DIGITAL COMMUNICATION

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QUANTIZATION AND PULSE SHAPING

Problems on SNR for transmission with Quantization Noise

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Problems on SNR for transmission with Quantization Noise

1) Let X be uniform over the range -10 to 10. If it is required that $\sigma_Q^2 < 0.2$ what Is the minimum N required. (By default we take Mid-riser quantizer only).

Sol:

Given : $\sigma_Q^2 < 0.2$

$$\frac{\Delta^2}{12} < 0.2$$

$$\Delta < \sqrt{2.4}$$

$$\Delta < 1.549.$$

Problems on SNR for transmission with Quantization Noise

$$\Delta = \frac{2A}{2^N} < 1.549$$

$$\frac{2\times(10)}{2^N} < 1.549$$

$$2^N > \frac{20}{1.549}$$

$$N > \log_2\left(\frac{20}{1.549}\right)$$

$$N > 3.69$$

$$\boxed{N \geq 4}$$

$\therefore$ minimum value of N required is $\underline{4}$.

Problems on SNR for transmission with Quantization Noise

2) Let X be uniform over the range [-A to A]. Find the SNR for N bit quantization (assume N is large).

Sol

$$SNR = \frac{\frac{(2A)^2}{12}}{\Delta^2/12} \qquad \left(SNR = \frac{\sigma_x^2}{\sigma_q^2} \right).$$

$$SNR = \frac{4A^2}{\Delta^2}$$

$$\text{w.k.t. } \Delta = \frac{2A}{2^N}.$$

$$\therefore SNR = \frac{4A^2}{4A^2/2^{2N}}.$$

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$$\boxed{SNR = 2^{2N}}$$

In dB we have

$$SNR_{dB} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_Q^2}\right).$$

$$= 10\log_{10}(2^{2N}).$$

$$SNR_{dB} = 20N\log_{10}(2).$$

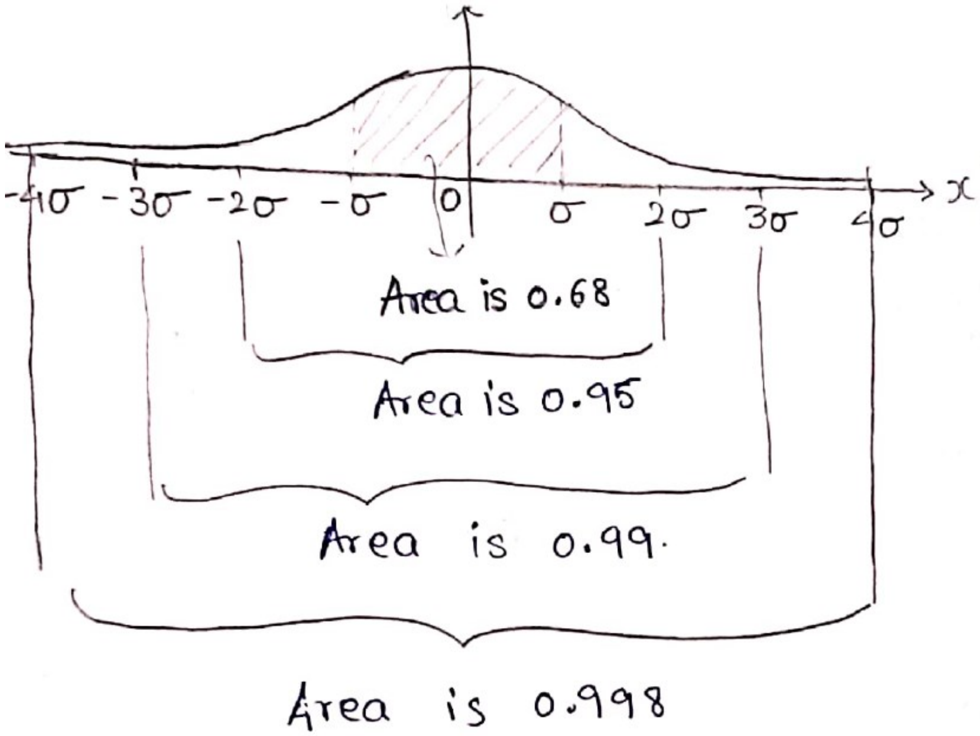
$$\boxed{SNR_{dB} = 6.02N \approx 6N}$$

Problems on SNR for transmission with Quantization Noise

3) ex: let $x \sim N(0, \sigma^2)$. (N → Normal distribution (Gaussian)).

let x be Gaussian with mean 0 and variance σ^2. Find SNR for N-bit Quantization.

let $A = 4\sigma$. (∵ peak for Gaussian distribution is at ∞).



* Hence for all practical purposes the value of x(n) can be take between -4σ & 4σ.

Problems on SNR for transmission with Quantization Noise

Sol w.r.t. for Gaussian distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

here $\mu = 0$.

$$\therefore f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\sigma_Q^2 = \frac{\Delta^2}{12} = \frac{(2 \times 4\sigma_X)^2}{12 \times 2^{2N}} = \frac{64\,\sigma_X^2}{12 \times 2^{2N}} = \frac{16}{3}\frac{\sigma_X^2}{2^{2N}}$$

$$SNR = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{\sigma_X^2}{\frac{16}{3}\frac{\sigma_X^2}{2^{2N}}}$$

Problems on SNR for transmission with Quantization Noise

$$\boxed{SNR = \frac{3}{16} 2^{2N}}$$

$$\therefore SNR_{dB} = 10\log_{10}\left(\frac{3}{16} 2^{2N}\right)$$

$$\boxed{SNR_{dB} = 6N - 7.269}$$

Problems on SNR for transmission with Quantization Noise

4) let $x(n) = A\cos\omega_0 n$. find SNR in terms of N.

Sol

since $x(n) = A\cos\omega_0 n$. i.e.. $x(n)$ is deterministic

$$\boxed{SNR = \frac{\text{Avg. Power of Input signal}}{\sigma_Q^2} = \frac{P_x}{\sigma_Q^2}}$$

w.k.t $P_x = \frac{A^2}{2}$ when $x(n) = A\cos\omega_0 n$.

Problems on SNR for transmission with Quantization Noise

$$SNR = \frac{A^2}{2} \Big/ \left(\Delta^2/12\right).$$

$$= \frac{A^2}{2} \Big/ \left(\frac{\cancel{4}A^2}{\cancel{12}\,3 \times 2^N}\right)$$

$$\therefore \boxed{SNR = \frac{3}{2}\, 2^N}$$

$$\boxed{SNR_{dB} = 6N + 1.76}$$

$$\boxed{P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2} \rightarrow ①$$

* when $x(n)$ is periodic

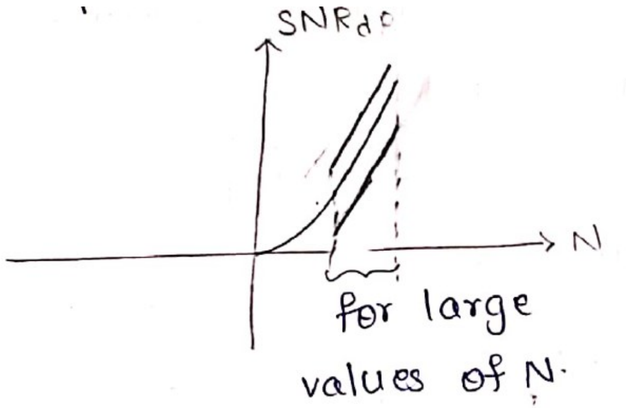
$$\boxed{P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2} \rightarrow ②$$

use ① has we don't know weather $x(n)$ is periodic (or) not & express $x(n) = A\cos\omega_0 n$ as $\frac{A}{2}\left(e^{j\omega_0 n} + e^{-j\omega_0 n}\right)$.

Observations from the previous examples

From the above results we can summarize that:

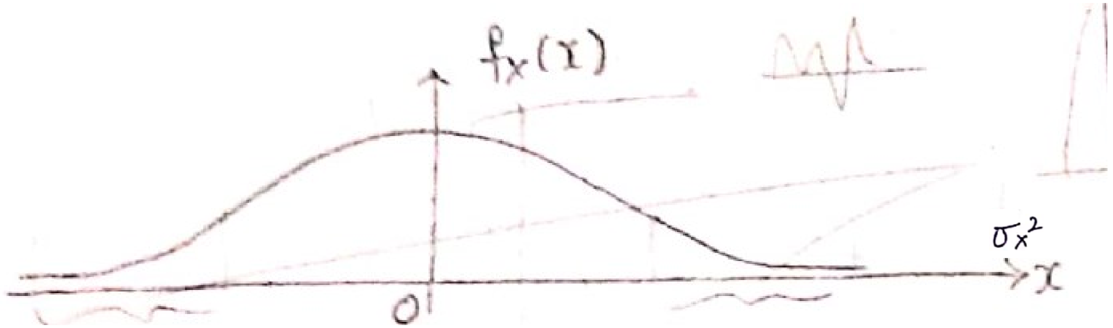
1) SNR depends upon input signal's pdf.
2) $SNR_{dB} = 6N + C$ is is an incrementally linear function of N with a slope of 6dB/bit.
3) For every additional bit, we get an improvement of 6dB in SNR. (From 2nd Result)
4) If the number of bits is increased by 1, $\sigma_Q{}^2$ decreases by a factor of 4.



The difference in performance between different signals with same peak to peak values is due to the change in signal power variation, i.e. $\sigma_X 2$

SNR for transmission with Quantization Noise

Consider the pdf $f_x(x)$:-



Here the value occurring near zero is more probable.

So there is more probability of values being close to zero than towards the extremes.

Thus the performance of a signal is affected by σ_x^2 and not the σ_Q^2.

So the differentiating factor for different SNR's is σ_x^2 and not the σ_Q^2.

THANK YOU

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