LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

VRINDA KAMATH
Department of Science and Humanities

Unit 3. Linear Transformations and Orthogonality

Topics

1. **Linear Transformations**
2. **Orthogonal vectors and Subspaces**
3. **Cosines and Projections onto lines**
4. **Projections and Least Squares**

Linear Transformations

Definition:

Let A be a matrix of order n. When A multiplies a n- dimensional vector x, it transforms x to a n-dimensional vector Ax. This happens at every x in R^n. The whole space R^n is ***transformed or mapped*** into itself by the matrix A. The matrix A induces a transformation of R^n.

Linear Transformations

Few Examples.....

1. $$A = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$

   If x = (x, y) then Ax = (cx, cy).

A multiple of the identity matrix A = *cI* ***stretches*** every vector by the scale factor c. The whole space expands or contracts.

2. $$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

   If x = (x, y) then Ax = (-y, x).

The matrix A ***rotates*** every vector about the origin through a right angle in the counter clockwise direction.

Linear Transformations

3. $$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

If x = (x, y) then Ax = (y, x).

The matrix A ***reflects*** every vector on the line y = x. It is also a permutation matrix.

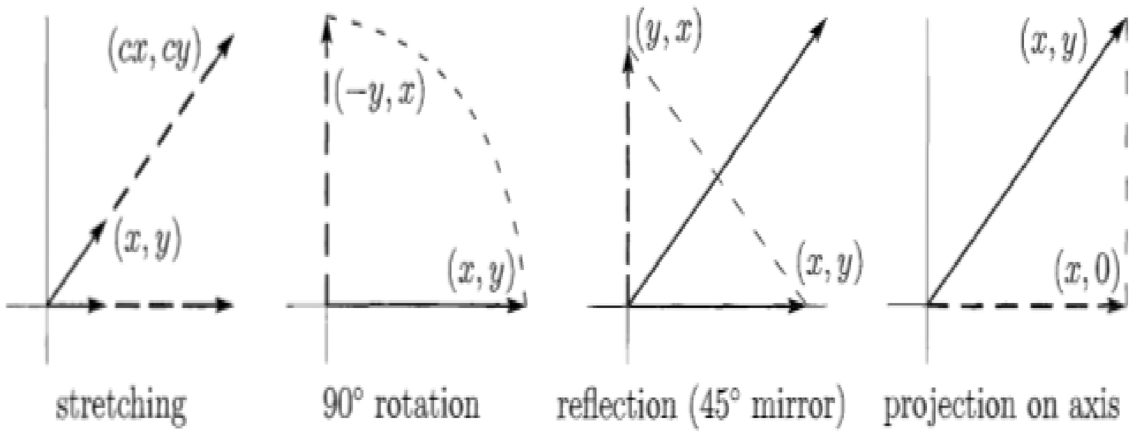
4. $$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

If x = (x , y) then Ax = (x , 0).

The matrix A ***projects*** every vector onto the x axis.

Unit 3. Linear Transformations and Orthogonality

Linear Transformations



Linear Transformations

Note

- A transformation can now be understood as a function (or a mapping) $f : A \to B$ defined by $f(x) = y$. In terms of matrices we have the rule

$A : R^n \to R^m$ defined by $A x = b$.

Linear Transformations

Definition :

A transformation T on R^n is said to be ***linear*** if

it satisfies the ***rule of linearity***

$$T (cx + dy) = c (Tx) + d (Ty)$$

for all scalars c, d and vectors x, y.

Note :

1. If T is linear then T (0) = 0 i.e T preserves origin. The converse may or may not be true.

2. If A is a matrix of order m x n then A induces a transformation from R^n to R^m.

Linear Transformations

Few examples.....

Let $v = (v_1, v_2)$. Then,

1. $T(v) = (v_2, v_1)$ is linear
2. $T(v) = (v_1, v_1)$ is not linear
3. $T(v) = (0, v_1)$ is not linear
4. $T(v) = (0, 1)$ is not linear
5. $T(v) = (v_1, v_2)$ is linear

Note :

If a transformation preserves origin it may or may not be linear!!

THANK YOU