Recap

Sampling (low-pass signals)

Topics for this session

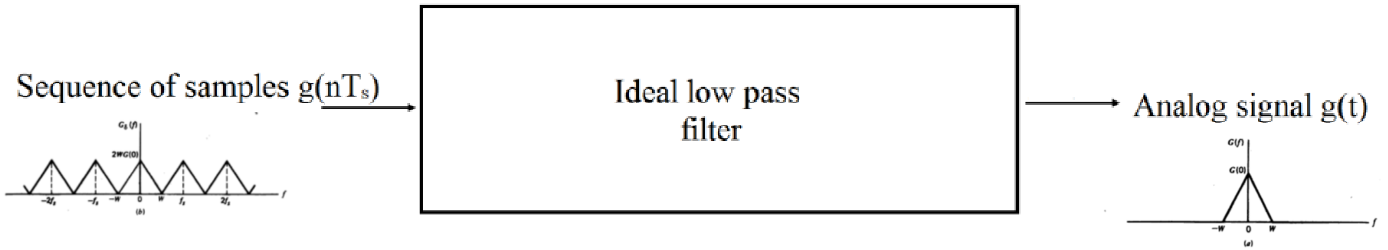
Reconstruction

Problems on sampling theorem

Reconstruction

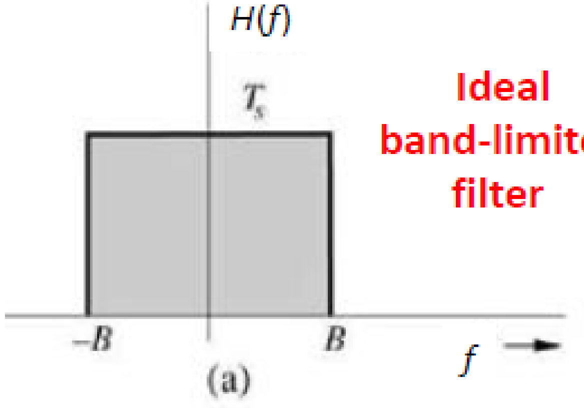
$$G(f) = G_\delta(f)H(f)$$

$$= \frac{1}{2W} G_\delta(f); -W \le f \le W$$

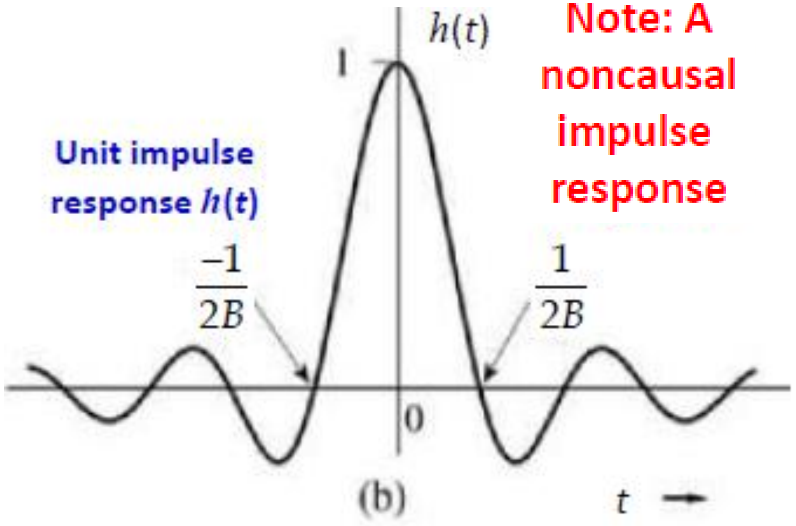


DIGITAL COMMUNICATION

Sampling Theorem



Ideal band-limited filter



Unit impulse response $h(t)$

Note: A noncausal impulse response

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Reconstruction

$$G_{\delta}(f) = \sum_{n=-\infty}^{\infty} g(nT_s)e^{-j2\pi f n T_s}$$

$$G_{\delta}(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(\frac{-j\pi n f}{W}\right)$$

$$G(f) = \frac{1}{2W}\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(\frac{-j\pi n f}{W}\right); -W \le f \le W$$

- IFT of G(f) is $g(t)=\int_{-\infty}^{\infty} G(f)e^{j2\pi ft}df$

$$g(t)=\frac{1}{2W}\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\int_{-W}^{W} e^{\frac{-j\pi nf}{W}} e^{j2\pi ft}df$$

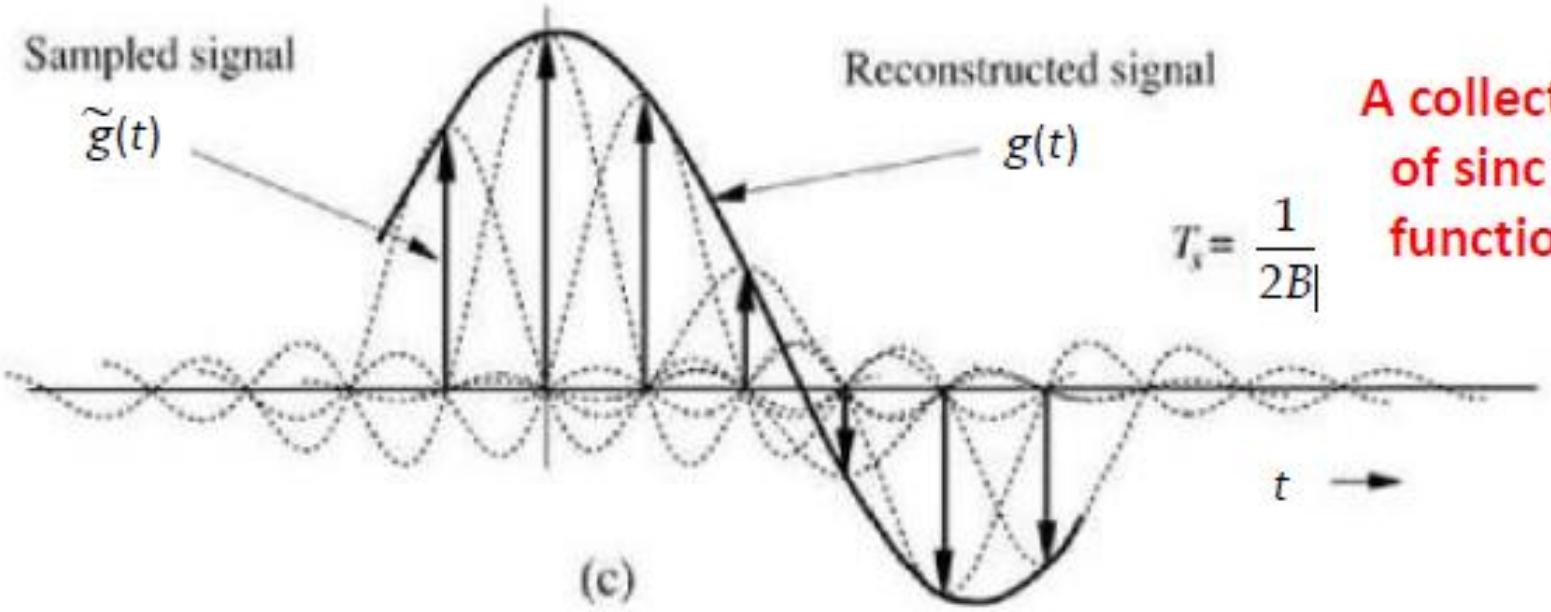
$$=\frac{1}{2W}\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\int_{-W}^{W} e^{j2\pi f\left(t-\frac{n}{2W}\right)}df$$

$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin\left(2\pi W\left(t - \frac{n}{2W}\right)\right)}{\pi\left(t - \frac{n}{2W}\right)}$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin\left(\pi\left(2Wt - n\right)\right)}{\pi\left(2Wt - n\right)}$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \sin c(2Wt - n)$$

Sampling Theorem



A collection of sinc (t) functions

Find Nyquist rates of

- 1) $\sin c(100t)$

$$FT\left\{A\,rect\left(\frac{t}{T}\right)\right\} \leftrightarrow AT\sin c(fT)$$

$$rect(t) \leftrightarrow \sin c(f)$$

From Duality property

$$\sin c(t) \leftrightarrow rect(f)$$

Using Time scaling property

$$\sin c(100t) \leftrightarrow \frac{1}{100}\,rect\left(\frac{f}{100}\right)$$

Band width W=50Hz

Nyquist rate = 2W = 100Hz

- 2) $\sin c(100t) + \sin c(1000t)$

$$\leftrightarrow \frac{1}{100} rect\left(\frac{f}{100}\right) + \frac{1}{1000} rect\left(\frac{f}{1000}\right)$$

Bandwidth = 500Hz

Nyquist rate = 1000Hz