CONTROL SYSTEMS

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CONTROL SYSTEMS

Introduction

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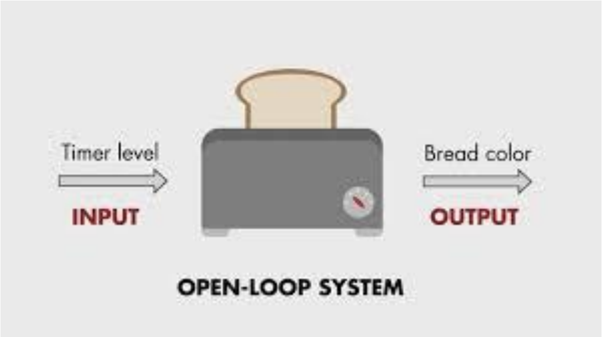
What is a system?

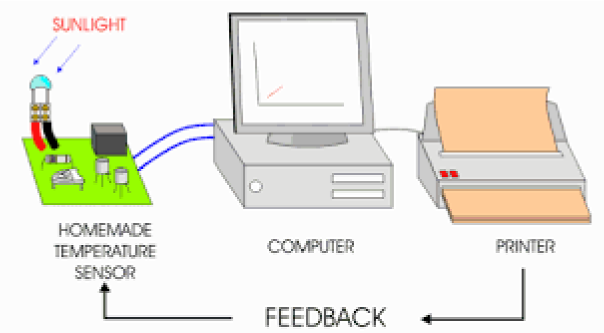
- **A system is..**
  - A set of things working together as parts of a mechanism or an interconnecting network to perform a function.
  - An interconnection of elements and devices for a purpose.
  - **Ex, Bus, two wheeler, fan etc.,**

What is a control system?

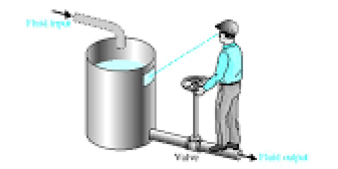
- A combination of components that act together to provide a desired system response.
- A **control system** manages, commands, directs, or regulates the behavior of other devices or **systems** using **control** loops. It provides the desired response by controlling the output.
- The control system maintain the actual system performance close to a desired set of performance specifications.

CONTROL SYSTEMS
Examples of Systems

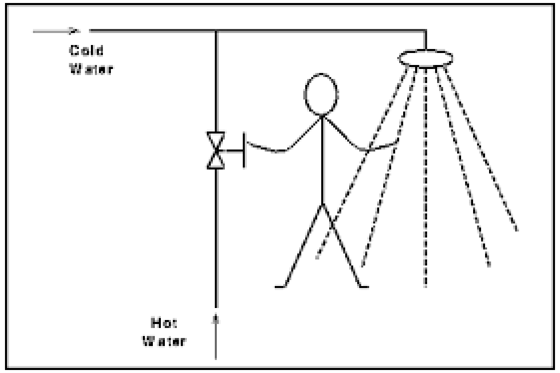


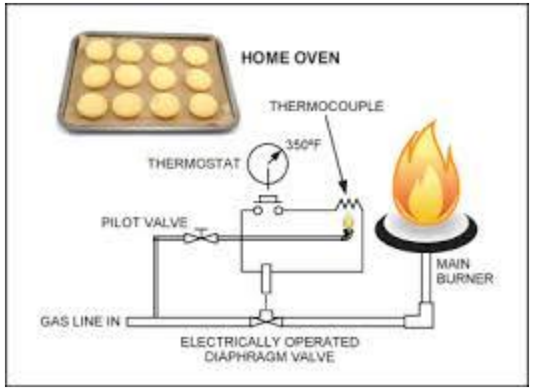


Example of Control Systems



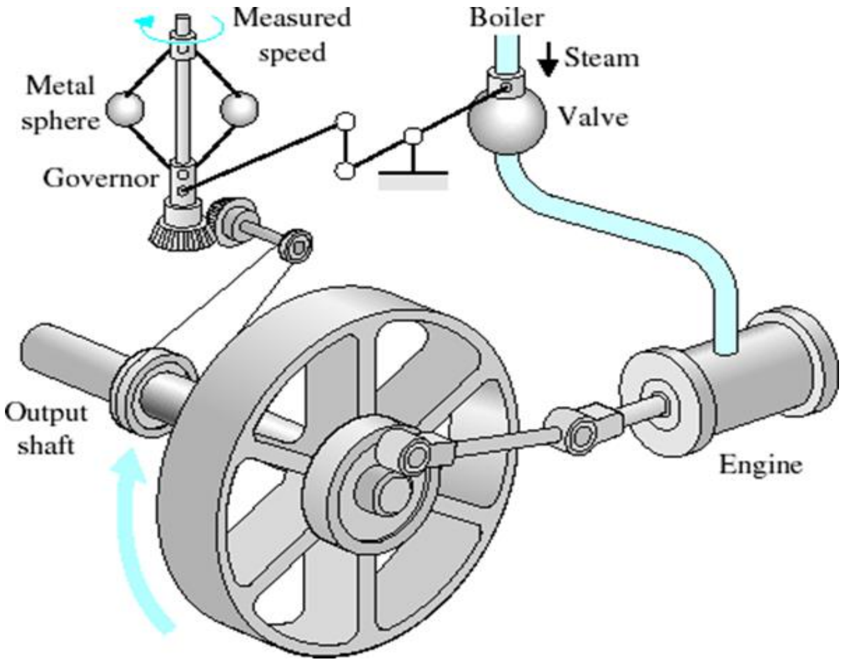
A manual control system for regulating the level of fluid in a tank by adjusting the output valve. The operator views the level of fluid through a port in the side of the tank.





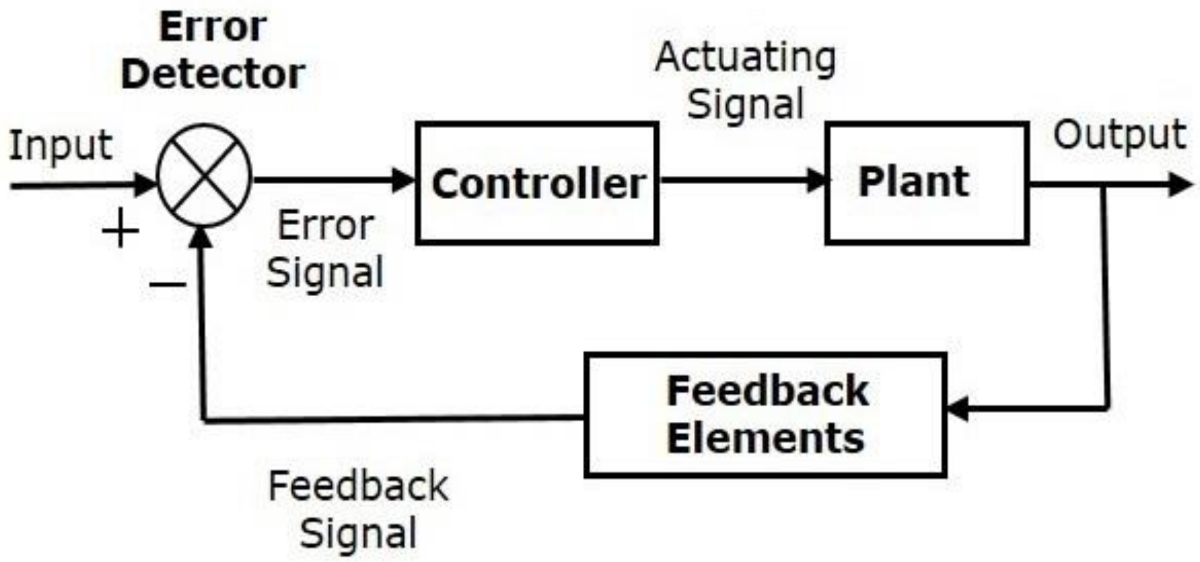
Why do we need control?

- Speed regulation(ex, Watt's Flyball governor, 1788) and maintain system response time
- Reduces the effect of variations in load(ex, Power systems)
- Maintain precision and robust(ex, Missile, Robotic arm)
- To handle huge and complex operations(ex, Airbus-A380)
- To ensure maneuverability and agility(ex, Fighter Aircraft)



Block Diagram of Control Systems

1. The Plant or process or system to be controlled
2. The *sensor* is a measurement device for measuring the output of the system.
3. The comparator and the controller determines the corrective action by comparing the actual output of the plant and the expected output/ reference.
4. The actuator takes the corrective action to return the system to its expected output.

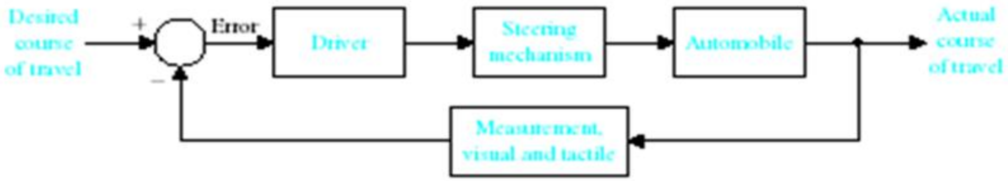


Examples of Control Systems

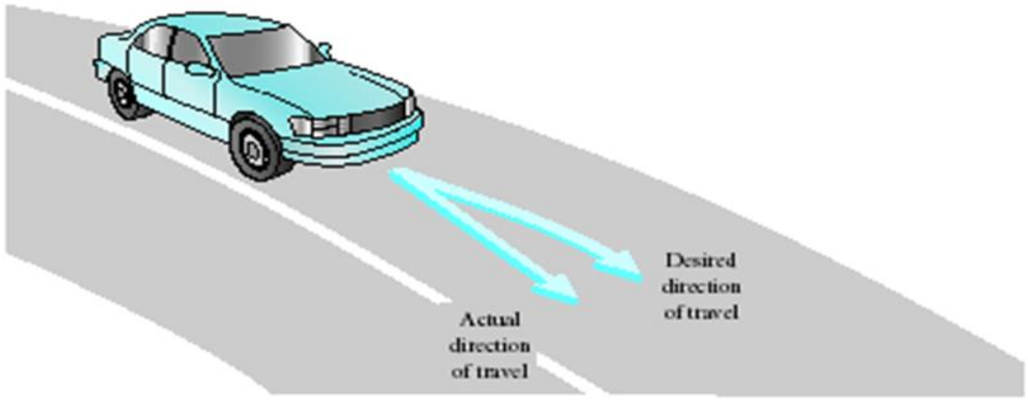
(a)Automobile steering control system.

(b)The driver uses the difference between the actual and the desired direction of travel to generate a controlled adjustment of the steering wheel.

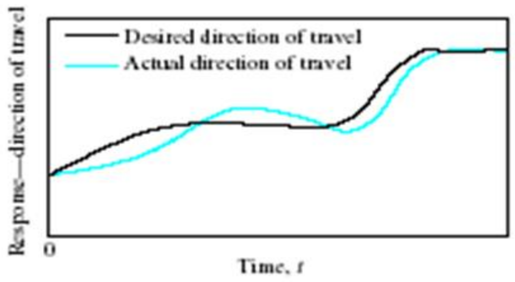
(c)Typical direction- of-travel response.



(a)



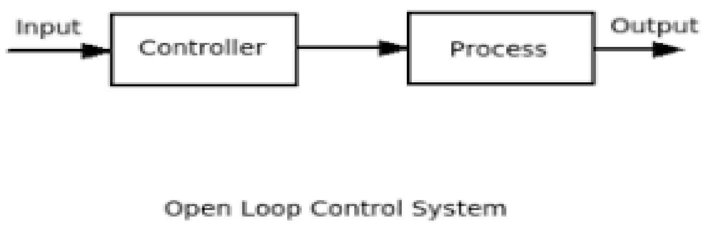
(b)

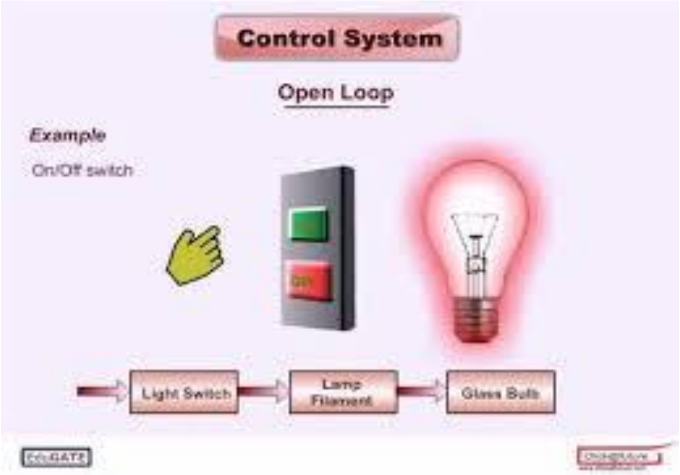


(c)

Classification of control systems

- **Open Loop**





- Systems in which the output has no influence or effect on the **control** action of the input signal , i.e output signal or condition is neither measured nor “fed back” for comparison with the input signal

- **Why Systems fail?**

How do you avoid Bread Blackening?

Causes: Timer failure (Hard Setting of Controller), Excess timer setting [Open Loop Control System]

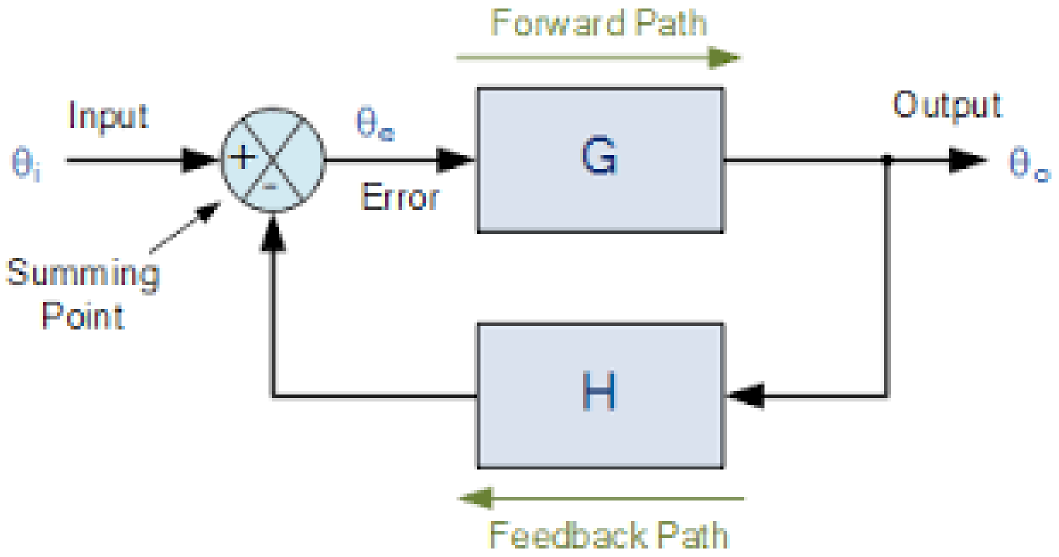
Solution: Bread Color Sensing. Use this measure to exercise restraint on turning on/off the heater. [Closed Loop or Feedback System]

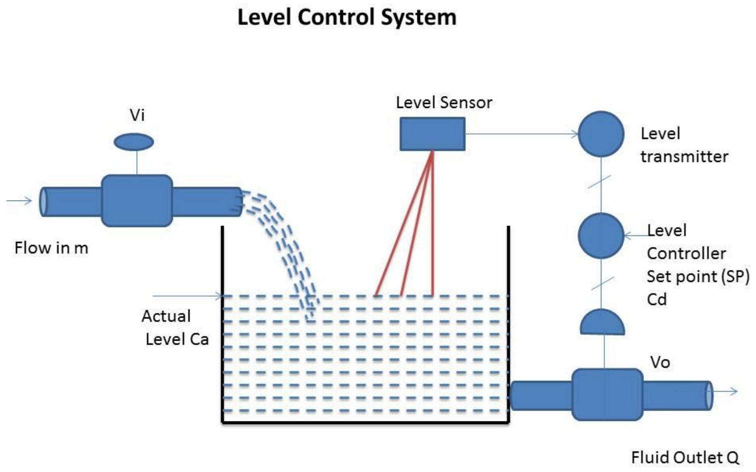
What failed here?

Cause: Depth not visible

Solution: Depth Sensing

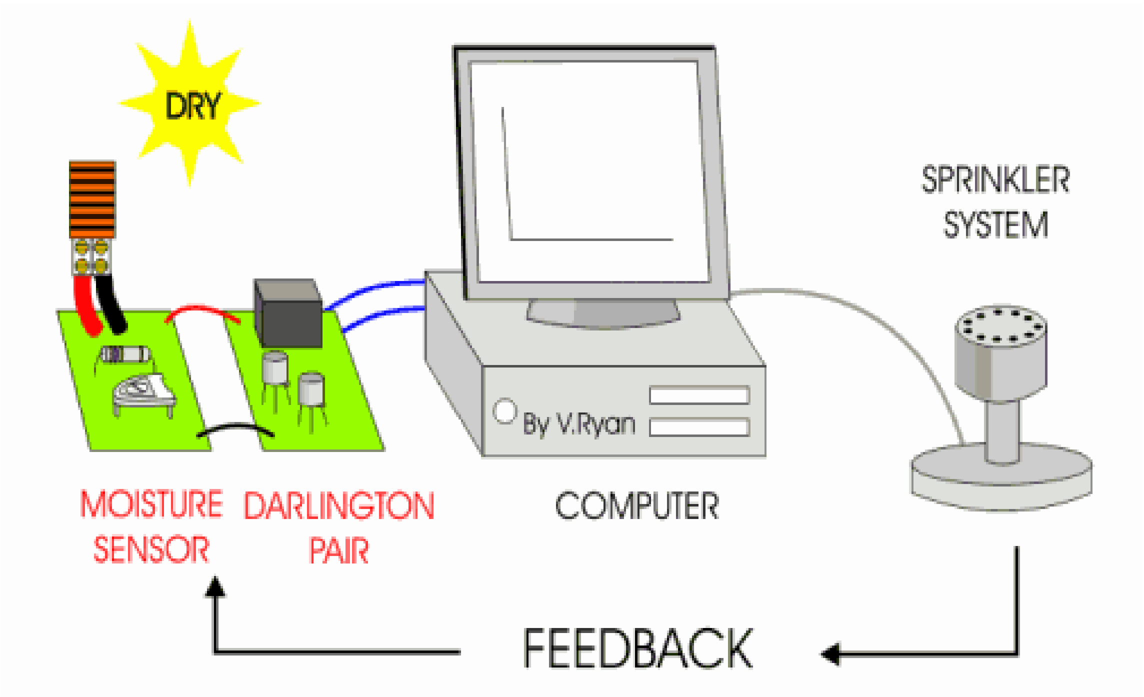
- **Closed Loop**





A **closed loop control** system considers the current output and alters it to the desired condition. The **control** action in these systems is based on the output.

- Example for Closed Loop



Course Contents

Unit 1: Mathematical Models of Systems

Introduction to control systems, Differential Equations of Physical Systems, Linear Approximations of Physical Systems, The Laplace Transform, The Transfer Function of Linear Systems, Block Diagram Models, Signal Flow Graph Model **12 Hours**

Unit 2: Feedback Control System Characteristics: Error signal analysis, Sensitivity of Control Systems to Parameter Variations, Control of the Transient Response of Control Systems, Disturbance Signals in a Feedback Control System, Steady State Error **10 Hours**

Unit 3: The Performance of Feedback Control Systems: Introduction, Test Input Signals, Performance of a Second Order System, Effects of a Third Pole and a Zero on the Second Order System Response, The s – Plane Root Location and the Transient Response, The Steady – State Error of Non unity Feedback Systems, Introduction to controllers, PD controller, PI and PID controllers **10Hours**

Course Contents

Unit 4: The Stability of Linear Feedback Systems: The Concept of Stability, the Routh – Hurwitz Stability Criterion, The Relative Stability of Feedback Control Systems, The Root Locus Method, Introduction, Concept and the Root Locus technique. Frequency Response methods: Introduction, Frequency Response Plots, Bode Diagram, and Performance Specifications in the Frequency Domain. **12Hours**

Unit 5: Stability in the Frequency Domain: Introduction, Mapping Contours in the s – Plane, the Nyquist Criterion, Relative Stability and the Nyquist Criterion. The Design of Feedback Control Systems: Introduction, Approaches to System Design, Cascade Compensation Networks, Phase – Lead Design Using the Bode Diagram, System Design Using Integration Networks, Phase-Lag Design Using Bode Diagram. **12Hours**

Text Book:

“Modern Control Systems”, Dorf, Richard C., and Robert H. Bishop, Pearson, 13th Edition 2017.

Reference Books:

- “Control Systems Engineering”, I J Nagrath, M Gopal, 6th Edition, New Age International, 2018.
- “Modern Control Engineering”, Ogata K & Yang Y., Pearson Education Asia, 5th Edition 2015.
- “Control Systems Engineering,” N. Nise, Wiley India, 2018

CONTROL SYSTEMS

Unit 1: Mathematical Models Of Physical Systems

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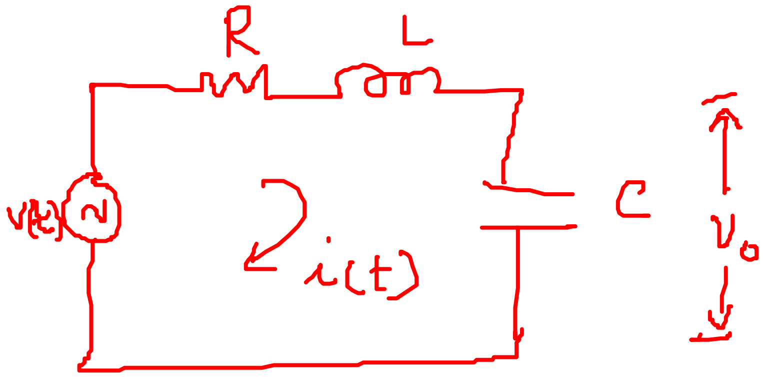
Mathematical Models Of Physical Systems

Example

▶ Simple System, RLC Circuit



KVL,

$$v(t) = R\,i(t) + L\frac{di}{dt} + \underbrace{\frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau}_{v_o(t)} \rightarrow (1)$$

$v(t) = V$

$\Rightarrow$ Integro - differential equation

$$v_o(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

(1) diff. w.r.t. t

$$V_o(s) = \frac{I(s)}{Cs}$$

$$0 = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i(t)}{C}$$

$$\Rightarrow L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i(t)}{C} = 0 \rightarrow (2)$$

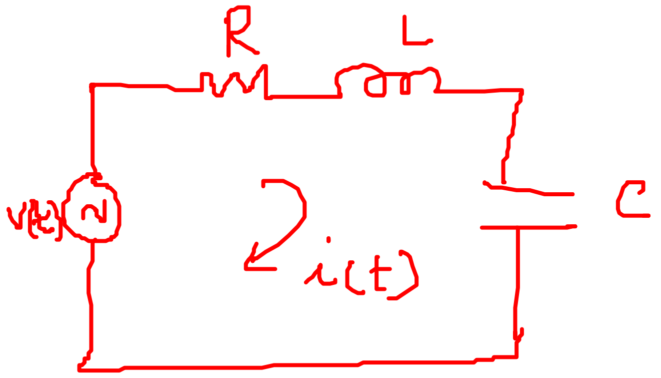
If order is higher

$\Rightarrow$ Applying LT (2) $\Rightarrow L\,s^2 I(s) + R\,s\,I(s) + \dfrac{I(s)}{Cs} = 0$

Mathematical Models Of Physical Systems

Example

- Simple System, RLC Circuit



① LT

$$V(s) = \left(R + Ls + \frac{1}{Cs}\right) I(s)$$

$$\frac{V_0(s)}{V(s)} = \frac{\frac{I(s)}{Cs}}{\left(R + Ls + \frac{1}{Cs}\right) I(s)}$$

$$= \frac{1}{RCs + LCs^2 + 1}$$ → mathematical model

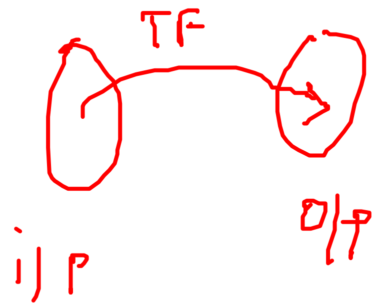
$$\frac{I(s)}{V(s)} = \frac{Cs}{LCs^2 + RCs + 1}$$ → algebraic function

Mathematical Models Of Physical Systems

Example

- Simple System, Mass Spring Damper

ex, modelling dynamics of car



→ state space ⇒ higher order s/m

↓

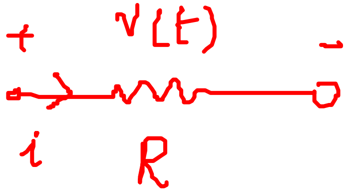
convert this into simple first order s/m

$$\frac{dx}{dt} = -ax + b$$ → r o diffs

Mathematical Models Of Physical Systems

Idealizing Assumptions

- Linear: The components are said to be linear if they follow super position principle.



$\Rightarrow v(t) = R\, i(t) \rightarrow$ ohm's law is true under certain idealizing assumptions

ex, tunnel diode

$u \rightarrow \boxed{\text{System}} \rightarrow y$

$u_1 \rightarrow y_1$

$u_2 \rightarrow y_2$

$u_1 + u_2 \rightarrow y_1 + y_2 \rightarrow$ additivity

$\alpha u \rightarrow \alpha y \rightarrow$ homogenity

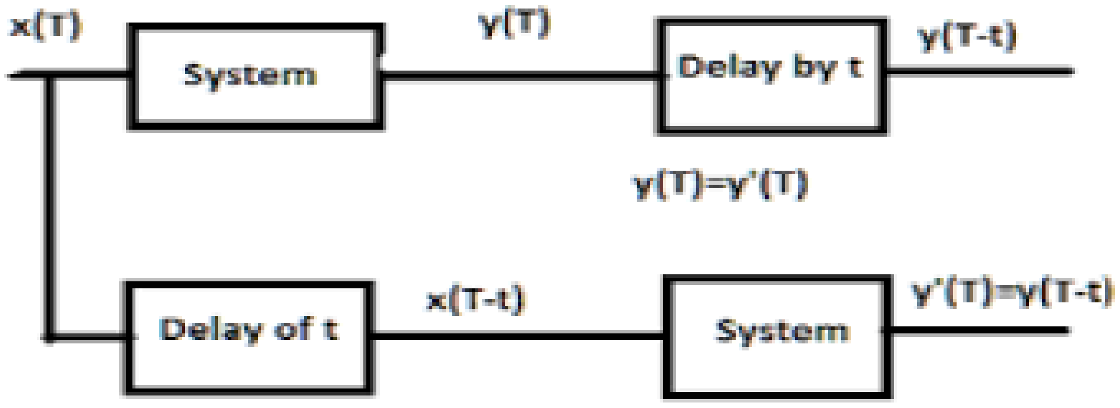
} principle of superposition

↳ linearity

Idealizing Assumptions

- Time Invariant:

This means the behavior of **system** does not depend on **time** at which input is applied.



Time invariant

PES UNIVERSITY ONLINE

- Lumped:

The components must be lumped ,which means the length of the device should be negligible compared to the wavelength of operation.

$f = 50\,Hz$, $\lambda = \frac{c}{f} = \frac{3\times10^8}{50} =$

MW , $f = 1$ to $1000\,GHz$

$\lambda = \frac{3\times10^8}{1000\times10^9} = 0.3$ – } distributed

VLSI

→ ODE , let time varying component

partial Diff equation

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Differential Equations Of Physical Systems

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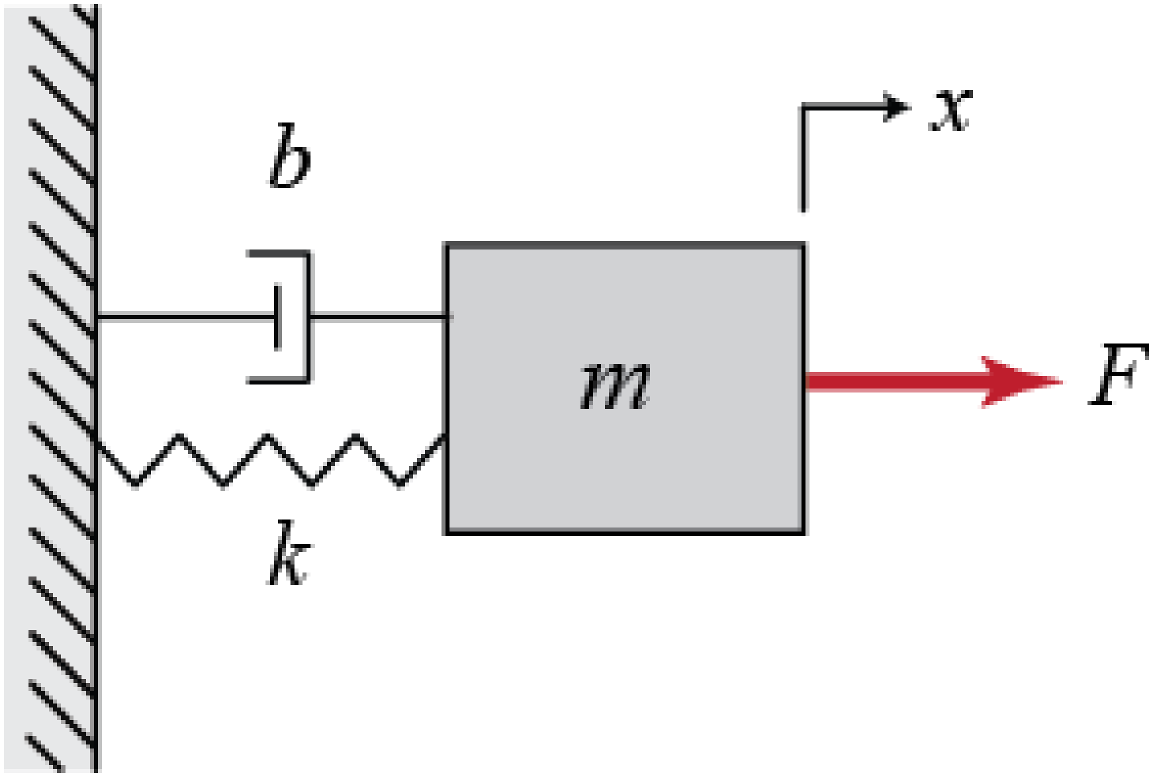
Differential Equations of Physical Systems

Mechanical Systems

- Systems that consist of spring , mass and damper. Ex, modelling of a car
- 2 types of mechanical system
  - Translational
  - Rotational



x – displacement

F - force

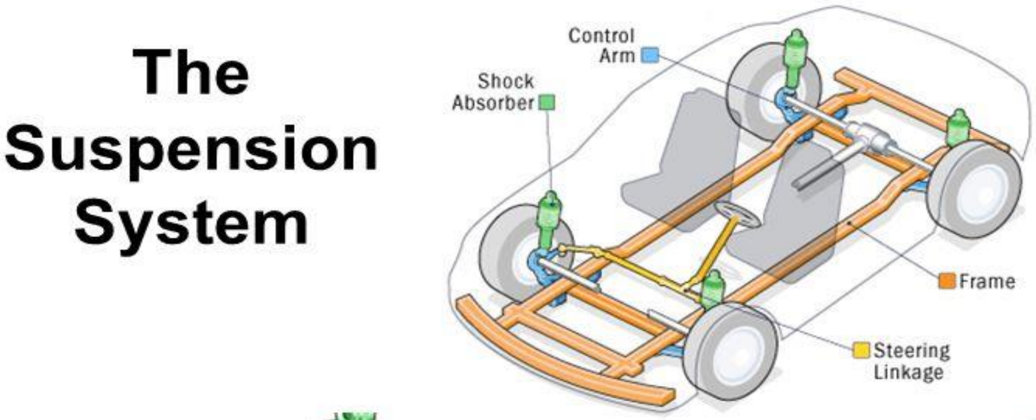
b – damping constant

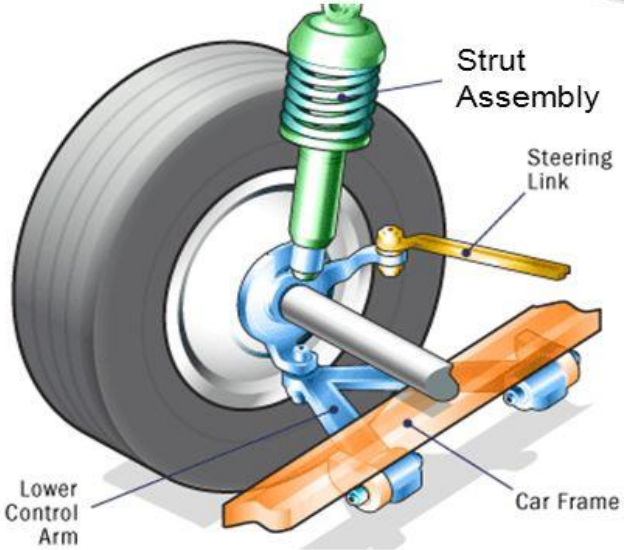
K – Spring Constant

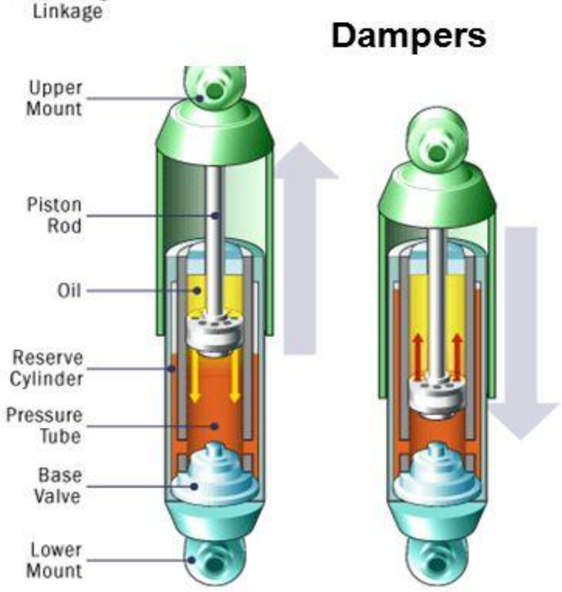
M - mass

Differential Equations of Physical Systems

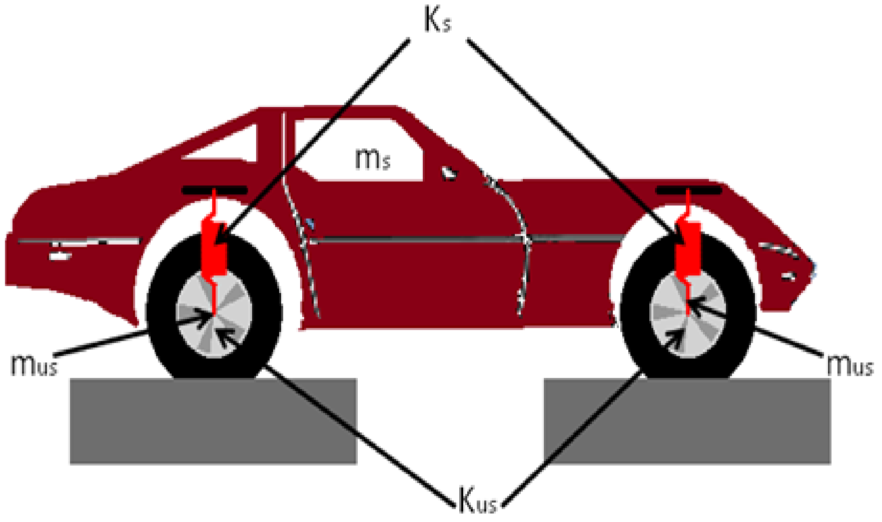
Mechanical Systems

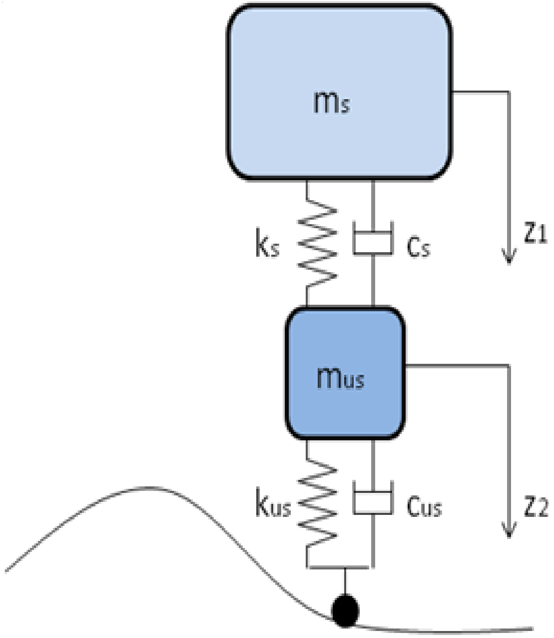






Differential Equations of Physical Systems
Mechanical Systems



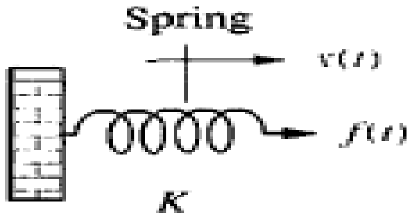
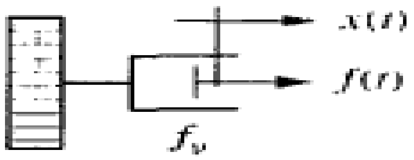
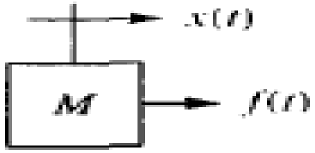


Differential Equations of Physical Systems

Translational Mechanical Systems

$i(t)$ $\quad$ $v(t)$ $\quad$ $v_i(t)$ $\quad$ $v_o(t)$

Ideal Elements: Through Variable, Across variable , Input Variable and Output Variable

Component	Force-velocity	Force-displacement
	$f(t) = K \int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$
	$f(t) = f_v v(t)$	$f(t) = f_v \dfrac{dx(t)}{dt}$
	$f(t) = M\dfrac{dv(t)}{dt}$	$f(t) = M\dfrac{d^2x(t)}{dt^2}$

Damping constant

→ B or D

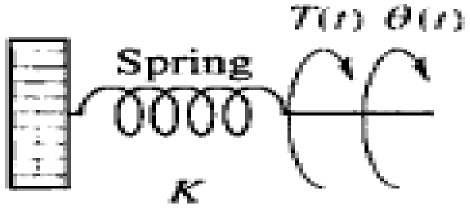
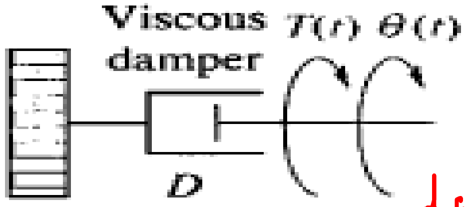
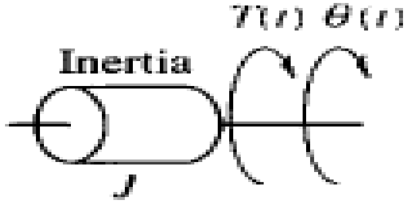
$v(t) = \dfrac{dx}{dt}$

$x(t) = \int_0^t v(\tau)\,d\tau$

friction → coulomb, viscous, stiction

$f(t) = \text{N (newtons)}$, $x(t) = \text{m (meters)}$, $v(t) = \text{m/s (meters/second)}$, $K = \text{N/m (newtons/meter)}$, $f_v = \text{N-s/m(newton-seconds/meter)}$, $M = \text{kg (kilograms = newton-seconds}^2\text{/meter)}$.

Differential Equations of Physical Systems
Rotational Mechanical Systems

Component	Torque-angular velocity	Torque-angular displacement
	$T(t) = K \int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$

damper | dashpot

$T(t)$ – N-m (newton-meters), $\theta(t)$ – rad(radians), $\omega(t)$ – rad/s(radians/second), K – N-m/rad(newton- meters/radian), D – N-m-s/rad (newton- meters-seconds/radian). J – kg-m^2(kilograms-meters^2 – newton-meters-seconds^2/radian).

$\theta(t) = \int_0^t \omega(\tau)d\tau$

$\omega = \dfrac{d\theta}{dt}$

θ - angular displacement

ω - angular velocity

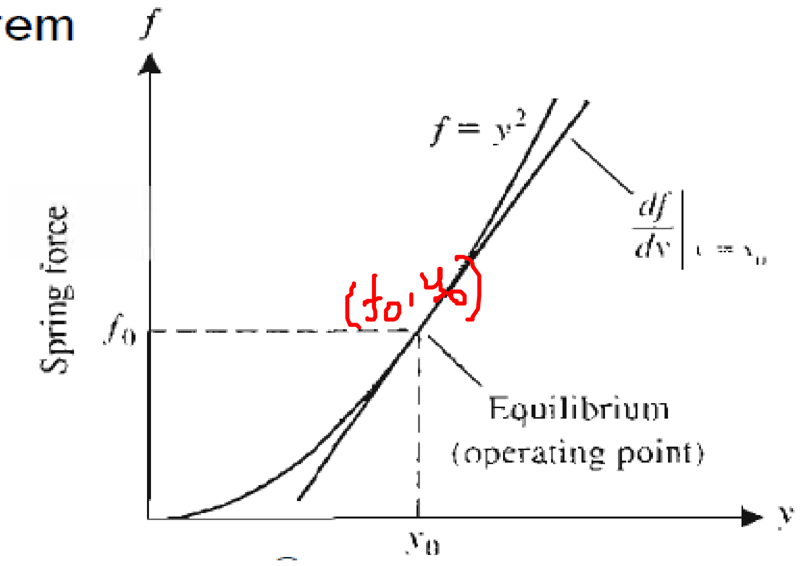
T - Torque

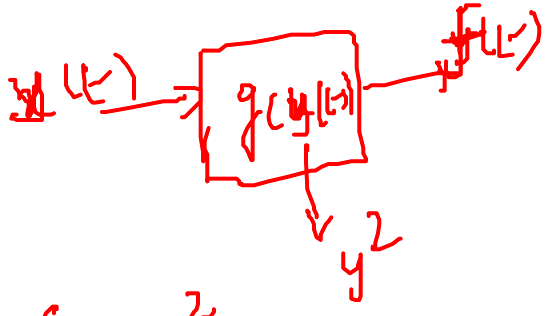
J - Moment of inertia

Differential Equations of Physical Systems
Linear Approximation of physical systems

- Most mechanical and electrical systems are linear for a large range of values
- Non Linear elements can be linearised by assuming small signal conditions
- Let $y(t) = g(x(t))$, where $x(t)$ is input applied to the system and $y(t)$ is output of the system

$f = y^2$





$f = y^2$

Differential Equations of Physical Systems
Linear Approximation of physical systems

$y_0 = x_0$

- Consider x_0 to be the operating point

Applying Taylor series expansion, over x_0,gives

$$y = g(x) = g(x_0) + \left.\frac{dg}{dx}\right|_{x=x_0} \frac{(x - x_0)}{1!} + \left.\frac{d^2g}{dx^2}\right|_{x=x_0} \frac{(x - x_0)^2}{2!} + \cdots .$$

- The slope at operating point x_0 ($\left.\frac{dg}{dx}\right|_{x=x_0}$) is a good approximation to the curve over a small range of $(x-x_0)$, the deviation from the operating point. Then, as a reasonable approximation, Taylor series becomes

$$y = g(x_0) + \left.\frac{dg}{dx}\right|_{x=x_0} (x - x_0) = y_0 + m(x - x_0),$$

$y = g(x)$

$$\Delta y = m\,\Delta x \rightarrow (x_0, y_0)$$

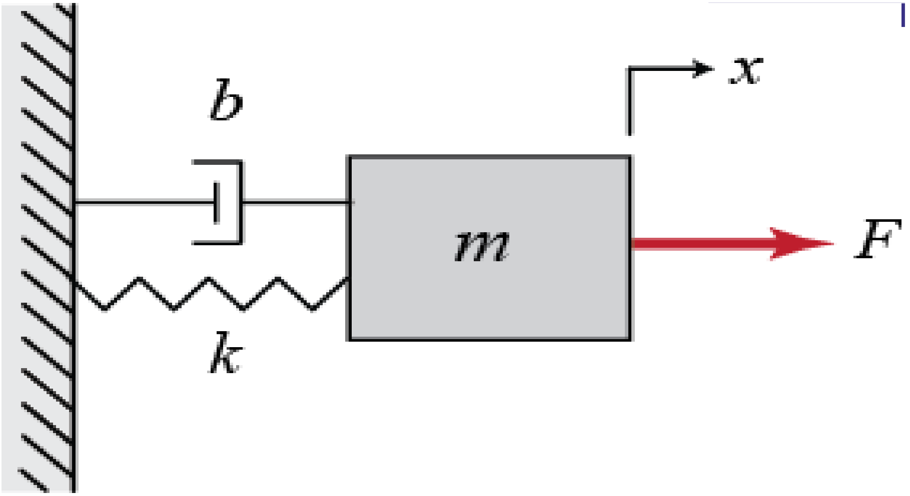
$y = y_0 + m(x - x_0) \Rightarrow y - y_0 = m(x - x_0)$

$\Delta y = m\,\Delta x$

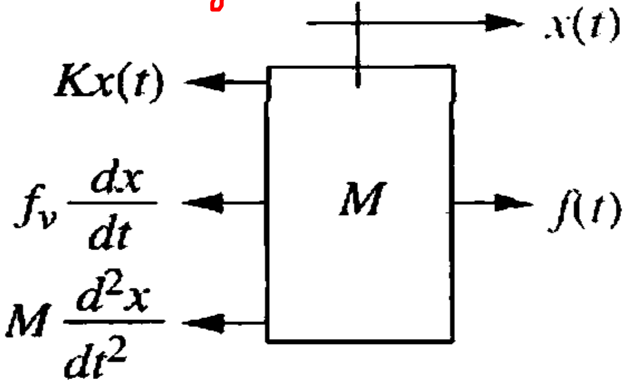
Differential Equations of Physical Systems
Examples of Mechanical Systems - Translational

- Systems that consist of spring , mass and damper.
- Force exerted in each element by considering x as displacement and F is force ,B or f is viscous damper, k is spring constant and M is mass
- $F = F_m + F_f + F_k$

$$= m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx$$



Free-body diagram

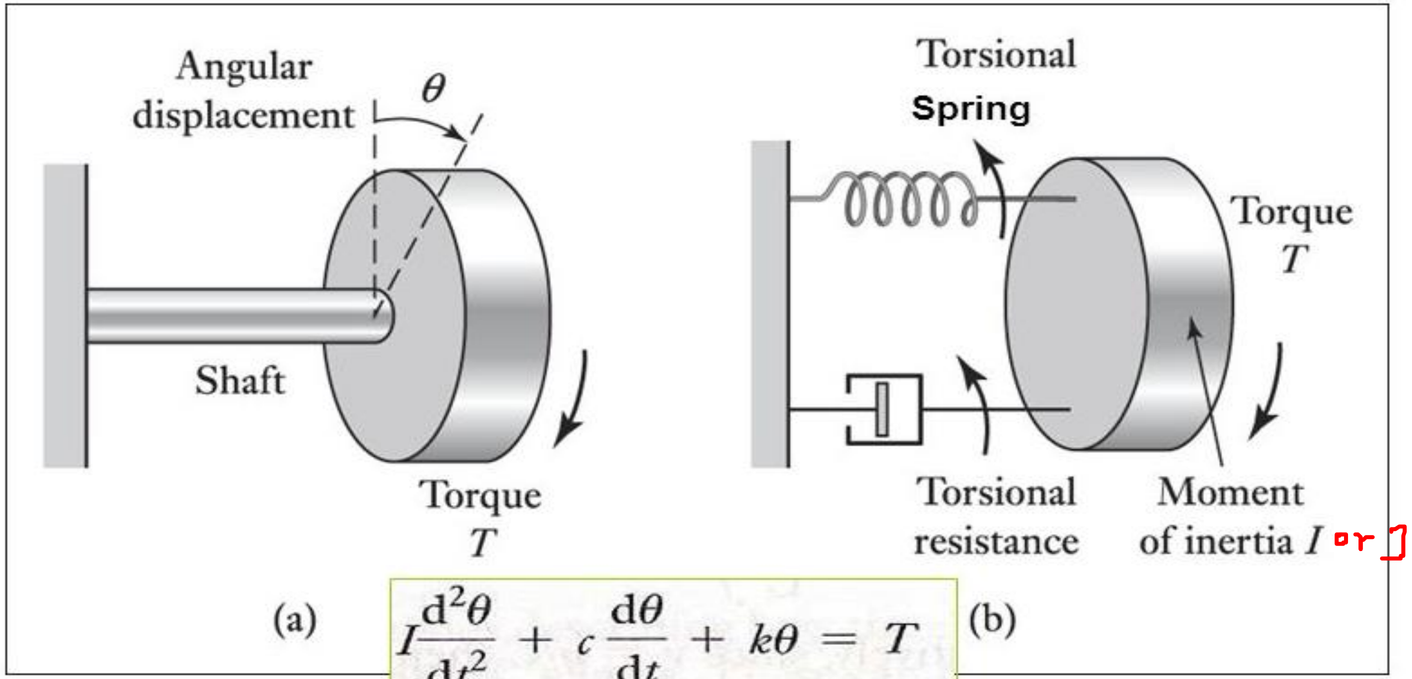


D'Alembert's Law

$\Sigma F = 0$

Differential Equations of Physical Systems
Examples of Mechanical Systems - Rotational

- Rotational : I or J as moment of inertia, c (D) or B as damping constant, T as Torque



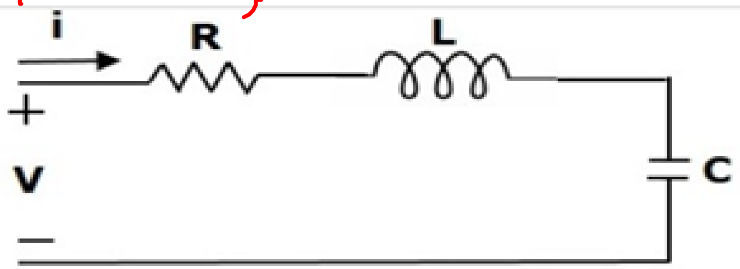
$$I\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + k\theta = T$$

Rotating a mass on the end of a shaft: (a) physical situation, (b) building block model

Differential Equations of Physical Systems

Electrical Systems

Series RLC,



Apply KVL,

Mesh equation for this circuit is

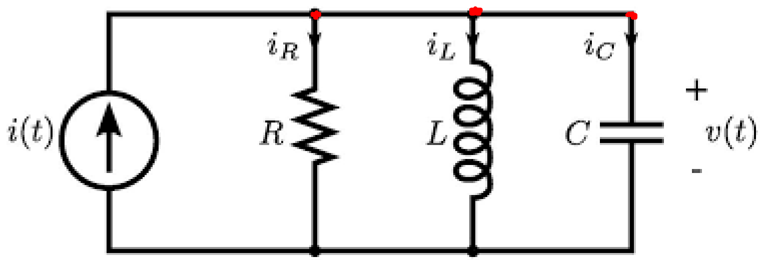
$$V = Ri + L\frac{di}{dt} + \frac{1}{c}\int i\,dt$$

Substitute, $i = \frac{dq}{dt}$ $\Rightarrow$ $q = \int i(t)\,dt$

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$\Rightarrow V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \left(\frac{1}{c}\right)q$$

parallel RLC



π = Pi

ϕ = phi

Apply KCL,

$\phi = flux\ linkage$

$$\text{i}= \frac{v}{R} + C\frac{dv}{dt} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau \text{ , v} = \frac{d\phi}{dt}$$

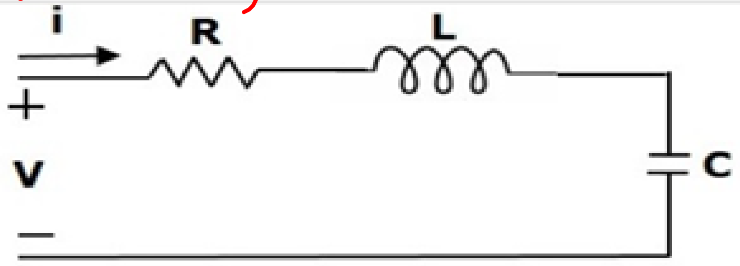
$$= \frac{1}{R}\frac{d\phi}{dt} + C\frac{d^2\phi}{dt^2} + \frac{\phi}{L}$$

$$i = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L}$$

Differential Equations of Physical Systems

Electrical Systems

Series RLC,



Apply KVL,

Mesh equation for this circuit is

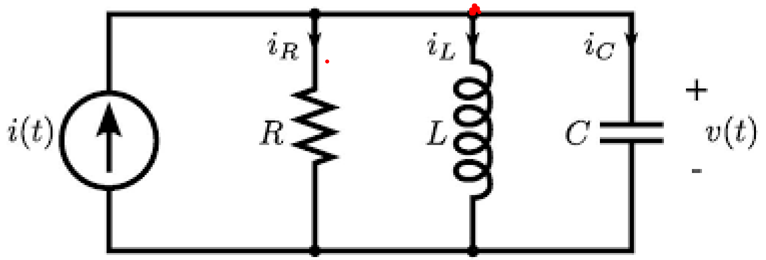
$$V = Ri + L\frac{di}{dt} + \frac{1}{c}\int i dt$$

Substitute, $i = \frac{dq}{dt}$ $\Rightarrow q = \int i(\tau) d\tau$ $\quad \frac{dv_c}{dt} = \frac{1}{C} i(t)$ $\Rightarrow i(t) = C\frac{dv_c}{dt}$

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$\Rightarrow V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \left(\frac{1}{c}\right) q$$

parallel RLC



$v_R = R\, i(t)$

$i(t) = \frac{v_R}{R}$

$\int v_L(t) = \int L\frac{di}{dt}$

$i(t) = \frac{1}{L}\int_0^t v_L(\tau) d\tau$

Apply KCL,

$i = i_R + i_L + i_C$

$\phi = flux\ linkage$

$$i = \frac{v}{R} + C\frac{dv}{dt} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau \text{ , } v = \frac{d\phi}{dt} \Rightarrow \phi = \int_{-\infty}^{t} v(\tau) d\tau$$

$$i = \frac{1}{R}\frac{d\phi}{dt} + C\frac{d^2\phi}{dt^2} + \frac{\phi}{L}$$

$$i = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L}$$

There are 2 ways :

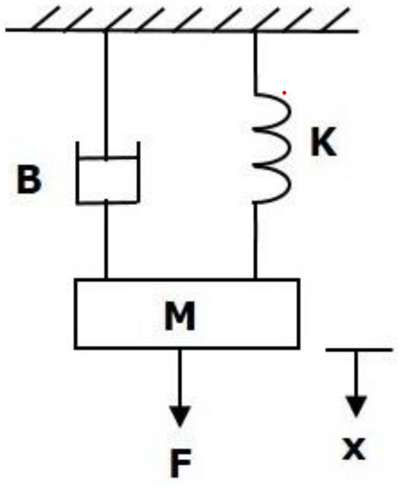
Force – Current Analogy

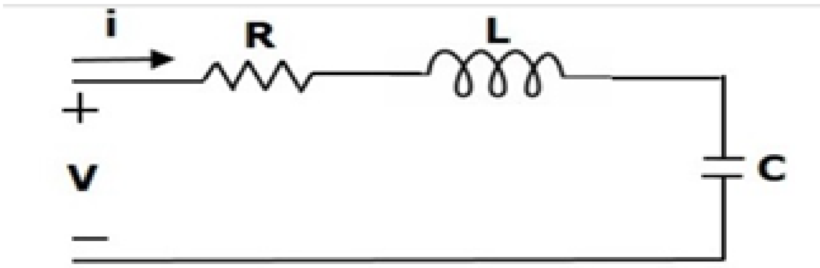
- In this case , force is analogous to current and velocity is analogous to voltage.

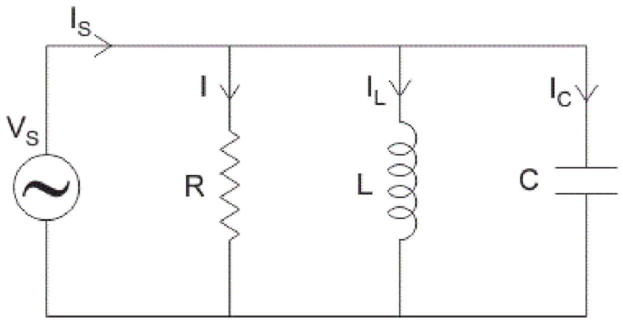
Force –Voltage Analogy

- In this case , force is analogous to voltage and velocity is analogous to current.

Converting a Translational Mechanical System to Electrical







$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \quad \text{—(1)}$$

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q \quad \text{—(2)}$$

$$i = C\frac{d^2\emptyset}{dt^2} + \frac{1}{R}\frac{d\emptyset}{dt} + \frac{\emptyset}{L}\ \text{, where } \phi = \psi \quad \text{—(3)}$$

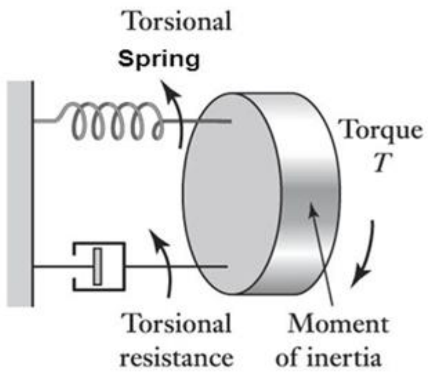
F - V (1) x (2)

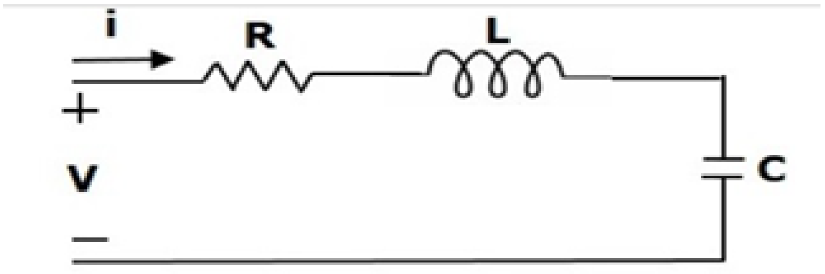
F - I → (1) x (3)

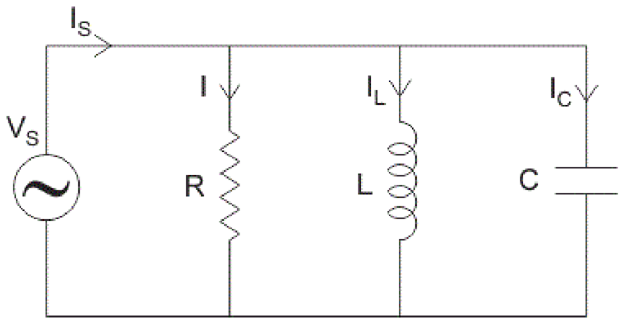
Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance $(\frac{1}{c})$
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Electrical System
Current(i)
Capacitance(C)
Reciprocal of Resistance$(\frac{1}{R})$
Reciprocal of Inductance$(\frac{1}{L})$
Magnetic Flux(ψ)
Voltage(V)

Converting a mechanical system to electrical system







$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q$$

$$i = C\frac{d^2\emptyset}{dt^2} + \frac{1}{R}\frac{d\emptyset}{dt} + \frac{\emptyset}{L}$$

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance ($\frac{1}{c}$)
Angular displacement(θ)	Charge(q)
Angular velocity(ω)	Current(i)

Electrical System
Current(i)
Capacitance(C)
Reciprocal of Resistance($\frac{1}{R}$)
Reciprocal of Inductance($\frac{1}{L}$)
Magnetic Flux(ψ)
Voltage(V)

PES UNIVERSITY ONLINE

Definition of Laplace Transform

$$F(s) = L\{f(t)\} = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

$s = \sigma + j\omega$

Sufficient conditions for existence of the Laplace transform

- $f(t)$ is piecewise continuous
- There exist M, α, t_0 such that

$$|f(t)| < M\ e^{\alpha t} \qquad for \quad t \geq t_0$$

Laplace Transform Table

$f(t)$	$\mathcal{L}[f(t)]$
unit step 1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$t \sin at$	$\frac{2as}{(s^2+a^2)^2}$
$t \cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$\frac{t^n}{n!}, n \in \mathbb{N}$	$\frac{1}{s^{n+1}}$
$e^{at} \cdot \frac{t^n}{n!}, n \in \mathbb{N}$	$\frac{1}{(s-a)^{n+1}}$

For first-order derivative:

$$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0)$$

$$= sF(s) - f(0^-)$$

For second-order derivative:

$$\mathcal{L}\{f''(t)\} = s^2\mathcal{L}\{f(t)\} - s\,f(0) - f'(0)$$

For third-order derivative:

$$\mathcal{L}\{f'''(t)\} = s^3\mathcal{L}\{f(t)\} - s^2 f(0) - s\,f'(0) - f''(0)$$

For n^th order derivative:

$$\boxed{\mathcal{L}\{f^n(t)\} = s^n\mathcal{L}\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{n-1}(0)}$$

Proof of Laplace Transform of Derivatives

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} f'(t)\,dt$$

$$L\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

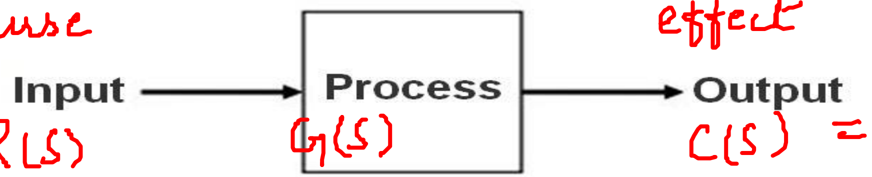
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions

- A control system consists of an output as well as an input signal. The output is related to the input through a function called **transfer function**.

excitation
cause

effect



$R(S)$ $G(S)$ $C(S) = G(s)\,R(s) \Rightarrow \frac{C(s)}{R(s)} = G(s)$

- In Laplace Transform, if the input/cause is represented by R(s) and output/effect is represented by C(s), then the transfer function will be
- $$C(S) = G(S)R(S)$$
- The **transfer function** of a control system is defined as the ratio of the Laplace transform of the output variable to Laplace transform of the input variable assuming **all initial conditions to be zero.**

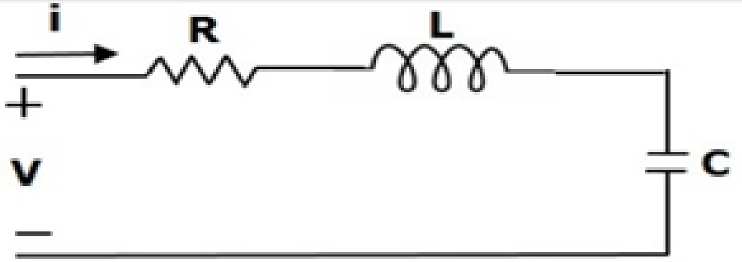
$\frac{C(s)}{R(s)}$ $f(0),\ f'(0),\ f''(0),\ \ldots$

- For any control system there exists a reference input termed as **excitation or cause** which operates through a transfer operation and produces an **effect** resulting in controlled **output** or response.

Transfer Functions Examples

- **Electrical systems:**

input | cause = V, output | effect = v_c



$\uparrow v_c \downarrow$

$$T.F = \frac{V_c(s)}{V(s)}, \quad v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$= \underbrace{\frac{1}{C}\int_{-\infty}^{0} i(\tau)d\tau}_{v_c(0) = 0} + \frac{1}{C}\int_{0}^{t} i(\tau)d\tau$$

$$V_c(s) = \frac{I(s)}{Cs}$$

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$V(s) = \left(R + Ls + \frac{1}{Cs}\right) I(s)$$

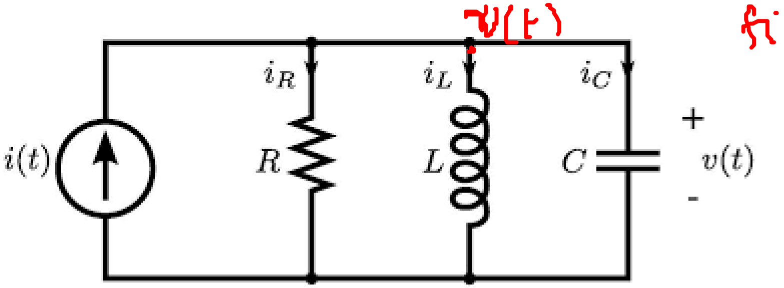
$$\frac{V_c(s)}{V(s)} = \frac{1/Cs}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1}$$

Transfer Functions Examples – H.W

- **Electrical systems:**

i/p | cause $= I(s)$, o/p | effect $= V(s)$

find T.F $\dfrac{V(s)}{I(s)}$



$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(\tau)\,d\tau$$

$$i(t) = \frac{v(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau + C\frac{dv}{dt}$$

Apply LT with zero initial conditions

$$I(s) = \frac{V(s)}{R} + \frac{V(s)}{Ls} + CsV(s)$$

$$= \left(\frac{1}{R} + \frac{1}{Ls} + Cs\right)V(s)$$

$$\Rightarrow \frac{V(s)}{I(s)} = \frac{1}{\frac{1}{R} + \frac{1}{Ls} + Cs} = \frac{RLs}{Ls + R + LCRs^2}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

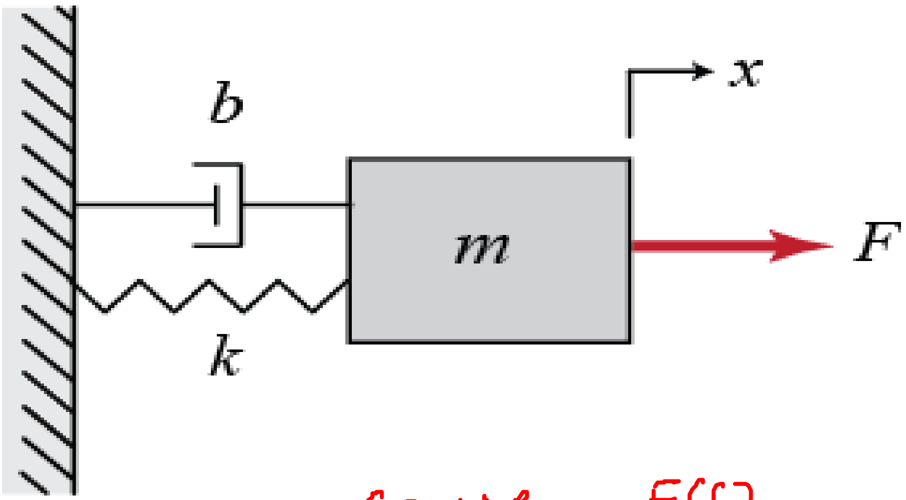
Transfer Functions Examples

energy storing elements

electrical → L & C

mechanical → K & M

- Cause – Force and Effect – displacement x



$$F = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx$$

$$= m\ddot{x} + b\dot{x} + kx$$

Apply LT with zero i.c

$$F(s) = ms^2X(s) + bsX(s) + kX(s)$$

$$= (ms^2 + bs + k)X(s)$$

Cause = $F(s)$

effect = $X(s)$

Find T.F $\frac{X(s)}{F(s)}$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} = \frac{N(s)}{D(s)} = \frac{\text{zero polynomial}}{\text{poles polynomial}}$$

roots

⇒ 2 finite poles
2 zeros at ∞

⇒ order of the system = highest degree of $D(s)$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions Examples

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

leading coefficient $= 1$

$\Rightarrow$ monic polynomial

$\deg N(s) \leq \deg D(s)$

$n \geq m$

$\Rightarrow$ strictly proper transfer function

- Cause – Force and Effect – displacement y_1 $\quad \frac{Y_1(s)}{F(s)}$

$$F(t) = M_1 \frac{d^2 y_1}{dt^2} + K_1 y_1 + b \frac{dy_1}{dt} + k_{12}(y_1 - y_2)$$

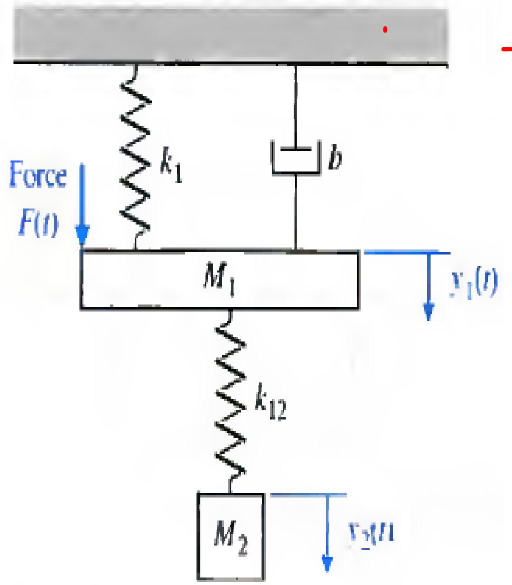
$$0 = M_2 \frac{d^2 y_2}{dt^2} + k_{12}(y_2 - y_1)$$

Apply LT with zero i.c

$$F(s) = (M_1 s^2 + bs + K_1 + K_{12}) Y_1(s) - K_{12} Y_2(s)$$

$$0 = (M_2 s^2 + K_{12}) Y_2(s) - K_{12} Y_1(s)$$

$$\frac{Y_1(s)}{F(s)}$$



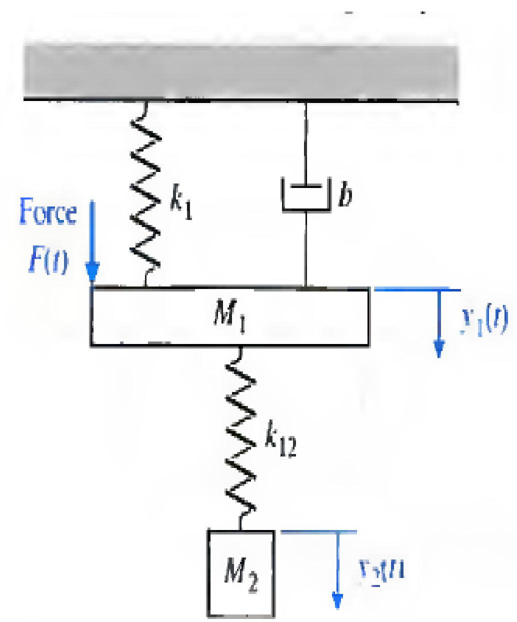
- Cause – Force and Effect – displacement y_1

$\frac{dy_1}{dt}$

$$M_1\ddot{y}_1 + k_{12}(y_1 - y_2) + b\dot{y}_1 + k_1 y_1 = F(t)$$

$$M_2\ddot{y}_2 + k_{12}(y_2 - y_1) = 0\ .$$

$\frac{d^2y_1}{dt^2} = \ddot{y}_1$



$$(M_1 s^2 + k_{12} + bs + k_1)Y_1(s) - \frac{k_{12}k_{12}Y_1(s)}{M_2 s^2 + k_{12}} = F(s)$$

$$\frac{Y_1(s)}{F(s)} = \frac{1}{(M_1 s^2 + k_{12} + bs + k_1) - \frac{k_{12}^2}{M_2 s^2 + k_{12}}}$$

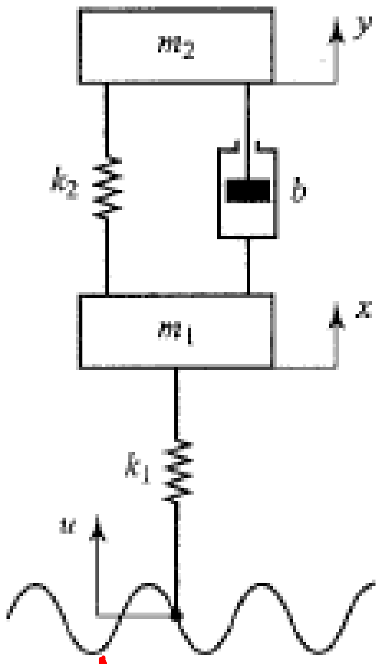
$\Rightarrow$ order of the system = 4

= no. of storage elements (k_1, M_1, k_{12}, M_2)

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions Examples – H.W

- Cause – u and Effect – displacement y , Find $\frac{Y(s)}{U(s)}$



$m_2\ddot{y} + b(\dot{y}-\dot{x}) + k_2(y-x) = 0$

$m_1\ddot{x} + k_1(x-u) + k_2(x-y) + b(\dot{x}-\dot{y}) = 0$

Apply LT with zero i.c

$(m_2s^2 + bs + k_2)Y(s) - (bs + k_2)X(s) = 0$

$(m_1s^2 + k_1 + k_2 + bs)X(s) - k_1U(s) - (k_2 + bs)Y(s) = 0$

$$\begin{pmatrix} -(bs+k_2) & m_2s^2+bs+k_2 \\ m_1s^2+k_1+k_2+bs & -(bs+k_2) \end{pmatrix}\begin{pmatrix} X(s) \\ Y(s) \end{pmatrix} = \begin{pmatrix} 0 \\ +k_1 \end{pmatrix} U(s)$$

apply cramer's rule

$$\frac{Y(s)}{U(s)} = \frac{\begin{vmatrix} -(bs+k_2) & 0 \\ (\ ) & k_1 \end{vmatrix}}{\begin{vmatrix} & \\ & \end{vmatrix}} =$$

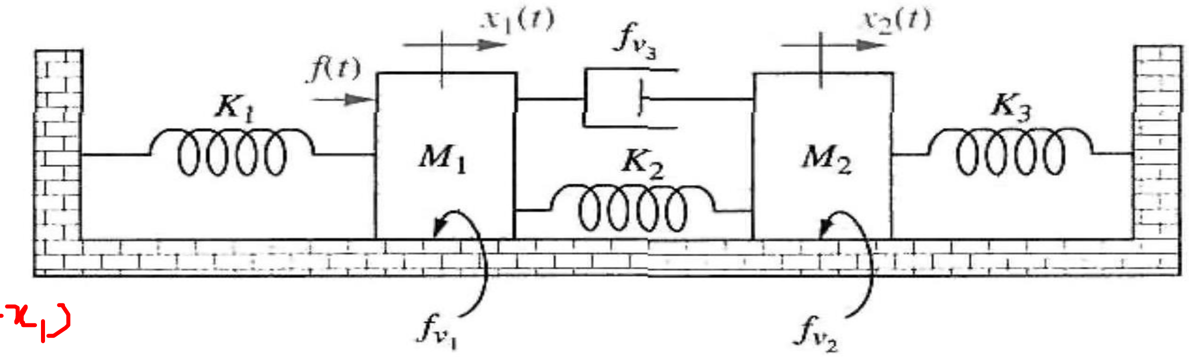
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions Examples

- Cause – Force F and Effect – displacement x_2 $\quad \frac{X_2(s)}{F(s)}$

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + K_2(x_1 - x_2) + f_{v_3}\frac{d}{dt}(x_1 - x_2) + f_{v_1}\frac{dx_1}{dt}$$

$$0 = M_2 \frac{d^2 x_2}{dt^2} + K_3 x_2 + K_2(x_2 - x_1) + f_{v_3}\frac{d}{dt}(x_2 - x_1) + f_{v_2}\frac{dx_2}{dt}$$



Apply LT with zero i.c

$$X_2(s) = \frac{\begin{vmatrix} & F(s) \\ & 0 \end{vmatrix}}{\Delta}$$

$$\left[M_1 s^2 (f_{v_1} + f_{v_3})s + (K_1 + K_2)\right] X_1(s) - (f_{v_3}s + K_2)X_2(s) = F(s)$$

$$-(f_{v_3}s + K_2)X_1(s) + \left[M_2 s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)\right] X_2(s) = 0$$

$$\Delta = \begin{vmatrix} [M_1 s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)] & -(f_{v_3}s + K_2) \\ -(f_{v_3}s + K_2) & [M_2 s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)] \end{vmatrix}$$

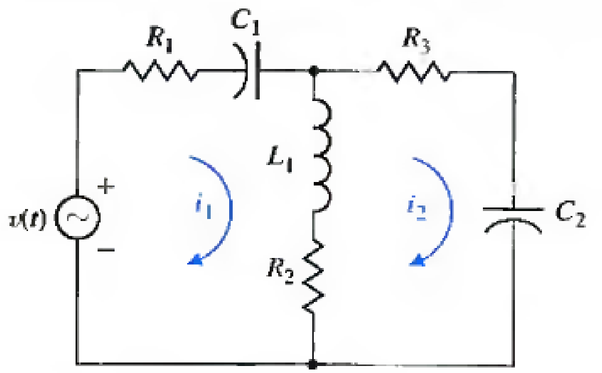
From this, the transfer function, $X_2(s)/F(s)$, is

$$\frac{X_2(s)}{F(s)} = G(s) = \frac{(f_{v_3}s + K_2)}{\Delta}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions Examples

- Cause – $\cancel{e_1}$ $v(t)$ and Effect – $\cancel{e_0}$ $i_2(t)$ Find, $\frac{I_2(s)}{V(s)}$



$$R_1 i_1 + \frac{1}{C_1}\int_{-\infty}^{t} i_1(\tau)\,d\tau + L_1 \frac{d}{dt}(i_1 - i_2) + R_2(i_1 - i_2) = v(t)$$

$$R_3 i_2 + \frac{1}{C_2}\int_{-\infty}^{t} i_2(\tau)\,d\tau + R_2(i_2 - i_1) + L_1 \frac{d}{dt}(i_2 - i_1) = 0$$

LT

$$\left(R_1 + \frac{1}{C_1 s} + L_1 s + R_2\right) I_1(s) - (L_1 s + R_2)\, I_2(s) = V(s)$$

$$-(R_2 + L_1 s)\, I_1(s) + \left(R_3 + \frac{1}{C_2 s} + R_2 + L_1 s\right) I_2(s) = 0$$

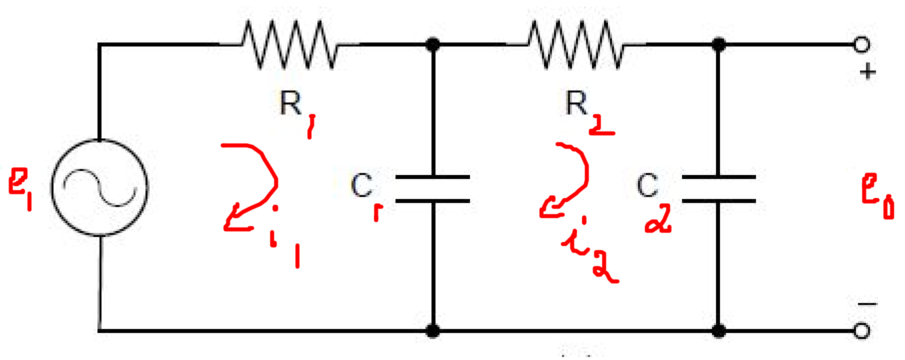
$$I_2(s) = \frac{\begin{vmatrix} & V(s) \\ & 0 \end{vmatrix}}{\Delta}$$

$$\Delta = \begin{vmatrix} & \end{vmatrix}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions Examples

- Cause – e_1 and Effect – e_0 , Find $\frac{E_0(s)}{E_1(s)}$



$$\frac{1}{C_1}\int (i_1 - i_2)\,dt + R_1 i_1 = e_i$$

$$\frac{1}{C_1}\int (i_2 - i_1)\,dt + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt = 0$$

$$\frac{1}{C_2}\int i_2\,dt = e_o$$

$$\frac{1}{C_1 s}[I_1(s) - I_2(s)] + R_1 I_1(s) = E_i(s)$$

$$\frac{1}{C_1 s}[I_2(s) - I_1(s)] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0$$

$$\frac{1}{C_2 s} I_2(s) = E_o(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{(R_1C_1 s + 1)(R_2C_2 s + 1) + R_1C_2 s}$$

$$= \frac{1}{R_1C_1R_2C_2 s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s + 1}$$

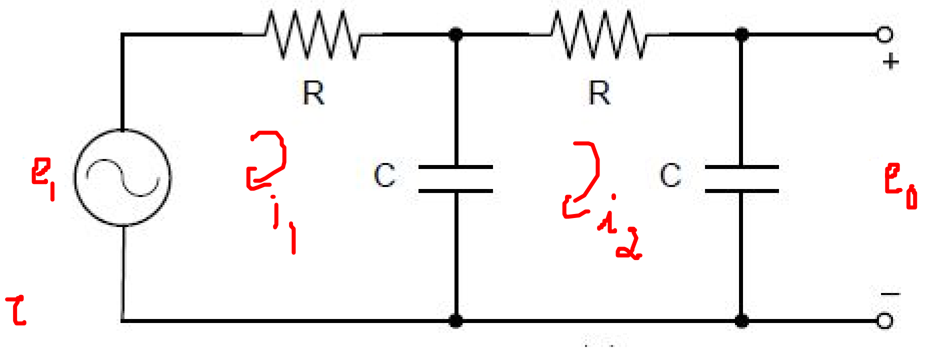
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions Examples

- Cause – e_1 and Effect – e_0

$$e_1 = R i_1 + \frac{1}{C}\int_{-\infty}^{t}(i_1 - i_2)\,d\tau$$

$$0 = R i_2 + \underbrace{\frac{1}{C}\int_{-\infty}^{t} i_2(\tau)\,d\tau}_{e_0} + \frac{1}{C}\int_{-\infty}^{t}(i_2 - i_1)\,d\tau$$



$$e_0(s) = \frac{I_2(s)}{Cs}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Analogous System Examples

F-V

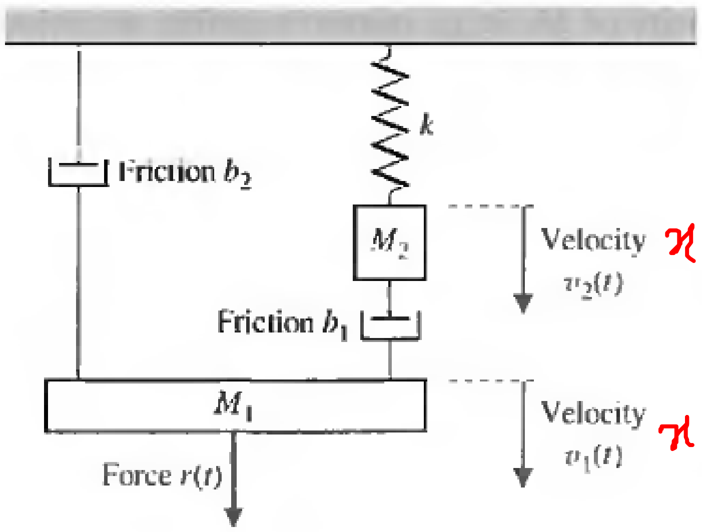
$M = L$, $b = R$, $k = \frac{1}{C}$

F-I, $M = C$, $b = \frac{1}{R}$, $k = \frac{1}{L}$

$$r(t) = M_1 \ddot{x}_1 + b_2 \frac{dx_1}{dt} + b_1 \frac{d(x_1 - x_2)}{dt}$$

$$= M_1 \frac{dv_1}{dt} + b_2 v_1 + b_1 (v_1 - v_2)$$

$$0 = M_2 \ddot{x}_2 + b_1(\dot{x}_2 - \dot{x}_1) + k x_2$$



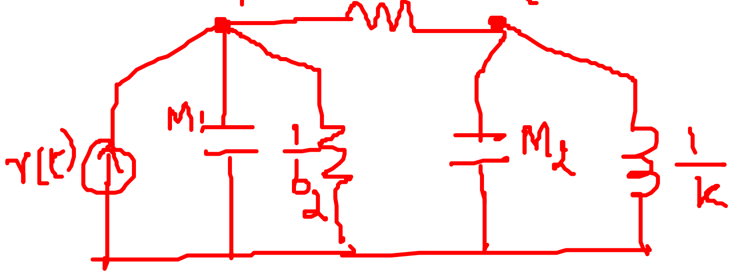
$x_2 \Rightarrow i_2 \quad v_2$

$x_1 \Rightarrow i_1 \quad v_1$

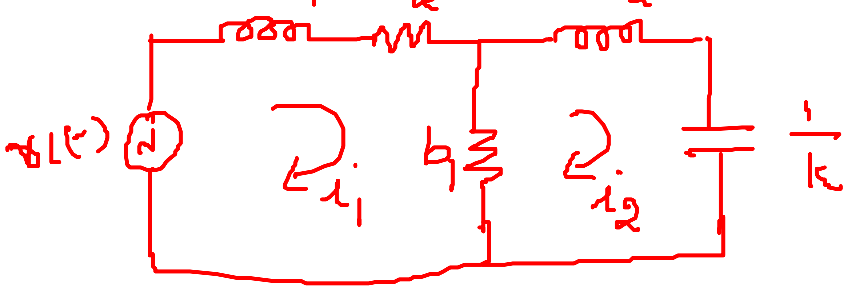
$C_1 = M_1$, $C_2 = M_2$, $L = 1/k$, $R_1 = 1/b_1$, $R_2 = 1/b_2$.

$\Downarrow$

F-I



F-V

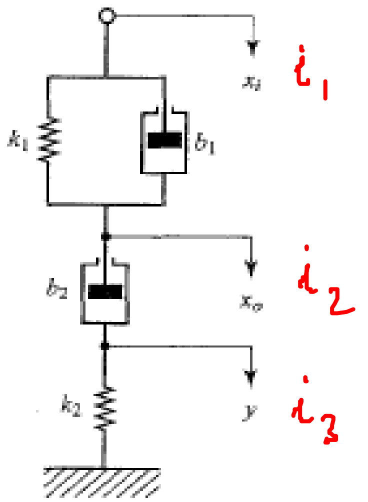


MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Analogous System Examples

$$b_1(\dot{x}_i - \dot{x}_o) + k_1(x_i - x_o) = b_2(\dot{x}_o - \dot{y})$$

$$b_2(\dot{x}_o - \dot{y}) = k_2 y$$

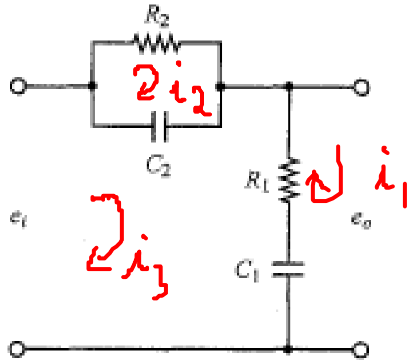


$$b_1[sX_i(s) - sX_o(s)] + k_1[X_i(s) - X_o(s)] = b_2[sX_o(s) - sY(s)]$$

$$b_2[sX_o(s) - sY(s)] = k_2 Y(s)$$

$$b_1[sX_i(s) - sX_o(s)] + k_1[X_i(s) - X_o(s)] = b_2 sX_o(s) - b_2 s\frac{b_2 sX_o(s)}{b_2 s + k_2}$$

$$(b_1 s + k_1)X_i(s) = \left(b_1 s + k_1 + b_2 s - b_2 s\frac{b_2 s}{b_2 s + k_2}\right)X_o(s)$$

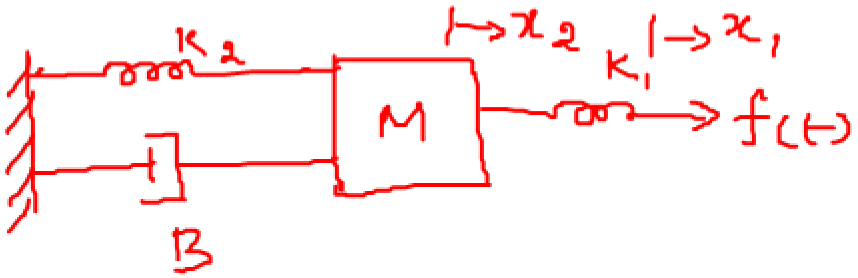


$$\frac{X_o(s)}{X_i(s)} = \frac{\left(\frac{b_1}{k_1}s + 1\right)\left(\frac{b_2}{k_2}s + 1\right)}{\left(\frac{b_1}{k_1}s + 1\right)\left(\frac{b_2}{k_2}s + 1\right) + \frac{b_2}{k_1}s}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Analogous System Examples

1)

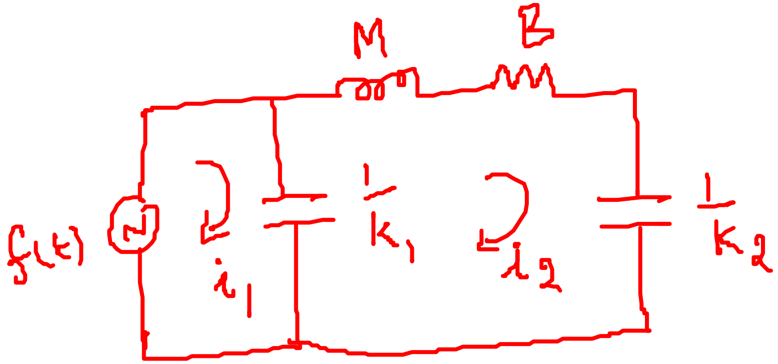


$$0 = M\frac{d^2x_2}{dt^2} + K_2x_2 + B\frac{dx_2}{dt} + \underbrace{K_1(x_2 - x_1)}$$

$$f(t) = K_1(x_1 - x_2)$$

f - V

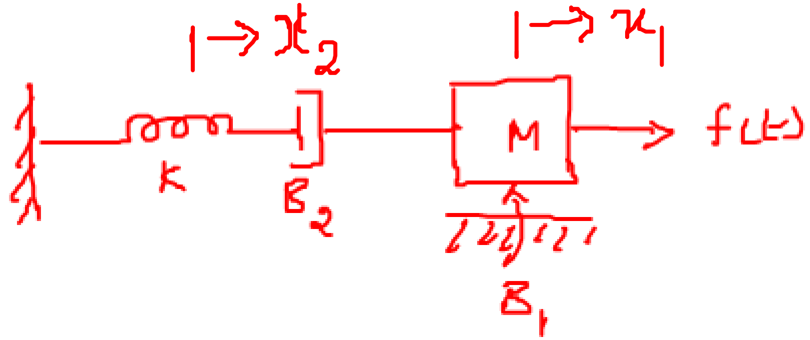
$M = L$, $B = R$, $K = 1/C$



f - I $\Rightarrow$ $M = C$, $B = \frac{1}{R}$, $K = \frac{1}{L}$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS
Transfer Functions Examples

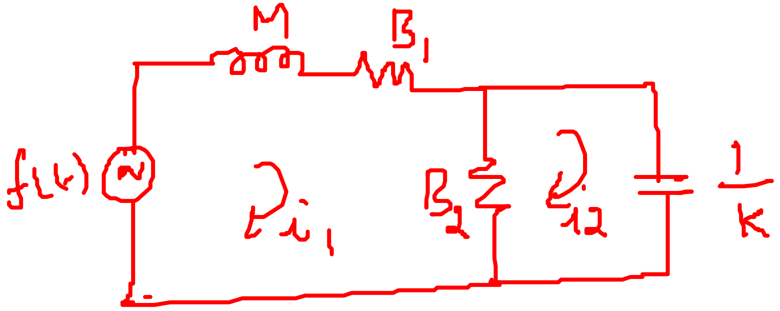
2)



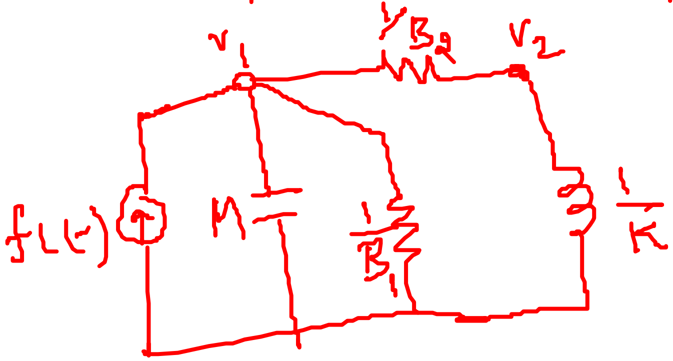
$$f(t) = M\frac{d^2x_1}{dt^2} + B_2\frac{d(x_1-x_2)}{dt} + B_1\frac{dx_1}{dt}$$

$$0 = Kx_2 + B_2\frac{d(x_2-x_1)}{dt}$$

$f-V \Rightarrow M = L,\ B = R,\ K = \frac{1}{C}$



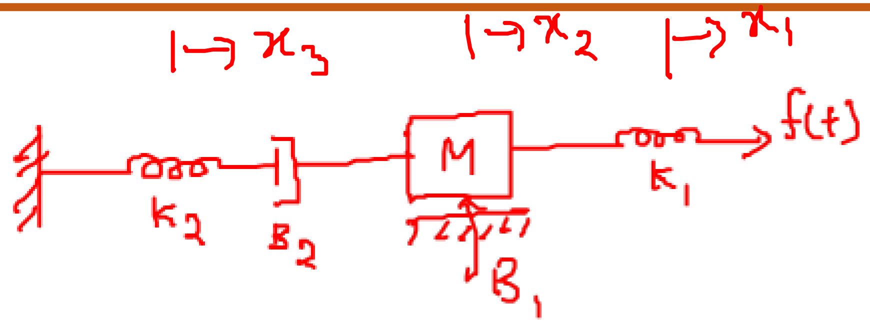
$f-I \Rightarrow M = C,\ B = \frac{1}{R},\ K = \frac{1}{L}$



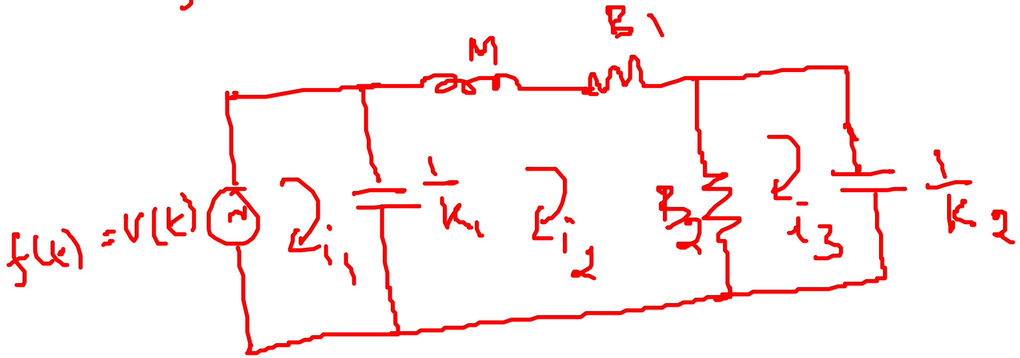
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Analogous System Examples

3)



f - V



$$f(t) = K_1(x_1 - x_2)$$

$$0 = M\frac{d^2x_2}{dt^2} + B_1\frac{dx_2}{dt} + B_2\frac{d(x_2 - x_3)}{dt} + K_1(x_2 - x_1)$$

f - I ~ H-W

$$0 = K_2x_3 + B_2\frac{d(x_3 - x_2)}{dt}$$

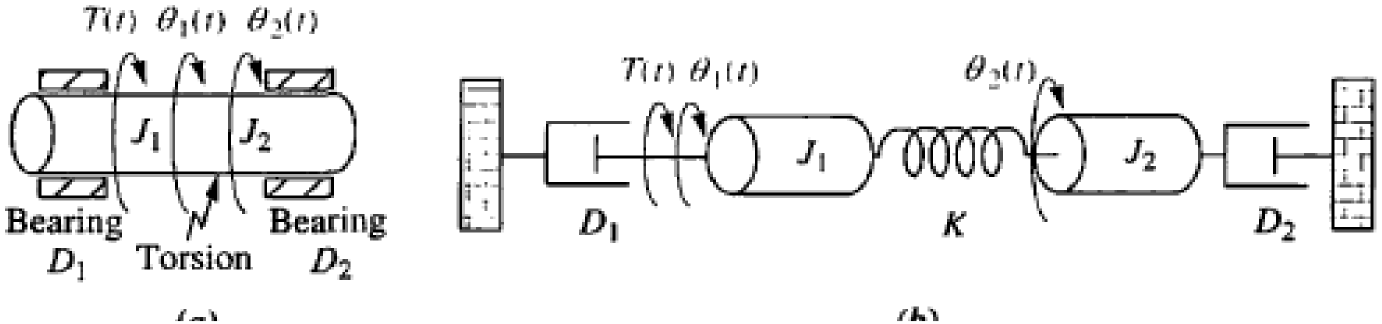
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Function Examples

$$\begin{bmatrix} J_1 s^2 + D_1 s + K & -K \\ -K & J_2 s^2 + D_2 s + K \end{bmatrix}\begin{pmatrix}\theta_1 \\ \theta_2\end{pmatrix} = \begin{pmatrix}T(s) \\ 0\end{pmatrix}$$

- Cause – T and Effect – θ_2

Find the transfer function, $\theta_2(s)/T(s)$, for the rotational system :



$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + D_1 \frac{d\theta_1}{dt} + K(\theta_1 - \theta_2)$$

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + D_2 \frac{d\theta_2}{dt} + K(\theta_2 - \theta_1)$$

$$(J_1 s^2 + D_1 s + K)\theta_1(s) \qquad - K\theta_2(s) = T(s)$$

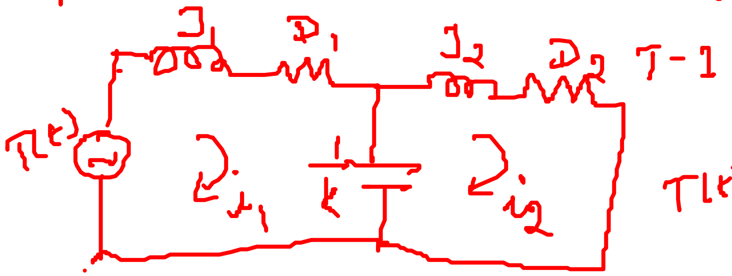
$$-K\theta_1(s) + (J_2 s^2 + D_2 s + K)\theta_2(s) = 0$$

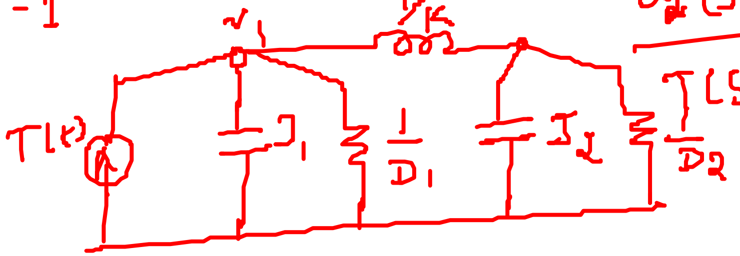
$$\theta_2(s) = \frac{\begin{bmatrix} J_1 s^2 + D_1 s + K & T(s) \\ -K & 0 \end{bmatrix}}{\Delta}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{K}{\Delta}$$

T – V

T – I



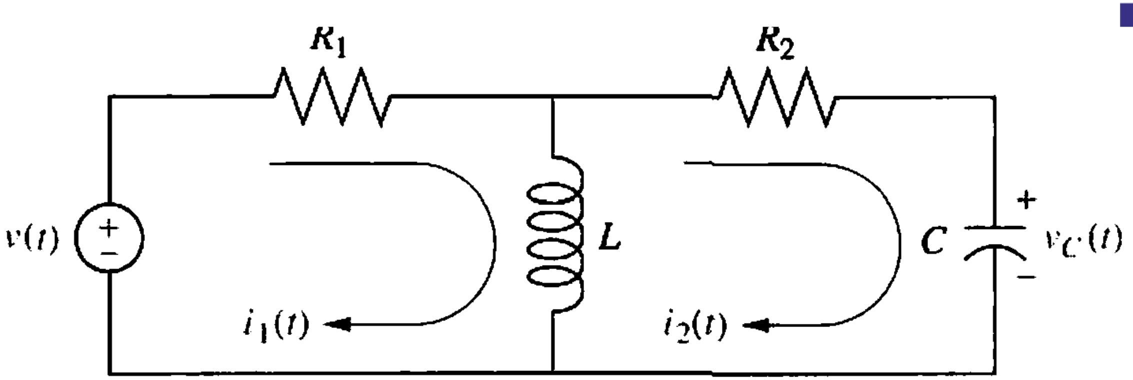


$$\frac{\theta_2(s)}{T(s)} = \frac{K}{(J_1 s^2 + D_1 s + K)(J_2 s^2 + D_2 s + K) - K^2}$$

$$\Delta = \begin{vmatrix} (J_1 s^2 + D_1 s + K) & -K \\ -K & (J_2 s^2 + D_2 s + K) \end{vmatrix}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS
Transfer Functions Examples

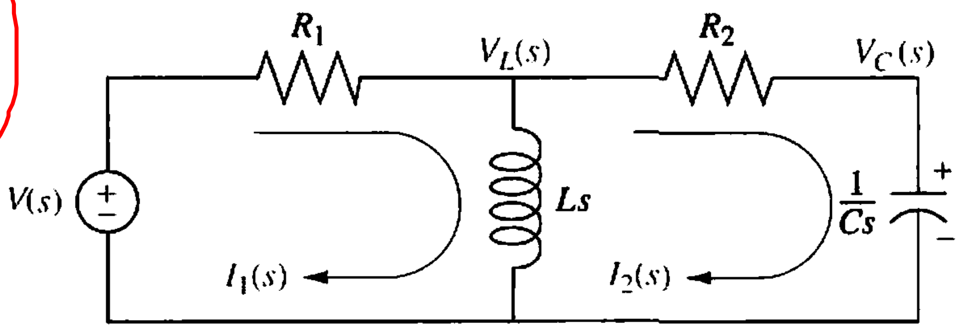
- Cause –v(t) and Effect – $i_2(t)$

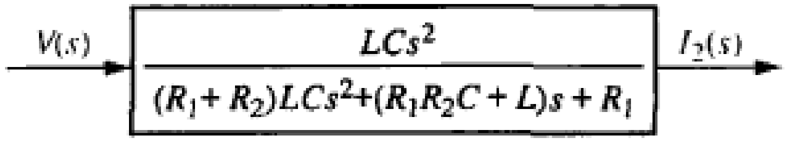


$$R_1 I_1(s) + LsI_1(s) - LsI_2(s) = V(s)$$

$$LsI_2(s) + R_2 I_2(s) + \frac{1}{Cs} I_2(s) - LsI_1(s) = 0$$

$$\begin{bmatrix} R_1 + Ls & -Ls \\ -Ls & Ls + R_2 + \frac{1}{Cs} \end{bmatrix} \begin{pmatrix} I_1(s) \\ I_2(s) \end{pmatrix} = \begin{pmatrix} V(s) \\ 0 \end{pmatrix}$$





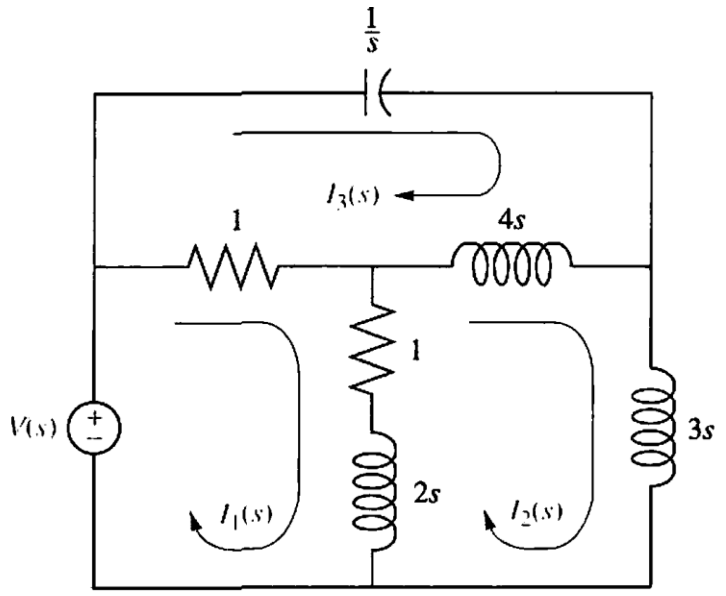
Transfer Functions Examples – H.W

- Cause –v(t) and Effect – $i_2(t)$

$$+(2s+2)I_1(s) - (2s+1)I_2(s) - I_3(s) = V(s)$$

$$-(2s+1)I_1(s) + (9s+1)I_2(s) - 4sI_3(s) = 0$$

$$-I_1(s) - 4sI_2(s) + \left(4s+1+\tfrac{1}{s}\right)I_3(s) = 0$$

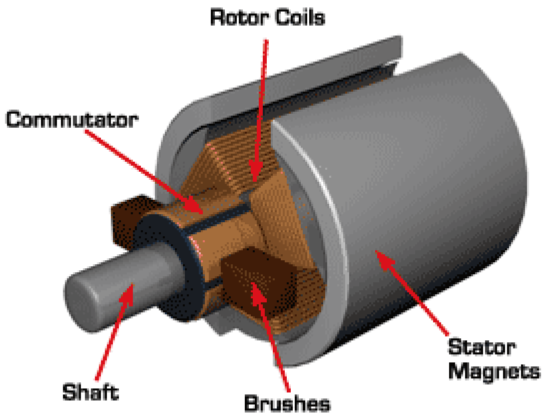


MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Electromechanical Systems

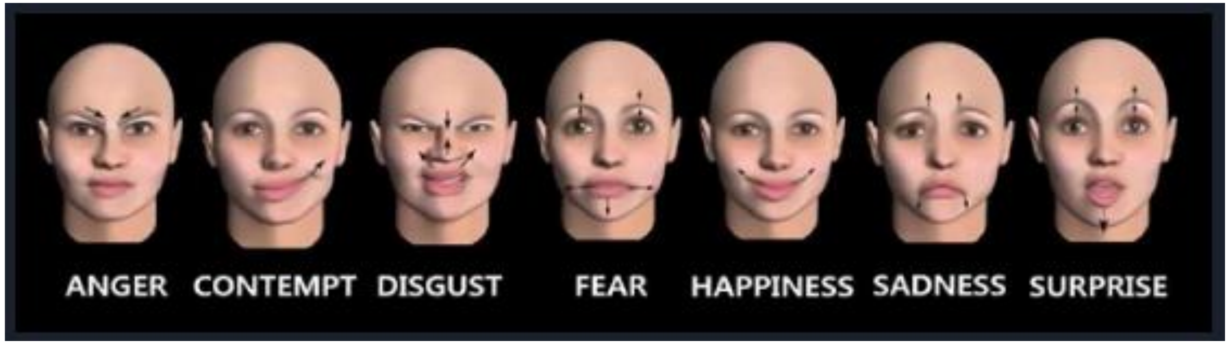
Karpagavalli S.
Department of Electronics and Communication Engineering

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS
Electromechanical System – DC Motor

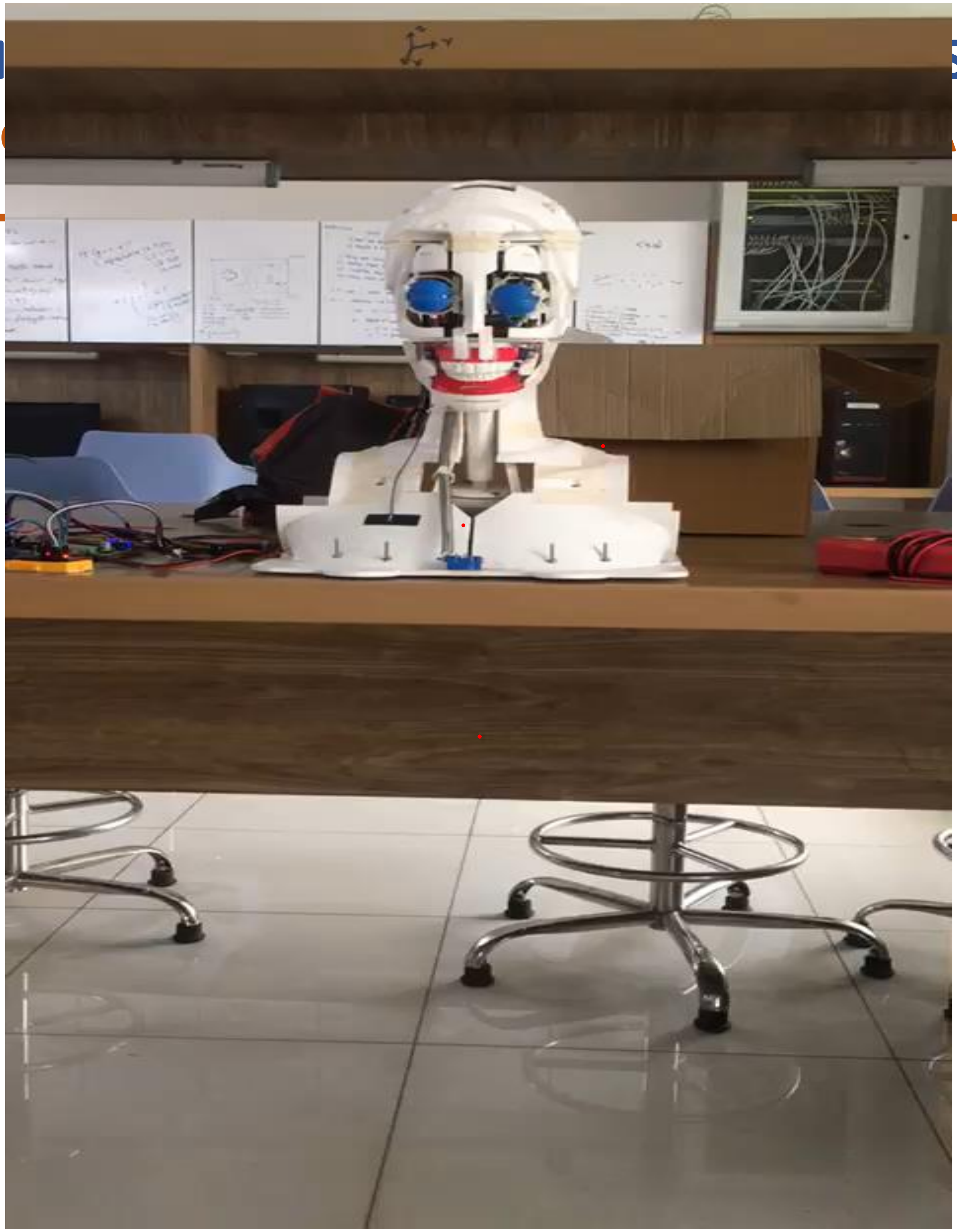


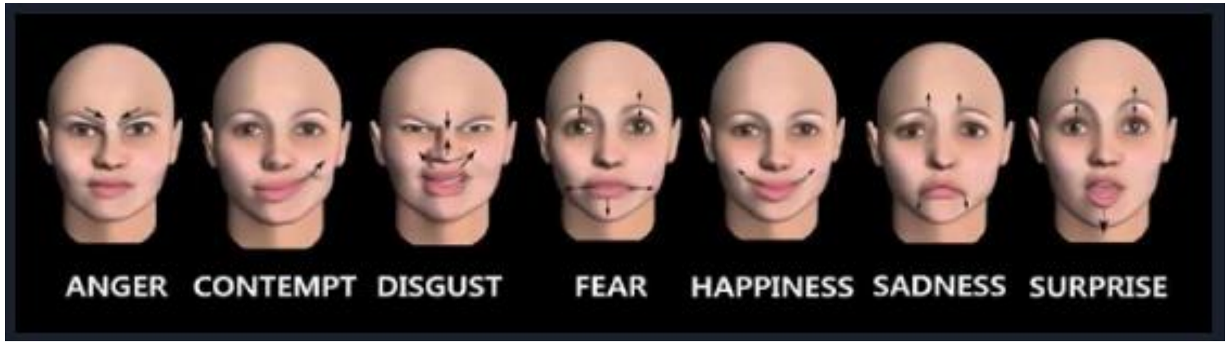
Principle of operation: whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force. The direction of this force is given by Fleming’s left hand rule

Electromechanical System – DC Motor Applications

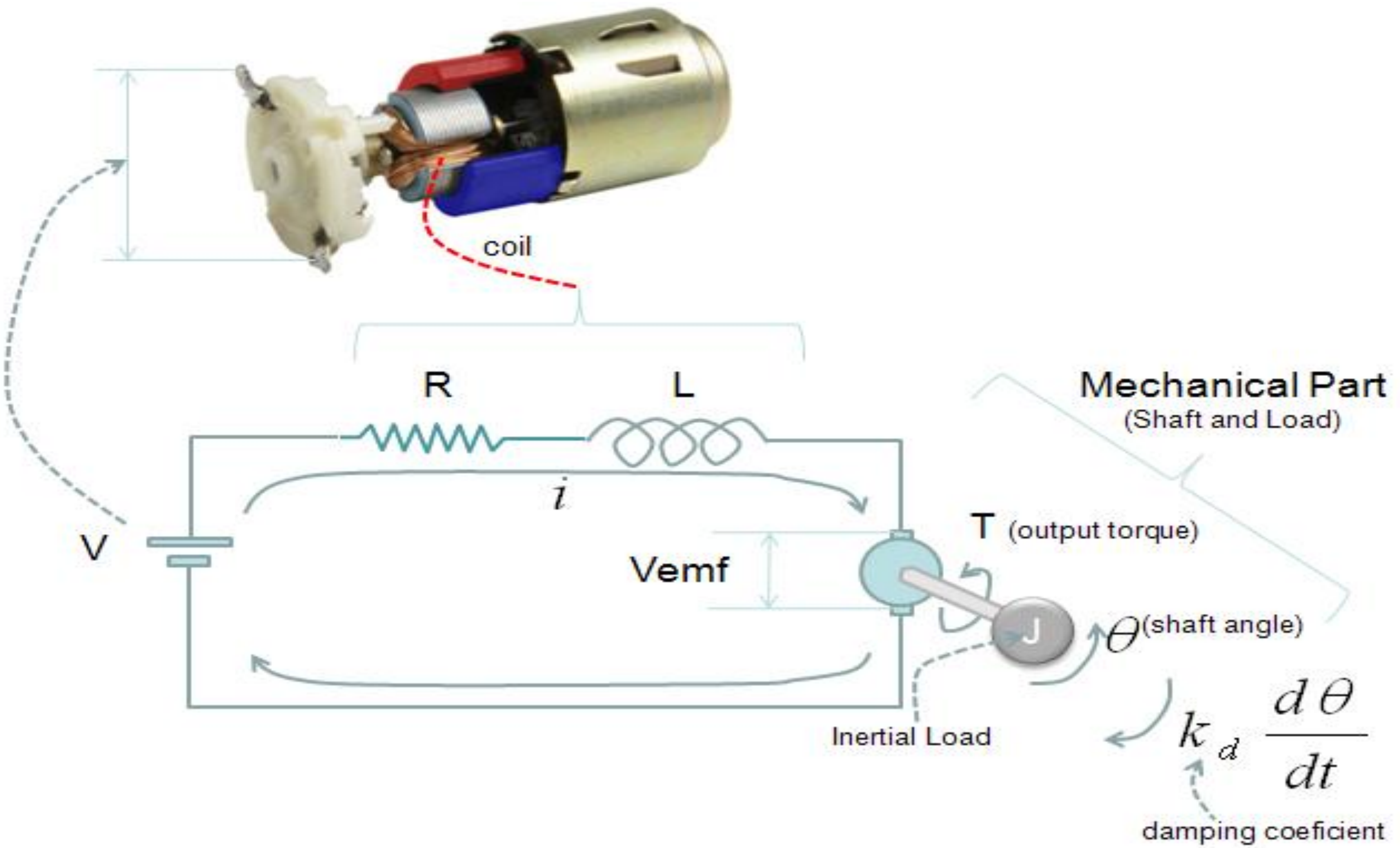


Stree : Humanoid Robot by Gopalakrishna et.al.,



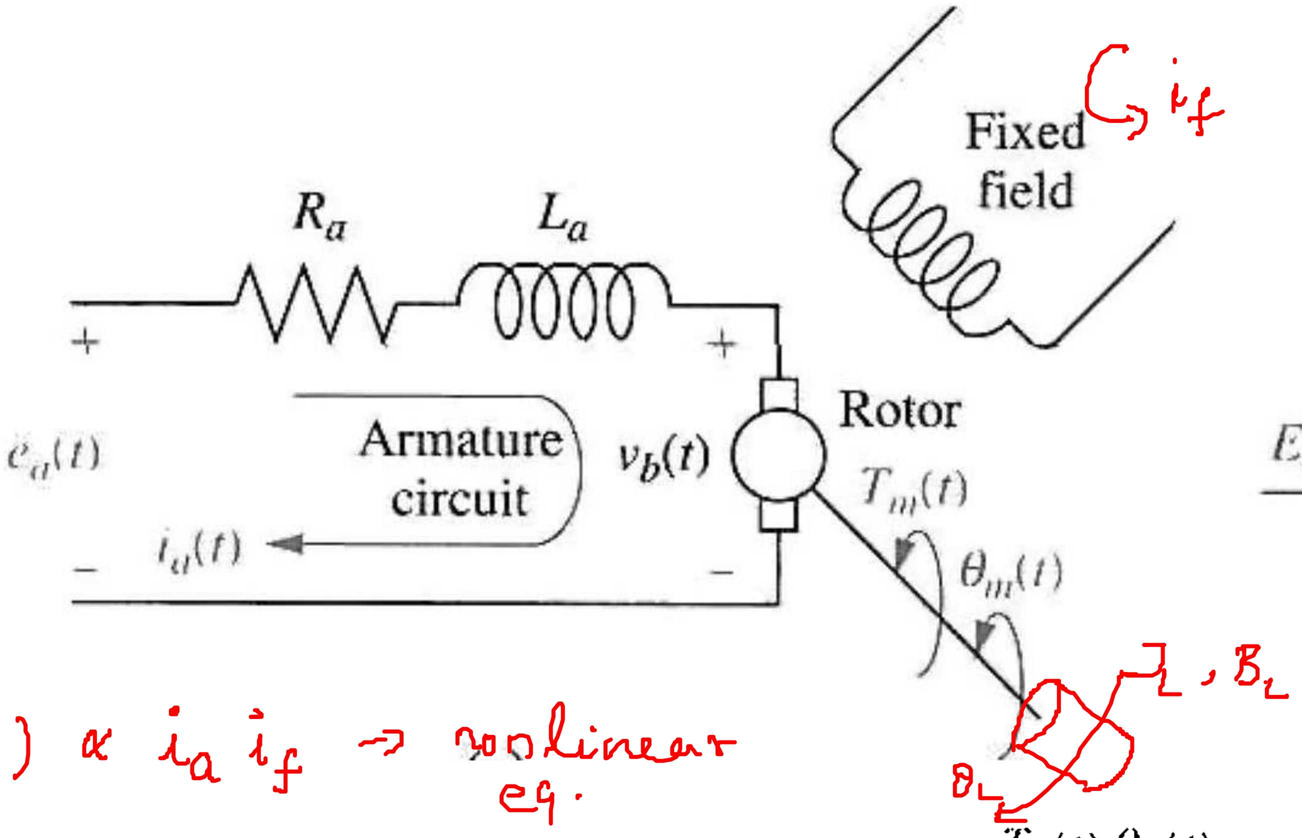

ANGER
CONTEMPT
DISGUST
FEAR
HAPPINESS
SADNESS
SURPRISE

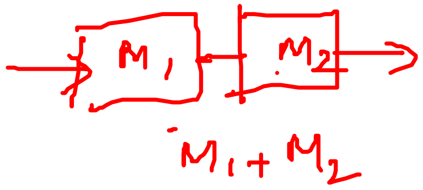
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS
Electromechanical System – DC Motor

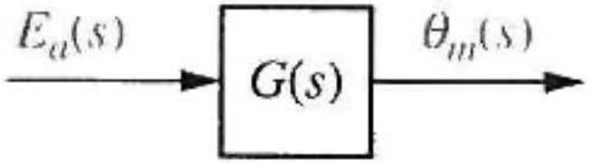


MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Electromechanical System – DC Motor



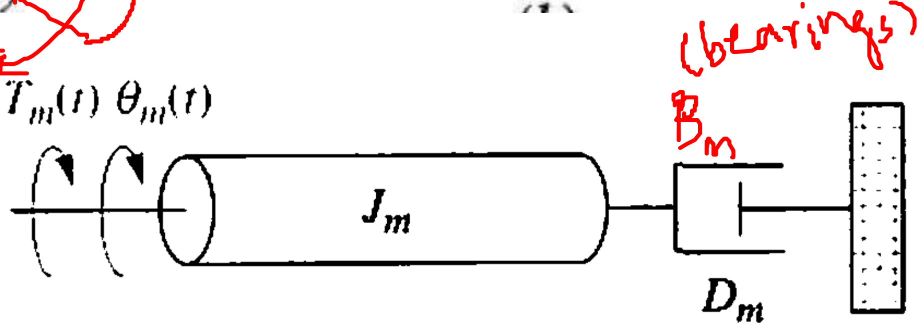




$T(t) \propto i_a i_f \rightarrow$ nonlinear eq.

$\rightarrow$ fix i_a, $i_f \uparrow\downarrow$ $\rightarrow$ field controlled

$\rightarrow$ fix i_f, $i_a \uparrow\downarrow$ $\rightarrow$ armature controlled



Suppose i_f is constant, then

$T(t) = K_1 i_a(t)$ → armature controlled dc motor

Back emf $v_b = K_2 \frac{d\theta}{dt}$

Applying KVL,

$$e_a(t) = R_a i_a + L_a \frac{di_a}{dt} + v_b$$

$$T(t) = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + T_L(t)$$

$$T_L(t) = J_L \frac{d^2\theta_L}{dt^2} + B_L \frac{d\theta_L}{dt} \quad , \quad \because \theta_m = \theta_L = \theta$$

To find T.F , i/p = E_a , o/p = θ

Applying LT,

$$E_a(s) = L_a s I_a(s) + R_a I_a(s) + K_2 s\theta(s)$$

$$= (L_a s + R_a) I_a(s) + K_2 s \theta(s)$$

$$T(s) = J_m s^2 \theta(s) + B_m s \theta(s) + J_L s^2 \theta(s) + B_L s \theta(s)$$

$$= \left[(J_m + J_L) s^2 + (B_m + B_L) s \right] \theta(s)$$

WKT,

$$T(s) = K_1 I_a(s)$$

$$K_1 I_a(s) = \left[(J_m + J_L) s^2 + (B_m + B_L) s \right] \theta(s)$$

order of the s/m = 3

Sub $I_a(s)$ in $E_a(s)$ $\Rightarrow$ $$\frac{\theta(s)}{E_a(s)} = \frac{K_1}{s\left[(L_a s + R_a)\left((J_m + J_L)s + (B_m + B_L) \right) + K_1 K_2 \right]}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Electromechanical System – DC Motor

$$\frac{\theta(s)}{E_a(s)} = \frac{K_1}{s\left\{(L_a s + R_a)\left[(J_m + J_L)s + (B_m + B_L)\right] + K_1 K_2\right\}} \approx \frac{K_1}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

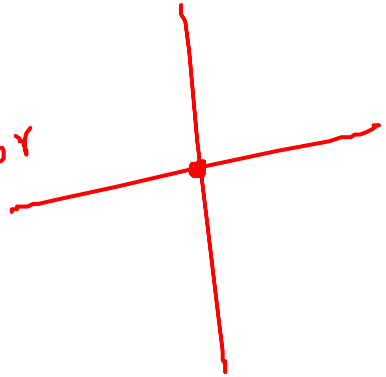
$\downarrow$

poles, $s = 0, -\zeta\omega_n \pm \sqrt{}$

$\Rightarrow$ 3rd order system (L_a, $J_m + J_L$)

$\Rightarrow$ armature controlled DC motor acts like integrator because there is a pole at the origin

$\Rightarrow \frac{1}{s} \Rightarrow$ integrator



MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Electromechanical System – DC Motor

$$I_a(s) = \frac{E_a(s) - K_2 s\,\theta(s)}{L_a s + R_a}$$

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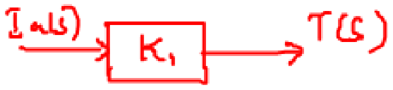
Formation of Block Diagram:

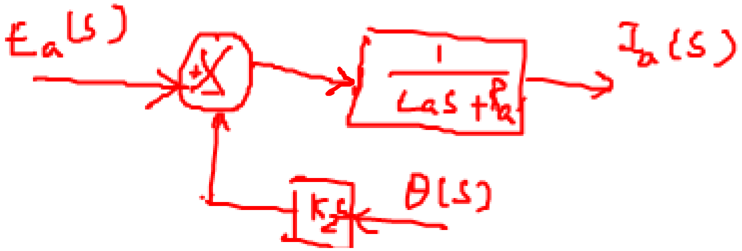
$$E_a(s) = (L_a s + R_a) I_a(s) + K_2 s\,\theta(s)$$

O/P i/p

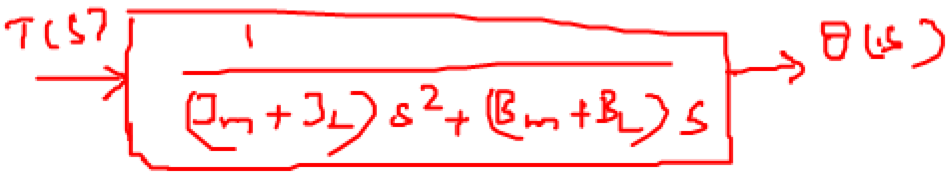
$$T(s) = K_1 I_a(s)$$

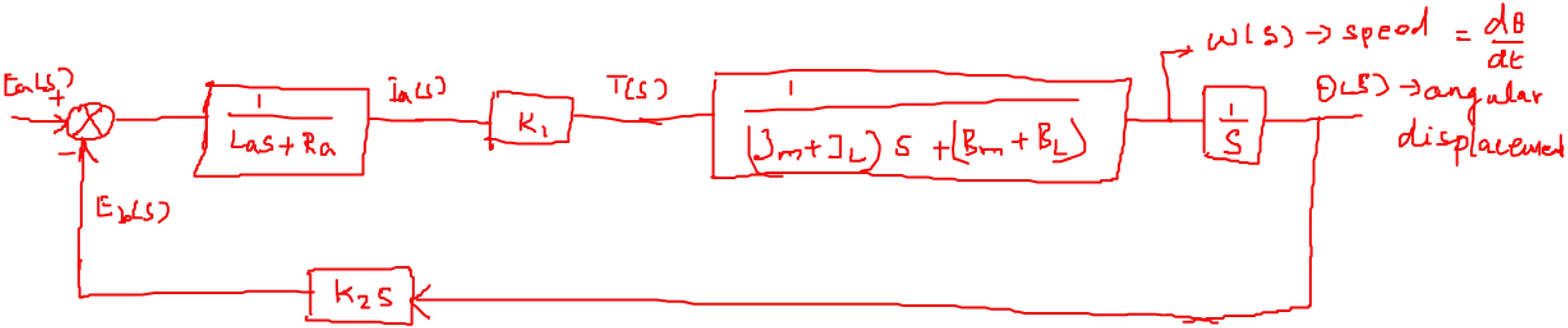
$$E_a(s) - K_2 s\,\theta(s) = I_a(s)$$





$$T(s) = \left[(J_m + J_L)s^2 + (B_m + B_L)s\right]\theta(s)$$

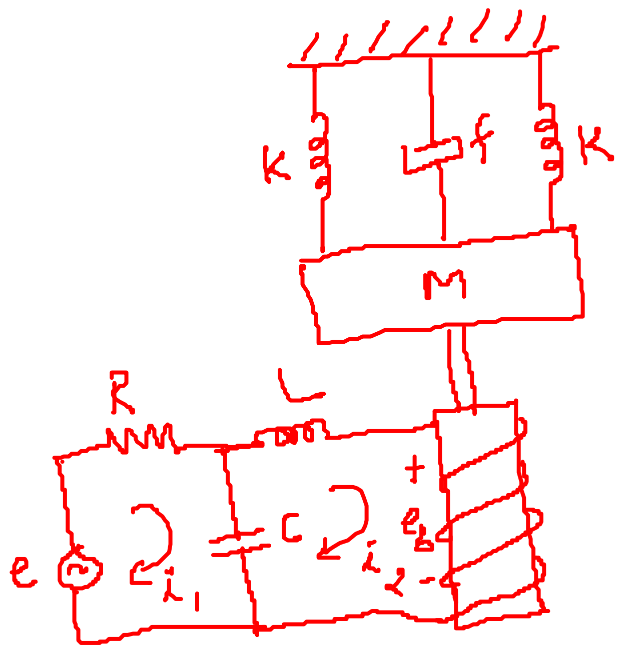




MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Electromechanical System – Example

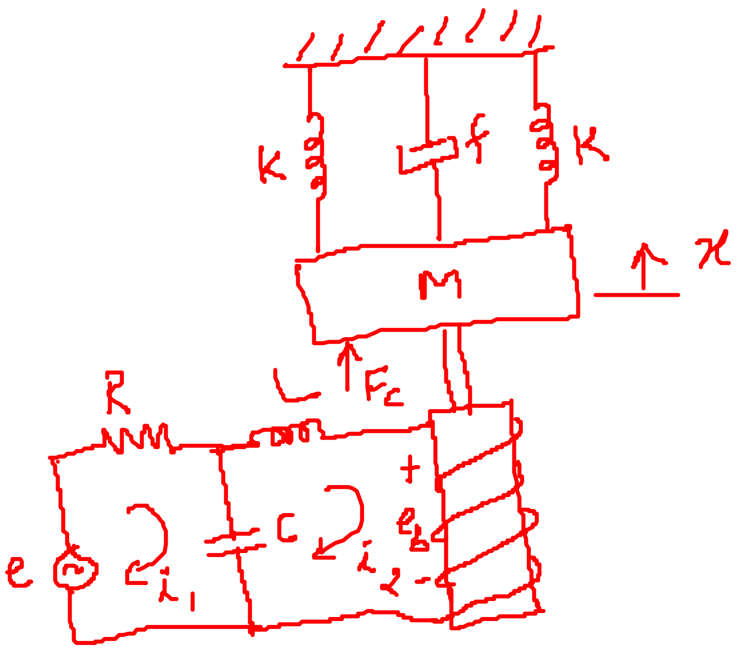
Given, $e_b = k_1 \frac{dx}{dt}$, $F_c = k_2 i_2$ on mass M



MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Electromechanical System – Example

Given, $e_b = k_1 \frac{dx}{dt}$, $F_c = K_2 i_2$ on mass M



O/P $= X(s)$, i/P $= E(s)$, T.F $= \frac{X(s)}{E(s)}$

$$e(t) = R i_1(t) + \frac{1}{C}\int_{-\infty}^{t} (i_1(\tau) - i_2(\tau))\, d\tau$$

$$0 = L\frac{di_2}{dt} + e_b + \frac{1}{C}\int_{-\infty}^{t} (i_2(\tau) - i_1(\tau))\, d\tau$$

$$F_c = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + 2Kx \Rightarrow K_2 i_2 =$$

Applying LT, $E(s) = R I_1(s) + \frac{1}{Cs}\left[I_1(s) - I_2(s)\right]$

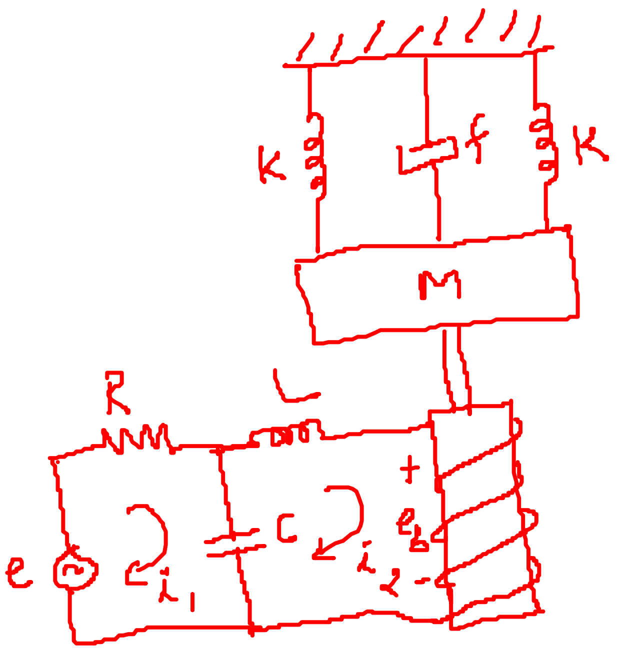
$$0 = Ls I_2(s) + K_1 s X(s) + \frac{1}{Cs}(I_2(s) - I_1(s))$$ $\Rightarrow$ find $I_1(s)$

sub $I_1(s)$ in $E(s)$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Electromechanical System – Example

Given, $e_b = k_1 \frac{dx}{dt}$, $F_c = k_2 i_2$ on mass M



$$k_2 I_2(s) = Ms^2X(s) + fsX(s) + 2kX(s)$$

$$I_2(s) = \frac{(Ms^2 + fs + 2K)}{K_2} X(s)$$

$$E(s) = (RLCs^2 + Ls + R)\frac{(Ms^2 + fs + 2k)}{K_2} X(s)$$

$$+ (k_1RCs^2 + k_1s) X(s)$$

$$\frac{X(s)}{E(s)} = \frac{K_2}{(RLCs^2 + Ls + R)(Ms^2 + fs + 2k) + K_2(s^2k_1RC + k_1s)}$$

→ order of the system = 4

= no. of storage elements (L, C, M, K)

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Block Diagram Reduction Techniques

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There are major two ways of obtaining a transfer function for the control system.

- **Block Diagram Reduction**
- **Signal Flow Graphs**

} → Theory

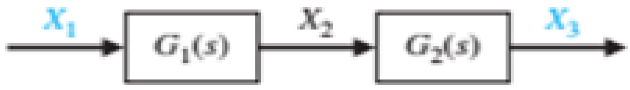
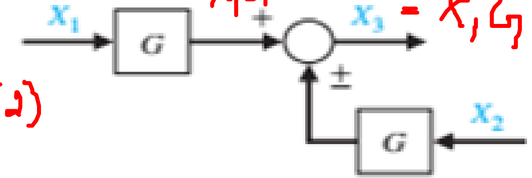
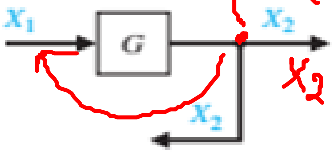
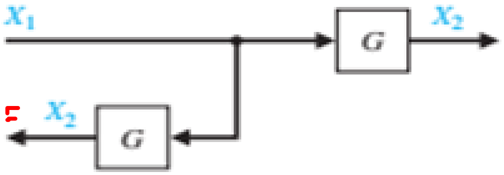
- State space model → Project

Block Diagram Reduction Technique:

- It is not convenient to derive a complete transfer function for a complex control system.
- Therefore the transfer function of each element of a control system is represented by a block diagram.
- Block diagram reduction techniques are applied to obtain the desired transfer function.
- Block diagram gives a pictorial representation of a control system.

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Block Diagram

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		

$X_2 = G_1 X_1$

$X_3 = G_2 X_2$

$X_1 \pm X_2$ $= G_2 G_1 X_1$

$X_3 = G(X_1 \pm X_2)$

$X_1 G$

$X_3 = X_1 G \pm G X_2$

pick-off or tap-off

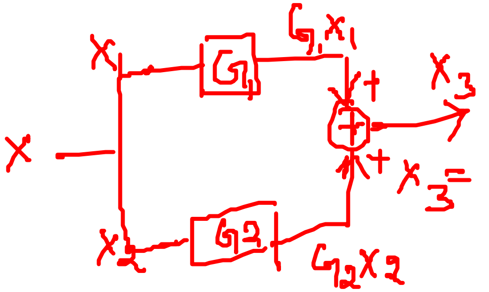
$X_2 = G X_1$

$G X_1 = X_2$

4. Moving a pickoff point behind a block

5. Moving a summing point ahead of a block

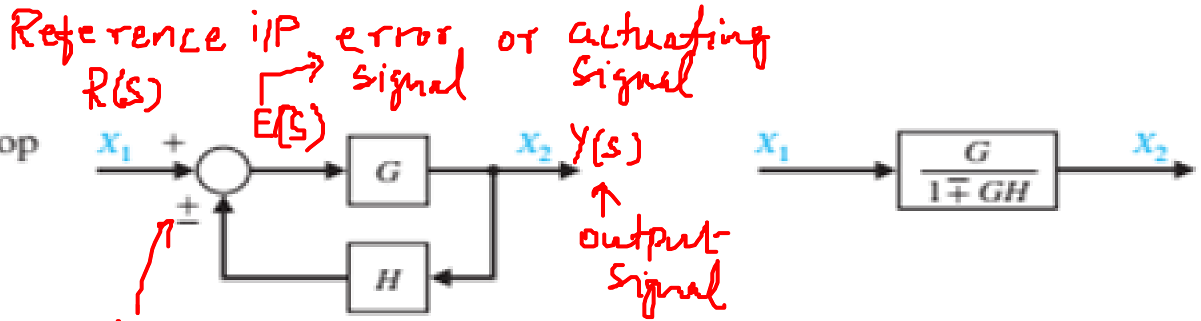
7) 2 blocks in parallel



$X_3 = G_1X_1 + G_2X_2$

$= (G_1+G_2)X$

6. Eliminating a feedback loop



$E(s) = X_1 \pm HX_2$

$X_2 = G\,E(s)$

$E(s) = X_1 \pm HGE(s)$

$E(s)(1 \mp HG) = X_1$

$E(s) = \frac{X_1}{1 \mp GH}$

$X_2 = G\,E(s)$

$= \frac{G X_1}{1 \mp GH}$

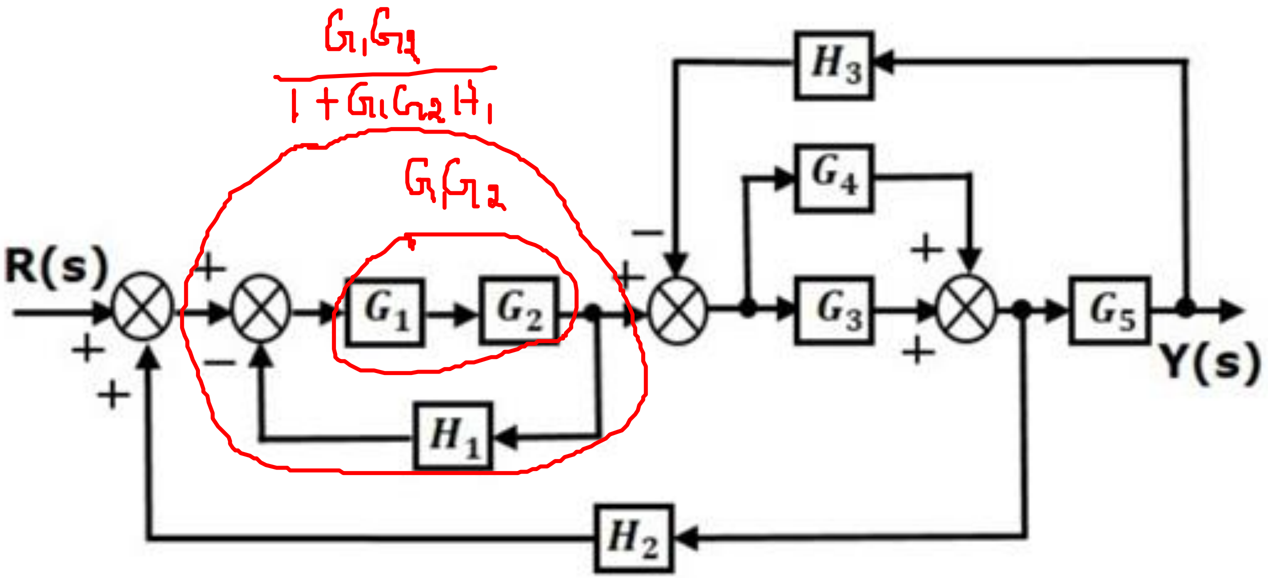
$\frac{X_2}{X_1} = \frac{G}{1 \mp GH}$

G – Forward path T.F

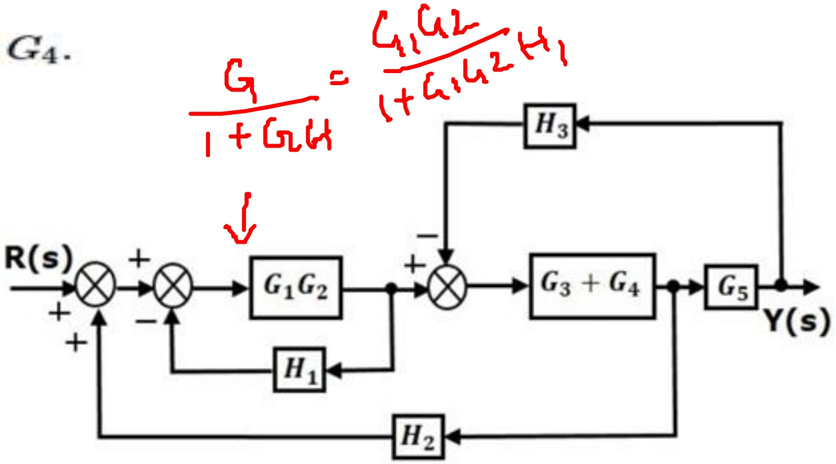
H – feedback path T.F

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Block Diagram – Example 1

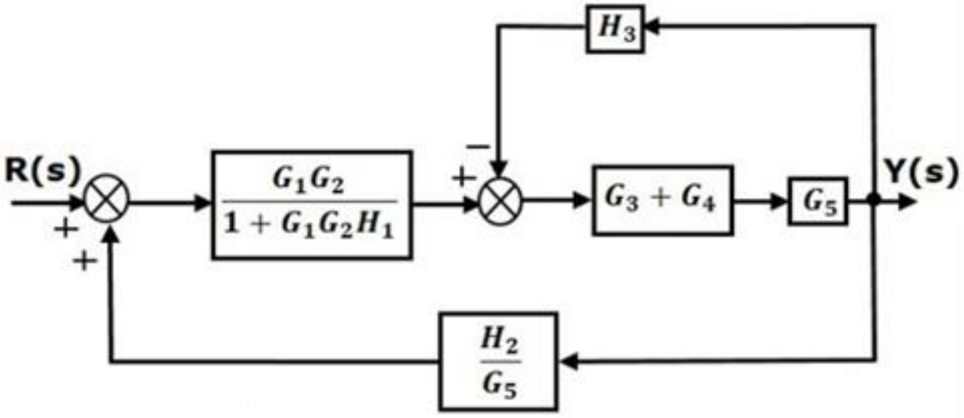


Step 1 – Use Rule 1 for blocks G_1 and G_2. Use Rule 2 for blocks G_3 and G_4. The modified block diagram is shown in the following figure.



Transfer Functions using Block Diagram - Example

Step 2 − Use Rule 3 for blocks G_1G_2 and H_1. Use Rule 4 for shifting take-off point after the block G_5. The modified block diagram is shown in the following figure.



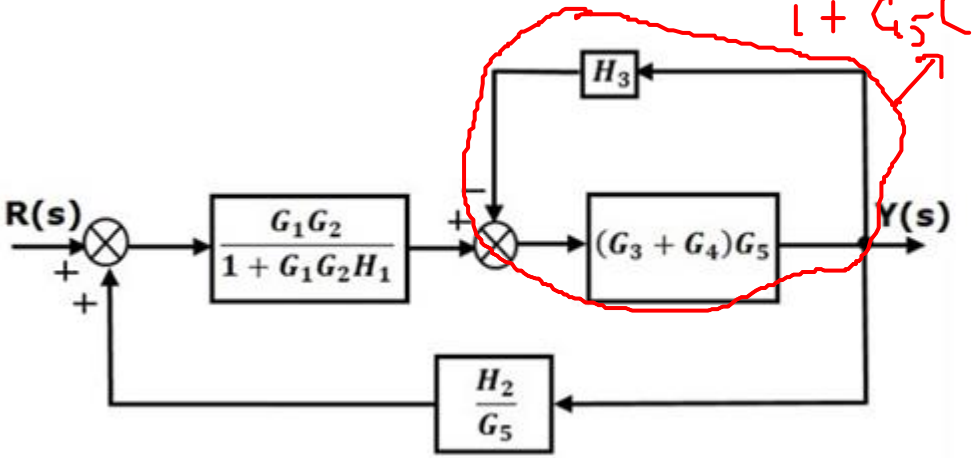
$$G = G_5(G_3+G_4)$$

$$H = H_3$$

$$= \frac{G}{1+GH}$$

$$= \frac{G_5(G_3+G_4)}{1+G_5(G_3+G_4)H_3}$$

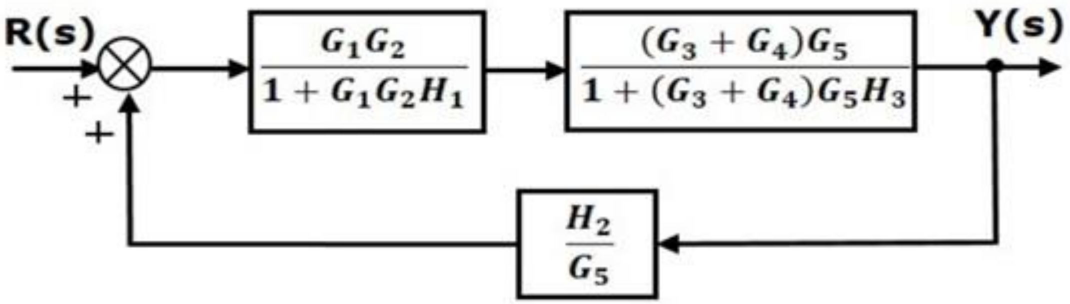
Step 3 − Use Rule 1 for blocks (G_3+G_4) and G_5. The modified block diagram is shown in the following figure.



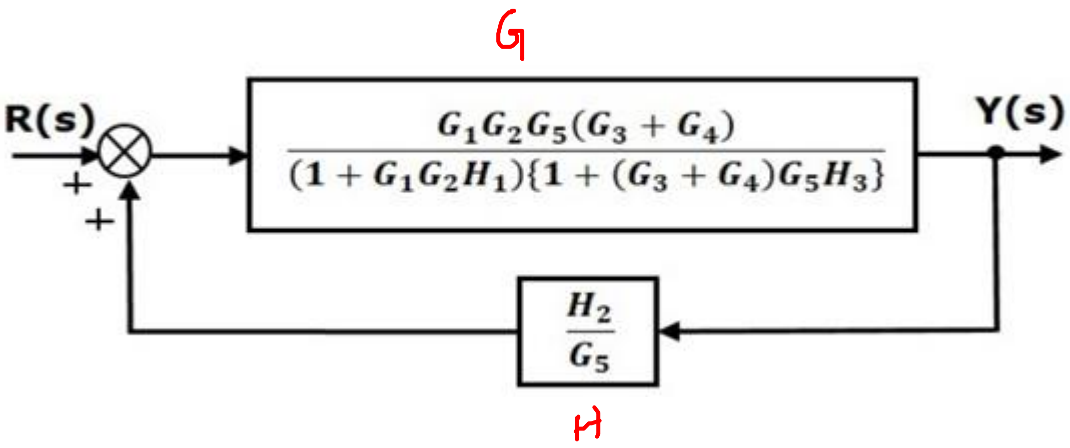
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Block Diagram - Example

Step 4 – Use Rule 3 for blocks $(G_3 + G_4)G_5$ and H_3. The modified block diagram is shown in the following figure.



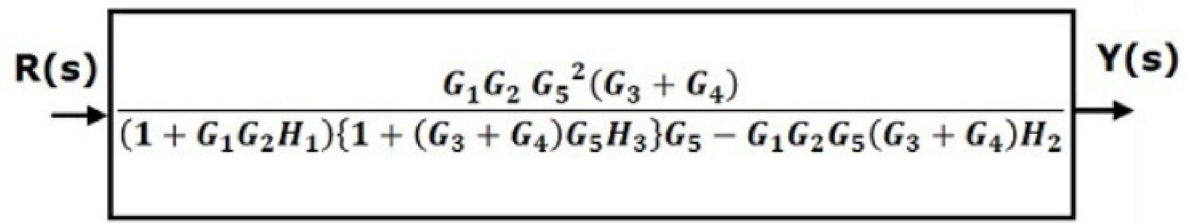
Step 5 – Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



$$\frac{Y(s)}{R(s)} = \frac{G}{1 - GH}$$

Step 6 – Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.

Transfer Functions using Block Diagram - Example

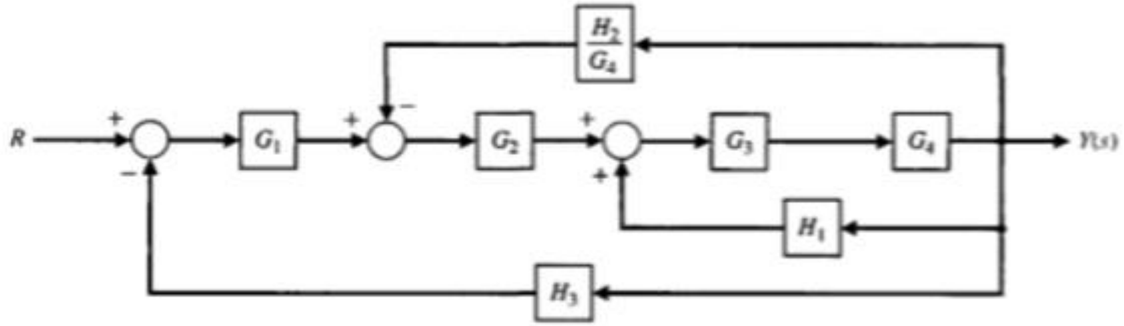


Therefore, the transfer function of the system is

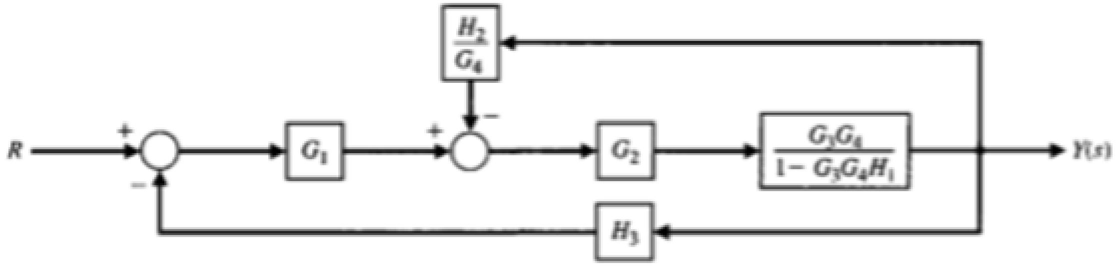
$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_5^2(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5-G_1G_2G_5(G_3+G_4)H_2}$$

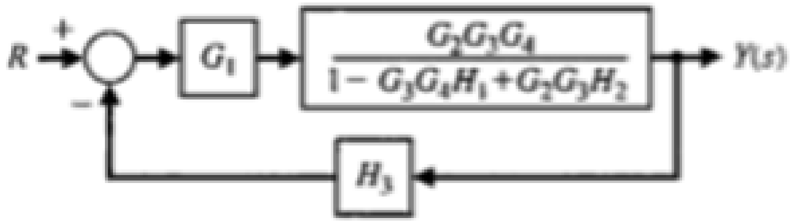
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Block Diagram – Example 2



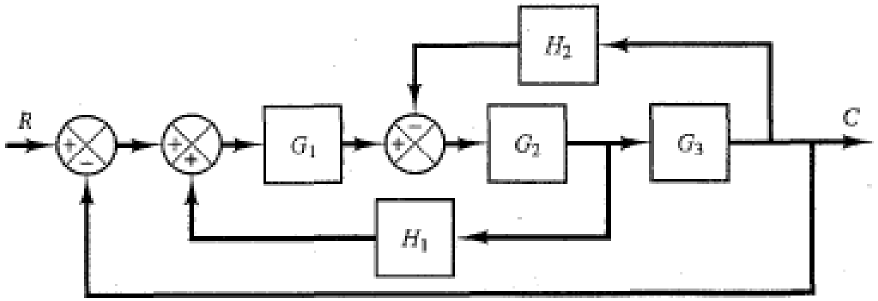
$$\frac{G}{1 + GH}$$



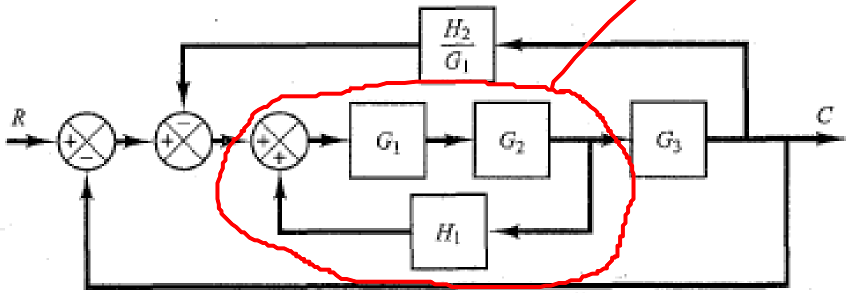


MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

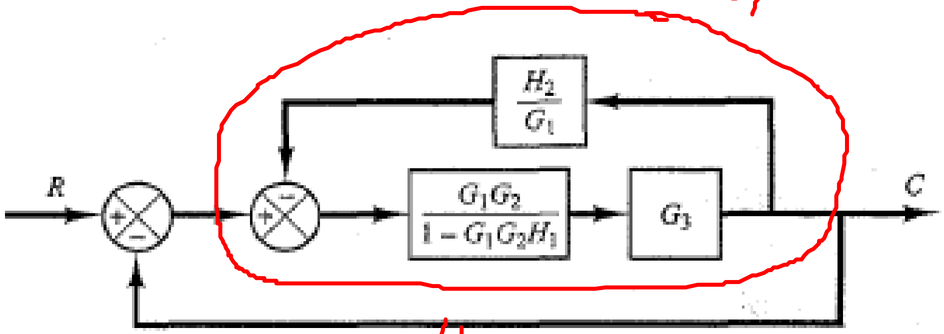
Transfer Functions using Block Diagram – Example 3



$$\frac{G_1G_2}{1-G_1G_2H_1}$$

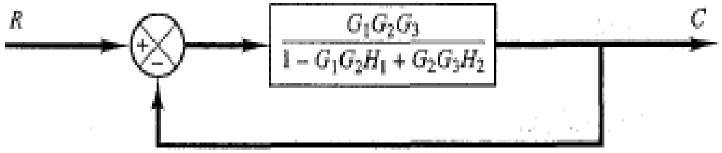


G_7

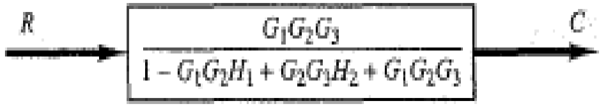


$H = 1$

$$\frac{\dfrac{G_1G_2G_3}{1-G_1G_2H_1}}{1+\dfrac{\cancel{G_1}G_2G_3}{1-G_1G_2H_1}\cdot\dfrac{H_2}{\cancel{G_1}}}$$

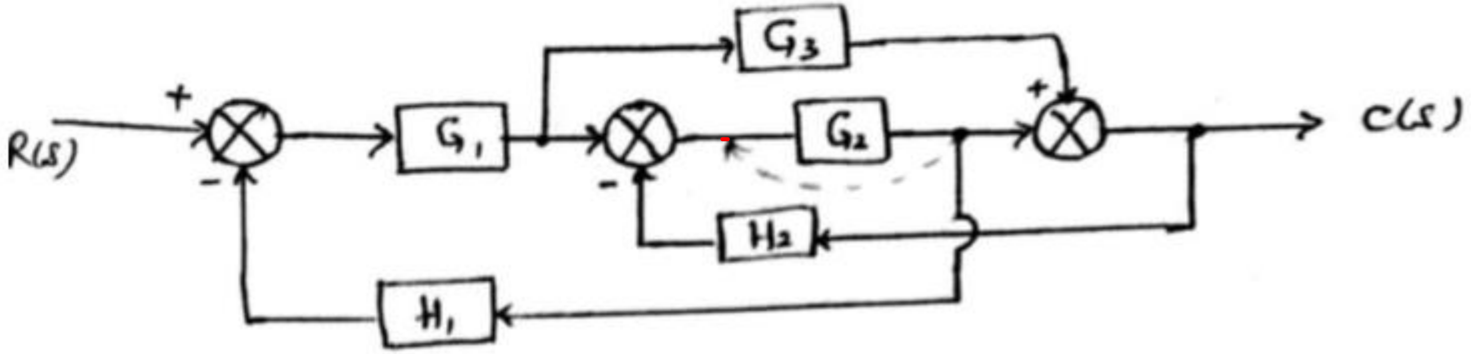


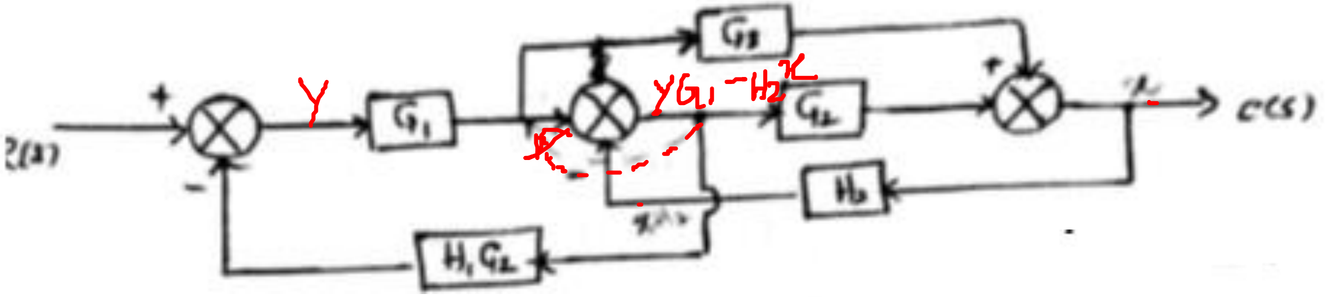
$$\frac{C}{R} = \frac{\dfrac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2}}{1+G_1G_2G_3/1-G_1G_2H_1+G_2G_3H_2} = \frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2}$$

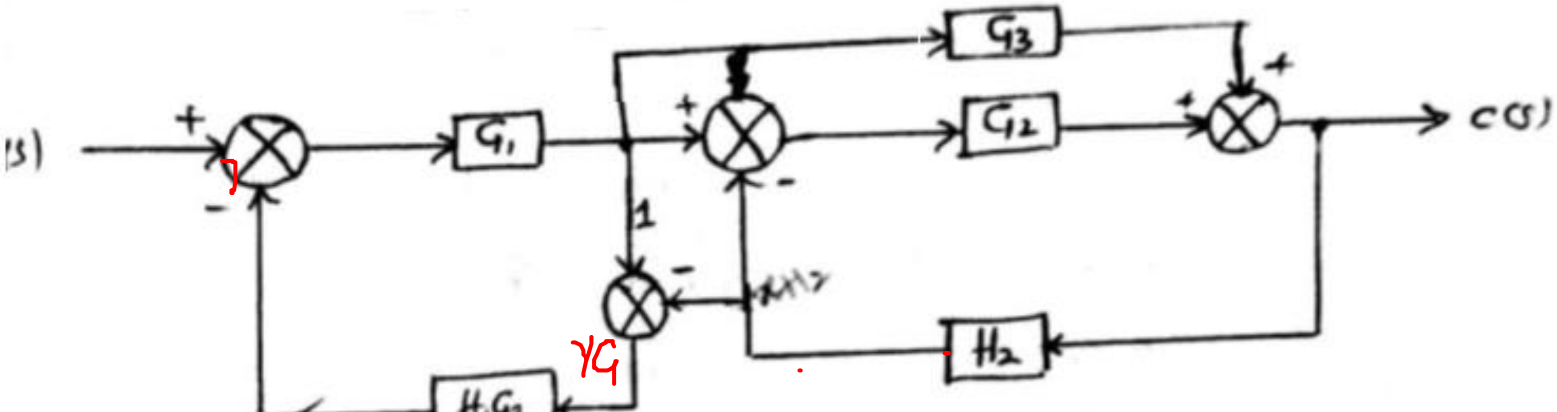


MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Block Diagram – Example 4

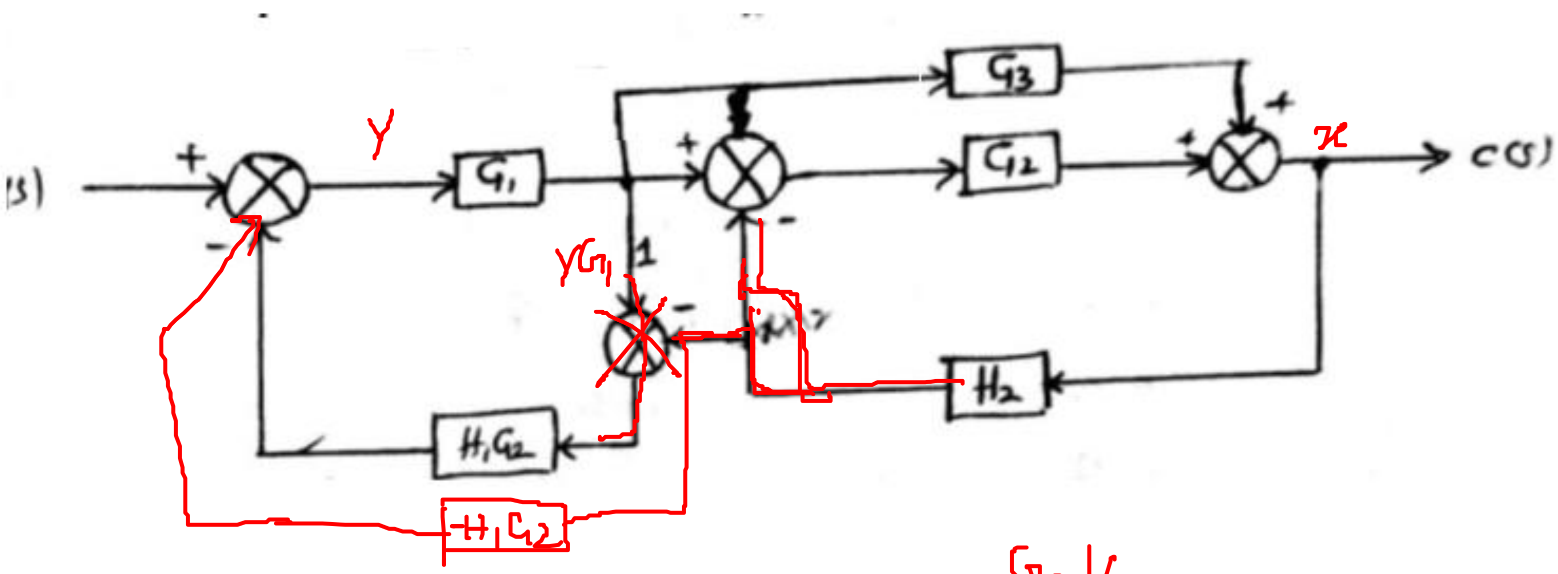


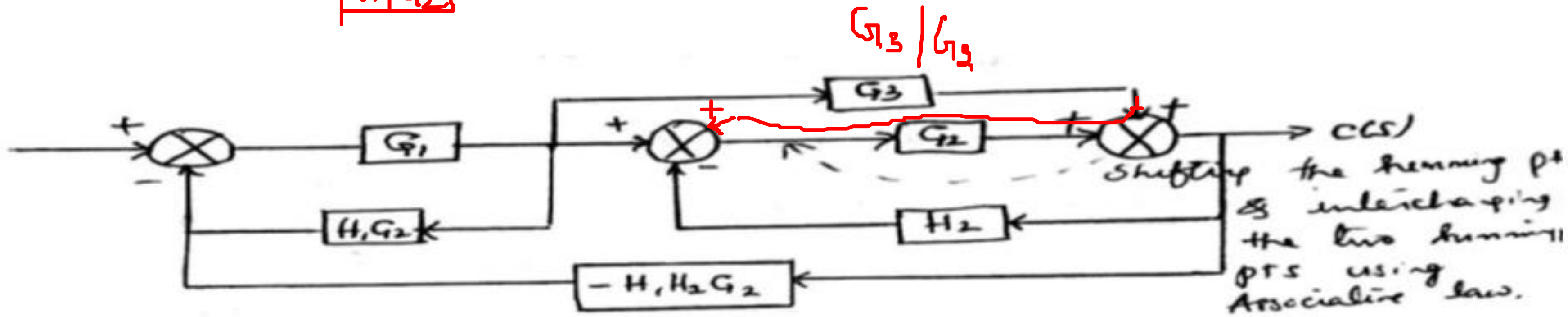




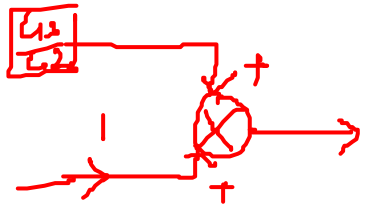
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

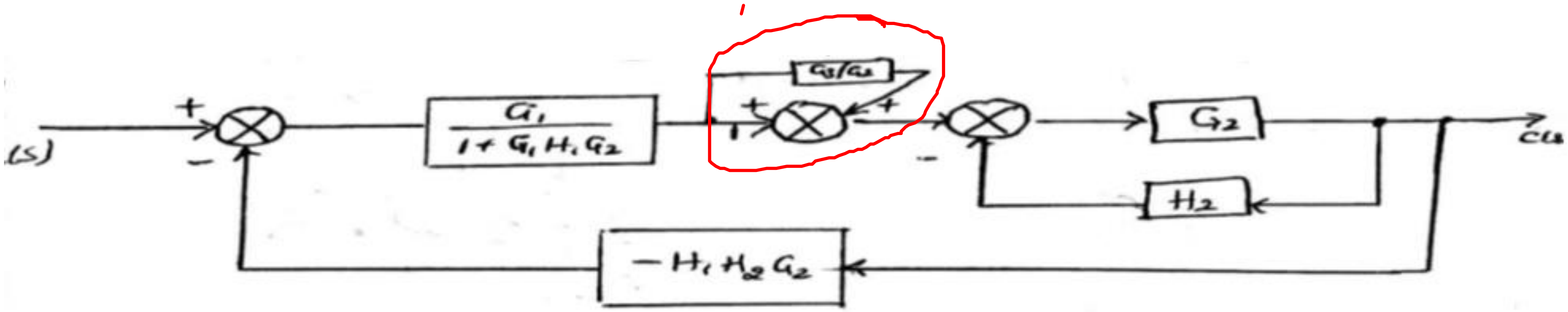
Transfer Functions using Block Diagram – Example 4

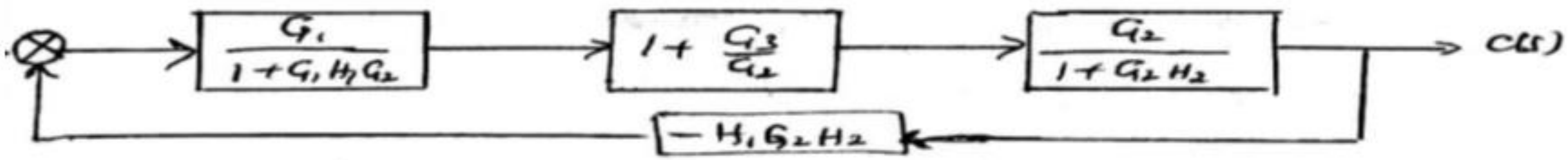




Transfer Functions using Block Diagram – Example 4



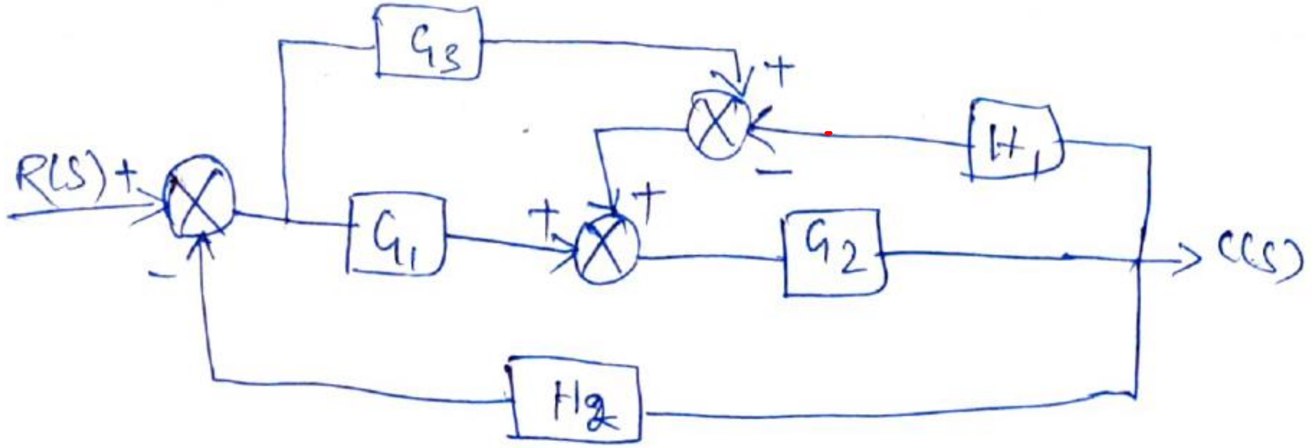


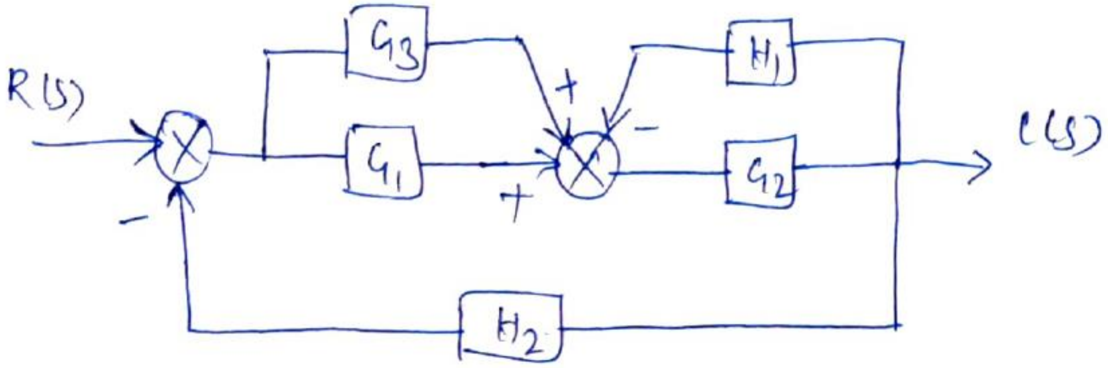


On simplification.

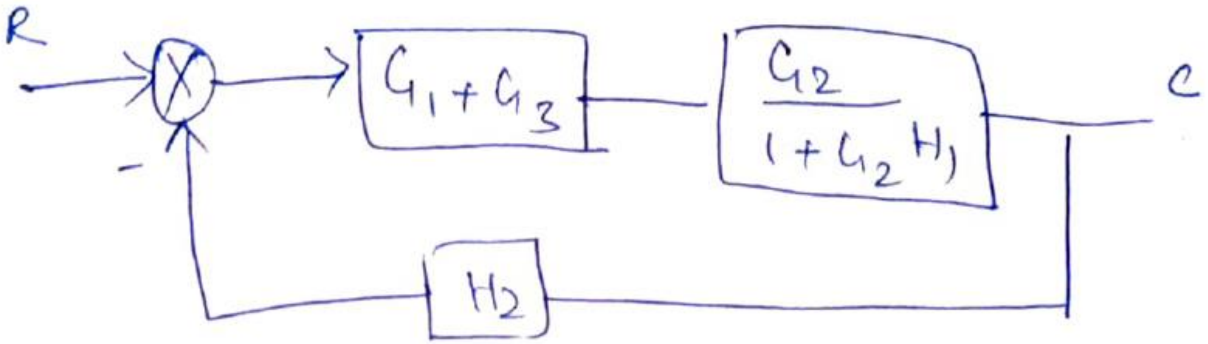
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1(G_2+G_3)}{1+G_2H_2+H_1G_1G_2-G_1G_2G_3H_1H_2}}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS
Transfer Functions using Block Diagram – Example 5





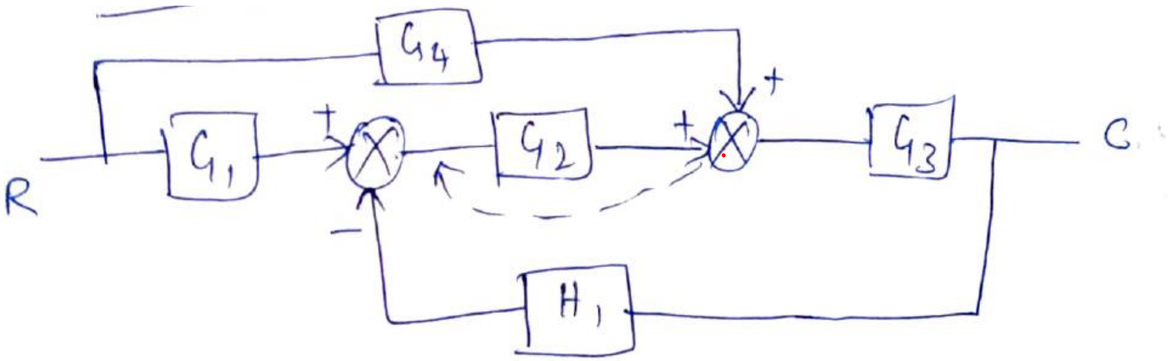
Transfer Functions using Block Diagram – Example 5

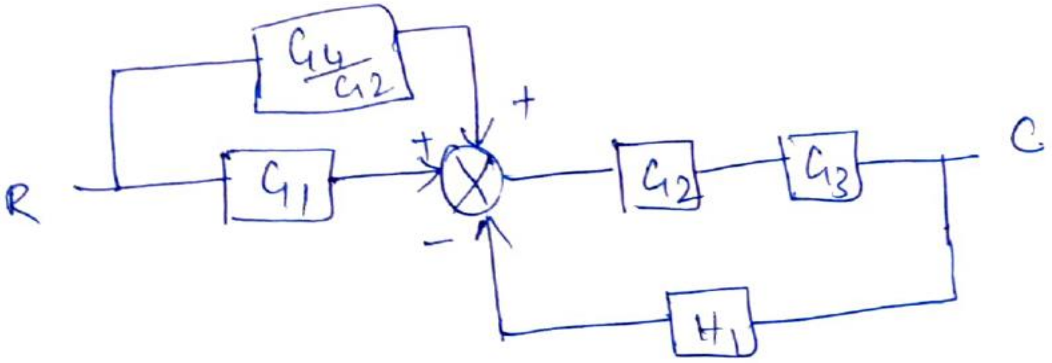


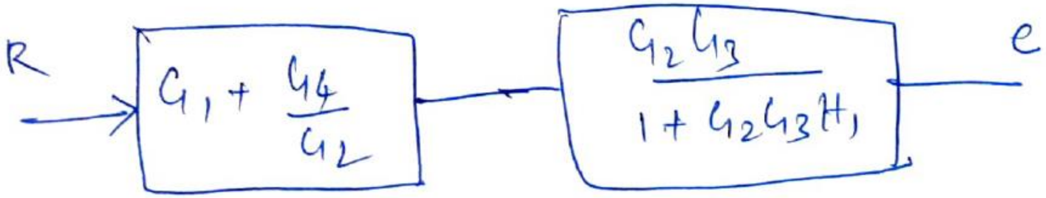
$$\frac{C}{R} = \frac{G_1 G_2 + G_2 G_3}{1 + G_2 H_1 + G_1 G_2 H_2 + G_2 G_3 H_3}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Block Diagram – Example 6





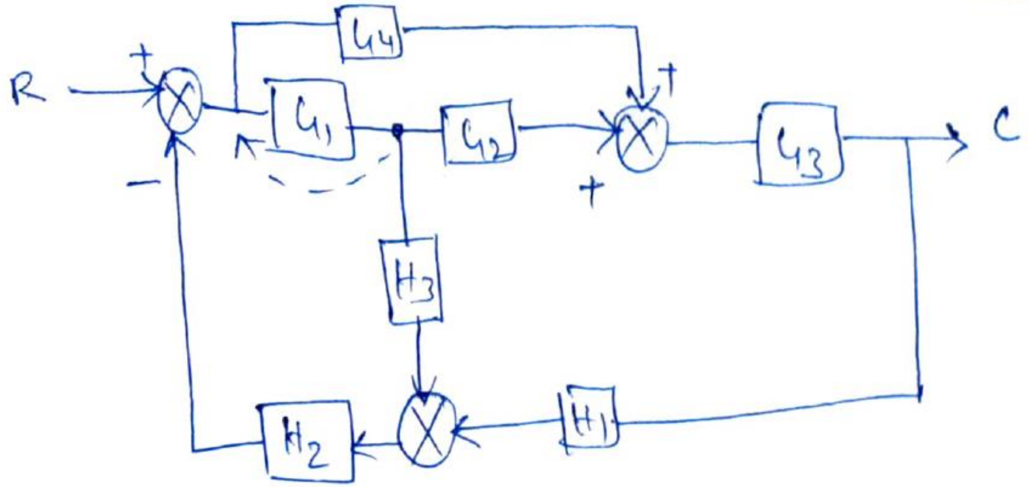


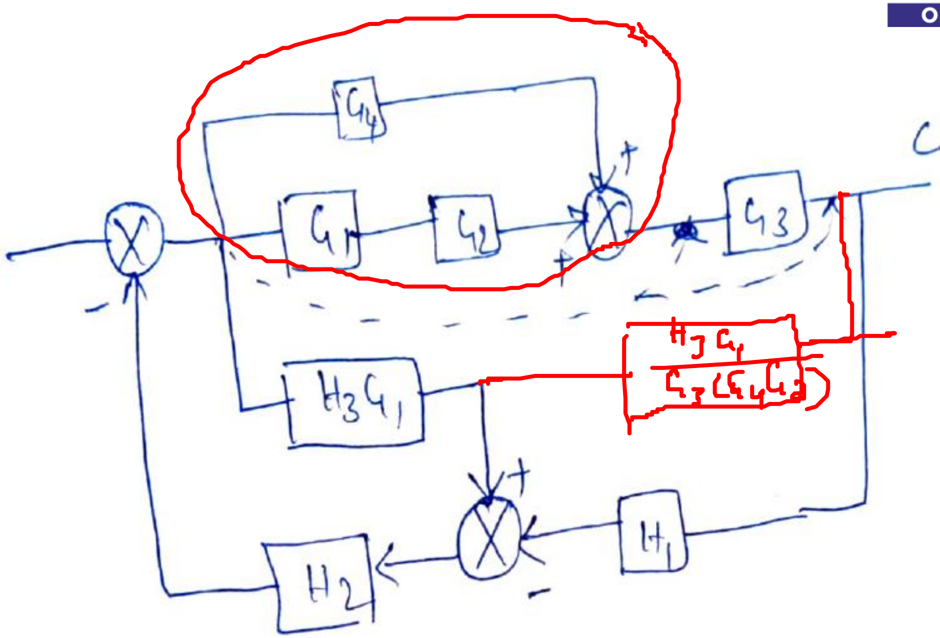
$$\frac{C}{R} = \frac{(G_1G_2 + G_4)\,G_3}{1 + G_2G_3H_1}$$

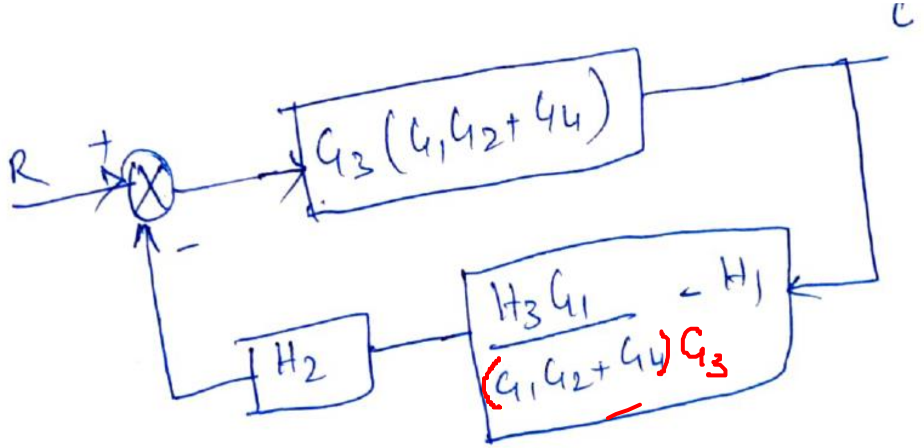
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Block Diagram – Example 7

$(G_4 + G_1 G_2) G_3$







$$\frac{C}{R} = \frac{G_3(G_1G_2+G_4)}{1+G_1H_2H_3 - H_1H_2G_3(G_1G_2+G_4)}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Signal Flow Graph

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Department of Electronics and Communication Engineering

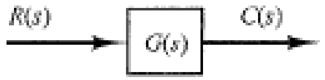
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

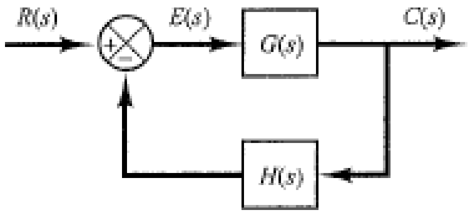
Transfer Functions using Signal Flow Graph

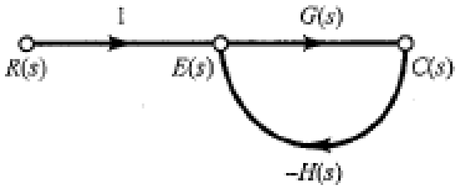
- An alternative method for determining the relationship between system variables has been developed by **S. J. Mason.**
- Adv: This method does not require any reduction techniques as such because of availability of Mason’s gain formula
- A **signal-flow graph** is a diagram consisting of nodes that are connected by several directed branches and is a graphical representation of a set of linear relations.

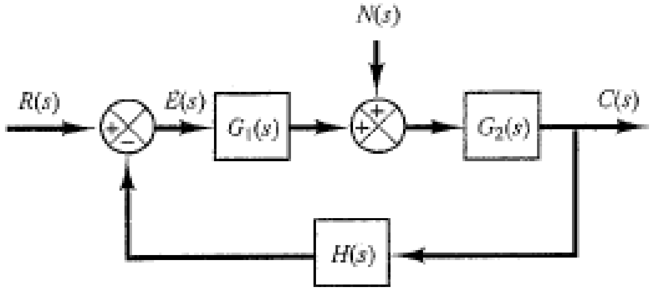
$$Y(s) = G(s)\,R(s)$$

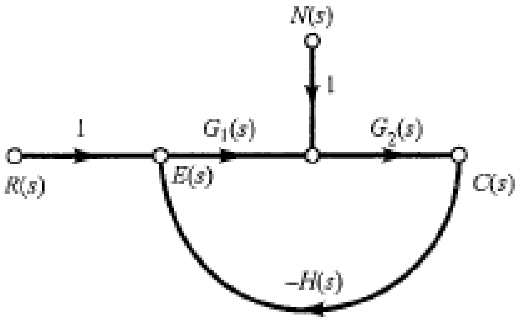
Block Diagram to Signal Flow Graph





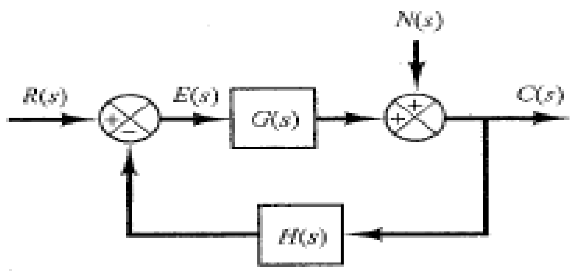


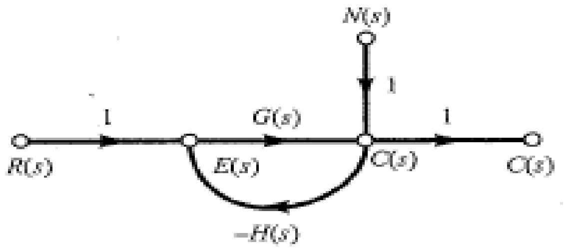


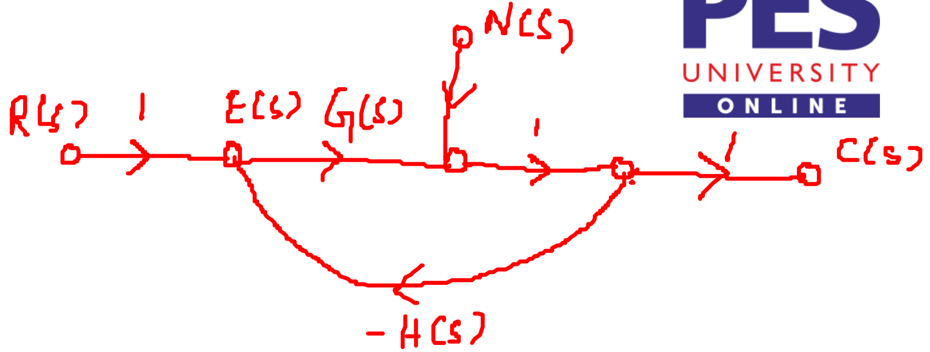


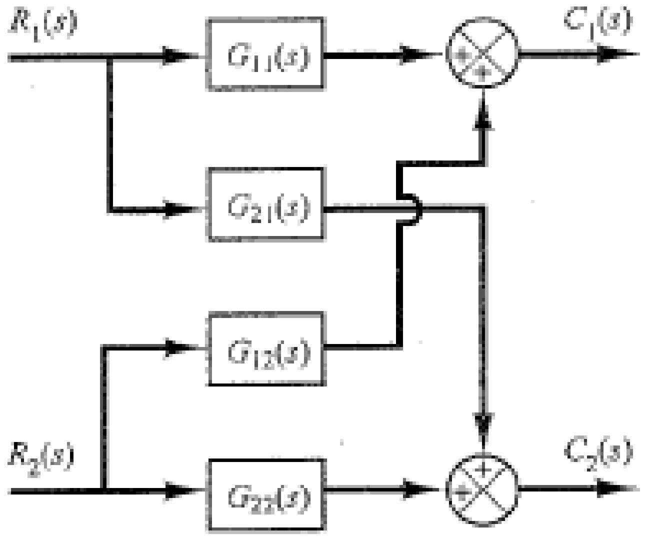
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

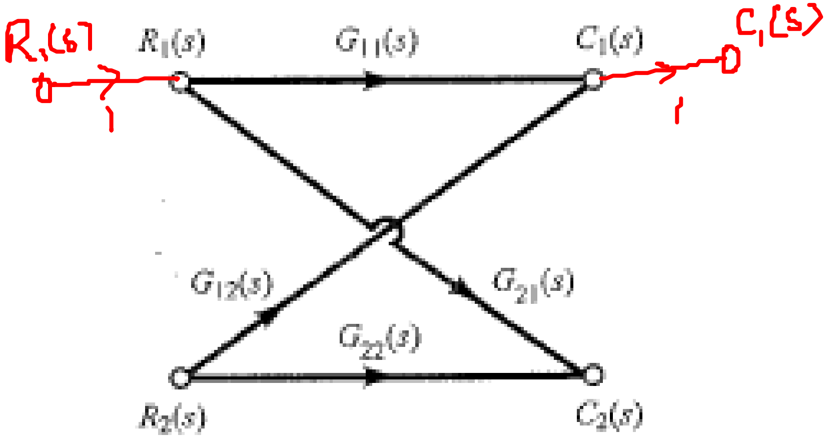
Block Diagram to Signal Flow Graph





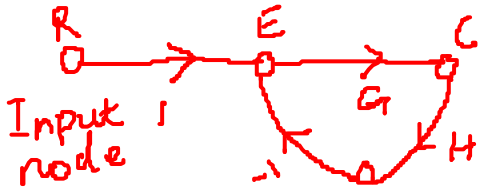


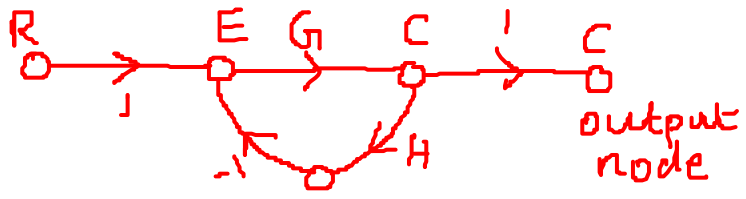


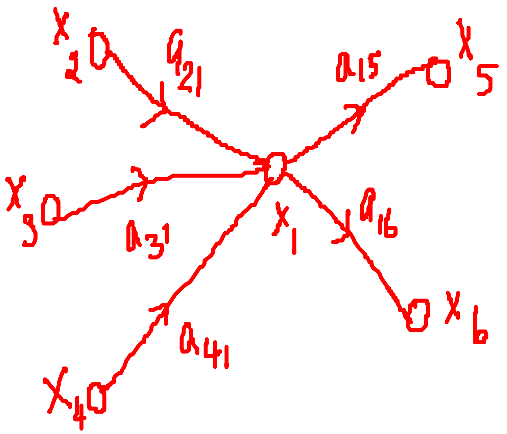


Terminology:

- **Branch:** A signal travels along a branch from one node to another in the direction indicated by the branch arrow and in the process gets multiplied by the gain or transmittance of the branch.
- For ex, signal traversing from node E to node C is given by GE







MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph

Terminology:

- **Nodes:** The input and output points or junctions are called nodes.

  $x_2, x_3, x_4, x_1, x_5, x_6$

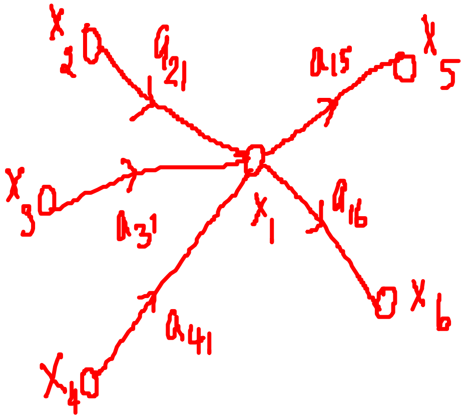
- **Node as a summing point:** sum of all incoming signals

  $$x_1 = a_{21}x_2 + a_{31}x_3 + a_{41}x_4$$

- **Node as a transmitting point:** transmitted through all branches outgoing from the node

  $$x_5 = a_{15}x_1$$

  $$x_6 = a_{16}x_1$$



- **Input node or source node:** node with only outgoing branches

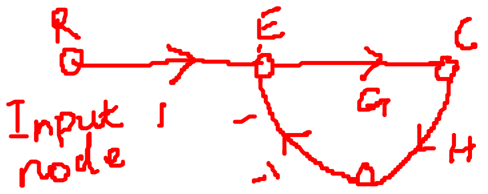
  x_2, x_3 & x_4

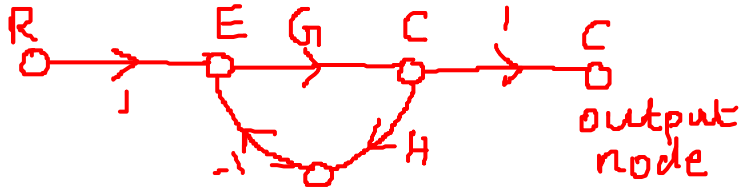
- **Output node or sink:** node with only incoming branches

  x_5 & x_6

Terminology:

- **A path** is a branch or a continuous sequence of branches that can be traversed from one signal (node) to another signal (node).
- **Forward path:** It is a path from the input node to the output node

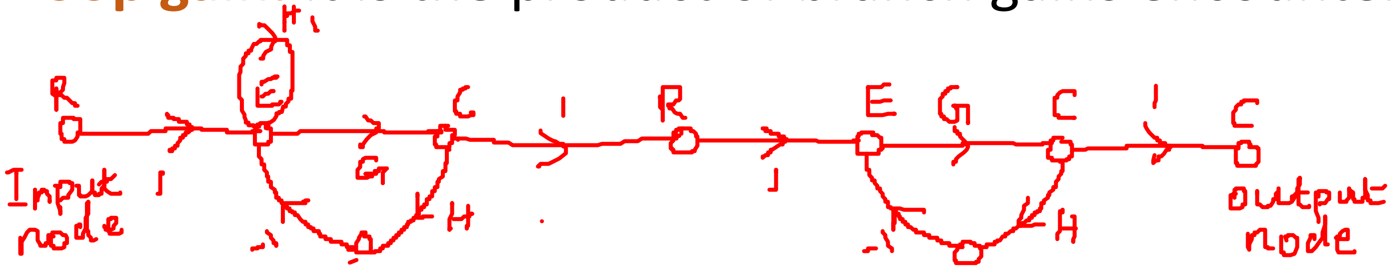




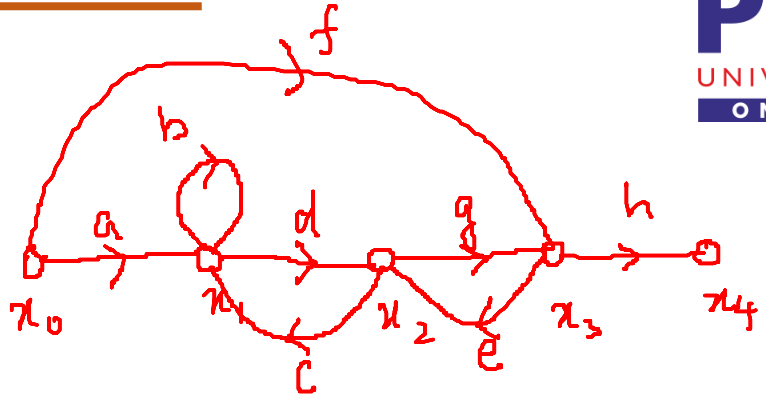
Terminology:

- **A loop** is a closed path that originates and terminates on the same node, with no node being met twice along the path.
- **Non-touching loops:** Two loops are said to be non-touching if they do not have a common node.
- **Forward path gain:** product of the branch gains , G^2
- **Loop gain:** It is the product of branch gains encountered in traversing the loop

$-GH$



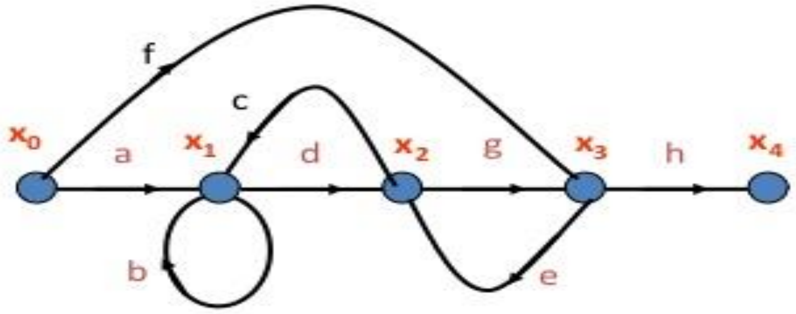
Construction of Signal Flow Graph:

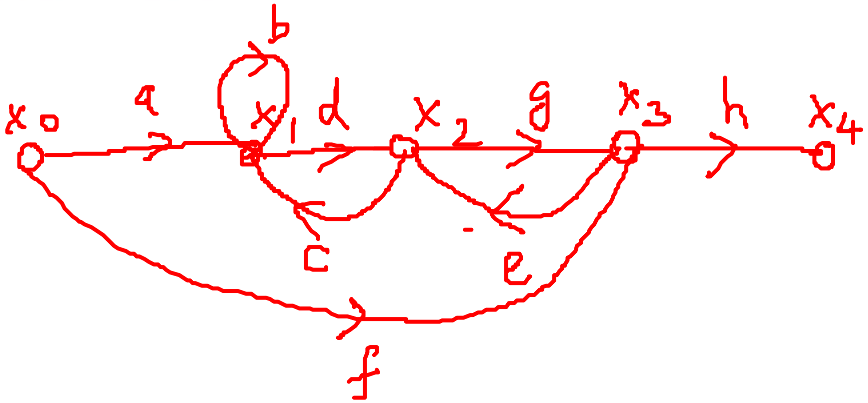


Signal-Flow Graph Models

x_o is input and x_4 is output

$$x_1 = ax_0 + bx_1 + cx_2$$
$$x_2 = dx_1 + ex_3$$
$$x_3 = fx_0 + gx_2$$
$$x_4 = hx_3$$





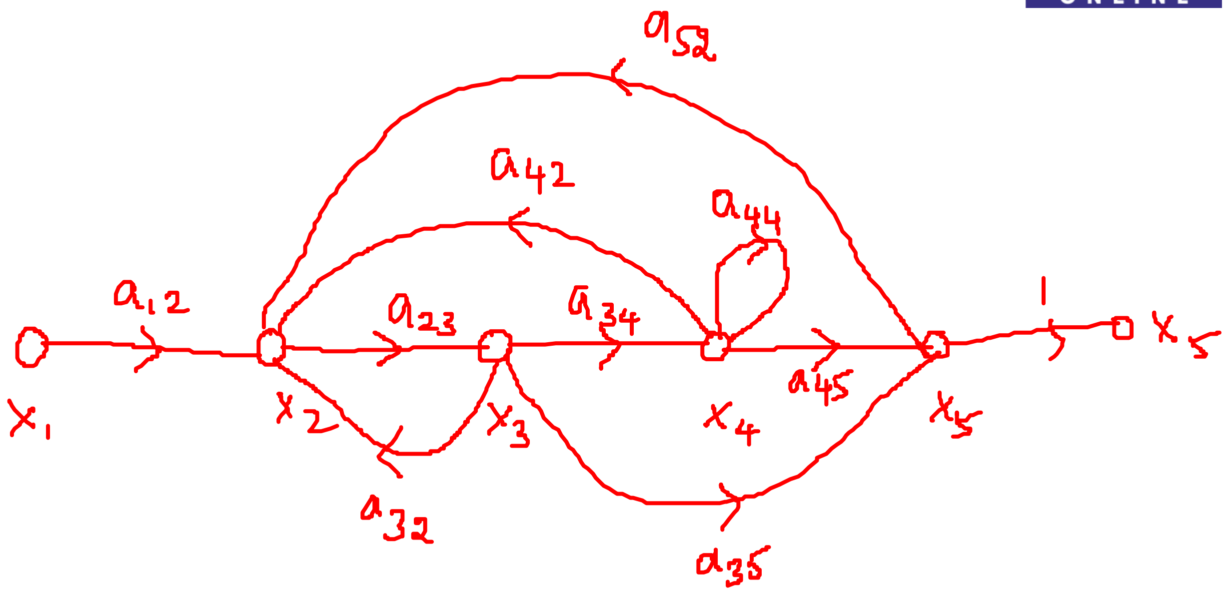
Construction of Signal Flow Graph:

$$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5$$

$$x_3 = a_{23}x_2$$

$$x_4 = a_{34}x_3 + a_{44}x_4$$

$$x_5 = a_{35}x_3 + a_{45}x_4$$



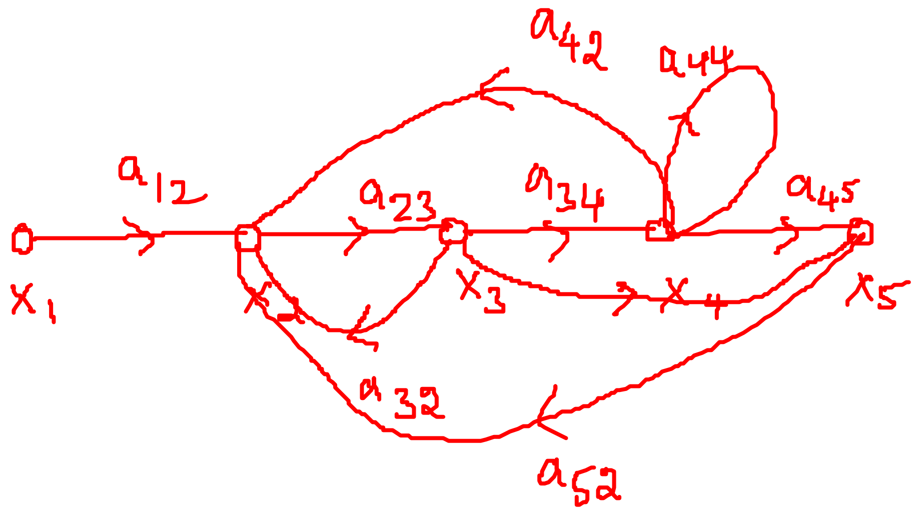
Construction of Signal Flow Graph:

$$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5$$

$$x_3 = a_{23}x_2$$

$$x_4 = a_{34}x_3 + a_{44}x_4$$

$$x_5 = a_{35}x_3 + a_{45}x_4$$



Mason's Gain Formula: The overall system gain is determined by the Mason's gain formula

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

Where, P_k = path gain of Kth forward path

Δ =determinant of the graph = 1-(sum of loop gains of all individual loops) + (sum of gain products of all possible combinations of two non-touching loops) - (sum of gain products of all possible combinations of three non-touching loops) + …,

Δ_k= the value of Δ for the part of the graph not touching the K-th forward path

T = overall gain of the system

Forward Paths:

$P_1 = X_1 X_2 X_3 X_4 X_5 = a_{12} a_{23} a_{34} a_{45}$

$P_2 = X_1 X_2 X_3 X_5 = a_{12} a_{23} a_{35}$

Loop Gains:

$L_1 = X_2 X_3 X_2 = a_{23} a_{32}$

$L_2 = X_2 X_3 X_4 X_2 = a_{23} a_{34} a_{42}$

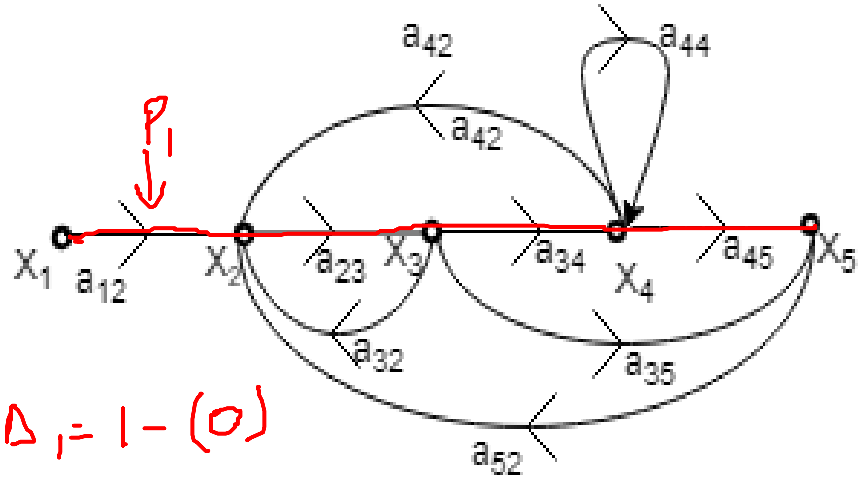
$L_3 = X_2 X_3 X_4 X_5 X_2 = a_{23} a_{34} a_{45} a_{52}$

$L_4 = X_4 X_4 = a_{44}$

$L_5 = X_2 X_3 X_5 X_2 = a_{23} a_{35} a_{52}$

Two Non-Touching loops

$L_1 L_4 = a_{23} a_{32} a_{44}$, $L_5 L_4 = a_{23} a_{35} a_{52} a_{44}$

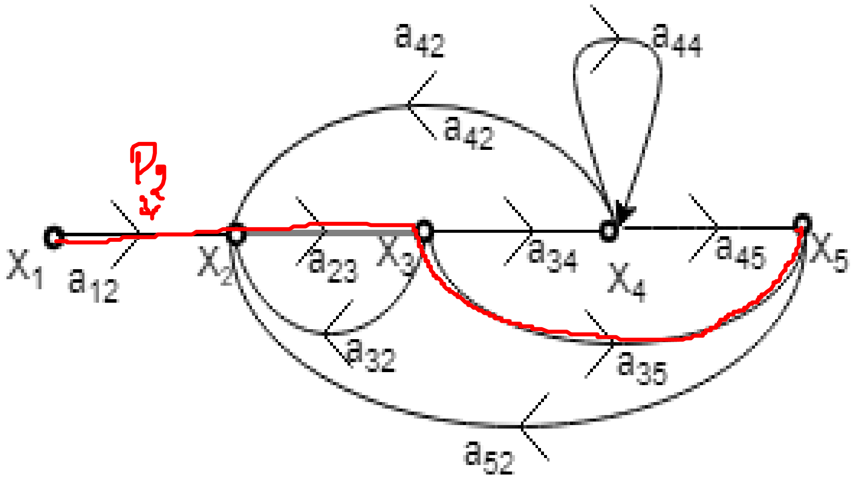


$\Delta_1 = 1 - (0)$
$= 1$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph - Example

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_4 + L_5 L_4)$$



$$\Delta_1 = 1 - 0 = 1$$

$$\Delta_2 = 1 - L_4$$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{a_{12}a_{23}a_{34}a_{45} + a_{12}a_{23}a_{35}(1 - a_{44})}{1 - (a_{23}a_{32} + a_{23}a_{34}a_{42} + a_{23}a_{34}a_{45}a_{52} + a_{44} + a_{23}a_{35}a_{52}) + (a_{23}a_{32}a_{44} + a_{23}a_{35}a_{52}a_{44})}$$

Transfer Functions using Signal Flow Graph – Example 2

Forward Paths:

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_5 G_6 G_7 G_8$

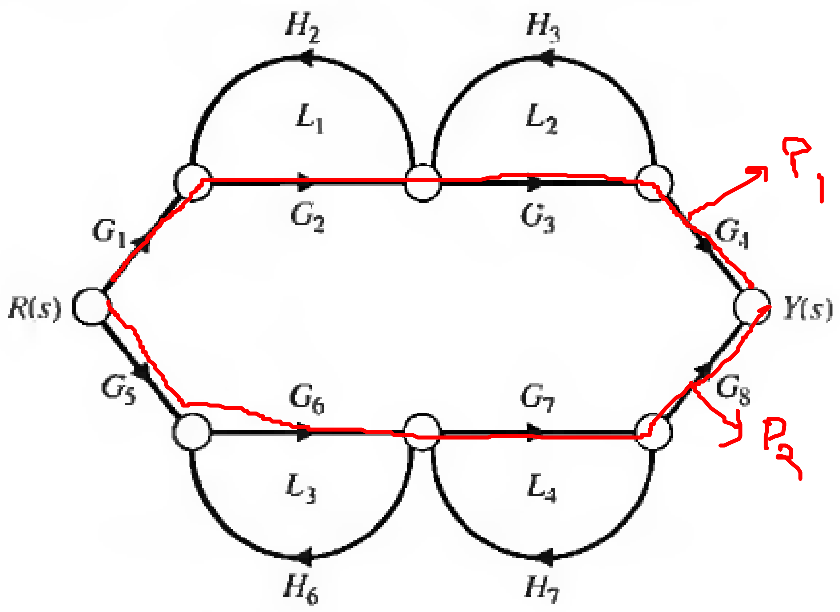
Loop Gains:

$L_1 = G_2 H_2$

$L_2 = G_3 H_3$

$L_3 = G_6 H_6$

$L_4 = G_7 H_7$



Two Non-Touching loops

$L_1 L_3, \ L_1 L_4, \ L_2 L_3, \ L_2 L_4$

$\Delta_1 = 1 - (L_3 + L_4) + (0)$

$\Delta_2 = 1 - (L_1 + L_2) + (0)$

Transfer Functions using Signal Flow Graph – Example 2

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3 + L_1 L_4 + L_2 L_3 + L_2 L_4)$

$$L_1 = L_2 = 0 \quad \text{and} \quad \Delta_1 = 1 - (L_3 + L_4).$$

Δ_1

$$\Delta_2 = 1 - (L_1 + L_2).$$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}.$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph – Example 3

Forward Paths:

P_1 $\quad P_1 = G_1G_2G_3G_4G_5G_6,$

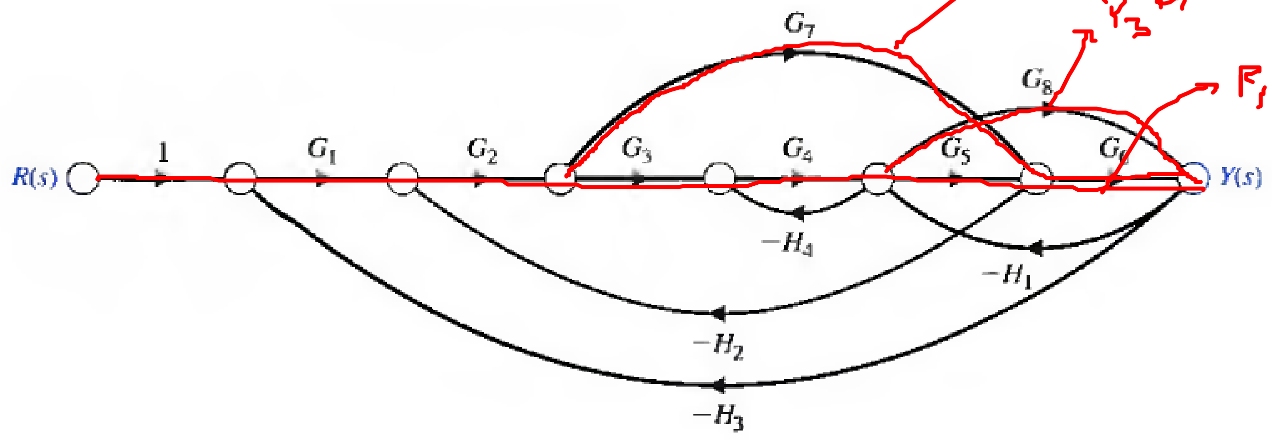
P_2 $\quad P_2 = G_1G_2G_7G_6,$

$P_3 = G_1G_2G_3G_4G_8.$

$P_1 \Rightarrow \Delta_1 = 1$

$P_2 \Rightarrow \Delta_2 = 1 - L_1$

$P_3 \Rightarrow \Delta_3 = 1$



Loop Gains:

$L_1 = -G_4H_4$

$L_2 = -G_5G_6H_1$

$L_3 = -G_2G_3G_4G_5H_2$

$L_4 = -G_1G_2G_3G_4G_5G_6H_3$

$L_5 = -G_2G_7H_2$

$L_6 = -G_8H_1$

$L_7 = -G_1G_2G_7G_6H_3$

$L_8 = -G_1G_2G_3G_4G_8H_3$

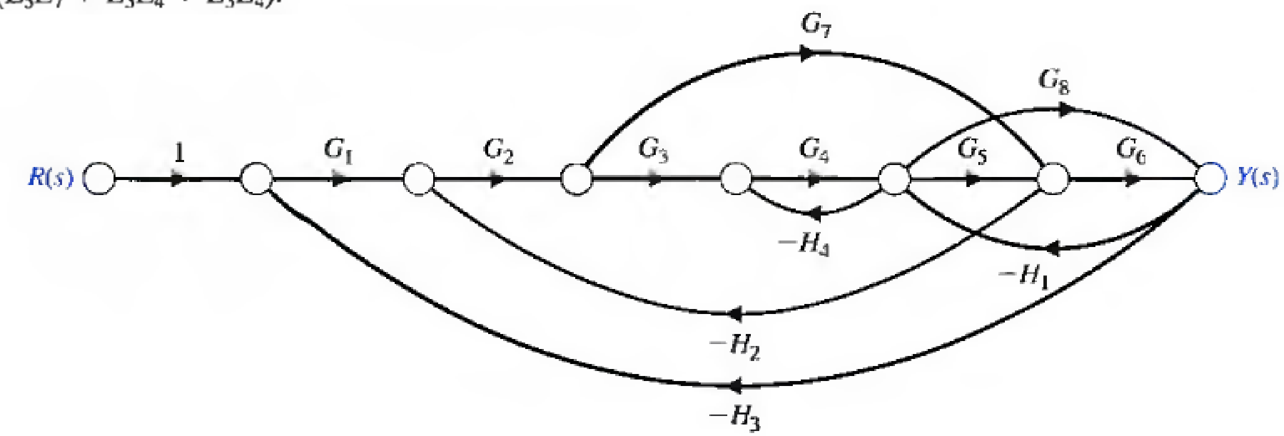
Two Non-Touching loops

$L_1L_7,\ L_1L_5,\ L_5L_6$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph – Example 2

Δ= $\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4).$



Δ_1 $\Delta_1 = \Delta_3 = 1$ and $\Delta_2 = 1 - L_5 = 1 + G_4H_4.$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}.$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

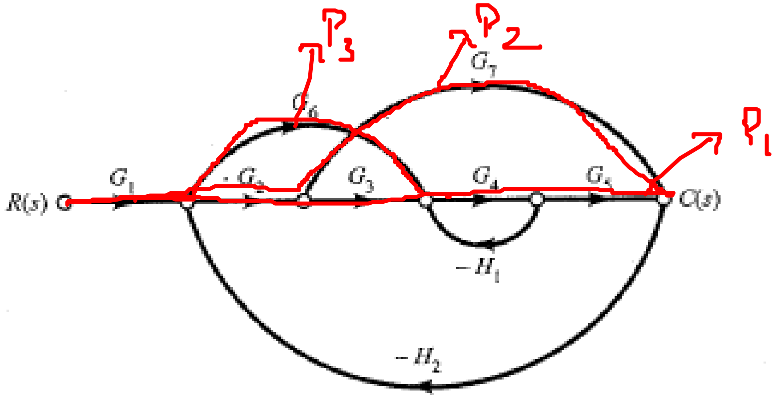
Transfer Functions using Signal Flow Graph – Example 4

forward paths

$P_1 = G_1G_2G_3G_4G_5$, $\Delta_1 = 1$

$P_2 = G_1G_2G_7$, $\Delta_2 = 1 - L_1$

$P_3 = G_1G_6G_4G_5$, $\Delta_3 = 1$



Loop gains

$L_1 = -G_4H_1$

$L_2 = -G_2G_3G_4G_5H_2$

$L_3 = -G_2G_7H_2$

$L_4 = -G_6G_4G_5H_2$

2- non touching loops

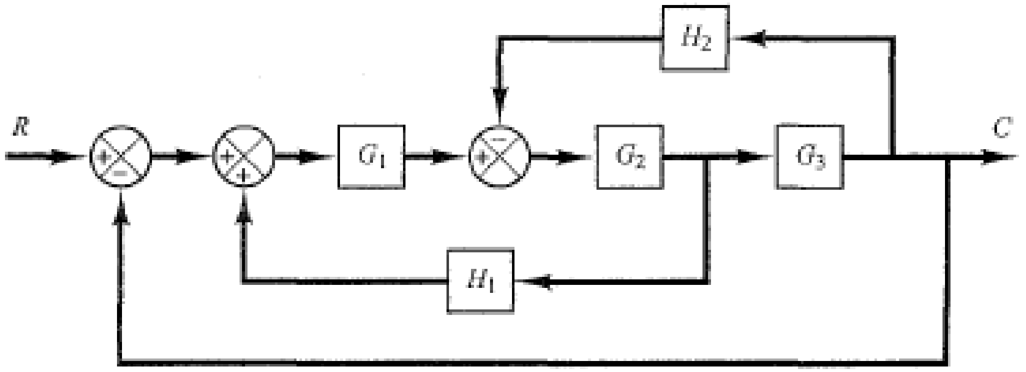
L_1L_3

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2$$

$$\frac{C(s)}{R(s)} = P = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3)$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7(1 + G_4H_1)}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_4H_1G_2G_7H_2}$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS
Transfer Functions using Signal Flow Graph – Example 5



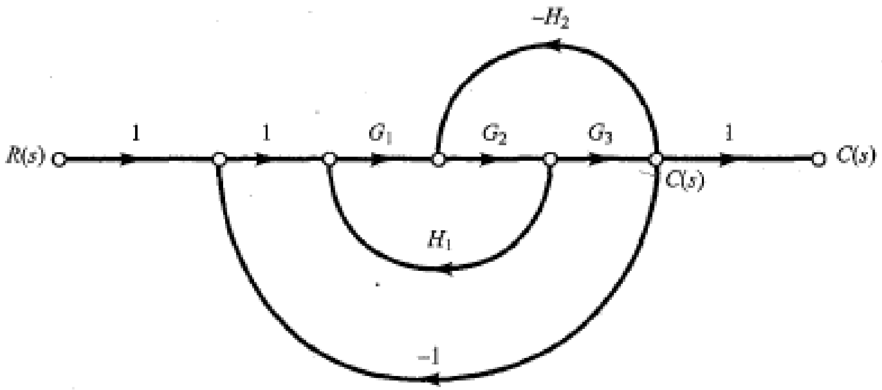
$$P_1 = G_1G_2G_3$$

$$L_1 = G_1G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_1G_2G_3$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$
$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

$$\Delta_1 = 1$$

$$\frac{C(s)}{R(s)} = P = \frac{P_1\Delta_1}{\Delta}$$
$$= \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

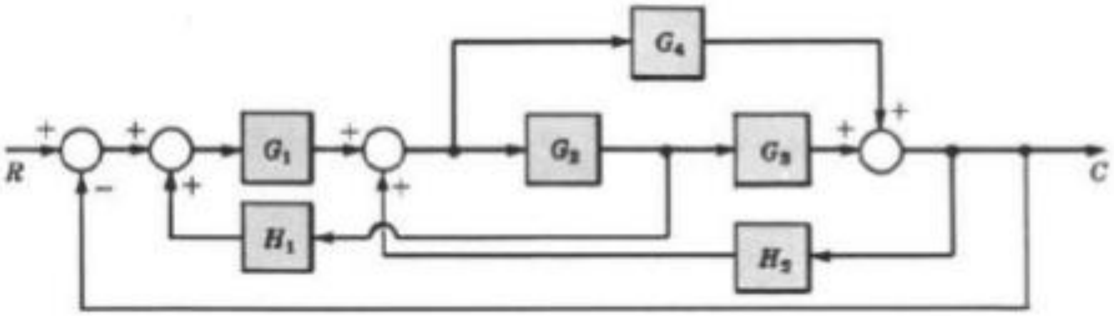


MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph – Example 6

The two forward path gains are

$P_1 = G_1G_2G_3$ and $P_2 = G_1G_4$



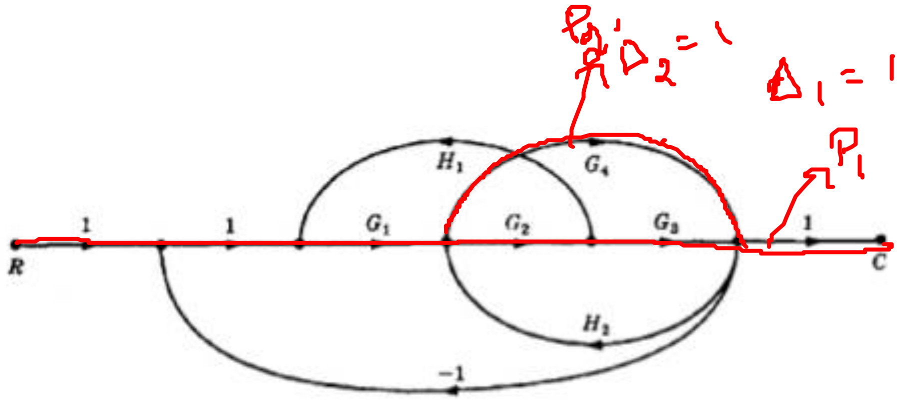
$L_1 = G_1G_2H_1$

$L_2 = G_2G_3H_2$

$L_3 = -G_1G_2G_3$

$L_4 = G_4H_2$

$L_5 = -G_1G_4$



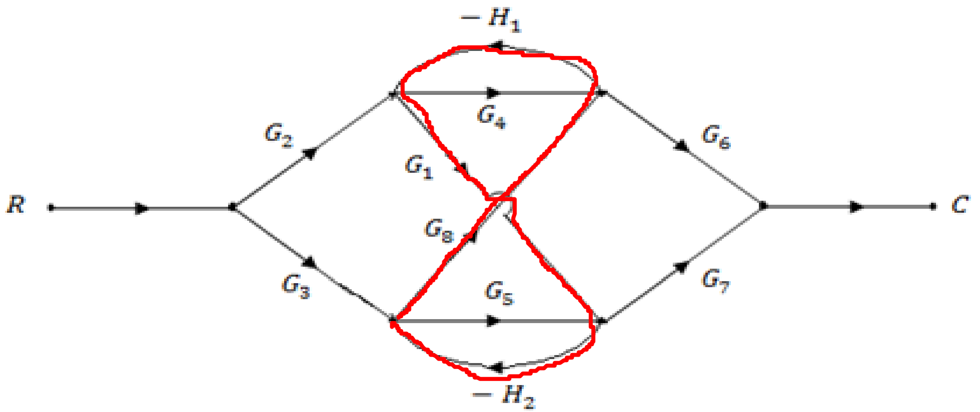
$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph – Example 7

Using Mason's gain formula, calculate the transfer function C/R of the signal flow graph shown below in the figure.



$P_1 = G_2 G_4 G_6$, $\Delta_1 = 1 - L_2$

$P_2 = G_3 G_5 G_7$, $\Delta_2 = 1 - L_1$

$P_3 = G_3 G_8 G_6$, $\Delta_3 = 1$

$P_4 = G_2 G_1 G_7$, $\Delta_4 = 1$

$P_5 = -G_2 G_1 H_2 G_8 G_6$, $\Delta_5 = 1$

$P_6 = -G_3 G_8 H_1 G_1 G_7$, $\Delta_6 = 1$

$L_1 = -G_4 H_1$

$L_2 = -G_5 H_2$

$L_3 = G_1 H_2 G_8 H_1$

Non-touching

$L_1 L_2$

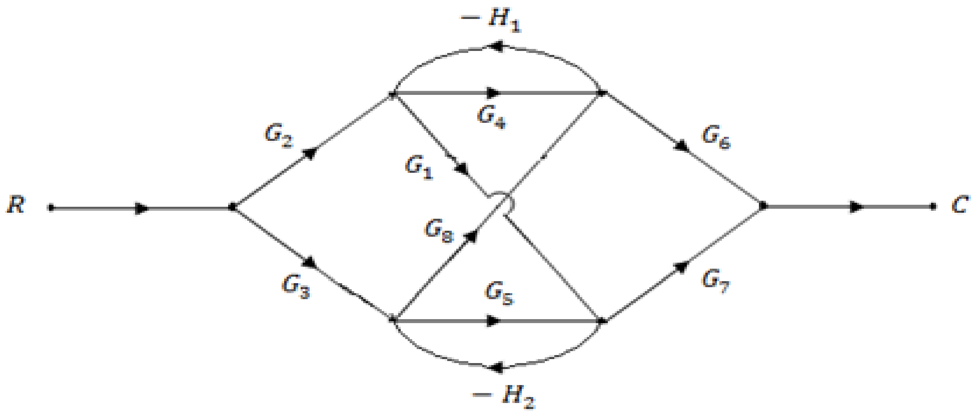
$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2 + \cdots + P_6 \Delta_6}{\Delta}$$

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2$$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph – Example 7

Using Mason's gain formula, calculate the transfer function C/R of the signal flow graph shown below in the figure.



Forward Path gains

$P_1 = G_2 G_4 G_6$

$P_2 = G_3 G_5 G_7$

$P_3 = G_2 G_1 G_7$

$P_4 = G_3 G_8 G_6$

$P_5 = -G_2 G_1 H_2 G_8 G_6$

$P_6 = -G_3 G_8 H_1 G_1 G_7$

Loop Gains:

$L_1 = -G_4 H_1$

$L_2 = -G_5 H_2$

$L_3 = G_1 H_2 G_8 H_1$

Non-touching loops

$L_1 L_2$

$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2$

$\Delta_1 = 1 - (-G_5 H_2)$

$\Delta_2 = 1 - (-G_4 H_1)$

$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

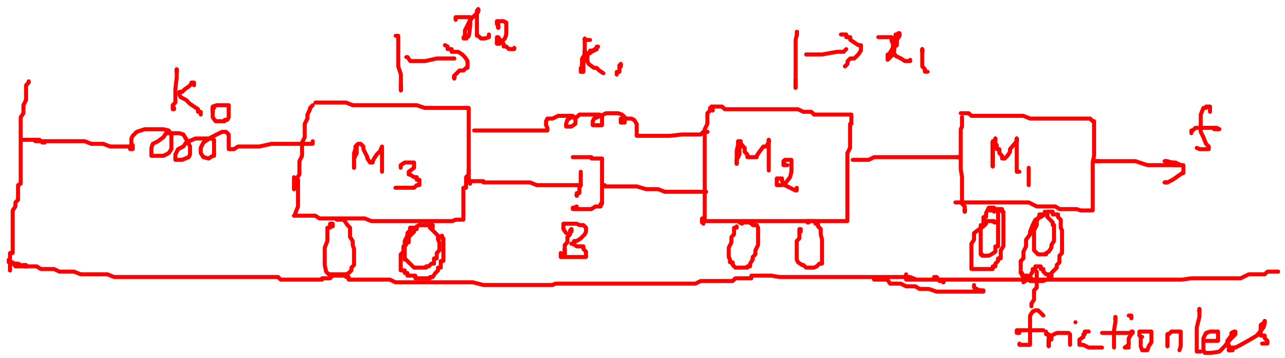
Revision on Unit 1

Karpagavalli S.

Department of Electronics and Communication Engineering

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Differential equation from Physical Systems



$$\frac{X_2(s)}{F(s)} \quad \& \quad \frac{X_1(s)}{F(s)}$$

$$f(t) = (M_1+M_2)\frac{d^2x_1}{dt^2} + K_1(x_1-x_2) + B\frac{d(x_1-x_2)}{dt}$$

$$0 = M_3\frac{d^2x_2}{dt^2} + K_1(x_2-x_1) + B\frac{d(x_2-x_1)}{dt} + K_0x_2$$

Taking LT

$$F(s) = (M_1+M_2)s^2X_1(s) + K_1(X_1(s)-X_2(s)) + Bs(X_1(s)-X_2(s))$$

$$0 = M_3s^2X_2(s) + K_1(X_2(s)-X_1(s)) + Bs(X_2(s)-X_1(s)) + K_0X_2(s)$$

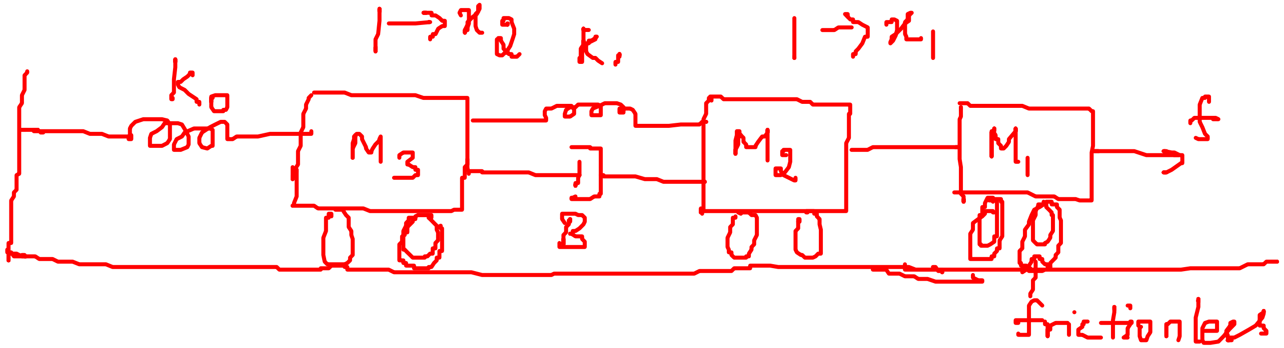
Differential equation from Physical Systems

$$\left((M_1 + M_2)s^2 + B s + K_1\right) X_1(s) - (Bs + K_1) X_2(s) = F(s)$$

$$(M_3 s^2 + K_0 + K_1 + Bs) X_2(s) - (K_1 + Bs) X_1(s) = 0$$

$$X_2(s) = \frac{\begin{vmatrix} (M_1+M_2)s^2 + Bs + K_1 & F(s) \\ -(K_1 + BS) & 0 \end{vmatrix}}{\begin{vmatrix} (M_1+M_2)s^2 + Bs + K_1 & -(BS + K_1) \\ -(K_1 + BS) & M_3 s^2 + K_0 + K_1 + BS \end{vmatrix}} = \frac{K_1 + Bs \cdot F(s)}{(M_1+M_2)s^2 (M_3 s^2 + Bs + K_1 + K_0) + (M_3 s^2 + K_0)(K_1 + Bs)}$$

order of the system = 4 $(K_0, K_1, M_3, M_1 + M_2)$



$$f = (M_1 + M_2)\frac{d^2x_1}{dt^2} + K_1(x_1 - x_2) + B\frac{d(x_1 - x_2)}{dt}$$

$$0 = M_3\frac{d^2x_2}{dt^2} + K_0 x_2 + K_1(x_2 - x_1) + B\frac{d}{dt}(x_2 - x_1)$$

f-V, $M = L$, $B = R$, $K = \frac{1}{C}$,

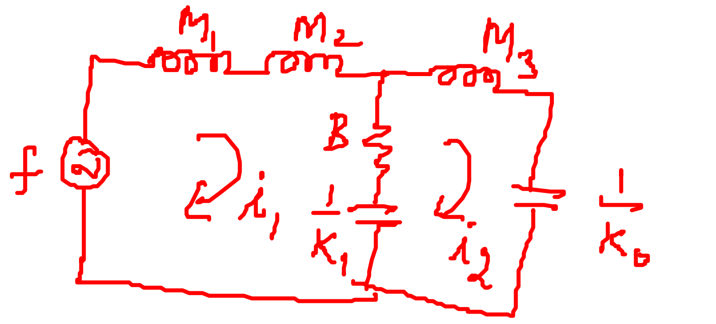
f-I

$M = C$, $B = \frac{1}{R}$, $K = \frac{1}{L}$

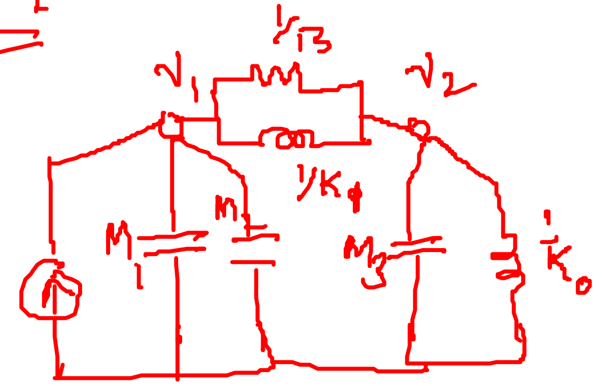
$x_1 = i_1$

$x_2 = i_2$

f-V



f-I



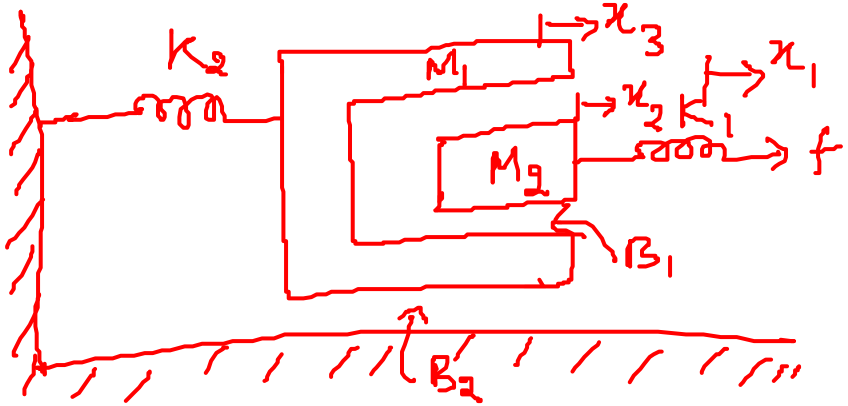
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Differential equation from Physical Systems

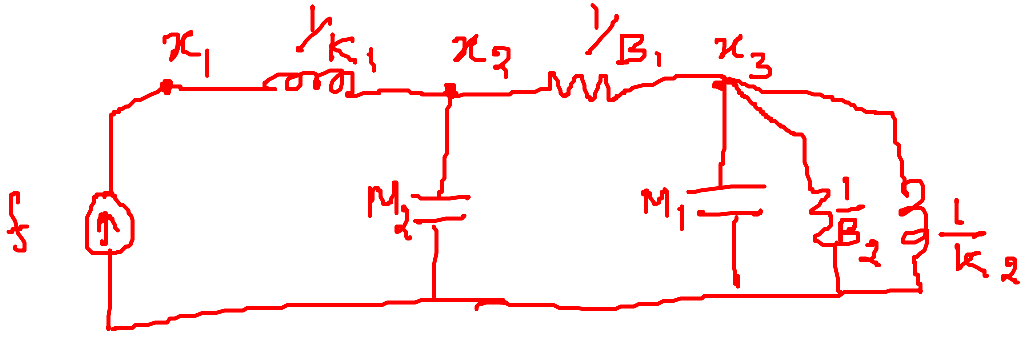
$$f(t) = K_1(x_1 - x_2)$$

$$0 = M_2 \frac{d^2 x_2}{dt^2} + K_1(x_2 - x_1) + B_1 \frac{d(x_2 - x_3)}{dt}$$

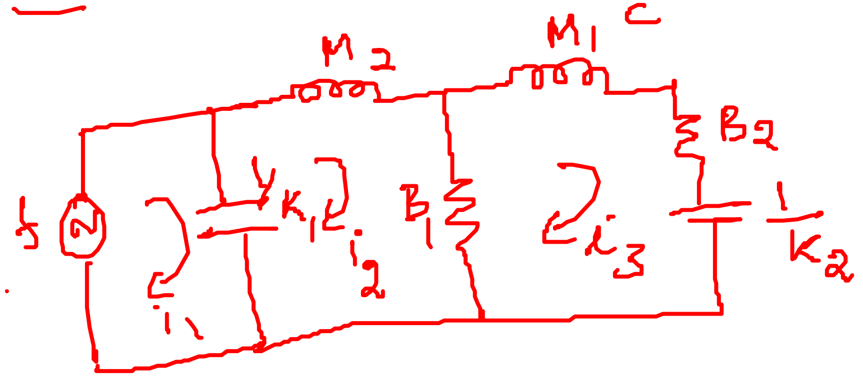
$$0 = M_1 \frac{d^2 x_3}{dt^2} + B_1 \frac{d(x_3 - x_2)}{dt} + B_2 \frac{dx_3}{dt} + K_2 x_3$$



F - i, $M = C$, $K = \frac{1}{L}$, $B = \frac{1}{R}$



F - V, $M = L$, $K = \frac{1}{C}$, $B = R$



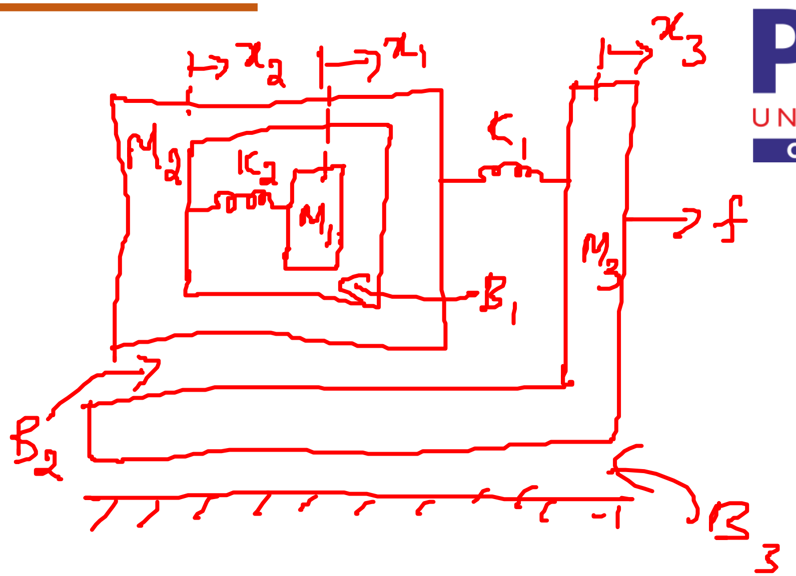
MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Differential equation from Physical Systems

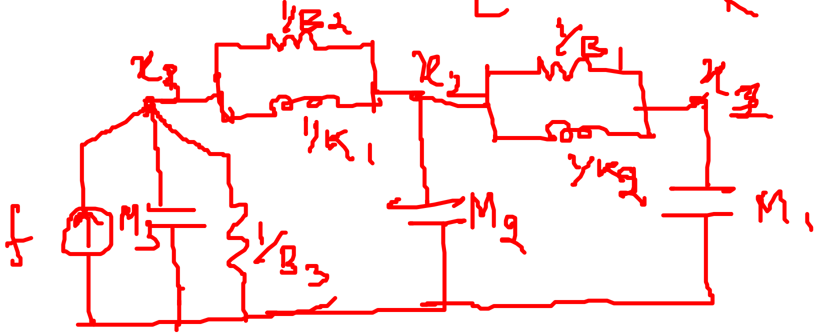
$$f(t) = M_3\ddot{x}_3 + B_3\dot{x}_3 + B_2(\dot{x}_3 - \dot{x}_2) + K_1(x_3 - x_2)$$

$$0 = M_2\ddot{x}_2 + B_2(\dot{x}_2 - \dot{x}_3) + K_1(x_2 - x_3) + B_1(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1)$$

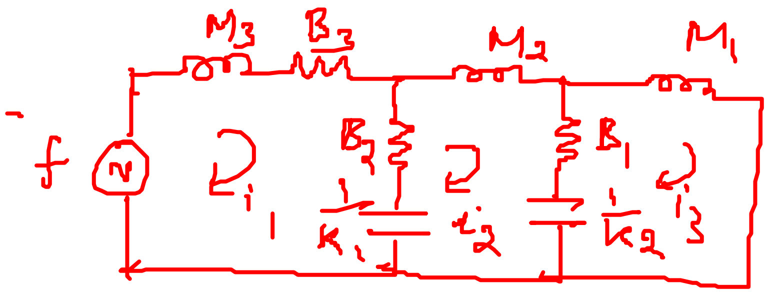
$$0 = M_1\ddot{x}_1 + B_1(\dot{x}_1 - \dot{x}_2) + K_2(x_1 - x_2)$$



F - i $\rightarrow$ $M = C$, $K = \frac{1}{L}$, $B = \frac{1}{R}$

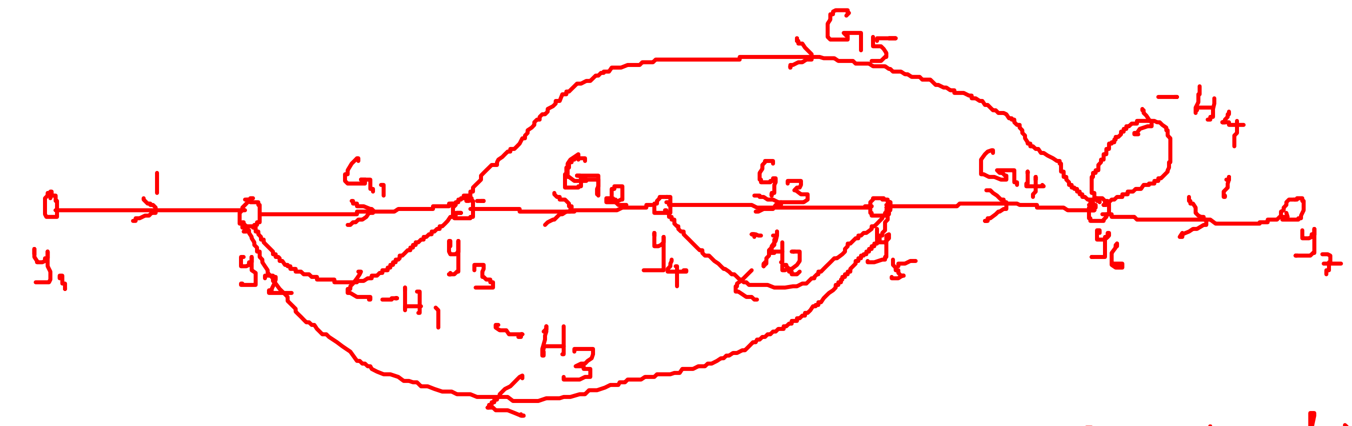


F - V $M = L$, $K = \frac{1}{C}$, $B = R$



MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Transfer Functions using Signal Flow Graph – Example



Find $\frac{y_7}{y_1}$, $\frac{y_7}{y_2}$

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_5$

$L_1 = -G_1 H_1$

$L_2 = -G_1 G_2 G_3 H_3$

$L_3 = -G_3 H_2$

$L_4 = -H_4$

Non-touching loops

$\underline{2}$: $L_1 L_3$, $L_1 L_4$, $L_3 L_4$, $L_2 L_4$

$\underline{3}$: $L_1 L_3 L_4$

$\Delta_1 = 1$

$\Delta_2 = 1 - L_3$

$$\frac{y_7}{y_1} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3 + L_1 L_4 + L_3 L_4 + L_2 L_4) - L_1 L_3 L_4$$

$$\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1} = \frac{\left.\frac{\sum P_k \Delta_k}{\Delta}\right| \text{from } y_1 \text{ to } y_7}{\left.\frac{\sum P_k \Delta_k}{\Delta}\right| \text{from } y_1 \text{ to } y_2}$$

$$= \frac{\sum P_k \Delta_k \,|\, \text{from } y_1 \text{ to } y_7}{\sum P_k \Delta_k \,|\, \text{from } y_1 \text{ to } y_2}$$

$$= P_1 \Delta_1 + P_2 \Delta_2$$

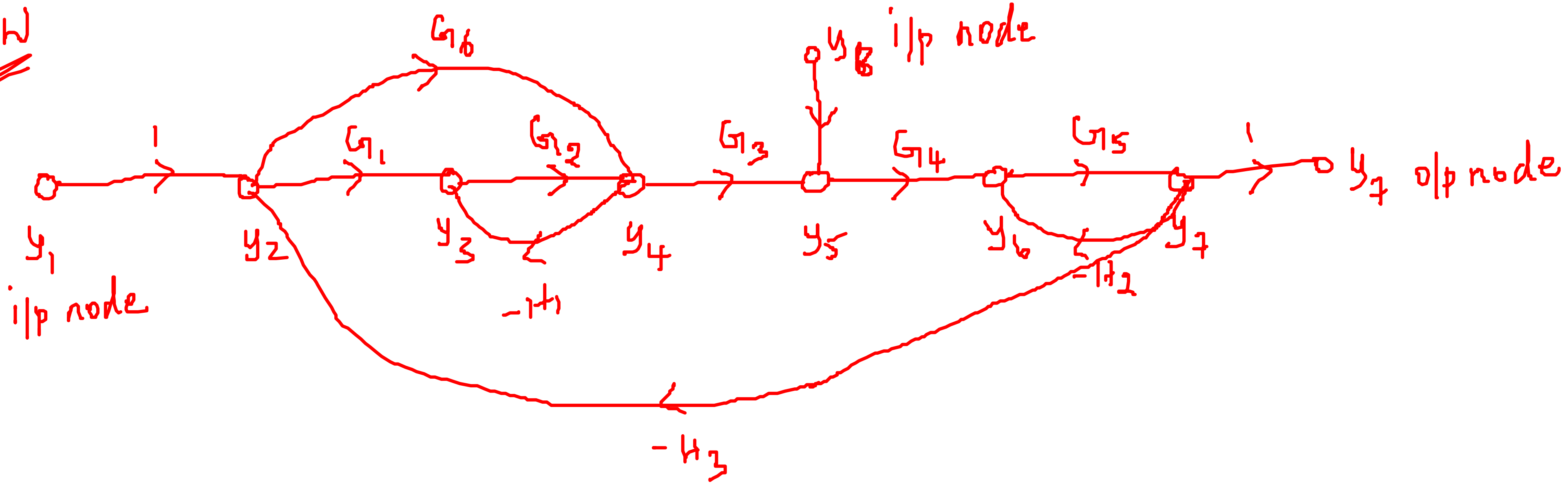
for $y_2/y_1 \Rightarrow P_1 = 1, \quad \Delta_1 = 1 - (L_3 + L_4) + L_3 L_4$

$$\frac{y_2}{y_1} = P_1 \Delta_1$$

$$\frac{y_7}{y_2} = \frac{P_1 \Delta_1 + P_2 \Delta_2 \,|\, \text{from } y_1 \text{ to } y_7}{P_1 \Delta_1 \,|\, \text{from } y_1 \text{ to } y_2}$$

$$= \frac{G_1 G_2 G_3 G_4 + G_1 G_5 (1 - L_3)}{1 - (L_3 + L_4) + L_3 L_4}$$

H. W



Find the trans functions

a) $\left.\frac{y_7}{y_1}\right|_{y_8=0}$

b) $\left.\frac{y_7}{y_8}\right|_{y_1=0}$

$\frac{y_7/y_1}{y_8/y_1}$

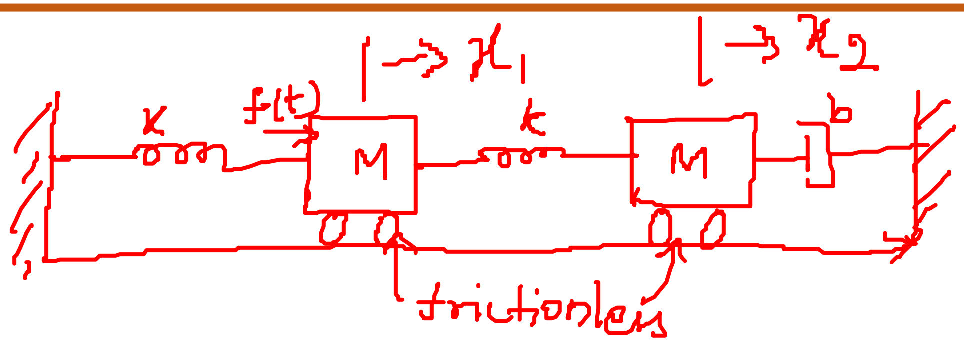
c) $\left.\frac{y_7}{y_4}\right|_{y_8=0}$

$\frac{y_7/y_1}{y_4/y_1}$

d) $\left.\frac{y_7}{y_4}\right|_{y_1=0}$

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Differential equation from Physical Systems



$$f(t) = M\ddot{x}_1 + K x_1 + K(x_1 - x_2)$$

$$0 = M\ddot{x}_2 + b\dot{x}_2 + k(x_2 - x_1)$$

THANK YOU

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