CONTROL SYSTEMS

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Introduction to Controllers

Controller Design

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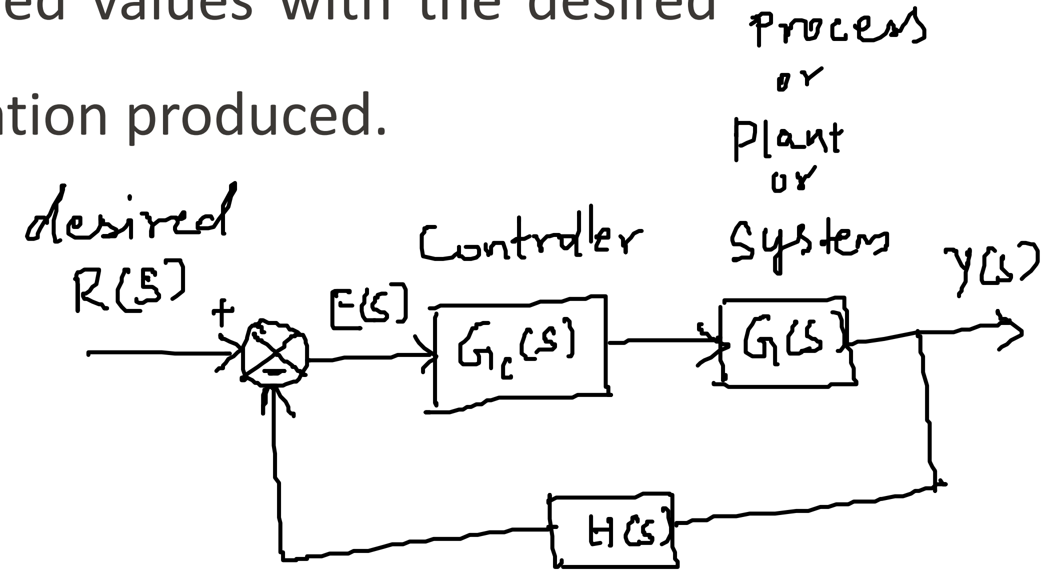
Introduction to Controllers

Controllers

- A controller is one which compares controlled values with the desired values and has a function to correct the deviation produced.
- There are three basic types of controllers:
  - Proportional controller { P controller }
  - Derivative controller { D controller}
  - Integral controller { I controller }

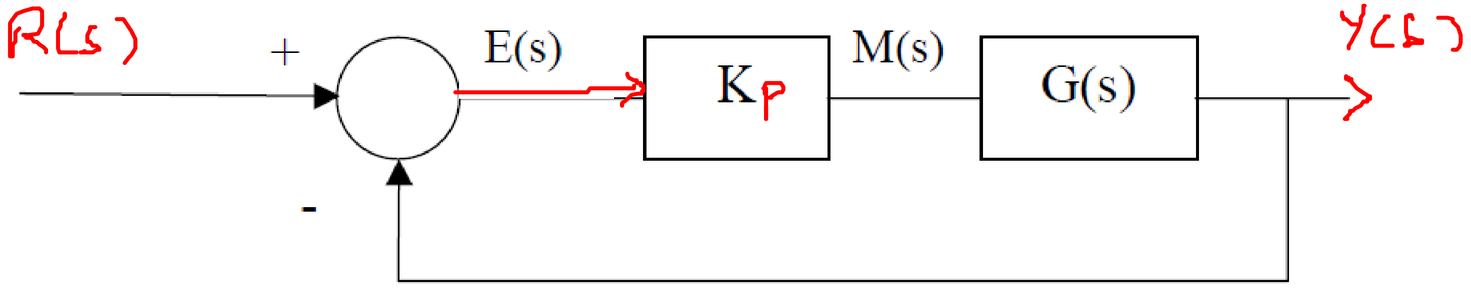


Introduction to Controllers

Proportional Controller

P controller: With proportional control, the actuator applies a corrective force that is proportional to the amount of error: Output = Kp × E

$E(s) = R(s) - Y(s)$



RH criterion

$C.E = 1 + K_p G(s) = 0$

or

Root locus

$M(s) = K_p E(s)$

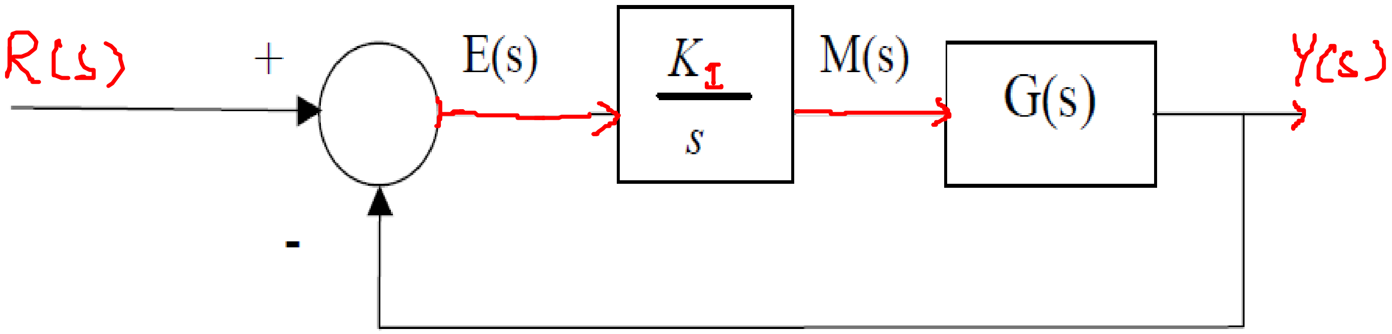
$$\frac{M(s)}{E(s)} = K_p \qquad m(t) = K_p \cdot e(t)$$

Adv: reduces overshoot as well as steady state error,

D.Adv : there is an offset

Integral Controller

I controller: adds a pole at the origin for the OLTF and TYPE of system is increased



$$M(s) = \frac{K_I}{s} E(s)$$

$$m(t) = K_I \int e(t)\, dt$$

$$\frac{M(s)}{E(s)} = \frac{K_I}{s} \qquad m(t) = K_I \int e(t)dt$$

$$C.E = 1 + \frac{K_I}{s} \cdot G(s) = 0$$

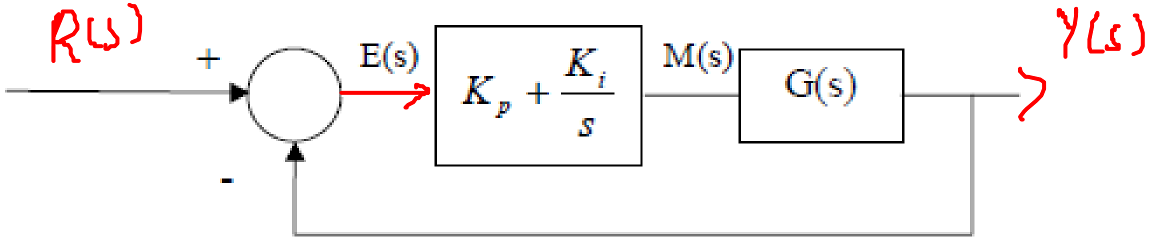
use RH criterion, to find K_I

Introduction to Controllers

Proportional + Integral Controller

PI controller: adds a pole at the origin and zero on real axis for the OLTF and increases system TYPE by 1, thus improves steady state error.

It is a low-pass filter.



Effects:

- increases rise time, decreases bandwidth, Improves GM, PM and resonant peak.

(Improves steady state response but rise time is increased)

Adv: improved damping, zero offset, no steady state error

$$\frac{M(s)}{E(s)} = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \qquad m(t) = K_p e(t) + \int K_i e(t) dt$$

A pole at the origin and a zero at $-\frac{K_i}{K_p}$ are added.

$K_p s + K_i = 0$

$s = -\frac{K_i}{K_p}$

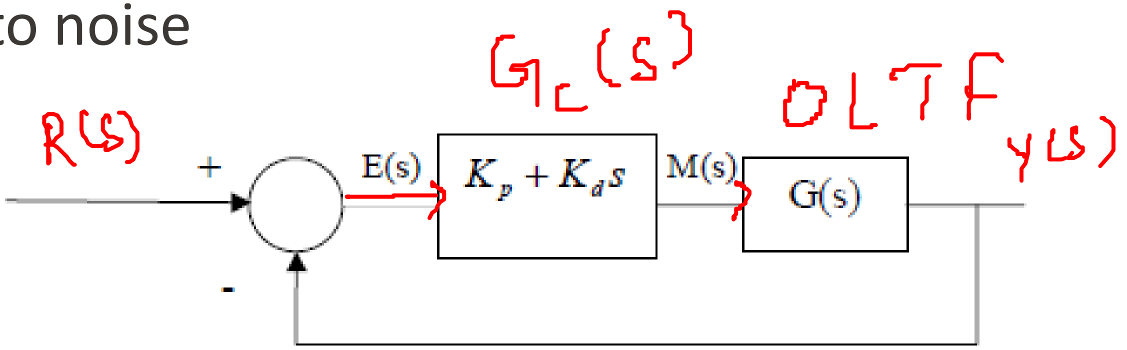
RH criterion

fix K_p, find K_i

Proportional + Derivative Controller

PD controller: adds zero on real axis for the OLTF.

- Root locus is pulled to the left, system becomes more stable and response faster.
- Differentiation makes the system sensitive to noise
- It is a high pass filter.



- Effects:
- Reduces overshoot, rise time
- Reduces settling time
- Increases bandwidth
- Improves GM, PM and resonant peak

$$\frac{M(s)}{E(s)} = K_p + K_d s \qquad m(t) = K_p e(t) + K_d \frac{de(t)}{dt}$$

$M(s) = K_p E(s) + K_d s E(s)$

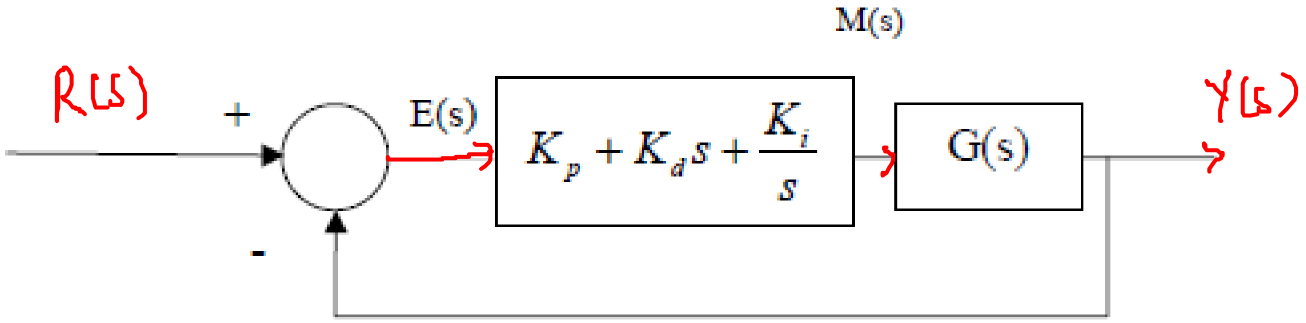
When ζ is low, PD is not effective in improving the stability

Introduction to Controllers

Proportional + Integral + Derivative Controller

PID controller: PI is connected in cascade with PD controller

- widely used in industry
- It is a band-pass or band-stop filter
- K_p decreases rise time,
- K_i eliminates e_{ss}
- K_D = decreases M_p and t_s



$$G_c(s) = \frac{M(s)}{E(s)} = K_p + K_d d + \frac{K_i}{s} \Rightarrow M(s) = \left(K_p + K_d s + \frac{K_i}{s}\right) E(s)$$

$$m(t) = K_p e(t) + K_d \frac{de(t)}{dt} + K_i \int e(t) dt$$

CONTROL SYSTEMS

Unit 5: Design of Feedback Control Systems

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Control system has two different facets:

- **Analysis:** refers to how the system works
- **Design:** aims at making the system work in a desired manner
- In this chapter we focus on design aspects of control systems

Introduction

- We have understood about the steady state performance of control systems (where does the system go) and
- Stability of control system (how does the system reach the steady state)
- The steady state and stability are specified in
  - Time domain: peak overshoot, rise time, settling time, steady state error etc.
  - Frequency domain: Gain margin, phase margin, bandwidth, peak resonance etc.

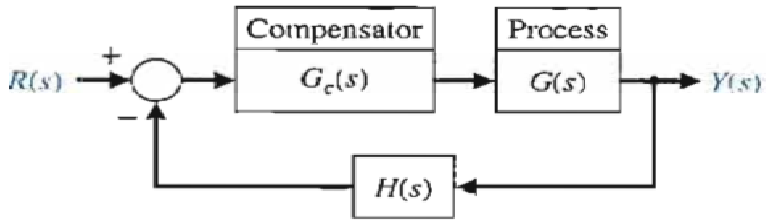
Introduction

- Knowing the performance specifications in time domain or frequency domain , the system can be designed using root locus or frequency response plots respectively.

- **Steps involved in Control System Design:**
- **Adjustment of gain**
- **Additional devices or components – known as compensation**
- **Compensators:**
  - **device introduced to satisfy the specifications**
  - **Introduce a pole and / or zero in OLTF**

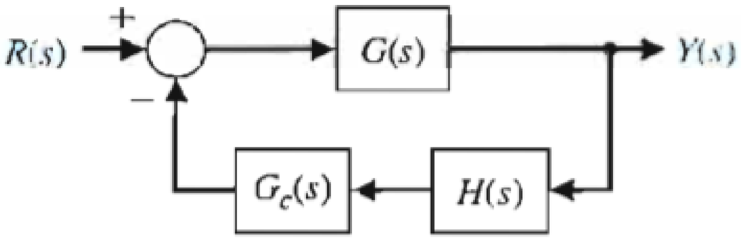
Design of Feedback Control Systems

Types of Compensation schemes



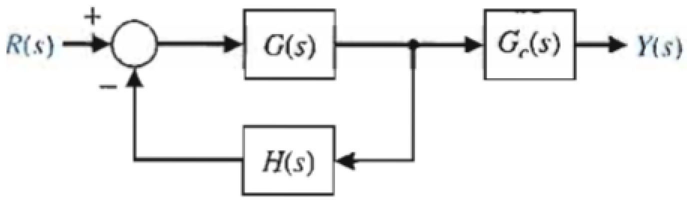
(a)

CASCADE COMPENSATION



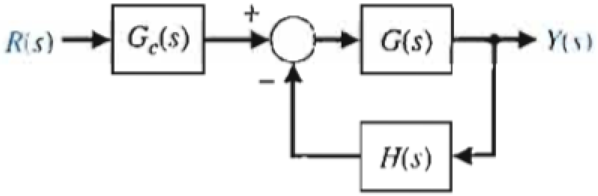
(b)

FEEDBACK COMPENSATION



(c)

OUTPUT/LOAD COMPENSATION



(d)

INPUT COMPENSATION

Design of Feedback Control Systems
Choice between compensation scheme

- Nature of signals in the system
- Power levels at various points
- Components available
- Designer's experience
- Economic considerations
- Compensating devices can be
  - Electrical
  - Mechanical
  - hydraulic etc.
    - But most electrical compensator are RC filter.

Types of compensators

- Lag compensator –

  used to reduce steady state error

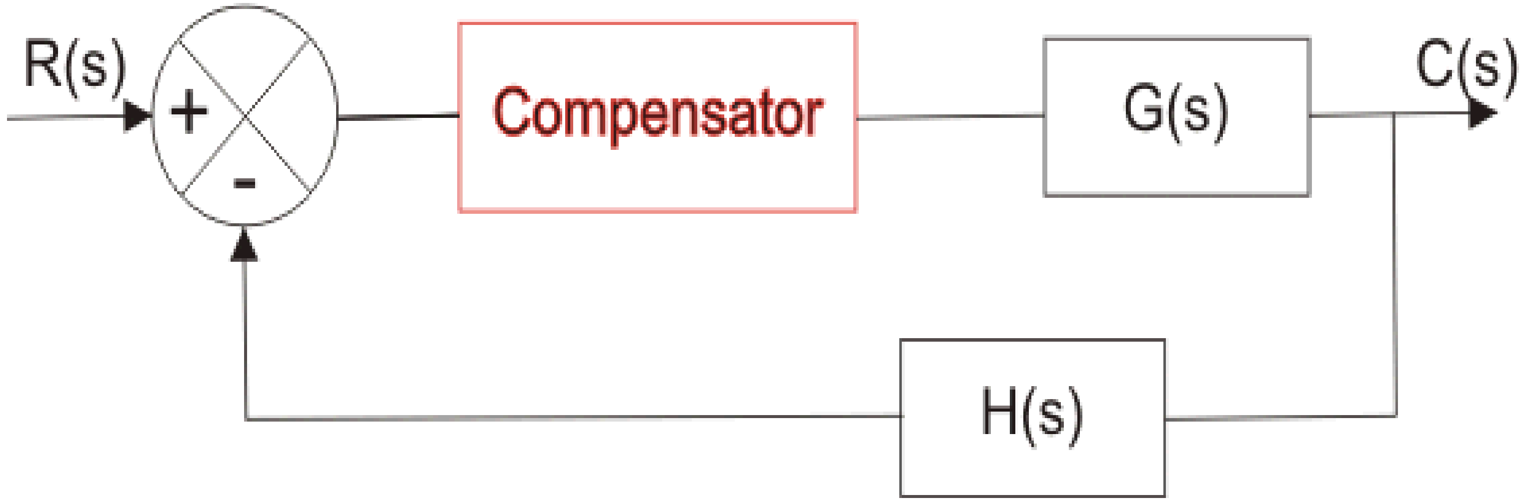
- Lead compensator –

  used to improve the transient response

- Lead-Lag compensator –

  used when both transient and steady state response are not satisfactory

Cascade/Series compensation



$G_c(s)$ – compensator transfer function

Phase Lead Compensation

- A system which has one pole and one dominating zero (the zero which is closer to the origin) is known as lead network.
- If we want to add a dominating zero for compensation in control system then we have to select lead compensation network.
- When the system is absolutely unstable:
  - Required to stabilize and to meet the desired performance
  - Ex: system with TYPE 2 and above –require lead compensation

Compensators are required

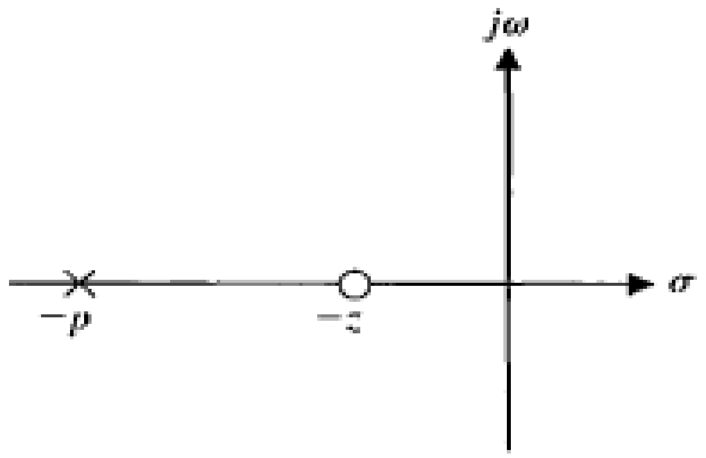
- When the system is stable:
    - To obtain desired performance
    - Ex TYPE 1 or 0 – stability can be achieved by adjusting the gain. Any of the three compensators can be used to improve the performance.

Compensator

- **Compensator transfer function:**

  - $G_c(s) = K\frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}$
  - First order compensator
  - $G_c(s) = K\frac{s+z}{s+p}$
  - Thus, selection of K, z and p completes the design of compensator

- When $|z| < |p|$, called phase lead network
- It acts as a differentiator network (when pole is negligible $|p| \gg |z|$ and the zero occur at origin)



Frequency response of phase lead network

- $G_c(s) = K\frac{s+z}{s+p}$ → First order compensator

- $G_c(j\omega) = K\frac{j\omega+z}{j\omega+p} = \frac{Kz/p(j(\omega/z)+1)}{(j(\omega/p)+1)} = \frac{K_1(1+j\omega\alpha\tau)}{1+j\omega\tau}$

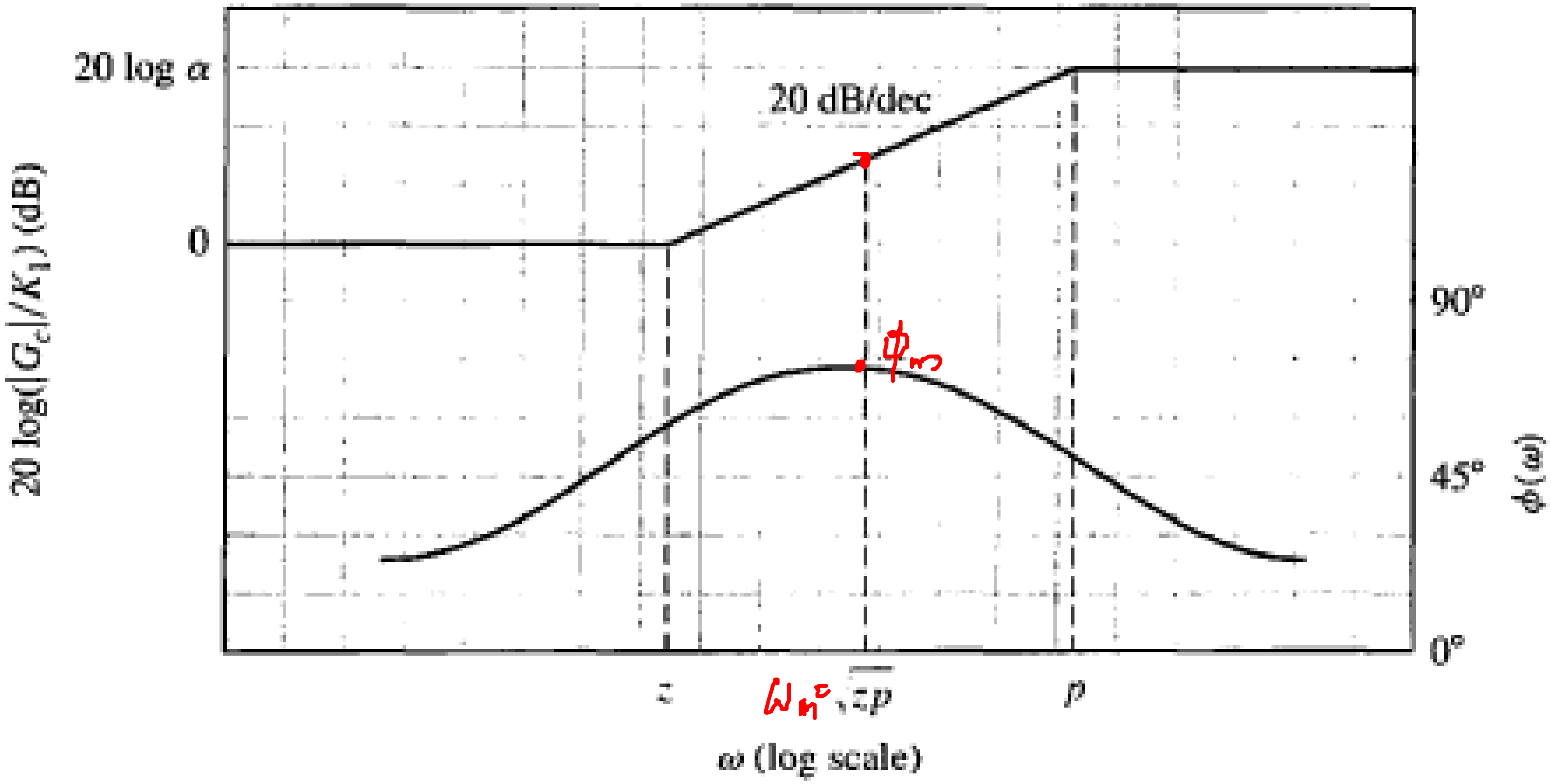
- **Where** $\boldsymbol{\tau = \frac{1}{p}, p = \alpha z, K_1 = {}^{K}/_{\alpha}}$, $p = \frac{1}{\tau}$, $z = \frac{1}{\alpha\tau}$

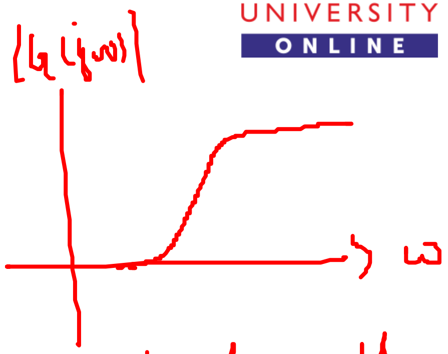
- $\emptyset(\boldsymbol{\omega}) = \tan^{-1}(\boldsymbol{\alpha\omega\tau}) - \tan^{-1}(\boldsymbol{\omega\tau})$

$$|G_c(j\omega)| = \frac{K_1\sqrt{1+(\omega\alpha\tau)^2}}{\sqrt{1+\omega^2\tau^2}}$$

Design of Feedback Control Systems

Frequency response of phase lead network

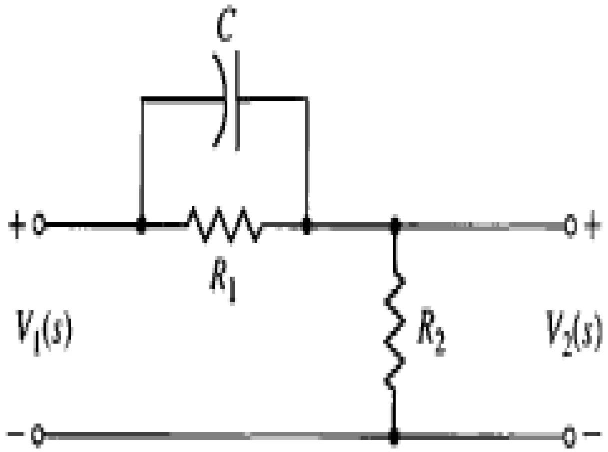




high-pass filter

Phase lead network

- $G_c(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \frac{R_1/(Cs)}{R_1+1/(Cs)}} = \frac{R_2}{R_1+R_2} \frac{R_1Cs+1}{\underbrace{\left[\frac{R_1R_2}{R_1+R_2}\right]}_{\tau}Cs+1}$



- Where $\tau = \left[\frac{R_1R_2}{R_1+R_2}\right]C$ and $\propto = \frac{R_1+R_2}{R_2}$
- $G_c(s) = \frac{1+\propto\tau s}{\alpha(1+\tau s)}$ => phase lead compensation transfer function
- $\boxed{p = \frac{1}{\tau} \; and \; z = \frac{1}{\propto\tau}}$

Phase lead network

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- **Maximum phase lead occurs at $\omega = \omega_m$**
- **ω_m can be obtained as**
  - $\frac{d\varphi}{d\omega} = 0$ at $\omega = \omega_m$ => $\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}$

$= \sqrt{\frac{1}{\alpha\tau} \cdot \frac{1}{\tau}} = \frac{1}{\tau\sqrt{\alpha}}$

  - **where,** $\emptyset(\omega) = \tan^{-1}(\alpha\omega\tau) - \tan^{-1}(\omega\tau)$

Phase lead network

- **To find maximum phase-lead angle,**

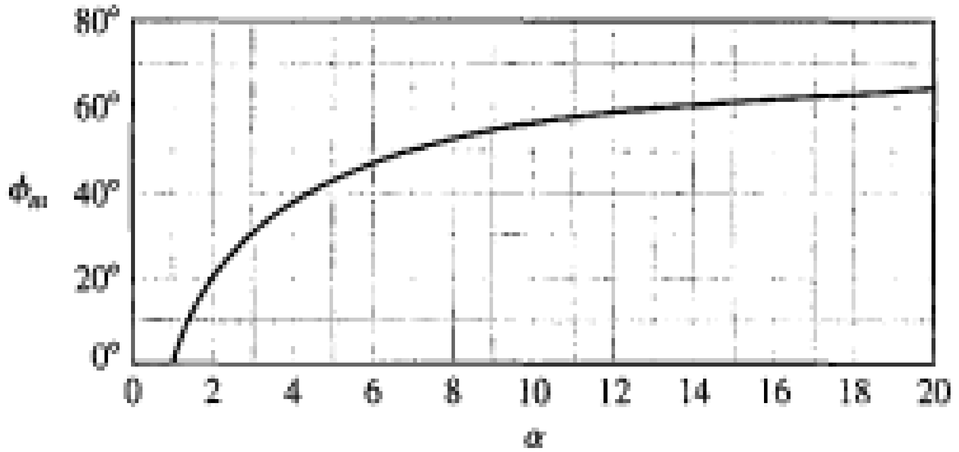
$$\emptyset = \tan^{-1}\frac{\alpha\omega\tau-\omega\tau}{1+(\omega\tau)^2\alpha} \; at\; \omega = \omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

$$\Rightarrow tan\, \emptyset_m = \frac{\alpha-1}{2\sqrt{\alpha}} \text{ using } sin\, \emptyset = \frac{tan\, \emptyset}{\sqrt{1+tan^2\emptyset}}$$

$$\Rightarrow sin\, \emptyset_m = \frac{\propto-1}{\propto+1}$$

$$\Rightarrow \boxed{\alpha = \frac{1+\sin\phi_m}{1-\sin\phi_m}}$$

$$\Rightarrow \text{Magnitude at } \omega = \omega_m = \frac{1}{\tau\sqrt{\alpha}} \Rightarrow |G(j\omega)| = \frac{1}{\sqrt{\alpha}}$$



Phase lead network

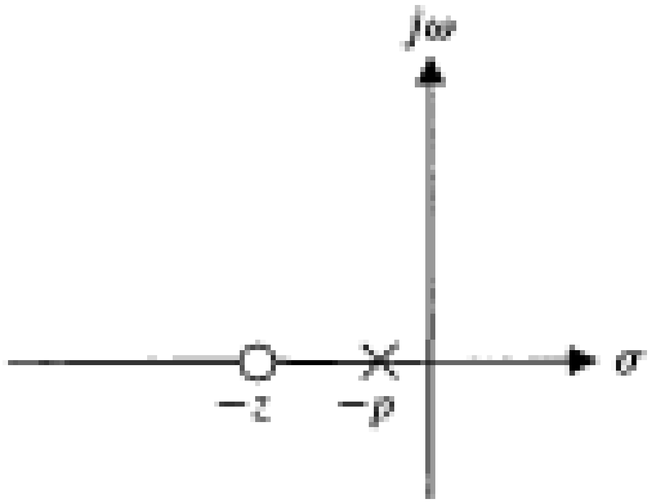
- **Effects of Phase Lead**
- **The velocity constant K_v increases.**
- **Phase margin increases.**
- **Response become faster.**
- **Steady state error decreases**

$$e_{ss} = \frac{A}{K_v}$$

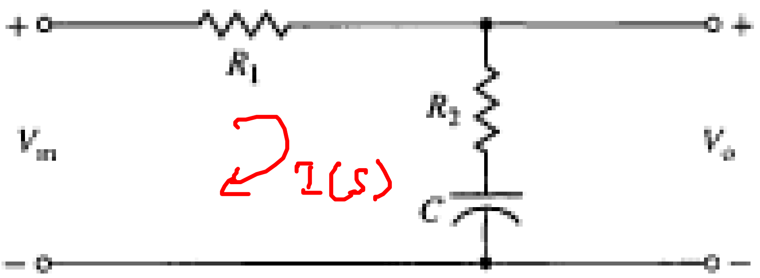
- **Adv**
- **The speed of the system increases because it shifts gain crossover frequency to a higher value.**
- **Maximum overshoot of the system decreases.**

Phase Lag Compensator

- When $|p| < |z|$, called phase lag network
- It acts as a integrating network



Phase Lag Network



$V_1(s) = \left(R_2 + \frac{1}{Cs}\right) I(s)$

$V_{in}(s) = \left(R_1 + R_2 + \frac{1}{Cs}\right) I(s)$

- $G_c(s) = \frac{V_0(s)}{V_{in}(s)} = \frac{R_2+1/(Cs)}{R_1+R_2+1/(Cs)} = \frac{R_2Cs+1}{(R_1+R_2)Cs+1}$
- Where $\tau = R_2C$ and $\propto = \frac{R_1+R_2}{R_2}$
- $G_c(s) = \frac{1+\tau s}{(1+\propto\tau s)} = \frac{1}{\propto}\frac{s+z}{s+p}$ phase lag compensation transfer function
- $\boxed{z = \frac{1}{\tau}\ and\ p = \frac{1}{\propto\tau}, \propto > 1}$

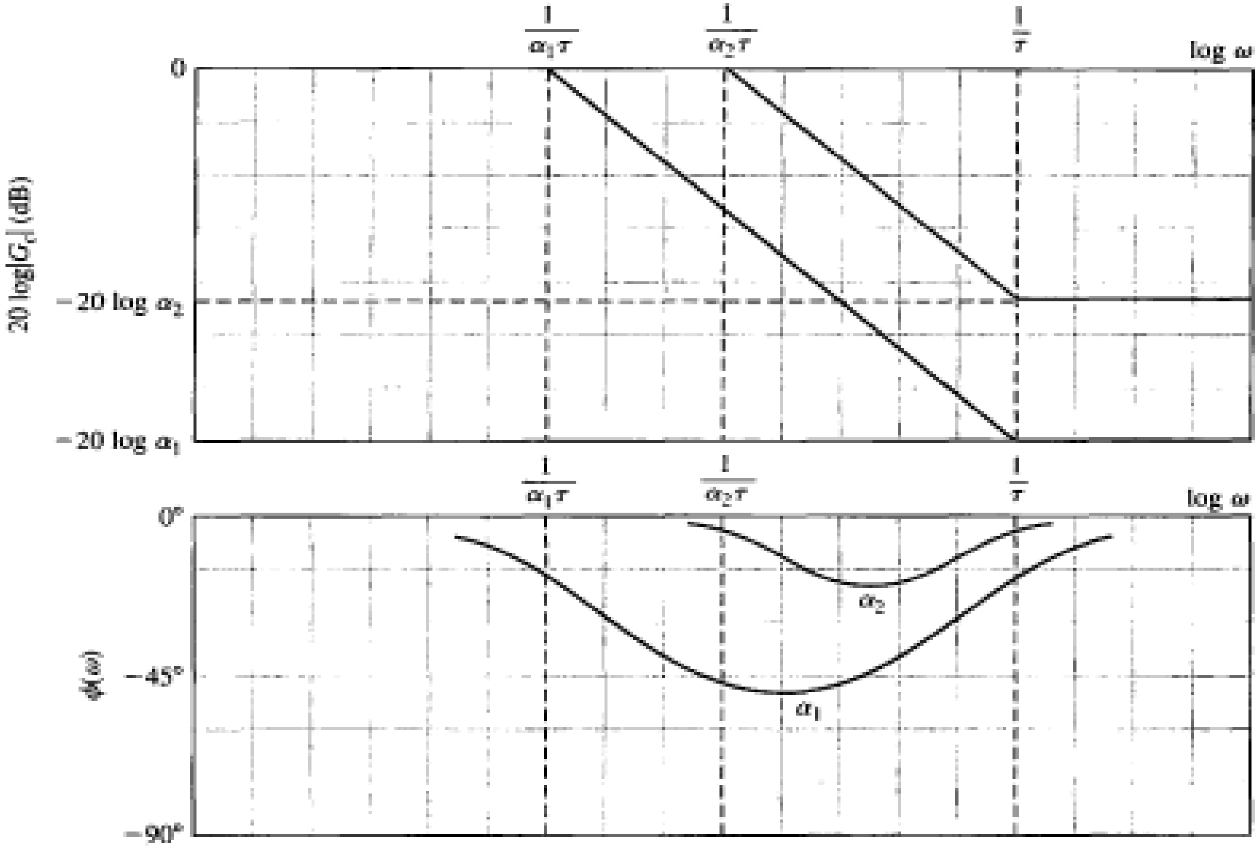
Design of Feedback Control Systems

Frequency Response of Phase Lag Network

- $G_c(j\omega) = \dfrac{1+j\omega\tau}{1+j\omega\alpha\tau}$

- Only difference is in attenuation and phase-lag angle instead of amplification and phase lead

- Therefore, maximum phase lag occurs at $\boldsymbol{\omega_m} = \sqrt{\boldsymbol{zp}}$



$z = \frac{1}{\tau}$, $p = \frac{1}{\alpha\tau}$, $\omega_m = \sqrt{\frac{1}{\tau}\cdot\frac{1}{\alpha\tau}} = \frac{1}{\tau\sqrt{\alpha}}$

Unit 5: Design of Feedback Control Systems

Phase-Lead design using Bode plot

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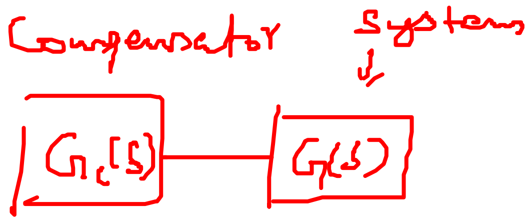
Design of Feedback Control Systems

Phase-Lead design using Bode Plot

- Design the Lead compensator such that system meets desired specifications.
- How? Frequency response of cascade compensation network is added to frequency response of the uncompensated system
- There are steps to design the Lead compensator



Phase-Lead design using Bode Plot

- Step 1: Evaluate the uncompensated system phase margin($\emptyset_{un}$) with error constants satisfied.
- Step 2: determine the necessary additional phase lead $\emptyset_m$ with a small amount of safety margin
  - $\emptyset_1 = desired\ PM\ - (\emptyset_{un})$
  - $\emptyset_m = \emptyset_1 + safety\ margin$
  - Safety margin = 5% of $\emptyset_1$, where % can be varied until you get the desired PM

- Step 3: Evaluate ∝ using the following equation
  - $\boldsymbol{sin\,\emptyset_m = \frac{\propto-1}{\propto+1}}$ or $\propto = \frac{1+\sin(\emptyset_m)}{1-\sin(\emptyset_m)}$
- Step 4: Evaluate $10\log_{10}\propto$ and determine the frequency where the uncompensated magnitude curve is equal to $-10\log_{10}\propto$ dB. (Because the compensation network provides a gain of $10\log_{10}\propto$ at $\boldsymbol{\omega_m}$ and this is the new 0dB cross over frequency)

Phase-Lead design using Bode Plot

- Step 5: Calculate the pole $p = \omega_m\sqrt{\propto}, z = \frac{p}{\propto}$
- Step 6: Draw the compensated frequency response, check the resulting phase margin and repeat the steps if necessary

$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$

$= \frac{p}{\sqrt{\alpha}} \Rightarrow p = \omega_m\sqrt{\alpha}$

$z = \frac{1}{\alpha\tau} = \frac{p}{\alpha}$

$G_c(s) = \frac{K(s+z)}{(s+p)}$, $L(s) = G_c(s)\,G(s)$

freq. response of $L(s)$, check PM of $L(s)$

Design of Feedback Control Systems

Example 1

- $G(s) = \frac{K_1}{s^2}$ and H(s) = 1, $T(s) = \frac{K_1}{s^2+K_1}$

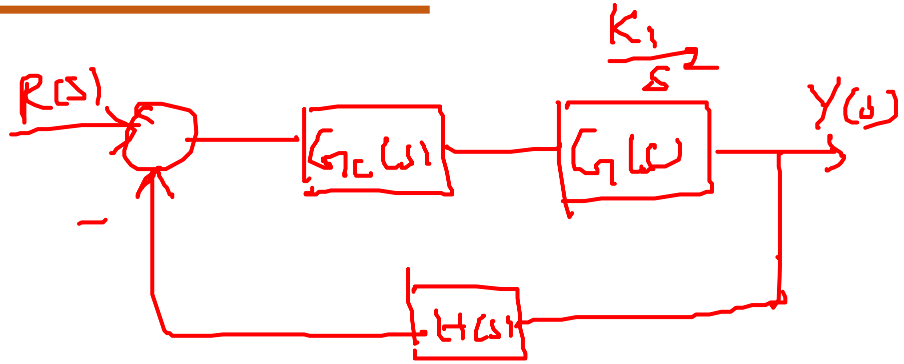
$K_1 = \omega_n^2$

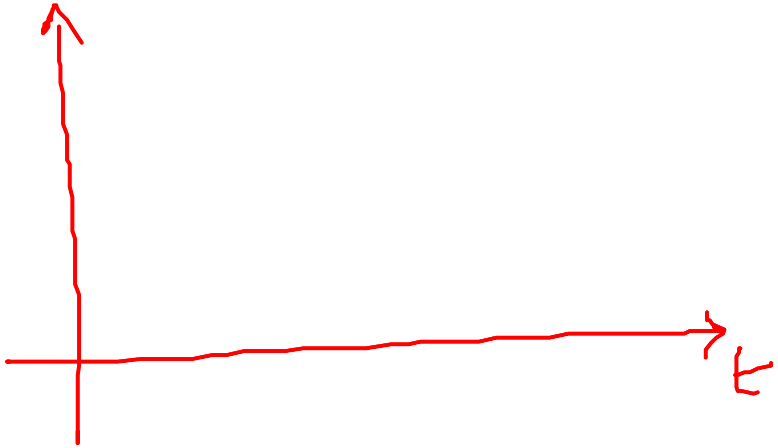
- Desired specifications:
  - Settling time, $t_s \leq 4\ s$
  - System damping constant, $\zeta\ \geq 0.45$

$$=> t_s = \frac{4}{\zeta \omega_n} = 4$$

$$=> \omega_n = \frac{1}{\zeta} = 2.22$$

$$=> K_1 = \omega_n^2 \approx 5$$





Example 1

- But we need specification in terms of phase margin
  - $\emptyset_{pm} = \frac{\zeta}{0.01} = 45\ degree$ ⇒ desired PM
- Step 1: Draw Bode plot for $G(j\omega) = \frac{K_1}{(j\omega)^2}$
- K_1=10 with safety margin as 5 or add in the step 2

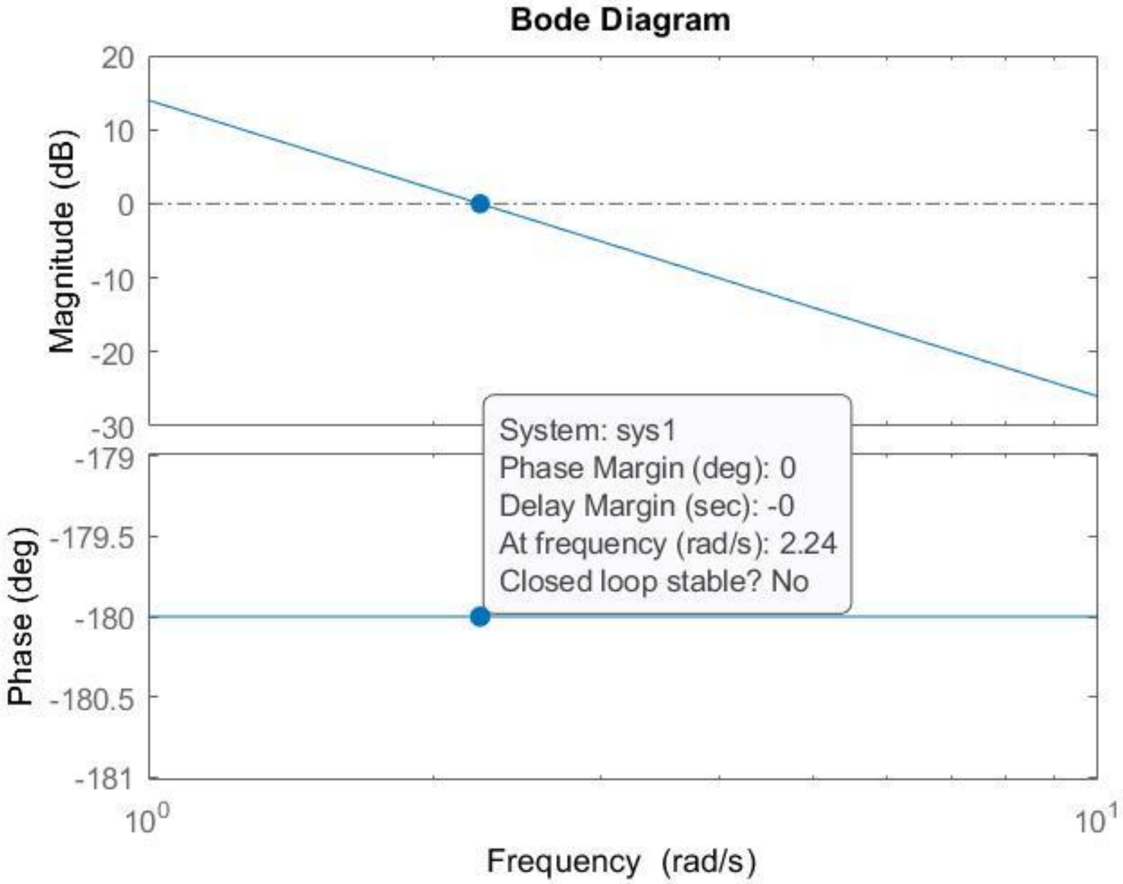
, $G(j\omega) = \frac{10}{(j\omega)^2}$

$|G(j\omega)| = 20\log 10 - 20\log \omega^2$, $\angle G(j\omega) = -\pi$

$\omega = 0.1$, $A = 60\ dB$

Design of Feedback Control Systems

Example 1



$PM = 180 + (-180^\circ) = 0^\circ$

Design of Feedback Control Systems

Example 1

- Step 2:

  45° 0°

  - $\boldsymbol{\emptyset_1 = desired\ PM\ - (\emptyset_{un})}$
  - $\boldsymbol{\emptyset_m = \emptyset_1 + safety\ margin}$
  - **Desired PM ,** $\emptyset_{pm}$**= 45 and** $\emptyset_{un} = 0$
  - $\emptyset_1 = 45$
  - $\emptyset_m = 45 + 0\ as\ safety\ margin = 45$

- Step 3: Evaluate ∝ using the following equation

  - $\boldsymbol{sin}\ \emptyset_m = \frac{\propto - 1}{\propto + 1}$ or $\propto = \frac{1+\sin(\emptyset_m)}{1-\sin(\emptyset_m)} = 5.8 \approx 6$

- Step 4: Evaluate $10\log_{10} \propto$ = 7.78 dB
  - frequency at which the uncompensated magnitude curve is equal to $-$ 7.78 dB is $\boldsymbol{\omega_m = 4.95\, rad/sec}$ (Because the compensation network provides a gain of $10\log_{10} \propto$ at $\boldsymbol{\omega_m}$ and this is the new 0dB cross over frequency)

- Step 5: Calculate the pole $\boldsymbol{p = \omega_m\sqrt{\propto} = 4.95\sqrt{6} = 12,\ z = \frac{p}{\propto} = \frac{12}{6} = 2}$

Design of Feedback Control Systems

Example 1

- Step 6: Draw the compensated frequency response,

  - $L(j\omega) = G_c(j\omega)G(j\omega) = \frac{10[\frac{j\omega}{2}+1]}{(j\omega)^2[\frac{j\omega}{12}+1]}$

  - Total DC loop gain must be raised by a factor of α in order to account for the factor $\frac{1}{\alpha}$ then the

  $G_c(s) = \frac{\frac{s}{z}+1}{\frac{s}{p}+1}$

  - $L(s) = G_c(s)G(s) = \frac{10[\frac{s}{2}+1]}{(s)^2[\frac{s}{12}+1]} = \frac{60(s+2)}{s^2(s+12)}$

  - $T(s) = \frac{60(s+2)}{s^3+12s^2+60s+120}$

  - check the resulting phase margin and repeat the steps if necessary

Design of Feedback Control Systems

Example 1

factor	C.F	slope (dB/dec)	Net slope (dB/dec)	freq. range (rad/sec)
$10/(j\omega)^2$	none	-40	-40	$\omega = 0.1$ to $\omega = 2$
$\left(\frac{j\omega}{2}+1\right)$	2	20	-20	$\omega = 2$ to $\omega = 12$
$\frac{1}{\frac{j\omega}{12}+1}$	12	-20	-40	$\omega = 12$ onwards

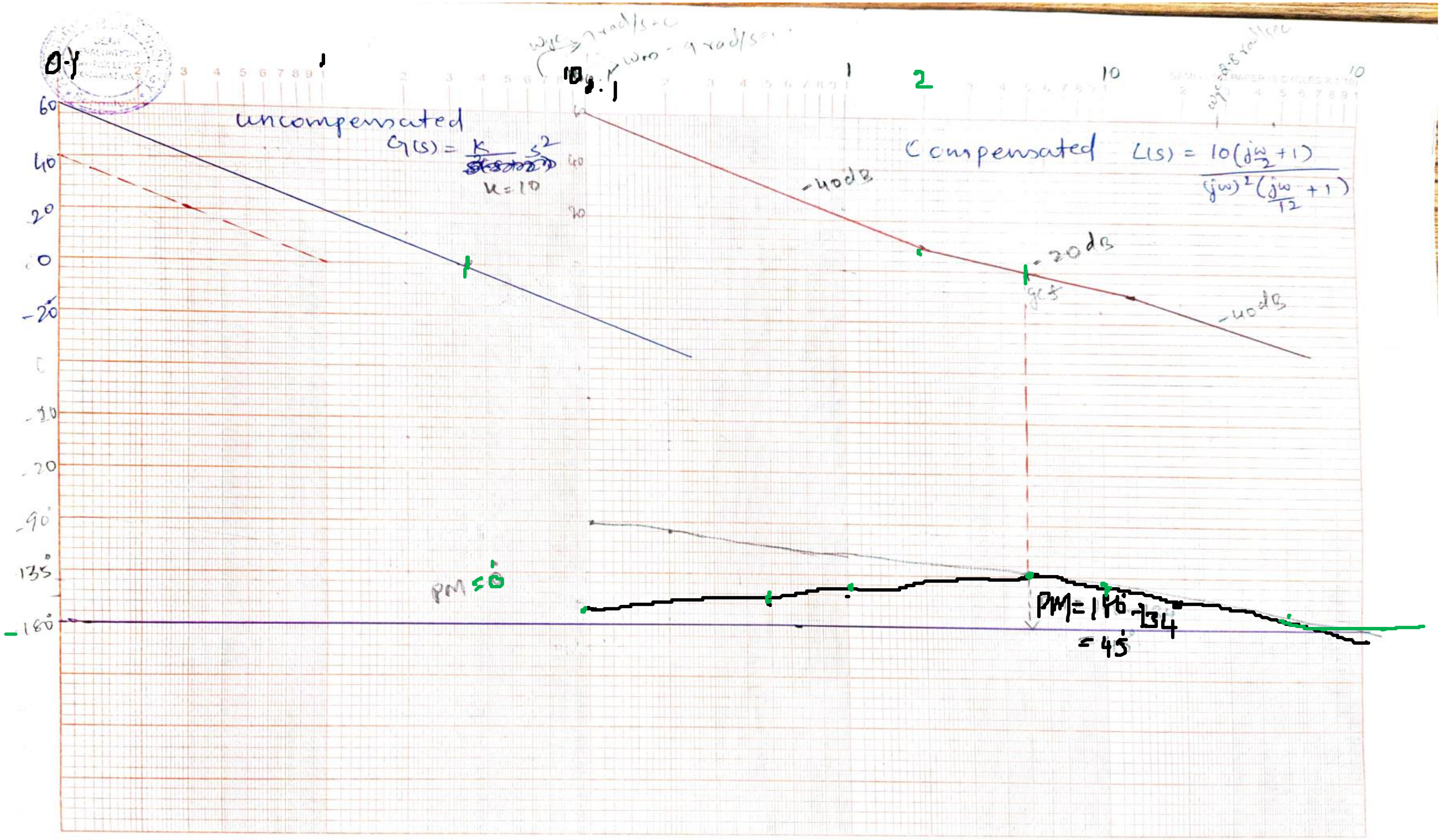
$$A = 20 \log \frac{10}{\omega^2} = 20\log 10 - 40 \log \omega$$

$$\angle G(j\omega) = \tan^{-1}\frac{\omega}{2} - 180^\circ - \tan^{-1}\frac{\omega}{12}$$

$$A \text{ at } \omega = 0.1 = 60\ dB$$

Design of Feedback Control Systems

Example 1



GCf , ω_{gc} = 5 rad/sec

desired PM = 45°

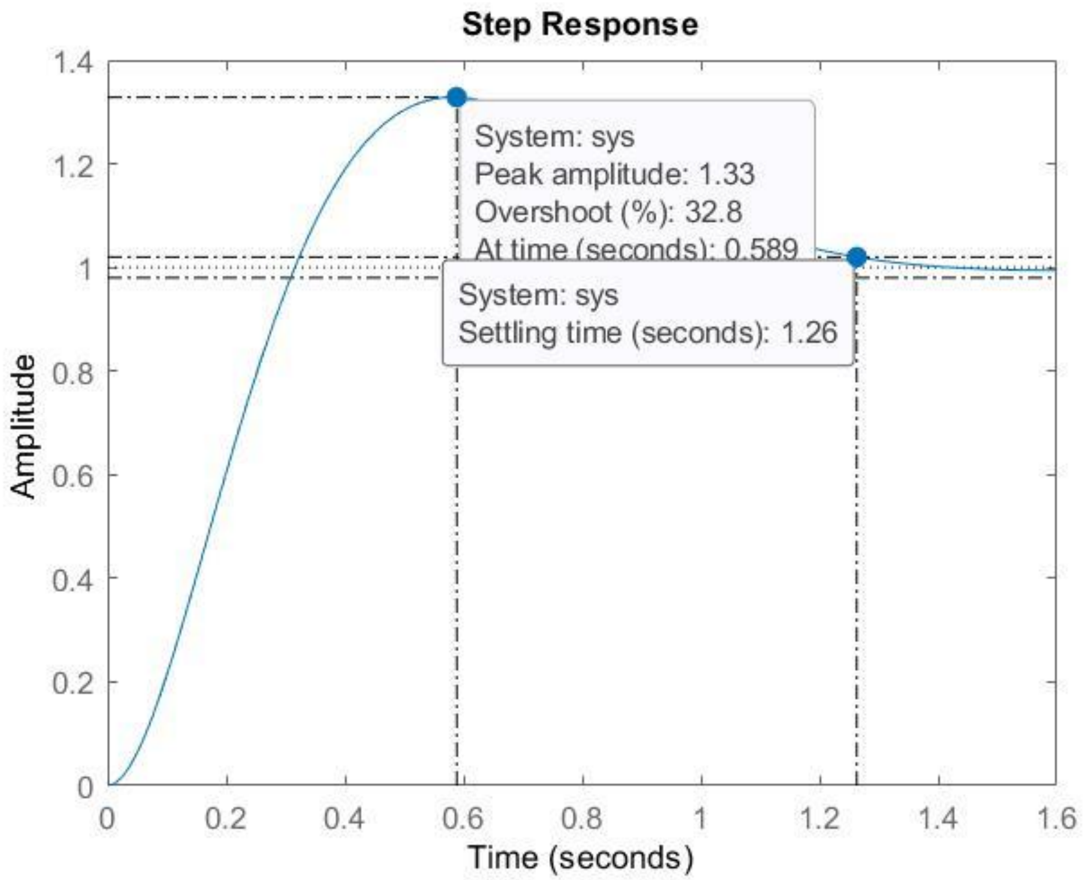
ω	0.1	0.5	1	5	10
∠G(jω)	-177	-168	-158	-134	-141

Example 1

- But the desired specifications are in terms of settling time damping constant
- Therefore, overall transfer function after compensating with lead compensator is
  - $T(s) = \frac{60(s+2)}{s^3+12s^2+60s+120}$
  - Step response of compensated system is used to check the specifications

Design of Feedback Control Systems

Example 1



Design of Feedback Control Systems

Example 2

- $G(s) = \frac{K}{s(s+2)}$ and H(s) = 1
- Desired specification
  - Phase margin = 45 degree
  - Steady state error for a ramp input equal to 5% of the velocity of the ramp
    - $e_{ss} = \frac{A}{K_v} => 0.05A = \frac{A}{K_v} => \ K_v = 20$
- Step 1: Draw Bode plot for $G(j\omega) = \frac{K_v}{j\omega(0.5j\omega+1)}$

$K_v = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{K}{\cancel{s}(s+2)}$

$K_v = \frac{K}{2}$

$2K_v = K$

$G(s) = \frac{2K_v}{s(s+2)} = \frac{\cancel{2}K_v}{\cancel{2}s(\frac{1}{2}s+1)} = \frac{K_v}{s(\frac{1}{2}s+1)}$

factor	C.F	slope	net slope	freq. range
$\frac{20}{j\omega}$	none	-20	-20	0.1 to 2
$\frac{1}{0.5j\omega+1}$	2	-20	-40	2 onwards

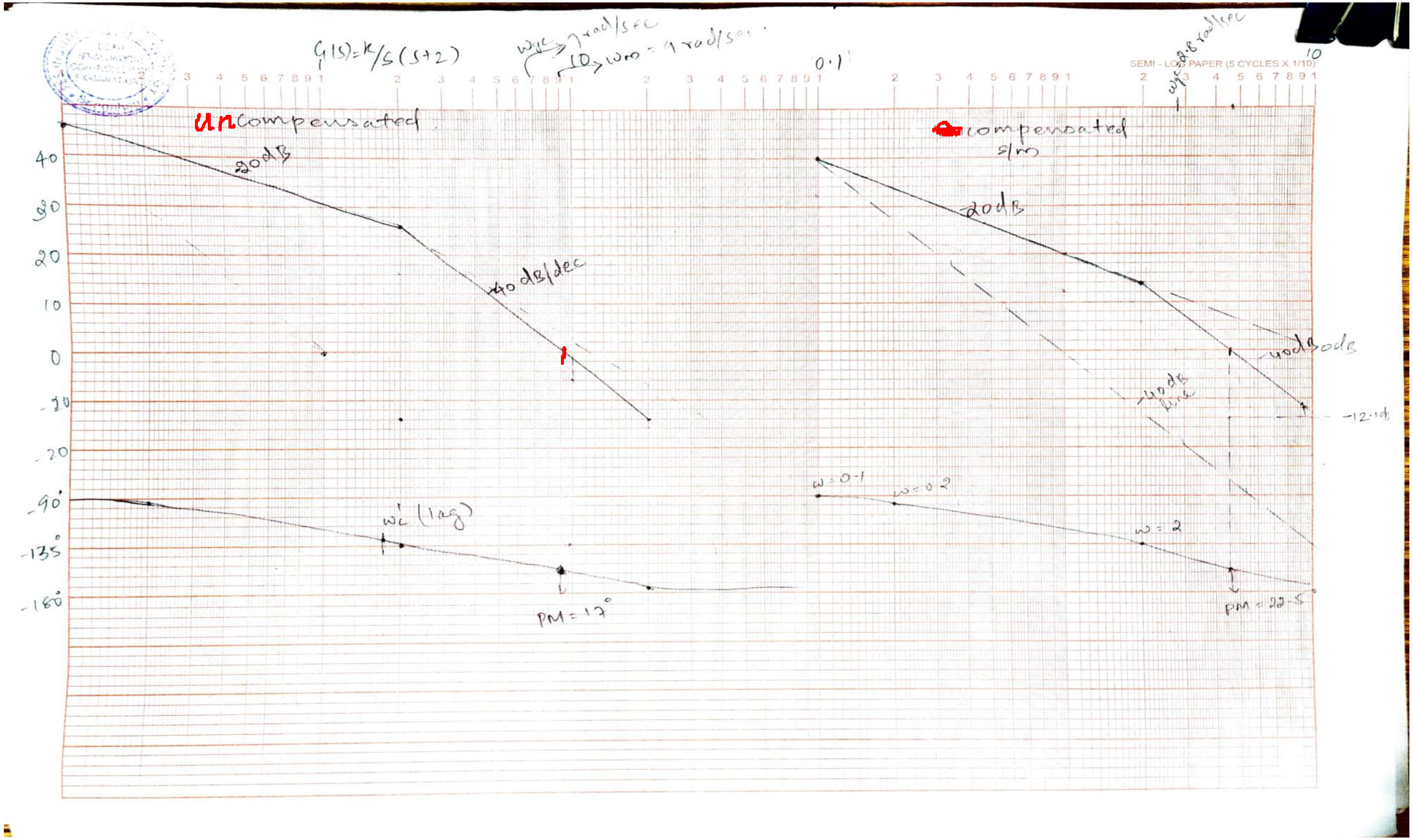
$\angle G(j\omega) = -90^\circ - \tan^{-1} 0.5\omega$

$A = 20\log\frac{20}{\omega} = 20\log 20 - 20\log\omega$

$= 46.02\ dB$

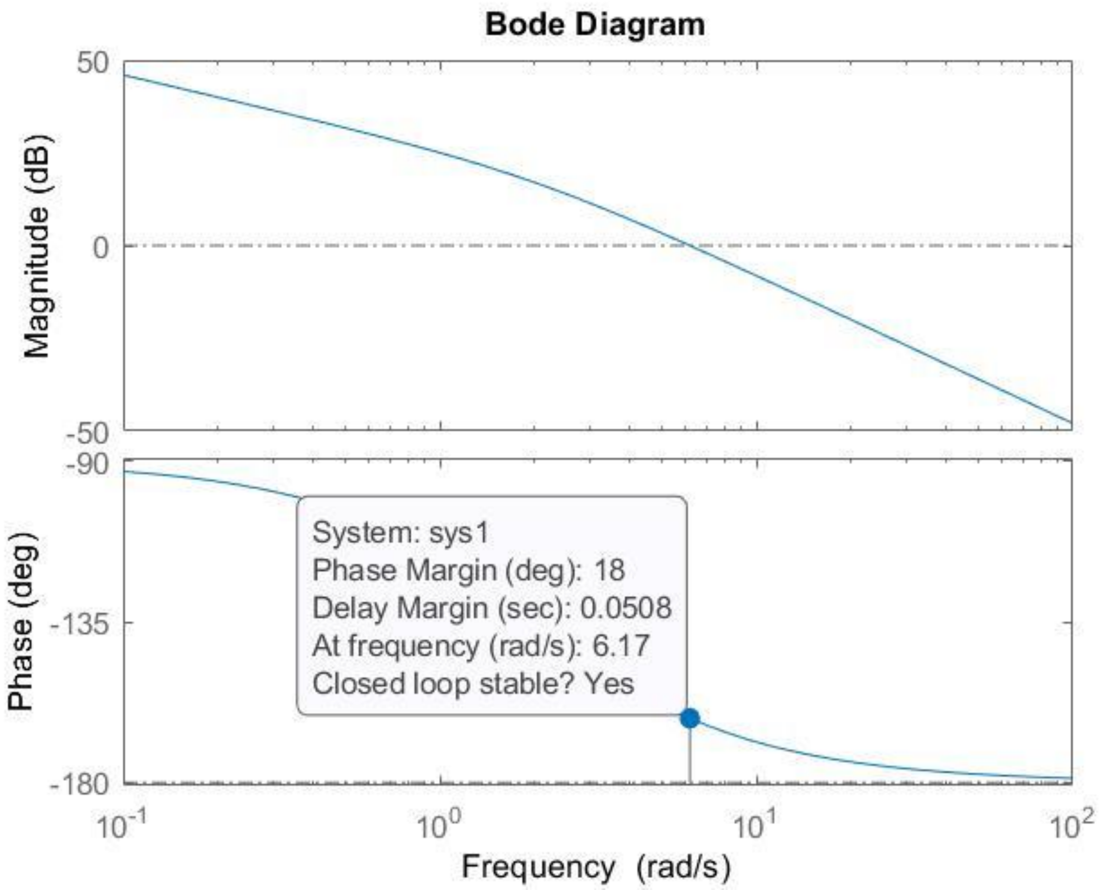
Design of Feedback Control Systems

Example 2



Design of Feedback Control Systems

Example 2



Design of Feedback Control Systems

Example 2

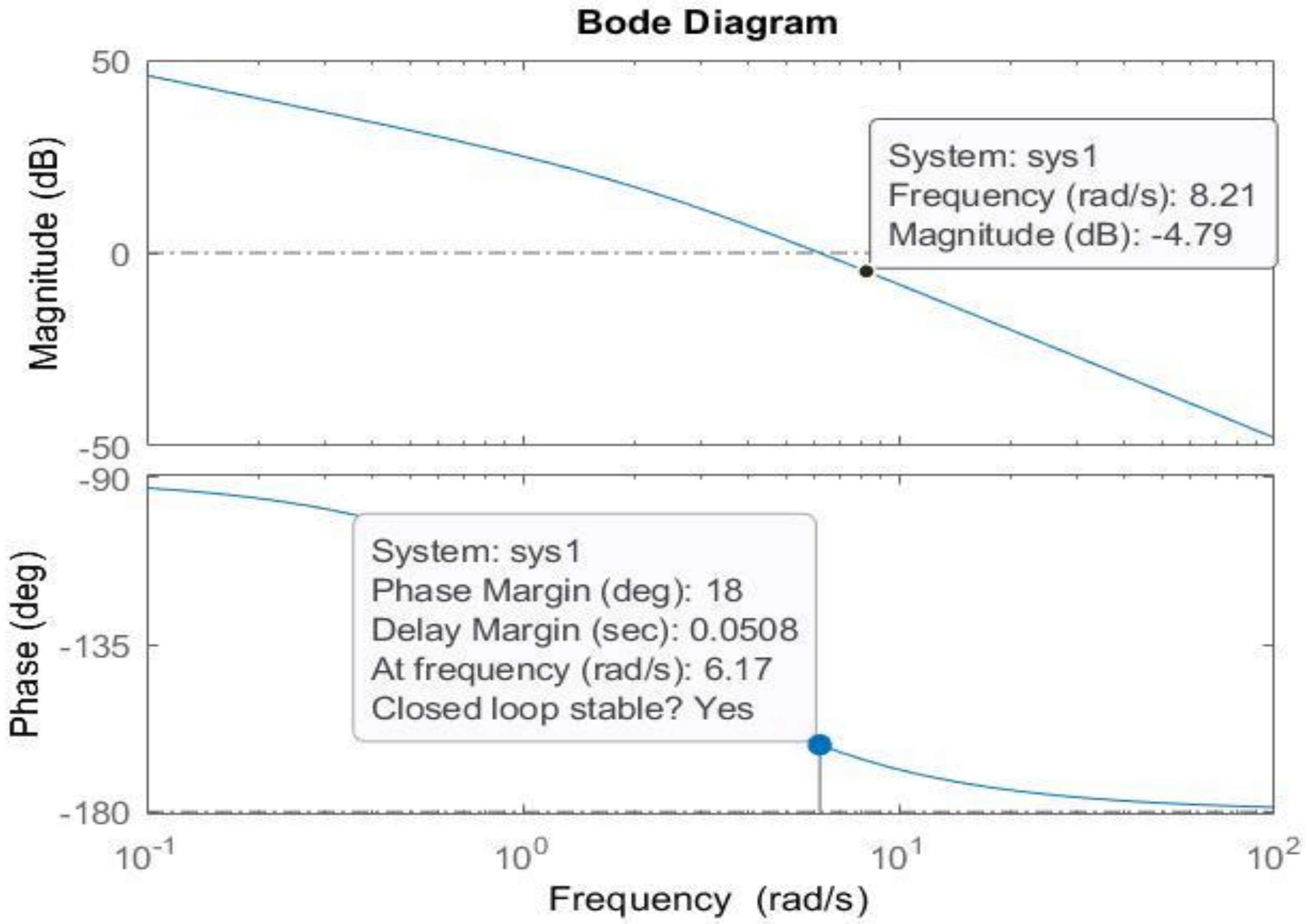
- Step 2:
  - $\emptyset_1 = \boldsymbol{desired\ PM} - (\emptyset_{un})$
  - $\emptyset_m = \emptyset_1 + \boldsymbol{safety\ margin}$
  - **Desired PM ,** $\emptyset_{pm}$**= 45 and** $\emptyset_{un} = 18$
  - $\emptyset_1 = 45\text{-}18 = 27$
  - $\emptyset_m = 27 + 3\ (10\%\ of\ \emptyset_1 as\ safety\ margin) = 30$
- Step 3: Evaluate ∝ using the following equation
  - $\sin\emptyset_m = \frac{\propto-1}{\propto+1}$ or $\propto = \frac{1+\sin(\emptyset_m)}{1-\sin(\emptyset_m)} = 3$

Example 2

- Step 4: Evaluate $10\log_{10}\propto$ = 4.8 dB
  - frequency at which the uncompensated magnitude curve is equal to -4.8 dB is $\omega_m = 8.21 rad/sec$ (Because the compensation network provides a gain of $10\log_{10}\propto$ at ω_m and this is the new 0dB cross over frequency)

Design of Feedback Control Systems

Example 2



Example 2

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- Step 5: Calculate $p = \omega_m\sqrt{\propto} = 8.21\sqrt{3} = 14.2201,$
  - $z = \frac{p}{\propto} = \frac{14.2201}{3} = 4.74$
- Step 6: Draw the bode plot for compensated system
  - $L(j\omega) = G_c(j\omega)G(j\omega) = \frac{20[\frac{j\omega}{4.74}+1]}{j\omega(0.5j\omega+1)[\frac{j\omega}{14.22}+1]}$
  - $L(s) = G_c(s)G(s) = \frac{20[\frac{s}{4.74}+1]}{s(0.5s+1)[\frac{s}{14.22}+1]}$
  - check the resulting phase margin and repeat the steps if necessary

$A = 20\log\frac{20}{\omega}$

$= 20\log 20 - 20\log\omega$

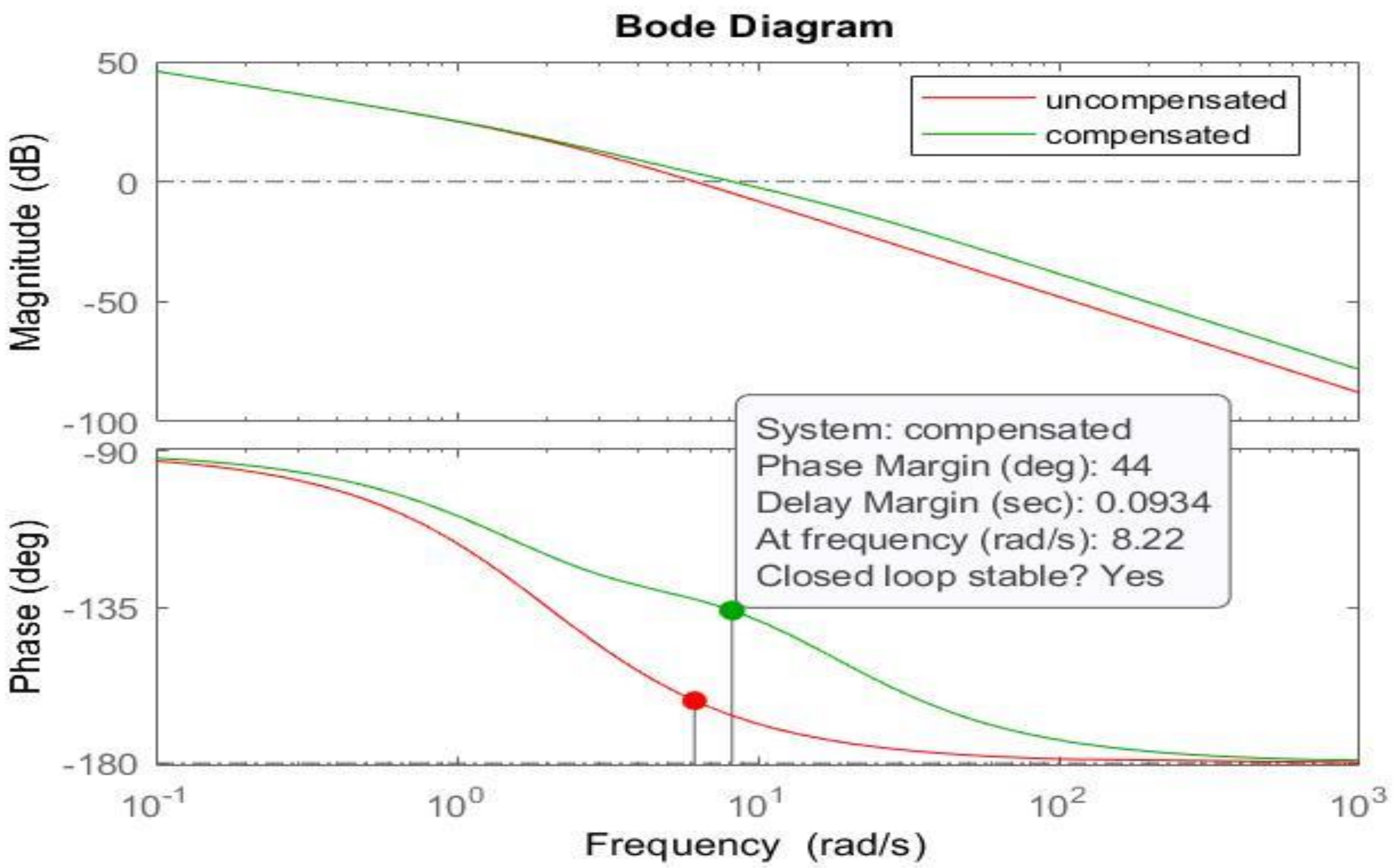
$A(\omega=0.1) = 46\ dB$

factor	C.F	slope	net slope	freq. range
$\frac{20}{j\omega}$	None	-20	-20	0.1 to 2
$\frac{1}{0.5j\omega+1}$	2	-20	-40	2 to 4.74
$\frac{j\omega}{4.75}+1$	4.74	20	-20	4.74 to 14.22
$\frac{j\omega}{14.22}+1$	14.22	-20	-40	14.22 onwards

$\angle L(j\omega) = \tan^{-1}\frac{\omega}{4.74} - 90° - \tan^{-1}0.5\omega - \tan^{-1}\frac{\omega}{14.22}$

Design of Feedback Control Systems

Example 2



Unit 5: Design of Feedback Control Systems

Phase – Lag using Bode Plot

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Design of Feedback Control Systems

Phase – Lag design using Bode Plot

- $G_c(j\omega) = \frac{1+j\omega\tau}{1+j\omega\alpha\tau}$ $p = \frac{1}{\alpha\tau}$, $z = \frac{1}{\tau}$

- Step1: Obtain the Bode plot for uncompensated system with the gain adjusted for the desired error constant

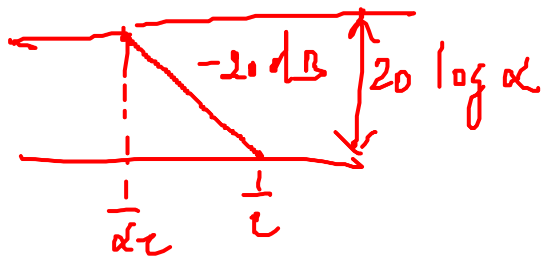
- Step2: obtain the phase margin of uncompensated system, if it is insufficient do the following,

- Step3: Determine the frequency where the PM requirement would be satisfied as ω_c'
  - i.e $\emptyset_1$ =desired PM + safety margin(10% or 5 deg of phase lag) then determine frequency at (180- $\emptyset_1$) on phase plot

Design of Feedback Control Systems

Phase – Lag design using Bode Plot

- Step4: Place the zero of the compensator one decade below the new crossover frequency ω_c'
  - $z = \frac{\omega_c'}{10}$



- Step5: Measure the necessary attenuation at ω_c' to ensure that the magnitude curve crosses at this frequency

- Step6: Calculate $\propto$ by noting that the attenuation introduced by the phase lag network is $20\log_{10}\propto$ at ω_c'

- Step7: Calculate $\boldsymbol{p} = \frac{1}{\propto\tau} = \frac{z}{\propto}$

Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 1

- $G(s) = \frac{K}{s(s+2)}$ and H(s) = 1
- Desired specification
  - Phase margin = 45 degree
  - Steady state error for a ramp input equal to 5% of the velocity of the ramp
    - $e_{ss} = \frac{A}{K_v} => 0.05A = \frac{A}{K_v} => K_v = 20$
- Step 1: Draw Bode plot for $G(j\omega) = \frac{K_v}{j\omega(0.5j\omega+1)}$

$$K_v = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} s \cdot \frac{K}{s(s+2)}$$

$$K_v = \frac{K}{2}$$

$$K = 2K_v$$

$$G(s) = \frac{2K_v}{s(s+2)} = \frac{2K_v}{2s(\frac{s}{2}+1)} = \frac{20}{s(s/2+1)}$$

$$|G(j\omega)| = \left.\frac{20}{\omega\sqrt{\omega^2/4+1}}\right|_{\omega=\omega_{gc}} = 1 \Rightarrow \omega_{gc} = 6.16 \text{ rad/sec}$$

$$20 = \omega_{gc}\sqrt{\frac{\omega_{gc}^2}{4}+1}$$

$$400 = \omega_{gc}^2\left(\frac{\omega_{gc}^2}{4}+1\right)$$

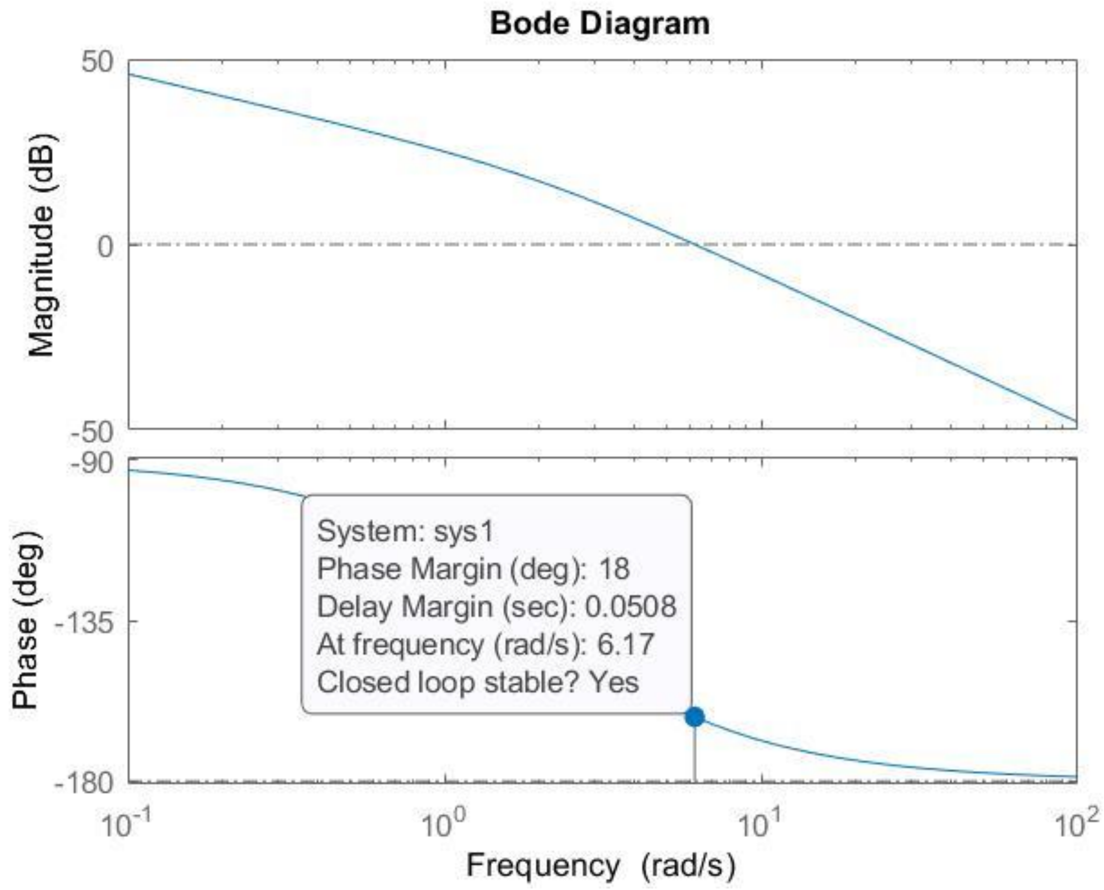
$$1600 = \omega_{gc}^4 + 4\omega_{gc}^2$$

$$\omega_{gc}^4 + 4\omega_{gc}^2 - 1600 = 0$$

$$\phi = -90 - \tan^{-1} 0.5\omega \,/\, \omega = \omega_{gc}$$
$$= -162^\circ$$

$$PM = 180 + \phi$$
$$= 180^\circ - 162^\circ$$
$$= 18^\circ$$

Phase – Lag design using Bode Plot – Example 1



$\phi = -90^\circ - \tan^{-1} 0.5\omega$

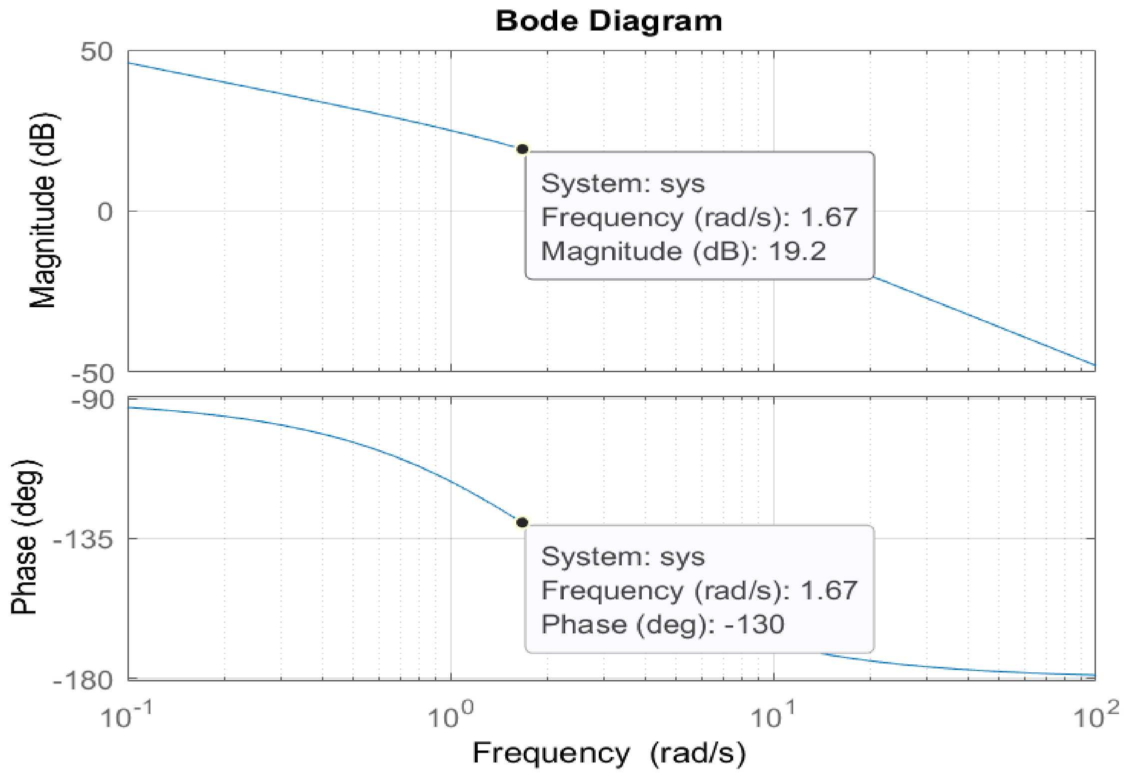
ω	0.1	1	5	10	∞
ϕ	-92.8	-116.5	-158	-168	-180°

- Since PM is 18, it is less than the desired, therefore we need to increase the PM

Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 1

- $\emptyset_1$ =desired PM + safety margin = 45° + 5° = 50°
- Determine the frequency at $-(180° - 50°) = -130°$ => $\omega_c' = 1.66$



Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 1

- Calculate $\propto$ by noting that the attenuation introduced by the phase lag network is $20\log_{10}\propto = Magnitude$ at ω_c'

$\Rightarrow \quad 20\log_{10}\alpha = 19.2$

$\log_{10}\alpha = \frac{19.2}{20} = 0.96$

$\alpha = 10^{0.96} = 9.22$

- $z = \frac{\omega_c'}{10} = \frac{1.66}{10} = 0.166$

- $p = \frac{1}{\propto\tau} = \frac{z}{\propto} = \frac{0.166}{9.2257} = \mathbf{0.0180}$

$G_c(s) = \frac{\frac{s}{z}+1}{\frac{s}{p}+1}$

- Compensated System

$G(j\omega)$

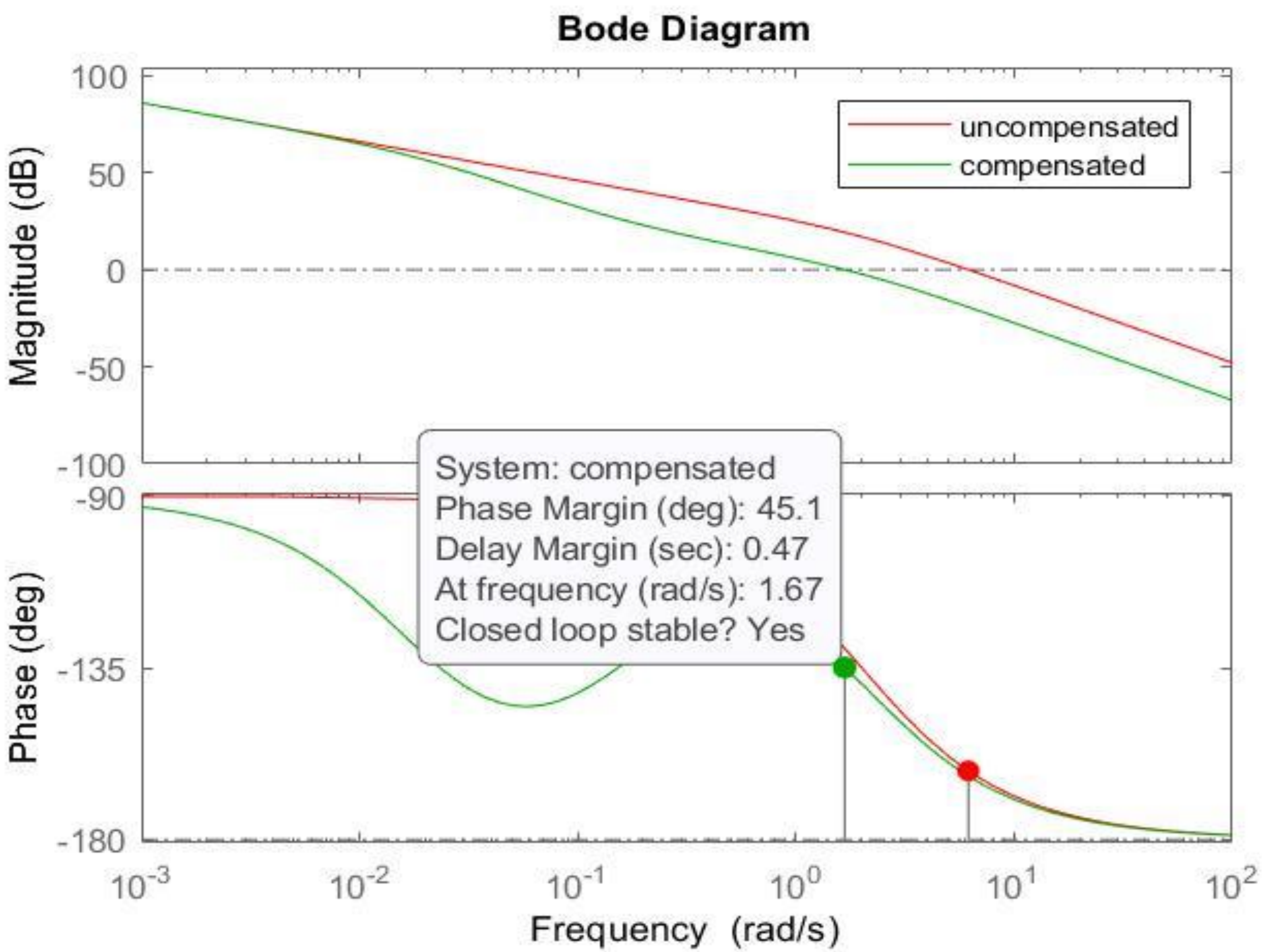
  - $L(j\omega) = G_c(j\omega)G(j\omega) = \frac{20[\frac{j\omega}{0.166}+1]}{j\omega(0.5j\omega+1)[\frac{j\omega}{0.0180}+1]}$

  - $L(s) = G_c(s)G(s) = \frac{20[6.0241s+1]}{s(0.5s+1)[55.5556\ s+1]}$

	C.F	Slope	Net	freq.
$\frac{20}{j\omega}$	none	-20	-20	0.001 to ~~0.0166~~ 0.01
$\frac{1}{\frac{j\omega}{0.018}+1}$	0.01	-20	-40	0.01 to 0.166
$\frac{j\omega}{0.166}+1$	0.166	20	-20	0.166 to 2
$\frac{1}{0.5j\omega+1}$	2	-20	-40	2 onwards

Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 1



$$A = 20 \log \frac{20}{\omega}$$

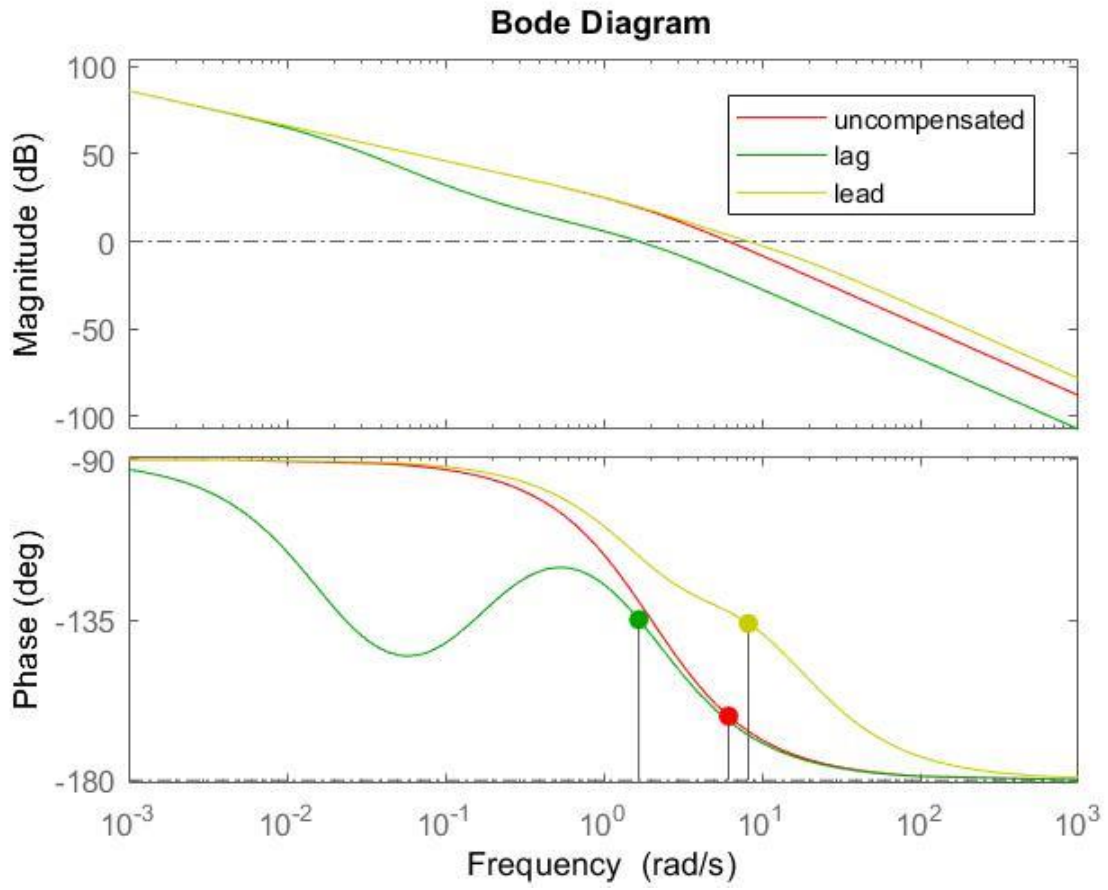
$$= 20 \log 20 - 20 \log \omega$$

at $\omega = 0.001$, $A = 86.02\,dB$

$$\angle L(j\omega) = \tan^{-1}\frac{\omega}{0.166} - 90^\circ - \tan^{-1}0.5\omega - \tan^{-1}\frac{\omega}{0.018}$$

Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 1



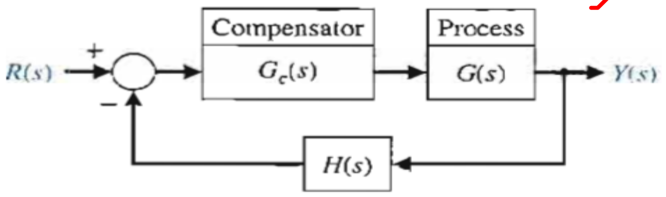
Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 1

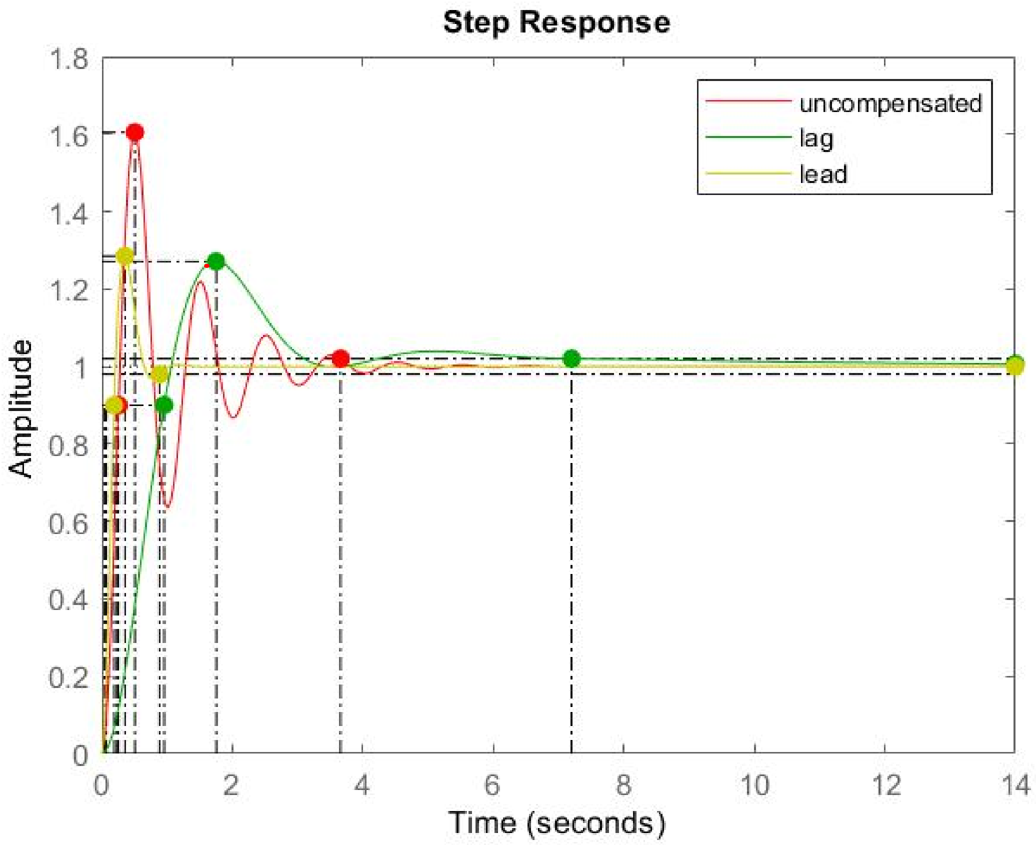
Lead & Lag

$\frac{K}{s(s+2)}$

$G_c(s) = K\frac{s+2}{s+p}$



(a)



CLTF of uncompensated

(T(s) = G(s)/(1+G(s)) and

compensated system

(T(s) = L(s)/(1+L(s))

where, L(s) = Gc(s)G(s)

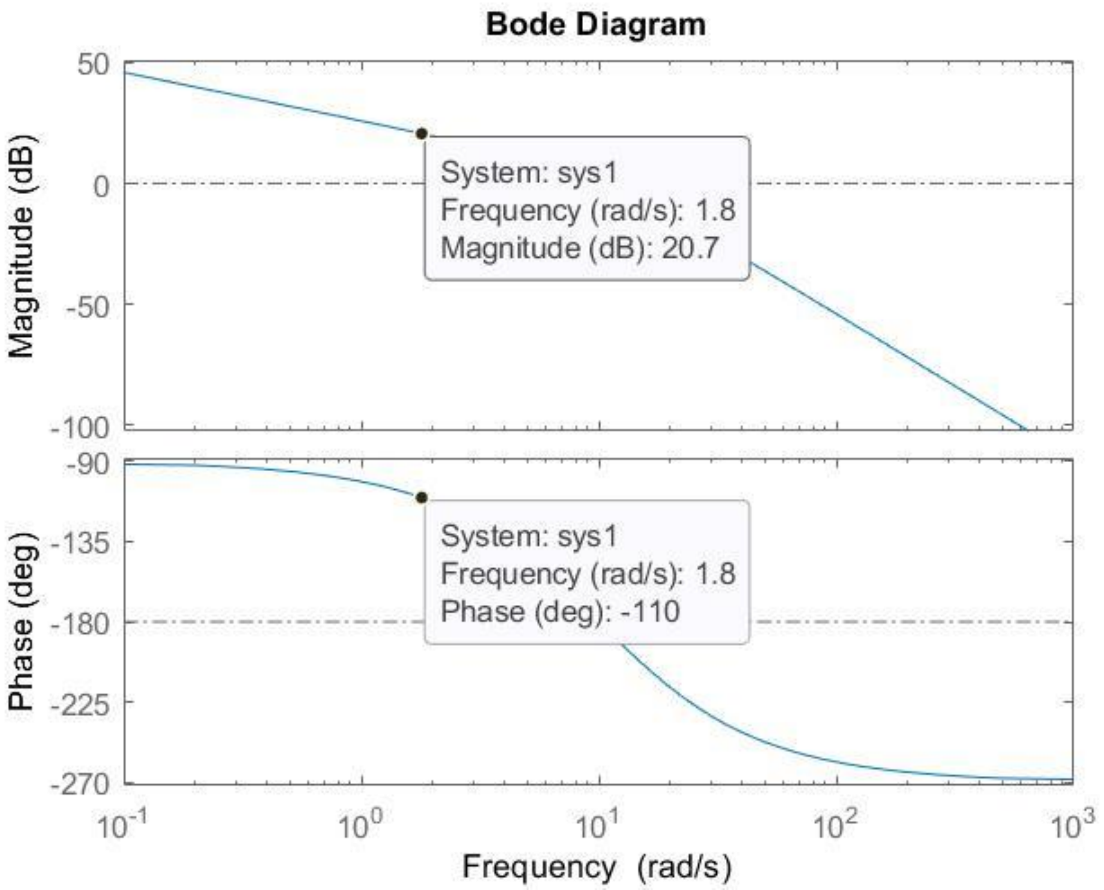
Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 2

- $G(s) = \frac{K}{s(s+10)^2}$ and H(s) = 1
- Desired specification
  - Velocity constant of $K_v = 20$
  - $\zeta = 0.707$
    - $\emptyset_{pm} = \frac{\zeta}{0.01} = 65°$(since 65 is max)
- $G(j\omega) = \frac{K}{j\omega(j\omega+10)^2} = \frac{K_v}{j\omega(0.1j\omega+1)^2}$
- $K_v = \frac{K}{100}$
- Draw Bode plot of $G(j\omega) = \frac{20}{j\omega(0.1j\omega+1)^2}$

Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 2



Phase – Lag design using Bode Plot – Example 2

- $\emptyset_1$ =desired PM + safety margin = 65° + 5° = 70°
- Determine the frequency at $-(180° - 70°) = -110°$ => $\omega_c' = 1.8$
- $-20\log_{10}\propto = Magnitude$ at ω_c'
- $20\log_{10}\propto = 20.7$
- => $\propto = 10.8393$

Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 2

- $z = \frac{\omega_c'}{10} = \frac{1.8}{10} = 0.18$

- $p = \frac{1}{\propto\tau} = \frac{z}{\propto} = \frac{0.18}{10.8393} = 0.0166$

- Compensated System

  - $L(j\omega) = G_c(j\omega)G(j\omega) = \frac{20[\frac{j\omega}{0.18}+1]}{j\omega(0.1j\omega+1)^2(\frac{j\omega}{0.0166}+1)}$

    G(jω)

  - $L(s) = G_c(s)G(s) = \frac{20[5.5556s+1]}{s(0.1j\omega+1)^2(60.2410\, s+1]}$

from Matlab

num = $[\frac{20}{0.18} \quad 20]$

den = $[\, d_2 \,]$

bode (num, den)

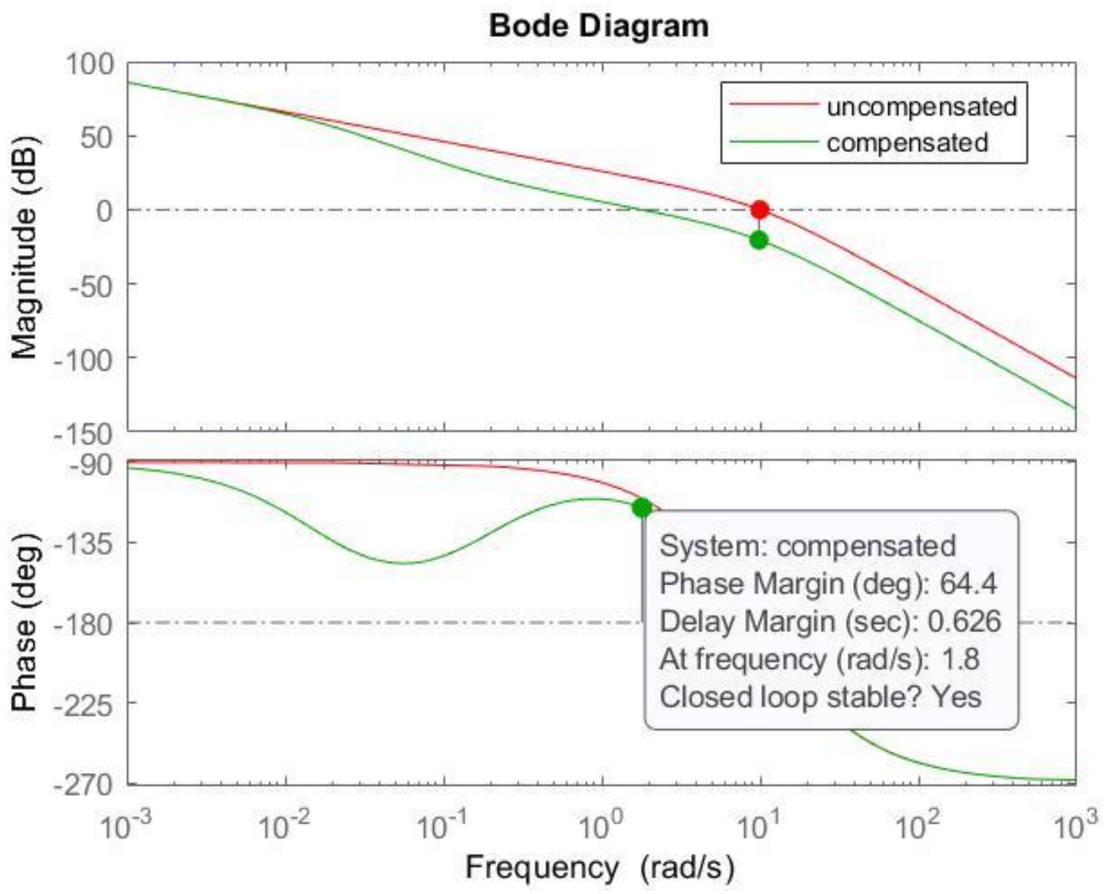
to multiply 2 poly

$d_1 = \text{conv}[(1 \;\; 0) \;\; ((0.1)^2 \;\; 2\times0.1 \;\; 1)]$

$d_2 = [d_1 \;\; (1/0.0166 \;\; 1)]$

Design of Feedback Control Systems

Phase – Lag design using Bode Plot – Example 2



Bode

How to Find GM and PM manually?

- For the given $G(s) = \frac{10}{s(s+10)}$, determine PM.
- Sol: calculate ω_{gc},

  $|G(j\omega)|\ at\ \omega = \omega_{gc}$ is equal to one

  $$\left|\frac{1}{\omega\sqrt{(\frac{\omega^2}{100}+1)}}\right| = 1 \Rightarrow \omega_{gc} = 1 rad/sec$$

  $$\emptyset = -90 - \tan^{-1}\frac{1}{10} = -95.71$$

  PM = 180 + Ø

Ans: PM = 84.29deg

$\Rightarrow \angle G(j\omega) = -90^\circ - \tan^{-1}\frac{\omega}{10}$

$1 = \omega\sqrt{\left(\frac{\omega^2}{100}+1\right)}$

$1 = \omega^2\left(\frac{\omega^2}{100}+1\right)$

$100 = \omega^4 + 100\omega^2$

$\omega^4 + 100\omega^2 - 100 = 0$

$\omega^2 = \quad \Rightarrow \omega$

How to Find GM and PM manually?

- Given $G(s) = \frac{1}{s(1+2s)(1+s)}$, Determine GM.
- Sol:
  - Find ω_{pc} => $\angle G(j\omega)\ at\ \ \omega = \omega_{pc}$ is equal to -180
    - -180 = $-90 - \tan^{-1} 2\omega_{pc} - \tan^{-1} \omega_{pc}$
    - $\tan^{-1} \frac{2\omega_{pc}+\omega_{pc}}{1-2\omega_{pc}*\omega_{pc}} = 90$ => $\omega_{pc} = \sqrt{\frac{1}{2}}$ rad/sec
  - GM(dB) = -20log($|G(j\omega)|\ at\ \omega = \omega_{pc}$)
  - GM(dB) = $-20\log\left|\frac{1}{\omega_{pc}\sqrt{(4\omega_{pc}{}^2+1)}\sqrt{(\omega_{pc}{}^2+1)}}\right|$
  - Ans : 3.5dB

$\frac{2\omega_{pc}+\omega_{pc}}{1-2\omega_{pc}^2} = \frac{1}{0}$

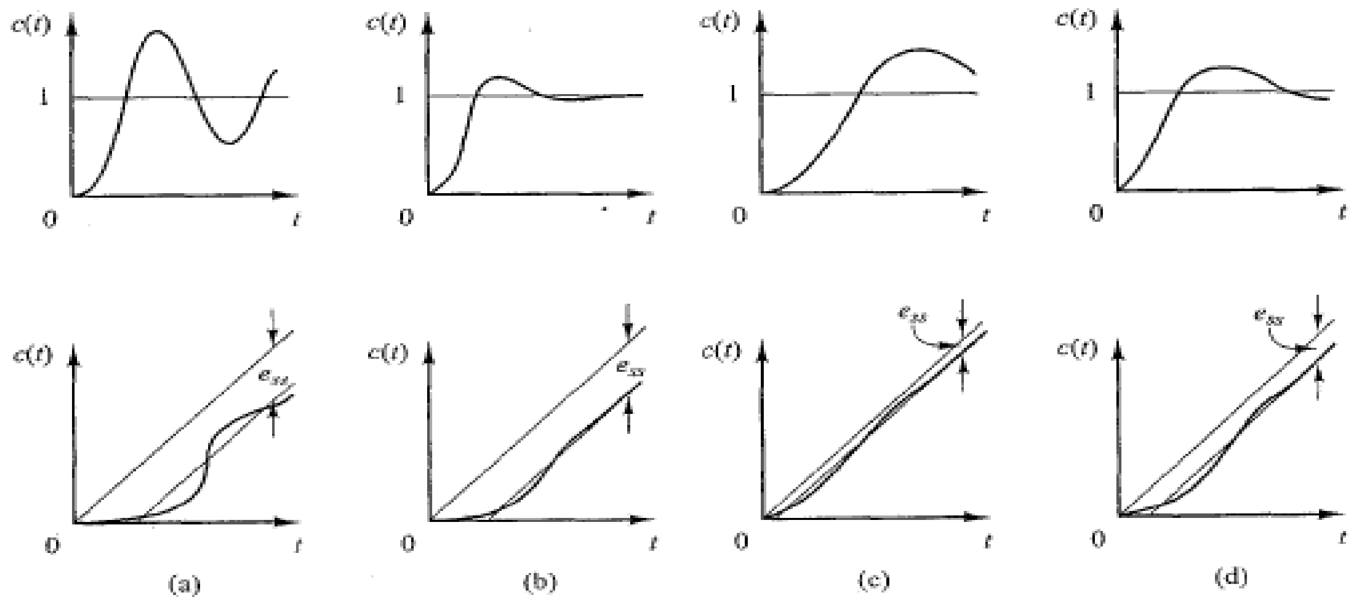
$1-2\omega_{pc}^2 = 0$

$\omega_{pc}^2 = \frac{1}{2}$

$\omega_{pc} = \sqrt{1/2}$

Comparison of Lead, Lag and Lag-Lead Compensators

- Graphical comparison



- Fig(a) shows a unit step response and unit ramp response of an uncompensated system
- Fig(b),(c) and (d) shows unit step response and unit ramp response of compensated using lead, lag and lag-lead compensator respectively

Comparison of Lead, Lag and Lag-Lead Compensators

- The system with lead compensator exhibits the fastest response while that with the lag compensator exhibits the slowest response, but with marked improvements in the unit ramp response.
- The system with lag-lead compensator will give a compromise; reasonable improvements in both the transient response and steady state response

Ex 3 : Lag Compensators

- $G(s) = \frac{1}{s(s+1)(0.5s+1)}$ and H(s) = 1
- Desired specification
  - Phase margin = 40 degree
  - Velocity error constant, $K_v = 5/sec$
  - Gain margin is at least 10dB

Ex 3 : Lag Compensators

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- If K is not given in G(s), include K in the numerator and find K such that velocity error constant, K_v is satisfied.

  - $K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{K}{s(s+1)(0.5s+1)} = 5 \Rightarrow K = 5$
  - => $G(s) = \frac{5}{s(s+1)(0.5s+1)}$ => $G(j\omega) = \frac{5}{j\omega(j\omega+1)(0.5j\omega+1)}$

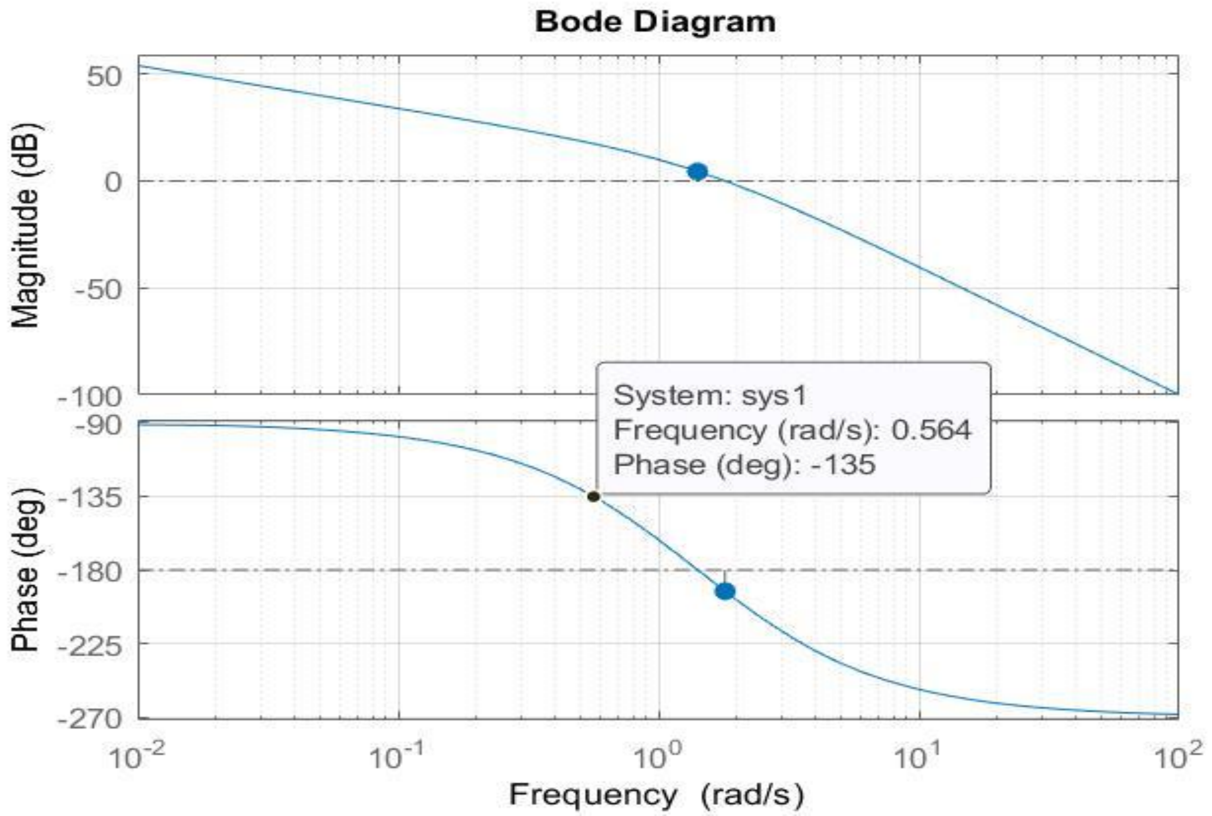
- Step 1: Draw Bode plot for

- $G(j\omega) = \frac{5}{j\omega(j\omega+1)(0.5j\omega+1)}$

- find K in terms of Kv
- sub. K in G(s)
- convert into time-constant form

Ex 3 : Lag Compensators

- $\emptyset_1$ =desired PM + safety margin = $40°$ + $5°$ = $45°$
- Determine the frequency at $-(180° - 45°) = -135°$ => $\omega_c' = 0.562$



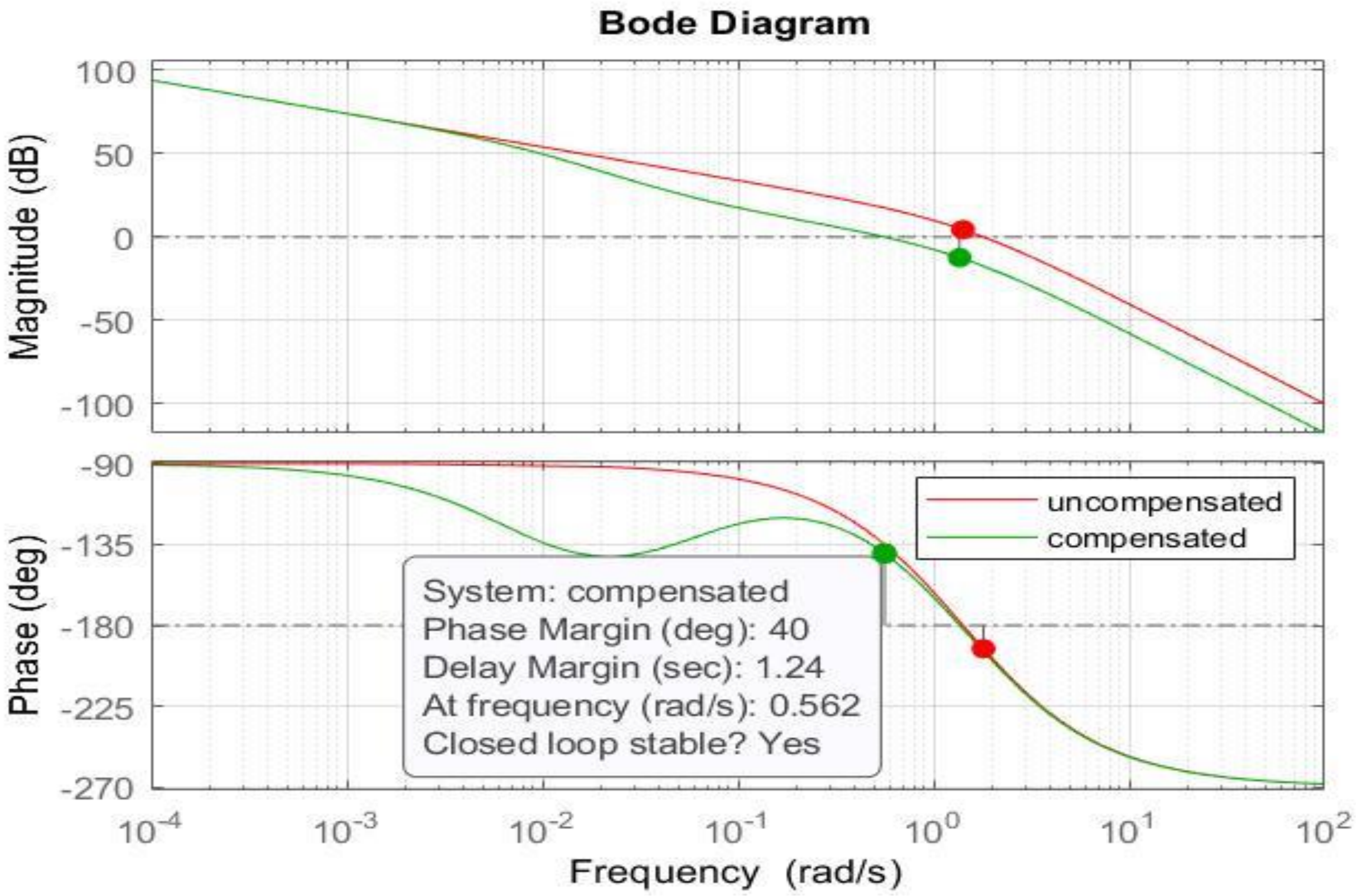
Design of Feedback Control Systems

Ex 3 : Lag Compensators

- $-20\log_{10}\propto = Magnitude$ at ω_c'
- $20\log_{10}\propto = 17.5$
- => $\propto = 7.498$
- $z = \frac{\omega_c'}{10} = \frac{0.562}{10} = 0.0562$
- $\boldsymbol{p = \frac{1}{\propto\tau} = \frac{z}{\propto} = \frac{0.0562}{7.498} = 0.0075}$
- Compensated System
  - $L(j\omega) = G_c(j\omega)G(j\omega) = \frac{5[\frac{j\omega}{0.0562}+1]}{j\omega(0.5j\omega+1)[\frac{j\omega}{0.0075}+1](j\omega+1)}$
  - $L(s) = G_c(s)G(s) = \frac{5[17.7936s+1]}{s(0.5s+1)[133.4331s+1](1+s)}$

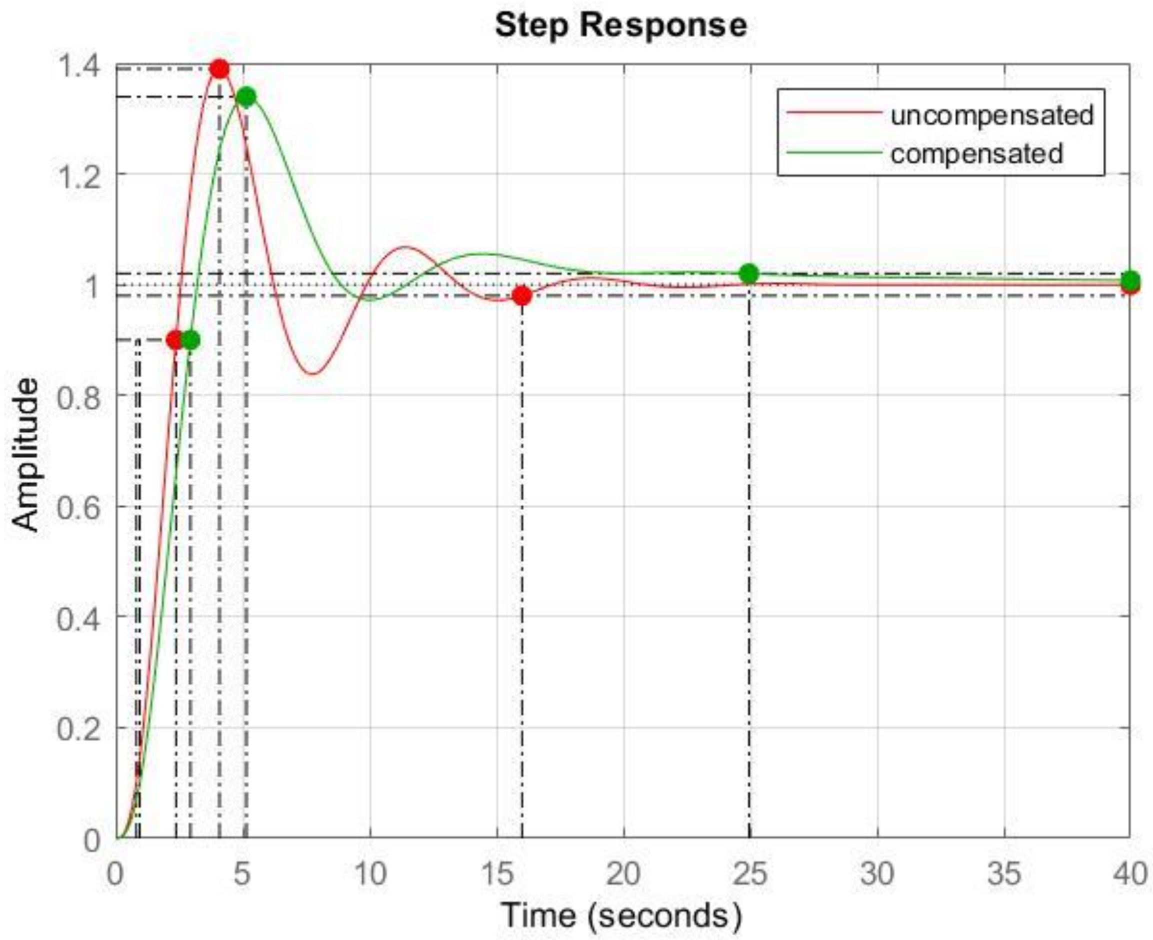
Design of Feedback Control Systems

Ex 3 : Lag Compensators

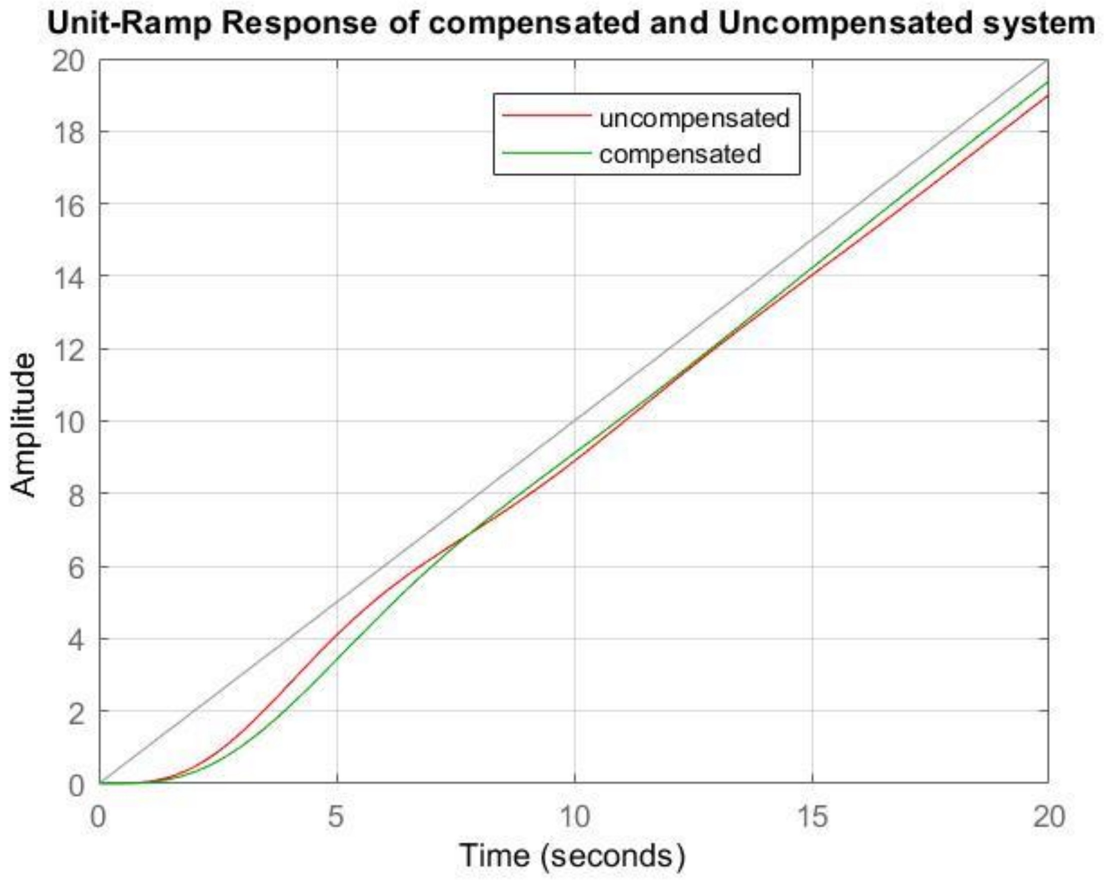


Design of Feedback Control Systems

Ex 3 : Lag Compensators

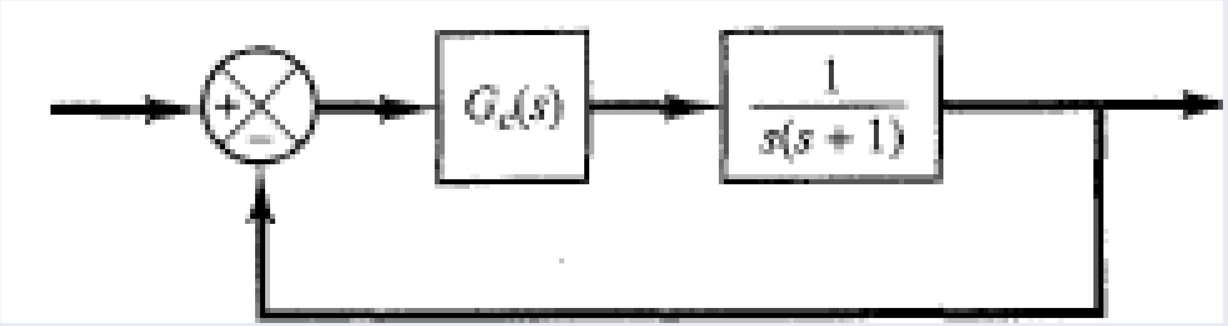


Ex 3 : Lag Compensators



Ex 4 : Lead Compensators

Consider the system shown in figure.



Design a compensator such that the static velocity error constant K_v is 50/sec, phase margin is 50 degree, and gain margin not less than 8 dB.

Ex 4 : Lead Compensators

- $K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{K}{s(s+1)} = 50 \Rightarrow \frac{K}{1} = 50 \Rightarrow K = 50$
- => $\mathrm{G(s)} = \frac{50}{s(s+1)}$ => $G(j\omega) = \frac{50}{j\omega(j\omega+1)}$
    - Draw Bode plot for $G(j\omega) = \frac{50}{j\omega(j\omega+1)}$=>$\emptyset_{un} = 8$

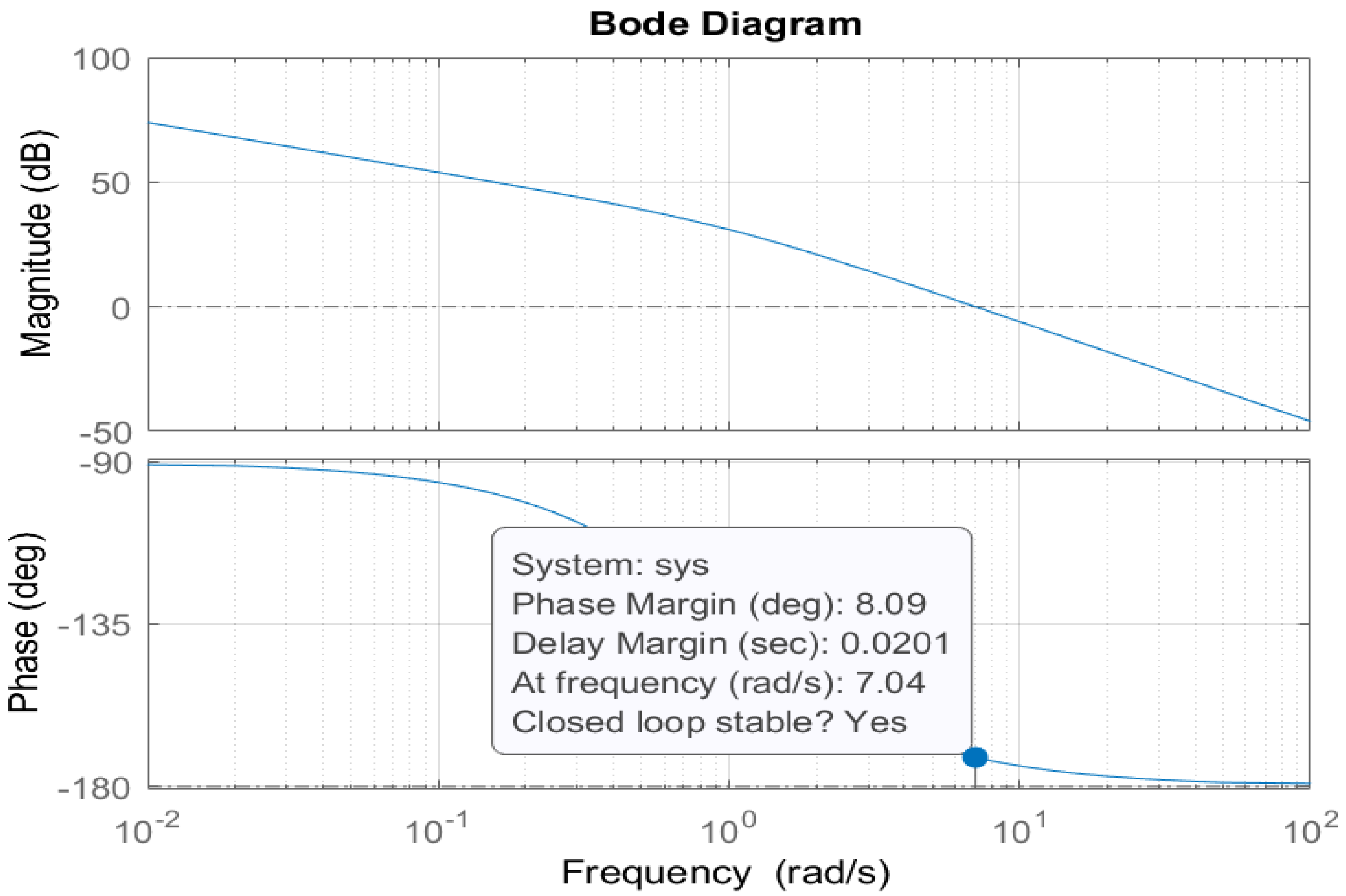
	none	slope	Net slope		
$\frac{50}{j\omega}$		-20	-20	$\omega = 0.1$ to 1	
$\frac{1}{j\omega+1}$	1	-20	-40	$\omega = 1$ onwards	

$A = 20\log\frac{50}{\omega}$

$= 20\log 50 - 20\log\omega$

Design of Feedback Control Systems

Ex 4 : Lead Compensators



$\phi = \angle G(j\omega) = -90^\circ - \tan^{-1}\omega$

$\omega = 0.1 \quad 0.5 \quad 1 \quad 10$

$\phi =$

$|G(j\omega)|_{\omega=\omega_{gc}} = \dfrac{50}{\omega\sqrt{\omega^2+1}} = 1$

$\omega_{gc}\sqrt{\omega_{gc}^2+1} = 50$

$\omega_{gc}^2(\omega_{gc}^2+1) = 2500$

$\omega_{gc}^4 + \omega_{gc}^2 - 2500 = 0$

Ex 4 : Lead Compensators

- $\boldsymbol{\emptyset_1 = desired\ PM \ - (\emptyset_{un})}$
- $\boldsymbol{\emptyset_m = \emptyset_1 + safety\ margin}$
- **Desired PM ,** $\emptyset_{pm}$**= 50 and** $\emptyset_{un} = 8$
- $\emptyset_1 = $ 50-8 = 42
- $\emptyset_m = 42 + 3\ (10\%\ of\ \emptyset_1 as\ safety\ margin) = 45$
- $\boldsymbol{sin\ \emptyset_m = \frac{\propto -1}{\propto +1}}$ or $\propto = \frac{1+\sin(\emptyset_m)}{1-\sin(\emptyset_m)} = 5.828$

Ex 4 : Lead Compensators

In lag compensator we use $20 \log_{10} \alpha$ = magnitude at ω_c'

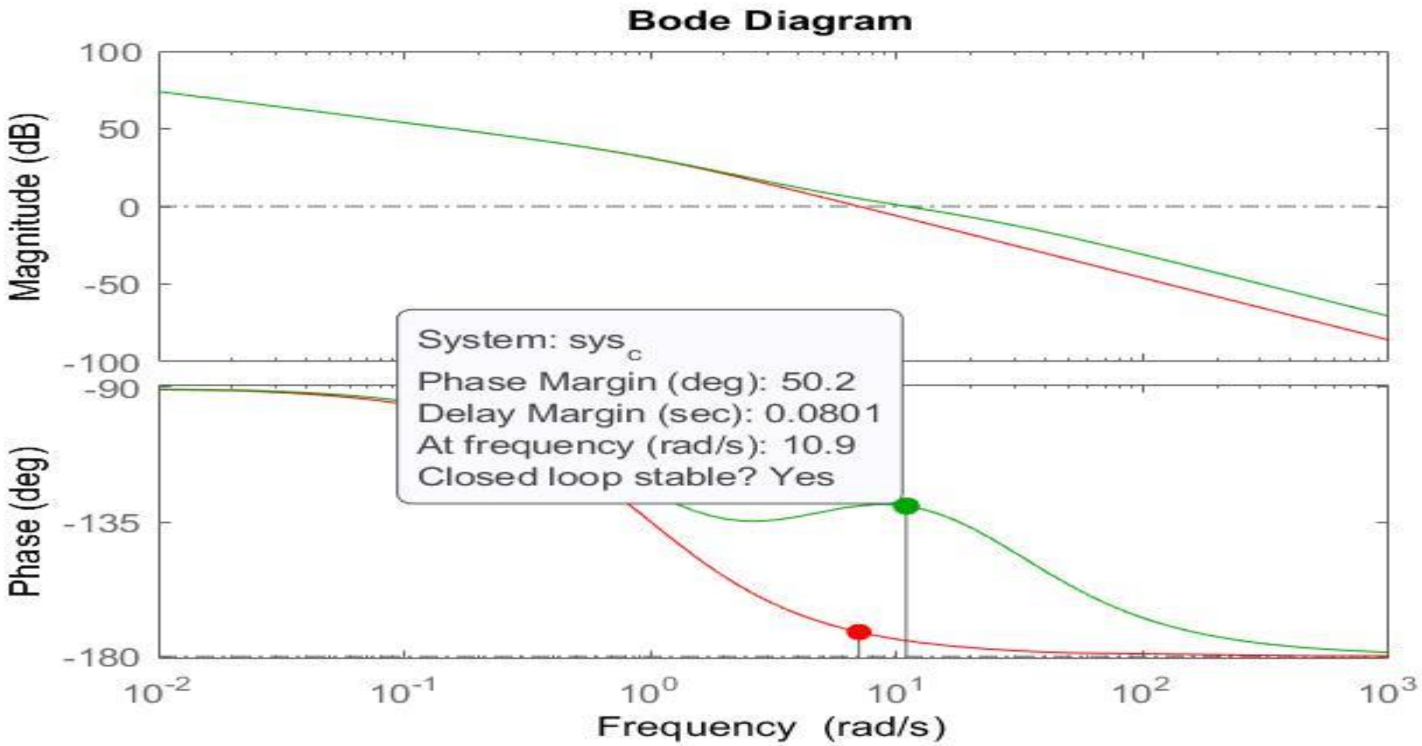
- Evaluate $10\log_{10} \propto$ = 7.66 dB
- frequency at which the uncompensated magnitude curve is equal to -7.66 dB is $\boldsymbol{\omega_m = 11\, rad/sec}$ (Because the compensation network provides a gain of $10\log_{10} \propto$ at $\boldsymbol{\omega_m}$ and this is the new 0dB cross over frequency)
- $\boldsymbol{p = \omega_m\sqrt{\propto} = 11\sqrt{5.828} = 26.556}$,
- $\boldsymbol{z = \frac{p}{\propto} = 4.556}$

Ex 4 : Lead Compensators

- Draw the bode plot for compensated system
- $L(j\omega) = G_c(j\omega)G(j\omega) = \dfrac{\cancel{2}5 0[\frac{j\omega}{z}+1]}{j\omega(j\omega+1)\ \ (\frac{j\omega}{p}+1)}$

$z = 4.556$

$p = 26.556$



Ex 5 : Lead Compensators

- Consider the system transfer function

$$G(s) = \frac{K}{s^2(0.2s + 1)}$$

$K_a = 10$

- Design a lead compensator such that the static acceleration error constant is greater than equal to 10/sec^2, and desired phase margin is 35 degree.

$$K_a = \lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} \cancel{s^2} \frac{K}{\cancel{s^2}(0.2s+1)} = K$$

$K = 10$

Draw Bode plot for $G(j\omega) = \dfrac{10}{(j\omega)^2(0.2j\omega+1)}$

		slope	net slope	
$\dfrac{10}{(j\omega)^2}$	none	-40	-40	$\omega = 0.1$ to $\omega = 5$
$\dfrac{1}{0.2j\omega+1}$	$1/0.2 = 5$	-20	-60	$\omega = 5$ onwards

$$A = 20\log\frac{10}{\omega^2}$$

$$= 20\log 10 - 20\log_{10}\omega^2$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}(0.2\omega)$$

PM = -33° , desired PM = 35°

$\phi_1 = 35^\circ - (-33^\circ) = 68^\circ$

$\phi_m = 68^\circ +$ safety margin

$= 68^\circ + 10^\circ = 78^\circ$ (ϕ_m max 65°)

Since $\phi_m > 65^\circ$, $\frac{\phi_m}{2} = \frac{78}{2} = 39$

$$\alpha = \frac{1 + \sin 39^\circ}{1 - \sin 39^\circ} = 4.395$$

$$-10 \log_{10} \alpha = 10 \log_{10} (4.395) = -6.43 \text{ dB}$$

Design of Feedback Control Systems

Ex 5 : Lead Compensators

since we are designing 2 compensators in cascade

we have the total gain $= -(6.43 + 6.43)\,dB$

$= -12.8\,dB$

freq. at $-12.8\,dB$ is $\underline{\omega_m = 5.44}$ rad/sec

$P = \omega_m\sqrt{\alpha} = 11.4046$

$Z = \dfrac{P}{\alpha} = 2.5949$

Design of Feedback Control Systems

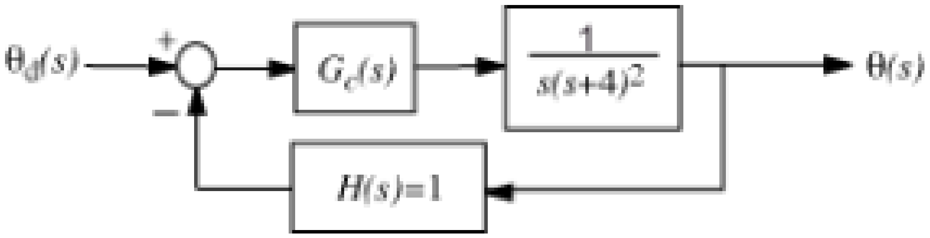
Ex 5 : Lead Compensators

$$L(s) = \frac{10}{s^2(0.2s+1)} \left(\frac{\frac{s}{2.5949}+1}{\frac{s}{11.4046}+1} \right) \left(\frac{\frac{s}{2.5949}+1}{\frac{s}{11.4046}+1} \right)$$

$$PM = 30.5$$

Ex 6 : Lag Compensators

- Consider the system shown in figure



- Design a compensator such that the static velocity error constant is greater than equal to 10/sec, and P.O. less than or equal to 20%

THANK YOU

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