Digital Signal Processing

Ms. Ashwini

Department of Electronics and Communication.

Digital Signal Processing

Properties of DFT

Ms. Ashwini
Department of Electronics and Communication.

Properties of DFT
Numericals

I $x_1(n) = \cos \frac{2\pi n}{N}$ $\qquad x_2(n) = \sin \frac{2\pi n}{N}$

$$X_1[k] = \frac{N}{2}\left[\delta(k-1) + \delta(k+1)\right]$$

$$X_2[k] = \frac{N}{2j}\left[\delta(k-1) - \delta(k+1)\right]$$

a) Conv $\rightarrow$ $Y[k] = X_1[k] \cdot X_2[k] \leftarrow x_1(n) \circledast_N x_2(n)$

$$= \frac{N^2}{4j}\left[\delta(k-1) - \delta(k+1)\right]$$

$$y(n) = \frac{N}{2} \sin \frac{2\pi n}{N}$$

Properties of DFT
Numericals

b) Cos → A· Cos·

$R_{x_1 x_1}(k) \rightarrow \frac{N}{2} \cdot \frac{N}{2} [\delta(k-1) + \delta(k+1)]$

IDFT

$r_{x_1 x_1}(l) = \frac{N^2}{2} \cos \frac{2\pi l}{N}$

$R_{x_2 x_2}(k) \rightarrow \frac{+N}{2j} \cdot \frac{+N}{2j} [\delta(k-1) + \delta(k+1)]$

IDFT

$= \frac{N^2}{4} [\delta(k-1) + \delta(k+1)]$

$r_{x_2 x_2}(l) = \frac{N}{2} \cos \frac{2\pi l}{N}$

Properties of DFT
Numericals

C. Cor. $\rightarrow$ $R_{x_1 x_2}(k) = X_1(k)\, X_2^*(k)$

$$= -\frac{N^2}{4j}\left[\delta(k-1) - \delta(k+1)\right]$$

$\xrightarrow{\text{IDFT}}$

$$r_{x_1 x_2}(l) = -\frac{N}{2} \sin\frac{2\pi l}{N} \qquad 0 \le l \le N-1$$

Properties of DFT
Numericals

II $X[k] = [1, j, -1, -j]$ $\quad H[k] = [0, 1, -1, 1]$

$x(n)$ $\quad h(n)$

a) $x(n-1)_4 \xrightarrow{D} W_4^{k \cdot 1} X(k)$

$= [W_4^0, W_4^1, W_4^2, W_4^3] [1, j, -1, -j] = [1, 1, 1, 1]$

Properties of DFT
Numericals

b) $x[n+3]_4 \xrightarrow{D} W_4^{-3 \cdot k} X[k]$

$[W_4^0 \cdot W_4^{-3} \cdot W_4^{-2}, W_4^{-1}] [1, j, -1, -j] = [1, 1, 1, 1]$

c) $Y[k] = H[k] \cdot X[k]$

$[0, 1, -1, 1] [1, j, -1, -j] = [0, j, 1, -j]$

Properties of DFT
Numericals

d) $(-1)^n x(n) \rightarrow \sum_{n=0}^{3} x(n)\left(e^{-j\frac{2\pi}{4}}\right)^{kn}$

$$\sum_{n=0}^{3} (-1)^n x(n)\, e^{-j\frac{\pi}{2}kn} = \sum_{n=0}^{3} e^{-j\pi n}\, x(n)\, e^{-j\frac{\pi}{2}kn}$$

$$\sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}(k+2)n} = X[k+2]_4 = [-1, -j, 1, j]$$

Properties of DFT

Numericals

e) $j^n x(n) \xrightarrow{D} \sum_{n=0}^{3} \left(e^{-j3\pi/2}\right)^n x(n)\, W_4^{kn}$

$$= \sum_{n=0}^{3} W_4^{3n}\, x(n)\, W_4^{kn} = \sum_{n=0}^{3} W_4^{(k+3)n}\, x(n)$$

$$= X[k+3]_4 = [-j, 1, j, -1]$$

Properties of DFT
Numericals

f) $x(-n)_4 \xrightarrow{D} X[-k]_4 = [1, -j, -1, j]$

g) $x(2-n)_4 \xrightarrow{D} [1, j, -1, -j]$

THANK YOU

Ms. Ashwini

Department of Electronics and Communication

ashwinib@pes.edu

+91 80 6666 3333 Ext 741