



DIGITAL COMMUNICATION

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POWER SPECTRUM OF A DISCRETE PAM SIGNAL

Polar NRZ Spectrum

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Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Polar NRZ



i NRZ Polar

Let b_k indicate the k^{th} bit. We assume that 0 and 1 occur with equal probability. (Same as in Unipolar).

To find $R_A(0)$:

b_k	A_k	P_r
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0	-a	1/2
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1	a	1/2
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$$\therefore R_A(0) = E[A_k^2] = a^2 \cdot \frac{1}{2} + a^2 \cdot \frac{1}{2} = a^2$$

Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Polar NRZ



To find $R_A(1)$:

b_k	b_{k-1}	A_k	A_{k-1}	P_r	$A_k A_{k-1}$
0	0	-a	-a	1/4	a^2
0	1	-a	a	1/4	$-a^2$
1	0	a	-a	1/4	$-a^2$
1	1	a	a	1/4	a^2

$$\therefore R_A(1) = E[A_k \cdot A_{k-1}] = a^2 \cdot \frac{1}{4} + (-a^2) \cdot \frac{1}{4} + (-a^2) \cdot \frac{1}{4} + a^2 \cdot \frac{1}{4} = 0$$

We can see that $R_A(n) = 0$ for any $n \neq 0$ as it behaves identical to how it does for $n = 1$.

$$\therefore R_A(n) = a^2 \delta(n) \quad (1)$$

Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Polar NRZ

Substituting $R_A(n)$

$$\begin{aligned} S_X(f) &= \frac{T_b^2 \text{sinc}^2(fT_b)}{T_b} \sum_{n=-\infty}^{\infty} a^2 \delta(n) e^{-j2\pi fn T_b} \\ &= T_b \text{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} a^2 \delta(n) e^{-j2\pi fn T_b} \end{aligned}$$

$$\boxed{\therefore S_X(f) = a^2 T_b \text{sinc}^2(fT_b)} \quad (2)$$

(2) is the expression for power spectral density for NRZ polar scheme. Its plot is as shown in Figure(1)

- ▶ Like, Unipolar, Polar also has a bandwidth of R_b Hz.
- ▶ As there is no impulse in the PSD, it has no DC Content.

Power Spectrum of a Discrete PAM Signal

Finding $S_A(f)$: Polar NRZ

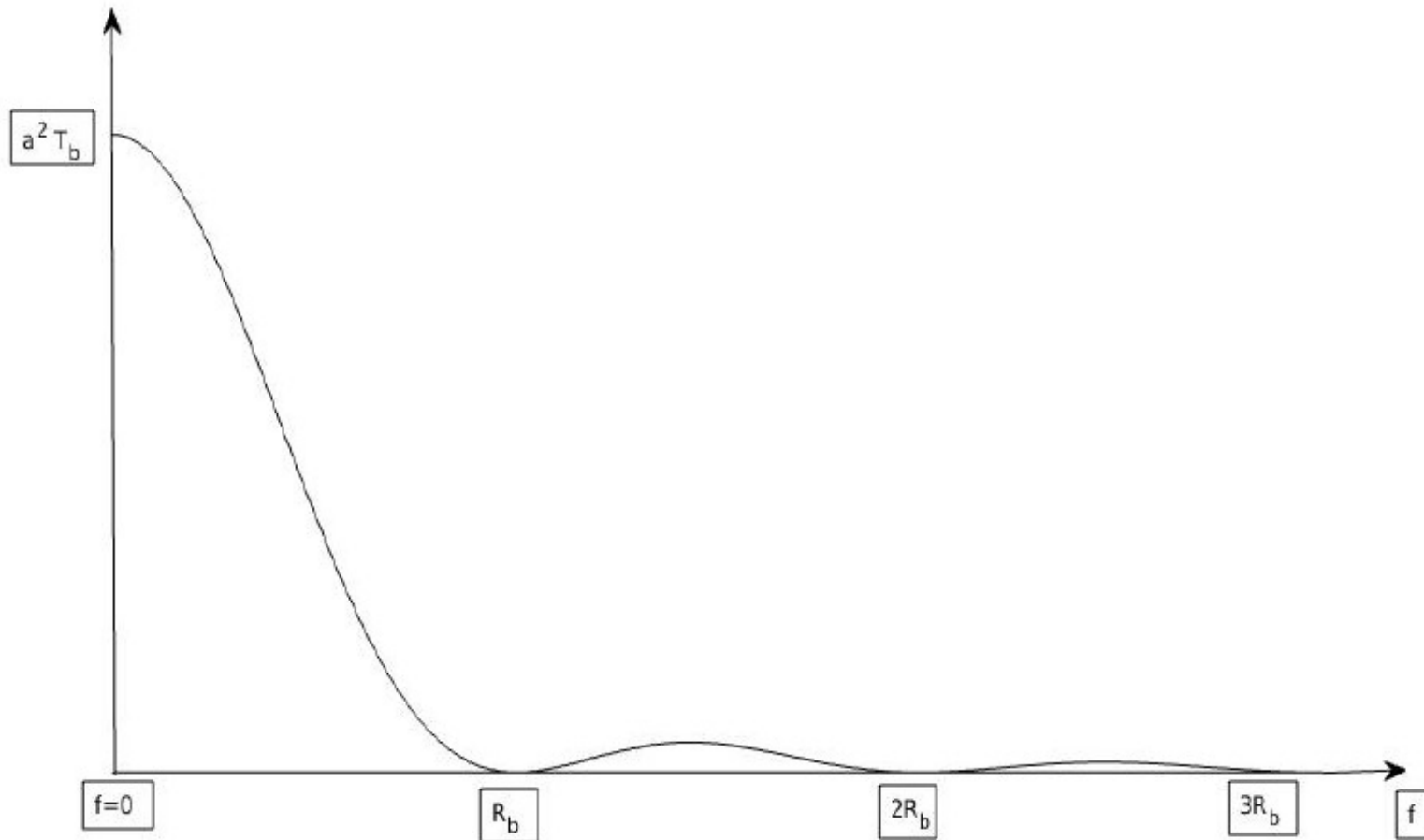
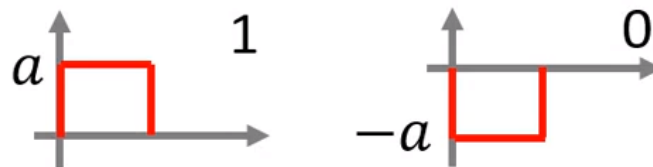


Figure: Power Spectral Density for Polar NRZ function

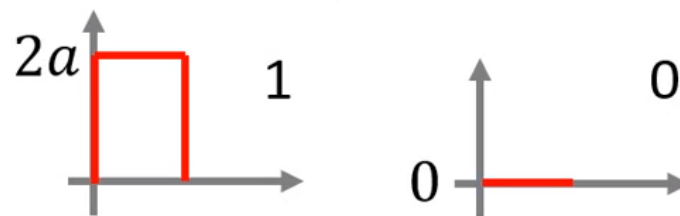
Power Spectrum of a Discrete PAM Signal

Polar NRZ Vs Unipolar NRZ

- Note that in polar NRZ, the energy per bit (whether 0 or 1) is $E_b = a^2 T_b$



- Average power in polar NRZ, $P = E_b / T_b = a^2$
- To achieve the same error performance (that is, same gap between represented values for bits 0 and 1), we need the following signals for unipolar NRZ



- Average power in unipolar NRZ

$$P = 1/2 \times 4a^2 T_b / T_b + 1/2 \times 0 / T_b = 2a^2$$

- Unipolar NRZ needs twice the power. The reason is the presence of DC component



THANK YOU

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