DIGITAL COMMUNICATION

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POWER SPECTRA OF PAM

Manchester Coding

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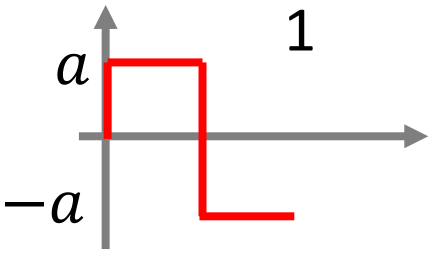
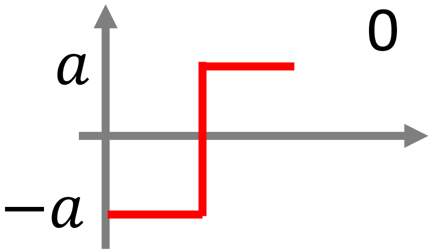
POWER SPECTRUM
Manchester Coding

- Let b_k be the k^{th} bit. We assume that bits 0 and 1 occur with equal probability
- Observe that A_k for Manchester coding is the same as that of polar NRZ
- Therefore, $R_A(n)$ and $S_A(f)$ are the same as that of polar NRZ
- However, note that $v(t)$ is different

POWER SPECTRUM

Manchester Coding





$$V(f) = \int_0^{T_b/2} e^{-j2\pi f t}\,dt + \int_{T_b/2}^{T_b} -e^{-j2\pi f t}\,dt$$

$$= \frac{1}{-j2\pi f}\left(e^{-j2\pi f t}\Big|_0^{T_b/2}\right) + \frac{1}{j2\pi f}\left(e^{-j2\pi f t}\Big|_{\frac{T_b}{2}}^{T_b}\right)$$

$$= \frac{e^{-j\pi f T_b} - 1}{-j2\pi f} + \frac{e^{-j2\pi f T_b} - e^{-j\pi f T_b}}{j2\pi f}$$

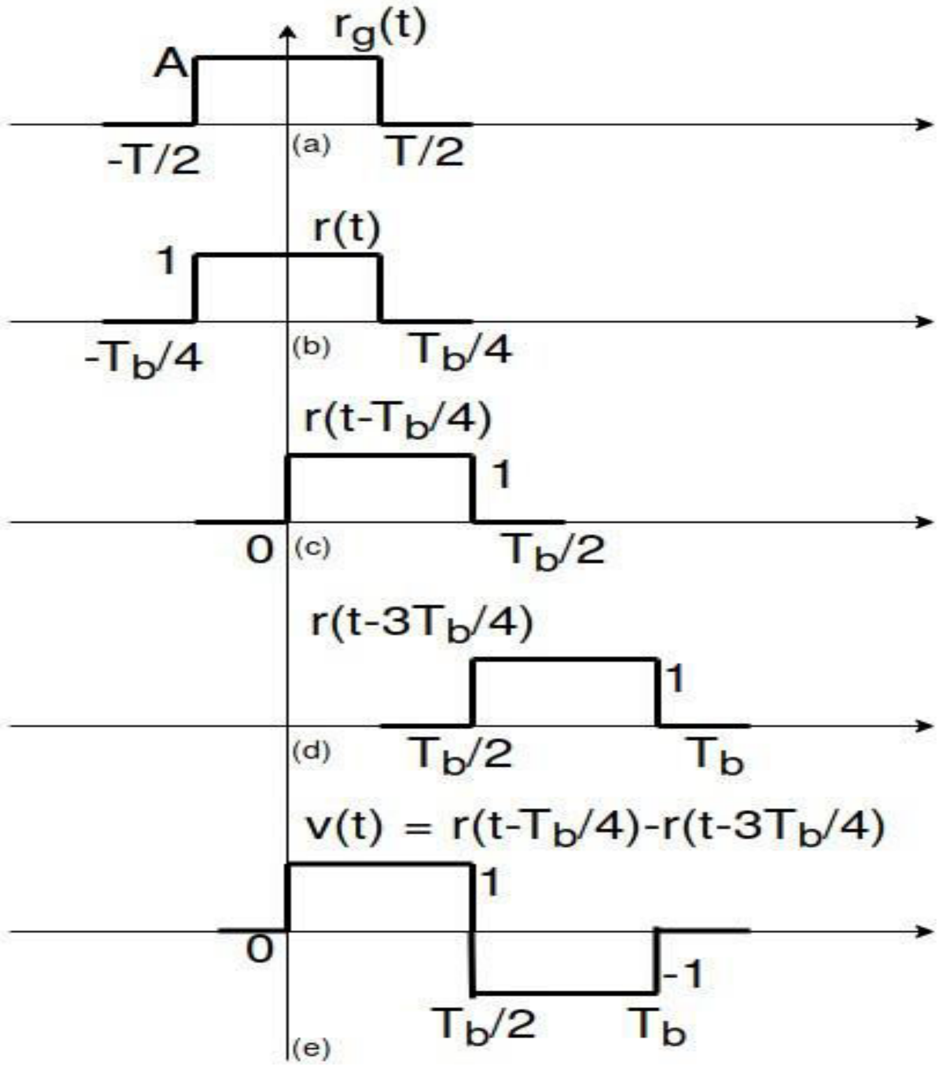
$$= \left\{ \begin{array}{l} \dfrac{e^{-j\frac{\pi}{2} f T_b}}{-j2\pi f}\left(e^{-j\frac{\pi}{2} f T_b} - e^{j\frac{\pi}{2} f T_b}\right) \\ \qquad + \\ \dfrac{e^{-j\pi f T_b}}{j2\pi f}\left(e^{-j\pi f T_b} - 1\right) \end{array} \right.$$

$$= \frac{e^{-j\frac{\pi}{2} f T_b}}{\pi f}\sin\left(\frac{\pi}{2} f T_b\right) - \frac{e^{-j\frac{3\pi}{2} f T_b}}{\pi f}\sin\left(\frac{\pi}{2} f T_b\right)$$

$$\boxed{V(f) = \frac{T_b}{2}\,\mathrm{sinc}\left(f\frac{T_b}{2}\right)\cdot e^{-j\frac{\pi}{2} f T_b}\left(1 - e^{-j\pi f T_b}\right)}$$

POWER SPECTRUM

Manchester Coding – Alternate Approach – May be Skipped



$r(t - \frac{T_b}{4}) - r(t - 3\frac{T_b}{4})$.

So,

$$v(t) = r(t - \frac{T_b}{4}) - r(t - 3\frac{T_b}{4})$$

$$r(t) \overset{F.T}{\longleftrightarrow} \frac{T_b}{2} sinc(f \frac{T_b}{2})$$

$$r(t - \frac{T_b}{4}) \overset{F.T}{\longleftrightarrow} e^{-j2\pi f \frac{T_b}{4}} \frac{T_b}{2} sinc(f \frac{T_b}{2})$$

$$\therefore r(t - \frac{T_b}{4}) \overset{F.T}{\longleftrightarrow} e^{-j\pi f \frac{T_b}{2}} \frac{T_b}{2} sinc(f \frac{T_b}{2})$$

$$r(t - 3\frac{T_b}{4}) \overset{F.T}{\longleftrightarrow} e^{-j2\pi f \frac{3T_b}{4}} \frac{T_b}{2} sinc(f \frac{T_b}{2})$$

$$\therefore r(t - 3\frac{T_b}{4}) \overset{F.T}{\longleftrightarrow} e^{-j3\pi f \frac{T_b}{2}} \frac{T_b}{2} sinc(f \frac{T_b}{2})$$

$$\therefore r(t - \frac{T_b}{4}) - r(t - 3\frac{T_b}{4}) \overset{F.T}{\longleftrightarrow} \frac{T_b}{2} sinc(f \frac{T_b}{2}) \cdot \left(e^{-j\pi f \frac{T_b}{2}} - e^{-j3\pi f \frac{T_b}{2}}\right)$$

$$\boxed{\therefore v(t) \overset{F.T}{\longleftrightarrow} \frac{T_b}{2} sinc(f \frac{T_b}{2}) e^{-j\pi f T_b} \cdot \left(e^{j\pi f \frac{T_b}{2}} - e^{-j\pi f \frac{T_b}{2}}\right)}$$

POWER SPECTRUM

Manchester Coding – Alternate Approach – May be Skipped

$$Let\ v(t) \xleftrightarrow{F.T} V(f)$$

$$\therefore V(f) = \frac{T_b}{2} sinc\left(f \frac{T_b}{2}\right) e^{-j\pi f T_b} \cdot \left(e^{j\pi f \frac{T_b}{2}} - e^{-j\pi f \frac{T_b}{2}} \right)$$

$$= jT_b sinc\left(f \frac{T_b}{2}\right) e^{-j\pi f T_b} \frac{e^{j\pi f \frac{T_b}{2}} - e^{-j\pi f \frac{T_b}{2}}}{2j}$$

$$\therefore V(f) = jT_b sinc\left(f \frac{T_b}{2}\right) e^{-j\pi f T_b} sin\left(\pi f \frac{T_b}{2}\right)$$

$$\therefore |V(f)| = T_b sinc\left(f \frac{T_b}{2}\right) sin\left(\pi f \frac{T_b}{2}\right)$$

$$\boxed{\therefore |V(f)|^2 = T_b^2 sinc^2\left(f \frac{T_b}{2}\right) sin^2\left(\pi f \frac{T_b}{2}\right)}$$

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- Substituting in the formula for $S_X(f)$

$$\boxed{V(f) = T_b j\, e^{-j2\pi f \frac{T_b}{2}} \operatorname{sinc}\left(\frac{fT_b}{2}\right) \sin\left(\frac{\pi f T_b}{2}\right).}$$

$$\boxed{\frac{|V(f)|^2}{T_b} = T_b \operatorname{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$

$$\boxed{S_X(f) = \frac{|V(f)|^2}{T_b} S_A(f) = T_b a^2 \operatorname{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$

- Observe that there is no DC content
- The BW of Manchester coding is $2R_b = {}^{2}/_{T_b}$
- The synchronization of clock at the receiver comes with a cost of excess BW of R_b

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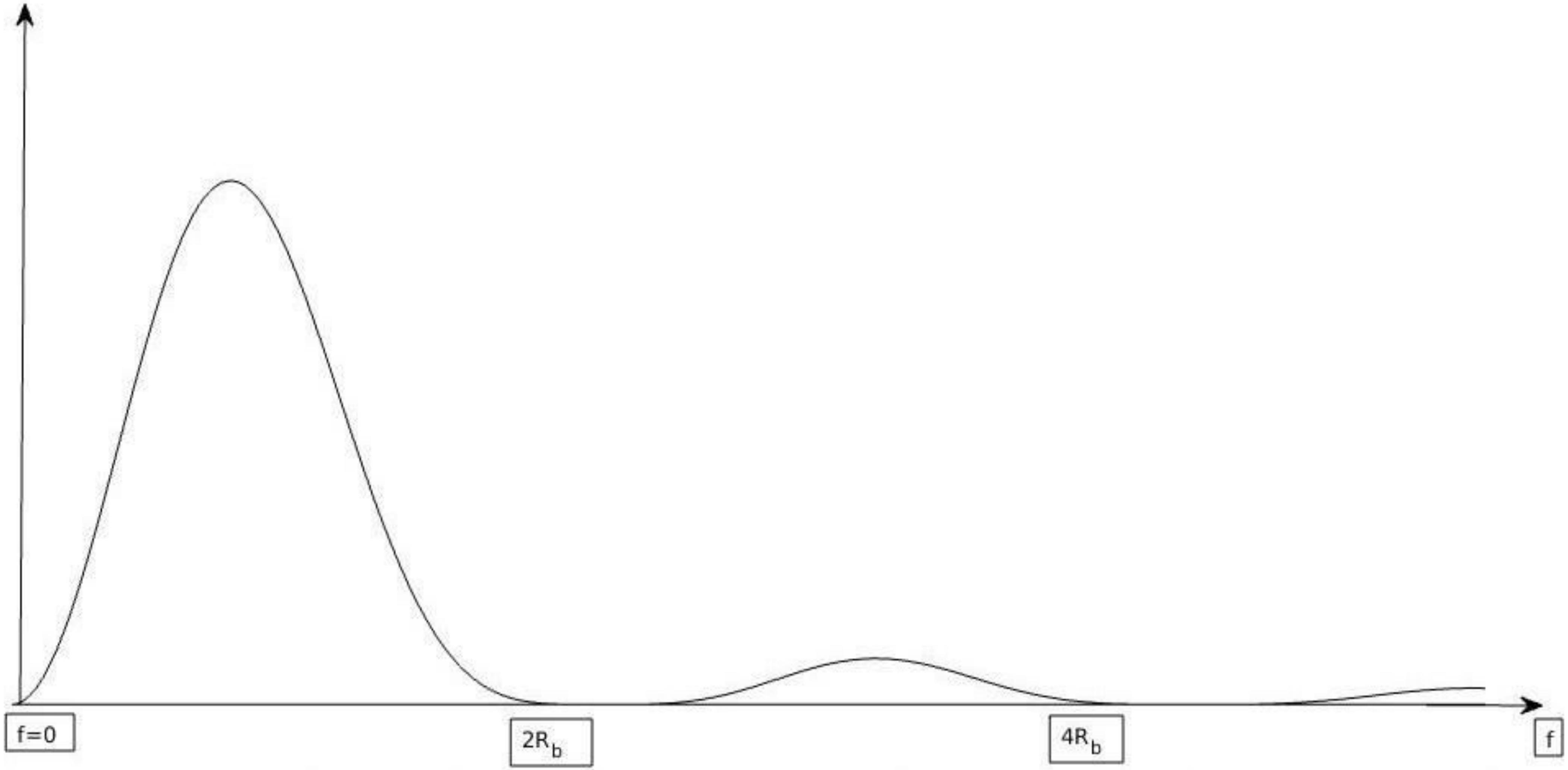

THANK YOU

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