DIGITAL COMMUNICATION

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POWER SPECTRUM OF A DISCRETE PAM SIGNAL

Unipolar NRZ Spectrum

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Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$ for different schemes

We will find $S_X(f)$ for each of the following three cases:

i NRZ Unipolar

ii NRZ Polar

Iii NRZ Bipolar.

Iv Manchester Coding

Note: To obtain $S_X(f)$, we first find $S_A(f)$ from $R_A(n)$ and substitute in

$$S_X(f) = \frac{|V(f)|^2}{T_b} S_A(f)$$

Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Unipolar NRZ

- NRZ Unipolar

Let b_k indicate the k^{th} bit. We assume that 0 and 1 occur with equal probability.

To find $R_A(0)$:

b_k	A_k	P_r
0	0	1/2
1	a	1/2

The above table has been obtained from equation (2)

$$\therefore R_A(0) = E[{A_k}^2] = 0^2.\frac{1}{2} + a^2.\frac{1}{2} = \frac{a^2}{2}$$

Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Unipolar NRZ

To find $R_A(1)$:

b_k	b_{k-1}	A_k	A_{k-1}	P_r	$A_k A_{k-1}$
0	0	0	0	1/4	0
0	1	0	a	1/4	0
1	0	a	0	1/4	0
1	1	a	a	1/4	a^2

$$\therefore R_A(1) = E[A_k.A_{k-1}] = 0.\tfrac{1}{4} + 0.\tfrac{1}{4} + 0.\tfrac{1}{4} + a^2.\tfrac{1}{4} = \tfrac{a^2}{4}$$

We can see that $R_A(n) = a^2/4$ for any $n \neq 0$ as it behaves identical to how it does for $n = 1$.

$$\therefore R_A(n) = \begin{cases} \frac{a^2}{2} & n = 0 \\ \frac{a^2}{4} & n \neq 0 \end{cases}$$

$$\therefore R_A(n) = \frac{a^2}{4} + \frac{a^2}{4}\delta(n) \qquad (13)$$

Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Unipolar NRZ

Substituting $S_A(f)$ using (12) and $\left|V(f)\right|^2$ using (8) in (5), we get:

$$\boxed{S_X(f) = \frac{{T_b}^2 sinc^2(fT_b)}{T_b} \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi fnT_b}} \tag{14}$$

$$= T_b sinc^2(fT_b) \sum_{n=-\infty}^{\infty} \left\{ \frac{a^2}{4} + \frac{a^2}{4}\delta(n) \right\} e^{-j2\pi fnT_b}$$

$$\therefore S_X(f) = \frac{a^2 T_b}{4} sinc^2(fT_b) + \frac{a^2 T_b}{4} sinc^2(fT_b) \sum_{n=-\infty}^{\infty} e^{-j2\pi fnT_b} \tag{15}$$

Finding $S_A(f)$: Unipolar NRZ

We can show that

$$\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \tag{16}$$

Hence,

$$S_X(f) = \frac{a^2 T_b}{4} sinc^2(fT_b) + \frac{a^2 T_b}{4} sinc^2(fT_b) \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \tag{17}$$

Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Unipolar NRZ

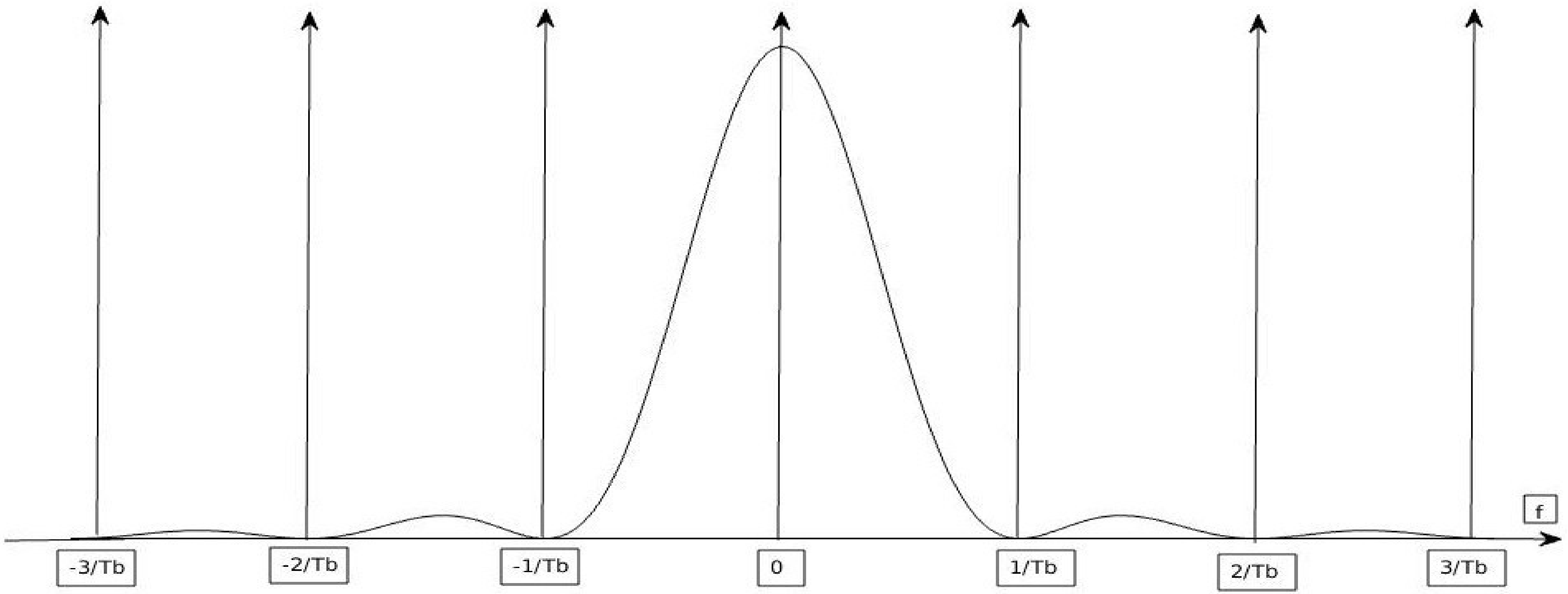

Figure:Illustration of multiplication of $sinc^2(fT_b)$ with dirac delta function

The multiplication in the second term in eqn (17) is as illustrated in Fig. As it is seen here, the multiplication will yield a non-zero value only for f=0. Everywhere else, the dirac delta coincides with the null of the $sinc^2$ function.

Finding $S_A(f)$: Unipolar NRZ

$$S_X(f) = \frac{a^2 T_b}{4} sinc^2(fT_b) + \frac{a^2}{4}\delta(f) \tag{18}$$

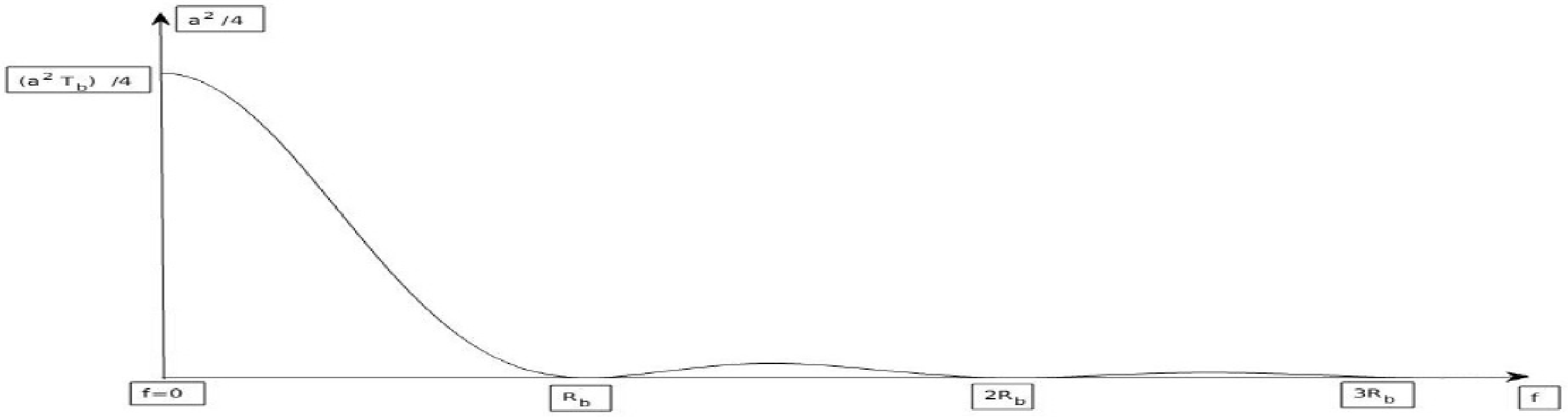

Figure:Power Spectral Density for Unipolar NRZ function

- By Considering the rst non DC null as the Bandwidth, we find that the Unipolar NRZ has a BW of R_b Hz.
- The impulse as f=0, indicates that unipolar NRZ has a DC component that accounts for half the signal power.

THANK YOU

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