DIGITAL COMMUNICATION

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QUANTIZATION

Differential Pulse Coded Modulation

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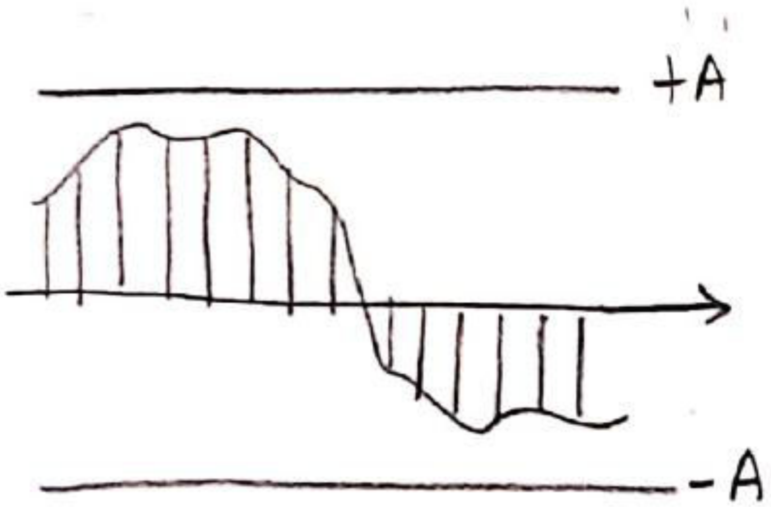
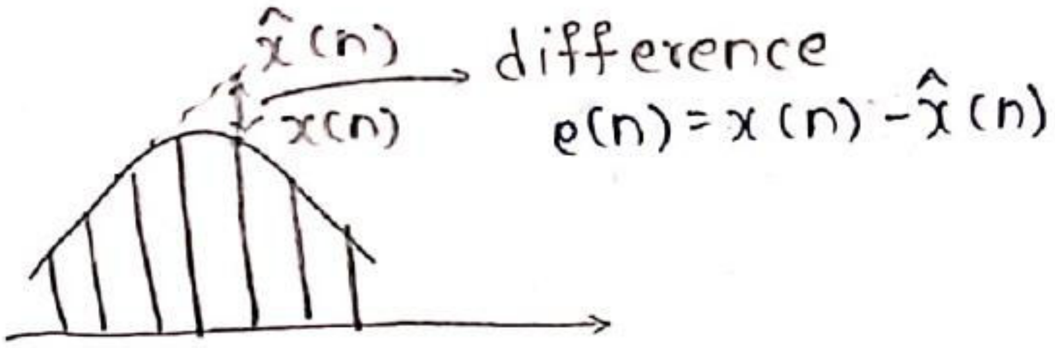
DIFFERENTIAL PULSE-CODED MODULATION

What and Why?

- To get a better SNR, we need less σ_Q^2, which occurs when Δ is less. This is achieved when A is small or N is large
- Moreover, the signal $x(n)$ can be correlated in time
- The idea in DPCM is to quantize the difference between the amplitude values in the samples instead of the regular amplitude values
- This decreases A and therefore increases SNR





DIFFERENTIAL PULSE-CODED MODULATION

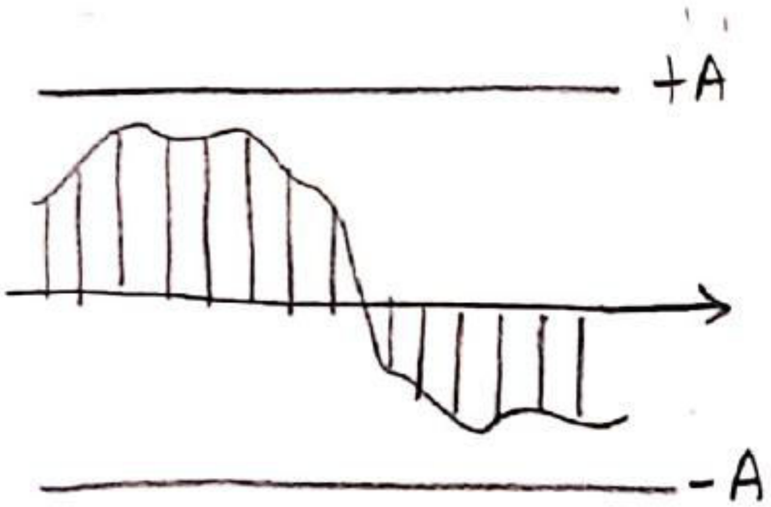
Prediction and Quantization

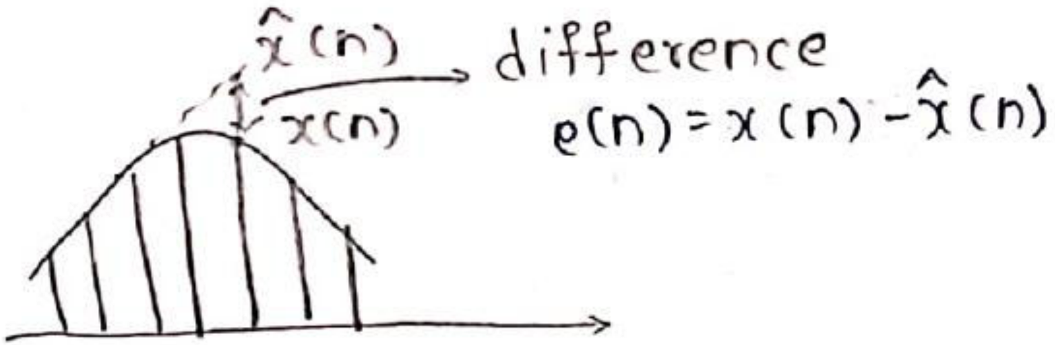
- Let $\hat{x}(n)$ be the estimate of $x(n)$ based on previous samples
- Let $e(n) = x(n) - \hat{x}(n)$ be the difference signal that has to be quantized
- Let $x_q(\mathrm{n})$ and $e_q(\mathrm{n})$ be the quantized versions of $x(n)$ and $e(n)$ respectively

$$x_q(\mathrm{n}) = x(n) + q(n)$$

$$e_q(\mathrm{n}) = e(n) + q(n)$$

- $q(n)$ is the quantization error





BLOCK DIAGRAM

Transmitter

$$x_q(\mathrm{n}) = x(n) + q(n)$$

$$e_q(\mathrm{n}) = e(n) + q(n)$$

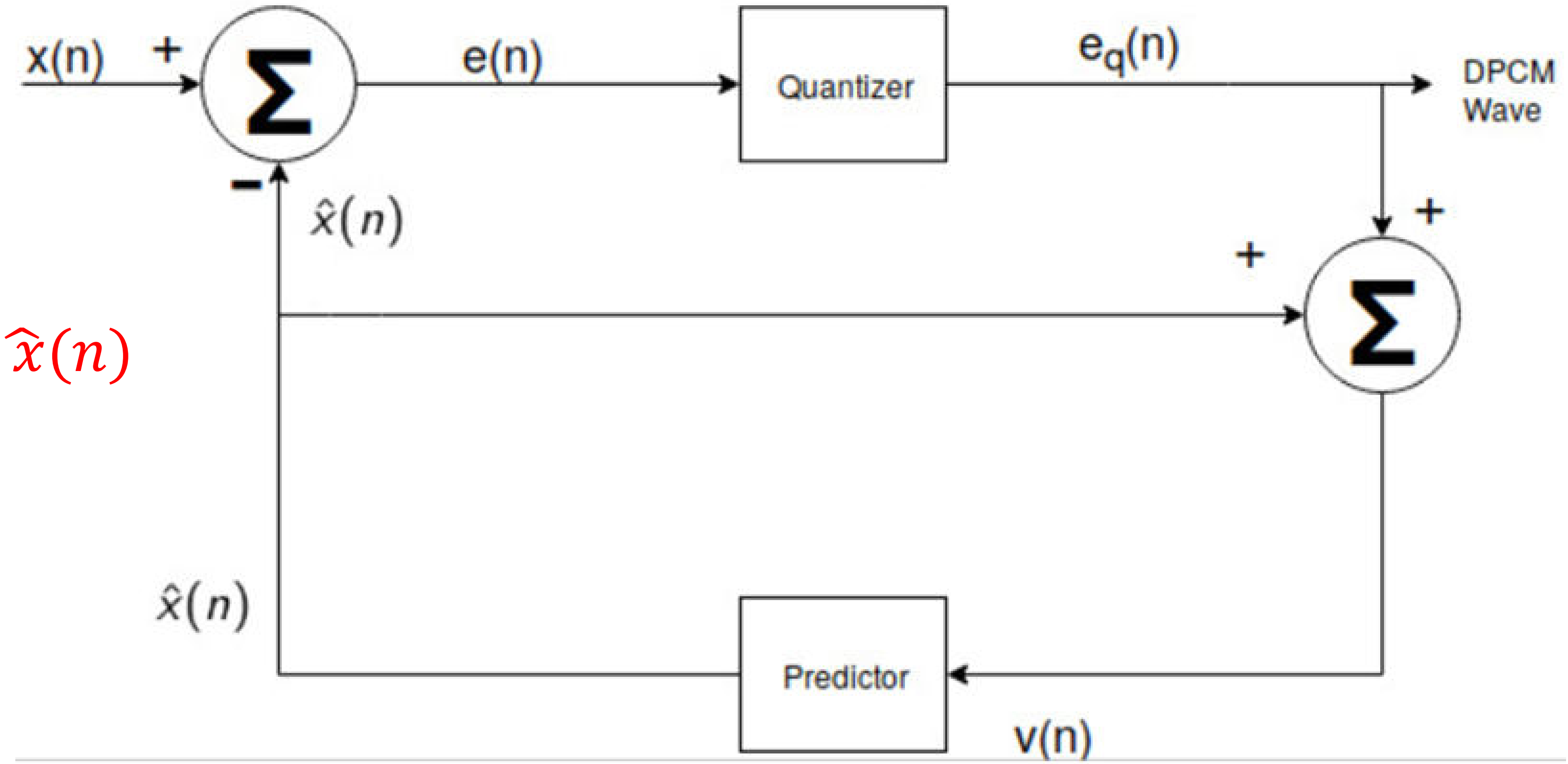
$$v(n) = \hat{x}(n) + e_q(n)$$

$$= \mathrm{e(n) + q(n)} + \hat{x}(n)$$

$$= \mathrm{x(n)} - \hat{x}(n) + \mathrm{q(n)} + \hat{x}(n)$$

$$= x(n) + \mathrm{q}(n)$$

$$v(n) = x_q(n)$$

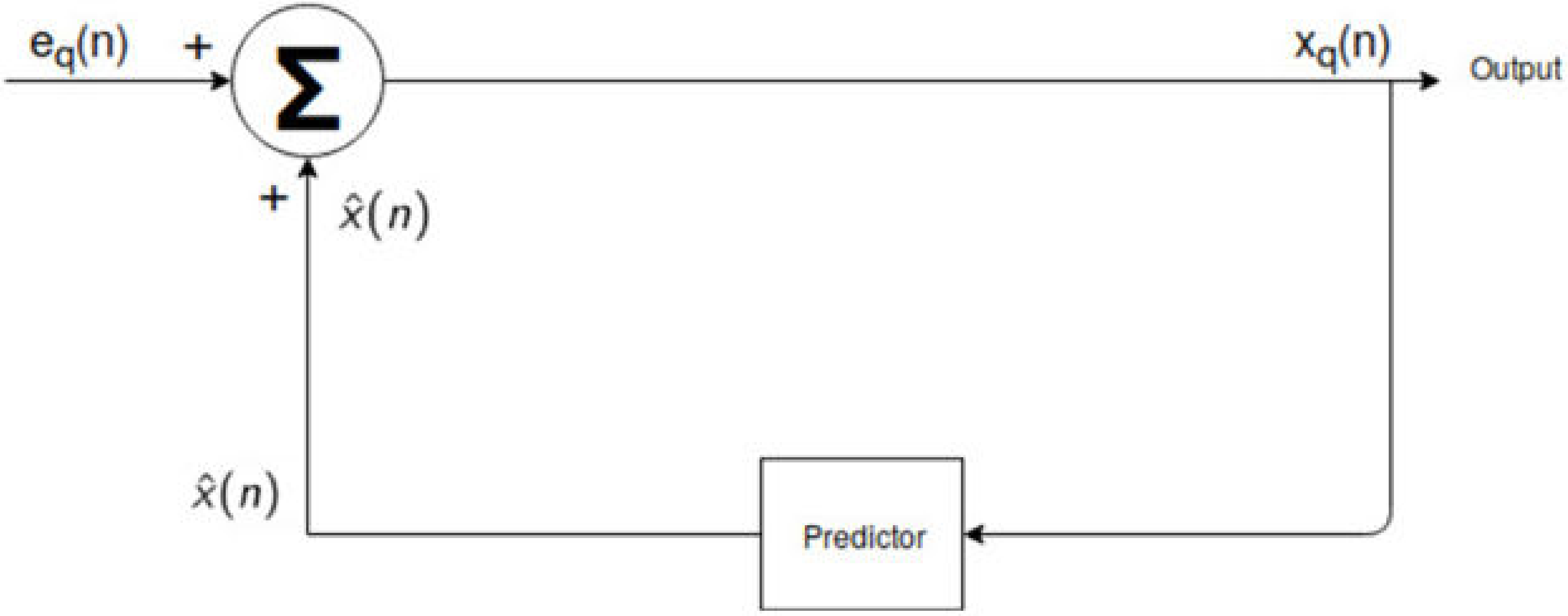


If predictor is just a delay,

$$\hat{x}(n) = x_q(\mathrm{n}-1)$$

BLOCK DIAGRAM

Receiver



- With the above exercise, we get the desired quantized signal $x_q(n)$
- Same feedback loop for prediction can be used at both transmitter and receiver
- As usual, we will not be able to recover $x(n)$

SNR Gain

- Recall that the SNR is defined as $SNR = \frac{{\sigma_X}^2}{{\sigma_Q}^2}$
- We can write the above expression of SNR as

$$SNR = \frac{{\sigma_X}^2}{{\sigma_E}^2} \times \frac{{\sigma_E}^2}{{\sigma_Q}^2}$$

- Here, $SNR_P = \frac{{\sigma_E}^2}{{\sigma_Q}^2}$ is the SNR of predictor, which is same as PCM model
- $G_P = \frac{{\sigma_X}^2}{{\sigma_E}^2}$ is called the prediction gain
- In dB scale, we get

$$SNR_{db} = 10log_{10}SNR_P + 10log_{10}G_P = (6N + c) + 10log_{10}\, G_P$$

DIFFERENTIAL PULSE-CODED MODULATION

Problem 1

A DPCM uses a 6-bit quantizer. If $\Delta_E = 0.25\,\Delta_x$, find the SNR in dB. Here, Δ_E and Δ_x are the step sizes of $e(n)$ and $x(n)$, respectively.

Assume that $e(n)$ and $x(n)$ are uniformly distributed

w.k.t. * $SNR = SNR_P G_P$

$$= \frac{\sigma_E^2}{\sigma_Q^2} \cdot \frac{\sigma_x^2}{\sigma_E^2}$$

$$= 2^{2N} \times \frac{\Delta_x^2}{(0.25\,\Delta_x)^2}$$

$$SNR_P = \frac{\sigma_E^2}{\sigma_q^2} = 2^{2N}$$

as both $e(n)$ & $q(n)$ are uniform

$\therefore$ $\boxed{SNR = 2^{12} \times 16 \doteq 65536}$

* $SNR_{dB} = 6N + 10\log_{10}16$

$SNR_{dB} = 6\times6 + 12.04$

$\therefore$ $\boxed{SNR_{dB} \approx 48.04}$

DIFFERENTIAL PULSE-CODED MODULATION

Problem 2

Let $x(n)$ be a WSS process with zero mean and autocorrelation function $R_x(k) = 0.9^{|k|}$, which is the input to a DPCM system. If the SNR is 8, find G_p. Assume that the predictor is just a delay.

w.k.t. $SNR = \frac{\sigma_x^2}{\sigma_Q^2}$.

$\sigma_x^2 = E[X^2] = R_x(0) = 1$

$\frac{1}{\sigma_Q^2} = 8$.

$\boxed{\sigma_Q^2 = \frac{1}{8}}$

$e(n) = x(n) - \hat{x}(n)$.

$e(n) = x(n) - x_q(n-1)$

$e(n) = x(n) - x(n-1) - q(n-1)$

$var(X+Y+Z) = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 + 2C_{xy} + 2C_{yz} + 2C_{xz}$

$var(E) = \sigma_x^2 + \sigma_x^2 + \sigma_Q^2 - 2R_x(1) + 0$.

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$$\sigma_E^2 = 1 + 1 + \frac{1}{8} - 2 \times 0.9$$

$$\sigma_E^2 = 0.325$$

$$\boxed{G_p = \frac{\sigma_x^2}{\sigma_E^2} = \frac{1}{0.325} = 3.07}$$

* variance for $x(n)$ & $x(n-1)$ is same.
* C_{xy} for $x(n)$ & $x(n-1)$ is $R_x(1) = E[X(n)X(n-1)]$
* zero mean
  ⇒ co-relation = covariance.
* Noise is independent of signal

THANK YOU

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