DIGITAL IMAGE PROCESSING-1

Unit 3: Lecture 36

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Unit 3: Image Enhancement

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Last Session

- Spatial Operations
  - Spatial Sharpening filters Cont..
  - Unsharp masking
  - Highboost filters
  - First order filters
  - Detection of edge using first order filters

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This Session

- Image Enhancement: Frequency domain methods
  - 2D DFT and its properties
  - Gaussian Filters
  - Smoothing using frequency domain filters
  - Correspondence between filtering in spatial and frequency domains

Types of Enhancement Techniques

Image Enhancement

- **Point Operations**
  - Contrast Stretching
  - Histogram Modeling
  - Noise Clipping
  - Window Slicing
- **Spatial Operations**
  - Noise Smoothing
  - Median Filtering
  - Unsharp Masking
- **Transform Operations**
  - Linear filtering
  - Root Filtering
  - Homomorphic Filtering
- **Pseudocoloring**
  - False Coloring
  - Pseudocoloring

Spatial Domain

Discrete Fourier Transform

- The Fourier transform is a staple of frequency-domain processing.
- For 1D discrete time signal f(x) , using the notation *x* and *y* for image coordinate variables and u and v for frequency variables, where these are integers

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \qquad u = 0, 1, 2, \ldots, M-1$$

and

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \qquad x = 0, 1, 2, \ldots, M-1$$

- Extending this to 2D signal f(x,y)

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

and

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)}$$

where $f(x, y)$ is a digital image of size $M \times N$, u = 0, 1, 2, ... , M − 1 and v = 0, 1, 2, ... , N − 1
Also, x = 0, 1, 2, ... , M − 1 and y = 0, 1, 2, ... , N − 1.

$f(x,y) \Leftrightarrow F(u,v)$; 2-D *discrete Fourier transform pair*

Find F(0,0)

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

$$F(0,0) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y),$$

Average value:

$$\bar{f} = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) = \frac{1}{MN} F(0,0)$$

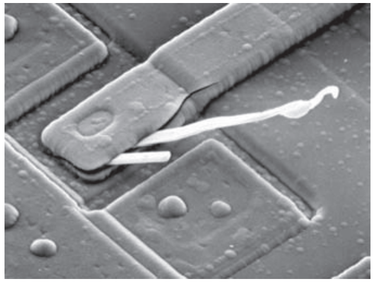
which is the DC value of the signal

Filtering in Frequency Domain

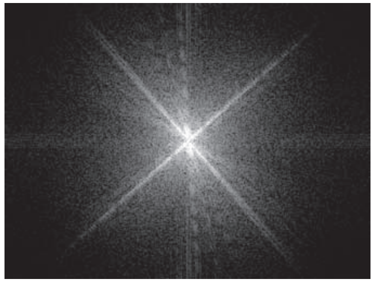
- The filtered signal is: $g(x,y) = f(x,y)*h(x,y)$

- In frequency domain:

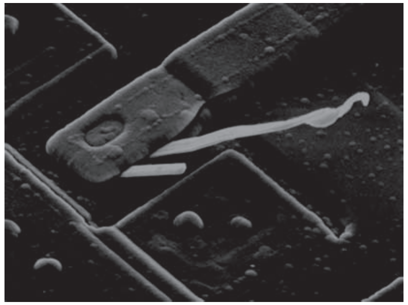
$$G(u,v) = F(u,v)\, H(u,v)$$



SEM image of a damaged integrated circuit.



Its Fourier spectrum



Result of filtering the image with a filter transfer function that sets to 0 the dc term, $F(P\ 2, Q\ 2)$, in the centered Fourier transform, while leaving all other transform terms unchanged.

Properties of DFT

1. Differentiation property

$$F\left\{\frac{d^n f(x)}{dx^n}\right\} = (j2\pi u)^n F(u)$$

f(x,y) $\longrightarrow$ F(u,v)

In spatial domain:

The Laplacian $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ $\longrightarrow$ $(j2\pi u)^2 F(u,v) + (j2\pi v)^2 F(u,v)$

$\longrightarrow$ $-4\pi^2 (u^2+v^2)F(u,v)$

$\therefore H(u,v) = -4\pi^2 (u^2+v^2)$ is the Laplacian in frequency domain

2. Frequency shifting property

$$x(n)e^{j\frac{2\pi}{N}k_0 n} \leftrightarrow X(k-k_0)$$

$$f(x,y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0)$$

Let $u_0 = M/2$ and $v_0 = N/2$
Then

$$f(x,y)e^{j\pi(x+y)} \leftrightarrow F(u-\frac{M}{2}, v-\frac{N}{2})$$

$$f(x,y)(-1)^{(x+y)} \leftrightarrow F(u-\frac{M}{2}, v-\frac{N}{2})$$ → Origin of F(u,v) shifts from (0,0) to (M/2, N/2)

Properties of DFT

- It is a common practice to multiply f(x,y) by $(-1)^{x+y}$ prior to computing DFT (to align the start to center pixel)

- Transfer function of Laplacian now becomes

$$H(u,v) = -4\pi^2\left[\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2\right]$$

$$= -4\pi^2 D^2(u,v)$$

where D(u,v) is distance of point (u,v) from origin (0,0)➔(M/2,N/2)

Consists of the following steps:

1. Given an input image $f(x,y)$ of size $M \times N$, zero pad to appropriate size to get $f_p(x, y)$ of size $P \times Q$ using zero-, mirror-, or replicate padding
2. Multiply $f_p(x, y)$ by $(-1)^{x+y}$ to center the Fourier transform on the $P \times Q$ frequency rectangle
3. Compute the DFT, $F(u,v)$, of the image from Step 3
4. Construct a real, symmetric filter transfer function, $H(u,v)$, of size $P \times Q$ with
   center at $(P/2, Q/2)$
6. Form the product $G(u,v) = H(u,v)F(u,v)$ using elementwise multiplication;that is,
   $G(i,k)=H(i,k)F(i,k)$ for $i=0,1,2,\ldots,M-1$ and $k=0,1,2,\ldots,N-1$.
7. Obtain the filtered image (of size $P \times Q$) by computing the IDFT of $G(u, v)$ taking real part

$$g(x,y) = \text{Real}\left\{\Im^{-1}\left[H(u,v)F(u,v)\right]\right\}$$

8. Multiply the result by $(-1)^{x+y}$ *(to cancel the multiplication of input* by $(-1)^{x+y}$)

$$g_p(x,y) = \left(\text{real}\left[\Im^{-1}\left\{G(u,v)\right\}\right]\right)(-1)^{x+y}$$

H(u,v) is a filter that suppresses certain frequencies in the transform while leaving others unchanged

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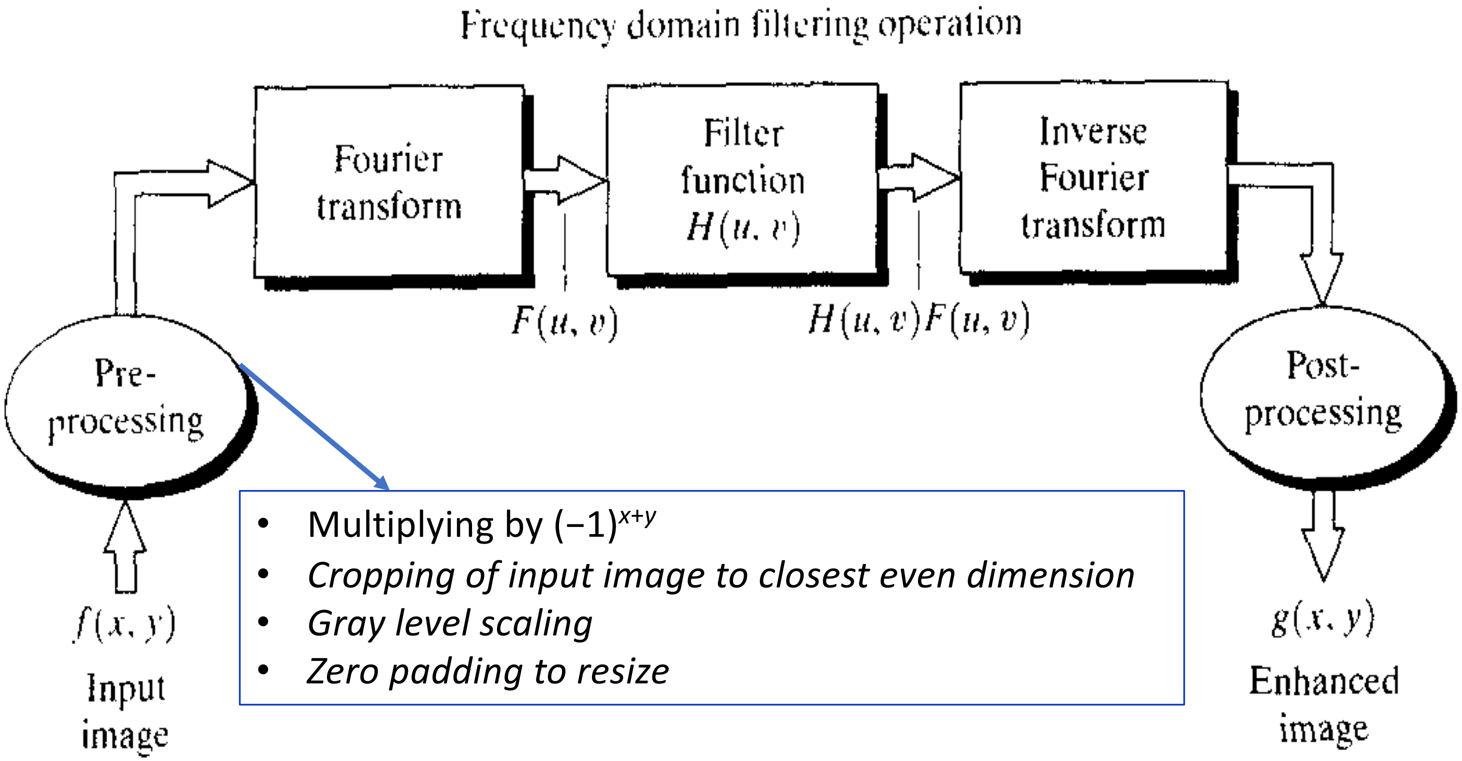
Filtering in Frequency Domain

$$g(x,y) = \text{Real}\left\{\Im^{-1}\left[H(u,v)F(u,v)\right]\right\}$$



- Multiplying by $(-1)^{x+y}$
- *Cropping of input image to closest even dimension*
- *Gray level scaling*
- *Zero padding to resize*

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Filtering in Frequency Domain

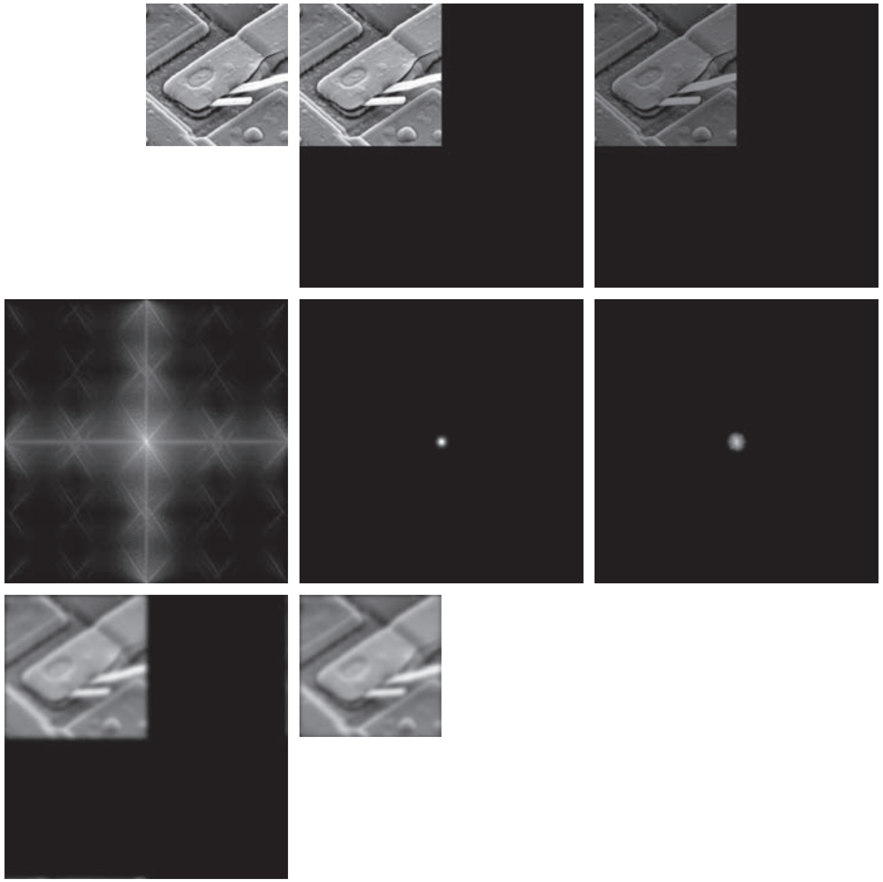

FIGURE 4.35 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F. (e) Centered Gaussian lowpass filter transfer function, H, of size $P \times Q$. (f) Spectrum of the product HF. (g) Image g_p, the real part of the IDFT of HF, multiplied by $(-1)^{x+y}$. (h) Final result, g, obtained by extracting the first M rows and N columns of g_p.

- Forcing average value to zero
- Average value is obtained by F(0,0): DC value
- Set F(0,0)/MN to 0 and take inverse Fourier transform
- Assuming transform is centered, we multiply F(u,v) with

$$F(0,0) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y),$$

$$H(u,v) = \begin{cases} 0 & ;\, if\ (u,v) = (\frac{M}{2},\frac{N}{2}) \\ 1 & ;\, Otherwise \end{cases}$$

- **This will set F(0,0) to zero and leave all other frequency component untouched**

Smoothing Frequency Domain Filters

Basic Model: $G(u,v) = H(u,v)\,F(u,v)$

- Smoothing is achieved in frequency domain by high frequency attenuation(LPF)
- Three types:
  - Ideal (Very sharp, not practical)
  - Butterworth (Flexible, controlled by order, varies from ideal (high order) to Gaussian (low order)
  - Gaussian (very smooth)

Smoothing Frequency Domain Filters

- Ideal Lowpass Filter (ILPF)

Here

$$H(u,v) = \begin{cases} 1 & ; if\ D \le D_0 \\ 0 & ; if\ D > D_0 \end{cases}$$

& D(u,v) is distance point (u,v) from origin (0,0)➔(M/2,N/2)

$$D(u,v) = \sqrt{(u - M/2)^2 \ + (v - N/2)^2}$$

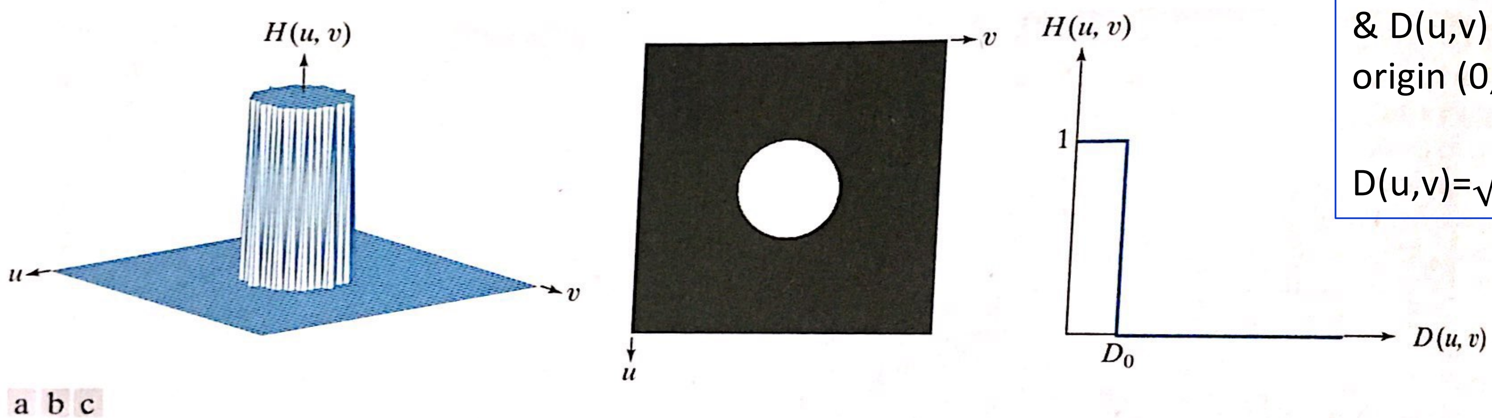

FIGURE 4.39 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Function displayed as an image. (c) Radial cross section.

Some Basic Filters

- Guassian LPF (GLPF)

  - Important because shapes are easily specified
  - FT and IFT of Gaussian functions are real gaussian functions
  - Consider 1D Gaussian

    H(u) = A$e^{-u^2/2\sigma^2}$

    h(x) = $\sqrt{2\pi}\,\sigma$A$e^{-2\pi^2\sigma^2x^2}$
  - Both are real (we don't have to deal with complex numbers)
  - These functions behave reciprocally w.r.t one another

Some Basic Filters

- 2D Guassian LPF (GLPF)

Here

H(u,v) = $e^{-D^2(u,v)/2\sigma^2}$

If σ =D_0 then

H(u,v) = $e^{-D^2(u,v)/2D_0}$

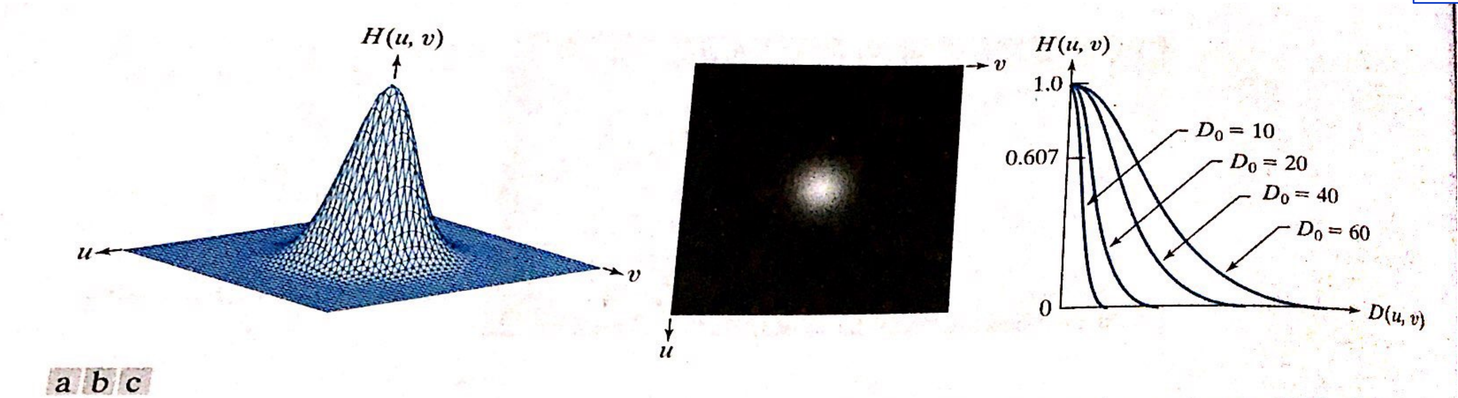

FIGURE 4.43 (a) Perspective plot of a GLPF transfer function. (b) Function displayed as an image. (c) Radial cross sections for various values of D_0.

Some Basic Filters

- Butterworth LPF (BLPF)

Here

$$H^2(u,v) = \frac{1}{1+\left(\frac{D(u,v)}{D_0}\right)^{2n}}$$

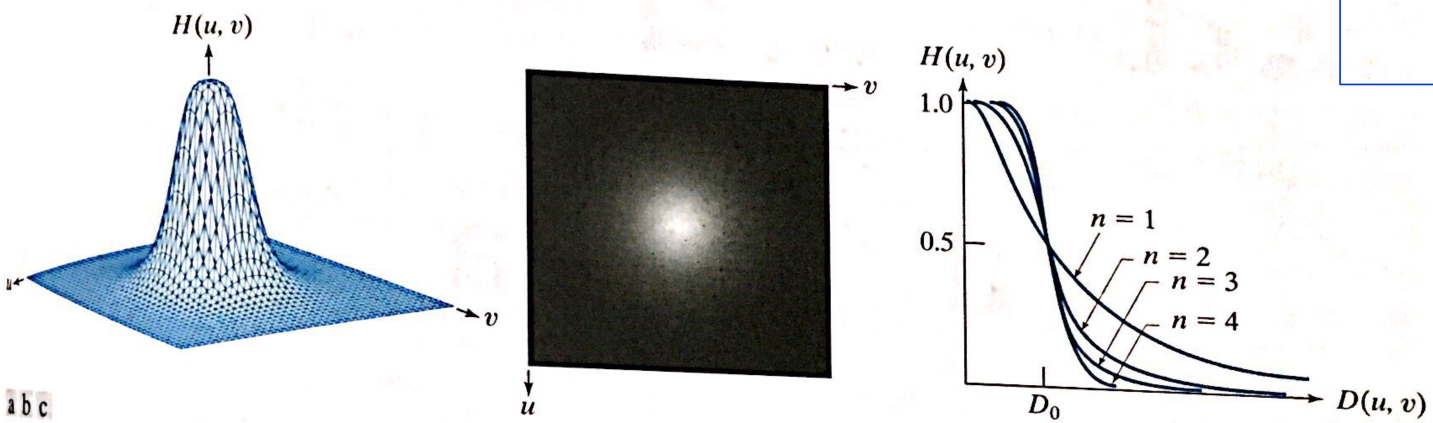

FIGURE 4.45 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Function displayed as an image. (c) Radial cross sections of BLPFs of orders 1 through 4.

Effect of Increasing Cut Off

a	b	c
d	e	f

(a) Original image of size 688 x 688 pixels. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.40(b). The power removed by these filters was 13.1, 7.2, 4.9, 2.4, and 0.6% of the total, respectively. We used mirror padding to avoid the black borders characteristic of zero padding

Effect of Increasing Cut Off

a b c

FIGURE 4.49 (a) Original 785×732 image. (b) Result of filtering using a GLPF with $D_0 = 150$. (c) Result of filterin using a GLPF with $D_0 = 130$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Comparision of Smoothing Filters

- BLPF function can be controlled to approach the characteristics of ILPF using higher values of n and GLPF using lower values of order 'n'
  - It provides smooth transition from low to high

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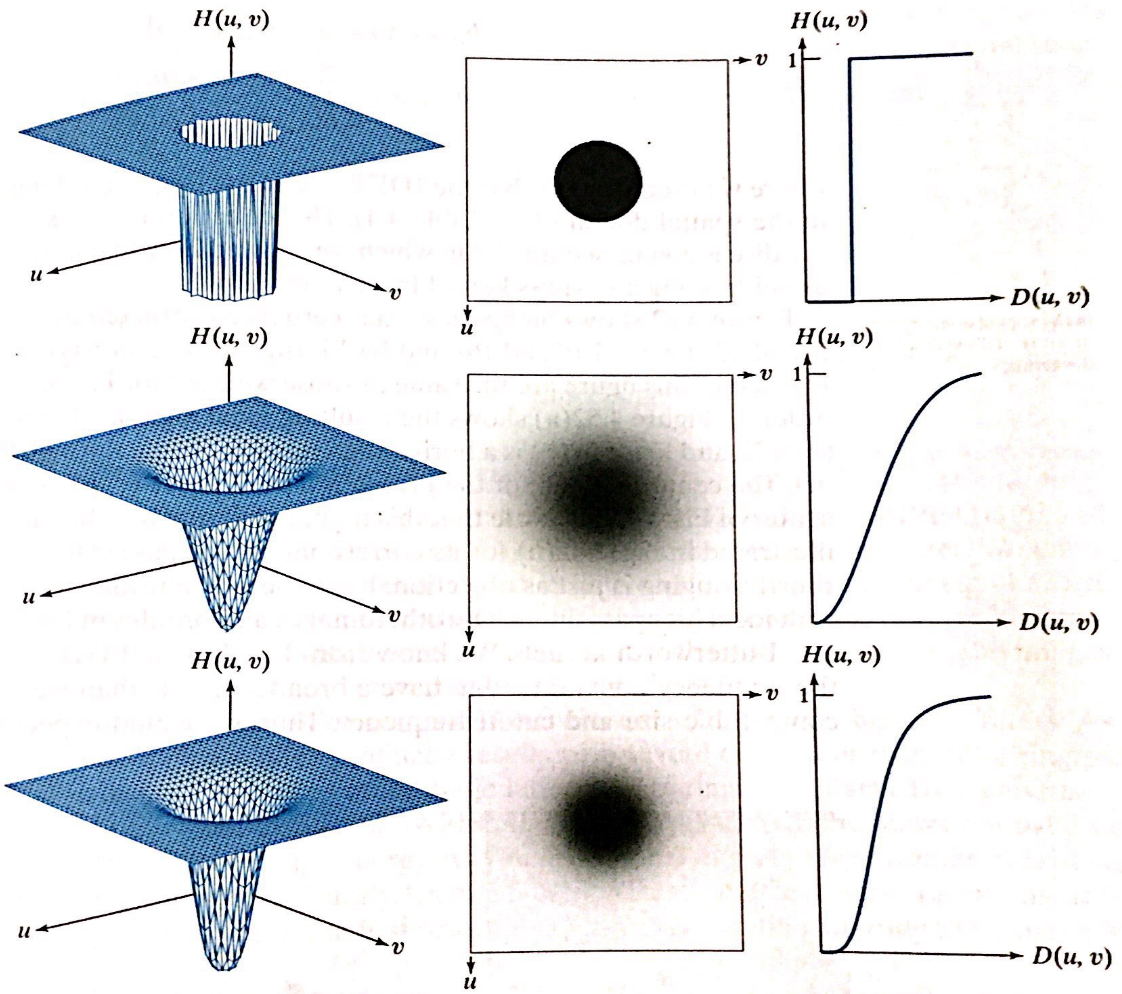
Some Basic Filters

Ideal Highpass filter

Guassian HP filter

Butterworth HP filter



$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2} \quad (4\text{-}120)$$

a b c

FIGURE 4.31 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with zero padding. Compare the vertical edges in (b) and (c).

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Filtering in Frequency Domain

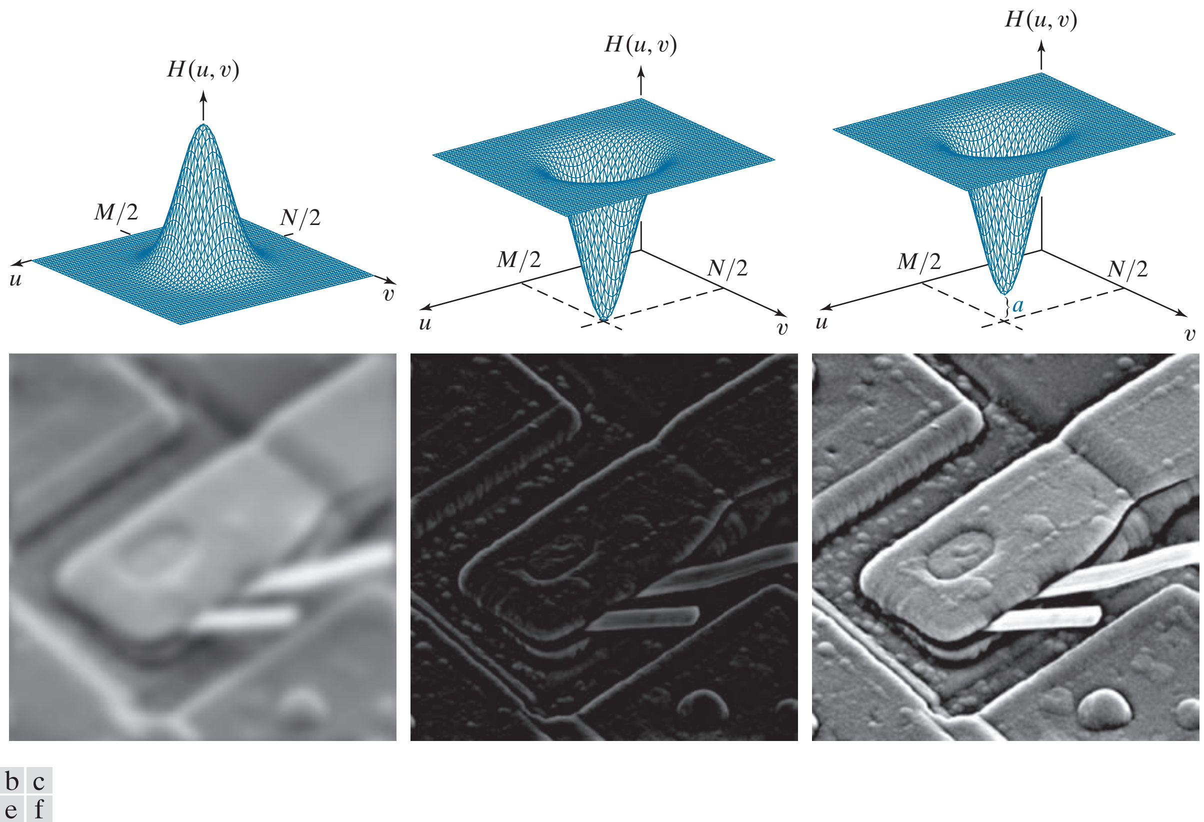


FIGURE 4.30 Top row: Frequency domain filter transfer functions of (a) a lowpass filter, (b) a highpass filter, and (c) an offset highpass filter. Bottom row: Corresponding filtered images obtained using Eq. (4-104). The offset in (c) is $a = 0.85$, and the height of $H(u,v)$ is 1. Compare (f) with Fig. 4.28(a).

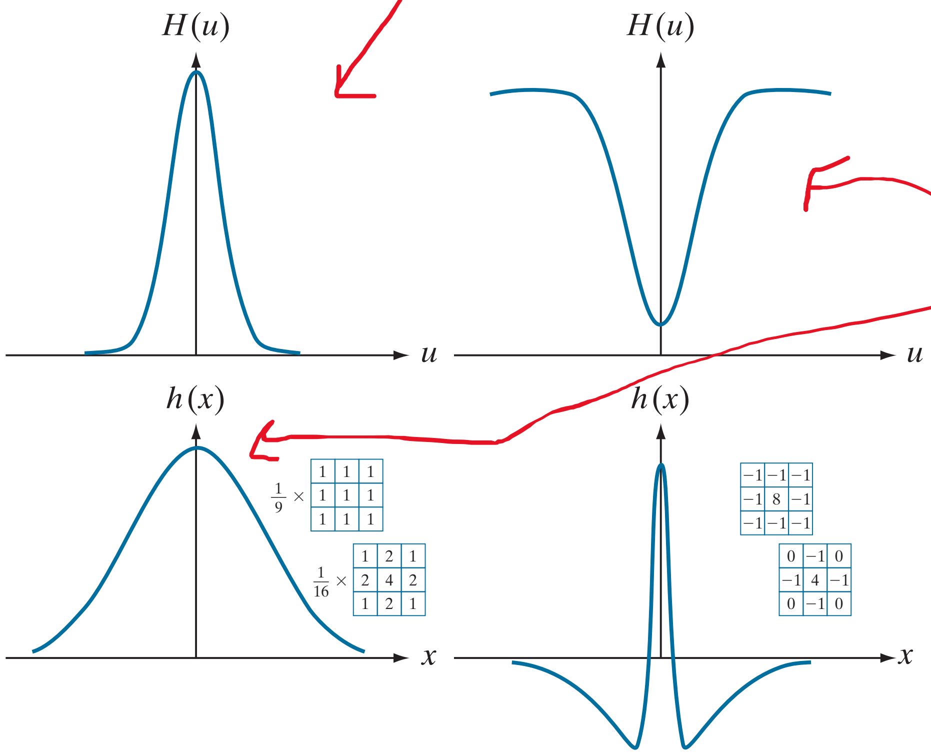
Correspondence between filtering in spatial and frequency domains

- The link between filtering in the spatial and frequency domains is the convolution theorem.
- Filtering in the frequency domain is elementwise product of a filter transfer function, $H(u,v)$, and $F(u,v)$, the Fourier transform of the input image.
- The two filters form a Fourier transform pair:

$$h(x,y) \Leftrightarrow H(u,v)$$ where $h(x, y)$ is the spatial kernel

Because this kernel can be obtained from the response of a frequency domain filter to an impulse, $h(x, y)$ sometimes is referred to as the *impulse response* of $H(u,v)$

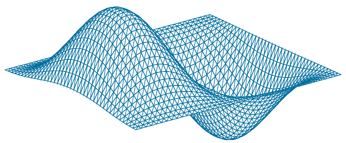
Correspondence between filtering in spatial and frequency domains



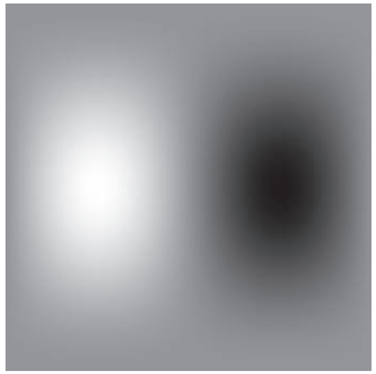
(a) A 1-D Gaussian lowpass transfer function in the frequency domain.
(b) Corresponding kernel in the spatial domain
(c) Gaussian highpass transfer function in frequency domain
(d) (d) Corresponding kernel. The small 2-D kernels shown are kernels we used

Correspondence between filtering in spatial and frequency domains

−1	0	1
−2	0	2
−1	0	1



A spatial kernel and perspective plot of its corresponding frequency domain filter transfer function.



Transfer function shown as an image.

Result of filtering the same image in the spatial domain with the kernel in spatial and frequency domain

The results are identical.

Next Session

- Sharpening in Frequency domain Cont..

THANK YOU

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