DIGITAL COMMUNICATION

Bharathi V Kalghatgi.
Department of Electronics and Communication Engineering

DELTA MODULATION

Delta Modulation & Problems

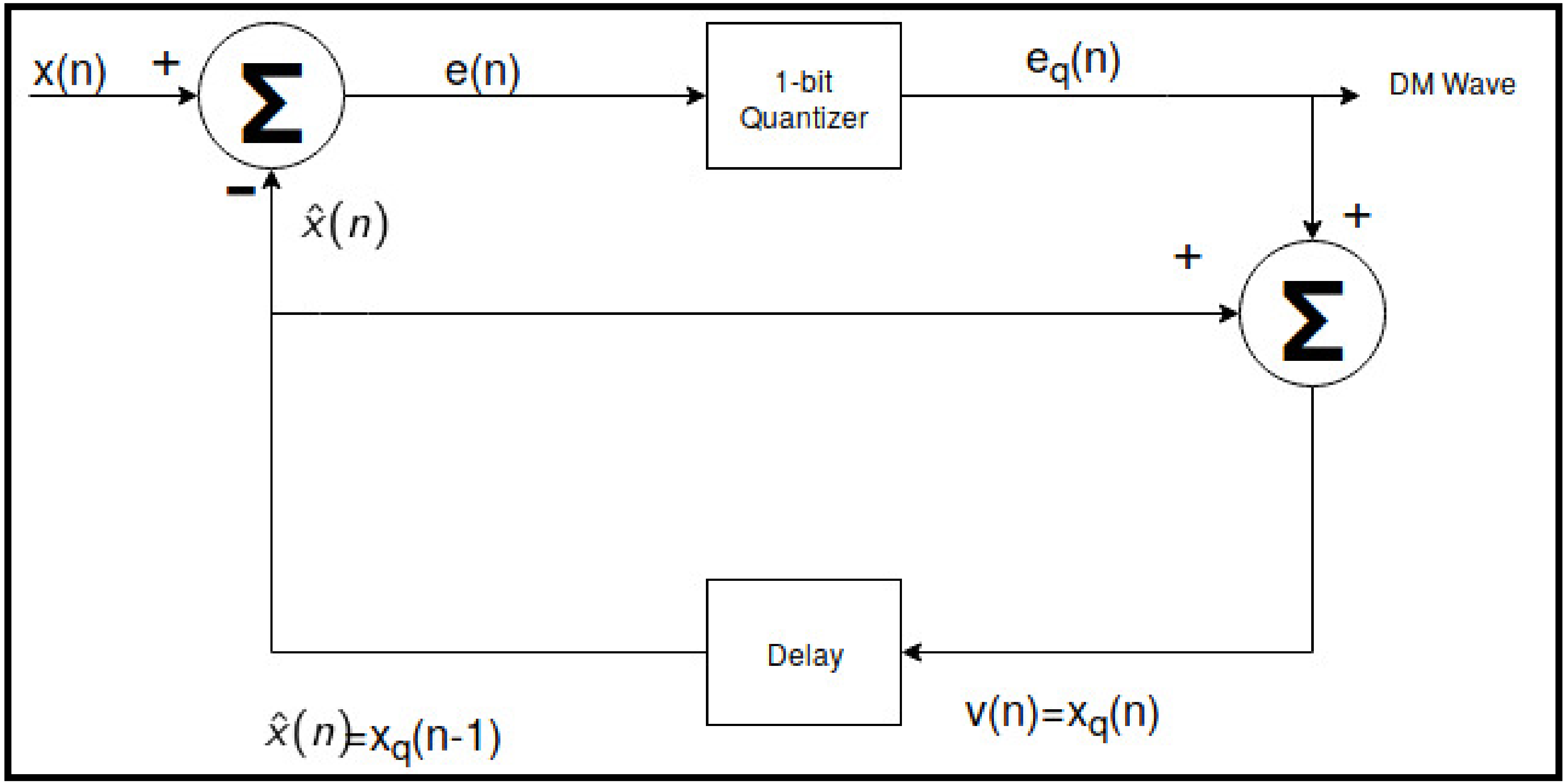
Bharathi V Kalghatgi.
Department of Electronics and Communication Engineering

DM's relation with DPCM: What is DM?

- DM is a special case of DPCM.
- DM is the one bit or version of DPCM.
- DM quantizes the difference between a sample and it's latest approximation using only one bit of quantization.
- If there is only one bit of quantization, the differences are being coded into two levels only.
- Let it be $+\delta$ and $-\delta$ respectively corresponding to positive or negative difference.

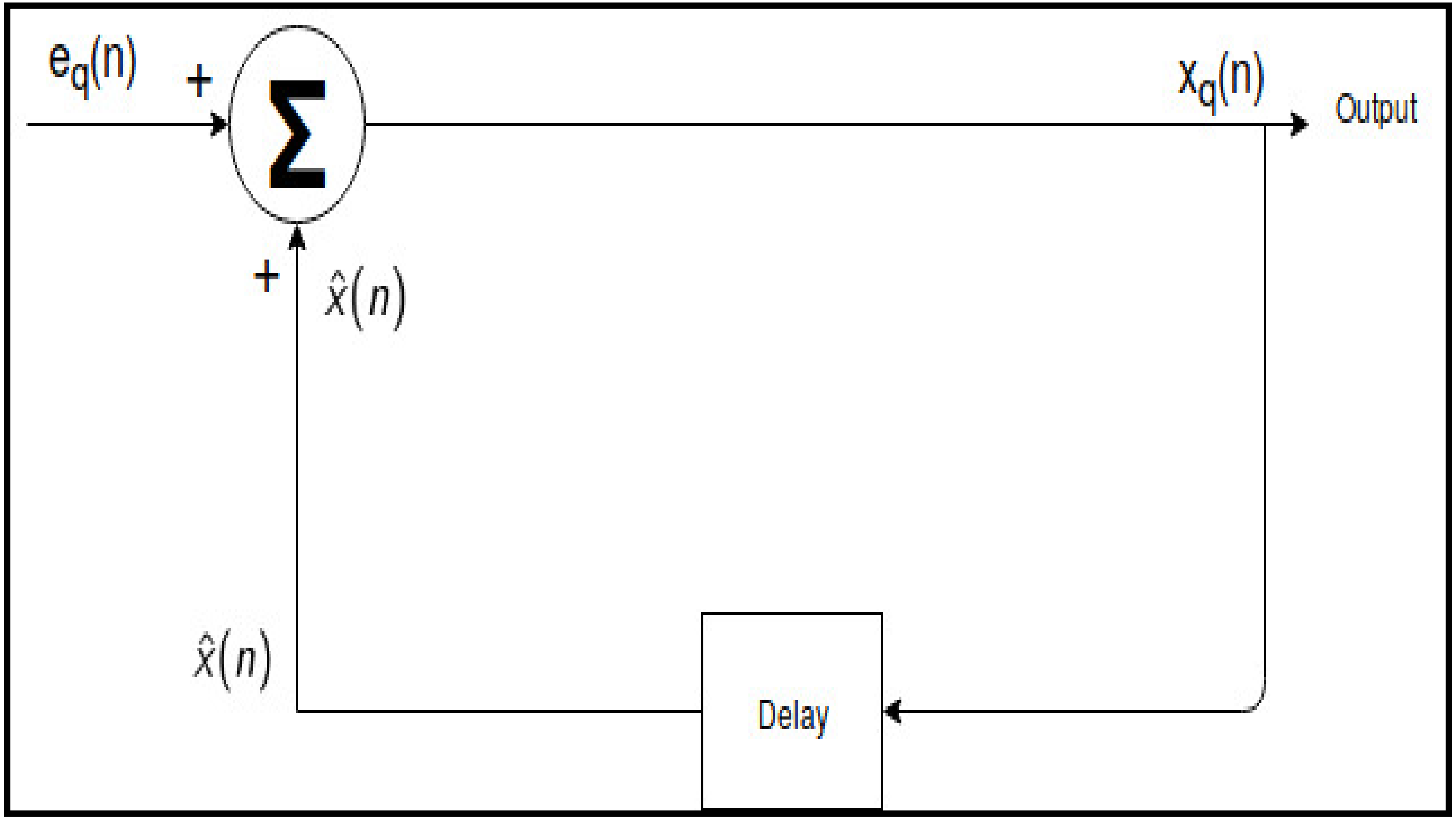
Delta Modulation

DM Transmitter Block Diagram



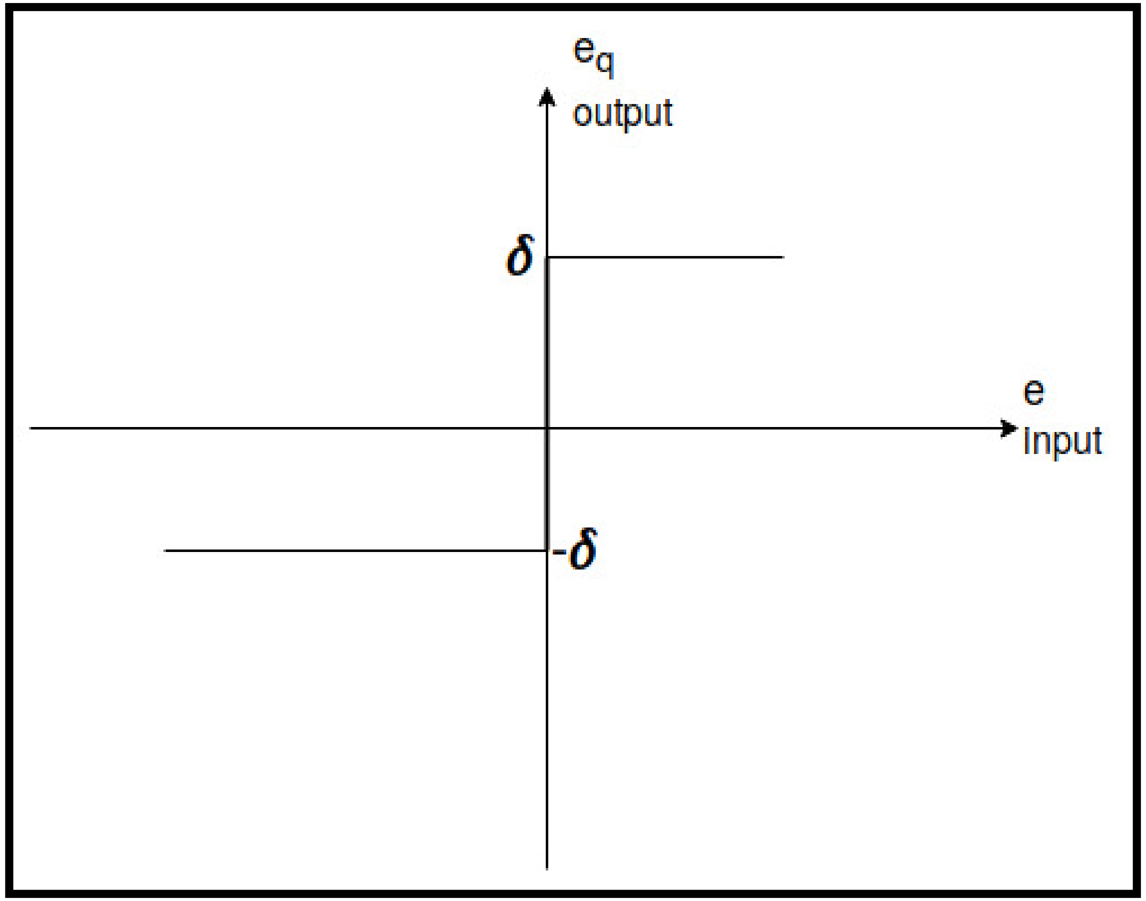
Delta Modulation

DM Receiver Block Diagram



Delta Modulation

DM Quantizer Transfer Characteristics



- The step size Δ of the quantizer is

$$\Delta = 2\delta$$

Delta Modulation
DM Transmitter

From the Transmitter block diagram

$$\mathrm{v}(n) = x_q(n) = e_q(n) + \hat{x}(n)$$

$$= e_q(n) + x_q(n-1)$$

$$= e_q(n) + e_q(n-1) + \hat{x}(n-1)$$

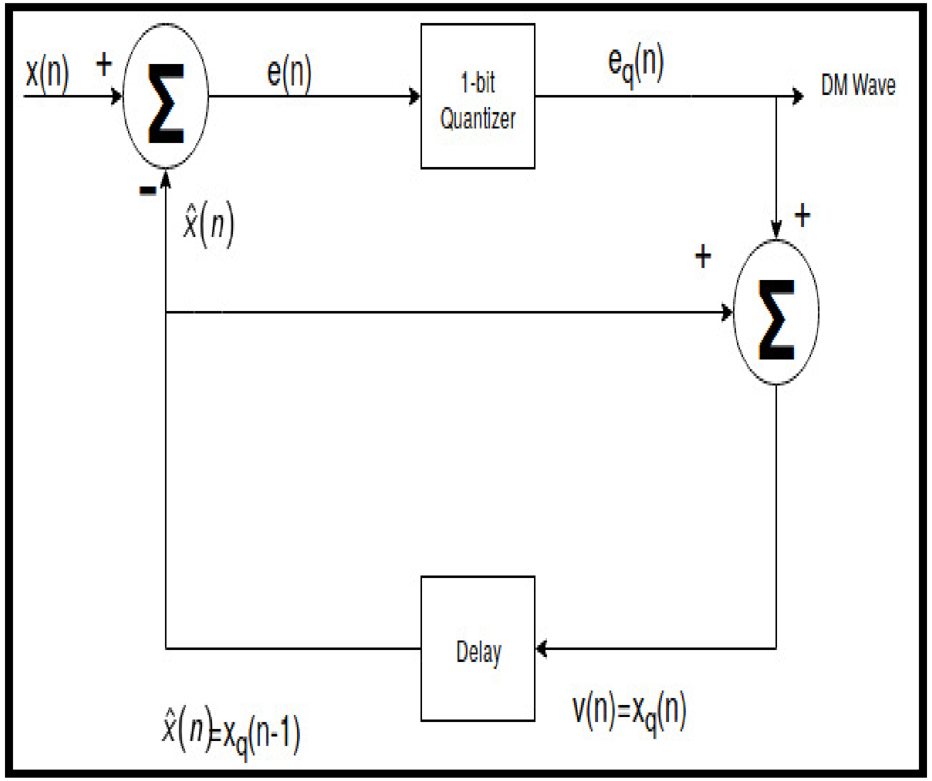
$$= e_q(n) + e_q(n-1) + x_q(n-2)$$

$$= e_q(n) + e_q(n-1) + e_q(n-2) + \hat{x}(n-2)$$

.

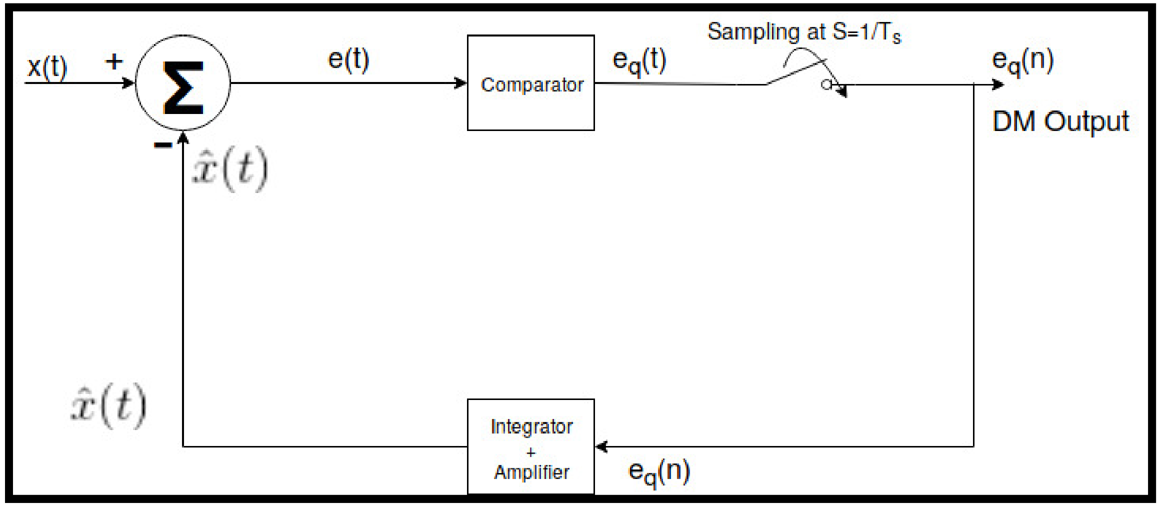
If $\hat{x}(0) = 0$, then

$$x_q(n) = \sum_{k=1}^{n} e_q(k)$$



Delta Modulation
DM Practical Implementation

- The quantized sample is obtained as an accumulation of the quantized error samples (previous equation)
- Thus at each sampling instant, the accumulator increments the approximation to the input signal by $\pm\delta$, depending on the binary output of the modulator. A practical implementation of the same is as show in Figure



DM Practical Implementation - Cont.

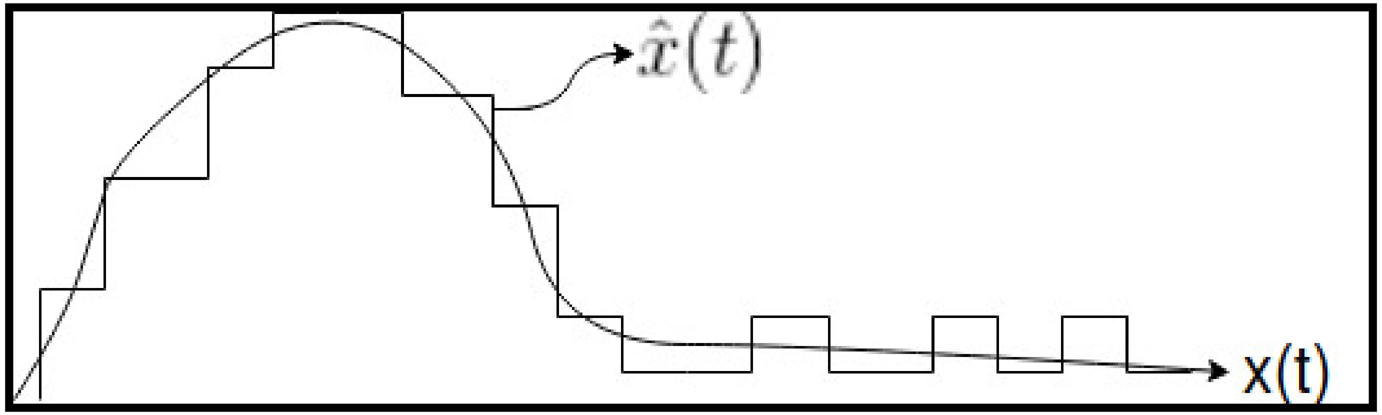
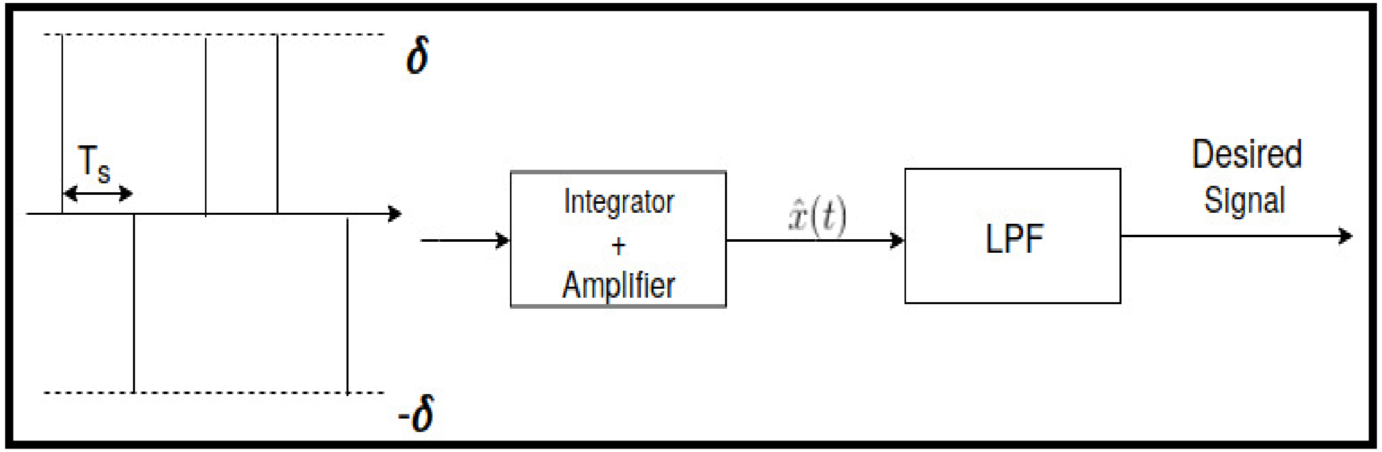


Illustration of DM working using Integrator and Comparator

- In this case, we have only two possible voltage levels: 0 or 1, i.e. $0V$ or $5V$



Delta Modulation

DM Practical Implementation - Cont.

- An LPF is used to remove step variations and get a smooth reconstructed message signal $x(t)$. It also rejects out of band quantizing noise in the high frequency staircase approximation.
- Also,

$$BW_{LPF} = BW\ of\ \ x(t)$$

Delta Modulation

Notes on DM

Note 1

Delta Modulation offers two unique features:

- A one bit codeword for the output which eliminates the need for word framing.
- Simplicity of design for both transmitter and receiver.

Note 2

The key to effective use of DM lies in the intelligent choice of two parameters:

- Staircase step size δ
- Sampling rate f_s

Note 3

- To account for fastest possible change in the signal, both sampling frequency f_s and step size δ must be increased or one of them must be increased.
- Increasing f_s requires more BW and increasing δ increases quantization error.

Quantization Noise (QN) in DM

Delta Modulation systems are subjected to two kinds of quantization error.

(i) Slope Overload Distortion.

(ii) Granular Noise.

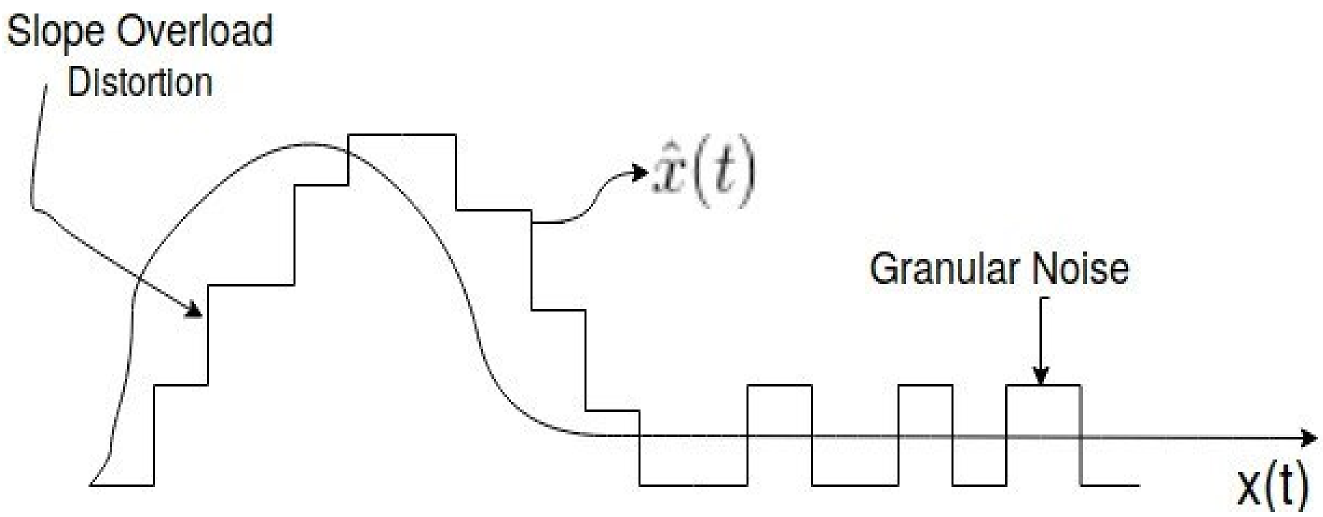


Figure :Illustration of the Quantization Errors in DM

QN in DM — Slope Overload Distortion (SOD)

$\Delta = 2\delta$ is too small for the staircase approximation to follow steep segment of input waveform $x(t)$.

As a result $\hat{x}(t)$ falls behind $x(t)$. This condition is called **slope overload distortion (noise)**.

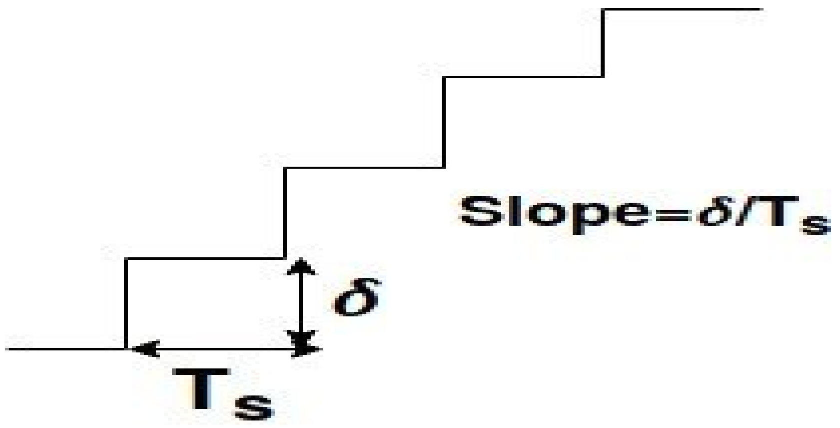


Figure :Slope Representation

In order to avoid slope overload, we require that $\boxed{\frac{\delta}{T_s} \geq max.\frac{dx(t)}{dt}.}$

QN in DM — Granular Noise (GN)

Granular Noise occurs when the step size $\Delta = 2\delta$ is too large relative to the local slope characteristics of the input waveform $x(t)$.

Here signal will not change rapidly enough compared to the changes in $\hat{x}(t)$.

This is analogous to the quantization noise in PCM system

Granular Noise can be reduced by reducing the step size.

EX 1: Let $x(t) = A\cos 2\pi f_o t$. For a given δ and T_s, what is the maximum value of A that we can have in order to avoid slope overload distortion? Also find the maximum signal power.

Sol:

$$\text{Given, } x(t) = A\cos 2\pi f_o t.$$

$$\Rightarrow \frac{dx(t)}{dt} = (-A\sin 2\pi f_o t.)(2\pi f_o)$$

$$\Rightarrow \frac{dx(t)}{dt} = -2A\pi f_o \sin 2\pi f_o t.$$

$$\therefore \max \left| \frac{dx(t)}{dt} \right| = 2A\pi f_o$$

$$\text{w.k.t. } \frac{\delta}{T_s} \geq \max \left| \frac{dx(t)}{dt} \right|$$

$$\Rightarrow \frac{\delta}{T_s} \geq 2A\pi f_o$$

$$\boxed{\therefore A \leq \frac{\delta}{T_s} \frac{1}{2\pi f_o}}$$

$$\boxed{\therefore A = \frac{\delta f_s}{2\pi f_o}} \text{ is the maximum value of A.}$$

This property of the maximum amplitude is useful in speech signal coding because A increases with decrease in f_o and vice-versa.

Hence the maximum permissible value of output signal power is

$$P_{max} = \frac{A^2}{2}$$

$$\boxed{P \le \frac{\delta^2}{8\pi^2} \frac{f_s^2}{f_o^2}}$$

Sigma Delta Modulation

DM is widely used for speech signal since the amplitude spectrum of speech signal decays with increasing frequency and hence the slope overload is not a concern.

The integrator in the receiver can result in noise enhancement. To avoid this the integrator can be moved to the transmitter itself.

This is called **Sigma- Delta Modulation** and it is widely used in practice.

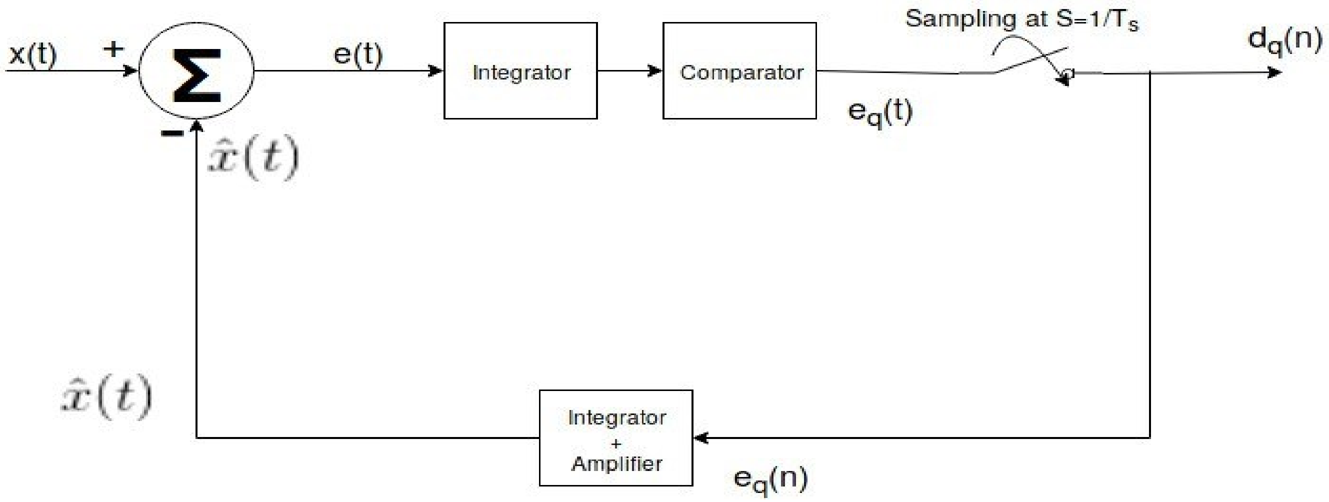


Figure :Sigma Delta Modulation Block Diagram

EX 2: A message signal has a BW of W Hz and is sampled at Nyquist Rate. The samples are quantized using an 8-bit PCM system. If the resulting bit rate is 40 Mbps. Find W.

Sol:

Bit rate is 40 Mbps.

No. of words/samples per second $= \frac{40 \times 10^6}{8}$

$= 5 \times 10^6$.

$\therefore f_s = 5\,MHz$

$\therefore$ Hence $\boxed{W = \frac{f_s}{2} = 2.5\,MHz.}$ (Since $f_s = 2W$ i.e.. Nyquist Rate Sampling)

EX 2: A message signal has a BW of W Hz and is sampled at Nyquist Rate. The samples are quantized using an 8-bit PCM system. If the resulting bit rate is 40 Mbps. Find W.

Sol:

Bit rate is 40 Mbps.

No. of words/samples per second $= \frac{40 \times 10^6}{8}$.

$= 5 \times 10^6$.

$\therefore f_s = 5\,MHz$

$\therefore$ Hence $\boxed{W = \frac{f_s}{2} = 2.5\,MHz.}$ (Since $f_s = 2W$ i.e., Nyquist Rate sampling)

EX 3: A signal $x(t) = 5\cos 10^5 \pi t$ is sampled at twice the Nyquist rate. It is to be quantized so that the SNR is atleast 43dB. Find the minimum bitrate and the corresponding step size for the quantizer.

Sol:

$SNR_{dB} \geq 43dB.$

$6N + 1.76 \geq 43dB.$

$N \geq 6.87$

$N \geq 7$

$f_s = 4f_o.$

$f_o = 5 \times 10^4$ (50kHz).

$f_s = 200$ kHz.

$\therefore$ Minimum possible bitrate is 7×200 kHz

$= \underline{1.4}$Mbps.

EX 4: A signal $x(t) = 5\cos 10^5\pi t + 10\cos 10^4\pi t$ is sampled at twice the Nyquist rate. It is to be quantized so that the SNR is atleast 43dB. Find the minimum bitrate and the corresponding step size for the quantizer.

Sol

$SNR_{dB} \geq 43\,dB$.

Signal power $(P_x) = \frac{5^2}{2} + \frac{10^2}{2} = \frac{125}{2}$

since the signals are orthogonal
$\because f_1 = 50\,kHz$; $f_2 = 5\,kHz$
Signals are orthogonal

$\Delta = \frac{2 \times 15}{2^N} = \frac{30}{2^N}$.

$\sigma_q^2 = \frac{\Delta^2}{12} = \frac{900}{12 \times 2^{2N}}$.

Delta Modulation
Problems

$\therefore \quad SNR = \dfrac{P_x}{\sigma_Q^2} = \dfrac{62.5}{900} \times 12 \times 2^{2N}$.

$SNR = \dfrac{5}{6} \times 2^{2N}$

$SNR_{dB} = 6N - 0.79$.

$SNR_{dB} \geq 43\,dB$.

$6N - 0.79 \geq 43$.

$6N \geq 43.79$

$N \geq 7.29$

$\underline{N} \geq 8$.

$f_s = 4 f_{max}$

$f_s = 4 \times 50\,kHz$.

$\boxed{f_s = 200\,kHz.}$

$\therefore$ Minimum possible bitrate is $8 \times 200\,kHz$.

$= \underline{1.6}\,Mbps$

THANK YOU

Bharathi V Kalghatgi
Department of Electronics and Communication Engineering

BharathiV.Kalghatgi@pes.edu