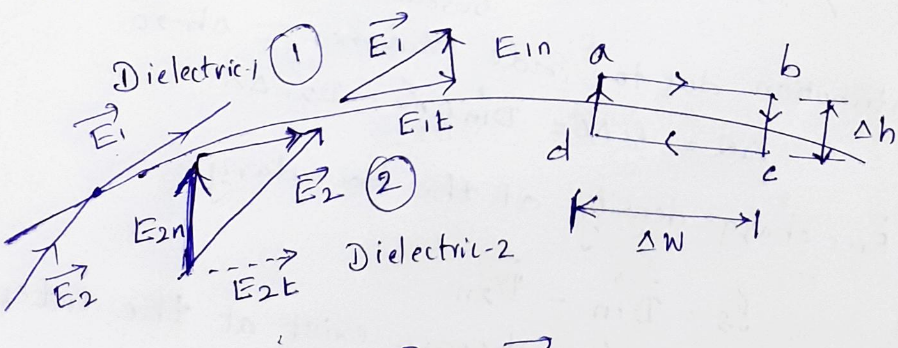
Boundary Conditions

So far, we have considered the existence of the electric field in a homogeneous medium. If the field exist in a region consisting of two different media, the conditions that the field must satisfy at the interface seperating the media are called boundary conditions. There are three types boundary conditions.

- Dielectric (ε_{r1}) and Dielectric (ε_{r2}) Boundary Conditions
- Conductor - Dielectric Boundary conditions
- Conductor - Free Space Boundary Conditions

Dielectric - Dielectric Boundary Conditions



$$\vec{E_1} = \vec{E_{1t}} + \vec{E_{1n}}$$

$$\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}}$$

Apply $\oint \vec{E} \cdot \vec{dl} = 0$ to the loop abcda

$$E_{1t}\Delta W - \cancel{E_{1n}\frac{\Delta h}{2}} - \cancel{E_{2n}\frac{\Delta h}{2}} - E_{2t}\Delta W$$

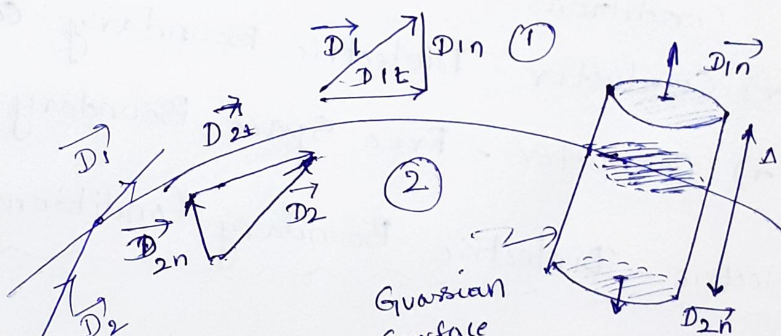
$$+ \cancel{E_{2n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}} = 0$$

$$\vec{E_{1t}}\,\Delta W = \vec{E_{2t}}\,\Delta W$$

$$\boxed{\vec{E_{1t}} = \vec{E_{2t}}}$$

$$\frac{\vec{D_{1t}}}{\varepsilon_0 \varepsilon_{r_1}} = \frac{\vec{D_{2t}}}{\varepsilon_0 \varepsilon_{r_2}}$$

or
$$\frac{\vec{D_{1t}}}{\varepsilon_1} = \frac{\vec{D_{2t}}}{\varepsilon_2}$$



Contribution due to side vanishes $\because \Delta h \to 0$

$\therefore$ $\Delta Q = \rho_s \Delta S = \vec{D_{1n}}\,\Delta S - \vec{D_{2n}}\,\Delta S$

ρ_s = Free charge density at the boundary

$$\rho_s = \vec{D_{1n}} - \vec{D_{2n}}$$

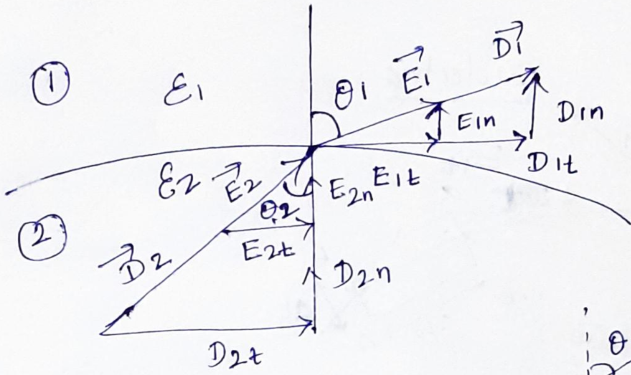
If no ~~charge~~ free charges exist at the interface

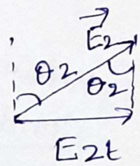
then $\rho_s = 0$

$$0 = \vec{D_{1n}} - \vec{D_{2n}}$$

$$\vec{D_{1n}} = \vec{D_{2n}}$$

$$\varepsilon_1 \vec{E_{1n}} = \varepsilon_2 \vec{E_{2n}}$$





$$E_{1t} = E_1 \sin\theta_1$$

$$E_{1t} = E_{2t}$$

$$E_{2t} = E_2 \sin\theta_2$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

$$D_{1n} = D_{2n}$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

$$\frac{\cancel{E_1} \sin\theta_1}{\varepsilon_1 \cancel{E_1} \cos\theta_1} = \frac{\cancel{E_2} \sin\theta_2}{\varepsilon_2 \cancel{E_2} \cos\theta_2}$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

[Law of Refraction of Electric field at a boundary free of charge.]
assumed at the interface

Conductor - Dielectric Boundary conditions

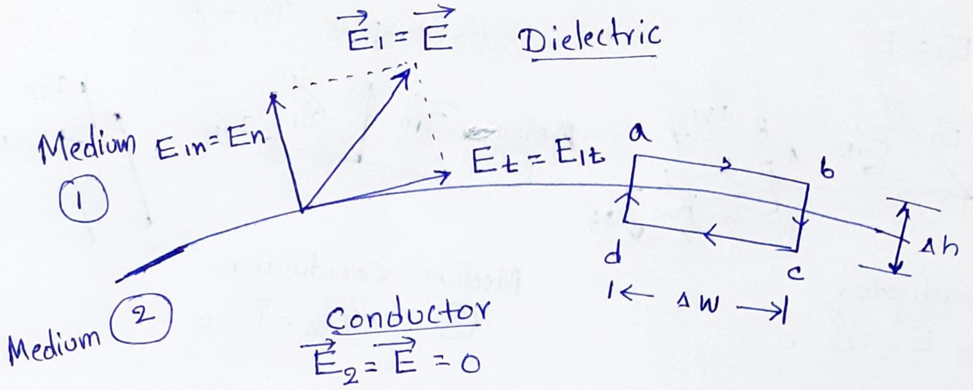


Figure shows the case of conductor - dielectric boundary conditions. The conductor is assumed to be a perfect conductor (i.e $\sigma \to \infty$ or $\rho_c = 0$). To find the boundary conditions for a conductor - dielectric interface.

In ②, Let the electric field inside the conductor $\vec{E}_2 = \vec{E} = 0$, $E_{2t} = 0$ $E_{2n} = 0$

In ①, the electric field outside the conductor i.e in dielectric $\vec{E_1} = \vec{E}$

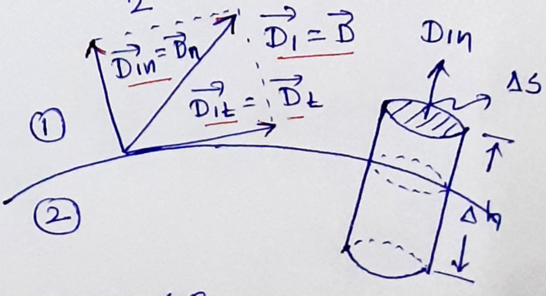
In ①, The tangential component of $\vec{E_1} = \vec{E_{1t}} = \vec{E_t}$

In ① The normal component of $\vec{E_1} = \vec{E_{1n}} = \vec{E_n}$

Applying $\oint \vec{E} \cdot d\vec{l} = 0$ to abcda

$$+ E_{1t} \Delta W - E_{1n} \frac{\Delta h}{2} + 0 + E_{1n} \frac{\Delta h}{2} = 0$$

$\therefore$ $\boxed{E_t = 0}$ $\boxed{E_{1t} = 0}$



Let, inside the conductor $\vec{D_2} = \vec{D} = 0$, $D_{2t} = 0$, $D_{2n} = 0$

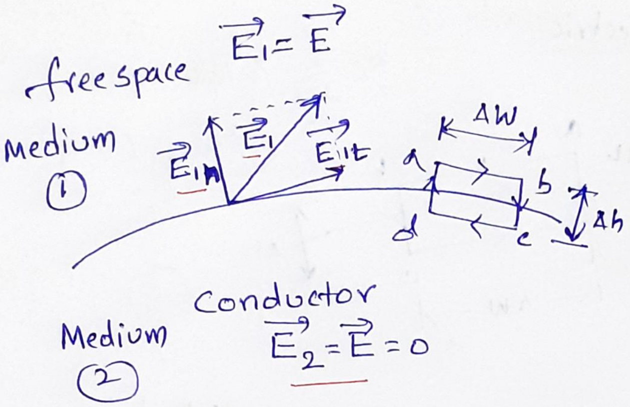
In the dielectric $\vec{D_1} = \vec{D}$

$\vec{D_{1t}} = \vec{D_t}$ $\quad$ $D_{1n} = D_n$

$\therefore$ $D_{1n} \Delta s - 0 \cdot \Delta s = \Delta Q \Rightarrow D_{1n} = \frac{\Delta Q}{\Delta s}$

$\therefore$ $\boxed{D_{1n} = D_n = \rho_s}$

Conductor - free space Boundary conditions



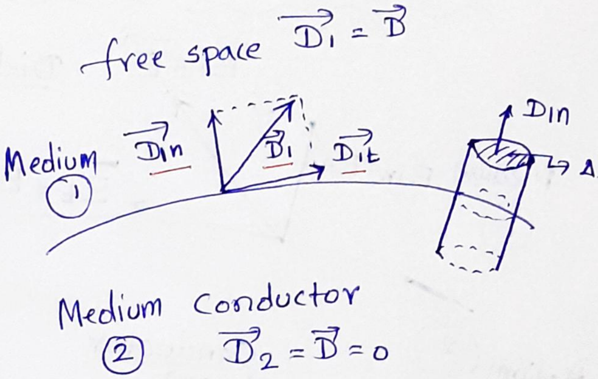


Figure shows the conductor - free space Boundary conditions

In medium ① $\vec{E_1} = \vec{E}$, $\vec{E_{1t}} = \vec{E_t}$, $\vec{E_{1n}} = \vec{E_n}$

In medium ② $\vec{E_2} = 0$ $\quad E_{2t} = 0$ $\quad E_{2n} = 0$

Applying $\oint \vec{E} \cdot \vec{dl} = 0$ to abcda

$$E_{1t}\Delta W - E_{1n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} = 0 \Rightarrow \boxed{E_{1t} = 0} \therefore \boxed{E_t = 0}$$

In medium ① $\vec{D_1} = \vec{D}$ $\quad \vec{D_{1t}} = \vec{D_t}$ $\quad \vec{D_{1n}} = \vec{D_n}$

In medium ② $\vec{D_2} = 0$ $\quad \vec{D_{2t}} = 0$ $\quad \vec{D_{2n}} = 0$

$\therefore$ $D_{1n}\Delta S - 0 = \Delta Q$

$\therefore$ $D_{1n} = \frac{\Delta Q}{\Delta S} = D_n = \rho_s$

$\therefore$ $\boxed{D_{1n} = D_n = \rho_s}$

1. Two extensive homogeneous isotropic dielectrics meet on plane $z = 0$. For $z > 0$, $\varepsilon_{r_1} = 4$ and for $z < 0$, $\varepsilon_{r_2} = 3$. A uniform electric field $\vec{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m exist for $z \geq 0$. Find

(a) $\vec{E}_2$ for $z \leq 0$

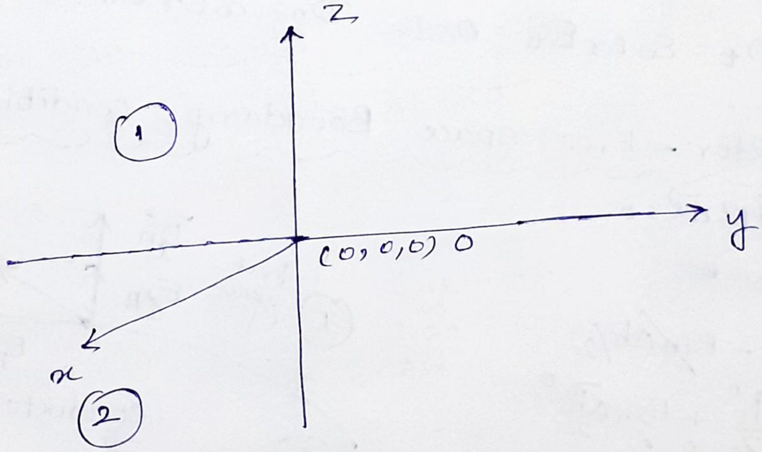
(b) The angles $\vec{E}_1$ and $\vec{E}_2$ make with interface

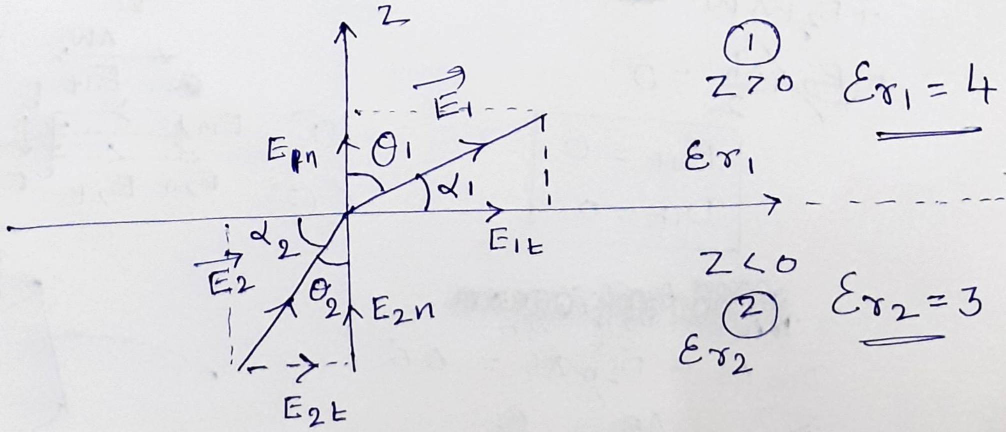
(c) The energy densities (in J/m^3) in both dielectrics

(d) The energy within a cube of side 2m centered at $(3, 4, -5)$

Solution:-

(a)





It is a case of dielectric to dielectric

$\therefore \quad \vec{E_{1t}} = \vec{E_{2t}}$

$\vec{D_{1n}} = \vec{D_{2n}} \Rightarrow \varepsilon_0 \varepsilon_{r1} \vec{E_{1n}} = \varepsilon_0 \varepsilon_{r2} \vec{E_{2n}}$

$\vec{E_1} = \vec{E_{1t}} + \vec{E_{1n}}$

$\vec{E_1} = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z \ \text{kV/m}$

$\therefore \quad \vec{E_{1n}} = 3\hat{a}_z$

$\vec{E_{1t}} = 5\hat{a}_x - 2\hat{a}_y$

$\therefore \quad \vec{E_{2t}} = 5\hat{a}_x - 2\hat{a}_y \rightarrow (3)$ kV/m

$\cancel{\varepsilon_0} \varepsilon_{r2} \vec{E_{2n}} = \cancel{\varepsilon_0} \varepsilon_{r1} \vec{E_{1n}}$

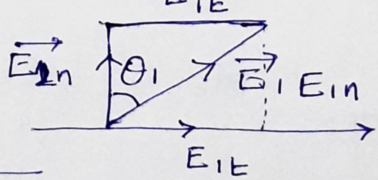
$\vec{E_{2n}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \vec{E_{1n}} = \frac{4}{\cancel{3}} \times \cancel{3}\hat{a}_z = 4\hat{a}_z \rightarrow (4)$ kV/m

But $\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}} = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z \rightarrow (5)$

$$\boxed{\vec{E_2} = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z} \text{ for } z \leq 0 \ \text{kV/m}$$

(b) We know for dielectric - dielectric boundary condition $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\cancel{\varepsilon_0}\varepsilon_{r1}}{\cancel{\varepsilon_0}\varepsilon_{r2}}$



$\tan\theta_1 = \frac{E_{1t}}{E_{1n}}$

$\tan\theta_1 = \frac{|5\hat{a}_x - 2\hat{a}_y|}{|3\hat{a}_z|} = \frac{\sqrt{5^2+2^2}}{\sqrt{3^2}}$

$\tan\theta_1 = \frac{\sqrt{29}}{3} \Rightarrow \theta_1 = \tan^{-1}\left(\frac{\sqrt{29}}{3}\right)$

$\theta_1 = 60.9^\circ$

$\alpha_1 = 90 - \theta_1$

$\alpha_1 = 90 - 60.9 = 22.1^\circ$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon r_1}{\varepsilon r_2}$$

$$\tan\theta_1 \times \frac{\varepsilon r_2}{\varepsilon_1} = \tan\theta_2$$

$$\tan\theta_2 = \frac{\varepsilon r_2}{\varepsilon r_1} \times \tan\theta_1$$

$$= \frac{\cancel{3}}{4} \times \frac{\sqrt{29}}{\cancel{3}} = 1.3462$$

$$\theta_2 = \tan^{-1}(1.3462) = 53.4^\circ$$

$$\alpha_2 = 90 - \theta_2 = 90 - 53.4 = 36.6^\circ$$

(c) The energy densities are given by

$$W_{E1} = \frac{dW_{E1}}{dV} = \frac{1}{2}\varepsilon_1 |E_1|^2 = \frac{1}{2}\varepsilon_0 \varepsilon r_1 |E_1|^2$$

$$\frac{dW_{E1}}{dV} = \frac{1}{2} \times \frac{10^{-9}}{36\pi} \times 4 \times |5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z|^2$$ It is in KV

$$\frac{dW_{E1}}{dV} = \frac{1}{2} \times \frac{10^{-9}}{36\pi} \times 4 \times \left(\sqrt{(5^2 + 2^2 + 3^2) \times 10^6}\right)^2$$

$$\frac{dW_{E1}}{dV} = \frac{1}{2} \times \frac{10^{-9}}{36\pi} \times 4 \times [25 + 4 + 9] \times 10^6$$

$$= 0.672 \times 10^{-3}\ J/m^3 = 672\ \mu J/m^3$$

$$W_{E2} = \frac{dW_{E2}}{dV} = \frac{1}{2}\varepsilon_2 |E_2|^2 = \frac{1}{2}\varepsilon_0 \varepsilon r_2 |E_2|^2$$

$$= \frac{1}{2} \times \frac{10^{-9}}{36\pi} \times 3 [5^2 + 2^2 + 4^2] \times 10^6$$

$$= 597\ \mu J/m^3$$

(d) $$W_E = \int_{x=2}^{4}\int_{y=3}^{5}\int_{z=-6}^{-4} W_{E2}\, dz\, dy\, dx = 597 \times 10^{-6} \times 2 \times 2 \times 2$$

$$= 4.776\ mJ$$

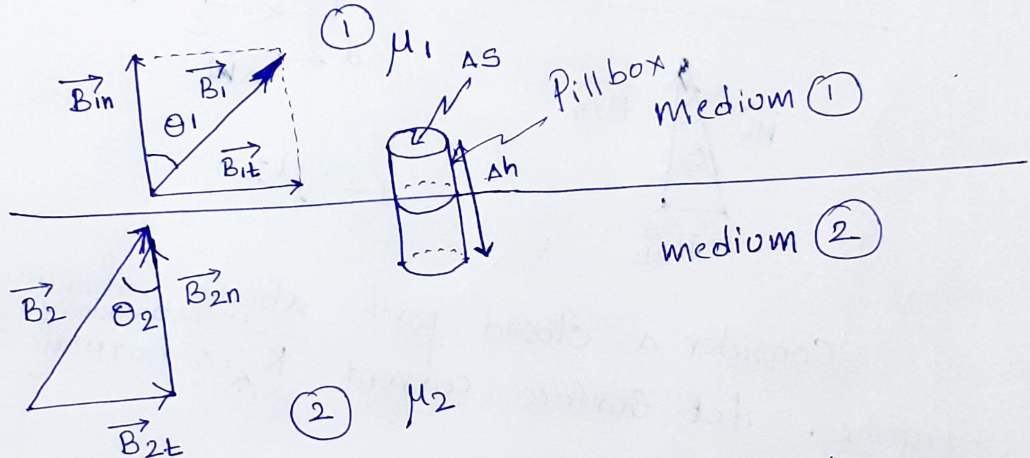
8.7 Magnetic Boundary Conditions

We define magnetic boundary conditions as the conditions that $\vec{H}$ (or $\vec{B}$) fields must satisfy at the boundary between two different media.

Here we will make use of

~~Magn~~ Gauss's law for magnetic fields $\oint_S \vec{B}\cdot\vec{ds} = 0$

and Amper's law or Ampere's circuit law $\oint_L \vec{H}\cdot\vec{dl} = I$



Consider the boundary between two magnetic media ① and ②, characterized by μ_1 and μ_2 respectively

Consider Gauss's law for magnetic fields

$$\oint_S \vec{B}\cdot\vec{ds} = 0 \rightarrow (1)$$

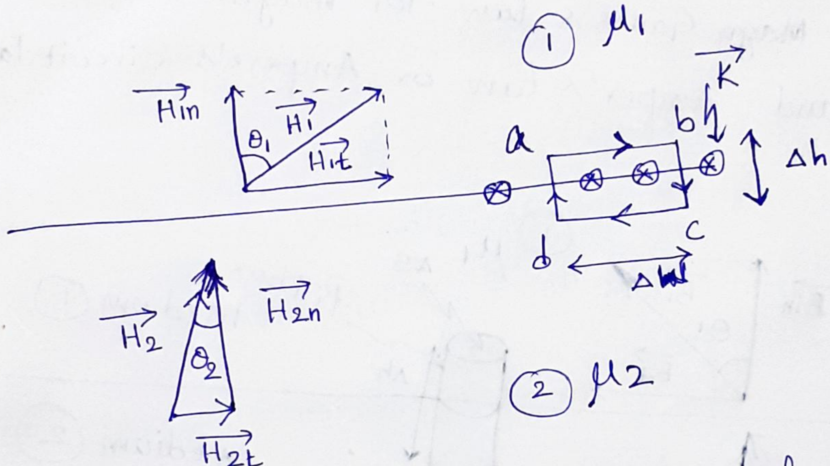
Applying equation ① to the pillbox (Gaussian surface) and allowing $\Delta h \rightarrow 0$

$$\vec{B_{1n}}\Delta S - \vec{B_{2n}}\Delta S = 0 \Rightarrow \boxed{\vec{B_{1n}} = \vec{B_{2n}}} \rightarrow (2)$$

$$\therefore \boxed{\mu_1 \vec{H_{1n}} = \mu_2 \vec{H_{2n}}} \rightarrow (3)$$

$$\boxed{\vec{B}_{1n} = \vec{B}_{2n}} \quad \text{or} \quad \boxed{\mu_1 \vec{H}_{1n} = \mu_2 \vec{H}_{2n}} \rightarrow (3)$$

Equation (3) shows that the normal component of $\vec{B}$ is continuous at the boundary. It also shows that the normal component of $\vec{H}$ is discontinuous at the boundary. $\vec{H}$ undergoes some changes at the interface.



Consider a closed path abcda shown in figure. Let surface current $\vec{K}$ on the boundary is normal to the surface

$$\vec{K} = K\hat{a}_n \ \text{A/m}$$

$$\therefore K\Delta W = H_{1t}\Delta W + H_{1n}\frac{\Delta h}{2} + H_{2n}\frac{\Delta h}{2}$$
$$- H_{2t}\Delta W - H_{2n}\frac{\Delta h}{2} - H_{1n}\frac{\Delta h}{2}$$

As $\Delta h \rightarrow 0$

$$K\Delta W = H_{1t}\Delta W - H_{2t}\Delta W$$

$$H_{1t}\cancel{\Delta W} - H_{2t}\cancel{\Delta W} = K\cancel{\Delta W}$$

$$H_{1t} - H_{2t} = K \rightarrow (4) \qquad \frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K \rightarrow (5)$$

In general case equation (4) becomes

$$(\vec{H_1} - \vec{H_2}) \times \hat{a}_{n12} = \vec{K}$$

where $\hat{a}_{n12}$ is unit vector normal to the interface and is directed from medium (1) to medium (2).

If the boundary is free of current or the ~~medium~~ media are not conductors, then

$$K = 0$$

$\therefore$ Equation (4) becomes

$$H_{1t} - H_{2t} = K$$

$$H_{1t} - H_{2t} = 0$$

$$\therefore \vec{H_{1t}} = \vec{H_{2t}}$$

$$\frac{\vec{B_{1t}}}{\mu_1} = \frac{\vec{B_{2t}}}{\mu_2} \rightarrow (6)$$

Thus the tangential component of $\vec{H}$ is continuous while that of $\vec{B}$ is discontinuous at the boundary.

If the fields make an angle θ with the normal to the interface

$$B_{1n} = B_1 \cos\theta_1 = B_{2n} = B_2 \cos\theta_2$$

$$\therefore B_{1n} = B_{2n}$$

$$B_1 \cos\theta_1 = B_2 \cos\theta_2 \rightarrow (7)$$

$$H_{1t} = H_{2t}$$

$$H_1 \sin\theta_1 = H_2 \sin\theta_2$$

$$\Rightarrow \frac{B_1}{\mu_1} \sin\theta_1 = \frac{B_2}{\mu_2} \sin\theta_2 \rightarrow (8)$$

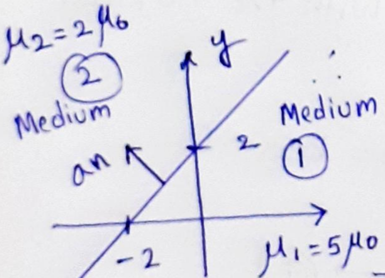
$$\frac{⑧}{⑦} \quad \frac{\frac{B_1}{\mu_1}\sin\theta_1}{B_1\cos\theta_1} = \frac{\frac{B_2}{\mu_2}\sin\theta_2}{B_2\cos\theta_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

8.8) Example 8.8

Given that $\vec{H_1} = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$ A/m in region $y - x - 2 \le 0$, where $\mu_1 = 5\mu_0$ calculate $\vec{H_2}$ and $\vec{B_2}$ in region $y - x - 2 \ge 0$, where $\mu_2 = 2\mu_0$

Solution :-



$H_{1n} = H_1 \cos\theta_1$

$\vec{H_{1n}} = (\vec{H_1} \cdot \hat{a}_n)\,\hat{a}_n$

$-x + y \ge 2$

$$\vec{H_{1n}} = \left[(-2, 6, 4) \cdot \frac{(-1, 1, 0)}{\sqrt{2}}\right] \frac{(-1, 1, 0)}{\sqrt{2}}$$

$$\vec{H_{1n}} = \left[\frac{+2+6}{\sqrt{2}}\right]\left[\frac{(-1, 1, 0)}{\sqrt{2}}\right]$$

$$\vec{H_{1n}} = -4\hat{a}_x + 4\hat{a}_y$$

$$\vec{H_1} = \vec{H_{1n}} + \vec{H_{1t}}$$

$$\vec{H_{1t}} = \vec{H_1} - \vec{H_{1n}} = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z - (-4\hat{a}_x + 4\hat{a}_y)$$

$$\vec{H_{1t}} = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z \text{ A/m}$$

$$\vec{B}_{1n} = \vec{B}_{2n}$$

$$\mu_1 \vec{H}_{1n} = \mu_2 \vec{H}_{2n}$$

$$\vec{H}_{2n} = \frac{\mu_1}{\mu_2} \vec{H}_{1n} = \frac{5}{2}\left(-4\hat{a}_x + 4\hat{a}_y\right)$$

$$\vec{H}_{2n} = -10\hat{a}_x + 10\hat{a}_y$$

$$\vec{H}_{1t} = \vec{H}_{2t}$$

$$\therefore \vec{H}_{2t} = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z \text{ A/m}$$

$$\vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n}$$

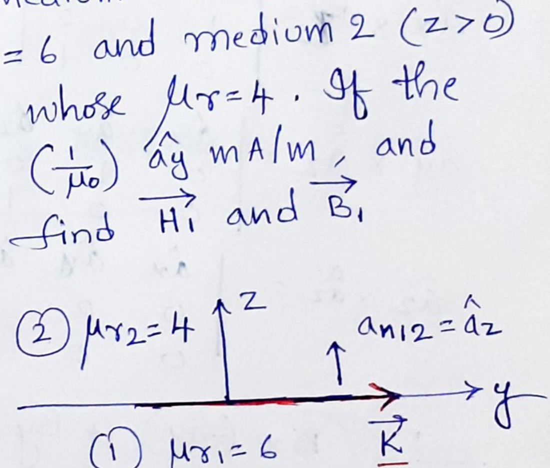
$$\vec{H}_2 = -8\hat{a}_x + 12\hat{a}_y + 4\hat{a}_z \text{ A/m}$$

$\vec{B}_2 = \mu_2 \vec{H}_2 = \mu_0 \mu_{r2} \vec{H}_2$ ($4\pi \times 10^{-7}$, 2)

$\vec{B}_2 = -20.11\hat{a}_x + 30.16\hat{a}_y + 10.05\hat{a}_z$ μWb/m²

8.9) Example 8.9 : The xy-plane serves as the interface between two different media. Medium 1 ($z<0$) is filled with a material whose $\mu_r = 6$ and medium 2 ($z>0$) is filled with a material whose $\mu_r = 4$. If the interface carries current $\left(\frac{1}{\mu_0}\right)\hat{a}_y$ mA/m, and $\vec{B}_2 = 5\hat{a}_x + 8\hat{a}_z$ mWb/m² find $\vec{H}_1$ and $\vec{B}_1$

Solution :



$$B_{1n} = B_{2n}$$

$$8\hat{a}_z = 8a_z$$

$$\boxed{B_z = 8} \quad \boxed{B_{1n} = 8\hat{a}_z} \text{ (Normal component) mWb/m}^2$$

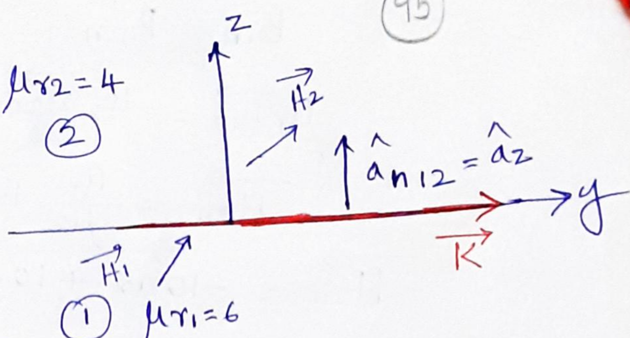
$$\vec{B}_2 = \left(5\hat{a}_x + 8\hat{a}_z\right) \text{mA/m}$$

$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = \frac{\left(5\hat{a}_x + 8\hat{a}_z\right) \text{mA/m}}{4\mu_0} = \frac{1}{4\mu_0}\left(5\hat{a}_x + 8\hat{a}_z\right) \text{mA/m}$$

$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{1}{6\mu_0}\left(B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z\right) \text{mA/m}$$

Tangential components can be found from the following

$$(\vec{H_1} - \vec{H_2}) \times \hat{a}_{n12} = \vec{K}$$



$$\vec{H_1} \times \hat{a}_{n12} = \vec{H_2}\, \hat{a}_{n12} + \vec{K}$$

$$\frac{1}{6\mu_0}(B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z) \times \hat{a}_z = \frac{1}{4\mu_0}(5\hat{a}_x + 8\hat{a}_z) \times \hat{a}_z + \frac{\hat{a}_y}{\mu_0}$$

$$\hat{a}_x \times \hat{a}_z = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\hat{a}_y$$

$$\hat{a}_y \times \hat{a}_z = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{a}_x$$

$$\hat{a}_z \times \hat{a}_z = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\frac{1}{6\mu_0}\left(B_x(-\hat{a}_y) + B_y \hat{a}_x + B_z(0)\right) = \frac{1}{4\mu_0}\left[5(-\hat{a}_y) + 8(0)\right] + \frac{\hat{a}_y}{\mu_0}$$

Equating the components

$$\frac{-B_x}{6} = -\frac{5}{4} + 1 \Rightarrow \cancel{-}\frac{B_x}{6} = \frac{\cancel{-}1}{4} \qquad \boxed{B_x = \frac{6}{4} = 1.5}$$

$$\boxed{B_y = 0} \qquad \boxed{B_z = 0}$$

$\therefore \vec{B_{1t}} = 1.5\hat{a}_x + 0\hat{a}_y$, $\vec{B_{1n}} = 8\hat{a}_z$ mWb/m^2

$$\therefore \boxed{\vec{B_1} = \vec{B_{1t}} + \vec{B_{1n}} = (1.5\hat{a}_x + 8\hat{a}_z)\ \text{mWb/m}^2}$$

$$\vec{H_1} = \frac{\vec{B_1}}{\mu_1} =$$

$$\vec{H_1} = \frac{(1.5\hat{a}_x + 8\hat{a}_z)}{6\mu_0} \text{ m}\cancel{A}/\text{m}$$

$$\vec{H_1} = \frac{1}{\mu_0}(0.25\hat{a}_x + 1.33\hat{a}_z) \text{ mA/m}$$

$$\vec{H_2} = \frac{\vec{B_2}}{\mu_0\mu_{r2}} = \frac{(5\hat{a}_x + 8\hat{a}_z)}{4\mu_0} \text{ m}$$

$$\vec{H_2} = \frac{1}{\mu_0}(1.25\hat{a}_x + 2\hat{a}_z) \text{ mA/m}$$

$$\vec{H_2} = \frac{1}{\mu_0}(1.25\hat{a}_x + 2\hat{a}_z) \text{ mA/m}$$

$$= (994.71\times10^3\,\hat{a}_x + 1.5915\times10^6\,\hat{a}_z) \text{ mA/m}$$

$$\vec{H_2} = (994.71\hat{a}_x + 1591.5\hat{a}_z) \text{ A/m}$$

$$\boxed{\vec{H_1} = 198.94\hat{a}_x + 1061.03\hat{a}_z} \text{ A/m}$$

8.8) Given that $\vec{H} = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$ A/m in region $y - x - 2 \leq 0$, where $\mu_1 = 5\mu_0$ calculate ~~M_1~~, M_1 and B_1 (97)

Solution:

$$M_1 = \chi_{m_1} H_1$$

$$M_1 = (\mu_r - 1)\vec{H}_1 \qquad (\chi_m + 1) = \mu_r$$

$$M_1 = [5-1]\vec{H}_1 \qquad \chi_m = \mu_r - 1$$

$$M_1 = 4[-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z]$$

$$M_1 = -8\hat{a}_x + 24\hat{a}_y + 16\hat{a}_z \text{ A/m.}$$