DIGITAL COMMUNICATION

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DPCM

Differential Pulse Code Modulation

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Differential Pulse Code Modulation (DPCM)

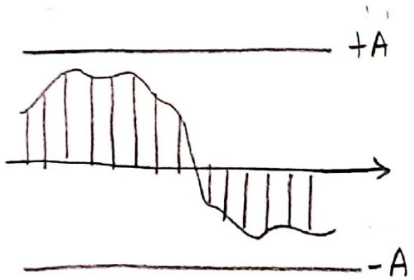
Why DPCM? What is DPCM?

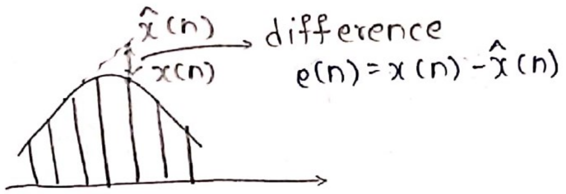
- In order to improve the performance of a PCM system, we need to either limit the peak value which introduces distortion or increase N which increases the bit rate. Hence alternatives for PCM had to be considered and DPCM was one of them.
- DPCM utilizes the fact that there is a correlation across samples in any signal.
- So, the previous samples are used to obtain an estimate/prediction of the current sample. The difference between the present sample and its estimate has a lot less amplitude and this difference value is quantized.

Differential Pulse Code Modulation (DPCM)

Why DPCM? What is DPCM?

- Quantize the difference between the amplitudes of samples instead of the amplitude of each sample.
- So $\Delta = \frac{2A}{2^N}$ decreases as A decreases and hence σ_Q^2 decreases. Therefore SNR increases.





- To improve the quantization performance one approach is to quantize the signal $e(n) = x(n) - \hat{x}(n)$.
- Where $\hat{x}(n)$ is the predicted value of x(n) based on the previous samples.

Differential Pulse Code Modulation (DPCM) --- Contd.

- Let $\hat{x}(n)$ be the estimate of $x(n)$ based on previous samples.
- Let $e(n) = x(n) - \hat{x}(n)$ be the difference signal that is quantized.
- Let $x_q(\mathrm{n})$ and $e_q(\mathrm{n})$ be the quantized versions of x(n) and e(n) respectively.

$$x_q(\mathrm{n}) = x(n) + q(n)$$

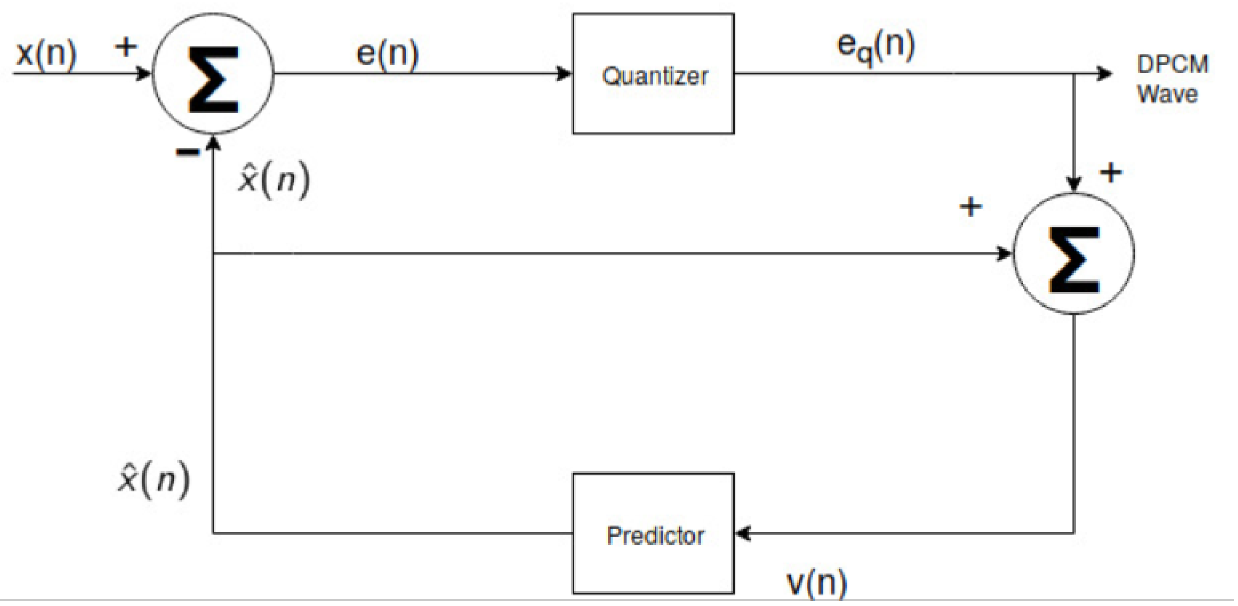
$$e_q(\mathrm{n}) = e(n) + q(n),$$

- $q(n)$ is the quantization error.

Differential Pulse Code Modulation (DPCM) --- Contd.

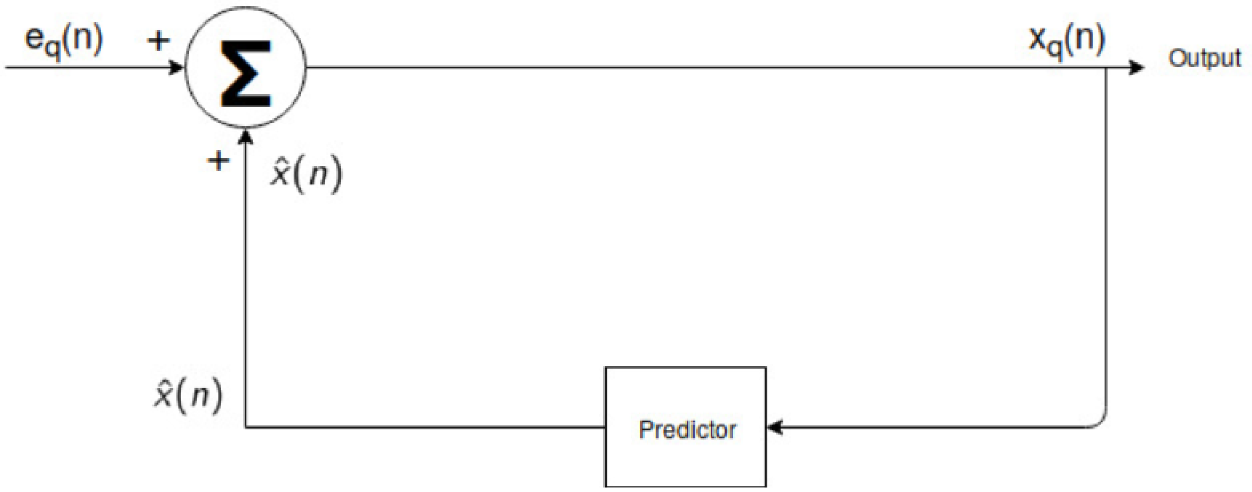
Block Diagram

DPCM Transmitter:



Differential Pulse Code Modulation (DPCM) --- Contd.

DPCM Receiver Block Diagram



Differential Pulse Code Modulation (DPCM) --- Contd.

DPCM Transmitter

From the Transmitter block diagram

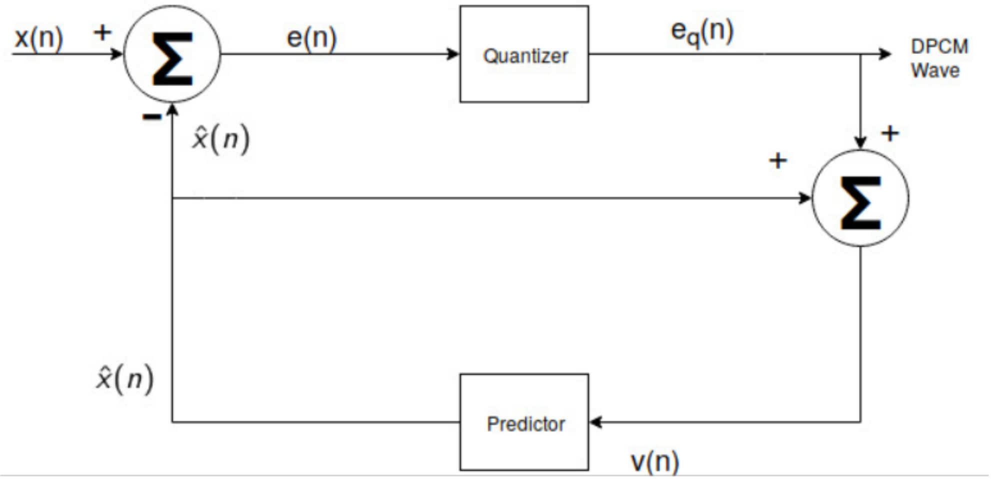
$$v(n) = \hat{x}(n) + e_q(n)$$

$= e(n) + q(n) + \hat{x}(n)$ $\qquad$ $(e(n) + \hat{x}(n) = x(n))$

$= x(n) - \hat{x}(n) + q(n) + \hat{x}(n)$

$= x(n) + q(n)$

$$v(n) = x_q(n)$$



Differential Pulse Code Modulation (DPCM) --- Contd.

DPCM Transmitter

- So, v(n) represents the quantized versions of x(n). i,e. Irrespective of the properties of the predictor, the quantized signal at the predictor input differs from the original input signal x(n) by the Quantization error.

- The above equation means that even if we don't quantize x(n), we get the signal $x_q(n)$. So, at the receiver end, we can't get back x(n). We will get $x_q(n)$.

Differential Pulse Code Modulation (DPCM) --- Contd.

DPCM Receiver

- In a noise free environment, the prediction filter at the transmitter and the receiver operate on the same sequence of samples $x_q(n)$.

- It is with this objective in mind that a feedback path is added to the quantizer in the transmitter.

- $\hat{x}(n)$ depends on previous value of $x_q(n)$ since predictor has delay. Hence the actual equation is

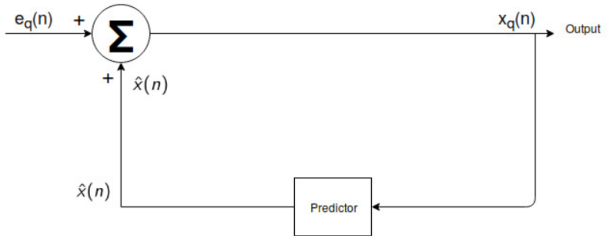
$$\hat{x}(n) = x_q(n-1)$$

Differential Pulse Code Modulation (DPCM) --- Contd.

DPCM Receiver

In the receiver model, consider

$$\hat{x}(n) + e_q(n) = \hat{x}(n) + e(n) + q(n)$$
$$= x(n) + q(n)$$
$$(\because x(n) = \hat{x}(n) + e(n))$$
$$= x_q(n)$$



In a noise free environment, the prediction filter at the transmitter and the receiver operate on the same sequence of samples $x_q(n)$. It is with this objective in mind that a feedback path is added to the quantizer in the transmitter.

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Differential Pulse Code Modulation (DPCM) --- Contd.

SNR for DPCM

- The output signal to noise ratio of a DPCM system is

$$SNR = \frac{{\sigma_X}^2}{{\sigma_Q}^2}$$

Where, ${\sigma_X}^2$ is the variance of the input signal $\mathrm{x(t)}$ assumed to be of zero mean.

${\sigma_Q}^2$ is the variance of the Quantization Error.

Differential Pulse Code Modulation (DPCM) --- Contd.

SNR for DPCM

$$SNR = \frac{{\sigma_X}^2}{{\sigma_E}^2}\frac{{\sigma_E}^2}{{\sigma_Q}^2}$$

Let $SNR_P = \frac{{\sigma_E}^2}{{\sigma_Q}^2}$ be called the SNR of predictor which is same as PCM model.

$G_P = \frac{{\sigma_X}^2}{{\sigma_E}^2}$ be called the gain due to prediction or the prediction gain.

$$SNR = G_P SNR_P$$

SNR for DPCM

Here SNR_P can be assumed to be of the same order as that of a PCM system. G_P indicates the gain of the DPCM system over the corresponding PCM system.

In dB, $SNR_{db} = 10 log_{10} SNR_P + 10 log_{10} G_P$

$$SNR_{db} = (6N \pm C) + 10 log_{10} G_P$$

DPCM

Example

Ex 1: A DPCM system uses a 6-bit quantizer. If $\Delta_E = 0.25\Delta_x$. Find SNR in dB. Her Δ_E and Δ_x are the step sizes of e(n) and x(n) respectively.

Sol: Assume both e(n) and x(n) are uniformly distributed

w.k.t. $SNR = SNR_p G_p$

$$= \frac{\sigma_E^2}{\sigma_Q^2} \cdot \frac{\sigma_x^2}{\sigma_E^2}$$

$$= 2^{2N} \times \frac{\Delta_x^2}{(0.25\Delta_x)^2}$$

$SNR_p = \frac{\sigma_E^2}{\sigma_q^2} = 2^{2N}$, as both e(n) & q(n) are uniform

$$\therefore \boxed{SNR = 2^{12} \times 16 \doteq 65536}$$

DPCM

Example

* $SNR_{dB} = 6N + 10\log_{10} 16.$

$SNR_{dB} = 6 \times 6 + 12.04$

∴ $\boxed{SNR_{dB} \approx 48.04}$

DPCM
Example

Ex 2: example: $x(n)$ is a WSS process with zero mean and $R_x(k) = 0.9^{|k|}$. It is input to a DPCM system. If the SNR is 8 find G_p. The predictor is just a delay.

sln

w.k.t. $SNR = \frac{\sigma_x^2}{\sigma_Q^2}$.

$\sigma_x^2 = E[X^2]$ ($\because x(n)$ has zero mean).

$\sigma_x^2 = R_x(0) = 1$.

$\therefore \frac{1}{\sigma_Q^2} = 8$.

$\boxed{\sigma_Q^2 = \frac{1}{8}}$

since the predictor is just a delay

$\hat{x}(n) = x_q(n-1)$

DPCM

Example

w.k.t. $e(n) = x(n) - \hat{x}(n)$.

$e(n) = x(n) - x_q(n-1)$

$e(n) = x(n) - x(n-1) - q(n-1)$

w.k.t. $\text{var}(X+Y+Z) = \sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 + 2C_{XY} + 2C_{YZ} + 2C_{XZ}$

$\text{var}(E) = \sigma_X^2 + \sigma_X^2 + \sigma_q^2 - 2R_X(1) + 0$.

* since $q(n)$ (noise) is independent of $x(n)$ and zero mean. C_{YZ} & $C_{XZ} = 0$

$\sigma_E^2 = 1 + 1 + \frac{1}{8} - 2 \times 0.9$

$\sigma_E^2 = 0.325$

$\therefore \boxed{G_p = \frac{\sigma_X^2}{\sigma_E^2} = \frac{1}{0.325} = 3.07}$

* variance for $x(n)$ & $x(n-1)$ is same.
* C_{XY} for $x(n)$ & $x(n-1)$ is $R_X(1) = E[X(n)X(n-1)]$
* zero mean
  $\Rightarrow$ co-relation = covariance.
* Noise is independent of signal

THANK YOU

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