Note Title 08-08-2022

(1). Consider any 2 vectors.

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$f(a,b) = a^T b = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \underline{a_1 b_1 + a_2 b_2 + a_3 b_3}$$

$$\text{|||}^{ly} \quad b^T a = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = b_1 a_1 + b_2 a_2 + b_3 a_3.$$

$$\frac{\partial a^T b}{\partial b} = \begin{bmatrix} \frac{\partial a^T b}{\partial b_1} \\ \frac{\partial a^T b}{\partial b_2} \\ \frac{\partial a^T b}{\partial b_3} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = a.$$

$$\frac{\partial b^T a}{\partial b} = \begin{bmatrix} \frac{\partial b^T a}{\partial b_1} \\ \frac{\partial b^T a}{\partial b_2} \\ \frac{\partial b^T a}{\partial b_3} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = a = \frac{\partial a^T b}{\partial b}$$

(2) Consider a symmetric matrix $P = \begin{bmatrix} p_1 & b \\ b & p_2 \end{bmatrix}$.

and a vector $a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$.

$$a^T P a = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} p_1 & b \\ b & p_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 p_1 + a_2 b & a_1 b + a_2 p_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$= \underline{a_1^2 \cdot p_1 + a_1 a_2 b + a_1 a_2 b + a_2^2 p_2}$$

$$f(a, p) = a_1^2 p_1 + 2 a_1 a_2 b + a_2^2 p_2 . \checkmark$$

$$\frac{\partial a^T P a}{\partial a} = \begin{bmatrix} \frac{\partial a^T P a}{\partial a_1} \\ \frac{\partial a^T P a}{\partial a_2} \end{bmatrix} = \begin{bmatrix} 2 a_1 p_1 + 2 a_2 b + 0 \\ 0 + 2 a_1 b + 2 a_2 p_2 \end{bmatrix}$$

$$= 2 \begin{bmatrix} a_1 p_1 + a_2 b \\ a_1 b + a_2 p_2 \end{bmatrix}$$

$$= 2 \cdot \begin{bmatrix} p_1 & b \\ b & p_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$= \underline{\underline{2}} \; P a$$