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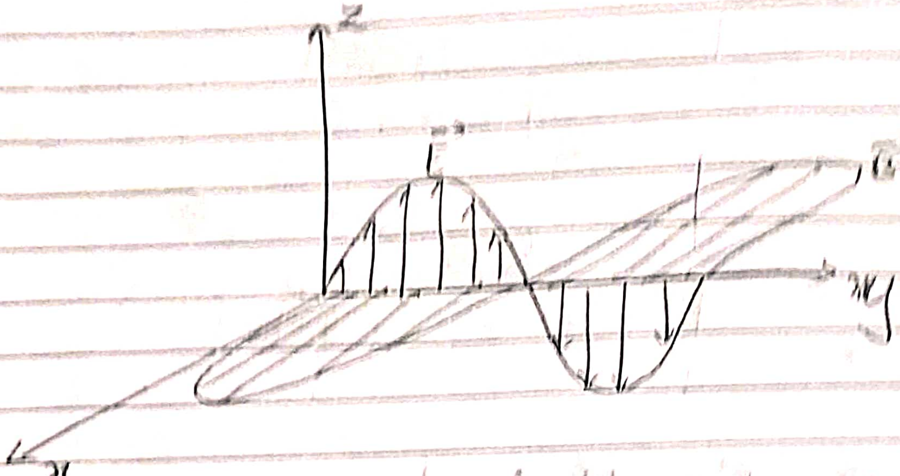
Unit 3

MAXWELL'S EQUATIONS FOR TIME VARYING FIELD



In electrostatics we have $\vec{E}$ as a func^n of (x,y,z) &
In magnetostatics we have $\vec{H}$ as a func^n of (x,y,z).

Time varying fields produces $\vec{E}$ as a func^n of (x,y,z,t) & $\vec{H}$ as a func^n of (x,y,z,t), due to alternating or time varying currents.

★ Faraday's Law

~~The EMF~~

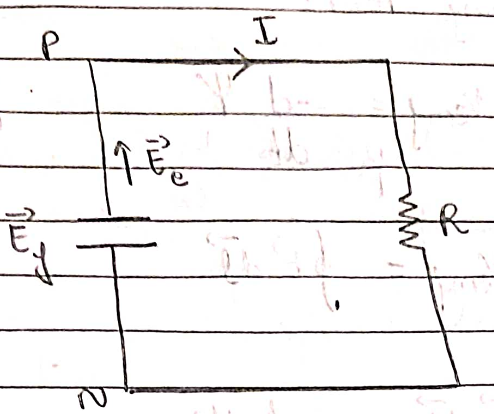
It states that the induced electromotive force V_{emf}, in any closed circuit is equal to rate of change of magnetic flux linkage by the circuit.

$$\boxed{V_{emf} = \frac{-d}{dt}\lambda = -N\frac{d}{dt}\Psi}$$

where

$\lambda = N\Psi =$ flux linkage
& $N =$ no. of turns in the circuit
$\Psi =$ flux through each turn

⇒ -ve sign shows that the induced voltage acts in such a way as to oppose the flux producing it. This behaviour is known as Lenz's law, which emphasises that the direc^n of current flow in the circuit is such that induced magnetic field produced by the induced current will oppose the change in original magnetic field.



$\Rightarrow$ $\vec{E}_e$ is conservative:

$$\oint_L \vec{E}_e \cdot d\vec{l} = 0$$

$\Rightarrow$ But $\vec{E}_f$ is non-conservative:

$$\oint_L \vec{E}_f \cdot d\vec{l} \neq 0$$

The electrochemical action of the battery produces an emf $\vec{E}_f$. Due to the accumulation of charges at the battery terminal an electrostatic field $\vec{E}_e$ $(= -\nabla V)$ also exists.

We have the foll. observations:

(i) An electrostatic field $\vec{E}_e$ can't maintain a steady current a closed circuit since:

$$\oint_L \vec{E}_e \cdot d\vec{l} = 0$$ [$\vec{E}_e$ is conservative]

(ii) An emf produced field $\vec{E}_f$ is non-conservative.

Transformer & Motional Electromotive Force

We know,

$$V_{emf} = \frac{-d}{dt} N\psi$$

assuming $N=1$:

$$V_{emf} = \frac{-d}{dt}\psi \qquad \text{[Faraday's law]}$$

also

$$V_{emf} = \oint \vec{E}\cdot d\vec{l}$$

$$\therefore \oint \vec{E}\cdot d\vec{l} = \frac{-d}{dt}\psi \qquad \Big|\ \text{give } \oint \vec{E}\cdot d\vec{l} \neq 0$$

But w.k.t $\psi = \int_S \vec{B}\cdot d\vec{S}$

$$\Rightarrow \boxed{\oint \vec{E}\cdot d\vec{l} = \frac{-d}{dt}\int_S \vec{B}\cdot d\vec{S}} \quad \text{—— (1)}$$

From the above eq^n (1), the variation of flux with time maybe caused in 3 ways:

(i) by having a stationary loop in a time varying $\vec{B}$ field.

(ii) by having time varying loop area in static $\vec{B}$ field

(iii) by having time varying looop area in time varying $\vec{B}$ field.

* We have to find V_{emf} for all the above 3 cases

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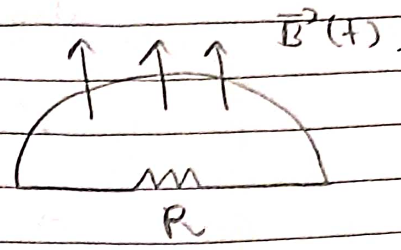
① Stationary loop in a time varying $\vec{B}$ field (Transformer emf)

* Case 1:



$$V_{emf} = \oint_L \vec{E} \cdot \vec{dl}$$

$$\Rightarrow V_{emf} = -\frac{d\lambda}{dt} = -\frac{dN\psi}{dt}$$

$$V_{emf} = -\frac{d\psi}{dt} = -\frac{d}{dt}\int_S \vec{B} \cdot \vec{dS}$$

$$= -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$$

$$\oint_L \vec{E} \cdot dl = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$$

Using Stoke's theorem

$$\Rightarrow \int_S \nabla \times \vec{E} \cdot \vec{dS} = \int_S -\frac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$$

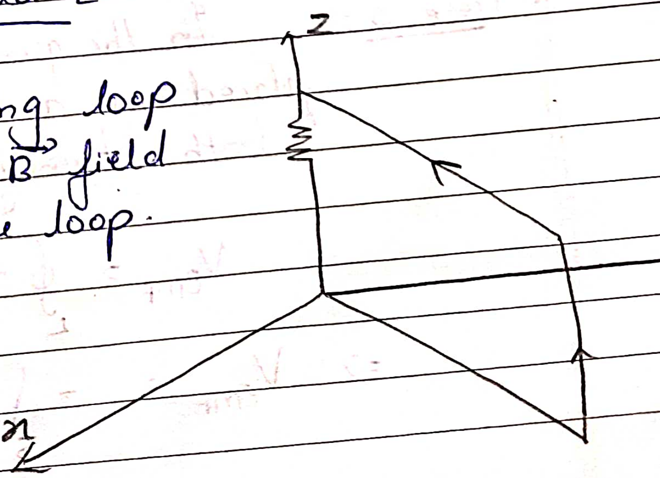
* Note: Here $\oint_L E \cdot dl \neq 0$ cos. it is not static.

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \Rightarrow \text{Maxwell's eq}^n \text{ for time varying field.}$$

~~Case 2~~

② Moving loop in static $\vec{B}$ field [Motional EMF]

* Case 2: When a conducting loop is moving in a static $\vec{B}$ field an EMF is induced in the loop.



$$V_{emf} = \oint \vec{E} \cdot \vec{dl} \rightarrow ①$$

w.k.t, $F_m = q[\vec{u} \times \vec{B}]$

also $E_m = \frac{F_m}{q} = \vec{u} \times \vec{B}$ (E_m: Motional electric field.)

$\therefore E = E_m$

$\Rightarrow V_{emf} = \oint \vec{E}_m \cdot \vec{dl}$

$$\boxed{V_{emf} = \oint (\vec{u} \times \vec{B}) \cdot \vec{dl}}$$

This type of EMF is known as motional EMF or Flux cutting EMF because it is due to motional action. This kind of EMF is found in electrical machines like motors & generators

Consider,

$$\oint E_m \, dl = \oint \vec{u} \times \vec{B} \cdot dl \qquad \text{(Motional EMF)}$$

Applying Stokes theorem:

$$\Rightarrow \int_S \nabla \times \vec{E}_m \, \vec{ds} = \int_S \nabla \times (\vec{u} \times \vec{B}) \, ds$$

$$\Rightarrow \boxed{\nabla \times \vec{E}_m = \nabla \times (\vec{u} \times \vec{B})}$$

③ Moving Loop in time varying Magnetic field

* Case 3: In the general case a moving conducting loop placed in a time varying magnetic field produces both transformer emf & motional emf.

$$V_{emf} = \oint_L \vec{E} \cdot \vec{dl}$$

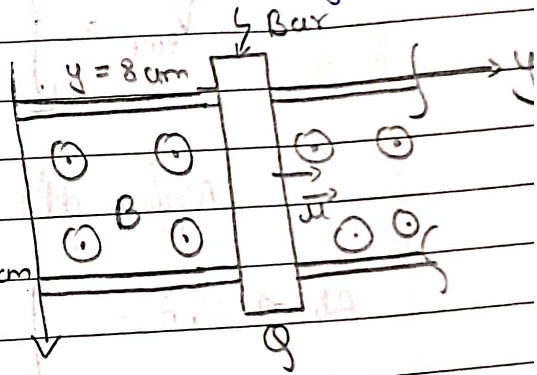
$$\Rightarrow V_{emf} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} + \int_L (\vec{u} \times \vec{B}) \, \vec{dl}$$

In this case:
$$\oint_L \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} + \int_L \vec{u}\times\vec{B}\cdot d\vec{l}$$

$$\Rightarrow \boxed{\int_S \nabla\times\vec{E}\cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} + \int_S \nabla\times\vec{u}\times\vec{B}\cdot d\vec{s}}$$

* **Note:**
① In moving loop placed in static $\vec{B}$ field, the integral of the eq^n $\oint_L \vec{u}\times\vec{B}\cdot dl = 0$ along the portion of the loop where $u=0$. Thus dl is taken along the portion of the loop that is cutting the field where $\vec{u}$ has non-zero value.
② The direc^n of the induced current is same as that of $\vec{u}\times\vec{B}$ (or E_m). The limits of integration in $\oint_L \vec{u}\times\vec{B}\cdot dl$ are selected in the direc^n opp. of induced current.

Q. A conducting bar can slide freely over 2 conducting rails as shown in fig. Calc. the induced voltage in the bar is stationary at $y = 8cm$ & $\vec{B} = 4\cos 10^6 t\,\hat{a}_z$ m Wb/m^2



$\Rightarrow$ It is the case of stationary loop placed in the varying magnetic field.

$$V_{emf} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$V_{emf} = -\int_{x=0}^{0.06}\int_{y=0}^{0.08} \frac{\partial}{\partial t}\left(4\cos 10^6 t\times 10^{-3}\right)\hat{a}_z\,dx\,dy\,\hat{a}_z$$

$$= +\int_{x=0}^{0.06}\int_{y=0}^{0.08} 4\sin 10^6 t\times 10^{-3}\times 10^6\,dx\,dy$$

$$= +4\times 0.06\times 0.08\times \sin 10^6 t\times 10^3$$

$$= +24\times 0.8\times \sin 10^6 t$$

$$\boxed{V_{emf} = +19.2\sin 10^6 t}$$

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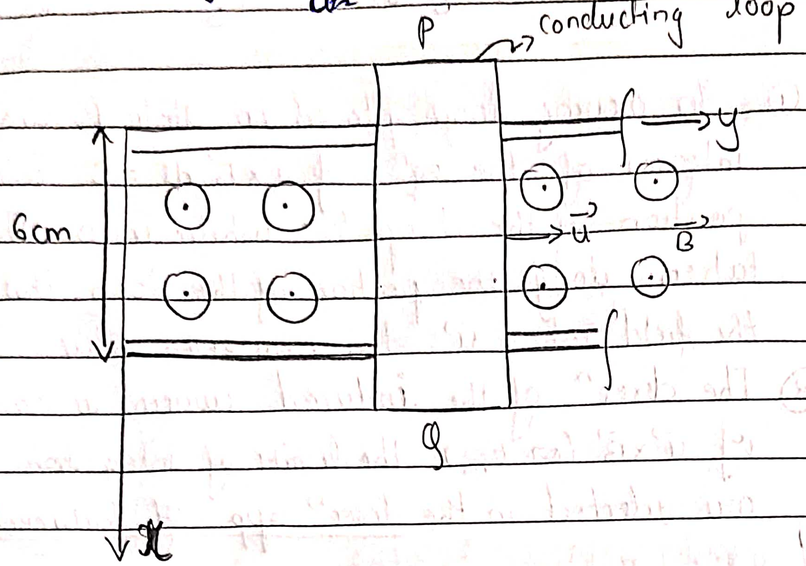
Q. A conducting bar can slide freely over two conducting rails as shown. Calc. the induced voltage in the bar if the bar slides at a velocity $\vec{u} = 20\hat{a}_y$ m/s & $\vec{B} = 4\hat{a}_z$ mWb/m^2.



=> It is a case of moving loop placed in a static field.

$$V_{EMF} = \oint_L \vec{u} \times \vec{B} \cdot \vec{dl}$$

Now, $\vec{dl} = dx\,\hat{a}_x$ (as it is along x-axis)

Also $\vec{u} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 20 & 0 \\ 0 & 0 & 4\times10^{-3} \end{vmatrix}$

$\Rightarrow \vec{u} \times \vec{B} = 80\times10^{-3}\,\hat{a}_x$

$\therefore$ Induced current will be along $+\hat{a}_x$.

=> The limits of the integration will be ~~oppos~~ in the direc^n opposite to $+\hat{a}_x$.

$$\therefore V_{EMF} = \int_{6\times10^{-2}}^{0} 80\times10^{-3}\,\hat{a}_x \cdot dx\,\hat{a}_x$$

$$= -480\times10^{-5}$$

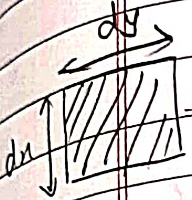
$$= \underline{\underline{-4.8\ mV}}$$

Q. In the above problem if the bar slides at a velocity $\vec{u} = 20\hat{a}_y$ m/s & $\vec{B} = 4\cos(10^6 t - y)\,\hat{a}_z$ mWb/m². Calc. the induced voltage.

=> It is a case of moving loop placed in time varying magnetic field

∴ Emf generated:

$$V_{emf} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot ds + \oint_L \vec{u}\times\vec{B}\,dl \longrightarrow (1)$$



$\rightarrow d\vec{s} = dx\,dy\,\hat{a}_z$

Now,

$$-\int_S \frac{\partial \vec{B}}{\partial t}\cdot ds = -\int_{x=0}^{6\times10^{-2}}\int_{y=0}^{y} \frac{\partial}{\partial t} 4\cos(10^6 t - y)\times10^{-3}\,\hat{a}_z\, dx\,dy\,\hat{a}_z$$

$$= \int_{x=0}^{6\times10^{-2}} \left\{ 4\cos(10^6 t - y)\times10^{+3}\Big|_{y=0}^{y} dx \right\}.$$

$$= \cancel{4x\cos(10^6 t - y)}$$

$$= 4x\cos(10^6 t - y)\times10^3 - 4x\cos(10^6 t)\times10^3 \Big|_0^{6\times10^{-2}}$$

$$= 4\times6\times10^{-2}\times\cos(10^6 t - y)\times10^3 - 24\times10\cos(10^6 t)$$

$$\Rightarrow -\int_S \frac{\partial \vec{B}}{\partial t}\cdot ds = 240\cos(10^6 t - y) - 240\cos(10^6 t) \longrightarrow (2)$$

Now,

$$\oint_L \vec{u}\times\vec{B}\,dl$$

$$\vec{u}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 20 & 0 \\ 0 & 0 & 4\cos(10^6t-y)\,\hat{a}_x\times10^{-3} \end{vmatrix}$$

$$\vec{u}\times\vec{B} = 0.08\cos(10^6t-y)\ \hat{a}_x$$

$$\&\ \vec{dl} = dx\,\hat{a}_x$$

$$\therefore \int_L \vec{u}\times\vec{B}\,dl = \int_{0.06}^{0} 0.08\cos(10^6t-y)\ \hat{a}_x\,dx\,\hat{a}_x$$

$$= 0.08x\cos(10^6t-y)\Big|_{0.06}^{0}$$

$$\Rightarrow \int_L \vec{u}\times\vec{B}\,dl = -4.8\cos(10^6t-y)\times10^{-3} \longrightarrow (3)$$

$$\therefore V_{emf} = Eq^n\ (2) + Eq^n\ (3) \qquad (\text{from } Eq^n\ (1))$$

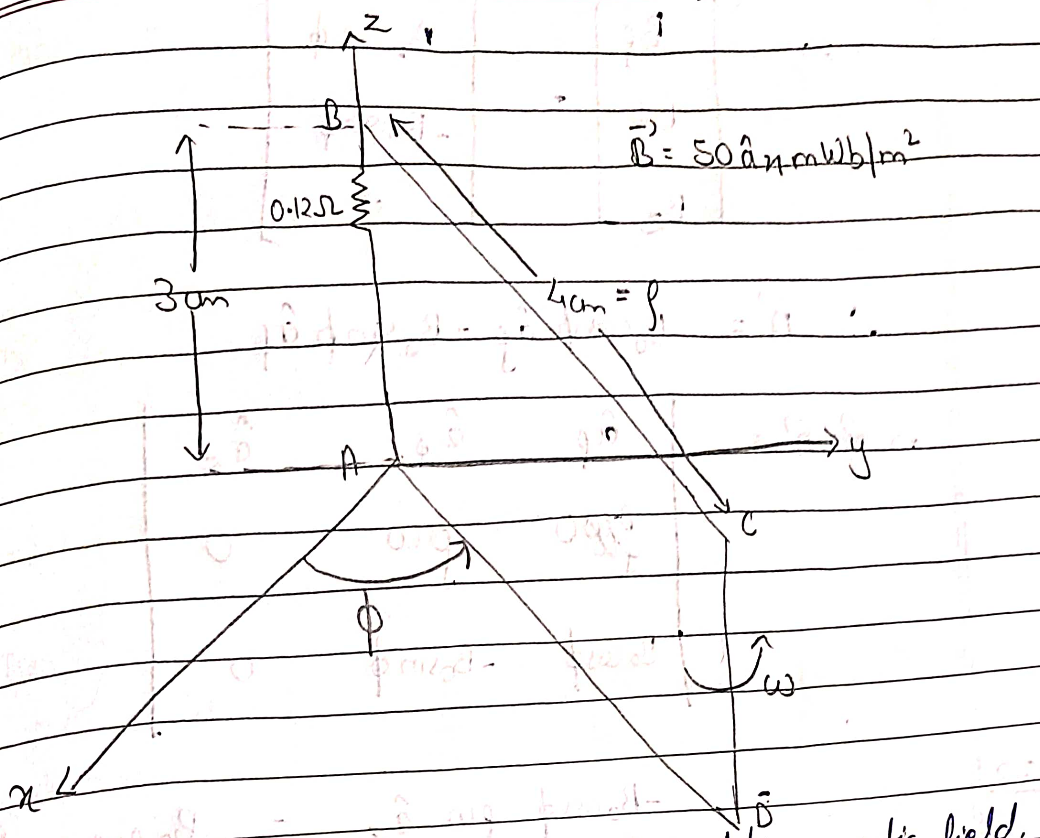
$$\Rightarrow V_{emf} = 240\cos(10^6t-y) - 240\cos(10^6t) - \underbrace{4.8\cos(10^6t-y)}_{\approx 0}\times10^{-3}$$

(negligible compared to rest)

$$\Rightarrow \boxed{V_{emf} = 480\sin\left(10^6t-\frac{y}{2}\right)\sin\frac{y}{2}\ V}$$

Q. The loop shown in fig. is inside a uniform magnetic field $\vec{B} = 50\,\hat{a}_x$ mWb/m². If the side DC of the loop cuts the flux lines at a freq. of 50Hz & the loop is at y-z plane at t = 0, find:
(a) the induced emf at t = 1ms
(b) the induced current at t = 3ms

Consider the foll. diagram:



⇒ It is a case of moving loop placed in static magnetic field.

∴ V_{emf} = motional emf

$$\Rightarrow V_{emf} = \int_L \vec{u} \times \vec{B}\, d\vec{l}$$

where $\vec{u} = \frac{d\vec{l}}{dt} = \rho\frac{d\phi}{dt}\hat{a}_\phi = \rho\omega\hat{a}_\phi$

~~& $d\vec{l} = \rho\omega\hat{a}_\rho$~~

& $d\vec{l} = dz\,\hat{a}_z$

Magnetic flux density $\vec{B}$ is given in rectangular co-ordinates & we have to calculate for cylindrical:

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} \cancel{d\vec{l} = dz\,\hat{a}} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} B_0 \cos\phi \\ -B_0 \sin\phi \\ 0 \end{bmatrix}$$

$$\therefore \vec{B} = B_0 \cos\phi \, \hat{a}_\rho - B_0 \sin\phi \, \hat{a}_\phi$$

$$\Rightarrow \vec{u} \times \vec{B} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & \rho\omega & 0 \\ B_0 \cos\phi & -B_0 \sin\phi & 0 \end{vmatrix}$$

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$$= -B_0 \cos\phi \, \rho\omega \, \hat{a}_z = B_0 \rho\omega \cos\phi \, (-\hat{a}_z)$$

$$\therefore V_{emf} = \int_{z=0}^{0.03} -B_0 \rho\omega \cos\phi \, \hat{a}_z \, dz \, \hat{a}_z$$

$$= -B_0 \rho\omega z \cos\phi \Big|_0^{0.03} \qquad \omega = 2\pi f$$

$$= -50 \times 10^{-3} \hat{a}_z \times 0.04 \times 2 \times \pi \times 50 \times [\cos\phi] \times (0.03)$$

$$\boxed{V_{emf} = -6\pi \cos\phi \, V}$$

But $\omega = \dfrac{d\phi}{dt}$

$\Rightarrow \omega dt = d\phi$

integrating on both sides:

$\Rightarrow \int d\phi = \omega \int dt$

$\Rightarrow \phi = \omega t + C_0$

at $t=0$

$$\phi = \omega(0) + C$$

$\Rightarrow$ $C = \frac{\pi}{2}$ (as at $t=0$ ϕ is along y-axis)

$\therefore$ $\phi = \omega t + \frac{\pi}{2}$

$\Rightarrow$ $V_{emf} = -6\pi \cos(\omega t + \frac{\pi}{2})$

(a)

$\Rightarrow$ $V_{emf}\Big|_{t = 1\times10^{-3}s} = -6\pi \cos\left(2\pi f \times 10^{-3} + \frac{\pi}{2}\right)$

$= 0.5825\,V$

(b) $I = \frac{V_{emf}}{R}\Big|_{t = 3\times10^{-3}s} = \frac{-6\pi \cos\left(2\pi f \times 3\times10^{-3} + \frac{\pi}{2}\right)}{0.01}$

$= 0.1520$

* Displacement Current

For static EM field:

$$\nabla \times \vec{H} = \vec{J}$$

and divergence of curl of any vector $= 0$

But, $\nabla\cdot(\nabla \times \vec{H}) = \nabla\cdot\vec{J} \neq 0$ —— (ii)

i.e, $\nabla\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$ (from continuity eq^n)

$\therefore$ $\nabla\cdot(\nabla\times\vec{H}) = \nabla\cdot\vec{J} + \nabla\cdot\vec{J}_d$

$\vec{J}_d$ = displacement current density

$\Rightarrow$ $0 = \nabla\cdot\vec{J} + \nabla\cdot\vec{J}_d$ (as divergence of curl of a vector $= 0$)

$$\Rightarrow \quad -\nabla\cdot\vec{J} = \nabla\cdot\vec{J}_d$$

$$\Rightarrow \quad -\left[-\frac{\partial \rho_v}{\partial t}\right] = \nabla\cdot\vec{J}_d$$

$$\Rightarrow \quad \frac{\partial}{\partial t}\rho_v = \nabla\cdot\vec{J}_d$$

$$\Rightarrow \quad \frac{\partial}{\partial t}(\nabla\cdot\vec{D}) = \nabla\cdot\vec{J}_d \qquad \Big|\ \rho_v = \nabla\cdot\vec{D}$$

$$\Rightarrow \quad \nabla\cdot\frac{\partial \vec{D}}{\partial t} = \nabla\cdot\vec{J}_d$$

$$\Rightarrow \quad \boxed{\vec{J}_d = \frac{\partial \vec{D}}{\partial t}} \longrightarrow \text{Displacement Current density.}$$

$$\Rightarrow \quad \boxed{(\nabla\times\vec{H}) = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \quad \text{(Maxwell's eq}^n\text{.)}$$

where ① $\vec{J}$ = conduc^n current density

$$\Rightarrow \quad \vec{J} = \sigma\vec{E}$$

② $\vec{J}_d$ = displacement current density

where displacement current is:

$$I_{disp.} = \int_S \vec{J}_d \cdot ds \qquad (= I_d)$$

$$\therefore \quad \boxed{I_d = \int_S \frac{\partial \vec{D}}{\partial t}\cdot ds} \longrightarrow \text{Displacement Current.}$$

This type of current exists when AC voltage is applied to the plates of the capacitor.

★ Maxwell's eq^n in Final form (Time varying field)

Sr.No.	Differential form	Integral form	Remarks
1.	$\nabla . \vec{D} = \rho_v$	$\oint_S \vec{D}.\vec{ds} = \int_V \rho_v dv$	Gauss's law
2.	$\nabla . \vec{B} = 0$	$\oint_S \vec{B}.\vec{ds} = 0$	Non-existence of isolated magnetic charge
3.	$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$	$\oint_L \vec{E}.\vec{dl} = -\int_S \frac{\partial \vec{B}}{\partial t} \vec{ds}$	Faraday's law
4.	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\int_L \vec{H}.\vec{dl} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \vec{ds}$	Ampere's law

⇒ For any field to qualify as em field, should satisfy all the 4 maxwell's eq^n.

★ Time Harmonic fields

A time harmonic field is one that varies periodically or sinusoidally with time. A sinusoidal variation is selected because it can be expressed in phasors.

The concept of phasors is as follows:

A phasor is a complex no. that contains the amplitude & phase of sinusoidal variations.

Consider a complex no.:

$$z = x + iy = r\angle\phi$$

$r = \sqrt{x^2+y^2}$

$\angle\phi = \tan^{-1}\left(\frac{y}{x}\right)$

① Now consider 2 complex nos.:

$$z_1 = x_1 + iy_1 \quad \& \quad z_2 = x_2 + iy_2$$
$$= r_1 \angle \phi_1 \qquad\qquad = r_2 \angle \phi_2$$

$$\Rightarrow \boxed{z_1 z_2 = r_1 r_2 \angle(\phi_1 + \phi_2)}$$

and $$\boxed{\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2}$$

② if $z = x + iy = r\angle\phi$

$$\Rightarrow \boxed{\sqrt{z} = \sqrt{r} \angle \phi/2}$$

③ if $z = x + iy$

$$\Rightarrow \boxed{z^* = x - iy} \longrightarrow (\text{Conjugate of } z)$$

In general, a phasor is a complex quantity & it could be a scalar or vector.

Consider, $\vec{A} = A_0 \cos(\omega t - \beta x)\, \hat{a}_y$

$$\Rightarrow \vec{A} = \mathrm{Re}\left[A_0 e^{j(\omega t - \beta x)}\, \hat{a}_y\right]$$
$$= \mathrm{Re}\left[A_0 e^{-j\beta x}\, \hat{a}_y \cdot e^{j\omega t}\right]$$
$$\Rightarrow \vec{A} = \mathrm{Re}\left[\vec{A}_s \cdot e^{j\omega t}\right] \quad \text{— ①}$$

where, $\boxed{\vec{A}_s = A_0 e^{-j\beta x}} \Longrightarrow$ Phasor

Now, differentiating eqⁿ ① w.r.t.

$$\frac{d\vec{A}}{dt} = \frac{\partial}{\partial t}\left[\mathrm{Re}\left[\vec{A}_s \cdot e^{j\omega t}\right]\right]$$

$$= Re\left[\vec{A_s}\frac{d}{dt}e^{j\omega t}\right]$$

$$\Rightarrow \frac{\partial \vec{A}}{\partial t} = Re\left[j\omega \vec{A_s} e^{j\omega t}\right] \quad \text{—— (2)}$$

$$\boxed{\begin{array}{l} \therefore \dfrac{\partial \vec{A}}{\partial t} \text{ can be obtained by:} \\ \quad \vec{A_s} \rightarrow j\omega \vec{A_s} \end{array}}$$

Now,

$$\vec{A} = Re\left[\vec{A_s} e^{j\omega t}\right]$$

$$\Rightarrow \int \vec{A}.dt = \int Re\left[\vec{A_s} e^{j\omega t}\right] dt$$

$$= Re\left[\vec{A_s} \int e^{j\omega t} dt\right]$$

$$= Re\left[\vec{A_s} \frac{e^{j\omega t}}{j\omega}\right]$$

$$\Rightarrow \int \vec{A}.dt = Re\left[\frac{\vec{A_s}}{j\omega} e^{j\omega t}\right] \quad \text{—— (3)}$$

$$\therefore \boxed{\begin{array}{l} \int \vec{A}.dt \text{ can be obtained by:} \\ \quad \vec{A_s} \rightarrow \dfrac{\vec{A_s}}{j\omega} \end{array}}$$

★ Maxwell's eq^n in Phasor form

Consider maxwell's eq^n:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{—— (1)}$$

And

$$\vec{E}(x,y,z;t) = Re\left[\vec{E_s}.e^{j\omega t}\right] \quad (\vec{E} \text{ in phasor form})$$

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$|||^{rly}$ $\vec{B}(x,y,z,t) = Re[\vec{B_s}e^{j\omega t}]$ ($\vec{B}$ in phasor form)

Consider,

$$E(x,y,z,t) = Re\left[\vec{E_s}e^{j\omega t}\right]$$

applying curl operator on both the sides:

$$\Rightarrow \nabla\times\vec{E} = \nabla\times\left[Re[\vec{E_s}e^{j\omega t}]\right]$$

$$\Rightarrow \nabla\times\vec{E} = Re\left[(\nabla\times\vec{E_s})e^{j\omega t}\right] \longrightarrow (2)$$

$\nabla\times\vec{E_s}$ will only have $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ & no $\frac{\partial}{\partial t} \Rightarrow e^{j\omega t}$ will act like a const.

Now, ~~$\Rightarrow \nabla\times\vec{E}$~~

$$\vec{B}(x,y,z,t) = Re\left[\vec{B_s}e^{j\omega t}\right]$$

differentiating w.r.t t:

$$\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}\left[Re[\vec{B_s}e^{j\omega t}]\right]$$

$$= Re\left[\vec{B_s}\frac{\partial}{\partial t}e^{j\omega t}\right]$$

$$\Rightarrow \frac{\partial\vec{B}}{\partial t} = Re\left[j\omega\vec{B_s}e^{j\omega t}\right] \longrightarrow (3)$$

Substituting eq^n (2) & (3) in (1):

$$Re\left[(\nabla\times\vec{E_s})e^{j\omega t}\right] = -Re\left[j\omega\vec{B_s}e^{j\omega t}\right]$$

Comparing both the sides:

$$\Rightarrow \boxed{\nabla\times\vec{E_s} = -j\omega\vec{B_s}}$$

w.k.t

$$\nabla \times \vec{H} = J + \frac{\partial}{\partial t}\vec{D}$$ (Maxwell's eq^n)

But,

$$\vec{H} = Re\left[\vec{H_s}\, e^{j\omega t}\right]$$
$$\vec{J} = Re\left[\vec{J_s}\, e^{j\omega t}\right]$$
$$\vec{D} = Re\left[\vec{D_s}\, e^{j\omega t}\right]$$

Phasor forms

Now consider,

$$\vec{H} = Re\left[\vec{H_s}\, e^{j\omega t}\right]$$

applying curl operator on both sides:

$$\nabla \times \vec{H} = \nabla \times Re\left[\vec{H_s}\, e^{j\omega t}\right]$$

$$\Rightarrow \nabla \times \vec{H} = Re\left[(\nabla \times \vec{H_s})\, e^{j\omega t}\right] \longrightarrow (6)$$

Now, $\vec{D} = Re\left[\vec{D_s}\, e^{j\omega t}\right]$

differentiating on both sides w.r.t t:

$$\Rightarrow \frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t} Re\left[\vec{D_s}\, e^{j\omega t}\right]$$

$$\Rightarrow \frac{\partial \vec{D}}{\partial t} = Re\left[j\omega \vec{D_s}\, e^{j\omega t}\right] \longrightarrow (7)$$

w.k.t

$$\nabla \times \vec{H} = J + \frac{\partial}{\partial t}\vec{D}$$

$$\Rightarrow Re\left[(\nabla \times \vec{H_s})\, e^{j\omega t}\right] = Re\left[\vec{J_s}\, e^{j\omega t}\right] + Re\left[j\omega \vec{D_s}\, e^{j\omega t}\right]$$

$$\Rightarrow \boxed{\nabla \times \vec{H_s} = \vec{J_s} + j\omega \vec{D_s}}$$

|||^{rly} all maxwell's eq^n's can be expressed in phasor form.

★ Time Harmonic Maxwell's eq^n assuming time factor $e^{j\omega t}$

Sr. No.	Point form :	Integral form
1.	$\nabla \cdot \vec{D_s} = \rho_{vs}$	$\oint_S \vec{D_s} \cdot d\vec{s} = \int_V \rho_{vs} dV$
2.	$\nabla \cdot \vec{B_s} = 0$	$\oint_S \vec{B_s} \cdot d\vec{s} = 0$
3.	$\nabla \times \vec{E_s} = -j\omega \vec{B_s}$	$\oint_L \vec{E_s} \cdot d\vec{l} = j\omega \int_S \vec{B_s} \cdot d\vec{s}$
4.	$\nabla \times \vec{H_s} = \vec{J_s} + j\omega \vec{D_s}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_S (\vec{J_s} + j\omega \vec{D_s}) \cdot d\vec{s}$

Q. The electric field & the magnetic field in free space are given by
$\vec{E} = \frac{50}{\rho} \cos(10^6 t + \beta z)\, \hat{a}_\phi$ V/m
$\vec{H} = \frac{H_0}{\rho} \cos(10^6 t + \beta z)\, \hat{a}_\rho$ A/m
Express these vectors in phasor form.

$\Rightarrow$ $$\vec{E} = Re\left[\vec{E_s}\, e^{j\omega t}\right]$$

where $$\vec{E_s} = E_0 e^{+j\beta z}\, \hat{a}_\phi$$ | $\vec{E} = E_0 \cos(\omega t - \beta x)\hat{a}_y$

$$\vec{E_s} = \frac{50}{\rho} e^{j\beta z}\, \hat{a}_\phi$$

$$\therefore \vec{E} = Re\left[\frac{50}{\rho} e^{j\beta z} \cdot e^{j10^6 t}\, \hat{a}_\phi\right]$$

$\Rightarrow$ $$\boxed{\vec{E_s} = \frac{50}{\rho} e^{j\beta z}\, \hat{a}_\phi} \longrightarrow \text{Phasor form of } \vec{E}$$

$|||^{rly}$ $\vec{H} = Re[\vec{H_s} \cdot e^{j\omega t}]$

$\Rightarrow \vec{H_s} = Re\left[\frac{H_0}{\rho} e^{j\beta z} e^{j10^6 t} \hat{a}_\rho\right]$

$\Rightarrow \vec{H} = Re\left[\frac{H_0}{\rho} e^{j\beta z} \hat{a}_\rho e^{j10^6 t}\right]$

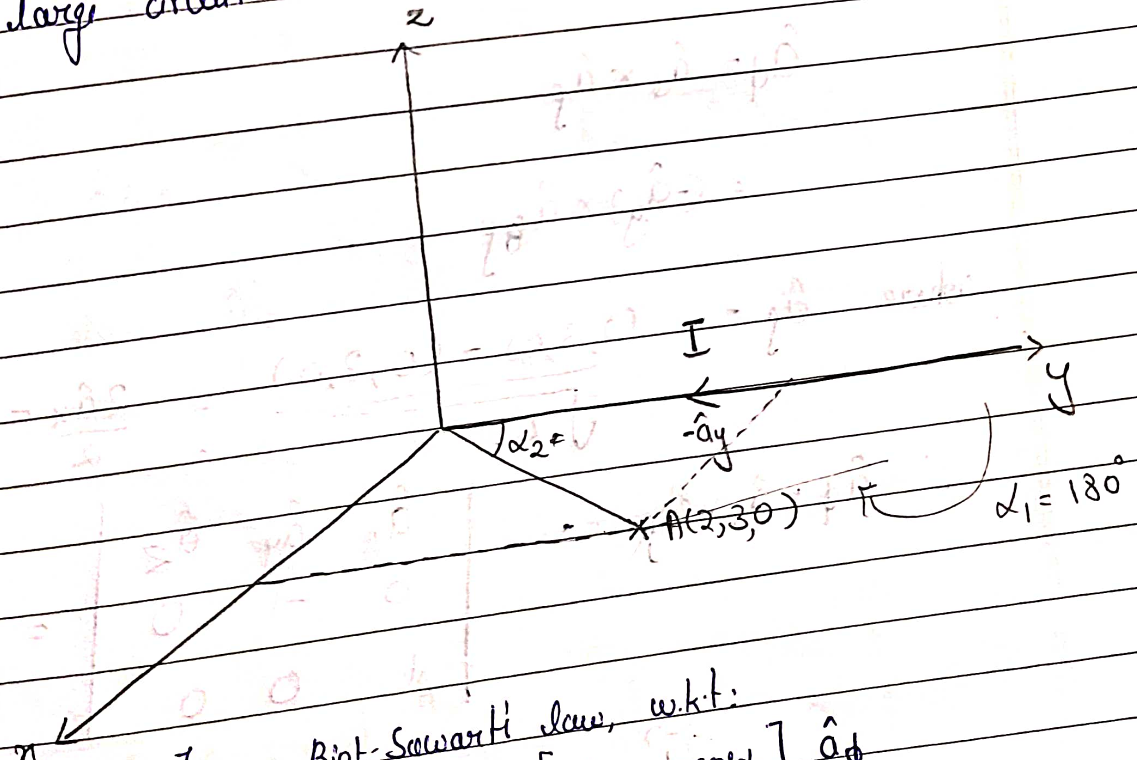
$\therefore \boxed{\vec{H_s} = \frac{H_0}{\rho} e^{j\beta z} \hat{a}_\rho} \rightarrow$ Phasor form of $\vec{H}$

Revision for ISA-2

20/9/22

Q. At +ve y-axis a semi infinite line w.r.t. origin carries a filamentary current of 2A in $-\hat{a}_y$ direcⁿ. Assume it is part of a large circuit. Find $\vec{H}$ at (i) A(2, 3, 0), (ii) B(3, 12, −4)

$\Rightarrow$



From Biot-Savart's law, w.k.t:

$\therefore \vec{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$

Here, $\rho = 2$

$\alpha_1 = 180°$

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23/09/22 .... Unit-3

Q. Electric field & magnetic field in free space are given as:

$$\vec{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\,\hat{a}_\phi \ V/m$$

$$\vec{H} = \frac{H_0}{\rho}\cos(10^6 t + \beta z)\,\hat{a}_\rho \ A/m$$

Express these in phasor form & determine the constant H_0 & β such the field satisfies maxwell's eq^n.

$$\Rightarrow \quad \vec{E} = Re\left[\vec{E_s}.e^{j\omega t}\right]$$

$$= Re\left[\left(\frac{50}{\rho}e^{j\beta z}\hat{a}_\phi\right)e^{j\omega t}\right]$$

$$\Rightarrow \boxed{\vec{E_s} = \frac{50}{\rho}e^{j\beta z}\hat{a}_\phi} \longrightarrow (1)$$

$$\vec{H} = Re\left[\vec{H_s}.e^{j\omega t}\right]$$

$$= Re\left[\left(\frac{H_0}{\rho}e^{j\beta z}\hat{a}_\rho\right)e^{j\omega t}\right]$$

$$\Rightarrow \boxed{\vec{H_s} = \frac{H_0}{\rho}e^{j\beta z}\hat{a}_\rho} \longrightarrow (2)$$

Since it satisfies maxwell's eq^n's:

(1) $\nabla.\vec{D_s} = \nabla\varepsilon\vec{E_s} = \rho_{vs} = 0$ (free space)

Now, $\nabla\varepsilon\vec{E_s} = \varepsilon\nabla.\vec{E_s}$

$$= \varepsilon\left[\frac{1}{\rho}\frac{\partial \vec{E_s}}{\partial\phi}\right]$$ (as it has only $\hat{a}_\phi$ component)

$$= \varepsilon.\left[\frac{1}{\rho}\frac{\partial}{\partial\phi}\left(\frac{50}{\rho}e^{j\beta z}\right)\right] = 0$$

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Ans (2) $\nabla \cdot \vec{B_s} = \nabla \cdot \mu_0 \vec{H}_s$

$= \mu_0 \nabla \cdot \vec{H_s}$

$= \mu_0 \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \vec{H_s}) \right]$

$= \mu_0 \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\cancel{\rho} \cdot \frac{H_0}{\cancel{\rho}} e^{j\beta z} \right) \right]$

$= 0$

(3) Consider $\nabla \times \vec{H_s} = \vec{J_s} + j\omega\varepsilon \vec{E_s}$

$= \cancelto{0}{\sigma} \vec{E_s} + j\omega\varepsilon \vec{E_s}$ ⟶ (3)

in free space, $\sigma = 0$, unless otherwise given.

$\Rightarrow \nabla \times \vec{H_s} = j\omega\varepsilon \vec{E_s}$

LHS: $\nabla \times \vec{H_s} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{H_0}{\rho} e^{j\beta z} & 0 & 0 \end{vmatrix}$

$\frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_{s\rho} & \rho H_{s\phi} & H_z \end{vmatrix}$ ⟶ in general

$\Rightarrow \frac{1}{\rho} \left[\frac{\partial}{\partial z} \left(\frac{H_0}{\rho} e^{j\beta z} \right) \hat{a}_\phi \rho - \cancelto{0}{\frac{\partial}{\partial \phi} \left(\frac{H_0}{\rho} e^{j\beta z} \right)} \hat{a}_z \right]$

$\Rightarrow \frac{1}{\rho} \left[j\beta \cancel{\rho} \frac{H_0}{\cancel{\rho}} e^{j\beta z} \right] \hat{a}_\phi$

$\Rightarrow \nabla \times \vec{H_s} = j\beta \frac{H_0}{\rho} e^{j\beta z} \hat{a}_\phi$ ⟶ (4)

RHS: $j\omega\varepsilon \vec{E_s} = j\omega\varepsilon \frac{50}{\rho} e^{j\beta z} \hat{a}_\phi$ ⟶ (5)

$\Rightarrow$ Equating eq^n (4) & (5)

$$\cancel{j}\beta \frac{H_0}{\cancel{\rho}} \cancel{e^{j\beta z}} \cancel{\hat{a}_\phi} = \cancel{j}\omega\varepsilon \frac{50}{\cancel{\rho}} \cancel{e^{j\beta z}} \cancel{\hat{a}_\phi}$$

$$\Rightarrow \quad \beta H_0 = 50\omega\varepsilon \longrightarrow (6)$$

$$\nabla \times \vec{E_s} = -j\omega\vec{B_s} = -j\omega\mu_0\vec{H_s}$$

$$\nabla \times \vec{E_s} = -j\omega\mu_0\vec{H_s} \longrightarrow (7)$$

LHS:
$$\nabla \times \vec{E_s} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ E_{s\rho} & E_{s\phi} & E_{sz} \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & \frac{50}{\rho} e^{j\beta z} & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[-\frac{\partial}{\partial z}\left(\cancel{\rho}\frac{50}{\cancel{\rho}} e^{j\beta z}\right)\hat{a}_\rho + \cancel{\frac{\partial}{\partial\rho}\left(\rho\frac{50}{\rho} e^{j\beta z}\right)\hat{a}_z}^{\,0}\right]$$

$$\nabla \times \vec{E_s} = \frac{1}{\rho}\left[-j\beta 50 e^{j\beta z}\,\hat{a}_\rho\right] \longrightarrow (8)$$

also

RHS: $$-j\omega\mu_0\vec{H_s} = -j\omega\mu_0 \frac{H_0}{\rho} e^{j\beta z}\,\hat{a}_\rho \longrightarrow (9)$$

Equating eq^n (8) & (9) :

$$\cancel{+}\frac{50}{\cancel{\rho}}\cancel{j}\beta\, \cancel{e^{j\beta z}}\,\cancel{\hat{a}_\phi} = \cancel{+j}\omega\mu_0\frac{H_0}{\cancel{\rho}}\cancel{e^{j\beta z}}\,\cancel{\hat{a}_\phi}$$

$\Rightarrow \quad 50\beta = \mu_0 H_0 \omega \longrightarrow$ (10)

and $\quad \beta H_0 = 50\omega\varepsilon_0 \longrightarrow$ (11)

Now, Dividing eq^n (10) & (11) :

$$\Rightarrow \frac{50\cancel{\beta}}{\cancel{\beta}H_0} = \frac{\mu_0 H_0 \cancel{\omega}}{50\cancel{\omega}\varepsilon_0}$$

$$\Rightarrow \frac{50^2 \times \varepsilon_0}{\mu_0} = H_0^2$$

$\varepsilon_0 = \frac{10^{-9}}{36\pi}$

$\mu_0 = 4\pi \times 10^{-7}$

$$\Rightarrow H_0^2 = 50\times 50 \times \frac{10^{-9}}{36\pi} \times \frac{1}{4\pi\times 10^{-7}}$$

$$\Rightarrow \boxed{H_0 = \pm 0.1326 \ A/m}$$

Now, $\quad 50\beta = \mu_0 H_0 \omega$

$$\Rightarrow \beta = \frac{4\pi\times 10^{-7} \times (\pm 0.1326) \times 10^{6}}{50}$$

$$\Rightarrow \beta = \pm 0.033 \times 10^{-1}$$

$$\Rightarrow \boxed{\beta = \pm 3.333 \times 10^{-3}}$$

Q. In a medium characterised by $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$ & $\vec{E} = 20\sin(10^8 t - \beta z)\,\hat{a}_y$ V/m. Calc. β & $\vec{H}$.

$\Rightarrow$ ~~$\vec{E} = \mathrm{Re}\{\vec{E}_s\, e^{j\omega t}\}$~~

~~$\Rightarrow \vec{E}_s =$~~

$\Rightarrow \vec{E} = \mathrm{Im}\left[\vec{E}_s\, e^{j\omega t}\right]$

$\Rightarrow \vec{E} = \mathrm{Im}\left[20 e^{-j\beta z}\,\hat{a}_y\, e^{j\omega t}\right]$

$\Rightarrow \vec{E}_s = 20 e^{-j\beta z}\,\hat{a}_y$

(1) Consider the maxwell's eq^n $\nabla \times \vec{E}_s = -j\omega \vec{B}_s$

$\Rightarrow \nabla \times \vec{E}_s = -j\omega \mu_0 H_s \longrightarrow (1)$

where

LHS: $\nabla \times \vec{E}_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 20 e^{-j\beta z} & 0 \end{vmatrix}$

$= -\frac{\partial}{\partial z}\left(20 e^{-j\beta z}\right)\hat{a}_x + \cancel{\frac{\partial}{\partial x}\left(20 e^{-j\beta z}\right)}\hat{a}_z$ [0]

$\Rightarrow \nabla \times \vec{E}_s = j\beta\, 20 e^{-j\beta z}\,\hat{a}_x \longrightarrow (2)$

~~RHS~~ ~~Eq~~ Equating this with RHS:

$\Rightarrow 20 e^{-j\beta z}\,\cancel{j}\beta\,\hat{a}_x = -\cancel{j}\omega \mu_0 H_s \longleftarrow (3)$

$\Rightarrow \vec{H}_s = -\frac{20\beta\, e^{-j\beta z}}{\mu_0 \omega}\,\hat{a}_x$

(2) Consider the maxwell's eq^n $\nabla.\vec{D_s} = \cancel{\rho_{vs}}^{0}$

$\nabla.\vec{D_s} = \nabla.\varepsilon\vec{E_s}$

$= \varepsilon\nabla\vec{E_s}$

as $\sigma = 0$
$\Rightarrow$ conductance $= 0$
hence
$\rho_{vs} = 0$

* T.P $\varepsilon\nabla\vec{E_s} = 0$:

$\varepsilon\nabla\vec{E_s} = \varepsilon\frac{\partial}{\partial y}(20e^{-j\beta z}\hat{a}_y)$

$= 0$

only $\hat{a}_y$ compont exists.

(3) Consider the maxwells' eq^n $\nabla.\vec{B_s} = 0$

$\Rightarrow \nabla.\mu_0\vec{H_s} = 0$

$\Rightarrow \mu_0\nabla\vec{H_s} = 0$

$\Rightarrow \nabla\vec{H_s} = 0$

T.P $\nabla\vec{H_s} = 0$:

$\cancel{\frac{\partial}{\partial x}\left(\frac{20\beta e^{-j\beta z}}{\mu_0\omega}\right)}^{0}$

$= 0$

only $\hat{a}_x$ component exists.

(4) Consider the maxwells eq^n $\nabla\times\vec{H_s} = \vec{J_s} + j\omega\vec{D_s}$

$(\sigma = 0)$

$\Rightarrow \nabla\times\vec{H_s} = \cancel{\vec{J_s}}^{0} + j\omega\varepsilon_0\vec{E_s} \longrightarrow (4)$

LHS:

where, $\nabla\times\vec{H_s} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{-20\beta e^{-j\beta z}}{\mu_0\omega} & 0 & 0 \end{vmatrix}$

$$-\left(-\frac{\partial}{\partial z}\left(\frac{-20}{\mu_0\omega}\beta e^{-j\beta z}\right)\hat{a}_y\right)-\left(\cancel{\frac{\partial}{\partial y}\left(\frac{-20\beta e^{-}}{}\right)}\right)$$

$$\Rightarrow \frac{20j\beta^2 e^{-j\beta z}}{\mu_0\omega}\hat{a}_y$$

$$\Rightarrow \nabla\times\vec{H_s} = \frac{20}{\mu_0\omega}\beta^2 j\, e^{-j\beta z}\,\hat{a}_y \longrightarrow (5)$$

RHS: $j\omega\varepsilon_0\vec{E_s} = j\omega\varepsilon_0\, 20\, e^{-j\beta z}\,\hat{a}_y \longrightarrow (6)$

Equating eq^n (5) & (6)

$$\Rightarrow \frac{\cancel{20}}{\mu_0\omega}\beta^2\cancel{j}\cancel{e^{-j\beta z}}\cancel{\hat{a}_y} = \cancel{j}\omega\varepsilon_0\cancel{20}\cancel{e^{-j\beta z}}\cancel{\hat{a}_y}$$

$$\Rightarrow \frac{\beta^2}{\mu_0\omega} = \omega\varepsilon_0$$

$$\Rightarrow \beta^2 = \mu_0\varepsilon_0\omega^2$$

$$\Rightarrow \beta = \pm\sqrt{\omega^2\mu_0\varepsilon_0}$$

$$\Rightarrow \beta = \pm\sqrt{\omega^2\times 4\cancel{\pi}\times 10^{-7}\times\frac{10^{-9}}{\cancel{36}\cancel{\pi}}}$$

$\omega = 10^8$

$\varepsilon_0 = \frac{10^{-9}}{36\pi}$

$\mu_0 = 4\pi\times 10^{-7}$

$$\Rightarrow \beta = \pm\frac{10^8\times 10^{-8}}{3}$$

$$\Rightarrow \boxed{\beta = \pm\frac{1}{3}}$$

And we got:

$$\vec{H_s} = \frac{-20\beta e^{-j\beta z}}{\mu_0\omega}\hat{a}_x$$

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$$\Rightarrow \vec{H_s} = \pm \frac{20 \times \frac{1}{3} \times e^{-j\beta z}}{10^8 \times 4\pi \times 10^{-7}} \hat{a}_x = \pm \frac{1}{6\pi} e^{-j\beta z} \hat{a}_x$$

w.k.t.

$$\vec{H} = I_m \left[\vec{H_s} e^{j\omega t} \right]$$

$$\Rightarrow \vec{H} = I_m \left[\pm \frac{1}{6\pi} e^{-j\beta z} e^{j\omega t} \right]$$

$$\Rightarrow \boxed{\vec{H} = \pm \frac{1}{6\pi} \sin\left(10^8 t \pm \frac{1}{3}\beta z\right) \hat{a}_x \; A/m}$$

Q A charged particle of mass 2kg & charge ~~an~~ 3 Coulumbs, starts at point (1, -2, 0) with velocity $(4\hat{a}_x + 3\hat{a}_z)$ m/s, in an electric field $(12\hat{a}_x + 10\hat{a}_y)$ V/m. At $t = 1s$, determine:

(a) the acceleration of the particle.

(b) its velocity

(c) its kinetic energy

(d) its position

$$\Rightarrow \vec{F} = m\vec{a} = 2\vec{E} = 3\left[12\hat{a}_x + 10\hat{a}_y\right]$$

$$m\vec{a} = 36\hat{a}_x + 30\hat{a}_y$$

$$\vec{a} = \frac{36}{2}\hat{a}_x + \frac{30}{2}\hat{a}_y = 18\hat{a}_x + 15\hat{a}_y \; m/s^2 \longrightarrow (1)$$

But w.k.t:

$$\vec{a} = \frac{d\vec{u}}{dt} = \frac{d}{dt}\left[u_x\hat{a}_x + u_y\hat{a}_y + u_z\hat{a}_z\right]$$

$$\Rightarrow \vec{a} = \frac{du_x}{dt}\hat{a}_x + \frac{du_y}{dt}\hat{a}_y + \frac{du_z}{dt}\hat{a}_z \longrightarrow (2)$$

~~= 18~~

Equate corresponding terms of 1 & 2:

$$\frac{du_x}{dt} = 18 \Rightarrow u_x = 18t + A$$

$$\frac{du_y}{dt} = 15 \Rightarrow u_y = 15t + B$$

$$\frac{du_z}{dt} = 0 \Rightarrow u_z = C$$

when, $t=0$:

$$\vec{u} = 4\hat{a}_x + 0\hat{a}_y + 3\hat{a}_z$$

$$\cancel{\Rightarrow u_x = 4 = 18t + A}$$

at $t=0$,

$$u_x = 4 = 18(0) = A$$

$$\Rightarrow \boxed{A=4}$$

$$u_y = 0 = 15(0) + B$$

$$\Rightarrow \boxed{B=0}$$

$$u_z = 3 = C$$

$$\Rightarrow \boxed{C=3}$$

$$\Rightarrow \vec{u} = (18t+4,\ 15t,\ 3)$$

at $t=1$,

$$\vec{u} = (18+4, 15, 3)$$

$$= (22, 15, 3) \underset{(b)}{=} \boxed{22\hat{a}_x + 15\hat{a}_y + 3\hat{a}_z \text{ m/s}}$$

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$$\vec{a} = \frac{d}{dt}\vec{u}$$

$$= \frac{d}{dt}\left[(18t+4, 15t, 3)\right]$$

$$\Rightarrow \boxed{\vec{a} = (18, 15, 0)} \Rightarrow (a) \boxed{\vec{a} = 18\hat{a}_x + 15\hat{a}_y.}$$

$$K.E = \frac{1}{2} m |u|^2$$

at $t = 1$sec:

$$\Rightarrow K.E = \frac{1}{2} \times 2 \times \sqrt{22^2 + 15^2 + 3^2}$$

(c) $\boxed{K.E = 718\,J}$

In order to find position:

$$\vec{u} = \frac{d}{dt}(x, y, z)$$

$$(18t+4, 15t, 3) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right).$$

$$\Rightarrow \frac{dx}{dt} = (18t+4) \Rightarrow x = \frac{18t^2}{2} + 4t + C_1$$

$$\frac{dy}{dt} = 15t \Rightarrow y = \frac{15t^2}{2} + C_2$$

$$\frac{dz}{dt} = 3 \Rightarrow z = 3t + C_3$$

at $t = 0$, $(1, -2, 0)$

$$x = \frac{18t^2}{2} + 4t + C_1$$

at $t=0, \; x=1$

$1 = 0+0+C_1 \Rightarrow \boxed{C_1 = 1}$

at $t=0, \; y=-2$

$-2 = 0 + C_2 \Rightarrow \boxed{C_2 = -2}$

at $t=0, \; z=0$

$\boxed{0 = C_3}$

At $t = 1$ secs

$x = \frac{18(1)}{2} + 4 + 1 = 14$

$y = \frac{15(1)}{2} + (-2) = 5.5$

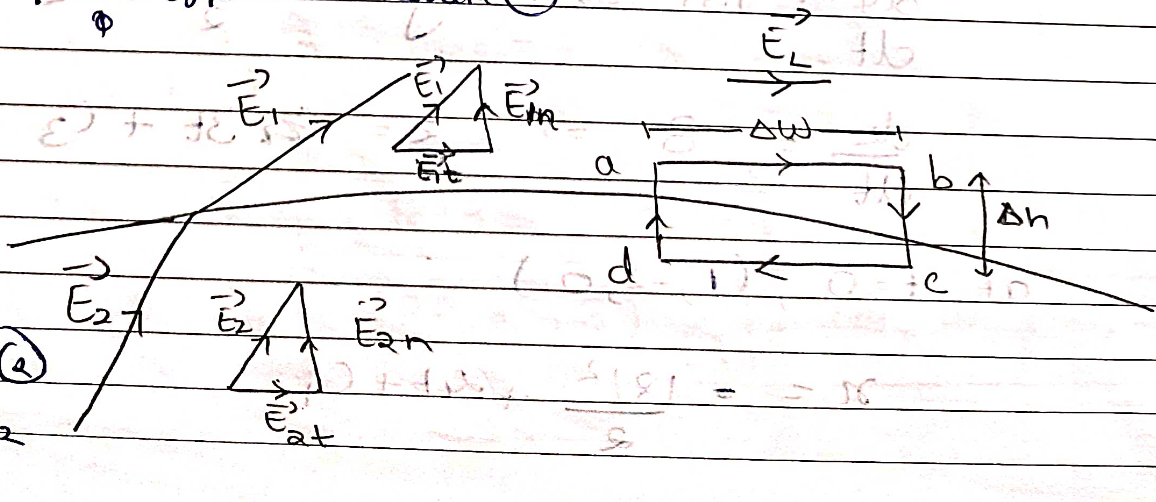
$z = 3(1) + 0 = 3$

$\boxed{\text{(d) position} = (14, 5.5, 3)}$

06/09/22

① Dielectric dielectric Boundary Conditions

1. $\varepsilon_1 = \varepsilon_0 \varepsilon_{r_1} \longrightarrow$ Medium ①



Medium ②
$\varepsilon_2 = \varepsilon_0 \varepsilon_{r_2}$

Apply $\oint_L \vec{E}\cdot d\vec{l} = 0$ to the path abcda

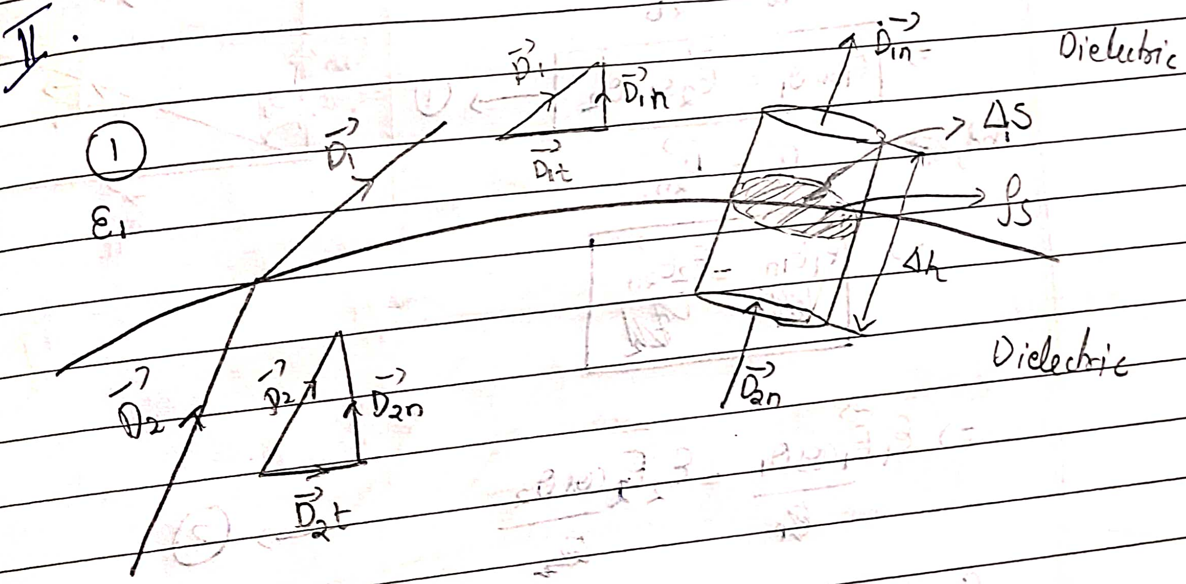
$$\Rightarrow E_{1t}\Delta w - \cancel{E_{1n}\frac{\Delta h}{2}} - \cancel{E_{2n}\frac{\Delta h}{2}} - E_{2t}\Delta w + \cancel{E_{2n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}} = 0$$

$$\Rightarrow E_{1t}\cancel{\Delta w} = E_{2t}\cancel{\Delta w}$$

$$\Rightarrow \vec{E}_{1t} = \vec{E}_{2t}$$

$$\Rightarrow \boxed{\frac{\vec{D}_{1t}}{\varepsilon_1} = \frac{\vec{D}_{2t}}{\varepsilon_2}}$$

II.



Applying Gauss's law:

$$\oint_S \vec{D}\cdot d\vec{s} = Q_{enc.}$$

$$\Rightarrow \vec{D}_{1n}\Delta S - \vec{D}_{2n}\Delta S = \rho_s \Delta S$$

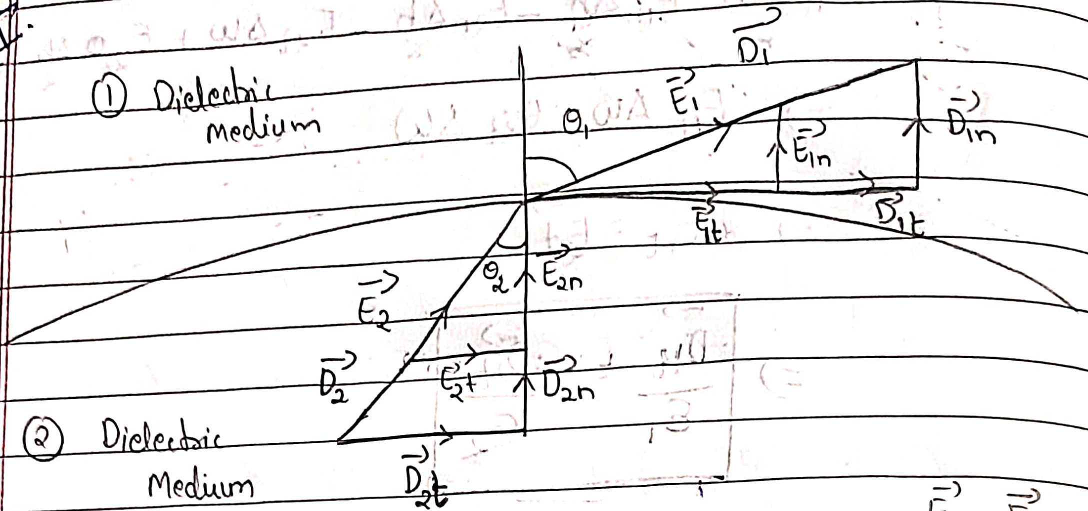
$$\Rightarrow (\vec{D}_{1n} - \vec{D}_{2n})\cancel{\Delta S} = \cancel{\rho_s}^{\,0}\cancel{\Delta S}$$

$$\Rightarrow \boxed{\vec{D}_{1n} = \vec{D}_{2n}}$$

unless you place a charge there is no charge present in the interface $\therefore \rho_s = 0$.

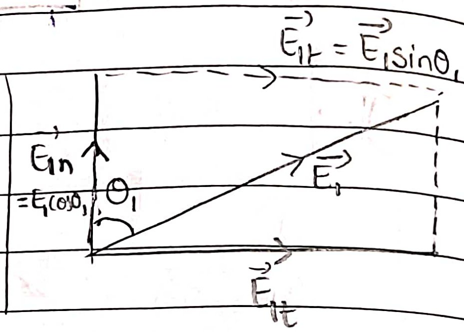
$$\Rightarrow \boxed{\varepsilon_1 \vec{E}_{1n} = \varepsilon_2 \vec{E}_{2n}}$$

III.



$$\vec{E}_{1t} = \vec{E}_{2t}$$

$$\Rightarrow \boxed{\vec{E}_1 \sin\theta_1 = \vec{E}_2 \sin\theta_2} \longrightarrow ①$$



And,

$$\vec{D}_{1n} = \vec{D}_{2n}$$

$$\Rightarrow \boxed{\varepsilon_1 \vec{E}_{1n} = \varepsilon_2 \vec{E}_{2n}}$$

$$\Rightarrow \varepsilon_1 \vec{E}_1 \cos\theta_1 = \varepsilon_2 \vec{E}_2 \cos\theta_2 \longrightarrow ②$$

Dividing eq^n's ①/②:

$$\Rightarrow \frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

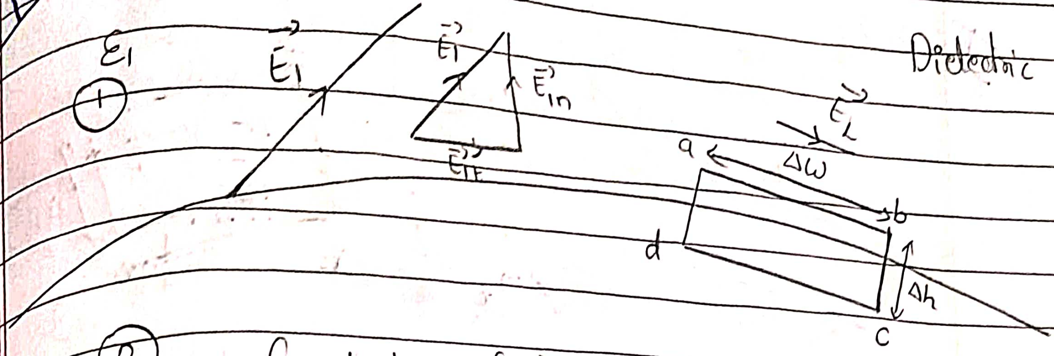
$$\Rightarrow \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}}$$

(It is known as law of refraction of Electric field at the boundary)

② Conductor - Dielectric Boundary Conditions

I.

①



② Conductor ($\vec{E}_2 = 0$)

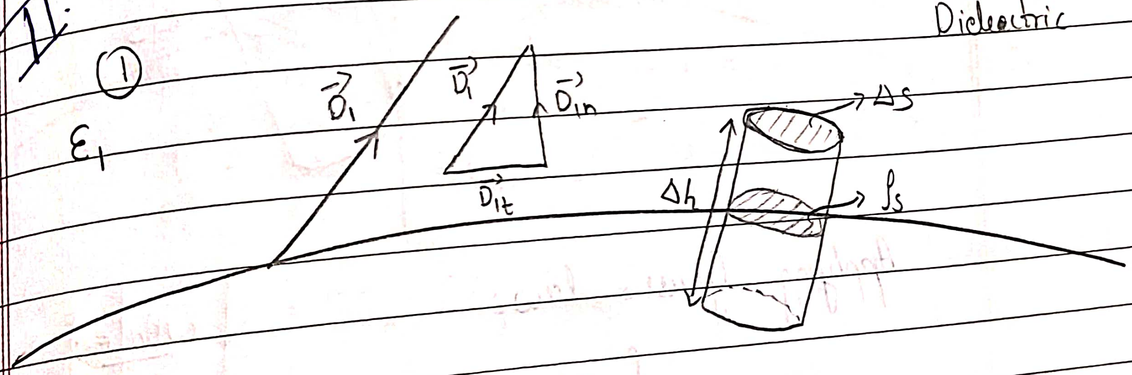
Apply, $\oint_L \vec{E}.\vec{dl} = 0$ to abcda:

$$E_{1t}\Delta w - \cancel{E_{1n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}} = 0$$

$$\Rightarrow \boxed{E_{1t} \cancel{\Delta w} = 0} \Rightarrow \boxed{D_{1t} = 0}$$

II.

①



② Conductor

Applying Gauss's law:

$$\oint_S \vec{D}.\vec{dS} = Q_{enc}$$

$$= \rho_s \Delta S$$

$D_{2n} = 0$, $\vec{E}_2 = 0$

$$\Rightarrow D_{1n}\Delta S - \cancelto{0}{D_{2n}}\Delta S = \rho_s \Delta S$$

$$\Rightarrow D_{1n}\cancel{\Delta S} = \rho_s \cancel{\Delta S}$$

$$\Rightarrow \boxed{D_{1n} = \rho_s} \Rightarrow \boxed{\varepsilon_1 \vec{E}_{1n} = \rho_s}$$

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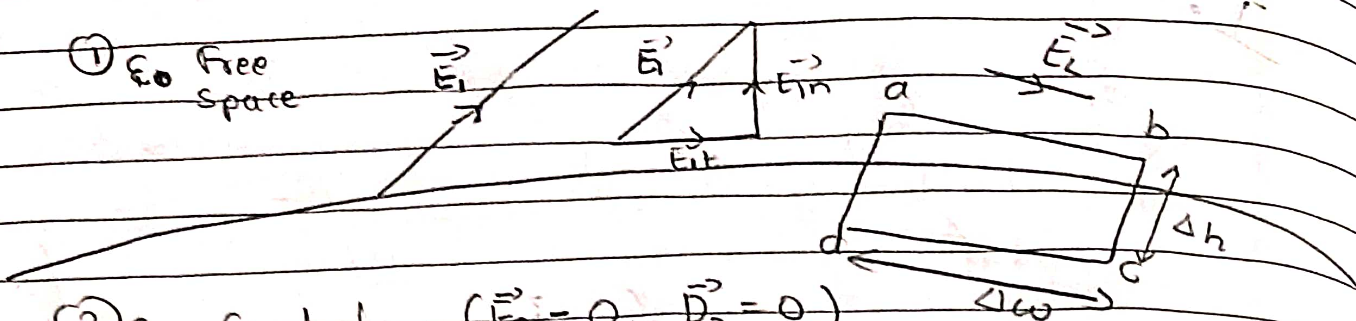
③ Conductor- Free Space Boundary conditions

I.



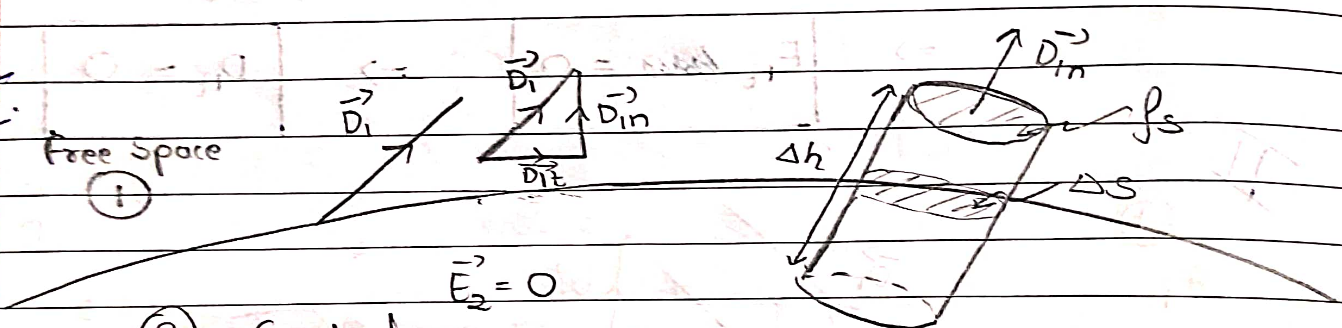
② ε_2 ~~Conductor ($\vec{E_2} = 0$, $\vec{D_2} = 0$)~~

Applying $\oint_L \vec{E} \cdot \vec{dl} = 0$ for abcda

$$\Rightarrow E_{1t}\Delta w - \cancel{E_{1n}\frac{\Delta h}{2}} + \cancel{E_{2n}\frac{\Delta h}{2}} = 0$$

$$\Rightarrow \boxed{E_{1t} = 0} \quad \text{or} \quad \boxed{D_{1t} = 0}$$

II.



② Conductor

Applying Gauss's law:

$$\oint_S \vec{D} \cdot \vec{dS} = Q$$

$$\Rightarrow \vec{D_{1n}}\Delta S - \cancel{\vec{D_{2n}}\Delta S}^{\,0} = \rho_s \Delta S$$

$$\Rightarrow \vec{D_{1n}}\Delta S = \rho_s \Delta S$$

$$\Rightarrow \boxed{\vec{D_{1n}} = \rho_s} \quad \text{or} \quad \boxed{\varepsilon_1 \vec{E_{1n}} = \rho_s}$$

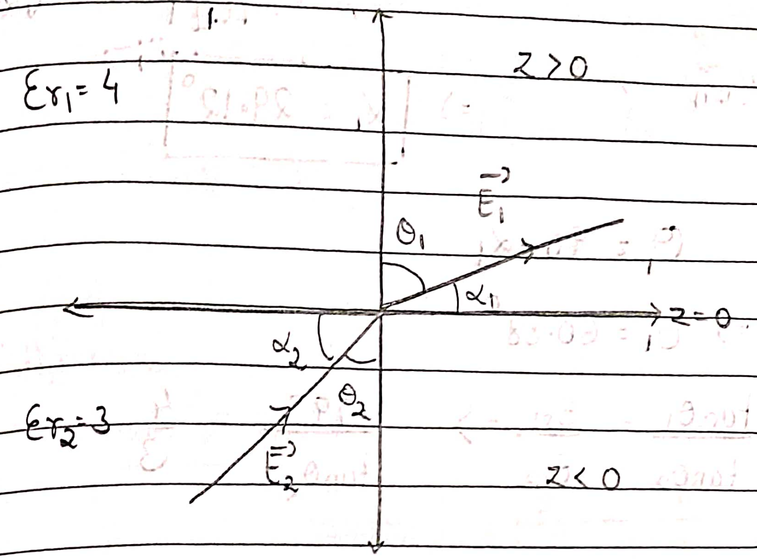
* Note:
$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$
for free space:
$\varepsilon_{r1} = 1$
$\Rightarrow \boxed{\varepsilon_1 = \varepsilon_0}$

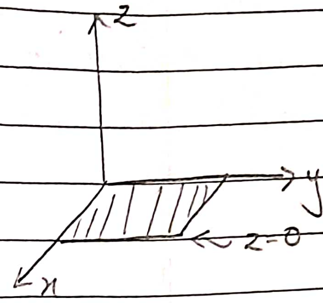
Q. 2 extensive, homogenius, isotropic dielectrics meet on $z=0$. For $z>0$, $\varepsilon_{r1}=4$ & for $z<0$, $\varepsilon_{r2}=3$. A uniform electric field $\vec{E_1} = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m, exists for $z>0$. Find:

(a) $\vec{E_2}$, for $z \le 0$

(b) the angles $\vec{E_1}$ & $\vec{E_2}$ make with interface.

(c) the energy densities (J/m³) in both dielectrics.

=> 



It is a case of dielectric-dielectric, the boundary cond^n's are:

$\vec{E}_{1t} = \vec{E}_{2t}$ or $\dfrac{\vec{D}_{1t}}{\varepsilon_1} = \dfrac{\vec{D}_{2t}}{\varepsilon_2}$	$\vec{D}_{1n} = \vec{D}_{2n}$ or $\varepsilon_1 \vec{E}_{1n} = \varepsilon_2 \vec{E}_{2n}$	$\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{\varepsilon_0\varepsilon_{r1}}{\varepsilon_0\varepsilon_{r2}}$

Given, $\vec{E_1} = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$

$\therefore \vec{E}_{1n} = 3\hat{a}_z$ & $\vec{E}_{1t} = 5\hat{a}_x - 2\hat{a}_y$

interface is $z=0$ & $\hat{a}_z$ is $\perp^r$ to it.

And, $\vec{E_2} = \vec{E}_{2t} + \vec{E}_{2n}$

by applying boundary cond^n: $(\vec{E}_{2t} = \vec{E}_{1t})$

$$\vec{E}_{2t} = 5\hat{a}_x - 2\hat{a}_y$$

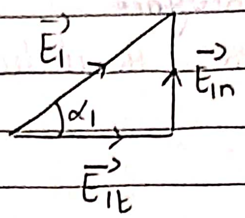
&

$$\vec{E}_{2n} = \frac{\varepsilon_1}{\varepsilon_2} \times \frac{\vec{E}_{1n}}{\cancel{E_{1n}}}$$

$$\vec{E}_{2n} = \frac{\cancel{\varepsilon_0}\varepsilon_{r1}}{\cancel{\varepsilon_0}\varepsilon_{r2}} \times \vec{E}_{1n} = \frac{4}{3} \times 3\hat{a}_z \Rightarrow \boxed{\vec{E}_{2n} = 4\hat{a}_z}$$

(a) $\therefore$ $\boxed{\vec{E_2} = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z \ \text{kV/m}}$

(b) We have to find α_1 & α_2 ($\angle$'s with the interface)



$\Rightarrow \tan\alpha_1 = \dfrac{|\vec{E_{1n}}|}{|\vec{E_{1t}}|} = \dfrac{3}{\sqrt{29}}$

$\Rightarrow \boxed{\alpha_1 = 29.12^\circ}$

$\theta_1 = 90 - \alpha_1$

$\Rightarrow \theta_1 = 60.88^\circ$

$\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_{r1}}{\varepsilon_{r2}} \Rightarrow \dfrac{1.795}{\tan\theta_2} = \dfrac{4}{3}$

$\Rightarrow \tan\theta_2 = 1.34625$

$\Rightarrow \theta_2 = 53.395^\circ$

$\therefore \alpha_2 = 90 - 53.395$

$\Rightarrow \boxed{\alpha_2 = 36.605^\circ}$

(C) Energy density : ** $\boxed{\dfrac{dW_{E_1}}{dV} = \dfrac{1}{2}\varepsilon_1 |E_1|^2}$ $\dfrac{dW_{E_2}}{dV} = \dfrac{1}{2}\varepsilon_2 |E_2|^2$

$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1} \Rightarrow \varepsilon_1 = \dfrac{10^{-9}}{36\pi} \times 4 \Rightarrow \varepsilon_1 = 0.0354 \times 10^{-9}$

and $|E_1| = \left(\sqrt{5^2+2^2+3^2}\right) \times 10^3 = \sqrt{38} \times 10^3$

$|E_1|^2 = 38 \times 10^6$

$\Rightarrow \boxed{\dfrac{dW_{E_1}}{dV} = 0.6726 \times 10^{-3} \ \text{J/m}^3}$

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III^{rdly} $\boxed{\dfrac{dW_{E2}}{dV} = 597 \mu\ J/m^3}$

★ Magnetic Boundary Conditions

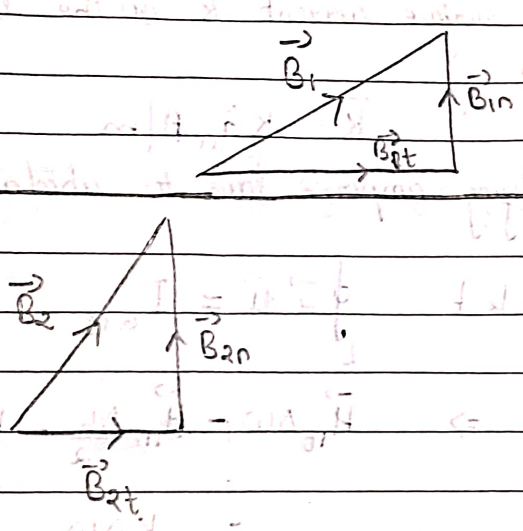
We define, magnetic Boundary cond^n's as the cond^n's that $\vec{H}$ (or $\vec{B}$) fields, must satisfy at the boundary b/w 2 diff. media.
Here we will make use of Gauss's law for magnetic field, i.e: $\oint_S \vec{B}.d\vec{S} = 0$ & Ampere's law or Ampere's circuit law, i.e: $\oint_L \vec{H}.d\vec{l} = I_{enc}$

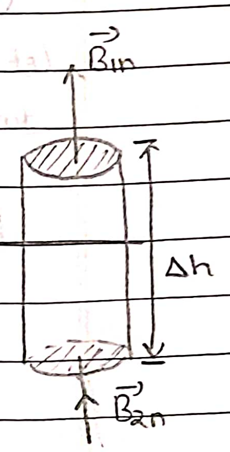
I. (1) $\mu_1 = \mu_0 \mu_{r_1}$

Medium (1)

Medium (2)

(2) $\mu_2 = \mu_0 \mu_{r_2}$





Applying Gauss's law for magnetic field:

$$\oint_S \vec{B}.d\vec{S} = 0$$

$$\Rightarrow \vec{B_{1n}}\Delta S - \vec{B_{2n}}\Delta S = 0$$

$$\Rightarrow \boxed{\vec{B_{1n}} = \vec{B_{2n}}}$$

or

$$\Rightarrow \mu_0 \mu_{r_1} \vec{H_{1n}} = \mu_0 \mu_{r_2} \vec{H_{2n}}$$

$$\Rightarrow \boxed{\mu_{r_1} \vec{H_{1n}} = \mu_{r_2} \vec{H_{2n}}}$$

$$\vec{H_{1n}} = \frac{\mu_{r_2}}{\mu_{r_1}} \vec{H_{2n}}$$

∴ the normal component of $\vec{B}$ is continuous at the boundary. It also shows that normal component of $\vec{H}$ is discontinuous at the boundary.

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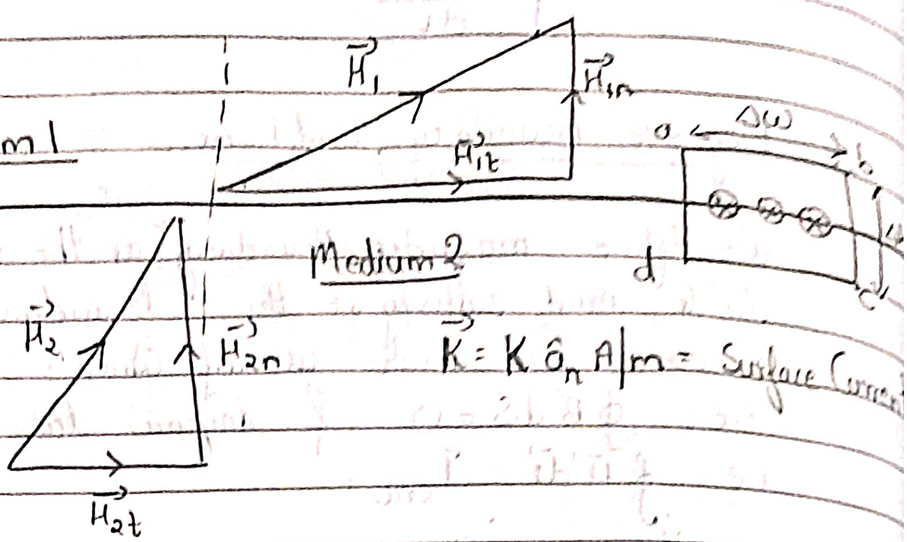
II.

① $\mu_1 = \mu_0 \mu_{r1}$

Medium 1



② $\mu_2 = \mu_0 \mu_{r2}$

$\vec{K} = K\hat{a}_n \ A/m$ = Surface Current

Consider a closed path abcda as shown in fig. Let the surface current $\vec{K}$ on the boundary is normal to the surface.

$\therefore \quad \vec{K} = K\hat{a}_n \ A/m$

Applying ampere's law to abcda:

w.k.t $\oint_L \vec{H}\cdot d\vec{l} = I_{ence}$ | $I_{enc} = K\Delta w$

$$\Rightarrow \vec{H}_{1t}\Delta w - \cancel{\vec{H}_{1n}\frac{\Delta h}{2}} - \cancel{\vec{H}_{2n}\frac{\Delta h}{2}} - \vec{H}_{2t}\Delta w + \cancel{\vec{H}_{2n}\frac{\Delta h}{2}} + \cancel{\vec{H}_{1n}\frac{\Delta h}{2}}$$

$$= K\Delta w$$

$$\Rightarrow \vec{H}_{1t}\cancel{\Delta w} - \vec{H}_{2t}\cancel{\Delta w} = K\cancel{\Delta w}$$

$$\Rightarrow \boxed{\vec{H}_{1t} - \vec{H}_{2t} = K} \quad \text{— ①}$$

$\vec{B} = \mu_0 \mu_r \vec{H}$

where $\mu_0 \mu_{r1} = \mu_1$, $\mu_0 \mu_{r2} = \mu_2$

$$\Rightarrow \boxed{\frac{\vec{B}_{1t}}{\mu_1} - \frac{\vec{B}_{2t}}{\mu_2} = K} \quad \text{— ②}$$

In the general case eq^n ① becomes:

$$\cancel{(\vec{H}_1 \times \vec{H}_2)} \ \boxed{(\vec{H}_1 - \vec{H}_2) \times \hat{a}_{n12} = \vec{K}}$$

where $\hat{a}_{n12}$ is unit vector normal to the interface & its directed from medium ① to medium ②.

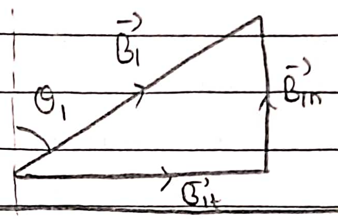
If the boundary is free of current or the media aren't conductors, then, $\underline{K = 0}$.

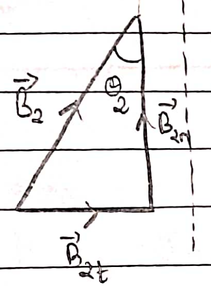
$$\therefore \boxed{H_{1t} = H_{2t}}$$

$$\Rightarrow \boxed{B_{1t}\mu_1 = B_{2t}\mu_2}$$

III

① $\mu_1 = \mu_0\mu_{r1}$





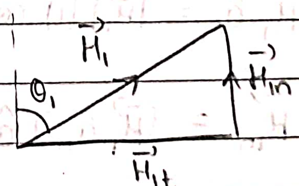
② $\mu_2 = \mu_0\mu_{r2}$

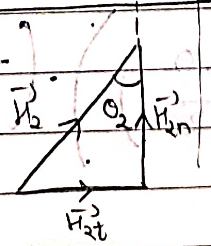
w.k.t. $\vec{B_{1n}} = \vec{B_{2n}}$

also, $\vec{B_{1n}} = B_1\cos\theta_1$ & $\vec{B_{2n}} = B_2\cos\theta_2$

$\Rightarrow B_1\cos\theta_1 = B_2\cos\theta_2$ —— ①

①





②

w.k.t $H_{1t} = H_{2t}$ for $K=0$

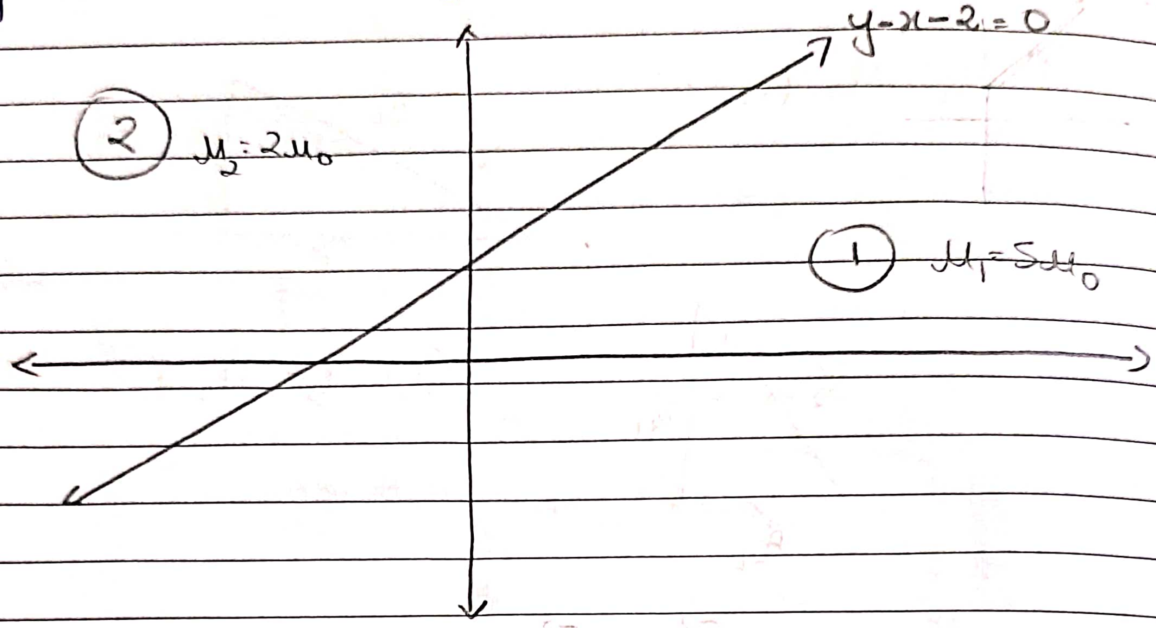
$\Rightarrow H_1\sin\theta_1 = H_2\sin\theta_2 \Rightarrow \dfrac{B_1}{\mu_1}\sin\theta_1 = \dfrac{B_2}{\mu_2}\sin\theta_2$ —— ②

Dividing as eqⁿ ② / ① :

$$\Rightarrow \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

Q Given $\vec{H_1} = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$ A/m in the region $y-x-2 \le 0$, where $\mu_1 = 5\mu_0$ & calc. $\vec{H_2}$ & $\vec{B_2}$ in the region $y-x-2>0$ where $\mu_2 = 2\mu_0$.

$\Rightarrow$



$$\vec{H}_{1n} = (H_1\cos\theta_1)\hat{a}_n$$

w.k.t $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\therefore \vec{H_1}\cdot\hat{a}_n = |\vec{H_1}|(1)\cos\theta_1$

$$\Rightarrow \vec{H}_{1n} = (\vec{H_1}\cdot\hat{a}_n)\hat{a}_n$$

$$= \left((-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z)\cdot\left(\frac{-1,1,0}{\sqrt{2}}\right)\right)\cdot\left(\frac{-1,1,0}{\sqrt{2}}\right)$$

$$= \left(\frac{2}{\sqrt{2}} + \frac{6}{\sqrt{2}}\right)\cdot\left(\frac{-1,1,0}{\sqrt{2}}\right)$$

$$= -\frac{8}{2}\hat{a}_x + \frac{8}{2}\hat{a}_y \Rightarrow \vec{H}_{1n} = -4\hat{a}_x + 4\hat{a}_y$$

$\therefore \quad \vec{H}_1 = \vec{H}_{1n} + \vec{H}_{1t}$

$\vec{H}_{1t} = \vec{H}_1 - \vec{H}_{1n}$

$= -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z + 4\hat{a}_x - 4\hat{a}_y$

$\Rightarrow \vec{H}_{1t} = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z$

and $\vec{H}_{2t} = \vec{H}_{1t}$ (for $k = 0$)

Now, $\vec{B}_{1n} = \vec{B}_{2n}$

$\Rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$

$\Rightarrow H_{2n} = \frac{\mu_1}{\mu_2} H_{1n}$

$\Rightarrow H_{2n} = \frac{5\cancel{\mu_0}}{2\cancel{\mu_0}}(-4\hat{a}_x + 4\hat{a}_y) \Rightarrow \vec{H}_{2n} = -10\hat{a}_x + 10\hat{a}_y$

$\therefore \vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n}$

$\Rightarrow \vec{H}_2 = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z - 10\hat{a}_x + 10\hat{a}_y$

$\Rightarrow \boxed{\vec{H}_2 = -8\hat{a}_x + 12\hat{a}_y + 4\hat{a}_z}$

$\vec{B}_2 = \mu_2 \vec{H}_2$