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#### **DFT**



To summrise DFT and IDFT

DFT: 
$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \qquad k = 0, 1, 2, ..., N-1$$

IDFT: 
$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}, \qquad n = 0, 1, 2, \dots, N-1$$

# Frequency domain sampling Relationship of DFT with other transform



Relationship to the Fourier series coefficients of a periodic sequence.

Relationship to the Fourier transform of an aperiodic sequence.

Relationship to the z-transform.

Relationship to the Fourier series coefficients of a continuous-time signal.

### Relationship of DFT with other transform



# Relationship to the Fourier series coefficients of a periodic sequence.

$$x_p(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi nk/N} \qquad -\infty < n < \infty$$

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi nk/N} \qquad k = 0, 1, \dots, N-1$$

DFT of 
$$x(n) = x_p(n)$$
,  $0 \le n \le N-1$ 

$$X(k) = Nc_k$$

### Relationship of DFT with other transform



### Relationship to the Fourier transform of an aperiodic sequence.

$$X(k) = X(\omega)|_{\omega=2\pi k/N} = \sum_{n=-\infty}^{\infty} x(n)e^{-j2\pi nk/N}$$
  $k = 0, 1, ... N-1$ 

DFT coefficients of periodic seq  $\longrightarrow x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$  of period N

### **Relationship of DFT with other transform**



$$\hat{x}(n) = \begin{cases} x_p(n), & 0 \le n \le N - 1 \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = \hat{x}(n) \qquad 0 \le n \le N - 1$$

Only when x(n) is of finite duration

### Relationship of DFT with other transform



# Relationship to the z-transform.

x(n) having z- transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

X(z) sampled at N equally spaced points on the unit circle  $z_k = e^{j2\pi k/N}$ 

$$X(k) \equiv X(z)|_{z=e^{j2\pi nk/N}}, \qquad k = 0, 1, \dots, N-1$$
$$= \sum_{n=0}^{\infty} x(n)e^{-j2\pi nk/N}$$

### Relationship of DFT with other transform



- •If x(n) has a finite duration of length N or less the sequence can be recovered from its N-point DFT.
- •Hence, its z-transform is uniquely determined by it s N-point DFT.

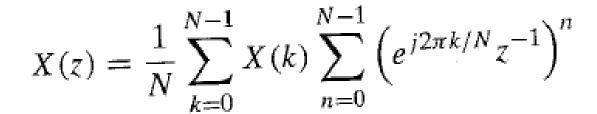
### Relationship of DFT with other transform



$$X(z) = \sum_{n=0}^{N-1} x(n)z^{-n}$$

$$X(z) = \sum_{n=0}^{N-1} \left[ \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \right] z^{-n}$$

#### Relationship of DFT with other transform



$$X(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1 - e^{j2\pi k/N} z^{-1}}$$

Polynomial interpolation formula for  $X(\omega)$ 

$$X(\omega) = \frac{1 - e^{-j\omega N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1 - e^{-j(\omega - 2\pi k/N)}}$$



### Relationship of DFT with other transform



### Relationship to the Fourier series coefficients of a continuous-time signal.

Continuous time periodic signal with fundamental period

$$T_p=1/F_0$$

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0}$$
Fourier series

Sample  $x_a(t)$  at a uniform rate Fs=N/Tp=1/T

### **Relationship of DFT with other transform**



### Continuous time periodic signal

# The discrete time sequence:

$$x(n) \equiv x_a(nT) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 nT} = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k n/N}$$

$$= \sum_{k=0}^{N-1} \left[ \sum_{l=-\infty}^{\infty} c_{k-lN} \right] e^{j2\pi k n/N}$$
DFT formula

### **Relationship of DFT with other transform**



$$X(k) = N \sum_{l=-\infty}^{\infty} c_{k-lN} \equiv N \tilde{c}_k$$

$$\tilde{c}_k = \sum_{l=-\infty}^{\infty} c_{k-lN}$$

Aliased version of the sequence  $\{c_k\}$ 



# **THANK YOU**

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