Principles of Digital Signal Processing

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DSP

Frequency domain sampling: DFT

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Frequency domain sampling and reconstruction of DT signals

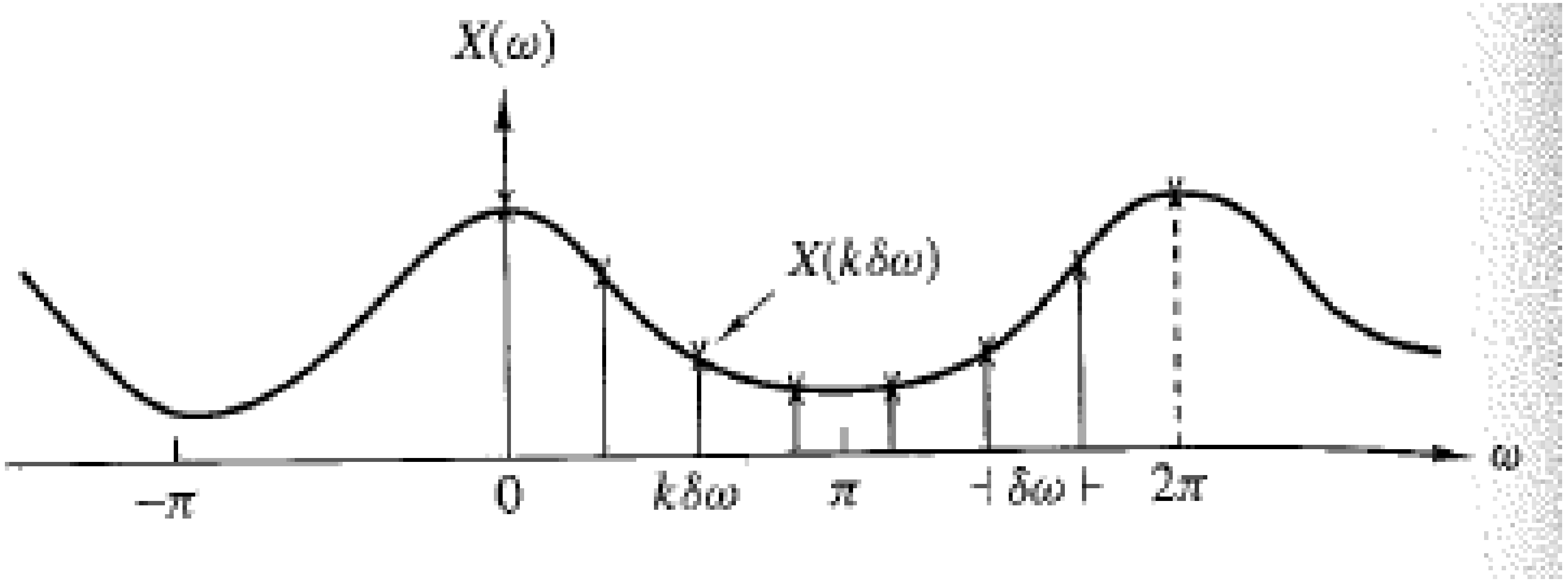
- Sampling of FT of an aperiodic discret e–time sequence

- Establish the relationship between the sampled FT and DFT

Frequency domain sampling

Frequency domain sampling and reconstruction of DT signals

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \qquad \text{-----(1)}$$



Frequency domain sampling

Frequency domain sampling and reconstruction of DT signals

If we evaluate at w=2πkn/N

$$X\left(\frac{2\pi}{N}k\right) = \cdots + \sum_{n=-N}^{-1} x(n)e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$$

$$+ \sum_{n=N}^{2N-1} x(n)e^{-j2\pi kn/N} + \cdots$$

$$= \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)e^{-j2\pi kn/N}$$

-----(2)

Frequency domain sampling and reconstruction of DT signals

Changing index of inner summation from n to n-lN

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\right]e^{-j2\pi kn/N} \qquad \text{-----(3)}$$

xp(n)

for $k = 0, 1, 2, \ldots, N-1$.

The signal

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN) \qquad \text{-----(4)}$$

Frequency domain sampling

Frequency domain sampling and reconstruction of DT signals

$$x_p(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}, \qquad n = 0, 1, \ldots, N-1$$ -----(5)

with Fourier coefficients

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi kn/N}, \qquad k = 0, 1, \ldots, N-1$$ -----(6)

$$c_k = \frac{1}{N} X\left(\frac{2\pi}{N}k\right), \qquad k = 0, 1, \ldots, N-1$$ -----(7)

Therefore,

$$x_p(n) = \frac{1}{N} \sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right) e^{j2\pi kn/N}, \qquad n = 0, 1, \ldots, N-1$$ -----(8)

Frequency domain sampling and reconstruction of DT signals

Relationship between $x_p(n)$ and $x(n)$

- Reconstruction of periodic signal

$x_p(n)$ from samples of the spectrum

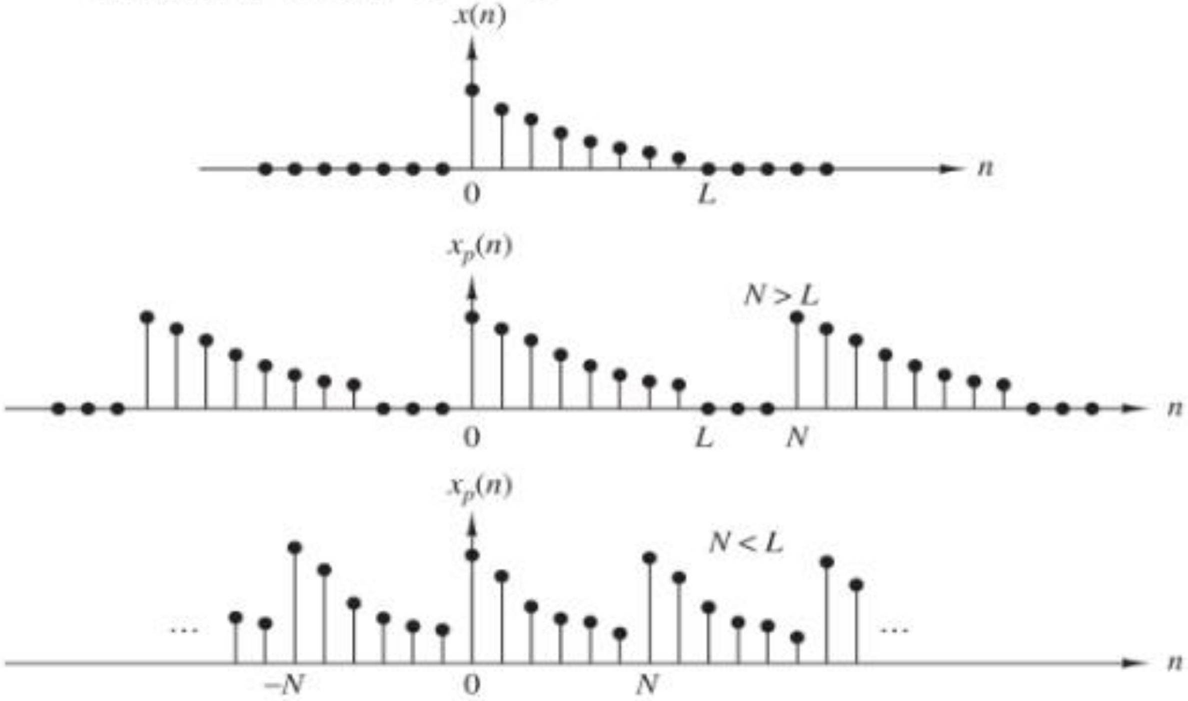
  – equation (8), *proved*

- However, reconstruction of $x(n)$ and $X(\omega)$ from the samples– yet to be done
- Hence, establish a relationship b/n $x_p(n)$ and $x(n)$

Frequency domain sampling

Frequency domain sampling and reconstruction of DT signals

- $x(n)$ can be recovered from its periodic repetition $x_p(n)$
  - If no aliasing in time−domain
    i.e., $x(n)$ is time limited to a period less than N of $x_p(n)$, say, $N \geq L$



Frequency domain sampling and reconstruction of DT signals

$$x(n) = x_p(n) \qquad 0 \le n \le N-1 \qquad \text{-----(9)}$$

We can conclude that the spectrum of aperiodic discrete–time signal with finite duration L can be exactly recovered from its samples at frequencies $\omega_k = 2\pi k/N$, if $N \geq L$

Frequency domain sampling

Frequency domain sampling and reconstruction of DT signals

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right) e^{j2\pi kn/N}, \qquad 0 \le n \le N-1$$

-----(10)

$$X(\omega) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right) e^{j2\pi kn/N}\right] e^{-j\omega n}$$

-----(11)

$$= \sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right)\left[\frac{1}{N}\sum_{n=0}^{N-1} e^{-j(\omega-2\pi k/N)n}\right]$$

Frequency domain sampling

Frequency domain sampling and reconstruction of DT signals

Basic interpolation function shifted by 2kπn/N

$$P(\omega) = \frac{1}{N}\sum_{n=0}^{N-1} e^{-j\omega n} = \frac{1}{N}\frac{1-e^{-j\omega N}}{1-e^{-j\omega}}$$

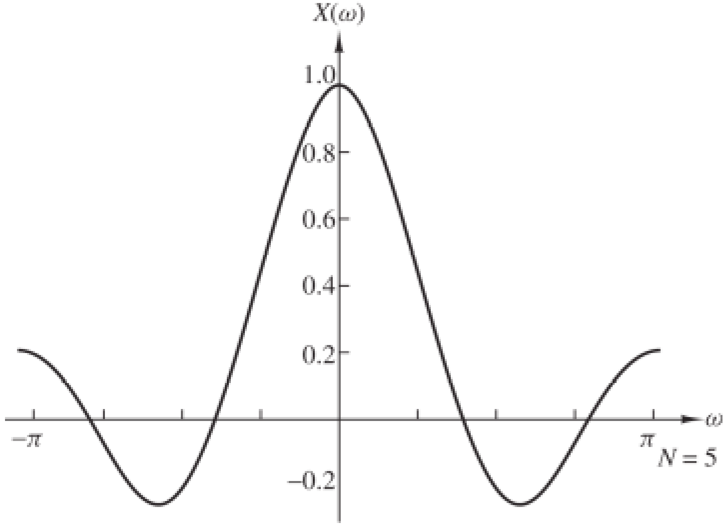
$$= \frac{\sin(\omega N/2)}{N\sin(\omega/2)} e^{-j\omega(N-1)/2} \qquad \text{-----(12)}$$

Frequency domain sampling

Frequency domain sampling and reconstruction of DT signals

$$X(\omega) = \sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right) P\left(\omega - \frac{2\pi}{N}k\right), \qquad N \geq L$$

-----(13)



- The linear interpolation formula in (13) gives exactly the sample values *X(2πk/N) for ω=2πk/N.*
- At all other frequencies equation (13) provides a properly weighted linear combination of the original spectral samples.

THANK YOU

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