DIGITAL COMMUNICATION

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Non-Uniform and Robust Quantization Companding

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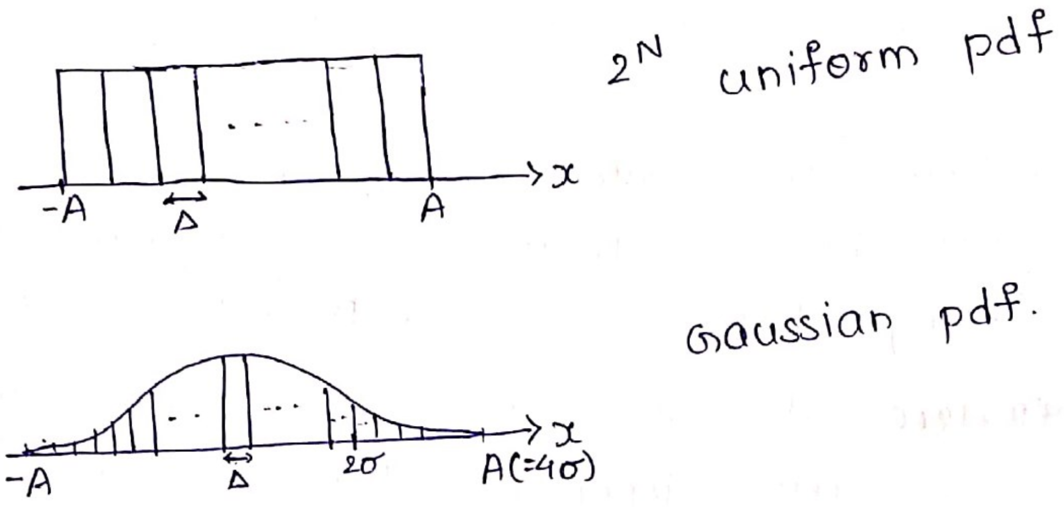
Non-uniform Quantization

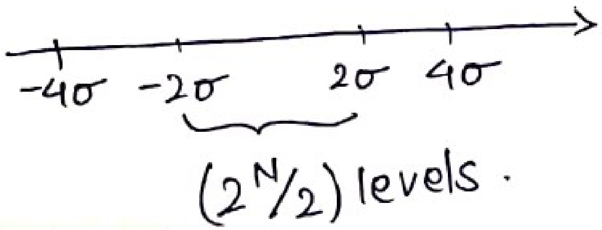


- In uniform pdf the probability of the input signal being in any level remains the same as the area of each interval remains constant.
- In Gaussian pdf the probability of occurrence of one level is higher than the other levels.

Non-uniform Quantization

- Between -2σ to 2σ the probability of occurrence is

$$P\{ -2\sigma < x < 2\sigma \} = 0.95$$



Half of the total number of levels ($2^N/2$) lies between the interval (-2σ , 2σ)

Non-uniform Quantization

With uniform quantization for input signal with uniform all the levels are equally likely to occur.

But for the Gaussian input the levels around the mean are much more likely to occur than the levels at the extremes.

Speech signal can be considered to have a Gaussian pdf.

Non-uniform Quantization

Human Perceptual considerations requisites the SNR to be constant across the different input power levels.

This means more levels with smaller step size have to be provided when signal power is low and the fewer levels with larger step size when signal power is high.

Such a Quantization scheme in which the SNR is almost same across the different input power levels is called **"Robust Quantization".**

Non-uniform Quantization

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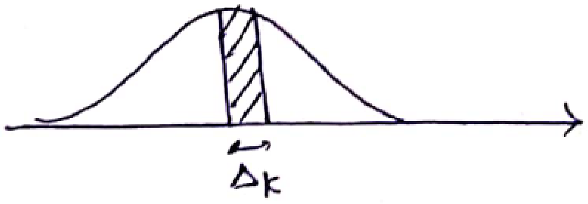
Non-uniform Quantization

Non Uniform Quantization has the same number of $L = 2^N$ values but each level can have different width.

If Δ_k is the width of the k^{th} level, then the quantization noise variance is given by :

$$\boxed{\sigma_Q^2 = \sum_{k=0}^{L-1} p_k \sigma_{Q_k}^2}$$

Where P_k is the probability of occurrence of the kth level and is given by



$$\therefore \boxed{p_k = \int_{b_k}^{b_{k+1}} f_X(x)\,dx}$$

$\sigma_{Q_k}^2 \rightarrow$ Quantization noise variance in the K^{th} interval

Non-uniform Quantization

When N is large, we have expression

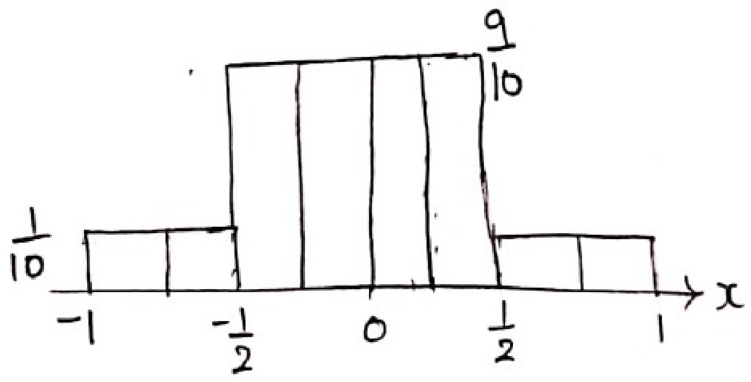
$$\sigma_Q{}^2 = \sum_{k=0}^{L-1} P_k \frac{\Delta_k^2}{12}$$

Consider the expression $\sigma_Q^2 = \sum_{k=0}^{L-1} p_k \frac{\Delta_k^2}{12}$

In case of uniform Quantization $\frac{\Delta_k^2}{12} = \frac{\Delta^2}{12}$

$\Rightarrow$ $\boxed{\sigma_Q^2 = \frac{\Delta^2}{12} \sum_{k=0}^{L-1} p_k = \frac{\Delta^2}{12}}$ $\rightarrow$ (as seen in case of uniform quanti-zation.

Ex 1: For the following pdf find the SNR 3 bit uniform quantizer



Sol:

$$\Delta = \frac{2A}{2^N} = \frac{2}{8} = 0.25$$

$$\sigma_x^2 = \int_{-1/2}^{1/2} x^2\left(\frac{9}{10}\right)dx + 2\int_{1/2}^{1} x^2\frac{1}{10}\,dx$$

$$= 2\cdot\frac{9}{10}\,\frac{x^3}{3}\Big|_0^{1/2} + \frac{2}{10}\,\frac{x^3}{3}\Big|_{1/2}^{1}$$

$$= \frac{3}{40} + \frac{7}{120}$$

$$\sigma_x^2 = \frac{2}{15}$$

$$SNR = \frac{\sigma_x^2}{\sigma_Q^2} = \frac{\frac{2}{15}}{\Delta^2/12} = \frac{2/15}{0.25^2/12}$$

$$\boxed{SNR = 25.6}$$

$$\boxed{SNR_{dB} = 14.08\,dB}$$

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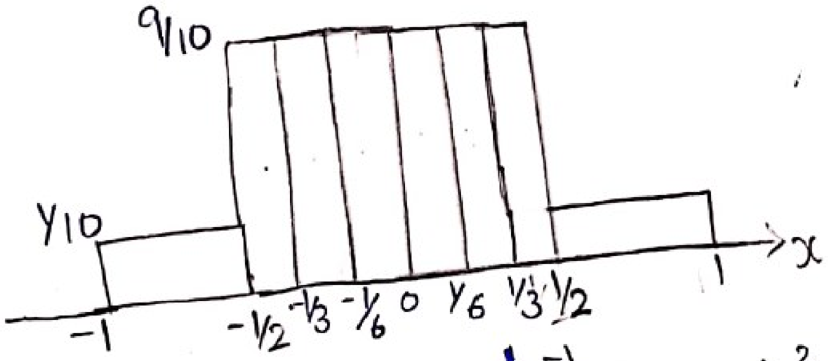
Problems

Ex 2: For the same pdf find the SNR for the following non-uniform quantization.

Sol:



w.k.t $\sigma_Q^2 = \sum_{k=0}^{L-1} p_k \frac{\Delta_k^2}{12}$

$$\sigma_Q^2 = \sum_{k=0}^{7} p_k \frac{\Delta_k^2}{12}.$$

for $k = 1$ to $6.$ $\quad p_k = \frac{9}{10} \times \frac{1}{6} = \frac{3}{20}.$

$$\Delta_k = \frac{1}{6}.$$

for $k = 0$ & $k = 7$ $\quad p_k = \frac{1}{20}$

$$\Delta_k = 0.5$$

$$\therefore \quad \sigma_Q^2 = 6 \times \frac{3}{20} \times \frac{1}{36} \times \frac{1}{12} + 2 \times \frac{1}{20} \times 0.25 \times \frac{1}{12}$$

$$\sigma_Q^2 = \frac{1}{240}.$$

$$\therefore \quad SNR = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{2/15}{1/240}$$

$$\boxed{\begin{array}{l} SNR = 32 \\ SNR_{dB} = 15.05 \end{array}}$$

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Robust Quantization

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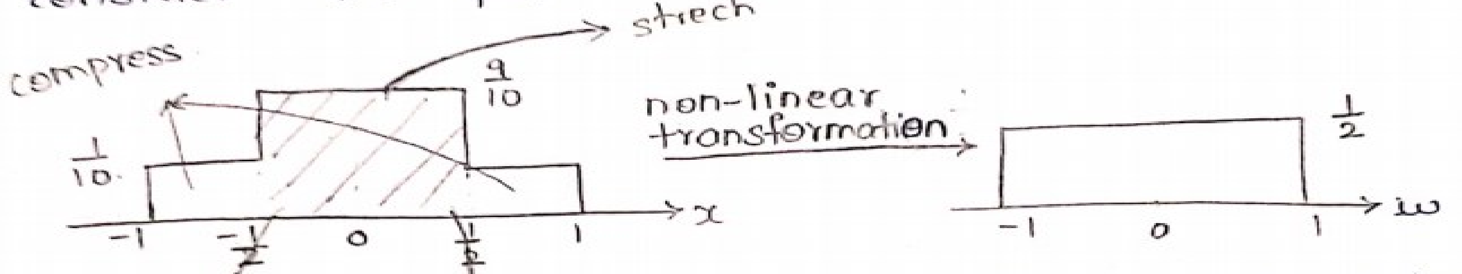
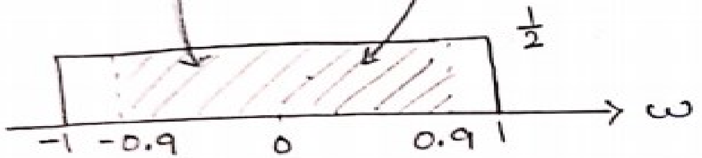
Robust Quantization

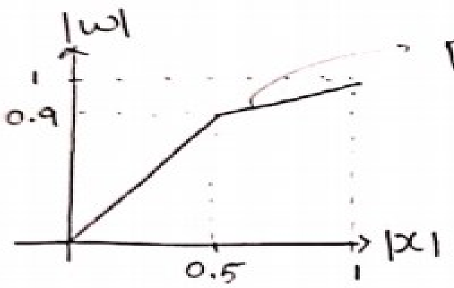
- It is one of the type of non uniform quantization where we select the step sizes so as to make the SNR almost same across all different power levels.
- In practice we first perform a non-linear transformation of the input signal and then apply a uniform quantizer. This transformation is called "compression".
- At the receiver we perform the inverse transformation called "Expansion".
- Together the process is called "companding".

* consider the pdf.



* In the compression operation the overload level should remain the same

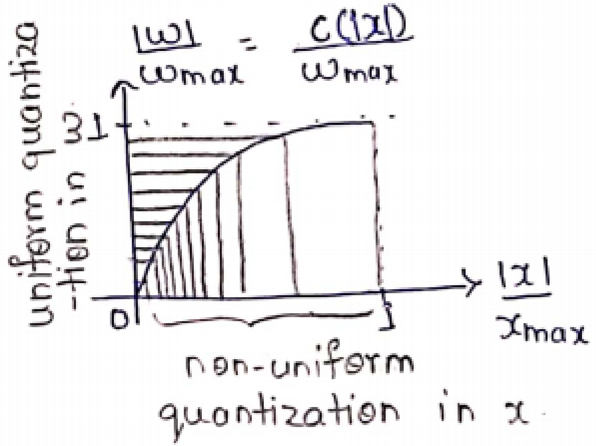




piecewise-linear function (its a non-linear function)

Ex: write the expression for the given transformation.

$$|\omega| = \begin{cases} \frac{9}{5}|x| & , 0 < |x| < \frac{1}{2} \\ \frac{1}{5}|x| + 0.8, & \frac{1}{2} < |x| < 1 \end{cases}$$



the compression function $|\omega|$ is called $c(|x|)$.

$$\boxed{\omega = c(|x|)}$$

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Companding Laws

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Companding Laws

There are two Companding laws:

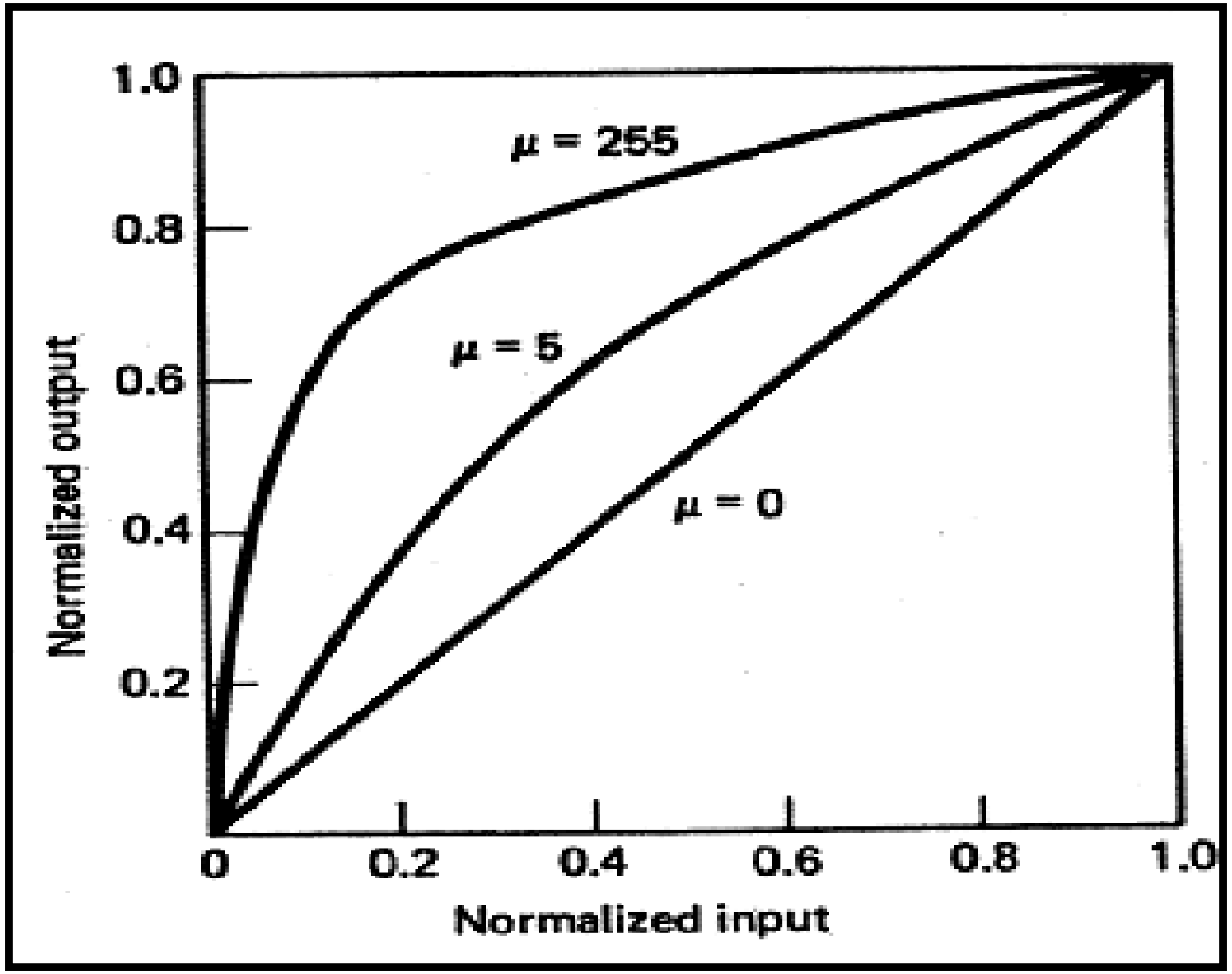
µ law companding:

$$\frac{c(|x|)}{x_{max}} = \frac{\ln(1 + \mu|x|/x_{max})}{\ln(1 + \mu)} \qquad 0 \leqslant \frac{|x|}{x_{max}} \leqslant 1$$

µ= 255 is practically used

µ=0 gives uniform quantization

Companding Laws: μ − Law



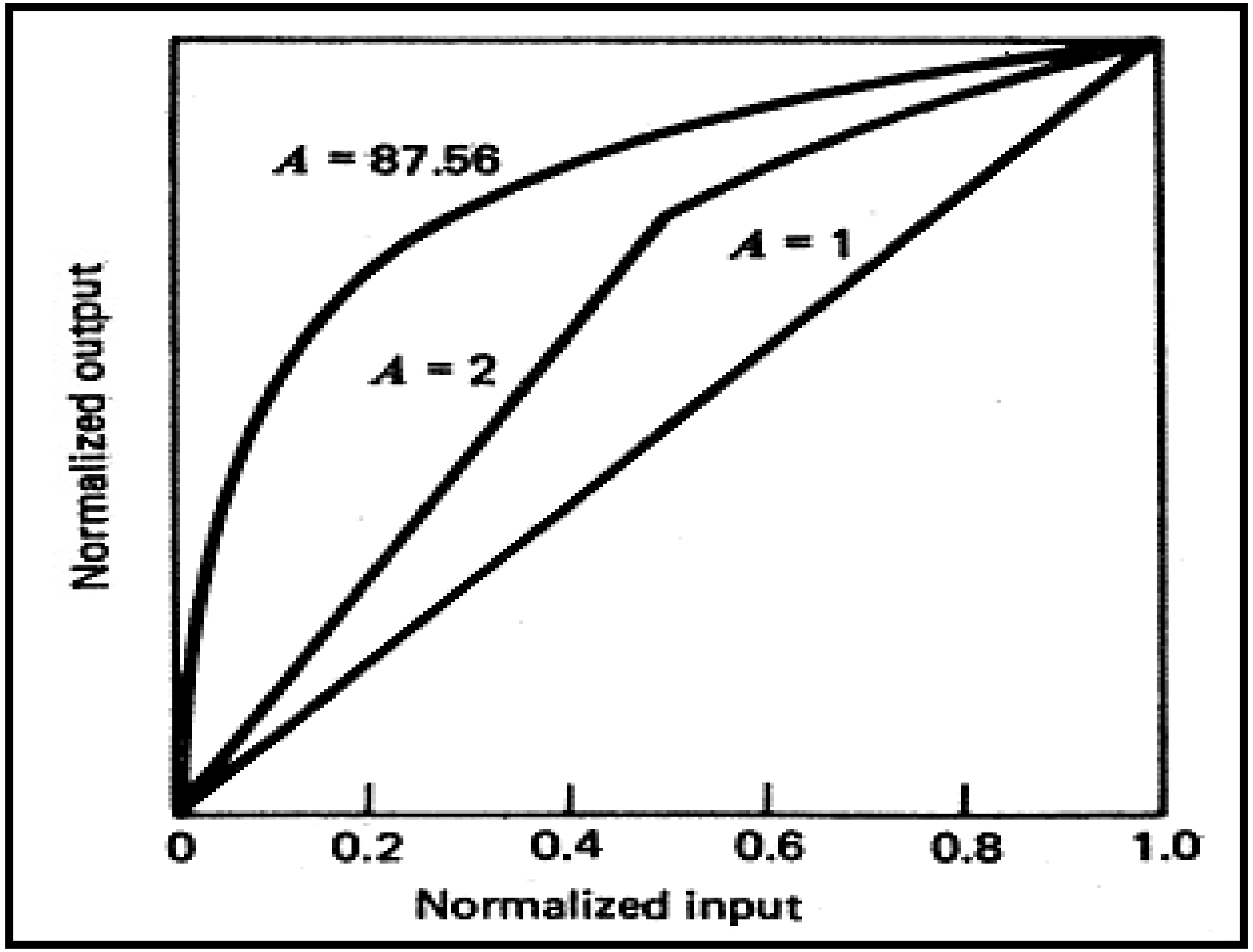
A law companding:

$$\frac{c(|x|)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1 + \ln A} & 0 \leqslant \dfrac{|x|}{x_{max}} \leqslant \dfrac{1}{A} \\ \dfrac{1 + \ln(A|x|/x_{max})}{1 + \ln A} & \dfrac{1}{A} \leqslant \dfrac{|x|}{x_{max}} \leqslant 1 \end{cases}$$

A= 1 gives uniform quantization

A=87.56 is practically used

Companding Laws: A Law



THANK YOU

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