DIGITAL IMAGE PROCESSING-1

Unit 4: Lecture 42-43

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Unit 4: Image Filtering and Restoration

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Last Session

- Estimation of noise parameters
- Restoration in the presence of Noise only (Spatial)

This Session

- Restoration in the presence of Noise only
  - Spatial domain
  - Frequency Domain
- Introduction to restoration in the presence of degradation

Restoration in the Presence of Noise Only (Spatial Filtering)

- Noise removal/ restoration models
  - Mean filter
    - Arithmetic mean filter
    - Geometric mean filters
    - Harmonic mean filter
    - Contraharmonic mean filter
  - Order Statistics filter
    - Median filter
    - Max and min filter
    - Midpoint filter
    - Alpha-Trimmed Mean Filter
  - Adaptive filter
    - Adaptive Local Noise Reduction Filter
    - Adaptive Median Filter

Filters Based on Order Statistics (Denoising)

- Median filter
  - Median represents the 50th percentile of a ranked set of numbers

$$\hat{f}(x,y) = \underset{(s,t)\in S_{x,y}}{\text{median}}\{g(s,t)\}$$

- Max and Min filter
  - Max filter uses the 100th percentile of a ranked set of numbers
    - Good for removing pepper noise

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

  - Min filter uses the 1 percentile of a ranked set of numbers
    - Good for removing salt noise

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

- Midpoint filter
  - Works best for noise with symmetric PDF like Gaussian or uniform noise

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Example

- Median filter

30	10	20
10	250	25
20	25	30

Sort →

10
10
20
20
25
25
30
30
250

Median =25

X	X	X
X	25	X
X	X	X

Example

- Max/Min filter

30	10	20
10	0	25
20	25	30

Max filter →

X	X	X
X	30	X
X	X	X

Removes pepper noise

30	10	20
10	250	25
20	25	30

Min filter →

X	X	X
X	10	X
X	X	X

Removes salt noise

Example

- Mid point filter

30	10	20
10	250	25
20	25	30

Mid point filter $\longrightarrow$

X	X	X
X	130	X
X	X	X

$$New\ value\ at\ (x,y) = {}^{1}/_{2}[\max\{g(s,t)\} + \min\{g(s,t)\}]$$

$$= {}^{1}/_{2}[250 + 10] = 130$$

Alpha-Trimmed Mean Filter (Denoising)

- Alpha-trimmed mean filter takes the mean value of the pixels enclosed by an m x n mask after deleting the pixels with the d/2 lowest and the d/2 highest gray-level values

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

  - $g_r(s,t)$ represents the remaining mn-d pixels

  - It is useful in situations involving multiple types of noise like a combination of salt-and-pepper and Gaussian

Example

- Alpha-Trimmed Mean Filter

30	10	20
10	250	25
20	25	30

$\xrightarrow{d=2}$

X	X	X
X	23	X
X	X	X

- Let d =2 remove d/2=1 min value & 1max value

~~10~~, 10, 20, 20, 25, 25, 30, 30, ~~250~~

$$New\ value\ at\ (x,y) = {}^{10+20+20+25+25+30+30}/_{7}$$
$$=23$$

For a high value of *d*,

- Let d =4 remove d/2=2 min value & 2 max value

~~10~~, ~~10~~, 20, 20, 25, 25, 30, ~~30~~, ~~250~~

$$New\ value\ at\ (x,y) = {}^{20+20+25+25+30}/_{5}$$
$$=24$$

X	X	X
X	24	X
X	X	X

Restoration in the Presence of Noise Only (Spatial Filtering)

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    - Adaptive Local Noise Reduction Filter
    - Adaptive Median Filter

Restoration in the Presence of Noise Only: Adaptive Filter

- The filters discussed so far are applied to an image without considering how image characteristics vary from one point to another.
- ***Adaptive* filters** are filters whose behavior changes based on statistical characteristics of the image inside the filter region defined by the $m \times n$ rectangular neighborhood S_{xy}
- They are capable of performance superior to that of the mean/median filters
- **It has improved filtering power with an increase in filter complexity.**

Adaptive Local Noise Reduction Filter

- The simplest statistical measures of a random variable are its mean and variance.
  - They are quantities closely related to the appearance of an image.
    - **The mean gives a measure of average intensity in the region over which the mean is computed**
    - **The variance gives a measure of image contrast in that region**
- Filter is to operate on a neighborhood, S_{xy}, centered on coordinates (x,y)
- The response of the filter at (x, y) is to be based on the following quantities:
  - $g(x, y)$, the value of the noisy image at (x, y)
  - σ_{η}^{2} , the variance of the noise **(Unknown)**
  - $\bar{z}_{S_{xy}}$, the local average intensity of the pixels in S_{xy}
  - and $\sigma_{S_{xy}}^{2}$, the local variance of the intensities of pixels in S_{xy} .

Adaptive Local Noise Reduction Filter

The behavior of the filter should be as follows:

1. If σ_η^2 is zero, the filter should return simply the value of *g* at (*x*, *y*). This is the trivial, zero-noise case in which *g* is equal to *f* at (*x*, *y*).
2. If the local variance $\sigma_{S_{xy}}^2$ is high relative to σ_η^2, the filter should return a value close to *g* at (*x*, *y*).
   - **A high local variance typically is associated with edges, and these should be preserved.**
3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in S_{xy} .
   - **This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced by averaging.**

Adaptive Local Noise Reduction Filter

- An adaptive expression for obtaining $\hat{f}(x, y)$ based on these assumptions may be written as

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_{\eta}^{2}}{\sigma_{S_{xy}}^{2}}\left[g(x, y) - \bar{z}_{S_{xy}}\right]$$

Assumption: The ratio of the two variances does not exceed 1 $\sigma_{\eta}^{2} \leq \sigma_{S_{xy}}^{2}$

- The only quantity that needs to be known a priori is σ_{η}^{2}, the variance of the noise corrupting image $f(x, y)$
- This is a constant that can be estimated from sample noisy images using

$$\sigma^{2} = \sum_{i=0}^{L-1}(z_i - \bar{z})^{2} p_S(z_i)$$

- The other parameters are computed from the pixels in neighborhood S_{xy} using

$$m_{S_{xy}} = \sum_{i=0}^{L-1} r_i p_{S_{xy}}(r_i) \qquad \sigma_{S_{xy}}^{2} = \sum_{i=0}^{L-1}(r_i - m_{S_{xy}})^{2} p_{S_{xy}}(r_i)$$

r is a random variable representing pixel intensity

Adaptive Local Noise Reduction Filter

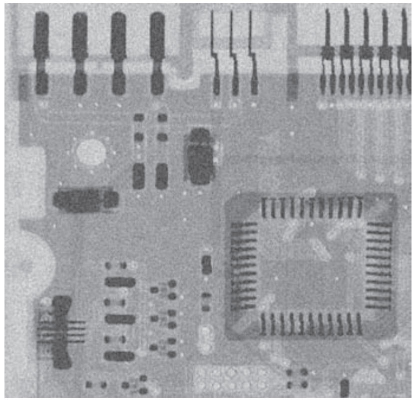
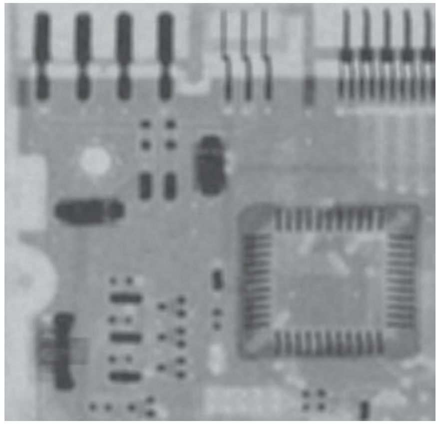
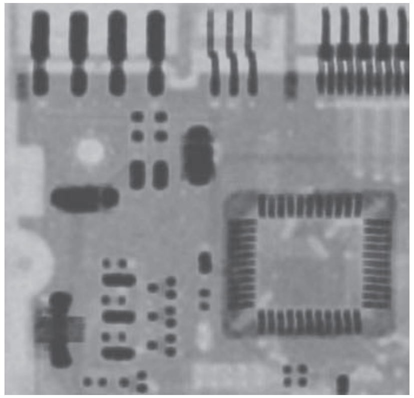


Image corrupted by additive Gaussian noise of zero mean and a variance of 1000.



Result of arithmetic mean filtering.

The noise was smoothed out, but at the cost of significant blurring.



Result of geometric mean filtering.

Similar to arithmetic filter. Only degree of blurring is different

Adaptive Local Noise Reduction Filter

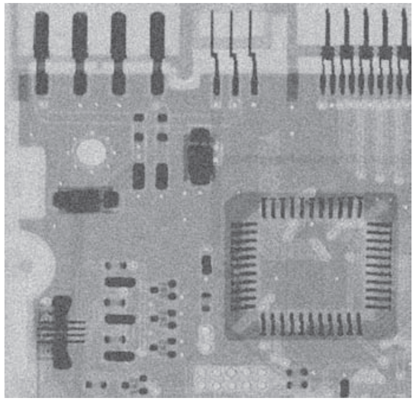
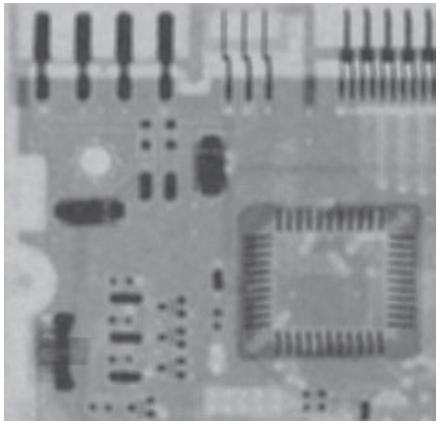
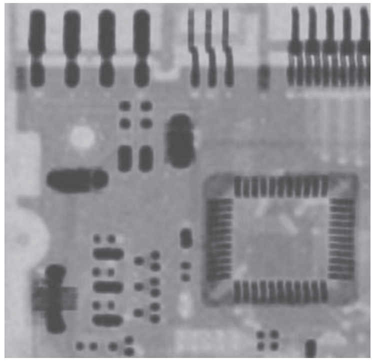


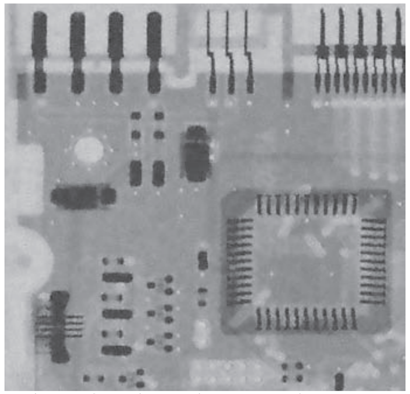
Image corrupted by additive Gaussian noise of zero mean and a variance of 1000.



Result of arithmetic mean filtering.



Result of geometric mean filtering.



Result of adaptive noise- reduction filtering with variance=1000.

The improvements in this result compared with the two previous filters are significant

Adaptive Local Noise Reduction Filter

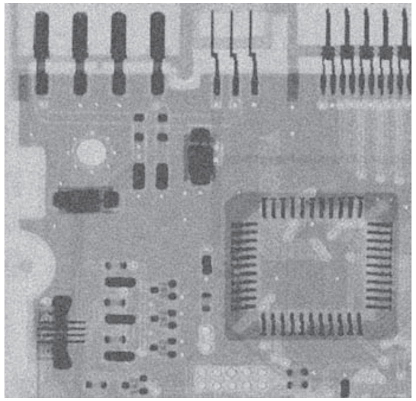
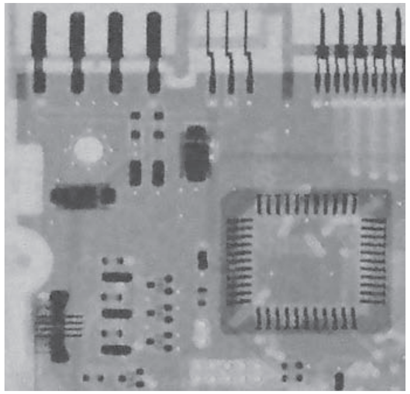


Image corrupted by additive Gaussian noise of zero mean and a variance of 1000.



Result of adaptive noise- reduction filtering with variance=1000.

- In terms of overall noise reduction, the adaptive filter achieved results similar to the arithmetic and geometric mean filters. However, the image filtered with the adaptive filter is much sharper.

Restoration in the Presence of Noise Only (Spatial Filtering)

- Noise removal/ restoration models
  - Mean filter
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  - Order Statistics filter
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  - Adaptive filter
    - Adaptive Local Noise Reduction Filter
    - Adaptive Median Filter

Adaptive Median Filter

- The median filter discussed earlier performs well if the spatial density of the salt-and- pepper noise is low (as a rule of thumb, P_s and P_p less than 0.2)
- Adaptive median filtering can handle noise with probabilities larger than these.
- An additional benefit is that it seeks to preserve detail while simultaneously smoothing non-impulse noise, something that the “traditional” median filter does not do.
- The adaptive median filter also works in a rectangular neighborhood, S_{xy}
- The output of the filter is a single value used to replace the value of the pixel at (x, y), the point on which region S_{xy} is centered at a given time.

Use the following notation:

z_{min} = minimum intensity value in S_{xy}

z_{max} = maximum intensity value in S_{xy}

z_{med} = median of intensity values in S_{xy}

z_{xy} = intensity at coordinates (x, y)

S_{max} = maximum allowed size of S_{xy}

Algorithm of Adaptive Median Filter

- The adaptive median-filtering algorithm uses two processing levels, denoted level *A* and level *B*, at each point (x, y) :

Level A : If $z_{\min} < z_{\text{med}} < z_{\max}$, go to Level B
Else, increase the size of S_{xy}
If $S_{xy} \leq S_{\max}$, repeat level A
Else, output z_{med}.

The values $z_{\min}$ and $z_{\max}$ are considered statistically by the algorithm to be "impulse-like" noise components in region S_{xy}, even if these are not the lowest and highest possible pixel values in the image.

Level B : If $z_{\min} < z_{xy} < z_{\max}$, output z_{xy}
Else output z_{med}.

Purpose of level *A* is to determine if the median filter output, z_{med} , is an impulse (salt *or* pepper) or not. If the condition $z_{\min} < z_{\text{med}} < z_{\max}$ holds, then z_{med} cannot be an impulse. In this case, we go to level *B* and test to see if the point in the center of the neighborhood is itself an impulse

Algorithm of Adaptive Median Filter

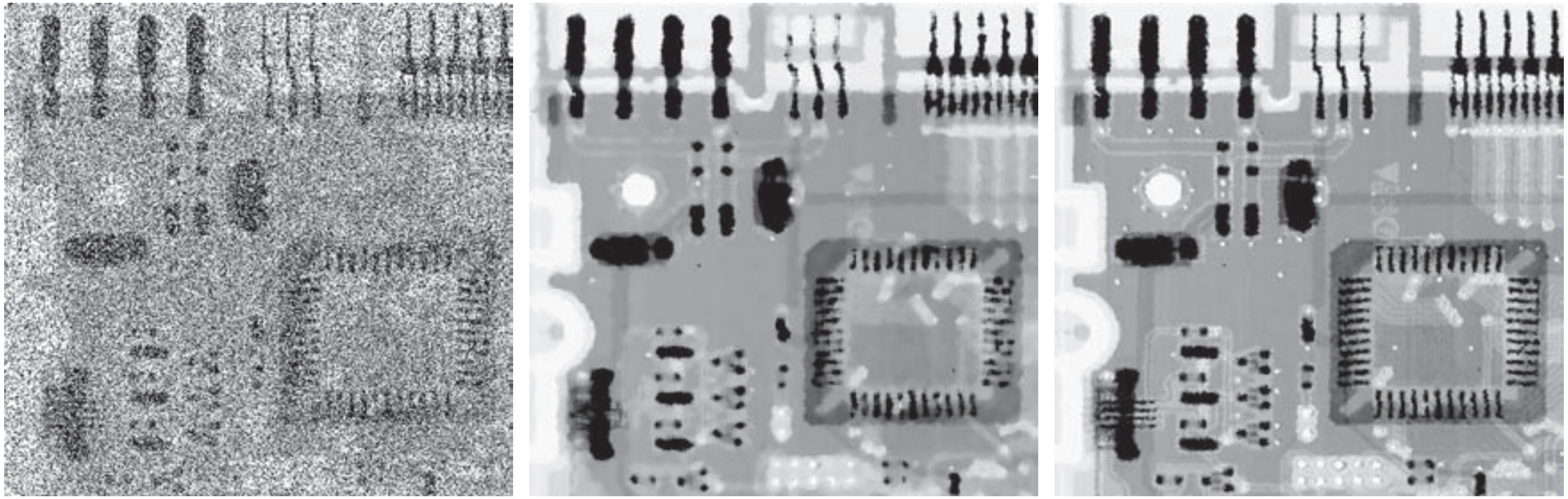
- The adaptive median-filtering algorithm uses two processing levels, denoted level *A* and level *B*, at each point (*x*, *y*) :

Level A: If $z_{\min} < z_{\text{med}} < z_{\max}$, go to Level B
Else, increase the size of S_{xy}
If $S_{xy} \le S_{\max}$, repeat level A
Else, output z_{med}.

Level B: If $z_{\min} < z_{xy} < z_{\max}$, output z_{xy}
Else output z_{med}.

Another option in the last step of level *A* is to output z_{xy} instead of z_{med}. This produces a slightly less blurred result, but can fail to detect salt (pepper) noise embedded in a constant background having the same value as pepper (salt) noise.

Algorithm of Adaptive Median Filter



(a) Image corrupted by salt-and-pepper noise with probabilities $P_s = P_s = 0.25$. (b) Result of filtering with a 7 x 7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Restoration in the Presence of Noise Only (Frequency domain Filtering)

- Periodic noise
  - Spatially dependent noise
  - Concentrated bursts of energy
  - Occurs due to electrical or electromagnetic interference
  - Analyzed & filtered using Frequency domain technique
  - Use of selective filters to isolate noise (Band-reject, Band-pass, & Notch filter)

Band Reject Filter

- Removes periodic noise from an image
- Involves removing particular range of frequencies from the image
- Transfer function of ideal band reject filter is:

$$H(u,v) = \begin{cases} 1, & D(u,v) < D_0 - \frac{W}{2} \\ 0, & D_0 - \frac{W}{2} \leq D(u,v) \leq D_0 + \frac{W}{2} \\ 1, & D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

Where W: Bandwidth, D(u,v):Distance from origin, D_0:Radial Centre

Band Reject Filter

- Butterworth band reject filter

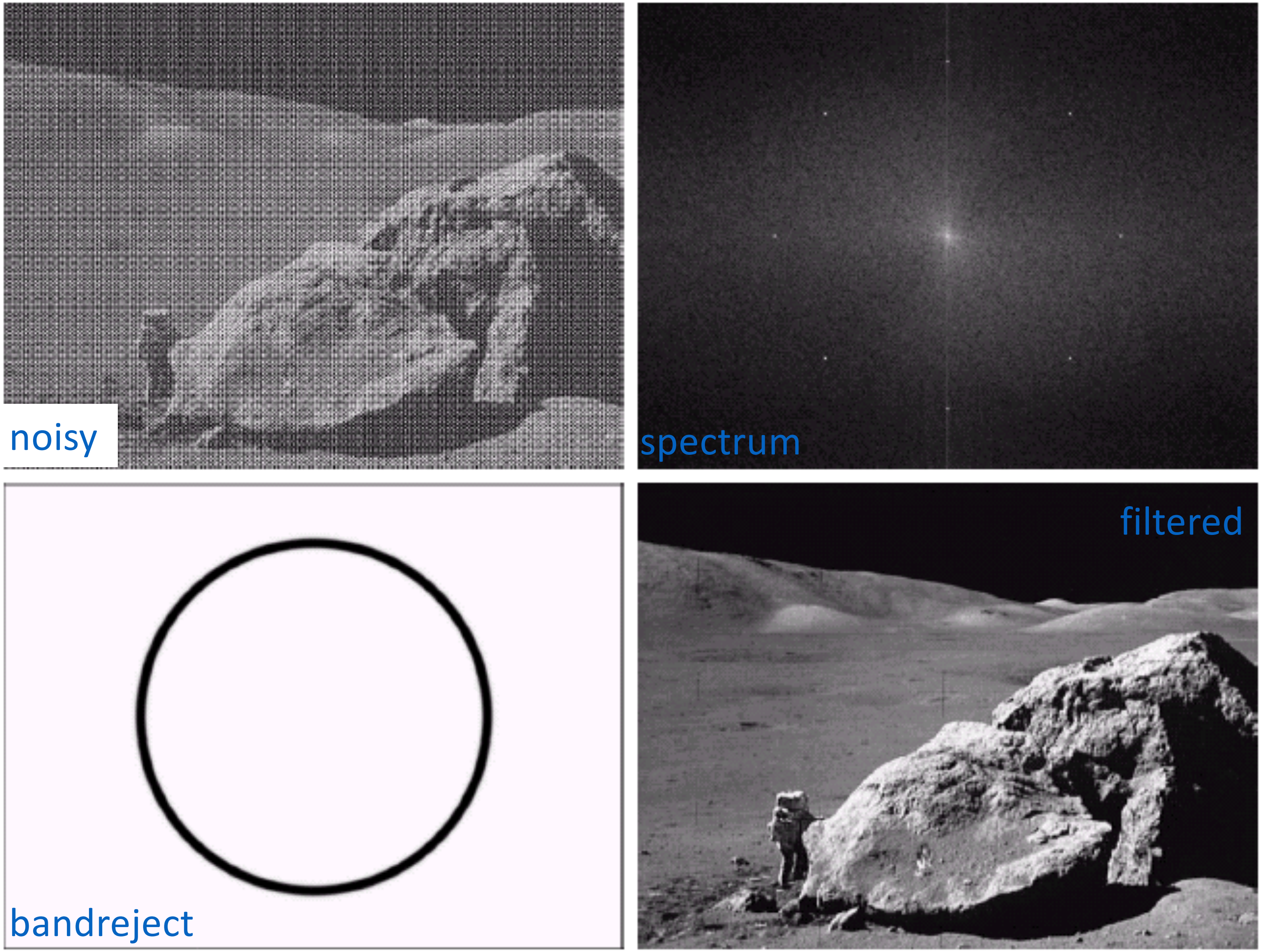
$$H(u,v) = \frac{1}{1 + \left\{ \dfrac{D(u,v)\, W}{D^2(u,v) - D_0^2} \right\}^{2n}}$$

- Gaussian band reject filter

$$H(u,v) = 1 - e^{-\left\{ \dfrac{D^2(u,v) - D_0^2}{D(u,v)\, W} \right\}}$$

Band Reject Filter

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.


noisy
spectrum
bandreject
filtered

Band Pass Filter

- A band-pass filter is the opposite of a band-reject filter
- It only passes frequency content within a range.
- A band-pass filter is obtained from a band-reject filter as:

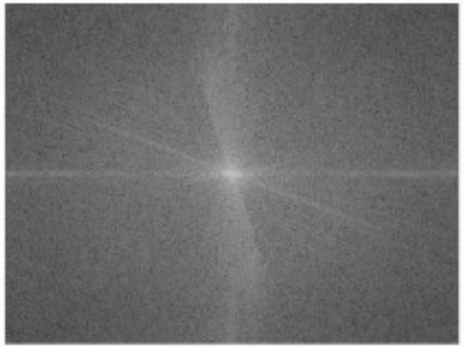
$$H_{BP}(u, v) = 1 - H_{BR}(u, v)$$

W: Bandwidth, D(u,v): distance from origin, : Radial Centre

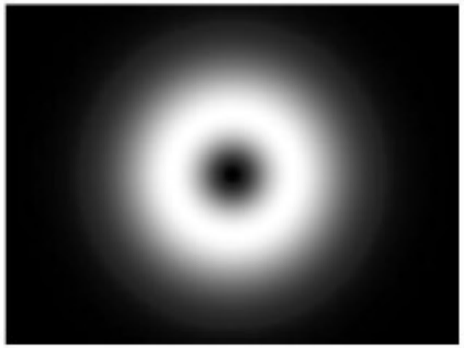
Band Pass Filter

Original Image

Fourier Spectrum of Image



Frequency Domain Filter Function Image



Bandpass Filtered Image

Notch Filter

- Reject(or pass) frequencies in predefined local neighborhoods about a center frequency
- Symmetrical pairs about the origin
- Transfer function of Ideal Notch filter is

$$H(u,v) = \begin{cases} 0, & D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1, & \text{otherwise} \end{cases}$$

$$D_1(u,v) = \left[(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2 \right]^{1/2}$$
$$D_2(u,v) = \left[(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2 \right]^{1/2}$$

are the distances from local co-ordinates , &,

The centre of frequency has been shifted to point (M/2,N/2)

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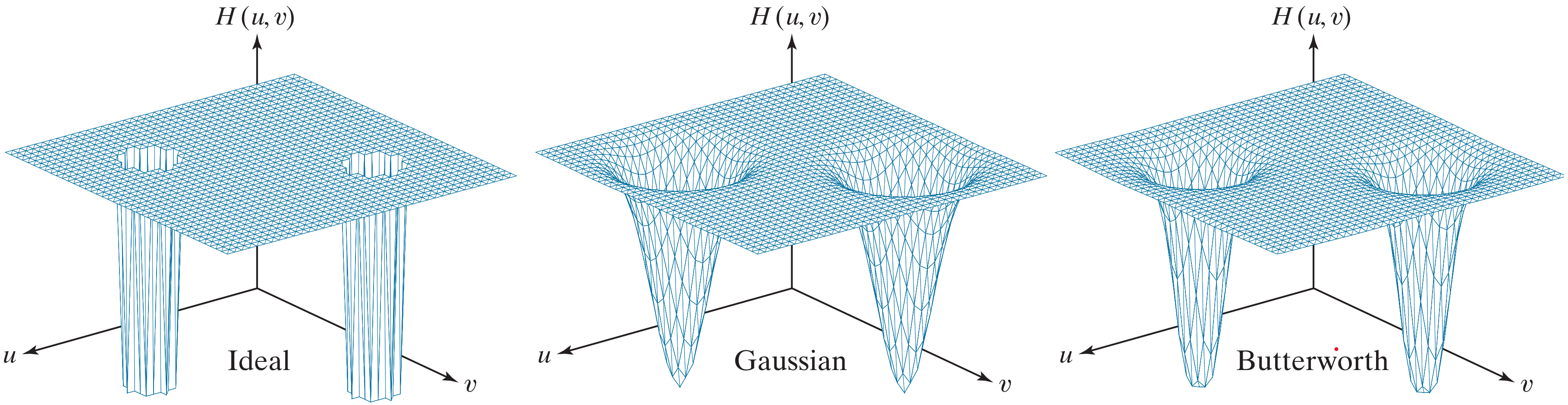
Notch Filter



a b c

FIGURE 5.15 Perspective plots of (a) ideal, (b) Gaussian, and (c) Butterworth notch reject filter transfer functions.

Notch Filter

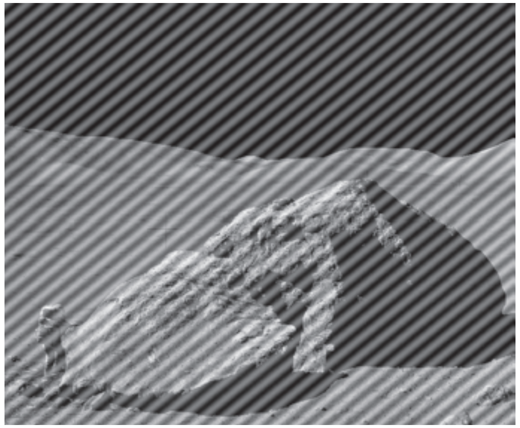
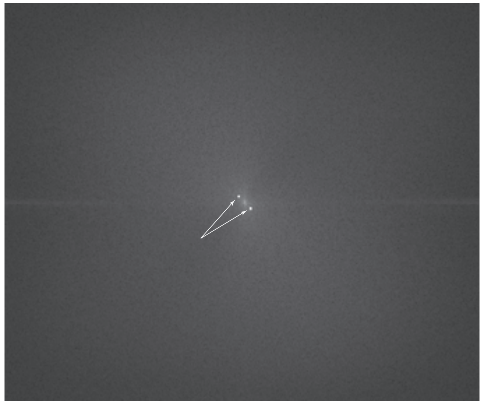
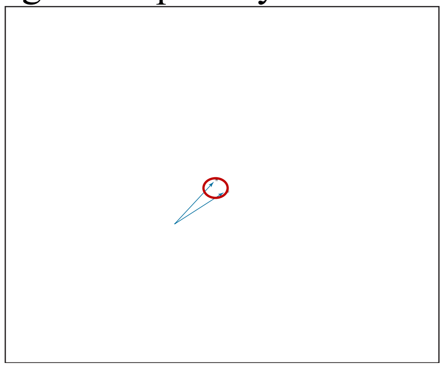


Image corrupted by sinusoidal interference.



Spectrum showing the bursts of energy caused by the interference

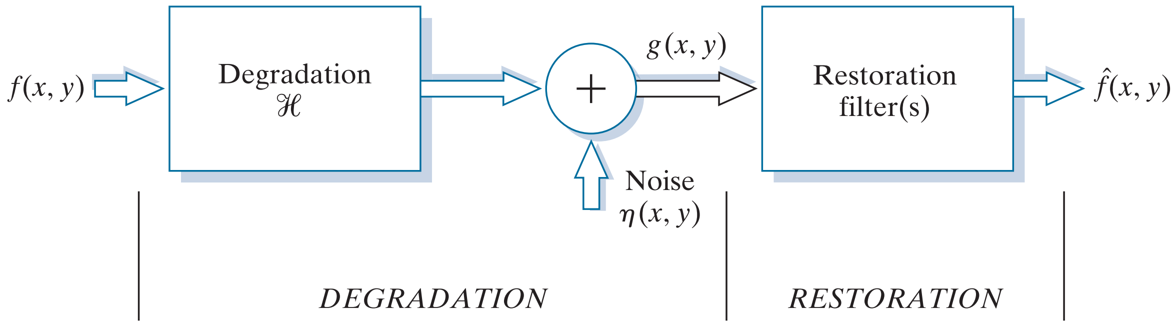


Notch filter (the radius of the circles is 2 pixels) used to eliminate the energy bursts.

Result of notch reject filtering.(Original image courtesy of NASA.)

Restoration in the Presence of Degradation Only

Degradation / Restoration Model



- Spatial Domain: $g(x,y) = h(x,y)*f(x,y) + \eta(x,y)$
- Frequency Domain: $G(u,v) = H(u,v).F(u,v) + N(u,v)$

Linear Position Invariant Degradation

- Now

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$Taking\ \ Fourier\ \ Transform\ \ on\ \ both\ \ sides$$

- G(u,v) =F(u,v)H(u,v) + N(u,v)
- If we assume N(u,v)=0 then

  G(u,v) = F(u,v).H(u,v)

  Or g(x,y) = f(x,y)*h(x,y)

Linear Position Invariant Degradation

- Degradation function ‘H’ satisfies following two properties:
  - Linearity
    - Superposition and homogeneity property
  - Shift Invariant (Position Invariant)
    - If $g(x,y) = H[f(x,y)]$

    Then $g(x-a,y-b)=H [f(x-a,y-b)]$ for any a , b & $f(x,y)$

- Any type of degradation can be approximated by LPI/LSI process, since degradations are modeled as a result of convolution.

- **Restoration seeks the filters performing reverse procedure: Deconvolution filters**

Linear Position Invariant Degradation

- The term *image* ***deconvolution*** is used frequently to signify linear image restoration.
- Similarly, the filters used in the restoration process often are called ***deconvolution filters*****.**

Estimation of Degradation Function

- Suppose that we are given a degraded image without any knowledge about the degradation function H.
- To restore image we need to estimate degradation function first
- Three methods:
  - By Observation
  - By Experimentation
  - By Mathematical modeling
- Once degradation function has been estimated, then **restoration is achieved by deconvolution**

$g(x,y) = f(x,y)*h(x,y)$

Next Session

- Image Restoration in presence of degradation cont..

THANK YOU

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