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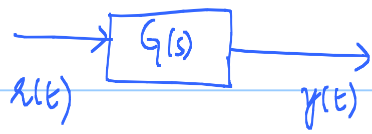


$$r(t) = A\sin(\omega t), \quad t \geq 0$$

$$R(s) = \frac{A\omega}{s^2+\omega^2}$$

$$Y(s) = G(s)\,R(s)$$

$$Y(s) = G(s)\,\frac{A\omega}{s^2+\omega^2}$$

$$Y(s) = \left[\frac{A_1}{s+j\omega} + \frac{A_2}{s-j\omega}\right]$$

$$A_1 = \frac{A\omega\,G(s)}{\cancel{(s+j\omega)}(s-j\omega)}\,\cancel{(s+j\omega)}\Bigg|_{s=-j\omega}$$

$$= \frac{A\omega\,G(-j\omega)}{-2j\omega}$$

$$A_2 = \frac{A\omega\,G(s)}{(s+j\omega)(s-j\omega)}\,(s-j\omega)\Bigg|_{s=j\omega}$$

$$A_2 = \frac{A\omega\,G(j\omega)}{2j\omega}$$

$$Y(s) = -\frac{A\omega G(-j\omega)}{2j\omega} \times \frac{1}{s+j\omega} + \frac{A\omega G(+j\omega)}{2j\omega} \times \frac{1}{s-j\omega}$$

$$G(j\omega) = |G(j\omega)|\,e^{j\angle G(j\omega)}$$

$$G(-j\omega) = |G(j\omega)|\,e^{-j\angle G(j\omega)} \qquad \phi$$

$$|G(-j\omega)|\,e^{j\angle G(-j\omega)} = |G(j\omega)|\,e^{-j\phi}$$

$$Y(s) = -\frac{A\omega\,|G(j\omega)|\,e^{-j\phi}}{2j\omega\,(s+j\omega)} + \frac{A\omega}{2j\omega}\,\frac{|G(j\omega)|\,e^{j\phi}}{(s-j\omega)}$$

$$y(t) = -\frac{A\omega |G(j\omega)| e^{-j\phi}}{2j\omega} e^{-j\omega t} + \frac{A\omega |G(j\omega)| e^{j\phi}}{2j\omega} e^{j\omega t}$$

$$y(t) = \frac{A\cancel{\omega}|G(j\omega)|}{\cancel{\omega}} \left[-\frac{e^{-j(\omega t+\phi)}}{2j} + \frac{e^{j(\omega t+\phi)}}{2j} \right]$$

$$y(t) = A|G(j\omega)| \sin(\omega t + \phi)$$

When the input to the system is a sinusoidal signal the output is always the sinusoidal signal with same freq. as input but different amplitude and phase

$G(j\omega)$ – Sinusoidal Transfer function
Frequency response

1. Bode plot
2. Nyquist plot

} – Stability analysis Techniques

$$G(s) = \frac{K \prod_{i=1}^{m} (s+z_i)}{s^N \prod_{j=1}^{n_1} (s+p_j) \prod_{j=1}^{n_2} (s^2 + 2\zeta\omega_j s + \omega_j^2)}$$

$s = j\omega$

$$G(j\omega) = \frac{K \prod_{i=1}^{m} (j\omega + z_i)}{(j\omega)^N \prod_{j=1}^{n_1} (j\omega + p_j) \prod_{j=1}^{n_2} \left((j\omega)^2 + 2\zeta\omega_j (j\omega) + \omega_j^2 \right)}$$

$$= \frac{K z_1 z_2 \cdots z_m \prod_{i=1}^{m} (j\omega T_i + 1)}{\omega_1^2 \cdots \omega_{n_2}^2 \, p_1 p_2 \cdots p_{n_1} (j\omega)^N \prod_{j=1}^{n_1} (j\omega T_j + 1) \prod_{j=1}^{n_2} \left(\left(\frac{j\omega}{\omega_j}\right)^2 + 2\zeta \left(\frac{j\omega}{\omega_j}\right) + 1 \right)}$$

Time constant form
$$\boxed{G(j\omega) = \frac{K' \prod_{i=1}^{m} (j\omega T_i + 1)}{(j\omega)^N \prod_{j=1}^{n_1} (j\omega T_j + 1) \prod_{j=1}^{n_2} \left(\left(\frac{j\omega}{\omega_j}\right)^2 + 2\zeta \left(\frac{j\omega}{\omega_j}\right) + 1 \right)}}$$

$$\left(\left(\frac{j\omega}{\omega_1}\right)^2 + 2\zeta j\frac{\omega}{\omega_1} + 1\right) = \left(-\frac{\omega^2}{\omega_1^2} + j\frac{2\zeta\omega}{\omega_1} + 1\right) = \left(1 - \left(\frac{\omega}{\omega_1}\right)^2 + j\,2\zeta\frac{\omega}{\omega_1}\right)$$

$$M = \sqrt{\left(1-\left(\frac{\omega}{\omega_1}\right)^2\right)^2 + \left(\frac{2\omega\zeta}{\omega_1}\right)^2}$$

$$|G(j\omega)| = \frac{K' \prod_{i=1}^{m} \sqrt{1+\omega^2 T_i^2}}{\omega^N \prod_{j=1}^{n_1} \sqrt{1+\omega^2 T_j^2} \prod_{j=1}^{n_2} \sqrt{\left(1-\left(\frac{\omega}{\omega_j}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_j}\right)^2}} \quad \text{--- } (1)$$

$$\angle G(j\omega) = \sum_{i=1}^{m} \tan^{-1}(\omega T_i) - 90 \times N - \sum_{j=1}^{n_1} \tan^{-1}(\omega T_j) - \underbrace{\sum_{j=1}^{n_2} \tan^{-1}\left(\frac{2\zeta\omega/\omega_j}{\left(1-\left(\frac{\omega}{\omega_j}\right)^2\right)}\right)}_{2\text{ poles}}$$

Apply $20\log_{10}$ on eq^n (1)

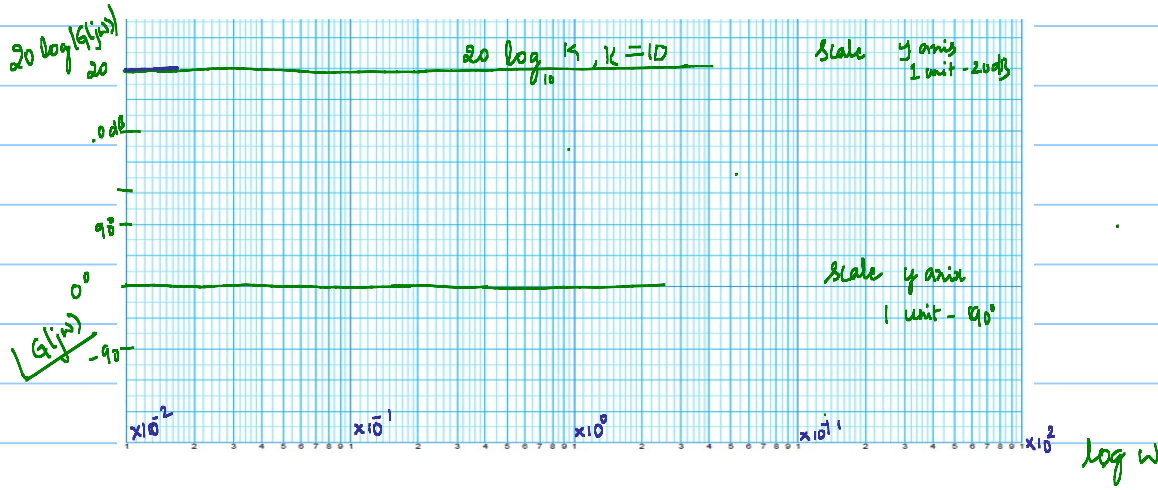
$$20\log_{10}|G(j\omega)| = 20\log_{10} K' + \sum_{i=1}^{m} 20\log_{10}\left(\sqrt{1+\omega^2 T_i^2}\right) - 20N\log_{10}\omega$$
$$- \sum_{j=1}^{n_1} 20\log_{10}\sqrt{1+\omega^2 T_j^2} - \sum_{j=1}^{n_2} 20\log\left(\sqrt{\left(1-\left(\frac{\omega}{\omega_j}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_j}\right)^2}\right)$$

1. The constant K

$$G(s) = K$$
$$s = j\omega$$
$$G(j\omega) = K$$

$$|G(j\omega)| = K \qquad \angle G(j\omega) = \tan^{-1}\left(\frac{0}{K}\right) = 0^\circ$$

$$20\log_{10}|G(j\omega)| = 20\log_{10} K$$



2. <u>Integral term:</u>

$$G(s) = 1/s$$

$$s = j\omega$$

$$G(j\omega) = \frac{1}{j\omega}$$

$$|G(j\omega)| = \frac{1}{\omega} \qquad \angle G(j\omega) = -90 \qquad -\tan^{-1}\left(\frac{\omega}{0}\right)$$

Magnitude $M = 20\log_{10}|G(j\omega)| = 20\log_{10}\frac{1}{\omega} = -20\log_{10}\omega$

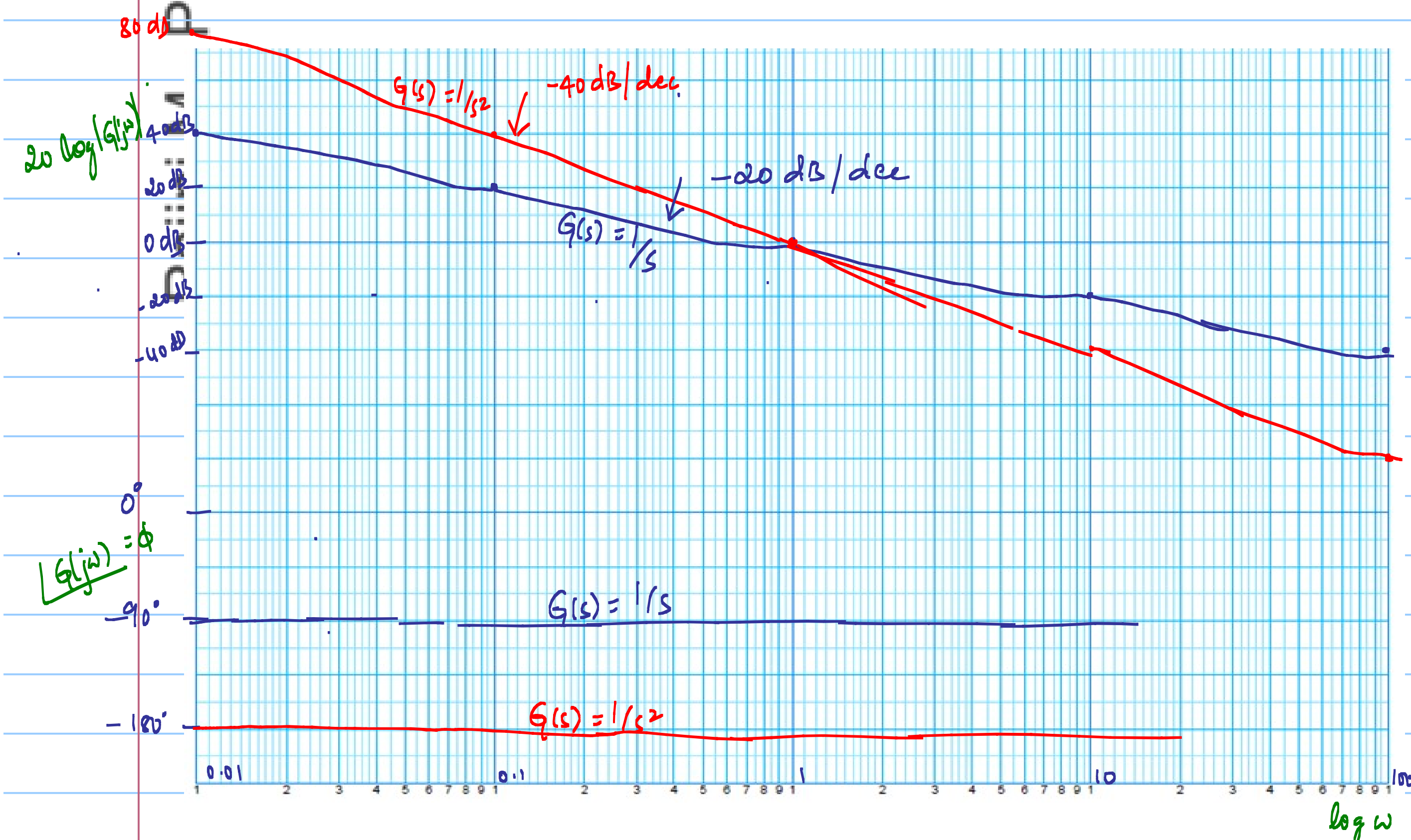
$\omega = 0.01$ $\quad M = -20\log 0.01 = -20\log 10^{-2} = +40\ dB$

$\omega = 0.1$ $\quad M = -20\log 0.1 = 20\ dB$

$\omega = 1$ $\quad M = -20\log 1 = 0\ dB$

$\omega = 10$ $\quad M = -20\ dB$

$\omega = 100$ $\quad M = -40\ dB$



$$G(s) = \frac{1}{s^N} \qquad G(j\omega) = \frac{1}{(j\omega)^N} \qquad |G(j\omega)| = \frac{1}{\omega^N} \qquad \angle G(j\omega) = -90 \times N$$

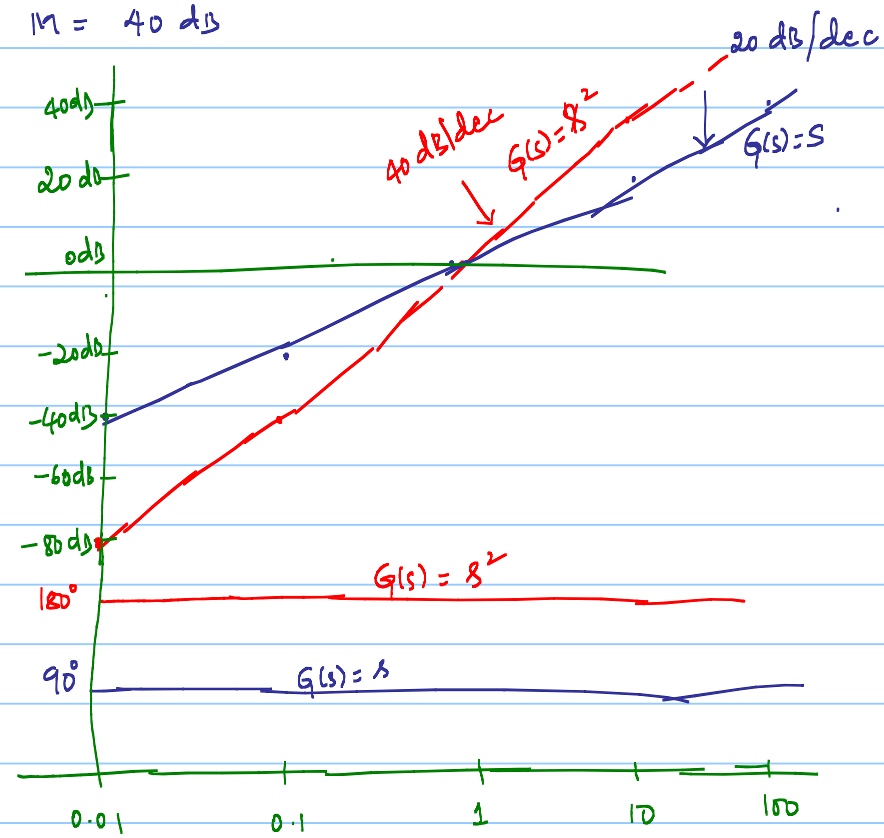
$$20\log_{10}|G(j\omega)| = 20\log_{10} 1/\omega^N = -20N\log_{10}\omega$$

3. $G(s) = s$

$G(j\omega) = j\omega$

$20 \log_{10} |G(j\omega)| = 20 \log_{10} \omega$ $\qquad \angle G(j\omega) = 90^\circ$

$\omega = 0.01$	$M = -40$ dB
$\omega = 0.1$	$M = -20$ dB
$\omega = 1$	$M = 0$ dB
$\omega = 10$	$M = 20$ dB
$\omega = 100$	$M = 40$ dB



4. $G(s) = \dfrac{1}{(1 + sT)}$ $\qquad$ T - constant $\qquad \dfrac{1}{s + p_j} = \dfrac{1}{p_j(s/p_j + 1)}$

$s = j\omega$

$G(j\omega) = \dfrac{1}{1 + j\omega T}$

$20 \log_{10} |G(j\omega)| = 20 \log_{10} \dfrac{1}{\sqrt{1 + \omega^2 T^2}}$

$= -20 \log_{10} \sqrt{1 + \omega^2 T^2}$

$= -10 \log_{10} (1 + \omega^2 T^2)$

when $\omega T \ll 1$ $\quad M \approx -10 \log_{10} 1 = 0$ dB $\quad$ — 0 dB line

$\omega T = 1$ $\quad M \approx -10 \log_{10} 2 = -3$ dB

$\omega T >> 1 \qquad M \approx -10 \log_{10} (\omega T)^2$

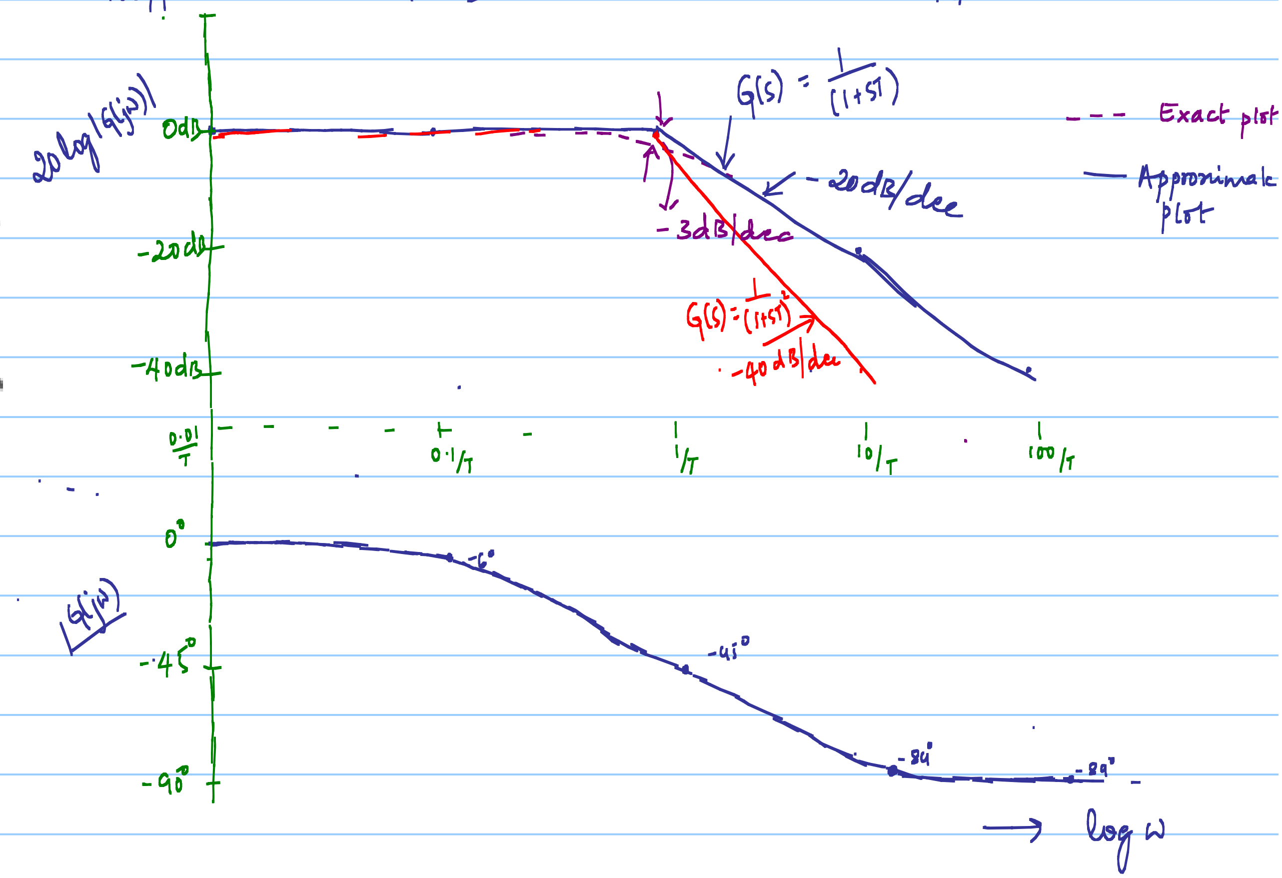
$= -20 \log_{10} \omega T$

a line with the slope -20 dB/dec

At $\omega T = 1 \Rightarrow \omega = 1/T$ — corner frequency

$\angle G(j\omega) = -\tan^{-1}(\omega T)$

	M	$\angle G(j\omega)$
$\omega = 0.01/T$	0 dB	0°
$\omega = 0.1/T$	0 dB	-5.7°
$\omega = 1/T$	0 dB (Exact plot: -3 dB)	-45°
$\omega = 10/T$	-20 dB	-84°
$\omega = 100/T$	-40 dB	-89.4°



5. $G(s) = (sT + 1)$

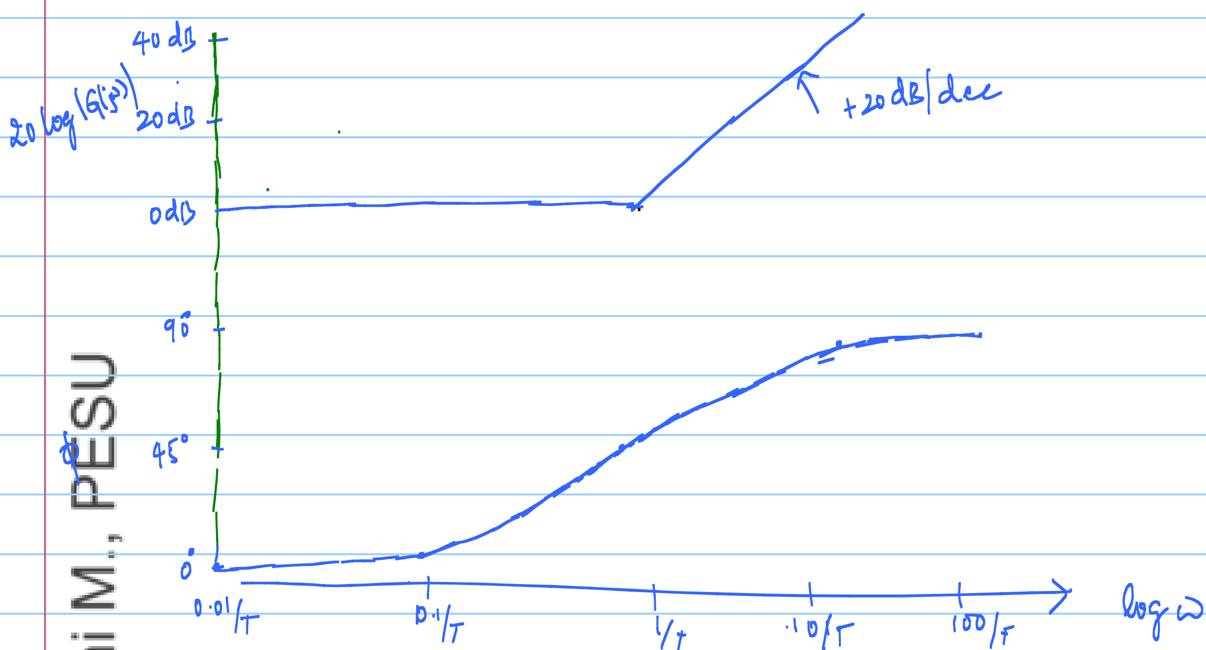
$G(j\omega) = \underline{(j\omega T + 1)}$

$20 \log |G(j\omega)| = 20 \log \sqrt{\omega^2 T^2 + 1} = 10 \log_{10} (\omega^2 T^2 + 1)$

$$M = \begin{cases} 0 \text{ dB} & \omega T << 1 \\ 3 \text{ dB} & \omega T = 1 \leftarrow \text{Exact plot} \\ 20 \log_{10} \omega T & \omega T >> 1 \end{cases}$$

$\underline{/G(j\omega)} = \tan^{-1}(\omega T)$

ω	M	$\underline{/G(j\omega)}$
$0.01/T$	0 dB	$0°$
$0.1/T$	0 dB	$5.7°$
corner frequency → $1/T$	0 dB	$45°$
$10/T$	+ 20 dB	$84.2°$
$100/T$	40 dB	$89.04°$



6. $$G(s) = \frac{1}{\left(\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\frac{s}{\omega_n} + 1\right)}$$

$$G(j\omega) = \frac{1}{\left(\left(j\omega/\omega_n\right)^2 + 2\zeta\, j\frac{\omega}{\omega_n} + 1\right)}$$

$$= \frac{1}{\left(1 - \underbrace{(\omega/\omega_n)}_{u}^2 + j\,2\zeta\frac{\omega}{\omega_n}\right)}$$

$$20\log|G(j\omega)| = -20\log_{10}\sqrt{(1-u^2)^2 + (2\zeta u)^2}$$

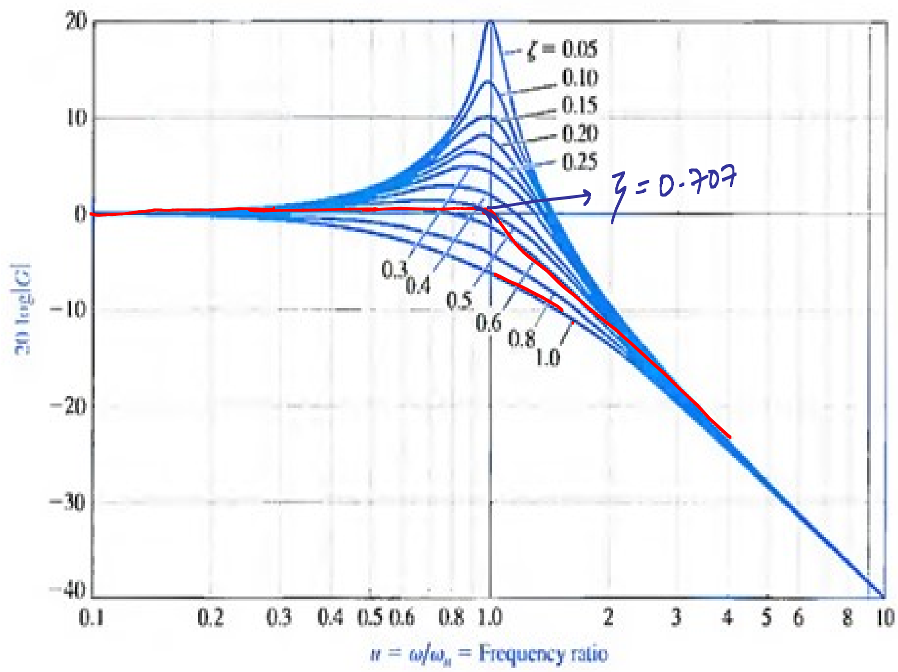
$$= -10\log_{10}\left[(1-u^2)^2 + \underbrace{4\zeta^2u^2}\right]$$

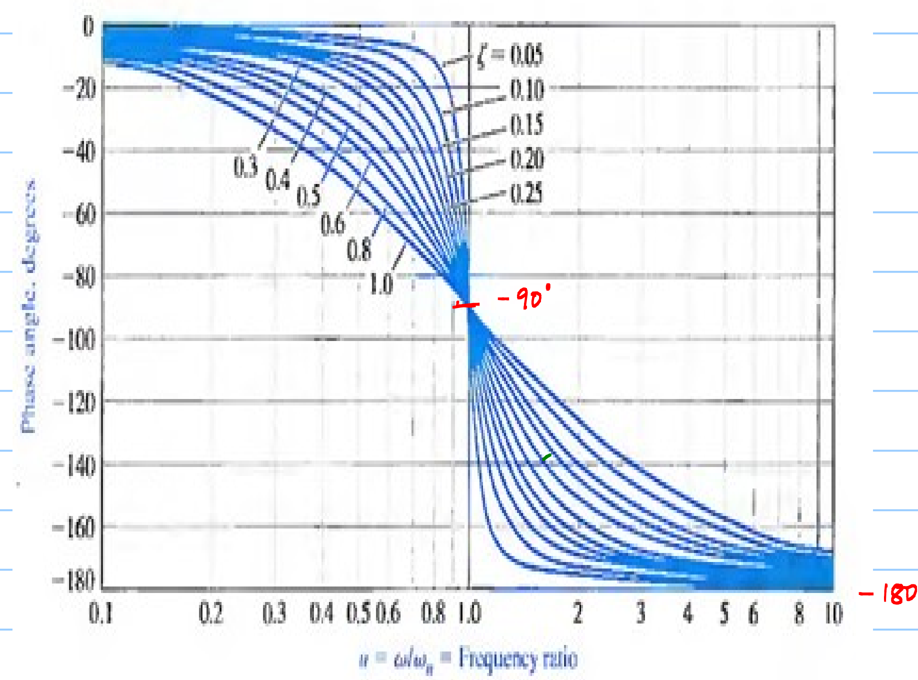
$$M = \begin{cases} 0\ \text{dB} & u \ll 1 \\ -20\log_{10} 2\zeta \quad 4\zeta^2 & u = 1 \\ -40\log_{10} u & u \gg 1 \end{cases}$$

↳ After the corner freq the slope of the line changes to -40 dB/dec

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{2\zeta u}{(1-u^2)}\right)$$

$u = 1 \qquad \frac{\omega}{\omega_n} = 1 \Rightarrow \omega = \omega_n$ —— corner frequency

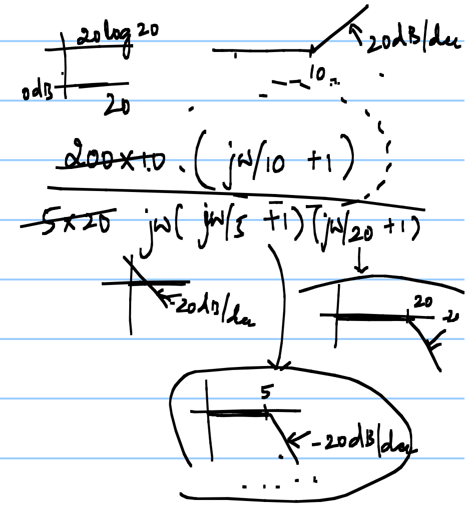




$$KG(s)H(s) = \frac{200\,(s+10)}{s\,(s+5)\,(s+20)}$$

$s = j\omega$

$$KG(j\omega)H(j\omega) = \frac{200\,(j\omega+10)}{j\omega\,(j\omega+5)\,(j\omega+20)} = \frac{\frac{200\times 10}{5\times 20}\,(j\omega/10+1)}{j\omega\,(j\omega/5+1)(j\omega/20+1)}$$



$$= \frac{20\,(j\omega/10+1)}{j\omega\,(j\omega/5+1)(j\omega/20+1)}$$

Corner frequencies: 5, 10, 20

poles zero

↓ Contribute from 10 rad/sec

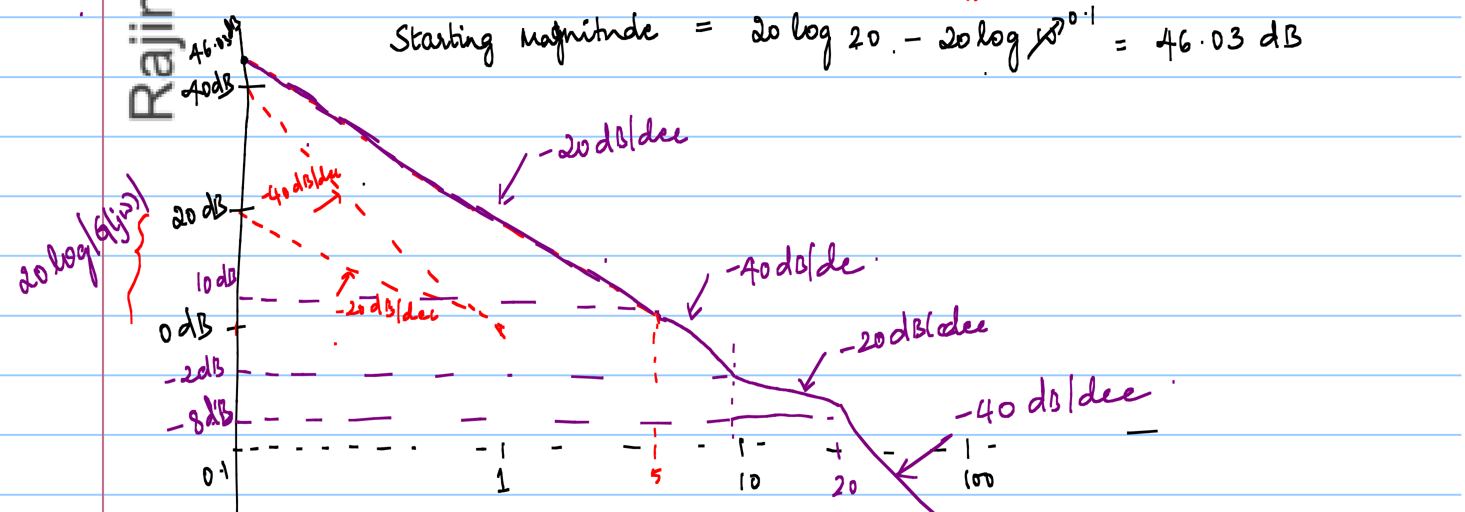
M → $20\log|G(j\omega)H(j\omega)| = \underline{20\log_{10} 20} + \underbrace{20\log_{10}\sqrt{\omega^2/100+1}} - \underline{20\log \omega}$

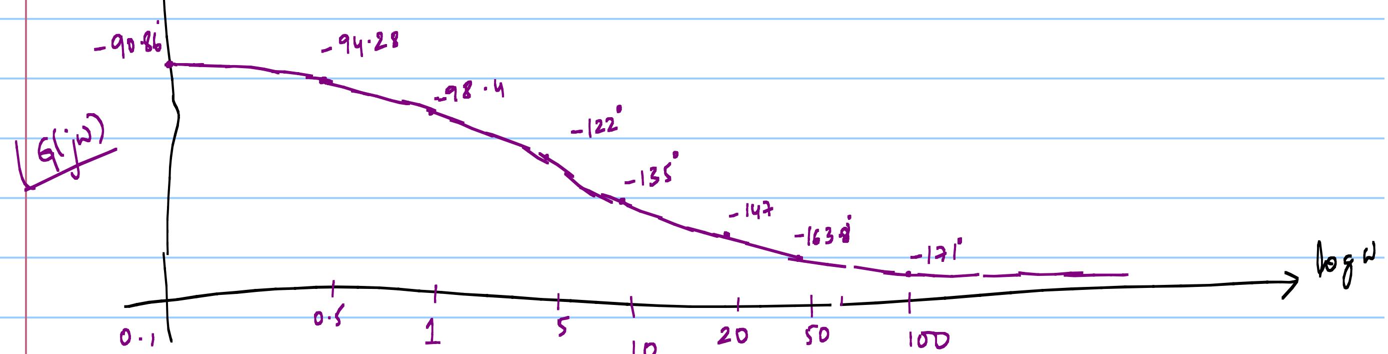
$- \underbrace{20\log_{10}\sqrt{\omega^2/5^2+1}}_{\downarrow \text{ from } 5\,r/s} - \underbrace{20\log_{10}\sqrt{\omega^2/20^2+1}} \rightarrow 20 \text{ rad/sec}$

$\underline{\angle G(j\omega)H(j\omega)} = \tan^{-1}(\omega/10) - \pi/2 - \tan^{-1}(\omega/5) - \tan^{-1}(\omega/20)$

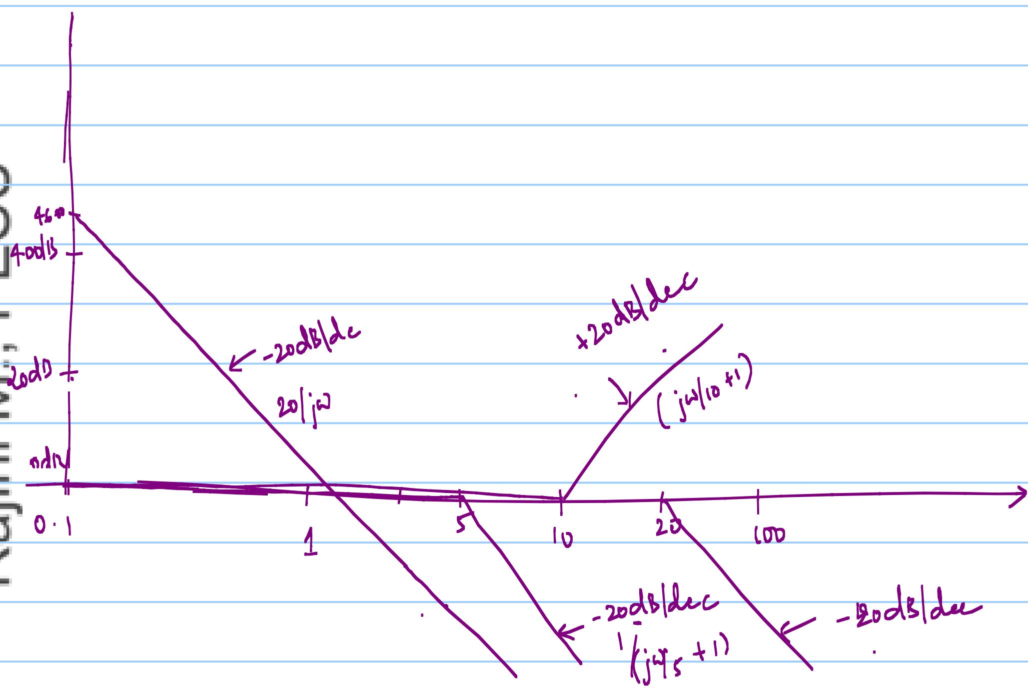
$\tan^{-1}(\omega/10)$: max 90°, min 0°; $\tan^{-1}(\omega/5)$: max 0°, min -90°; $\tan^{-1}(\omega/20)$: max 0°, min -90

Starting magnitude $= 20\log 20 - 20\log 0.1 = 46.03$ dB

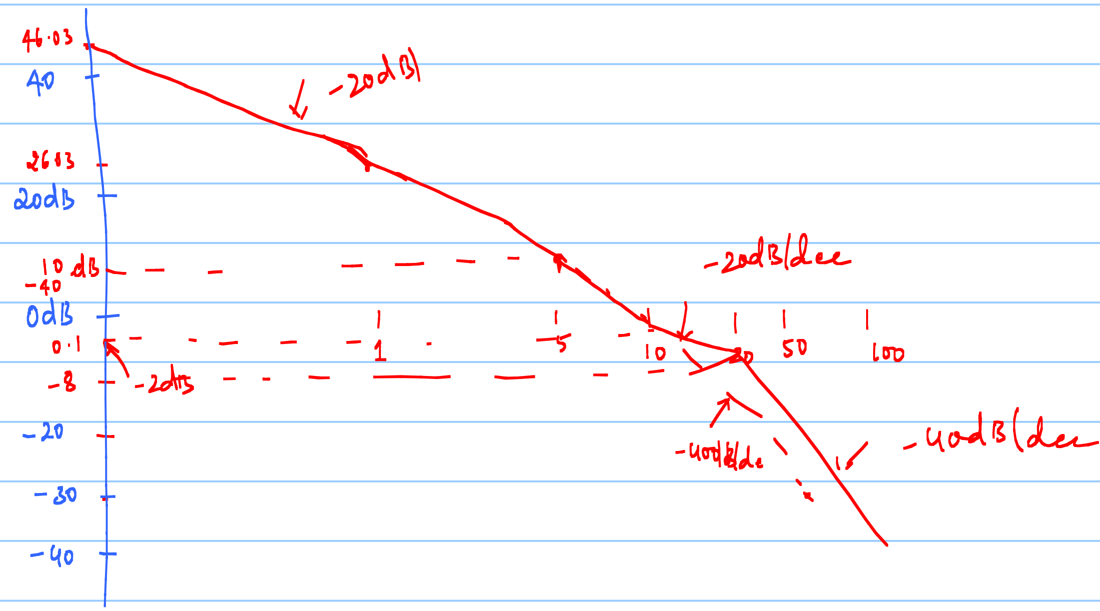




Factor	Corner freq	Slope contributed by each factor	Net slope	Frequency
$20/j\omega$	–	-20 dB/dec	-20 dB/dec	0.1 to 5
$1/(j\omega/5+1)$	5	-20 dB/dec (after 5 rad/sec)	-40 dB/dec	5 to 10
$j\omega/10+1$	10	$+20$ dB/dec (after 10 rad/sec)	-20 dB/dec	10 to 20
$1/(j\omega/20+1)$	20	-20 dB/dec (after 20 rad/sec)	-40 dB/dec	20 to ∞



ω	0.1	0.5	1	2	5	10	20	50	100	200
$\angle G(j\omega)$	-90.85	-94.28	-98.46	-106.2	-122.47	-135	-147.5	-164	-171	-175



(2)

$$G(s) = \frac{5(1+0.1s)}{s(1+0.5s)\left((s/50)^2 + 0.6(s/50)+1\right)}$$

$$s^2 + \underbrace{0.6\times 50s}_{2\zeta\omega_n} + \underbrace{50^2}_{\omega_n^2}$$

$s = j\omega$

$$G(j\omega) = \frac{5(1+j0.1\omega)}{j\omega(1+j0.5\omega)\left((j\omega/50)^2 + j0.6\left(\frac{\omega}{50}\right)+1\right)}$$

Corner freq: 2, 10, 50

Starting magnitude: $20 \log 5 - 20 \log \omega$

Let $\omega = 0.1$

$$20 \log 5 - 20 \log 0.1 = 33.98 \simeq 34\ dB$$

Factor	Corner freq	Slope contributed by each factor	Net slope	Frequency
$5/j\omega$	–	−20 dB/dec	−20 dB/dec	0.1 to 2
$1/(1+j\omega/2)$	2	−20 dB/dec (after 2 rad/s)	−40 dB/dec	2 to 10
$1+j\omega/10$	10	20 dB/dec (after 10 rad/sec)	−20 dB/dec	10 to 50
$1/\left((j\omega/50)^2 + j0.6\frac{\omega}{50}+1\right)$	50	−40 dB/dec (after 50 rad/sec)	−60 dB/dec	50 to ∞

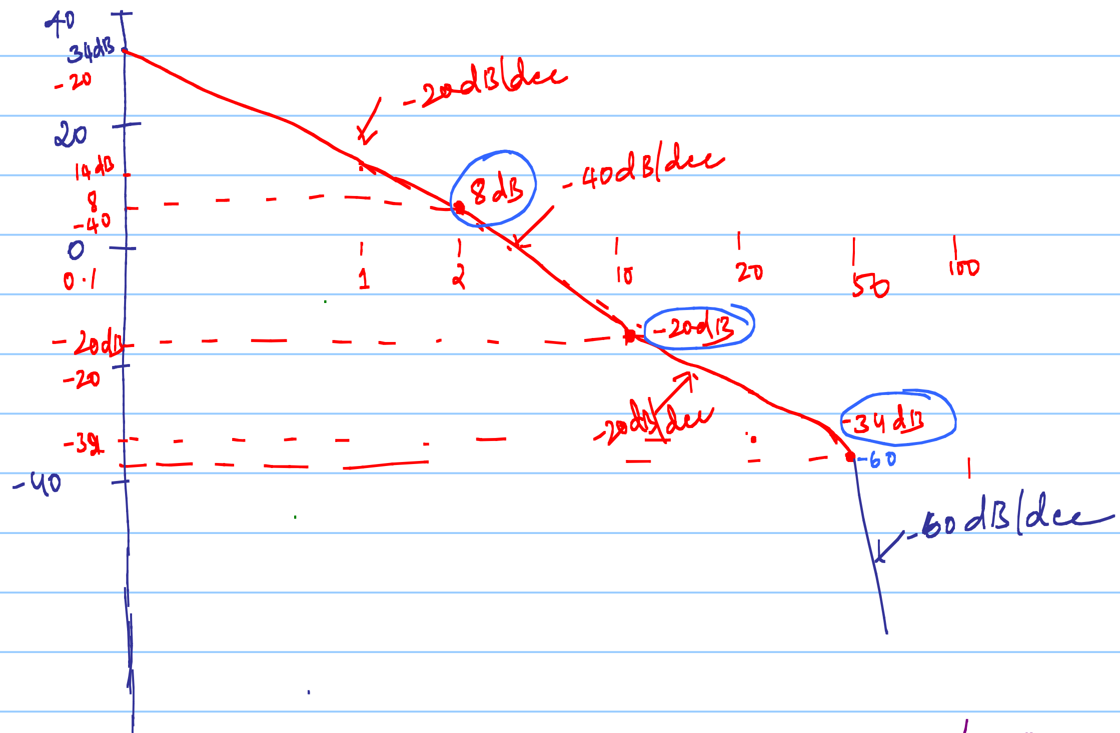
$$\angle G(j\omega) = \tan^{-1}(0.1\omega) - 90 - \tan^{-1}(0.5\omega) - \underbrace{\tan^{-1}\left(\frac{0.6\,\omega/50}{1-\omega^2/50^2}\right)}$$

ω	0.1	0.2	1	2	5	10	20	50	100	500	∞
$\angle G(j\omega)$	-92.35	-94.7	-111.5	-125	-135	-130.8	-126.8	-189	-252.76	-267	-270°

→ when $\omega > 50$

only last term in $\angle G(j\omega)$ $\quad -\left(\pi - \tan^{-1}\left(\frac{0.6\omega/50}{\omega^2/50^2 - 1}\right)\right)$

→ At corner freq 50 rad/sec of the second order factor, the phase is -90°.



$$\text{error} = 20\log 2\zeta = 20\log 2\times 0.3 = -4.4$$

$$\left(\frac{j\omega}{50}\right)^2 + 0.6\,j\frac{\omega}{50} + 1$$

$$= \frac{(j\omega)^2 + 0.6\times 50 j\omega + 50^2}{50^2}$$

$$\Rightarrow \underbrace{(j\omega)^2 + 0.6\times 50 j\omega + 50^2}_{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$2\zeta\omega_n = 0.6\times 50$

$\omega_n = 50$

$\Rightarrow \zeta = 0.3$

$GM = 33 - 4.4$

$\boxed{GM = 28.6\ dB}$

$\omega_{gcf} = 3.2$ rad/sec

$\omega_{pcf} = 47$ rad/sec.

$PM = 180 - 135^\circ$

$= 45^\circ$

→ The CLS stable PM & GM are positive

$\rightarrow$ $G(s) = \dfrac{10}{s(0.5s+1)(0.1s+1)}$. Find ω_{PCF} & GM, ω_{gcf} & PM

$$G(j\omega) = \frac{10}{j\omega(0.5j\omega+1)(0.1j\omega+1)}$$

$$\angle G(j\omega) = \left[-90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.1\omega)\right]\Big|_{\omega=\omega_{PCF}} = -180^\circ$$

$$-90^\circ - \tan^{-1}(0.5\omega_{PCF}) - \tan^{-1}(0.1\omega_{PCF}) = -180^\circ$$

$$+\tan^{-1}(0.5\omega_{PCF}) + \tan^{-1}(0.1\omega_{PCF}) = +90^\circ .$$

Note : $\tan^{-1}A \pm \tan^{-1}B = \tan^{-1}\left(\dfrac{A \pm B}{1 \mp AB}\right)$

$$\tan^{-1}\left(\frac{0.6\,\omega_{PCF}}{1 - 0.05\,\omega_{PCF}^2}\right) = 90^\circ$$

$$\tan 90 = \frac{1}{0}$$

$$\frac{0.6\,\omega_{PCF}}{1 - 0.05\,\omega_{PCF}^2} = \frac{1}{0}$$

$$1 - 0.05\,\omega_{PCF}^2 = 0$$

$$\omega_{PCF} = \sqrt{20} = 4.47 \text{ rad/sec}$$

$$GM\big|_{dB} = 0 - 20\log|G(j\omega)|\Big|_{\omega=\omega_{PCF}}$$

$$20\log|G(j\omega)| = 20\log 10 - 20\log\omega_{pcf} - 20\log\sqrt{1+0.5^2\omega_{PCF}^2} - 20\log\sqrt{1+0.1^2\omega_{PCF}^2}$$

$$= -1.57$$

$$GM = 0 - (-1.57)$$

$$= 1.57 \text{ dB}$$

To find ω_{gcf} $\quad 20\log|G(j\omega)|\Big|_{\omega=\omega_{gcf}} = 0\,dB$

$$20\log_{10}\left(\frac{10}{\omega_{gcf}\sqrt{1+0.5^2\omega_{gcf}^2}\sqrt{1+0.1^2\omega_{gcf}^2}}\right) = 0$$

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$$\frac{10}{\omega_{gcf}\sqrt{1+0.5^2\omega_{gcf}^2}\sqrt{1+0.1^2\omega_{gcf}^2}} = 1$$

$$\Rightarrow \omega_{gcf}^2(1+0.5^2\omega_{gcf}^2)(1+0.1^2\omega_{gcf}^2) = 100$$

$$0.5^2\times0.1^2\,\omega_{gcf}^6 + (0.5^2+0.1^2)\,\omega_{gcf}^4 + \omega_{gcf}^2 - 100 = 0$$

Solve for ω_{gcf} $\quad \omega_{gcf} = \pm j5.02,\ \pm 9.97j,\ \pm 4.07$.

$$\omega_{gcf} = 4.07 \text{ rad/sec.}$$

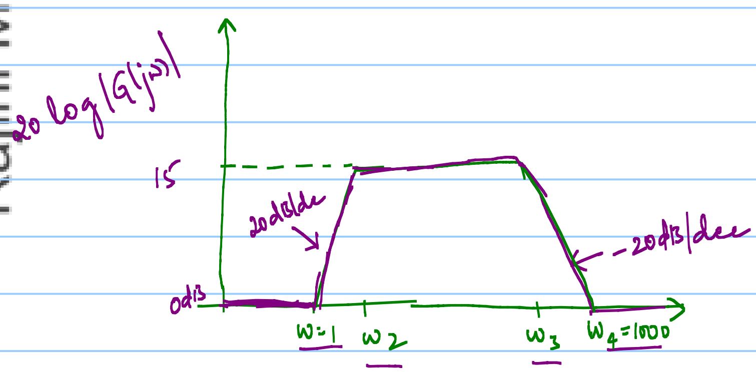
$$\left[-90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.1\omega)\right]\Big|_{\omega=4.07} = \angle G(j\omega)\Big|_{\omega=\omega_{gcf}} = -175.97$$

$$PM = 180^\circ + \angle G(j\omega)\Big|_{\omega=\omega_{gcf}} = \angle G(j\omega)\Big|_{\omega=\omega_{gcf}} - (-180^\circ)$$

$$\boxed{PM = 4.02^0}$$

→ $G(s) = \dfrac{10}{s+20}$ Find GM + PM

Reverse Bode plot



Initial slope : 0 dB/ dec , implies the factor K in G(s)

$$20\log K = 0\,dB$$

$$K = 10^{0/20} = 1$$

$$\boxed{K=1}$$

Freq range	Net slope	Slope contribution	Corner freq	factor
0.1 to ~~ω_1~~ 1	0 dB/dec	0 dB/ dec	—	K
ω_1 to ω_2	20 dB/dec	20 dB/dec	$\omega_1 = 1$	$(1+jw/\omega_1)$
ω_2 to ω_3	0 dB/dec	−20 dB/dec	ω_2	$\dfrac{1}{1+jw/\omega_2}$

ω_3 to ω_4	-20 dB/dec	-20 dB/dec	ω_3	$\frac{1}{1+j\omega/\omega_3}$
ω_4 to ∞	0 dB/dec	$+20$ dB/dec	$\omega_4=1000$	$(1+j\omega/\omega_4)$

$$G(j\omega) = \frac{K\,(1+j\omega/\omega_1)\,(1+j\omega/\omega_4)}{(1+j\omega/\omega_2)\,(1+j\omega/\omega_3)}$$

To find ω_2:

$y_2 = 15 \quad y_1 = 0 \quad x_1 = \omega_1 \quad x_2 = \omega_2 \quad m = +20$

$$y_2 - y_1 = m \log\left(\frac{x_2}{x_1}\right)$$

$$15 - 0 = 20 \log\left(\frac{\omega_2}{1}\right)$$

$$\omega_2 = 10^{15/20} = 5.62 \text{ rad/sec}$$

To find ω_3:

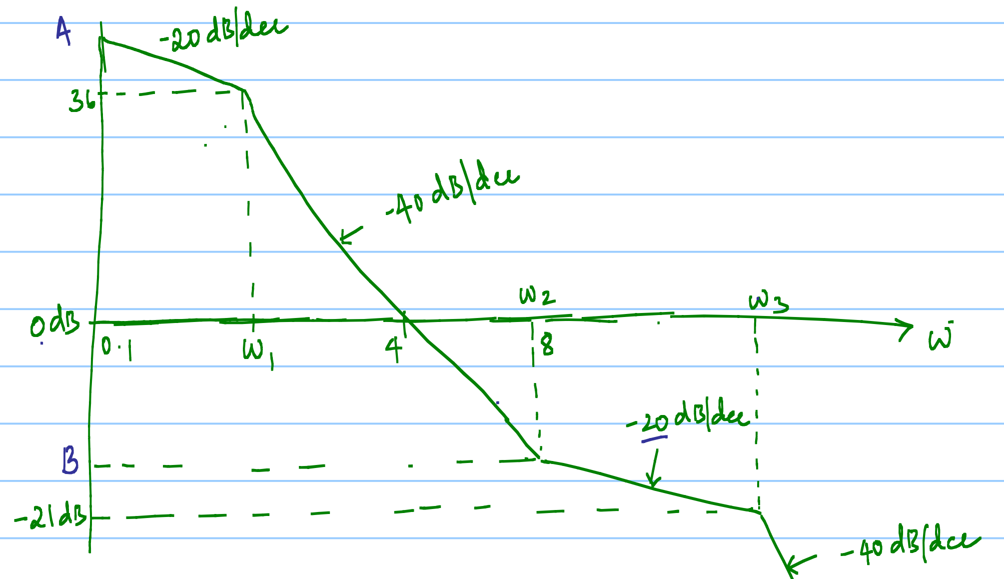
$y_2 = 0 \quad y_1 = 15 \quad x_1 = \omega_3 \quad x_2 = 1000 \quad m = -20$

$$0 - 15 = -20 \log\left(\frac{1000}{\omega_3}\right)$$

$$\Rightarrow \omega_3 = 177.82 \text{ rad/sec}$$

$$\boxed{G(s) = \frac{1\,(1+s/1)\,(1+s/1000)}{(1+s/5.62)\,(1+s/177.82)}}$$

2.



starting slope: -20 dB/dec, indicate the factor $K/j\omega$

Freq range	Net slope	Slope contribution	Corner freq	factor
0.1 to ω_1	−20 dB/dec	−20 dB/dec	—	$K/j\omega$
ω_1 to ω_2 (8)	−40 dB/dec	−20 dB/dec	ω_1	$\frac{1}{(1+j\omega/\omega_1)}$
ω_2 (8) to ω_3	−20 dB/dec	+20 dB/dec	$\omega_2 = 8$	$(1+j\omega/\omega_2)$
ω_3 to ∞	−40 dB/dec	−20 dB/dec	ω_3	$\frac{1}{(1+j\omega/\omega_3)}$

$$G(j\omega) \quad \frac{K(1+j\omega/8)}{j\omega(1+j\omega/\omega_1)(1+j\omega/\omega_3)}$$

To find ω_1:

$$0 - 36 = -40 \log\left(\frac{4}{\omega_1}\right)$$

$$\omega_1 = 0.503 \text{ rad/sec}$$

To find K:

$$A - 36 = -20 \log\left(\frac{0.1}{0.503}\right)$$

$$A = 50.03 \text{ dB}$$

$$50.03 = 20 \log K - 20 \log \cancel{\omega}\ 0.1$$

$$\boxed{K = 31.7}$$

To find ω_3:

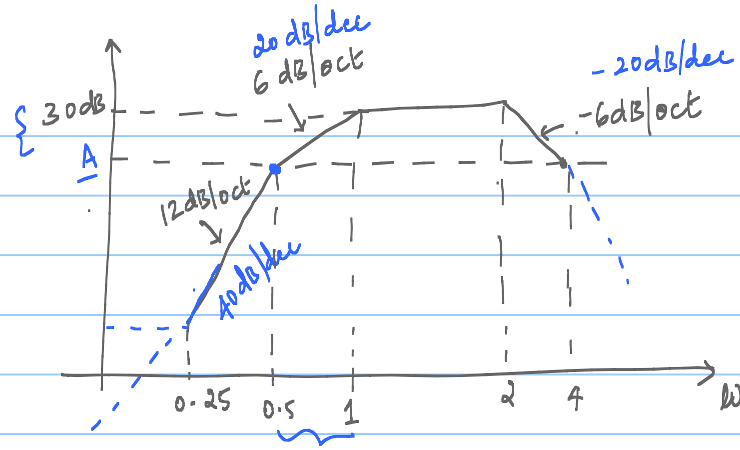
$$0 - B = -40 \log\left(\frac{4}{8}\right)$$

$$B = -12.04 \text{ dB}$$

$$-21 - (-12.04) = -20 \log\left(\frac{\omega_3}{8}\right)$$

$$\omega_3 = 22.5 \text{ rad/sec}$$

$$G(s) = \frac{31.7(1+s/8)}{s(1+s/0.5)(1+s/22.5)}$$



Initial slope 40 dB/dec indicates the factor $K s^2$ or $K(j\omega)^2$

$$G(s) = \frac{K s^2}{(1+s/0.5)(1+s/1)(1+s/2)}$$

$$30 - A = 20 \log_{10}\left(\frac{1}{0.5}\right)$$

$$A = 23.9 \text{ dB}$$

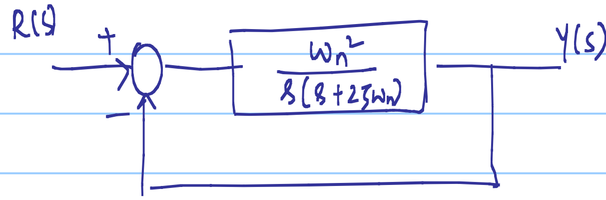
$$24 = 20 \log K \cancel{\omega^2}\, 0.5^2$$

$$K = \frac{10^{(24/20)}}{(0.5)^2}$$

$$K = 63.39$$

$$\boxed{G(s) = \frac{63.39\, s^2}{(1+2s)(1+s)(1+0.5s)}}$$

Frequency Domain Specification :



$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s = j\omega$$

$$T(j\omega) = \frac{Y(j\omega)}{R(j\omega)} = \frac{1}{(j\omega/\omega_n)^2 + j2\zeta\frac{\omega}{\omega_n} + 1}$$

$$T(j\omega) = \frac{1}{(1-\underbrace{(\omega/\omega_n)^2}_{u^2}) + j\,2\zeta\underbrace{\left(\frac{\omega}{\omega_n}\right)}_{u}}$$

Let $u = \omega/\omega_n$

$$\rightarrow M = \frac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}} \qquad \alpha = -\tan^{-1}\left(\frac{2\zeta u}{1-u^2}\right)$$

Valid for $0 < \zeta < 0.707$

$$\frac{dM}{du} = 0 = \frac{-1}{2}\frac{(4u^3 - 4u + 8u\zeta^2)}{((1-u^2)^2 + (2\zeta u)^2)^{3/2}}$$

$$\Rightarrow 4u^3 - 4u + 8u\zeta^2 = 0 \qquad 4u(u^2 - 1 + 2\zeta^2) = 0$$

$$u = 0,\ \pm\sqrt{1-2\zeta^2}$$

$$u = \sqrt{1-2\zeta^2}$$

$$\frac{\omega}{\omega_n} = \sqrt{1-2\zeta^2}$$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$$ Resonant frequency

$$M\Big|_{u=\frac{\omega_r}{\omega_n}} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega_r}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega_r}{\omega_n}\right)^2}}$$

$$\boxed{M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$ Resonant Peak Magnitude

To find Bandwidth

$$M = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1-u^2)^2 + 4\zeta^2u^2}}$$

$$\boxed{BW = \omega_n\left[(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}\right]^{1/2}}$$

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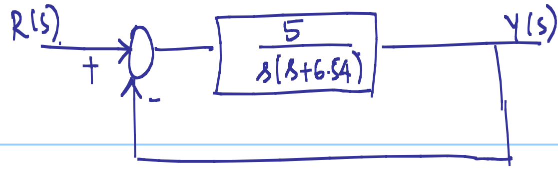

Find M_p, W_r, BW

$W_n = \sqrt{5}$ $\quad$ $2\zeta W_n = 6.54$

$\zeta = 1.46$

$\because \zeta > 0.707$ $\quad M_r = 1$ $\quad W_r = 0$ rad/sec

$$\boxed{BW = 0.867 \text{ rad/sec}}$$

<u>Refer to slides for missing topics and extra problems</u>