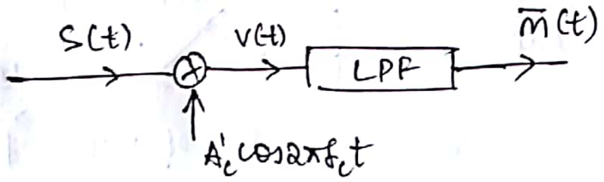
Coherent detection



$$v(t) = s(t) \cdot A_c' \cos 2\pi f_c t$$

$$= \left[\frac{A_c}{2} m(t) \cos 2\pi f_c t \mp \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t \right] A_c' \cos 2\pi f_c t$$

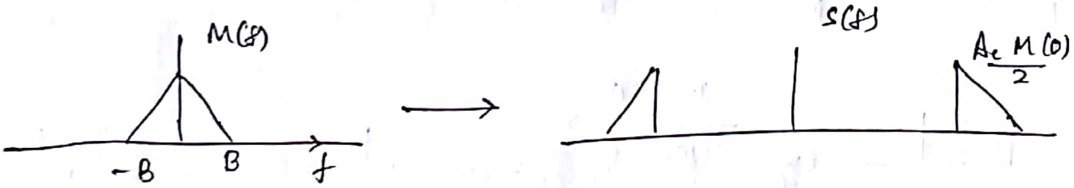
$$= \frac{A_c A_c'}{4} m(t) \left[1 + \cos 2\pi (2f_c) t \right] \mp \frac{A_c A_c'}{4} \hat{m}(t) \sin 2\pi (2f_c) t$$

After LPF,

$$\bar{m}(t) = \frac{A_c A_c'}{4} m(t).$$

Average power of s(t)

We have $S(f) = \frac{A_c}{2} M_+(f - f_c) + \frac{A_c}{2} M_-(f + f_c)$ (for USB)



Hence the Avg. power of SSB wave is half of that of DSBSC wave. Hence

$$P_s = \frac{A_c^2}{4} P_m.$$

Examples:

1. Find s(t) & plot spectrum of SSB-USB & LSB for $m(t) = A_m \cos 2\pi f_m t$ [single tone modulation]

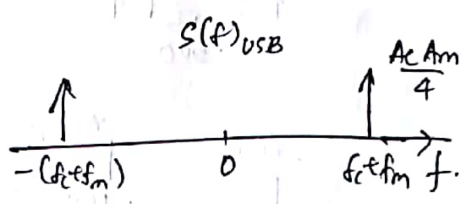
Ans: $s(t) = \frac{A_c}{2} [m(t) \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t]$

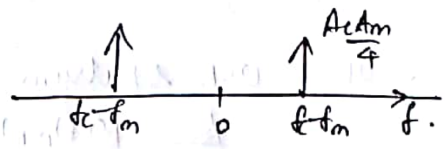
$$= \frac{A_c}{2} \left[A_m \cos 2\pi f_m t \cdot \cos 2\pi f_c t \mp A_m \sin 2\pi f_m t \sin 2\pi f_c t \right]$$

$= \frac{A_c}{\sqrt{2}} [$

$$= \frac{A_c A_m}{4}\left[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t \mp \{\cos 2\pi(f_c-f_m)t \mp \cos 2\pi(f_c+f_m)t\}\right]$$

$$= \begin{cases} \frac{A_c A_m}{2}\cos 2\pi(f_c+f_m)t & \text{USB} \\ \frac{A_c A_m}{2}\cos 2\pi(f_c-f_m)t & \text{LSB}. \end{cases}$$





Alternatively, you can find the DSBSC spectrum and then write the USB & LSB separately.

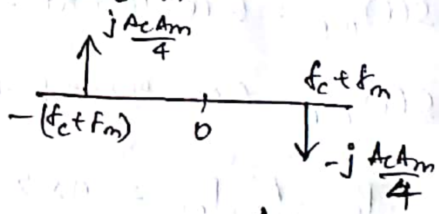
2. Find the SSB-USB & LSB signals for $m(t) = A_m \sin 2\pi f_m t$, & plot the spectrum.

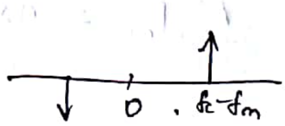
Ans: WKT, for $m(t) = A_m \sin 2\pi f_m t$, the DSBSC signal is

$$s(t) = \frac{A_c A_m}{2}\sin 2\pi(f_c+f_m)t - \frac{A_c A_m}{2}\sin 2\pi(f_c-f_m)t.$$

$$\therefore\ s(t)_{USB} = \frac{A_c A_m}{2}\sin 2\pi(f_c+f_m)t$$

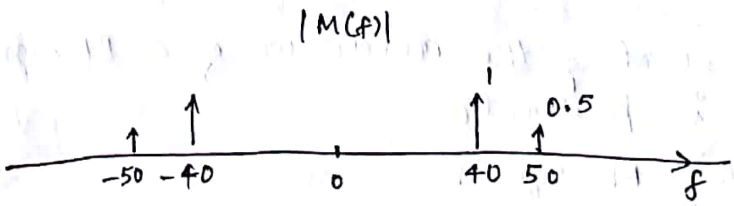
$$s(t)_{LSB} = -\frac{A_c A_m}{2}\sin 2\pi(f_c-f_m)t$$

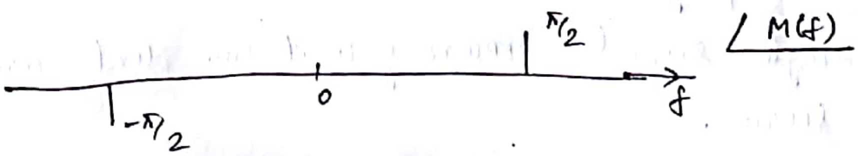




3. Let $m(t) = 2\cos 80\pi t - \sin 100\pi t$. The in-phase carrier is $c(t) = 3\cos 1000\pi t$. Find $s(t)_{USB}$ & $s(t)_{LSB}$ & plot their spectra. Also find P_s.

Ans: The spectrum of $m(t)$ is

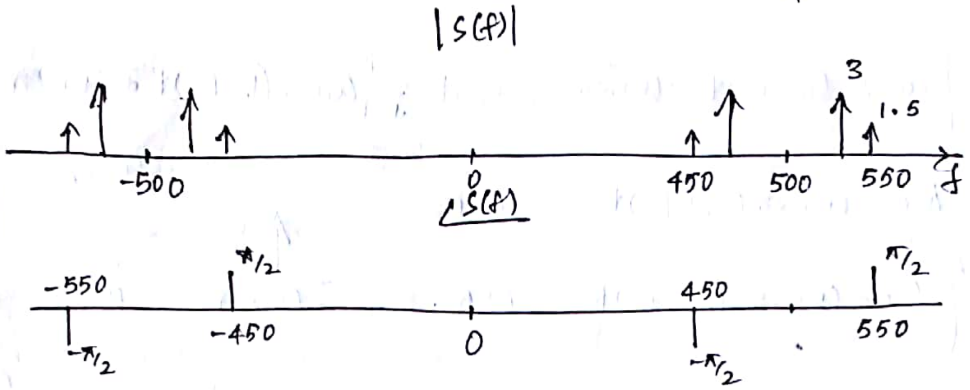




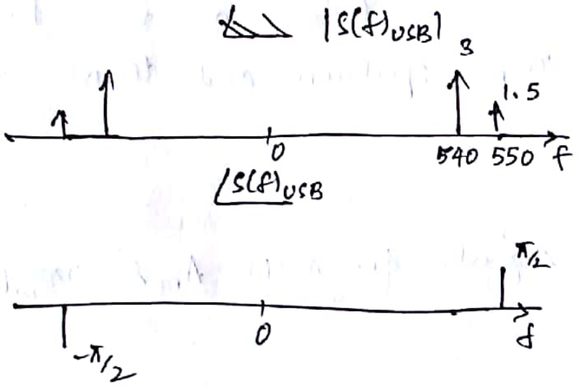
$\frac{A_c}{2} = 3 \Rightarrow A_c = 6\ V$

$\frac{A_c A_m}{4} = 3$

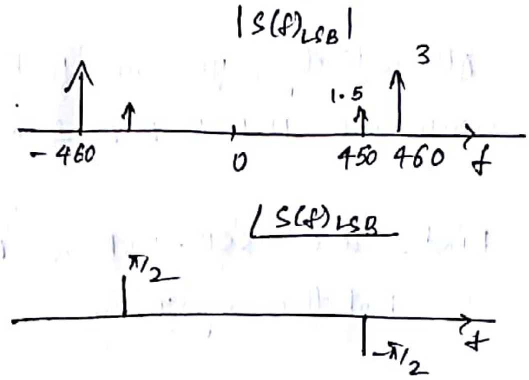
$\therefore$ The DSBSC spectrum is



$\therefore$ The USB spectrum is



The LSB spectrum is



$\therefore$ We have

$$s(t)_{USB} = 6\cos 1080\pi t - 3\sin 1100\pi t$$

$$s(t)_{LSB} = 6\cos 920\pi t + 3\sin 900\pi t$$

$$P_s = \frac{A_c^2}{4}\cdot P_m = 9\times 2.5 = 22.5W \quad \text{or} \quad \frac{6^2}{2} + \frac{3^2}{2} = 22.5W$$

Canonical representation of bandpass signals & systems

Here we consider the lowpass / baseband representation of bandpass signals & systems.

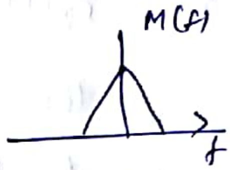
Motivation:

1) Directly evaluating the convolution of a BP signal with a BP system is problematic.
2) For sampling BP signals
3) A single signal processing unit can deal with multiple sources.

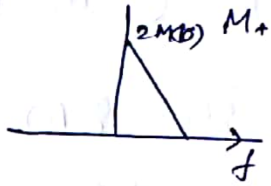
Pre-envelope:

Let $m(t)$ be a real signal. Its pre-envelope is defined as

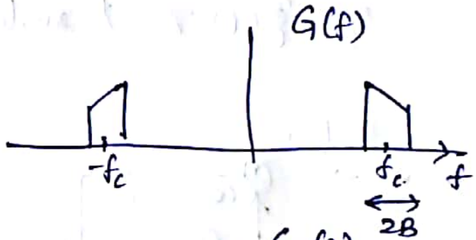
$$m_+(t) = m(t) + j\hat{m}(t)$$



$$\Rightarrow \quad M_+(f) = \begin{cases} 2M(f) & f > 0 \\ M(0) & f = 0 \\ 0 & f < 0. \end{cases}$$



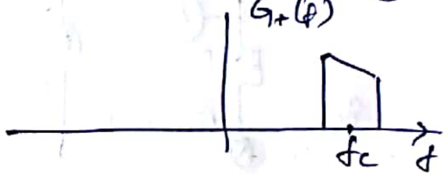
Let $g(t)$ be a real signal whose spectrum is restricted to $\pm f_c \pm B$ Hz



Its pre-envelope is

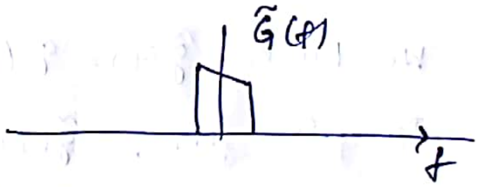
$$g_+(t) = g(t) + j\hat{g}(t) \longrightarrow (1)$$

where $\hat{g}(t)$ is the Hilbert transform of $g(t)$



We can express $g_+(t)$ as

$$g_+(t) = \tilde{g}(t)\, e^{j2\pi f_c t} \longrightarrow (2)$$



where $\tilde{g}(t)$ is a lowpass signal. It is called the "complex envelope" of $g(t)$. Since it is complex in general, we can write

$$\tilde{g}(t) = g_I(t) + j g_Q(t)$$

, where $g_I(t)$ & $g_Q(t)$ are baseband signals.

From (1), we have

$$g(t) = \text{Re}\{g_+(t)\}$$

$$= \text{Re}\{\tilde{g}(t)\, e^{j2\pi f_c t}\} \qquad (\text{from } (2))$$

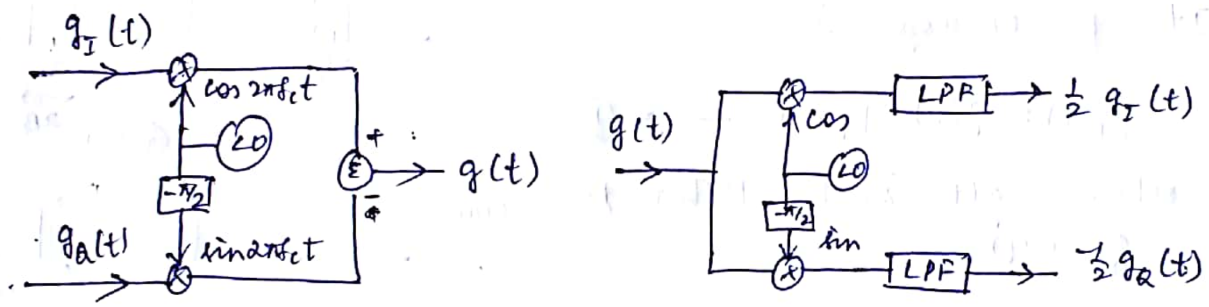
$$= \text{Re}\{[g_I(t) + j g_Q(t)][\cos 2\pi f_c t + j \sin 2\pi f_c t]\}$$

$\therefore$ we have

$$g(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t$$

This is the representation of a bandpass signal in terms of its baseband in-phase & quadrature components.

$\tilde{g}(t) = g_I(t) + j g_Q(t)$ is the baseband "equivalent" representation of $g(t)$. i.e, we can do all the processing of $\tilde{g}(t)$ & translate the final result back to passband.



we have $\tilde{g}(t) = g_I(t) + j g_Q(t)$.

In polar form, $\tilde{g}(t) = a(t)\, e^{j\phi(t)}$, where

$a(t) = \sqrt{g_I^2(t) + g_Q^2(t)}$ is the "natural envelope" or the "envelope" of $g(t)$

$$\phi(t) = \tan^{-1}\left[\frac{g_Q(t)}{g_I(t)}\right].$$

$\therefore$ we have
$$g_I(t) = a(t)\cos\phi(t)$$
$$g_Q(t) = a(t)\sin\phi(t)$$

and
$$g(t) = a(t)\cos\phi(t)\cdot\cos 2\pi f_c t - a(t)\sin\phi(t)\sin 2\pi f_c t$$
$$= a(t)\cos\left[2\pi f_c t + \phi(t)\right]$$

NOTE: If $G_+(f)$ is symmetric w.r.t f_c, then

* $\tilde{G}(f)$ is even symmetric
* $\tilde{g}(t)$ is real $\Rightarrow$ $\tilde{g}(t) = g_I(t)$
* $a(t) = g_I(t)$
* $\phi(t) = 0$
* $g(t) = g_I(t)\cos 2\pi f_c t = a(t)\cos 2\pi f_c t$ [DSBSC case]

Examples:

1. $g(t) = \cos 2\pi f_c t$. Find $g_+(t)$, $\tilde{g}(t)$ & $a(t)$.

Ans:

$$g_+(t) = g(t) + j\hat{g}(t) = \cos 2\pi f_c t + j\sin 2\pi f_c t = e^{j2\pi f_c t}.$$

$$\tilde{g}(t) = g_+(t)\, e^{-j2\pi f_c t} = e^{j2\pi f_c t} \cdot e^{-j2\pi f_c t} = 1$$

$$\therefore \tilde{g}(t) = a(t) = 1$$

2. $g(t) = \sin 2\pi f_c t$. Find $g_+(t)$, $\tilde{g}(t)$ & $a(t)$

$$g_+(t) = g(t) + j\hat{g}(t) = \sin 2\pi f_c t - j\cos 2\pi f_c t$$

$$= -j\left[\cos 2\pi f_c t + j\sin 2\pi f_c t\right]$$

$$= -j \cdot e^{j2\pi f_c t} = e^{j[2\pi f_c t - \pi/2]}$$

$$\tilde{g}(t) = g_+(t)\, e^{-j2\pi f_c t} = e^{j2\pi f_c t} \cdot e^{-j\pi/2} \cdot e^{-j2\pi f_c t} = e^{-j\pi/2}$$

$$= -j$$

$\Rightarrow$ $g_I(t) = 0$ & $g_Q(t) = -1$

$a(t) = 1$, $\phi(t) = -\pi/2$.

Angle Modulation

Angle modulation includes i) Frequency modulation, and ii) phase modulation.

* FM was motivated by the (wrong) belief that the transmission BW can be made arbitrarily small by varying the carrier frequency itself according to $m(t)$. [Carson]

* Later, Armstrong showed the noise resistant capabilities of FM, and built the FM system.

In amplitude modulation schemes, information is conveyed by the amplitude variation of the carrier. But in any communication system, additive noise cannot be avoided. It affects the amplitude. But if the carrier frequency is varied in accordance with the message signal, instead of amplitude, we can reduce the effect of noise.

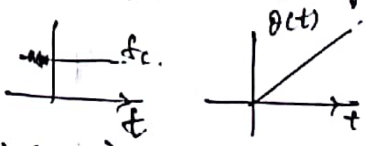
FM formulation

consider the signal $s(t) = A_c \cos[\theta(t)]$

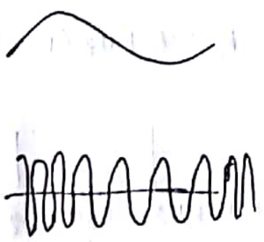
Typically, we consider $\theta(t) = 2\pi f_c t = \omega_c t$ (constant frequency).

In terms of phasor representation, we can call $\theta(t)$ "the angular distance covered" and ω_c as the "angular velocity".

$\therefore$ we have $\theta(t) = \int_0^t \omega_c \cdot dt = 2\pi \int_0^t f_c \cdot dt$



Now suppose the carrier frequency is varied in accordance with $m(t)$, then the frequency is no longer constant, but a fn. of time, & is called "instantaneous frequency", denoted by $f_i(t)$.



$\therefore$ we have

$$\theta(t) = 2\pi \int_0^t f_i(\tau) \cdot d\tau. \quad \rightarrow (1)$$

In FM, the instantaneous frequency is related to the message signal as

$$f_i(t) = f_c + K_f m(t)$$

where K_f is the "frequency sensitivity" ~~(volts/Hz)~~ (Hz/V).

$\therefore$ we have $\theta(t) = 2\pi \int_0^t [f_c + K_f\, m(\tau)]\, d\tau$

$$= 2\pi f_c t + 2\pi K_f \int_0^t m(\tau) \cdot d\tau .$$

$\therefore$ The modulated signal is

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau) \cdot d\tau\right]$$

Phase Modulation

In phase modulation, ~~m(t)~~ The phase of the carrier is modified in accordance with the message signal.

$$s(t)_{PM} = A_c \cos\left[2\pi f_c t + K_p m(t)\right]$$

K_p = phase sensitivity (rad/V)

$$\therefore \quad \theta(t) = 2\pi f_c t + K_p m(t)$$

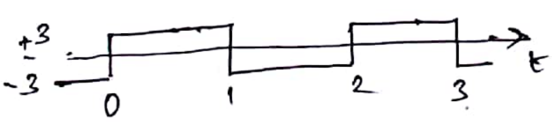
From (1), we have

$$f_i(t) = \frac{1}{2\pi} \frac{d\,\theta(t)}{dt}$$

$$= f_c + \frac{K_p}{2\pi} \frac{d\,m(t)}{dt}$$

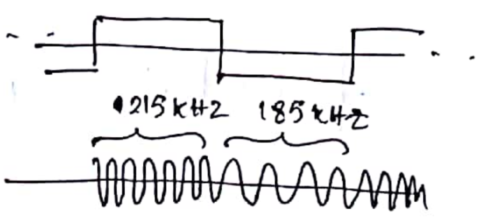
Examples:

1. plot the FM wave with $f_c = 200\,kHz$ & $K_f = 5\,kHz/V$, for the $m(t)$ shown.

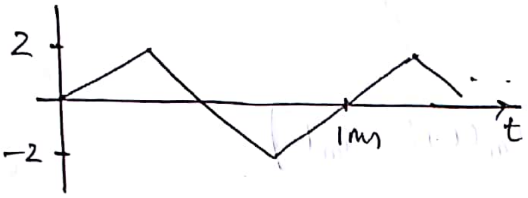


$f_i(t) = f_c + K_f m(t)$

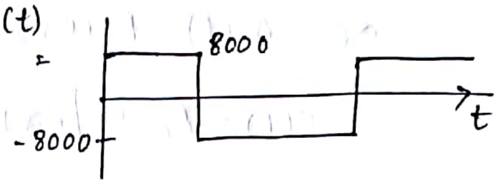
$= 200 \pm 15\,kHz$



2. plot the PM wave, with $f_c = 100\,kHz$ & $K_p = \pi$ rad/V.



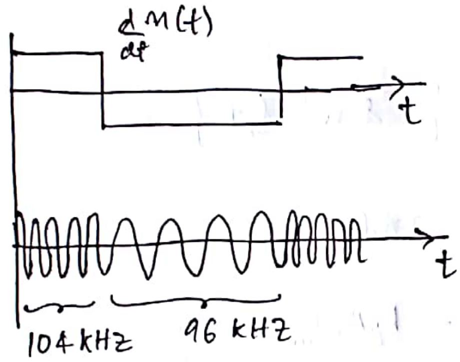
$\therefore \frac{d\,m(t)}{dt} =$



$$f_i(t) = f_c + \frac{K_p}{2\pi}\frac{d\,m(t)}{dt} = f_c \pm \frac{\pi\, 8000}{2\pi} = f_c \pm 4\,kHz.$$

$$= 100 \pm 4\ kHZ.$$

$\therefore$



<u>NOTE:</u> It is difficult to plot phase variation directly. $\therefore$ we find the frequency variation & plot it.

Single tone FM

$$m(t) = A_m \cos 2\pi f_m t$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m \tau \cdot d\tau\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{2\pi k_f A_m}{2\pi f_m} \cdot \sin 2\pi f_m t\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{K_f A_m}{f_m} \cdot \sin 2\pi f_m t\right]$$

Let $\Delta f = K_f A_m$. This is the maximum frequency deviation from f_c.

$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$ is defined as the "modulation index" of FM.

β is the max. phase deviation for FM (w.r.t the unmodulated carrier)

Single tone PM

SC $\quad m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p A_m \cos 2\pi f_m t\right]$$

$\beta = k_p A_m$ is the max. phase deviation

$$f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

$$= f_c + \frac{k_p}{\cancel{2\pi}} A_m \cancel{2\pi} f_m \{-\sin 2\pi f_m t\}$$

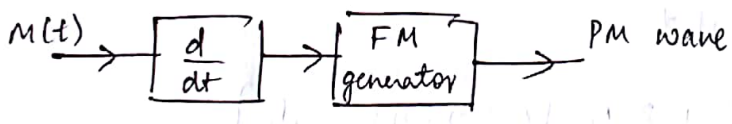
$$= f_c - k_p A_m f_m \sin 2\pi f_m t$$

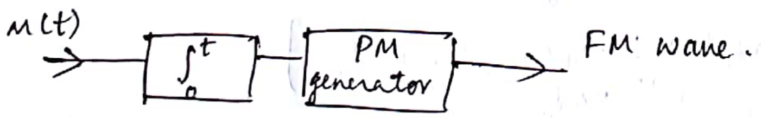
$\therefore$ max. freq. deviation : $\Delta f = k_p A_m f_m$

$$= \beta \cdot f_m .$$

Relationship between FM & PM

From the expressions for $s(t)$ & $f_i(t)$, we have





In single tone FM, depending on β, we consider two cases

i) $\beta << 1$: Narrowband FM

ii) $\beta >> 1$: Wideband FM

Narrowband FM

We have $s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$

$$= A_c\left[\cos 2\pi f_c t \cdot \cos\{\beta \sin 2\pi f_m t\} - \sin 2\pi f_c t \cdot \sin\{\beta \sin 2\pi f_m t\}\right]$$

Since $\beta << 1$, we can make the following approximations

$$\cos\{\beta \sin 2\pi f_m t\} \approx 1 \quad \& \quad \sin\{\beta \sin 2\pi f_m t\} \approx \beta \sin 2\pi f_m t .$$

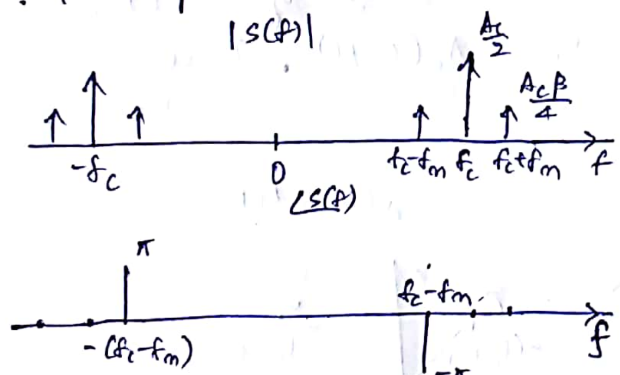
$\therefore$ we have

$$s(t) = A_c\left[\cos 2\pi f_c t - \sin 2\pi f_c t \cdot \beta \sin 2\pi f_m t\right]$$

$$= A_c \cos 2\pi f_c t - A_c\left[\frac{\beta}{2} \cos 2\pi (f_c - f_m)t - \frac{\beta}{2} \cos 2\pi (f_c + f_m)t\right]$$

$$\therefore s(t) = A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos 2\pi (f_c + f_m)t - \frac{A_c \beta}{2} \cos 2\pi (f_c - f_m)t .$$

$\therefore$ The spectrum is



compare with the AM expression

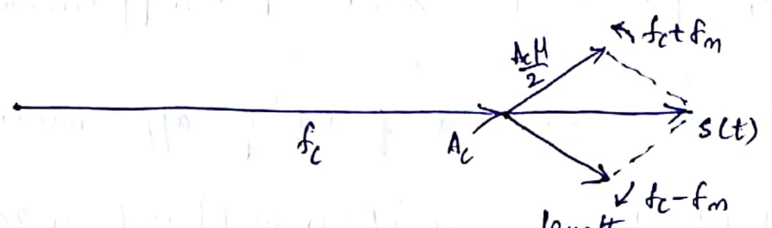
$$s(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m)t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m)t .$$

* In AM, the sidebands are in phase with the carrier. But in NBFM, the lower sideband has an offset of π rad w.r.t the carrier.

* The BW for NBFM is $2f_m$, the same as that for AM.

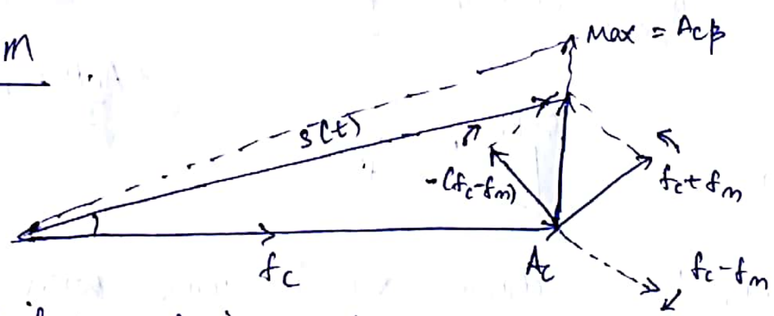
We can understand AM v/s NBFM better, by considering their phasor diagrams.

Consider the phasor diagram for AM



The phasor for the carrier has ~~amplitude~~ length A_c & rotates with an angular velocity $2\pi f_c$ rad/s. The phasor for the USB, with ~~amplitude~~ $\cancel{\frac{A_c\mu}{2}}$ length $\frac{A_c\mu}{2}$, rotates with angular velocity $2\pi(f_c+f_m)$ rad/s, and hence leads the carrier phasor. Similarly, the LSB phasor lags the carrier phasor by the same angle. The effective phasor is indicated by $s(t)$. We can recognize the amplitude variation of $s(t)$ between $S_{max} = A_c(1+\mu)$ & $S_{min} = A_c(1-\mu)$. There is no phase variation.

For NBFM



With a similar analysis as before, we find

* $S_{max} = \sqrt{A_c^2 + A_c^2\beta^2} = A_c\sqrt{1+\beta^2} \approx A_c$ since $\beta \ll 1$.
  $\therefore$ There is a very small variation in amplitude.
* There is phase variation. Its maximum value is $\phi_{max} = \tan^{-1}\frac{A_c\beta}{A_c} = \tan^{-1}\beta \approx \beta$ since $\beta \ll 1$.
  [Agrees with the fact that β is defined as the maximum phase deviation]

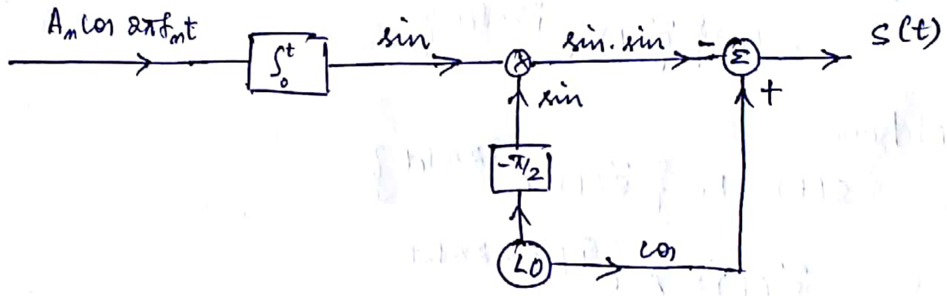
Instantaneous phase

$$\phi_i(t) = \tan^{-1} \frac{A_c \beta \sin 2\pi f_m t}{A_c} = \tan^{-1} \beta \sin 2\pi f_m t$$

$$\approx \beta \sin 2\pi f_m t \text{ . since } \beta << 1.$$

[recall $\tan^{-1}\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$]

Generation of single tone NBFM



Ex :

i) Find S_{max} & P_s for $m(t) = 2 \cos 20000\pi t$, with $k_f = 5\text{KHz/v}$ & $A_c = 1V$.

Ans: $\beta = \frac{2 \times 5000}{100000} = 0.1$ ($\Rightarrow$ NBFM) .

$$P_s = A_c^2 \left[1 + \frac{\beta^2}{2}\right] = 1 + \frac{0.01}{2} = 1.005 \text{ W}$$

$$S_{max} = A_c\sqrt{1+\beta^2} = \sqrt{1.01} \approx 1 \text{ V}.$$

Wideband FM

Consider $s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$

We can rewrite this as

$$s(t) = \mathrm{Re}\{A_c e^{j[2\pi f_c t + \beta \sin 2\pi f_m t]}\}$$

$$= \mathrm{Re}\{A_c \cdot e^{j\beta \sin 2\pi f_m t} \cdot e^{j 2\pi f_c t}\}$$

Recall the relation b/w a bandpass signal & its complex envelope.

$$g(t) = \mathrm{Re}\{g_+(t)\}$$

$$= \mathrm{Re}\{\tilde{g}(t) \cdot e^{j2\pi f_c t}\}$$

$\therefore$ we have

$$s(t) = \mathrm{Re}\{\tilde{s}(t) \cdot e^{j2\pi f_c t}\}$$

where $\tilde{s}(t) = A_c e^{j\beta \sin 2\pi f_m t}$.

$\tilde{s}(t)$ is periodic with fundamental frequency f_m. Hence we can expand it in terms of Fourier series.

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$$

where $C_n = \frac{1}{T}\int_{-T/2}^{T/2} \tilde{s}(t) e^{-j2\pi n f_m t} dt = f_m \int_{-1/2f_m}^{1/2f_m} A_c e^{j\beta \sin 2\pi f_m t} \cdot e^{-j2\pi n f_m t} dt$

$$= A_c \cdot f_m \cdot \int_{-1/2f_m}^{1/2f_m} e^{j[\beta \sin 2\pi f_m t - 2\pi \cdot n f_m t]} dt.$$

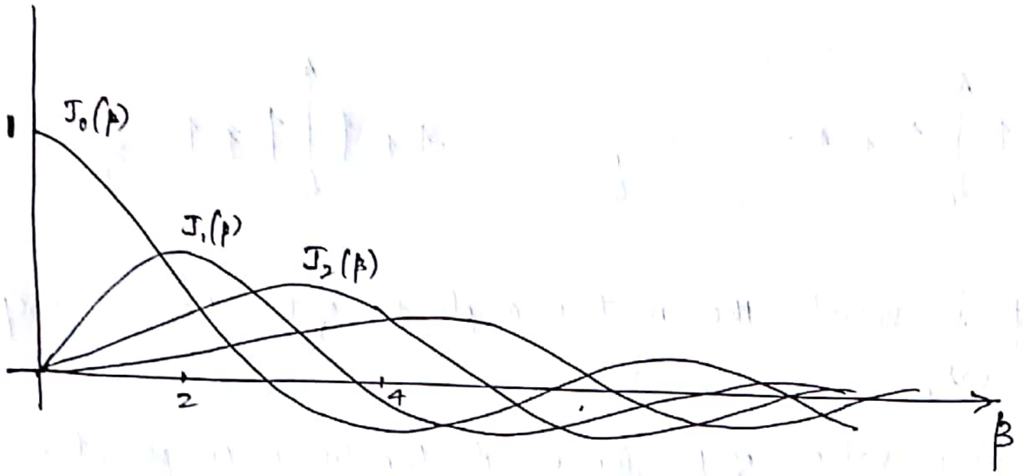
Let $x = 2\pi f_m t$.

$dx = 2\pi f_m \cdot dt$

$$\therefore \quad C_n = A_c \cdot \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} e^{j[\beta \sin x - nx]} dx$$

$$= A_c \cdot J_n(\beta),$$

where $J_n(\beta)$ is the n^{th} order Bessel function of the first kind.



properties

1. $J_n(\beta) = \begin{cases} J_{-n}(\beta) & \text{if } n \text{ even} \\ -J_{-n}(\beta) & \text{if } n \text{ odd.} \end{cases}$

2. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

3. For $\beta << 1$, $J_0(\beta) \approx 1$
$J_1(\beta) \approx \frac{\beta}{2}$ $\Rightarrow$ NBFM.
$J_n(\beta) \approx 0$ for $n \geq 2$

As β is varied, the relative amplitudes of $J_n(\beta)$ also vary.

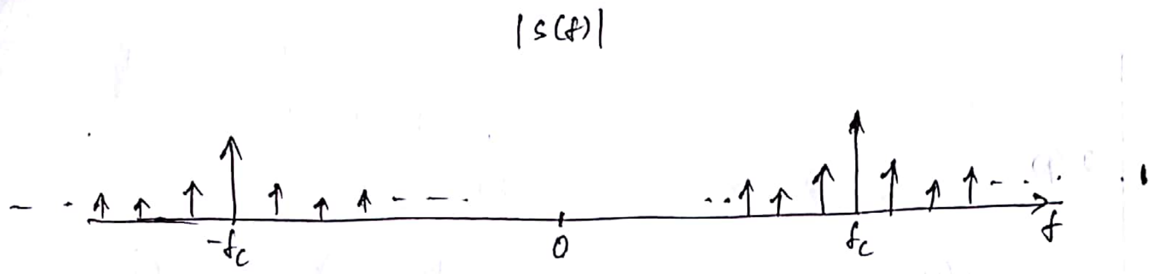
We have $C_n = A_c \cdot J_n(\beta)$

$$\therefore \quad \tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c \cdot J_n(\beta)\, e^{j2\pi n f_m t}$$

& $$s(t) = Re\left\{ \tilde{s}(t)\, e^{j2\pi f_c t} \right\}$$
$$= Re\left\{ A_c \sum_{n=\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t} \cdot e^{j2\pi f_c t} \right\}$$

$$\therefore \quad s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi \{f_c + n f_m\} t$$

& $$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - \{f_c + n f_m\}) + \delta(f + \{f_c + n f_m\}) \right]$$



* As β is varied, the relative amplitudes of the Fourier coefficients also vary.

* For some values of β, for example, the carrier component is zero.

* Theoretically, the FM wave has infinitely many sidebands.

* The total power is constant at $\frac{A_c^2}{2}$. It is redistributed among the different sidebands as β varies.

* In practice, it has been observed that most of the power is concentrated in $2(\beta+1)$ sidebands, around the carrier.

$$\therefore \quad BW = 2(\beta+1) f_m \qquad : \text{Carson's rule.}$$

$$= 2\left(\frac{\Delta f}{f_m}+1\right) f_m = 2(\Delta f + f_m)$$

Similarly, for PM,

$$BW = 2(\beta+1) f_m$$

$$= 2(k_p A_m + 1) f_m.$$

Ex: consider $s(t) = 10\cos[10^8\pi t + 8\sin 2000\pi t]$

i) Is it FM or PM?

Ans: Can't say.

ii) Assuming that it's FM,

a) find BW.

Ans: $\beta = 8$.

$$BW = 2(\beta+1) f_m = 2 \times 9 \times 1\,KHz = 18\,KHz$$

b) if A_m is doubled, what will be the BW?

Ans: $\beta = \frac{k_f A_m}{f_m} = 16$ if A_m is doubled.

$BW = 2(\beta+1) f_m = 34$ kHz.

c) if f_m is doubled, what will be the BW?

Ans: $\beta = \frac{k_f A_m}{f_m} = 4$, if f_m is doubled.

$BW = 2(\beta+1) f_m = 2 \times 5 \times 2$ kHz $= 20$ kHz.

d) if both A_m & f_m are doubled, what will be the BW?

~~BW~~ $\beta = \frac{k_f A_m}{f_m} = 8$, if both f_m & A_m are doubled.

$BW = 2(\beta+1) f_m = 2 \times 9 \times 2$ kHz $= 36$ kHz.

iii) Assuming that it is PM, repeat a to d

Ans: a $BW = 2(\beta+1) f_m = 18$ kHz

b $\beta = k_p A_m = 16$.

$\therefore BW = 2(\beta+1) f_m = 34$ kHz

c $\beta = 8$

$BW = 2(\beta+1) \cdot 2\text{kHz} = 36$ kHz

d $\beta = k_p A_m = 16$

$BW = 2(16+1) \cdot 2$ kHz $= 68$ kHz.

For a general message signal $m(t)$, with bandwidth B Hz, we have

$$\Delta f = k_f \cdot [m(t)]_{max}$$

"Deviation Ratio": $D = \frac{\Delta f}{B}$.

$BW = 2(D+1) B = \cancel{2(\Delta f + W)}\ 2(\Delta f + B)$

Typical FM transmission :

$B = 15$ kHz

$\Delta f = 75$ kHz.

$BW = 2(\Delta f + B) = 180$ kHz.

Ex: The instantaneous frequency of an FM wave is given by $f_i(t) = 10^7 + 10^5 \cos 20000\pi t$. Find the BW required.

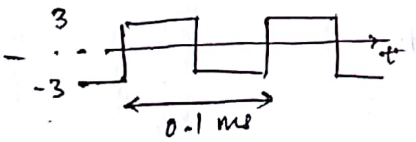
Ans: We have $f_i(t) = f_c + k_f m(t)$

$= f_c + k_f \cdot A_m \cos 2\pi f_m t$ for single tone.

$\therefore k_f A_m = 10^5 = \Delta f$. $\quad f_m = 10^4$ Hz

$BW = 2(\Delta f + f_m) = 2(10^5 + 10^4) = 220$ kHz.

Ex: For the m(t) shown below, the spectral content till the 5^{th} harmonic can be assumed to be significant. If $k_f = 10^5$ Hz/v, what is the BW required?



Ans: $T = 10^{-4}$ s $\Rightarrow f_0 = 10^4$ Hz.

$\therefore BW = 5 \cdot f_0 = 5 \times 10^4$ Hz.

$\Delta f = k_f \, |m(t)|_{max} = 10^5 \times 3$ Hz.

$BW = 2(\Delta f + B) = 2(3 \times 10^5 + 5 \times 10^4)$

$= 700$ kHz.

FM generation

1. Direct Method:

Here the FM generator is a voltage controlled oscillator (VCO). Either L or C of the oscillator is varied in accordance with m(t).

Ex: A reverse biased diode can act as a capacitor whose capacitance depends on the bias voltage.

$\therefore$ ~~$C(t) = C_0 - \alpha m(t)$~~ ~~$\alpha \rightarrow$ constant~~ $C(t) = C_0 - \alpha . m(t)$

~~$f_i(t) = \frac{1}{2\pi\sqrt{LC(t)}}$~~ $f_i(t) = \frac{1}{2\pi\sqrt{L\,C(t)}}$

Disadvantages:

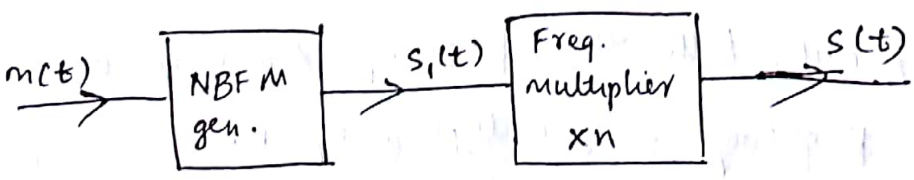
i) Difficult to get large variations in "C" value

ii) Difficult to maintain linear dependence of $f_i(t)$ on $C(t)$ over large ranges.

Hence it is difficult to generate wideband FM using this approach.

2. Indirect method:

Here, we first generate NBFM, and then use frequency multipliers to get NBFM.



$$s_1(t) = A_{c_1} \cos\left[2\pi f_{c_1} t + 2\pi k_{f_1} \int_0^t m(\tau).d\tau\right]$$

$$s(t) = A_c \cos\left[2\pi (n f_{c_1})t + 2\pi (n k_{f_1}) \int_0^t m(\tau)\, d\tau\right]$$

We can also have multiple frequency multiplier stages.

Freq. multiplier:

$x(t) \rightarrow$ NLD $\xrightarrow{y(t)}$ BPF $\rightarrow v(t)$

Ex: Let $y(t) = a \cdot x^2(t)$

Let $x(t) = \cos[\theta(t)]$

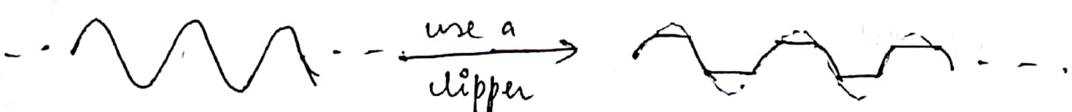
$$y(t) = a \cdot \cos^2[\theta(t)] = \frac{a}{2} + \frac{a}{2}\cos[2\theta(t)]$$

After BPF, $v(t) = \frac{a}{2}\cos[2\theta(t)]$

$$\Rightarrow \cos\left[2\pi f_{c_1} t + 2\pi k_{f_1} \int_0^t m(\tau)\,d\tau\right] \longrightarrow \cos\left[2\pi (2f_{c_1}) t + 2\pi (2k_{f_1}) \int_0^t m(\tau)\,d\tau\right]$$

For higher multiplication factors, we need to use higher order of non-linearity, followed by BPF to select the appropriate copy.

Another approach:



frequency: f_0

fundamental freq: f_0
& harmonics at $2f_0, 3f_0, \ldots$

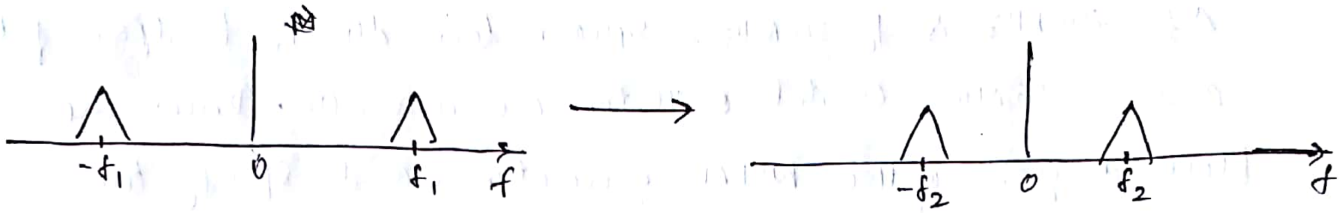
Select the n^{th} harmonic using the BPF

In this approach, both the frequency sensitivity and the carrier frequency are multiplied by n.

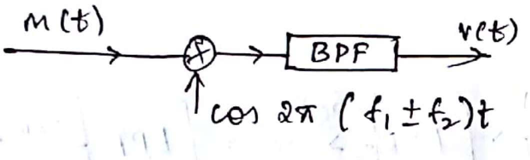
$\therefore$ we need to use a "mixer" to shift the signal's spectrum to the designated f_c.

Mixer :

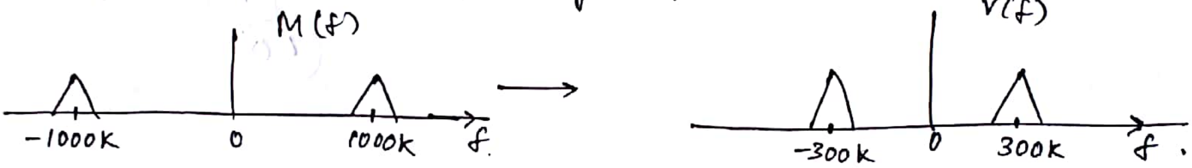
A mixer effects a frequency shift.



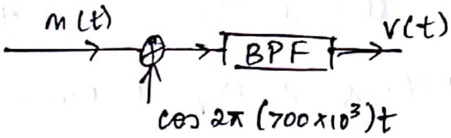
How to achieve this ?

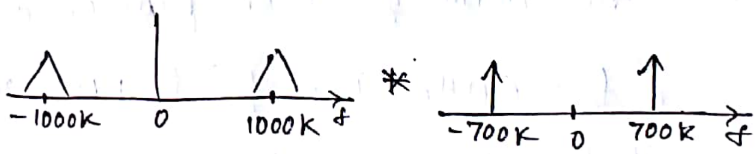


Ex: Suppose we need the following shift :



1st approach (using $f_1 - f_2$)





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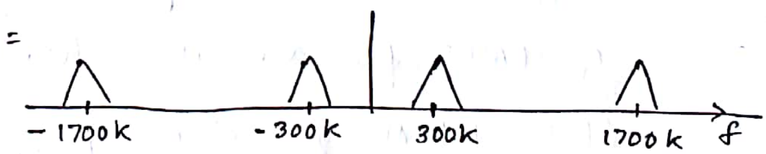
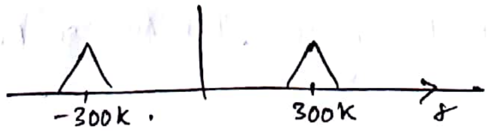
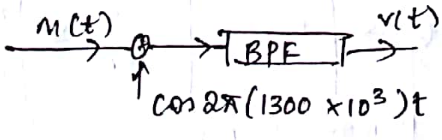
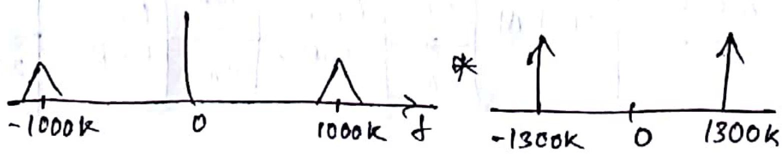


After BPF



2nd approach (using $f_1 + f_2$)





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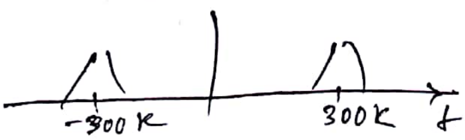
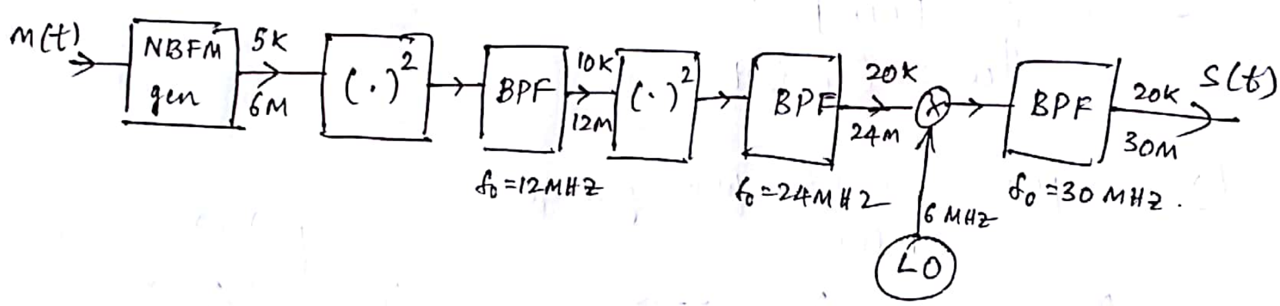
After BPF



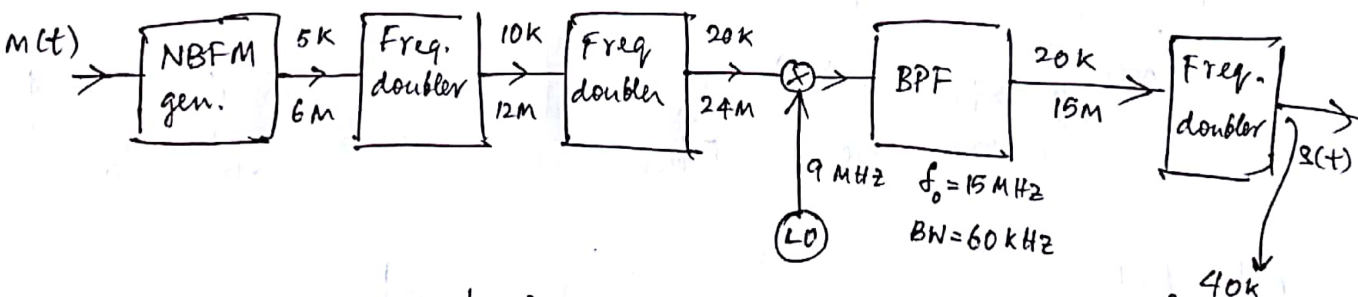
Ex: An NBFM generator produces an FM wave with $\Delta f_1 = 5\ kHz$ and $f_{c_1} = 6\ MHz$. The required WBFM wave should have $\Delta f = 20\ kHz$ & $f_c = 30\ MHz$. Square law devices, bandpass filters and a 6 MHz crystal oscillator are available. Draw the block diagram of the NBFM generator, and specify the center frequency of all the BPFs.

Ans:



Ex: An NBFM generator produces an FM wave with $\Delta f_1 = 5\ KHz$ & $f_{c_1} = 6\ MHz$. The final FM wave should have $\Delta f = 40\ kHz$ & $f_{c_1} = 30\ MHz$. Frequency doublers, BPFs & a crystal oscillator of 9 MHz freq. are available. Draw the block diagram of the WBFM generator. Specify the center frequency & BW of the BPF used in the mixer. The message BW is 10 kHz.

Ans:



BPF BW : $2(\Delta f' + B) = 2(20+10)$
$= 60\ kHz$

FM demodulation :

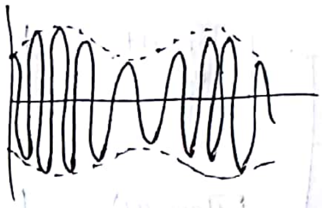
we have $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$

$$\therefore \frac{ds(t)}{dt} = A_c\left[2\pi f_c + 2\pi k_f m(t)\right] \sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau + \pi\right]$$

$$= 2\pi f_c A_c \underbrace{\left[1 + \frac{k_f}{f_c} m(t)\right]}_{AM} \underbrace{\sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau + \pi\right]}_{FM}$$

This is a hybrid AM-FM wave.

$\therefore$ An envelope detector can be used.

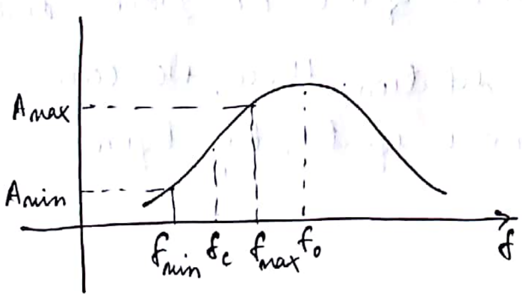


WKT $\frac{d}{dt} x(t) \xleftrightarrow{F} \underbrace{j2\pi f}_{} X(f)$

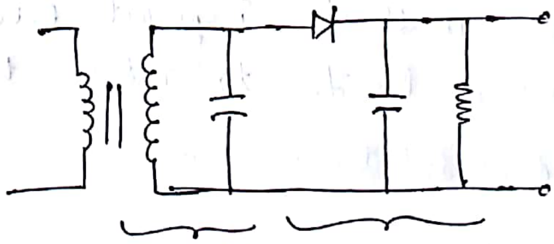
linear magnitude response.

$\therefore$ we need a "slope circuit" with linear response over the FM bandwidth: ~~$f_c \pm (\Delta f + B)$~~ $f_c \pm (\Delta f + B)$.

we can use a BPF whose transition band can provide the required linear response.



slope circuit :



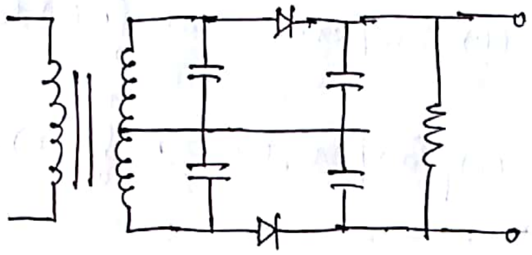
The problems with this approach are

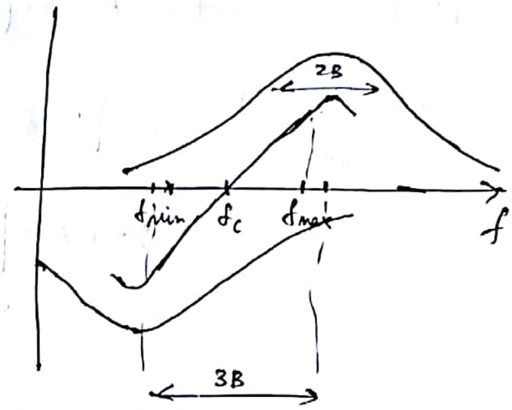
i) d.c is still present

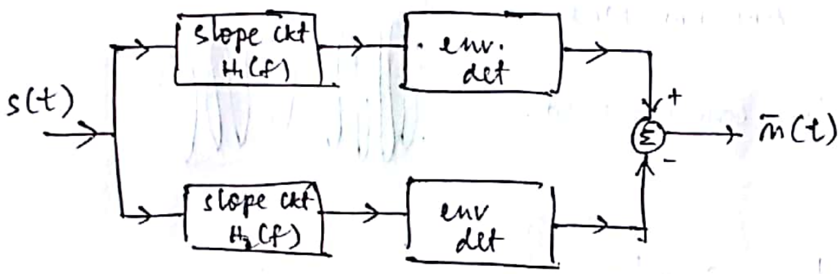
ii) difficult to ensure linear response over the whole FM bandwidth.

$\therefore$ we go for "balanced slope circuit".

Balanced slope circuit (Foster seeley discriminator)



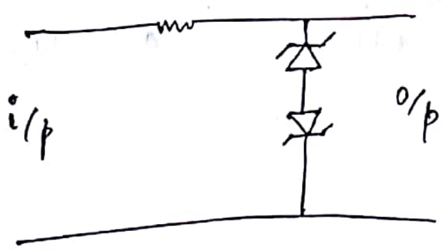




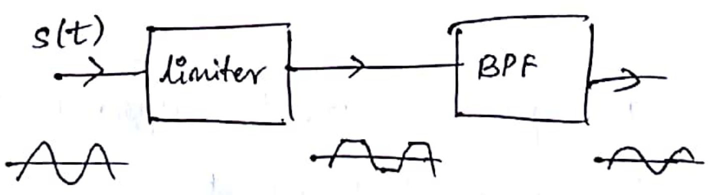
* Two slope circuits in opposition, followed by envelope detectors.

* For best results, we need a separation of 3B between the center frequencies of the two BPFs, where 2B is the 3-dB BW of the BPF.

Additive noise affects the amplitude of the FM wave, and can result in imperfect envelope detection. Hence, we can use a limiter to reduce the noise effect, ~~bef~~ before demodulation.



limiter

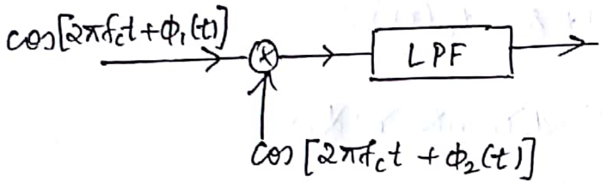


A BPF can be used to smoothen the limiter o/p.

Phase locked loop (PLL)

A PLL is a negative feedback system that can be used for FM demodulation. It consists of a VCO (FM modulator), a multiplier and a loop filter. It works on the principle that the phase difference between two sinusoids is constant iff they are of the same frequency.

Recall the phase discriminator.

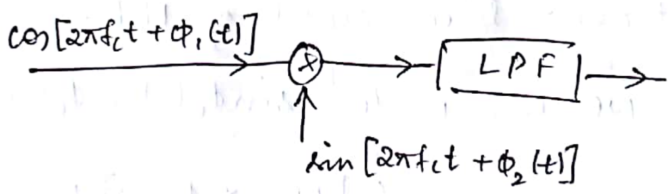


$\frac{1}{2}\cos[\phi_1(t) - \phi_2(t)]$

if $\phi_1(t) = \phi_2(t)$, o/p $= \frac{1}{2}$.

But we want the o/p to be zero if the phase difference is zero.

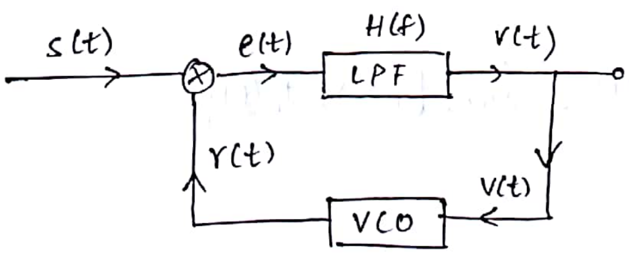
$\therefore$ Consider



$\frac{1}{2}\sin[\phi_1(t) - \phi_2(t)]$

$= 0$ if $\phi_1(t) = \phi_2(t)$

PLL operation



The free running frequency of the VCO (the frequency of VCO when the control voltage $v(t) = 0$) is set to f_c Hz. Since the VCO is an FM modulator, we can write

$$r(t) = \sin\left[2\pi f_c t + 2\pi k_v \int_0^t v(\tau)\,d\tau\right]$$

$$= \sin\left[2\pi f_c t + \phi_2(t)\right]$$

We have

$$s(t) = \cos\left[2\pi f_c t + \phi_1(t)\right]$$

The product signal $e(t)$ has both sum and difference frequencies. But since the LPF blocks the sum term, we consider only the difference term.

$$v(t) = h(t) * \sin\left[\phi_1(t) - \phi_2(t)\right]$$

Let $\phi_e(t) = \phi_1(t) - \phi_2(t)$

$$= \phi_1(t) - 2\pi k_v \int_0^t v(\tau)\, d\tau$$

$$= \phi_1(t) - 2\pi k_v \int_0^t \left[h(\tau) * \sin\phi_e(\tau)\right] d\tau$$

$$\therefore \cancel{\phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t \int_{-\infty}^{\infty} \sin\phi_e(\tau) h(t-\lambda) d\lambda}$$

$$\therefore \phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t \int_{-\infty}^{\infty} \sin\phi_e(\lambda) h(\tau - \lambda)\, d\lambda \cdot d\tau$$

This is the non-linear model of PLL.

If $\phi_e(t) << 1$, we can write $\sin\phi_e(t) \approx \phi_e(t)$

Then, $\phi_e(t) = \phi_1(t) - 2\pi K_v \int_0^t \left[h(\tau) * \phi_e(\tau)\right] d\tau$

$\rightarrow$ linear model of PLL

$$\therefore \frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_v \left[h(t) * \phi_e(t)\right]$$

Applying FT,

$$j2\pi f \Phi_e(f) = j2\pi f \Phi_1(f) - 2\pi k_v \cdot H(f)\, \Phi_e(f)$$

$$\therefore \Phi_e(f) = \frac{j2\pi f\, \Phi_1(f)}{j2\pi f + 2\pi k_v H(f)}$$

$$V(f) = H(f) \cdot \Phi_e(f)$$

$$= \frac{H(f) \cdot j2\pi f\, \Phi_1(f)}{j2\pi f + 2\pi k_v H(f)}$$

$$= \frac{j2\pi f \cdot \Phi_1(f)}{2\pi k_v + \frac{j2\pi f}{H(f)}}$$

Let $L(f) = \frac{H(f)}{jf}$. It is called the open-loop transfer function.

If $|L(f)| >> 1$, then

$$V(f) \simeq \frac{j2\pi f \Phi_1(f)}{2\pi k_v}$$

or $$v(t) = \frac{1}{2\pi k_v} \cdot \frac{d\phi_1(t)}{dt}$$

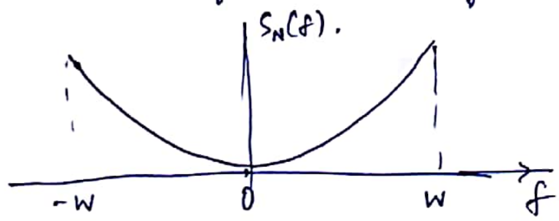
Recall $$\phi_1(t) = 2\pi k_f \int_0^t m(\tau) \cdot d\tau$$

$$\therefore \frac{d}{dt}\phi_1(t) = 2\pi k_f\, m(t)$$

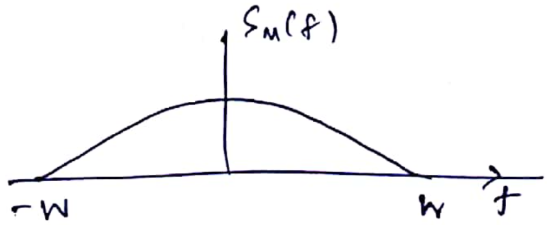
$$\therefore v(t) = \frac{2\pi k_f\, m(t)}{2\pi k_v} = \frac{k_f}{k_v} \cdot m(t).$$

Pre-emphasis and De-emphasis

It can be shown that the power spectral density of the noise at the output of the FM receiver ~~has~~ is proportional to square of the frequency.

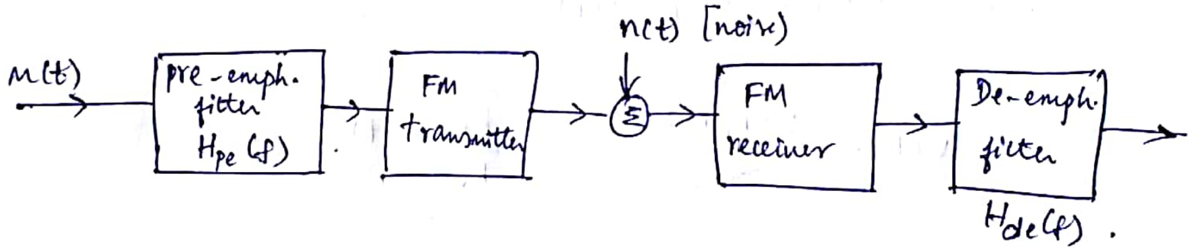


A typical message signal, on the other hand, has predominantly low frequency contents



Therefore, at the output of the receiver, the high freq. components of the message are lost in noise. In order to avoid this, we enhance / amplify the high frequency components of the message signal prior to modulation at the transmitter. This will make the message signal occupy the message bandwidth uniformly.

At the receiver, we perform the inverse operation of deemphasizing the high freq. components. This also improves the SNR at the receiver output by reducing the noise power at high frequencies.



We need

$$H_{de}(f) = \frac{1}{H_{pe}(f)}$$

Superheterodyne receiver

In any receiver, before the actual demodulation can take place, the following tasks need to be performed.

i) Tuning : selecting the desired station /channel.

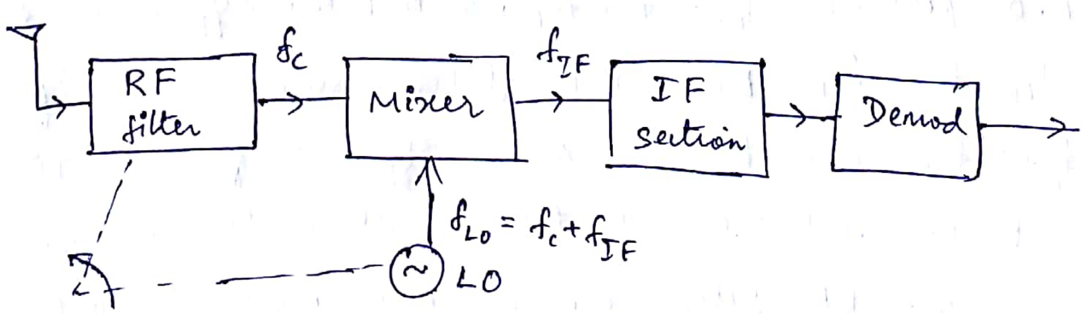
ii) Selectivity : Separating the desired channel from the neighboring channels

iii) Amplification : The received signal may have low power. Hence it may have to be amplified.

Tuning requires a tunable BPF. Selectivity requires a filter with sharp cut-offs. And amplification requires a filter with high gain.

The initial approach was to perform the three tasks using a single tunable filter (Tuned Radio Freq. Receiver). But this approach leads to many practical difficulties (stability issues at high frequencies & high gain).

The Superheterodyne receiver decouples tuning from selectivity & amplification.



* The RF filter is a tuned filter, to select the desired station. The filter need ~~to~~ not have a sharp transition band.

* The Local oscillator is ~~also~~ tuned together with the RF filter.

* It produces a sinusoid of frequency $f_{LO} = f_c + f_{IF}$, where f_{IF} is the "intermediate frequency".

* In the mixer, the input signal spectrum is shifted from f_c to f_{IF}. (The difference frequency copy is retained).

* The IF section has one or more BPF (with f_{IF} as center frequency) that provide ~~selectivity~~ selectivity & amplification. These are fixed (not tuned) filters & have sharp cut offs.

AM: f_c : 535 KHz to 1605 kHz

$f_{IF} = 455$ kHz

FM: $f_c = 88$ MHz to 108 MHz.

$f_{IF} = 10.7$ MHz.

Image frequency

When we tune to a station with carrier frequency f_c Hz, the L.O produces $f_{LO} = f_c + f_{IF}$, so that the difference frequency is $f_{LO} - f_c = f_{IF}$. But the station whose carrier freq. is $f_c + 2f_{IF}$ also has the same difference of f_{IF} with f_{LO}. Hence, it can interfere with the desired station's signal. Therefore, the RF filter, even tho' not required to be sharp, has to block the "image frequency" copy at $f_c + 2f_{IF}$.

Ex: In AM txn, find the image station for the station with $f_c = 1000$ kHz.

Ans: $f_c' = f_c + 2f_{IF} = 1000 + 2 \times 455 = 1910$ kHz.

Choice of f_{LO}.

We have $f_{LO} = f_c + f_{IF}$.

But we can also select $f_{LO} = f_c - f_{IF}$.

Consider $f_{LO} = f_c - f_{IF}$.

For AM, $f_{IF} = 455$ kHz.

$\therefore$ ~~f_{LO}~~ since ~~45~~ $535 \text{ kHz} \le f_c \le 1605 \text{ kHz}$

we have $80 \text{ kHz} \le f_{LO} \le 1150 \text{ kHz}$.

$$\Rightarrow \frac{\text{max. freq}}{\text{min freq}} = \frac{1150}{80} \approx 14.5$$

$\Rightarrow$ A very high tuning range.

Now **consider $f_{LO} = f_c + f_{IF}$.**

we have $990 \text{ kHz} \le f_{LO} \le 2060 \text{ kHz}$.

$$\frac{\text{max freq}}{\text{min freq}} = \frac{2060}{990} \approx 2.1$$

$\Rightarrow$ Much smaller tuning range.

$\therefore$ we prefer $f_{LO} = f_c + f_{IF}$