DIGITAL COMMUNICATION

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POWER SPECTRA OF PAM

Unipolar NRZ

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POWER SPECTRUM

Unipolar NRZ

- Let b_k be the k^{th} bit. We assume that bits 0 and 1 occur with equal probability
- Further, assume that the sequence of bits are independent
- We need to calculate the autocorrelation function $R_A(n)$
- Observe that $R_A(n) = \mathbb{E}(A_k A_{k-n})$
- It is easy to see that $\mathbb{E}(A_k) = 0 \times {}^1/_2 + a \times {}^1/_2 = \frac{a}{2}$
- Now, it can be seen that $R_A(0) = \mathbb{E}\left(A_k^2\right) = 0 \times {}^1/_2 + a^2 \times {}^1/_2 = \frac{a^2}{2}$
- Also, for any general n, $R_A(n) = \mathbb{E}(A_k A_{k-n}) = \mathbb{E}(A_k)\mathbb{E}(A_{k-n}) = \frac{a}{2} \times \frac{a}{2} = \frac{a^2}{4}$
- Combining the above results, we get

$$R_A(n) = \frac{a^2}{4} + \frac{a^2}{4}\delta(n)$$

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- Substituting in the formula for $S_X(f)$

$$S_X(f) = \frac{{T_b}^2 sinc^2(fT_b)}{T_b} \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}$$

$$= T_b sinc^2(fT_b) \sum_{n=-\infty}^{\infty} \left\{ \frac{a^2}{4} + \frac{a^2}{4}\delta(n) \right\} e^{-j2\pi f n T_b}$$

$$\therefore S_X(f) = \frac{a^2 T_b}{4} sinc^2(fT_b) + \frac{a^2 T_b}{4} sinc^2(fT_b) \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b}$$

- Now, we can write

$$\boxed{\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)}$$

$$F\left\{ \sum_{k=-\infty}^{\infty} \delta(t - nT_b) \right\} = \sum_{k=-\infty}^{\infty} F\left\{ \delta(t - nT_b) \right\} = \sum_{k=-\infty}^{\infty} e^{-j2\pi f n T_b}$$

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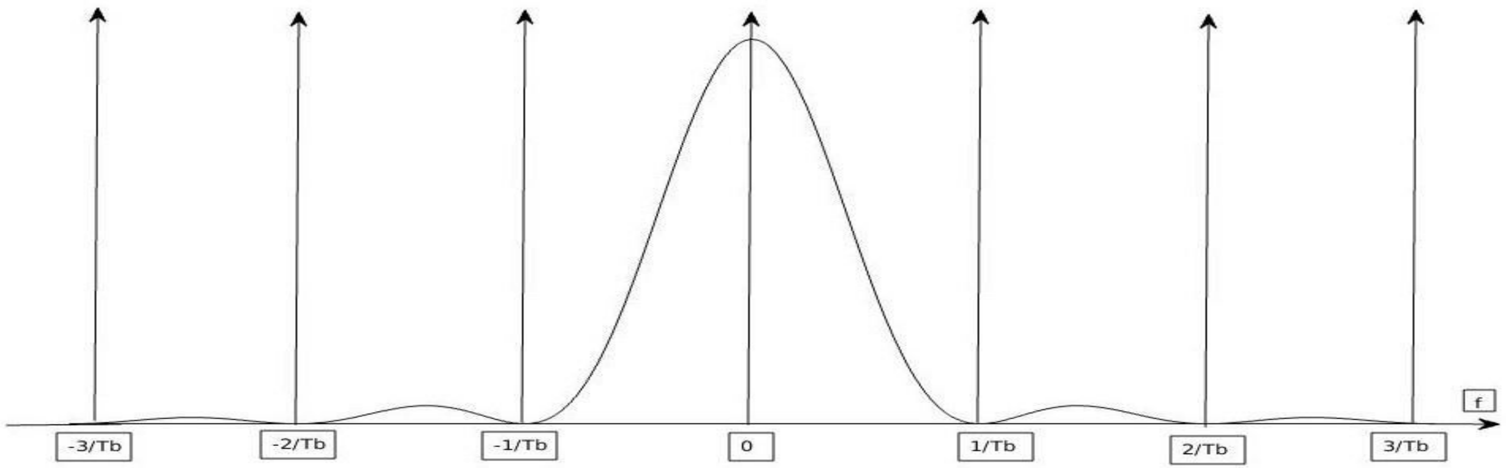
- Substituting and simplifying, we get

$$S_X(f) = \frac{a^2 T_b}{4} sinc^2(fT_b) + \frac{a^2 T_b}{4} sinc^2(fT_b) \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)$$



- Since the zeros of the sinc function coincide with the impulses, we can write

$$S_X(f) = \frac{a^2 T_b}{4} sinc^2(fT_b) + \frac{a^2}{4} \delta(f)$$

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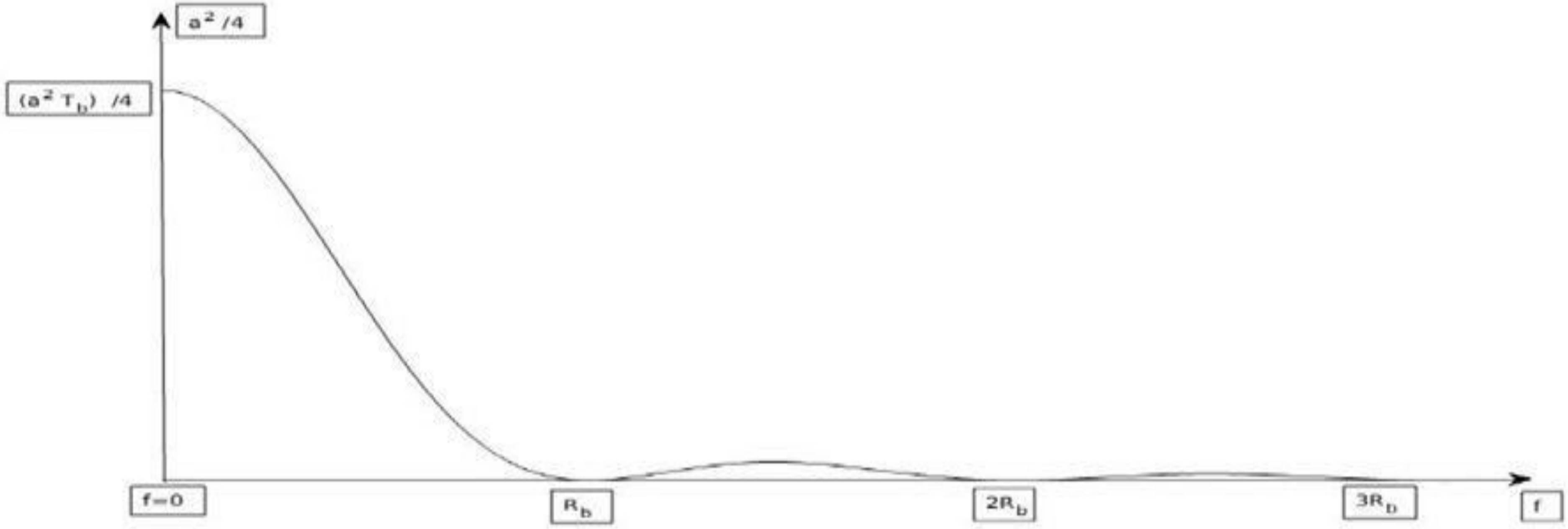

- Here $R_b = {}^{1}/_{T_b}$. By considering the first non-DC null as bandwidth, we have BW= R_b
- The total power is ${}^{a^2}/_{2}$
- There is a non-zero DC value in $S_X(f)$, which accounts for half of power i.e., ${}^{a^2}/_{4}$

THANK YOU

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