UE20EC254
DIGITAL COMMUNICATION

Department of Electronics and Communication Engg

Course Objectives

- Understand the principles of amplitude and angle modulation
- Learn the different sampling techniques
- Understand the performance of different waveform coding techniques
- Understand the idea of signal space
- Learn the different digital modulation techniques

Course Outcomes

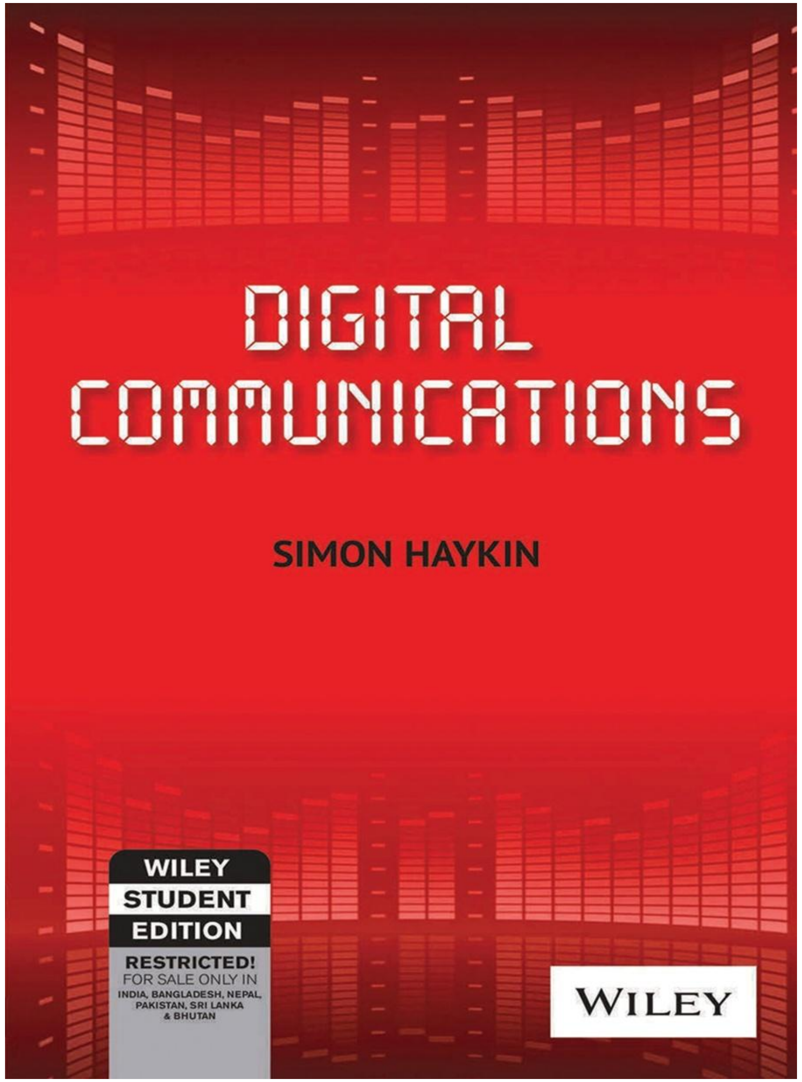
Students completing the course should be able to

- Analyze the different analog modulation techniques
- Analyze the different sampling techniques
- Design quantization and pulse shaping systems
- Develop detection rules for the given transmission scheme
- Analyze the different coherent and non-coherent digital modulation techniques

Units

- Amplitude and Angle Modulation
- FM and Sampling
- Quantization and Pulse Shaping
- Intersymbol Interference and Signal Space Representation
- Digital Modulation

Text Books


DIGITAL COMMUNICATIONS
SIMON HAYKIN
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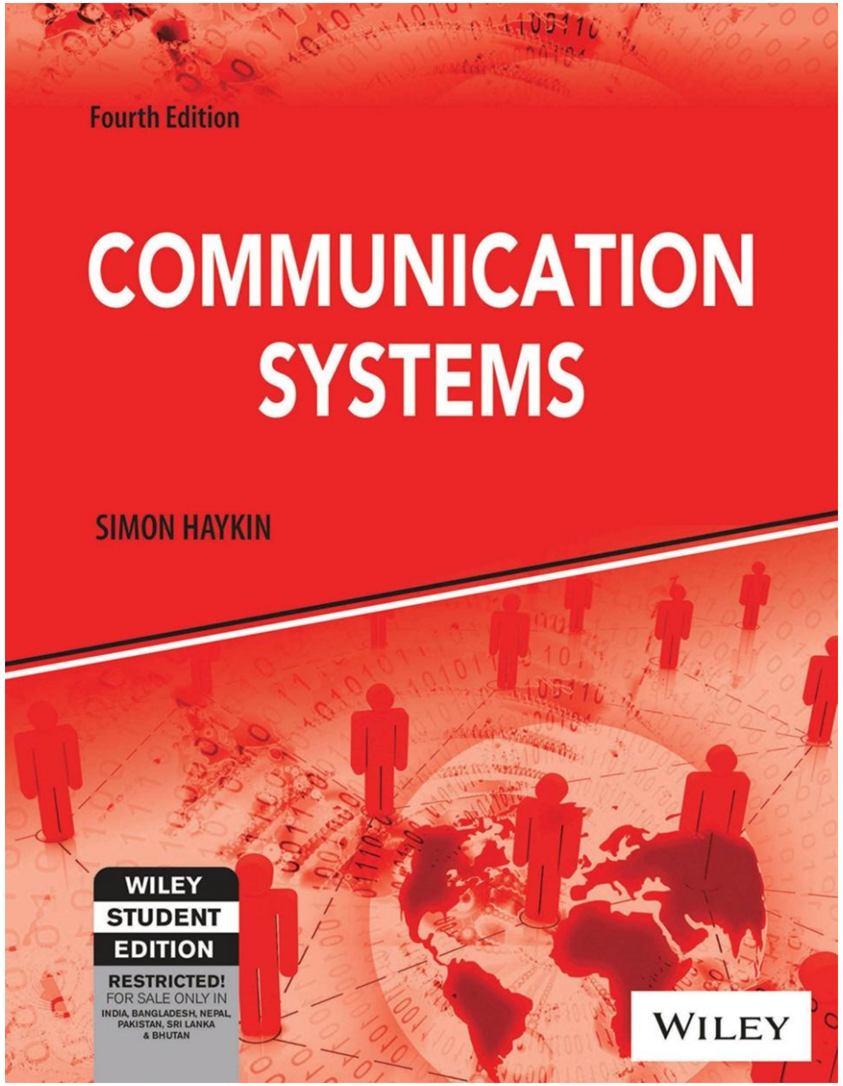
Fourth Edition
COMMUNICATION SYSTEMS
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INTRODUCTION
Motivation

Idea of communication : Transfer of information

Electrical Communication : How did it develop?

1. Telegraph
2. Telephone
3. Triode
4. TV
5. Shannon's Idea
6. transistor

INTRODUCTION

Basic Block Diagram of A Communication System

Why Analog Communication?

- Many communication systems still in use are analog

  e.g.: Broadcast Systems

- Helps in understanding digital communication
- Real time operations
- At very high frequencies ADC/DAC operations become difficult
- Analog communication is useful in such situations

  e.g.: 5G systems where hybrid analog/digital techniques are used

FOURIER TRANSFORM

Review

- Basic Definitions

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt, \qquad x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

- Properties
  - $x(t - t_0) \leftrightarrow X(f)\, e^{-j2\pi f t_0}$
  - $e^{j2\pi f_0 t} x(t) \leftrightarrow X(f - f_0)$
  - $x(-t) \leftrightarrow X(-f)$
  - $x^*(t) \leftrightarrow X^*(-f)$
  - $\frac{dx(t)}{dt} \leftrightarrow j2\pi f X(f)$
- Properties
  - $\int_{-\infty}^{\infty} x(\tau) d\tau \leftrightarrow \frac{X(f)}{j\,2\,\pi f} + \frac{1}{2} X(0)\delta(f)$
  - If $x(t) \leftrightarrow$ X(f), then $X(t) \leftrightarrow x(-f)$
  - Parseval's identity

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

MODULATION

What and Why?

- The information bearing signal is called message signal / modulating signal / baseband signal
- The term baseband refers to band of frequencies occupied by the message signal
- Modulation is a process by which some characteristic of carrier wave is varied in accordance with message signal
- The carrier is typically a sinusoid, the result of modulation is called the modulated signal

- Antenna height
- To avoid interference from other baseband sources, since ”separation in time” is not practical , so we employ “separation in frequencies”
- Efficient utilization of the available spectrum
- To avoid man made noise in baseband frequencies

MODULATION

Types

Depending on the varied parameter of the carrier wave

1. Amplitude modulation
2. Frequency modulation
3. Phase modulation

MODULATION

Need

- Antenna height
  - Height of an antenna should be at least $\lambda/4$
  - Recall that $\lambda = c/f$
  - Find the antenna height if $f_0 = 2$ kHz
  - Find the antenna height for a speech signal (300 Hz to 3 kHz)
  - Radio frequency transmission at high frequencies is helpful
- Avoiding interference with other baseband sources
  - Separation in time is not practical
  - Separation in frequency can be used

Amplitude Modulation

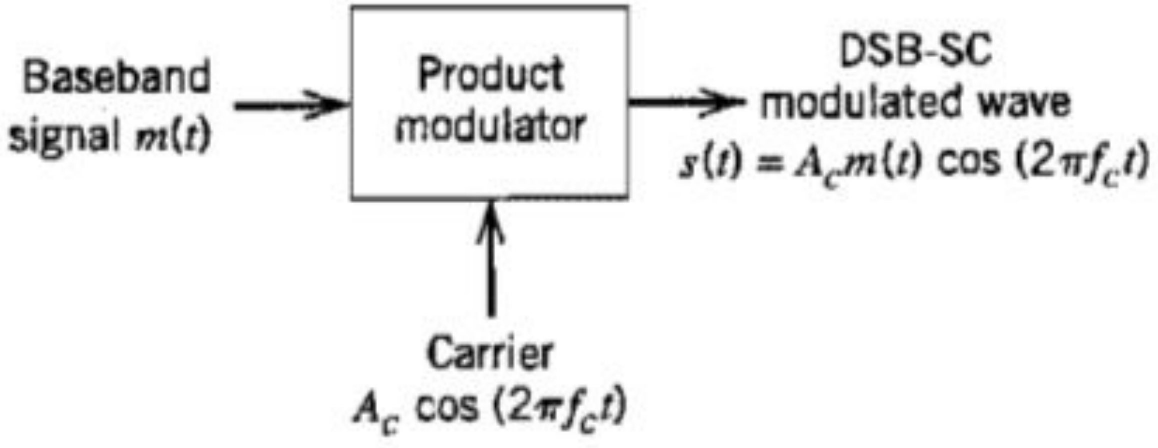
Types

- Standard Amplitude Modulation (AM)
  - Also called as Double-sideband full carrier (DSBFC) modulation
  - Transmit power is “wasted”
  - Channel bandwidth is “wasted”

- Double-sideband suppressed carrier (DSBSC) modulation
  - Transmit power is saved due to suppression of carrier wave

DSBSC
Modulation

- This technique is the direct application of frequency shift property of Fourier transform



- The message signal m(t) is multiplied with the carrier to obtain the modulated waveform s(t).

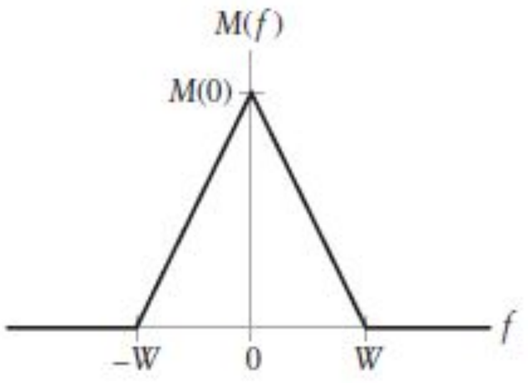
$$\begin{aligned} s(t) &= c(t)m(t) \\ &= A_c \cos(2\pi f_c t)m(t) \end{aligned}$$

- By frequency shift property of FT

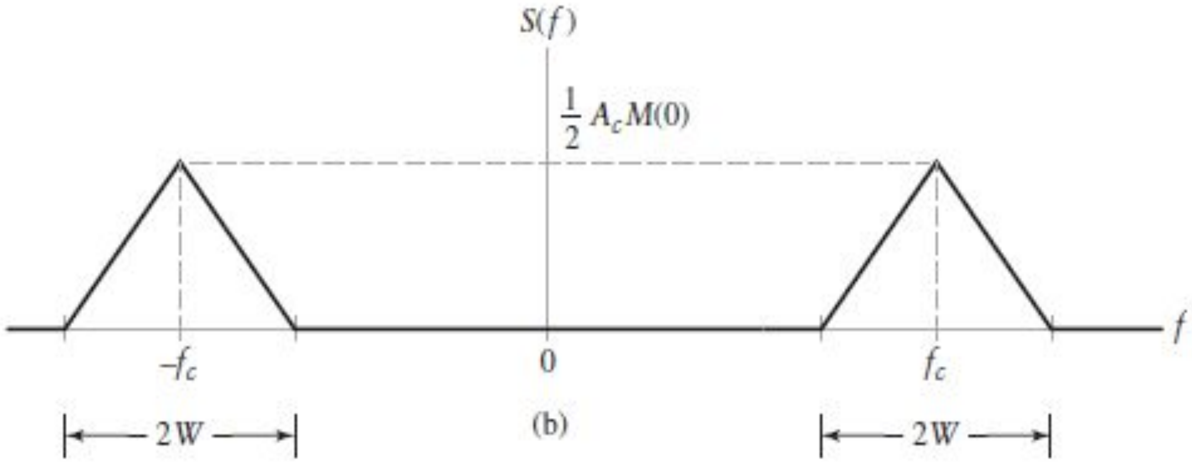
$$S(f) = \frac{A_c}{2}[M(f + f_c) + M(f - f_c)]$$

DSBSC

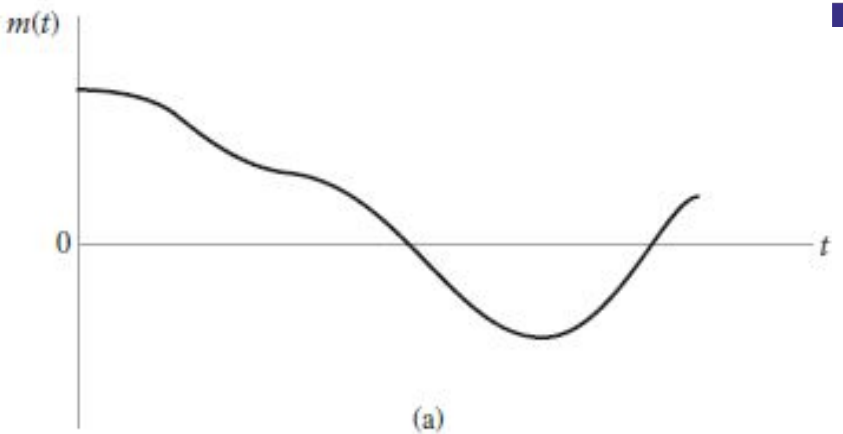
Frequency and Time Domain Representations



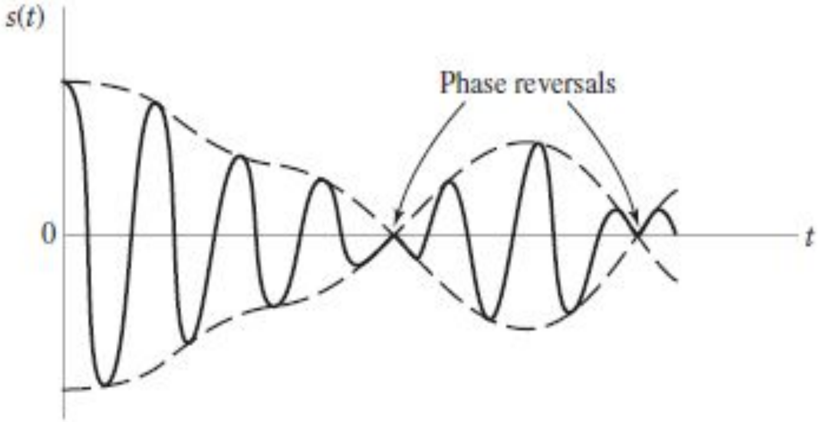
(a)



(b)



(a)



(b)

DSBSC

Side Note

- Deterministic signals do not contain information
- Only “Random” signals do
- Hence, we need to consider random process and their power spectra
- The above deterministic example is for illustration purpose only
- Eventually, we consider random signals and their spectra

Generation: Method 1 – Using Non-Linearity

Consider a device with characteristic

V_0 (t) = $V_i^2(t)$ ex – diode, transistor etc

Let V_i (t) = m(t) + c(t)   [c(t) = A_C Cos2πf_C t]

Therefore, V_0 (t) = $m^2(t) + c^2(t)$ + 2 m(t) c(t)

$$c^2(t) = A_c^2 cos^2(2\pi f_c t)$$
$$= \frac{A_c^2}{2}[1 + \cos(2\pi(2f_c)t)]$$
$$= \frac{A_c^2}{2}\delta(f) + \frac{A_c^2}{4}[\delta(f - f_c) + \delta(f + f_c)]$$

$$2m(t)c(t) = 2\frac{A_c^2}{2}[M(f - f_c) + M(f + f_c)]$$

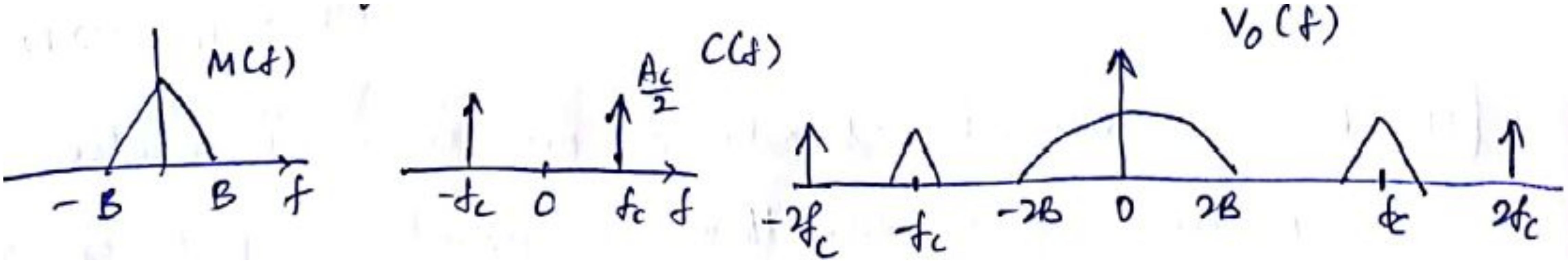
Generation: Method 1 – Using Non-Linearity

Consider a device with characteristic

V_0 (t) = $V_i^2(t)$ ex – diode, transistor etc

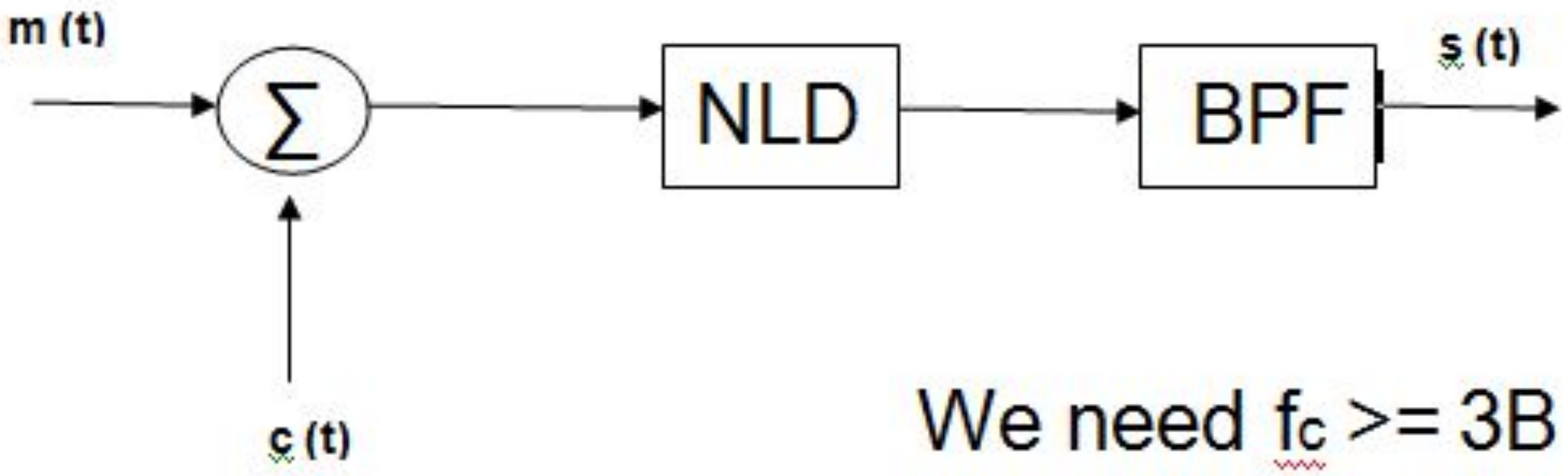
Let V_i (t) = m(t) + c(t) [c(t) = A_C Cos2πf_C t]

Therefore, V_0 (t) = $m^2(t)$ + $c^2(t)$ + 2 m(t) c(t)



Generation: Method 1 – Using Non-Linearity

- A BPF can be used to recover $m(t)\ c(t)$



We need $f_c >= 3B$

DSBSC

Generation: Method 1 – Using Non-Linearity

Suppose V_0 (t) = $a_1 V_i$ (t) + $a_2 V_i^2(t)$

Let V_i (t) = m (t) + c(t)

Then V_0 (t) = a_1 [m (t) + c (t)] + $\boldsymbol{a_2} [\boldsymbol{m^2}(\boldsymbol{t}) + \boldsymbol{c^2}(\boldsymbol{t}) + \mathbf{2\,m(t)\,c(t)}]$

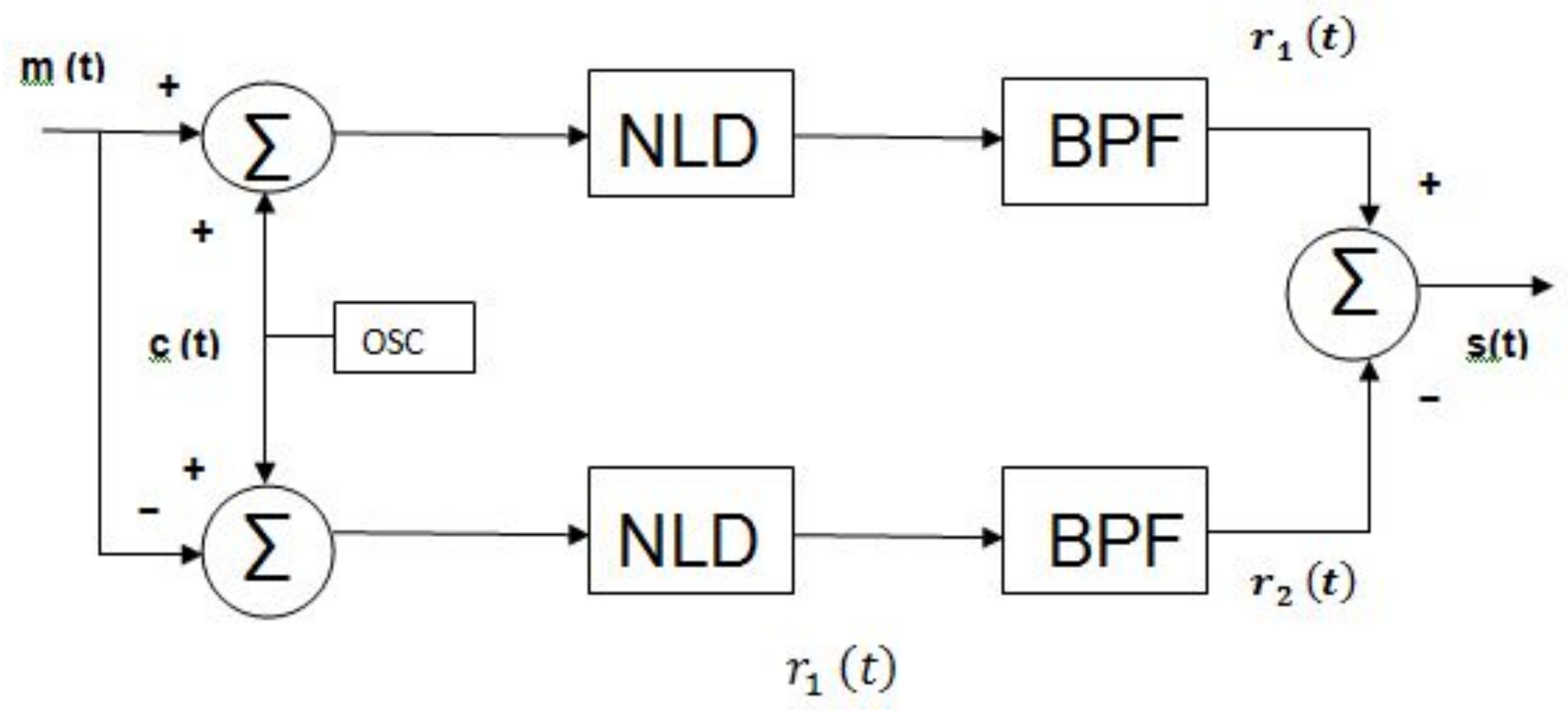
After BPF, we get $r_1(t) = a_1 C$(t) + $2a_2$ m (t) c(t)

Similarly, if V_i (t) = m(t) - c(t) ,

After BPF, we have $r_2(t) = a_1\ C$(t) - $2a_2$ m (t) c (t)

Generation: Method 1 – Using Non-Linearity

Therefore, we have the following structure



Average Power

- Let m(t) have an average power of P_m Watts

$$P_m = \int_{-\infty}^{\infty} |m(t)|^2 dt = \int_{-\infty}^{\infty} |M(f)|^2 df$$

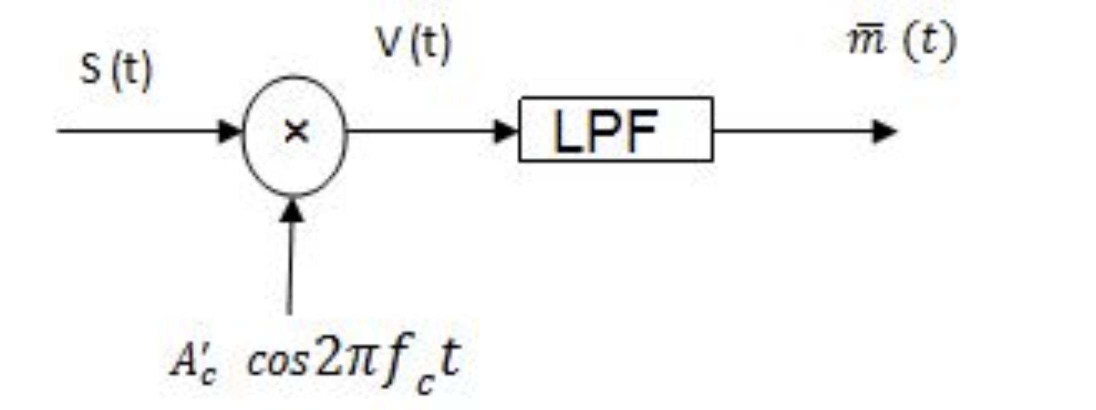
$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

$$P_s = \int_{-\infty}^{\infty} |M(f)|^2 df = \frac{A_c{}^2}{4}\left[\int_{-\infty}^{\infty} |M(f - f_c)|^2 df + \int_{-\infty}^{\infty} |M(f + f_c)|^2 df\right]$$

$$= \frac{A_c{}^2}{4}(P_m + P_m) = \frac{A_c{}^2}{2} P_m$$

Coherent Demodulation

Coherent => A Local copy of carrier (with the same phase) is used at the receiver



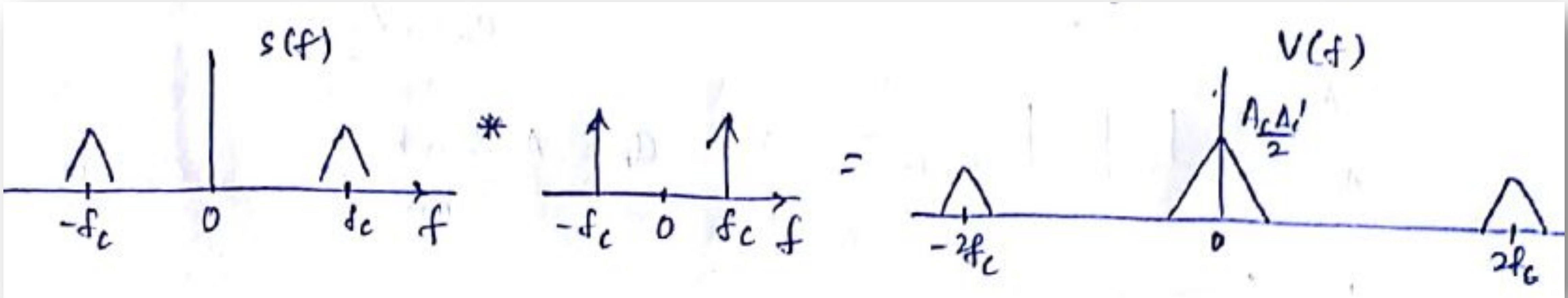
$$V(t) = s(t)\, A'_c \ cos2\pi f_c t$$

$$= A_c \ m(t) cos2\pi f_c t \times A'_c \ cos2\pi f_c t$$

$$= A_c \ A'_c \ m(t)\, Cos^2 2\pi f_c t$$

$$= \frac{A_c \ A'_c}{2} \ m(t)[1 + cos4\pi f_c t]$$

Coherent Demodulation



After LPF,

$$\bar{m}(t) = \frac{A_c \quad A_c'}{2} \; m(t)$$

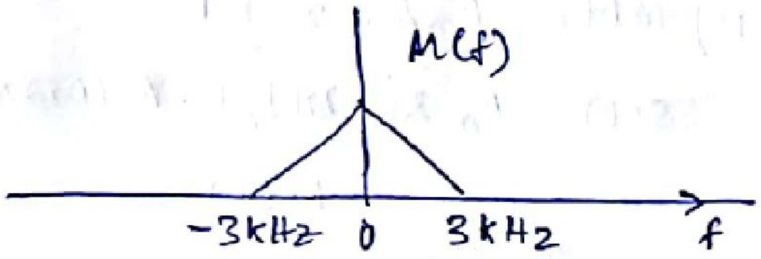
DSBSC
Example

consider the signal m(t) with the spectrum shown below. Find the required antenna height at the cutoff frequencies for

i) no modulation

ii) DSBSC with $f_c = 100$ kHz

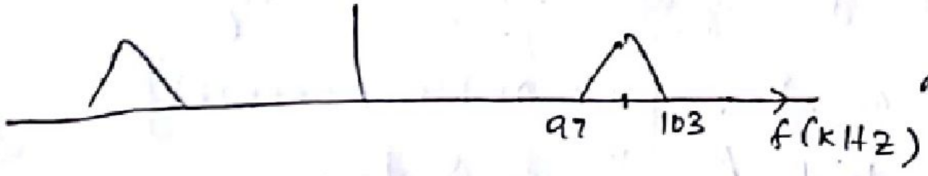
iii) DSBSC with $f_c = 1000$ kHz



Ans: i) no modulation

$$\frac{\lambda}{4} = \frac{3\times10^8}{4\times3\times10^3} = \frac{10^5}{4} = 25 \text{ km}$$

ii) $f_c = 100$ kHz



at 97 kHz, $\frac{\lambda}{4} = \frac{3\times10^8}{4\times97\times10^3} = 773$ m

at 103 kHz, $\frac{\lambda}{4} = \frac{3\times10^8}{4\times103\times10^3} = 728$ m

DSBSC

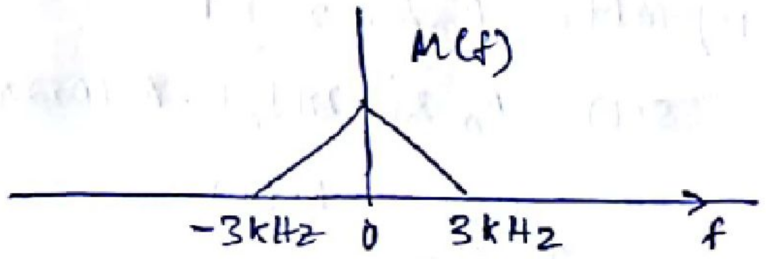
Example

Consider the signal $m(t)$ with the spectrum shown below. Find the required antenna height at the cutoff frequencies for

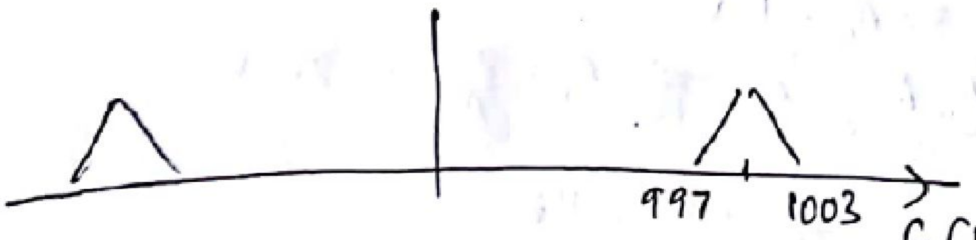
i) no modulation

ii) DSBSC with $f_c = 100$ kHz

iii) DSBSC with $f_c = 1000$ kHz



iii) $f_c = 1000$ kHz



at 997 kHz, $\frac{\lambda}{4} = \frac{3\times10^8}{4\times997\times10^3}$

$= 75.2$ m

at 1003 kHz,

$\frac{\lambda}{4} = \frac{3\times10^8}{4\times1003\times10^3} = 74.7$ m

Example: Single Tone

Draw the spectrum of the modulated wave for

i) $m(t) = A_m \cos 2\pi f_m t$

ii) $m(t) = A_m \sin 2\pi f_m t$

with f_c as the carrier frequency

$c(t) = A_c \cos 2\pi f_c t$.

Also find the power of $s(t)$

$$M(f) = \frac{A_m}{2}[\delta(f - f_m) + \delta(f + f_m)]$$

$$C(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

$$s(t) = c(t)m(t)$$

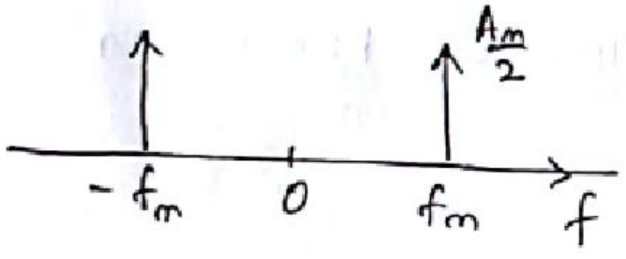
DSBSC

Example: Single Tone

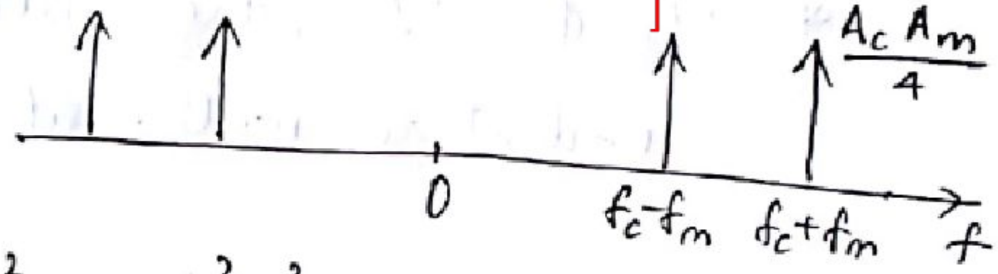
i) $m(t) = A_m \cos 2\pi f_m t$

$$\frac{A_m A_c}{4}[\delta(f - (f_c+f_m)) + \delta(f + (f_c+f_m))]$$

$$s(t) = A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t = \frac{A_m A_c}{2}\left[\cos 2\pi (f_c+f_m)t + \cos 2\pi (f_c-f_m)t\right]$$

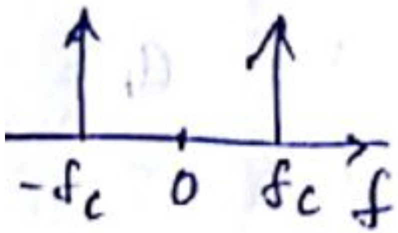


$$\frac{A_m A_c}{4}[\delta(f - (f_c-f_m)) + \delta(f + (f_c-f_m))]$$



Avg. power : $P_s = \frac{A_c^2}{2} \cdot P_m = \frac{A_c^2}{2} \cdot \frac{A_m^2}{2} = \frac{A_c^2 A_m^2}{4}$

or, from $S(f)$: $P_s = \left(\frac{A_c A_m}{4}\right)^2 \times 4 = \frac{A_c^2 A_m^2}{4}$

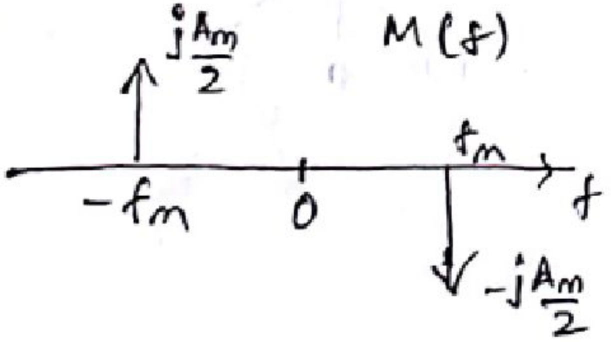


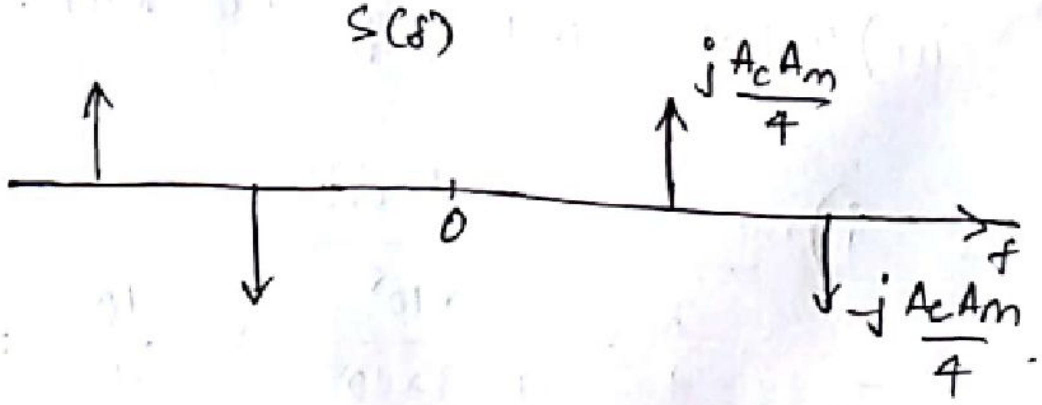
DSBSC

Example: Single Tone

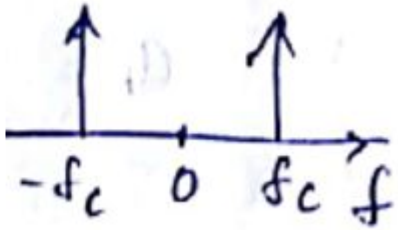
ii) $m(t) = A_m \sin 2\pi f_m t = \frac{A_m}{2j}[\delta(f - f_m) + \delta(f + f_m)] = \frac{jA_m}{2}[\delta(f + f_m) - \delta(f - f_m)]$

$s(t) = A_m \sin 2\pi f_m t \cdot A_c \cos 2\pi f_c t = \frac{A_c A_m}{2}\left[\sin 2\pi(f_c+f_m)t \mp \sin 2\pi(f_c-f_m)t\right]$





$P_s = \frac{A_c^2 A_m^2}{4}$.



$\frac{A_m A_c}{4j}[\delta(f - (f_c+f_m)) + \delta(f + (f_c+f_m))] = \frac{jA_m A_c}{4}[\delta(f + (f_c+ f_m)) - \delta(f - (f_c+ f_m))]$

$\frac{A_m A_c}{4j}[\delta(f - (f_c-f_m)) + \delta(f + (f_c-f_m))] = \frac{jA_m A_c}{4}[\delta(f + (f_c- f_m)) - \delta(f - (f_c- f_m))]$

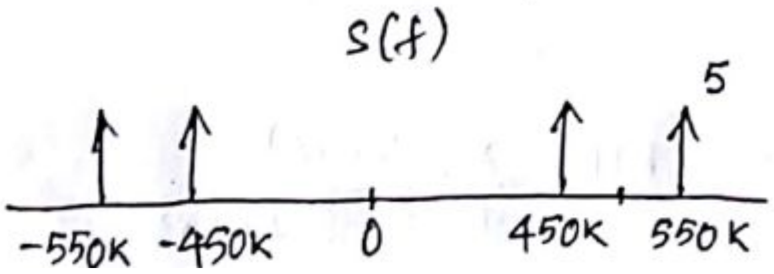
DSBSC

Example: Single Tone

The spectrum of the modulated signal is given by

$$S(f) = 5\left[\delta(f-550K) + \delta(f-450K) + \delta(f+450K) + \delta(f+550K)\right]$$

If $m(t)$ has power $P_m = 0.5\ W$, find A_c, P_s, f_c & f_m.



$m(t) = A_m \cos 2\pi f_m t \qquad P_m = \frac{A_m^2}{2} = 1/2$

$\Rightarrow A_m = 1\ V$

$\frac{A_m A_c}{4} = 5 \quad \Rightarrow A_c = \frac{20}{A_m} = 20\ V.$

$P_s = \frac{A_c^2}{2} \cdot P_m = 100W$

$f_c = 500\ KHz$

$f_m = 50\ KHz.$

AMPLITUDE MODULATION

Basics

- The amplitude of the carrier is modulated according to the message signal
- A simpler alternative to coherent demodulation is envelope detector
- Cannot be used in DSBSC, since the envelope in DSBSC is distorted
- Let carrier be denoted by $c(t) = A_c \cos(2\pi f_c t)$
- The AM wave is described as s(t) given below

$$s(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t)$$

- $K_a \rightarrow$ **Amplitude Sensitivity** $\left(\mathbf{Volt^{-1}}\right)$
- $\mu = |k_a m(t)|_{max}$ = **Modulation index**

AMPLITUDE MODULATION
Basics

-
- The AM wave is described as s(t) given below

$$s(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t)$$

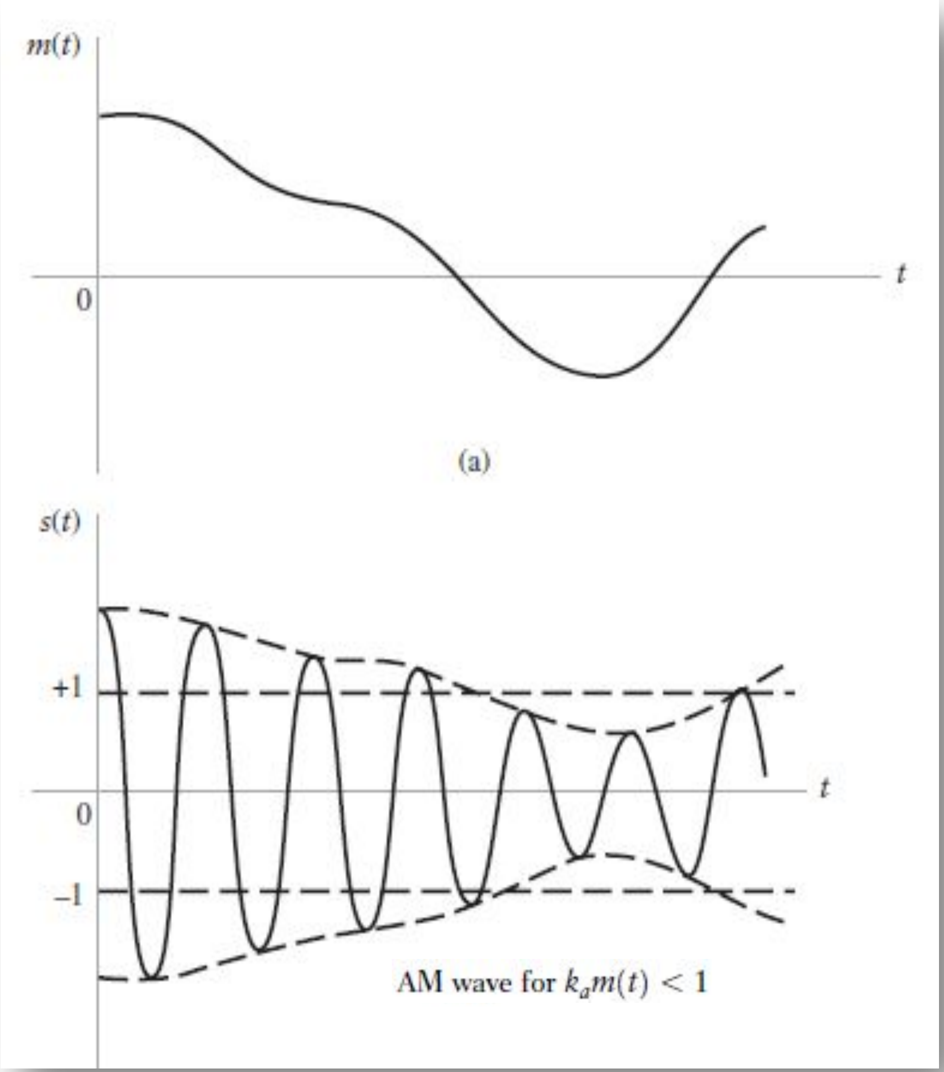
$K_a = 1$

$K_a = 0.25$

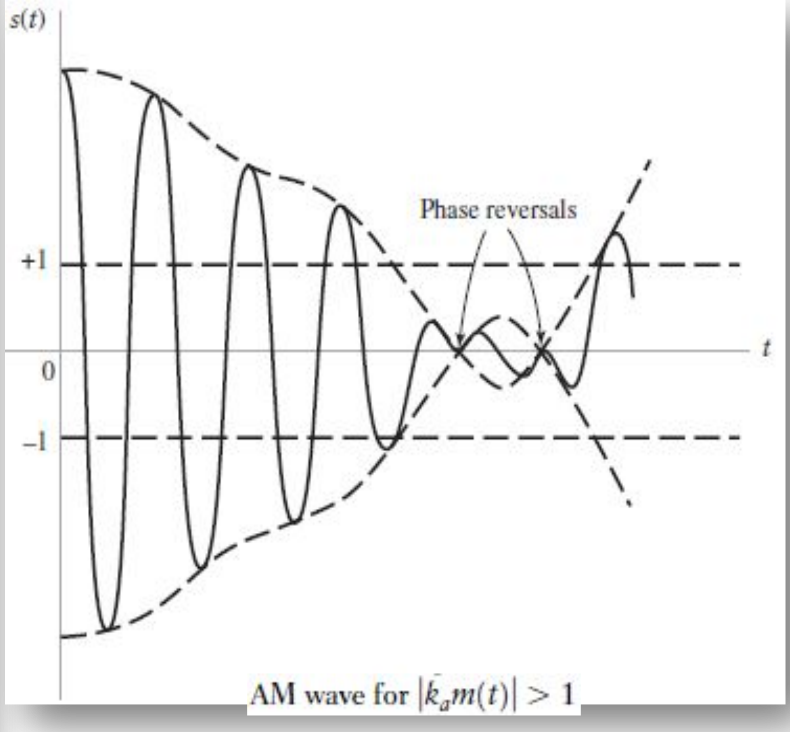
$K_a = 0.5$

AMPLITUDE MODULATION

Effect of Modulation Index



AM wave for $k_a m(t) < 1$



AM wave for $|k_a m(t)| > 1$

AMPLITUDE MODULATION

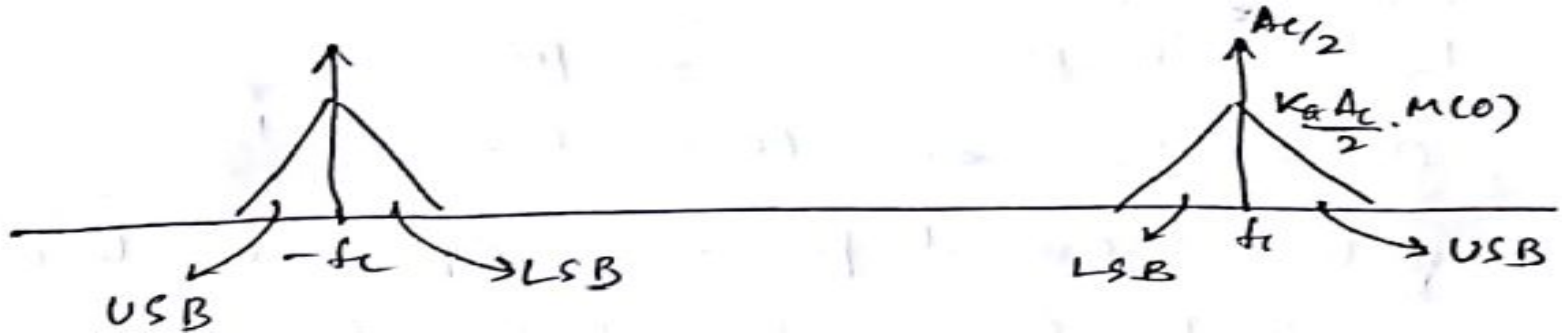
In Frequency Domain

- The modulated signal

$$S(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\textbf{Carrier Component}} + \underbrace{A_c K_a m(t) \cos(2\pi f_c t)}_{\textbf{Sidebands}}$$

- The Fourier transform gives

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{K_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$



AMPLITUDE MODULATION

Single Tone Modulation

The AM wave is given by

$$s(t) = A_c \ [1 + k_a A_m cos2\pi f_m t] \ cos2\pi f_c t$$

$$= A_c \ [1 + \mu cos2\pi f_m t] \ cos2\pi f_c t \ , \qquad \boxed{\text{Since } \mu = k_a A_m}$$

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2}\mu A_c \cos[2\pi(f_c - f_m)t]$$

$$s(t) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{\mu A_c}{4}\{\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))\} + \frac{\mu A_c}{4}\{\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))\}$$

AMPLITUDE MODULATION

Single Tone Modulation

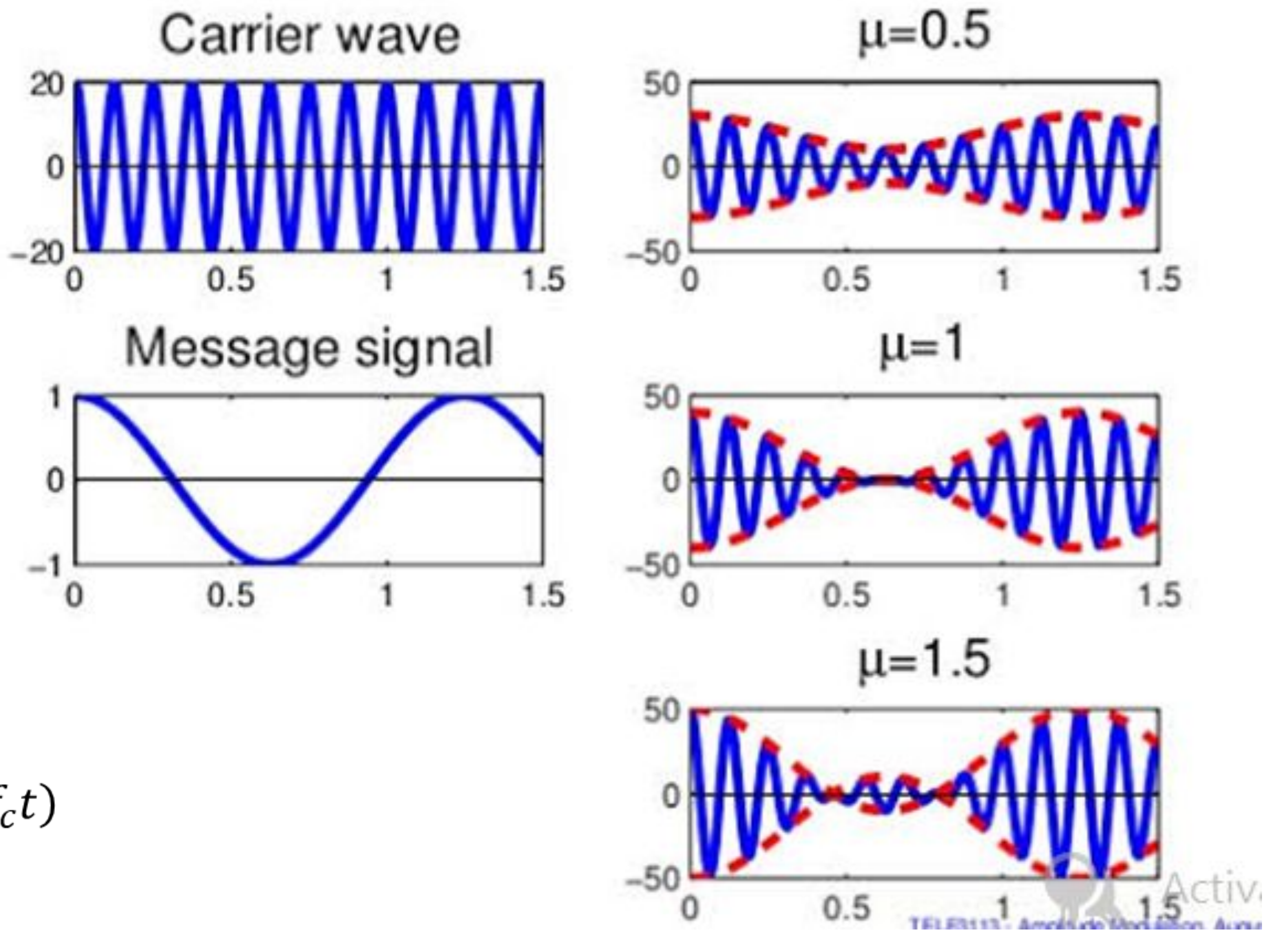
Envelope : a(t) =

a_{max} = $A_c(1 + \mu)$

a_{min} = $A_c(1 - \mu)$

or

$$s(t) = A_c[1 + K_a m(t)]\cos(2\pi f_c t)$$

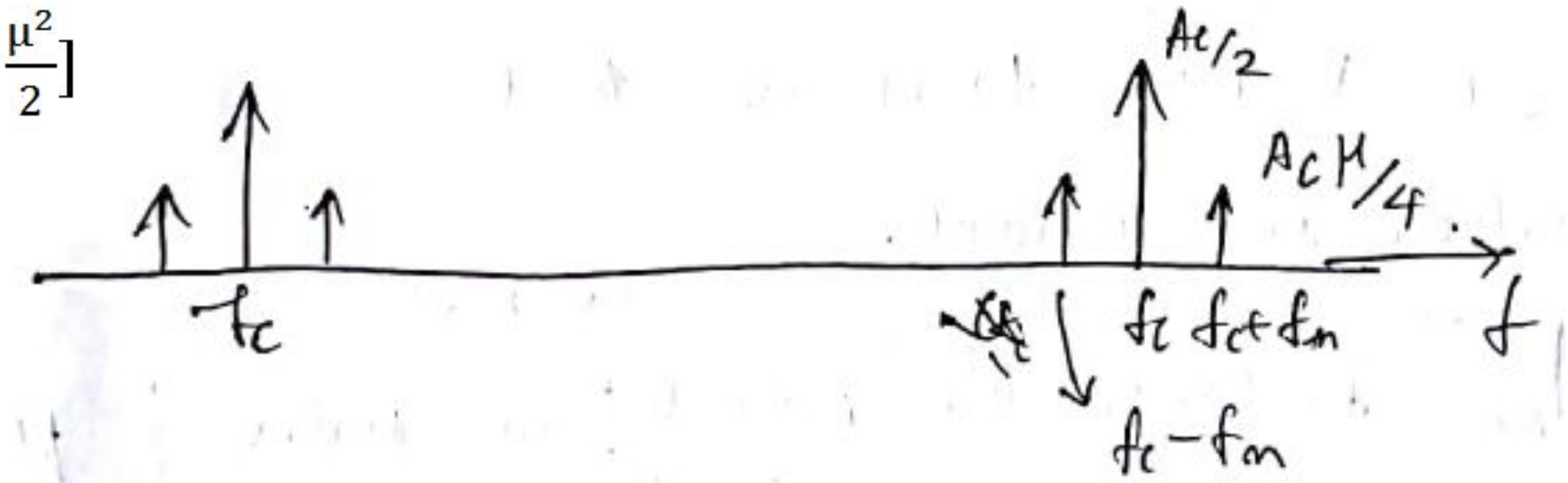


AMPLITUDE MODULATION

Single Tone Modulation

- Carrier Power : $P_c = \frac{A_c^2}{2}$
- Side band Power : $P_{SB} = \frac{A_c^2 \mu^2}{4}$
- Total Power : $P_T = P_c + P_{SB}$

$$= \frac{A_c^2}{2}[1 + \frac{\mu^2}{2}]$$



AMPLITUDE MODULATION

Modulation / Power Efficiency

$$\eta = \frac{P_{SB}}{P_T} = \frac{\mu^2}{2+\mu^2}$$

- For $0 \leq \mu \leq 1$, we have $0 \leq \eta \leq \frac{1}{3}$
- Most of the transmit power is spent on the carrier, that carries no information.

AMPLITUDE MODULATION

Generation: Method 1 – Squaring Generator

- Consider the NLD with

$$V_0(t) = a_1 V_i(t) + a_2 V_i^2(t)$$

Let $V_i(t) = m(t) + c(t)$

Cross term

Operate on the square law characteristic region

- After BPF $s(t) = a_1 C(t) + 2a_2 m(t)\, c(t)$

$$= a_1 A_c \cos 2\pi f_c t + 2a_2 A_c\, m(t) \cos 2\pi f_c t$$

$$= A_c\, a_1 [1 + \frac{2a_2}{a_1} m(t)] \cos 2\pi f_c t$$

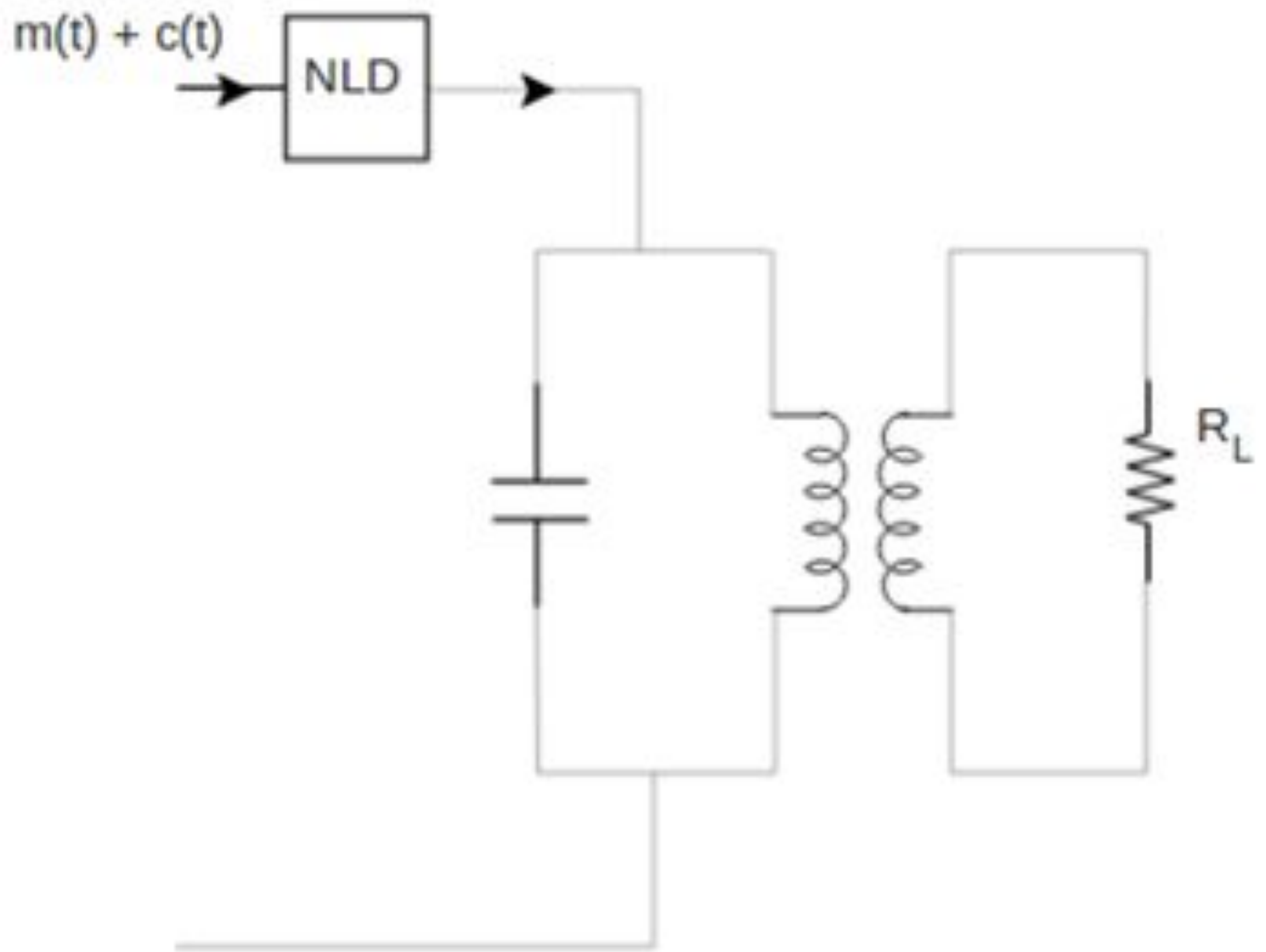
Compare with standard AM expression

$$K_a = \frac{2a_2}{a_1}$$

Amplitude sensitivity

AMPLITUDE MODULATION

Generation: Method 1 – Squaring Generator

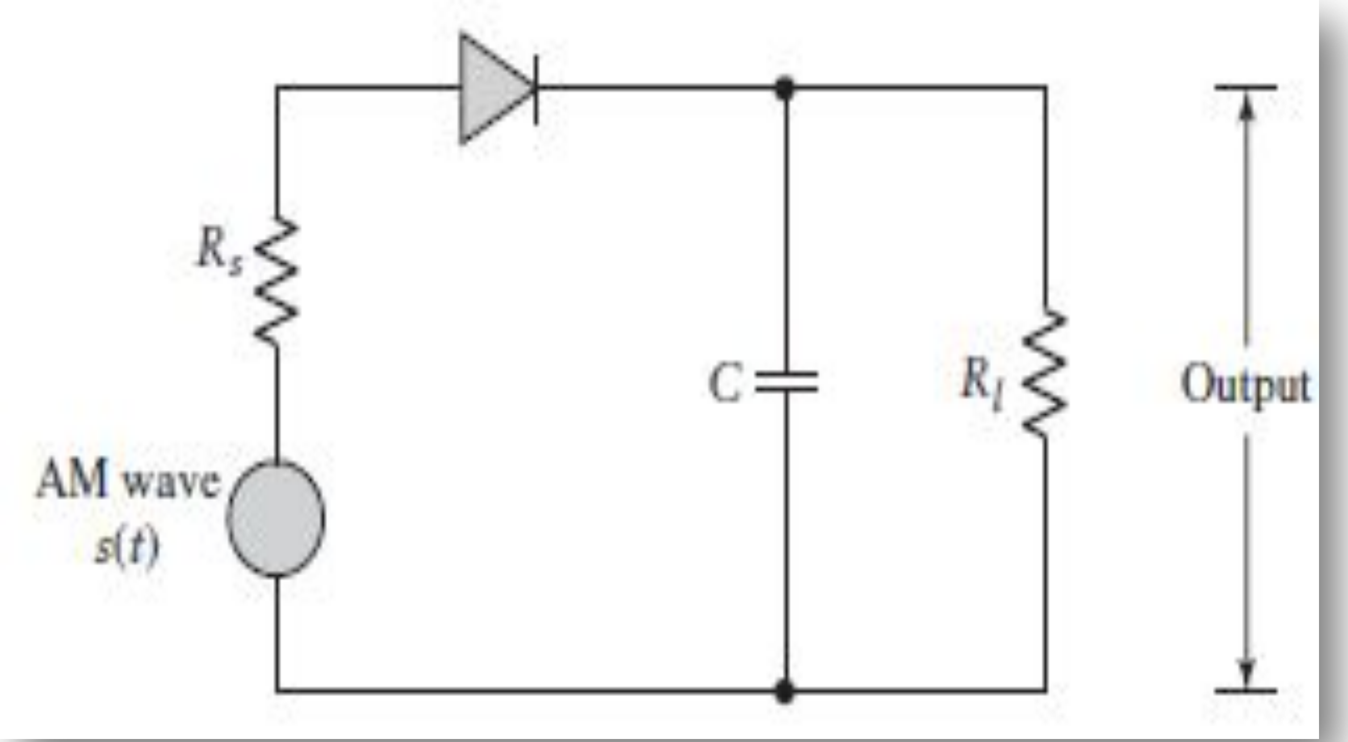


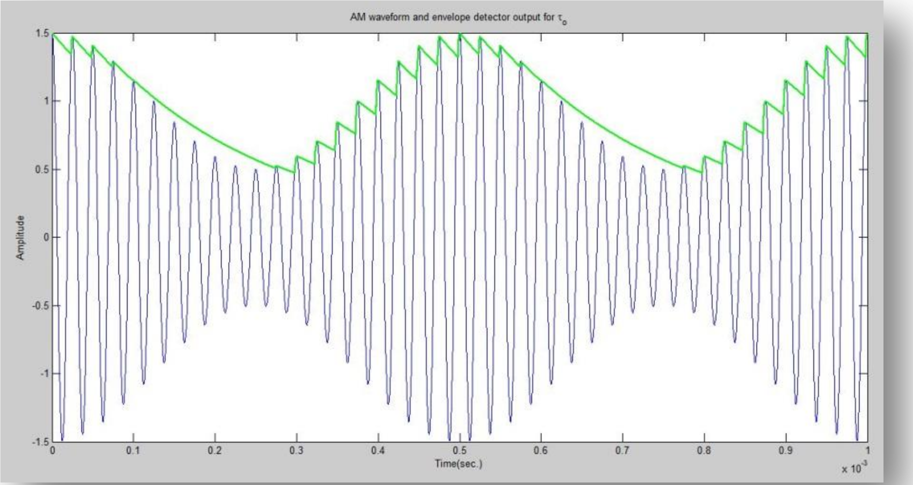
We need $f_c \geq 3B$

AMPLITUDE MODULATION

Detection: Envelope Detector

- For DSBSC we used – coherent demodulation.
- Broadcast point of view- we need a simple receiver.
- Millions of receiver
- Complexity at transmitter.





Works only if signal is not overmodulated i.e. modulation index<100, otherwise go for coherent demodulation

Detection: Envelope Detector

- Envelope detector can be used to demodulate the AM wave only if μ <=1
- If μ > 1, coherent demodulation needs to be used
- We assume ideal diode behavior
- It is assumed that he source resistance R_s is very small, and hence $R_s C << \frac{1}{f_c}$
- Hence during the positive half cycle of s(t) , (diode is forward biased) and the capacitor charges up almost instantaneously to peak voltage
- As the input signal decreases from the peak, the diode becomes reverse biased, and the capacitor discharges through R_L. R_L is chosen such $R_L C >> \frac{1}{f_c}$, hence the capacitor discharges slowly till next half cycle
- If the envelope varies w.r.t the time constant $R_L C$, then the capacitor discharge will too slow and many peaks may be missed. This is known as Diagonal clipping

AMPLITUDE MODULATION

Avoiding Diagonal Clipping

- To avoid this we need to ensure $R_L C << \frac{1}{B}$
- Overall we require $\frac{1}{f_c} << R_L C << \frac{1}{B}$
- The remaining ripple is removed by LPF

Problem 1 – Modulation Efficiency – Homework Problem

Recall that modulation efficiency of AM with a single tone is given by $\eta = \frac{\mu^2}{2+\mu^2}$.

For a general signal $m(t)$, recall that

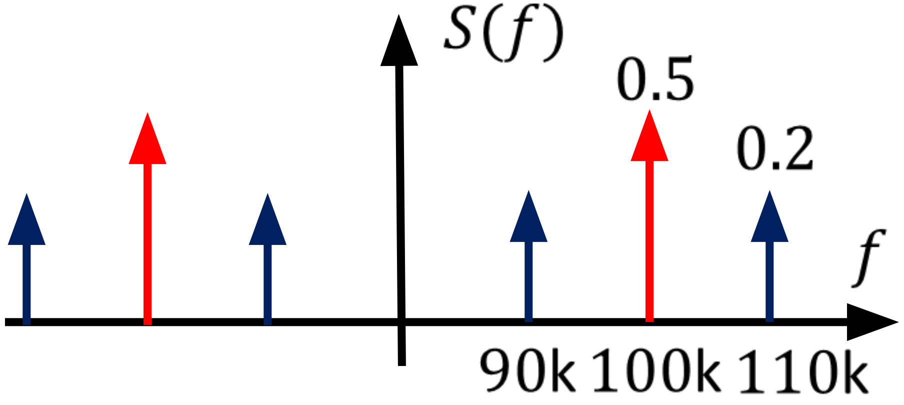
$$s(t) = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t) .$$

Let $m(t)$ be a bandlimited signal with power P_m. Find η and its maximum value, for $0 \leq \mu \leq 1$.

AM

Problem 2 – Modulation Efficiency

From the given spectrum $S(f)$, find $s(t), \mu, \eta$ and P_s.



$$\frac{A_c}{2} = 0.5 \Rightarrow A_c = 1$$

$$\frac{A_c \mu}{4} = 0.2 \Rightarrow \mu = 0.8$$

$$\eta = \frac{0.8^2}{2+0.8^2} = \frac{0.64}{2.64} = 0.24$$

$$s(t) = \left[1 + 0.8 \cos 2\pi 10^4 t\right] \cos 2\pi . 10^5 t$$

$$P_s = \frac{A_c^2}{2}\left[1+\frac{\mu^2}{2}\right] = 0.66\,W$$

Problem 2 – Modulation Efficiency

2. If P_c is 90% of P_T, find μ.

$$\eta = \frac{P_T - P_c}{P_T} = 0.1 = \frac{\mu^2}{2+\mu^2} \Rightarrow \mu = \frac{\sqrt{2}}{3}.$$

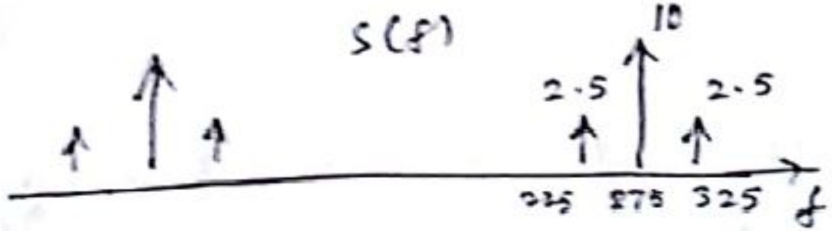
Problem 3 – Modulation Parameters

Let $s(t) = 5\cos(450\pi t) + 20\cos(550\pi t) + 5\cos(650\pi t)$. Find the following: f_c, f_m, A_c, μ, P_T and η. Also, draw the power spectrum.

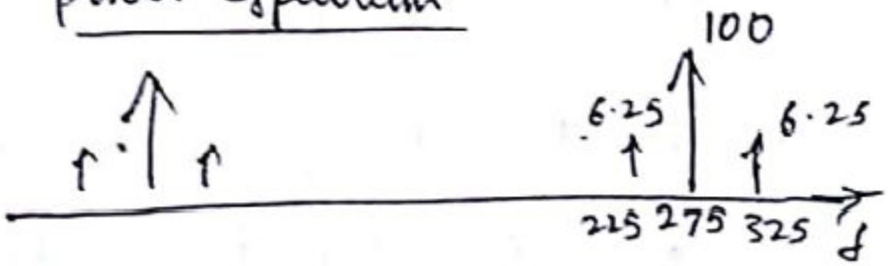
$$s(t) = 20\,[1 + 0.5\cos 100\pi t]\cos 550\pi t$$

$\therefore\ f_c = 275\,\text{Hz}, \quad f_m = 50\,\text{Hz}, \quad A_c = 20\,\text{V}, \quad \mu = 0.5,$ ~~$A_c = 10\,\text{V}$~~

$\eta = 1/9 \qquad P_T = 225\,\text{W}.$

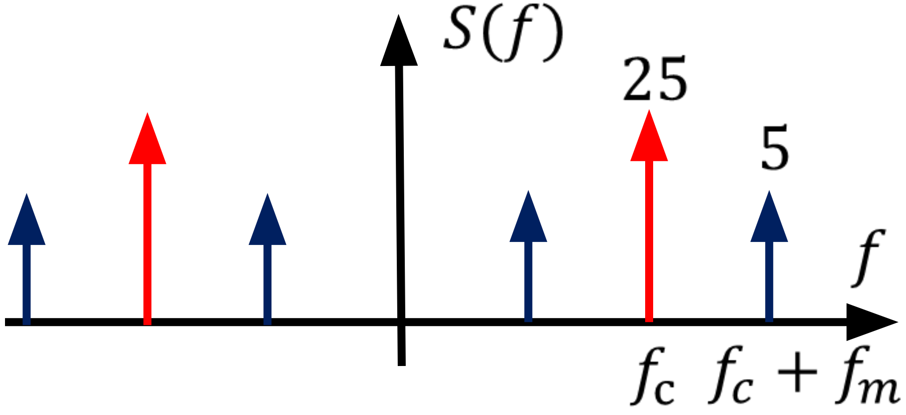


power spectrum



Problem 4 – Modulation Parameters

From the given power spectrum, find P_T, μ, and η.



From the figure, $P_c = 50W$ & $P_{SB} = 20\,W$

$P_T = 70W$, $\eta = \dfrac{P_{SB}}{P_T} = \dfrac{20}{70} = \dfrac{2}{7}$ $\qquad \dfrac{2}{7} = \dfrac{\mu^2}{2+\mu^2} \Rightarrow 4 + 2\mu^2 = 7\mu^2$ or $5\mu^2 = 4$, $\mu^2 = \dfrac{4}{5}$

$\mu = \dfrac{2}{\sqrt{5}}$.

Problem 5 – AM Wave as Voltage

Output voltage of an AM transmitter is given by

$$s(t) = 400(1 + 0.4\cos(2000\pi t))\cos(10^7\pi t)\,.$$

This voltage is fed to a load of 600 Ω. Find, f_c, f_m, P_C and P_T.

Problem 6 – Envelope Distortion

PES UNIVERSITY ONLINE

The tuned circuit of the oscillator in an AM transmission system employs a 40 μH coil & a 12nF capacitor. If the message signal is a sinusoid of frequency 5 kHz, find the lower & upper sideband frequencies & the transmission bandwidth.

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40\times10^{-6}\times12\times10^{-9}}} \approx 230 \text{ kHz}$$

$$\therefore \quad f_{USB} = 230+5 = 235 \text{ kHz}$$

$$f_{LSB} = 230-5 = 225 \text{ kHz}$$

BW = 10 kHz.

AM

Problem 7 – Envelope Distortion

A 400 W carrier is modulated to a depth of 80% by a sinusoid. Find P_T.

$$P_T = \frac{A_c^2}{2}\left[1+\frac{\mu^2}{2}\right] = P_c\left[1+\frac{\mu^2}{2}\right] = 400\left[1+\frac{0.8^2}{2}\right] = 528\text{ W}$$

Problem 8 – Envelope Distortion

The total current in an AM transmitter is 5A. If the modulation index is 0.6 (single tone modulation), find the antenna current when only the carrier is sent.

$$I_T = I_c \sqrt{1+\frac{\mu^2}{2}} \quad \therefore I_c = \frac{I_T}{\sqrt{1+\mu^2/2}} = \frac{5}{\sqrt{1+\frac{0.6^2}{2}}} = 4.6\ A.$$

HILBERT TRANFORM

Definition

- Not a "Transform"
- An LTI system with impulse response h(t) = $\frac{1}{\pi t}$

$$m(t) \longrightarrow \boxed{h(t)} \longrightarrow \hat{m}(t)$$

- $\hat{m}(t) = m(t) * \frac{1}{\pi t}$ = $\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau}\, d\tau$

$$\therefore\ \hat{M}(f) = M(f) \cdot F\left\{ \frac{1}{\pi t} \right\}$$

HILBERT TRANFORM

Definition

Right sided exponential

$$x(t) = e^{-at}u(t)$$
$$a>0$$

The time reversal property of Fourier transform states that if a function $x(t)$ is reversed in time domain, then its spectrum in frequency domain is also reversed,

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt = \int_{0}^{\infty} e^{-at}e^{-j2\pi ft}dt$$
$$= \frac{1}{a+j2\pi f}$$

Using reversal property

$$\frac{1}{a-j2\pi f}$$

$$x(t) = e^{-at}u(t) - e^{at}u(-t)$$
$$X(f) = \frac{1}{a+j2\pi f} - \frac{1}{a-j2\pi f}$$
$$= \frac{-j4\pi f}{a^2+4\pi^2 f^2}$$

$$\lim_{a\to 0} x(t) =$$

sgn(t)

$$sgn(t) \overset{F}{\longleftrightarrow} \frac{1}{j\pi f}$$

HILBERT TRANFORM

Definition

WKT, $\operatorname{sgn}(t) \xleftrightarrow{F} \frac{1}{j\pi f}$

or $j\operatorname{sgn}(t) \xleftrightarrow{F} \frac{1}{\pi f}$

$\therefore$ By duality

$$\frac{1}{\pi t} \xleftrightarrow{F} -j\operatorname{sgn}(f)$$

$$\therefore \hat{M}(f) = M(f)\{-j\operatorname{sgn}(f)\}$$

$x(t) \xleftrightarrow{F} X(f)$

$X(t) \xleftrightarrow{F} x(-f)$

sgn(-f) = -sgn(f) it is odd function

- $\hat{M}(f) = \begin{cases} -jM(f), & f > 0 \\ jM(f), & f < 0 \end{cases}$

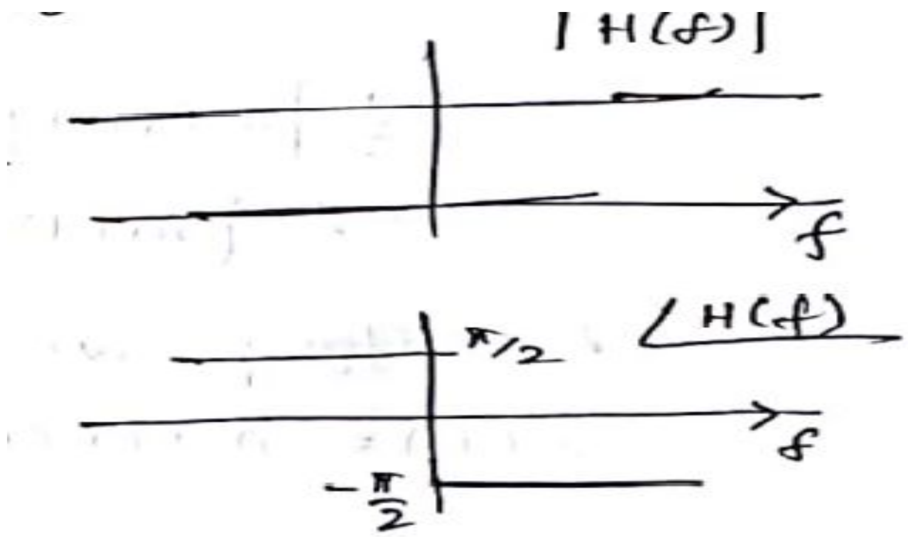
- $\hat{M}(f) = \begin{cases} M(f)e^{-j\frac{\pi}{2}}, & f > 0 \\ M(f)e^{j\frac{\pi}{2}}, & f < 0 \end{cases}$

$$= \begin{cases} |M(f)|\, e^{j[\theta(f) - \pi/2]} & f > 0 \\ |M(f)|\, e^{j[\theta(f) + \pi/2]} & f < 0 . \end{cases}$$

HILBERT TRANFORM

Defination

- NOTE: Magnitude spectrum remains unchanged
- Only the phase spectrum is modified
- Hilbert transform is a "Wideband " $-\frac{\pi}{2}$ phase shifter



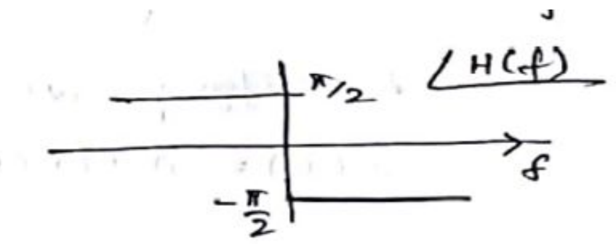
Properties

1. $\hat{m}(t)$ has the same magnitude spectrum as m(t) [except at f=0,

   since $\hat{M}(f) = 0$]

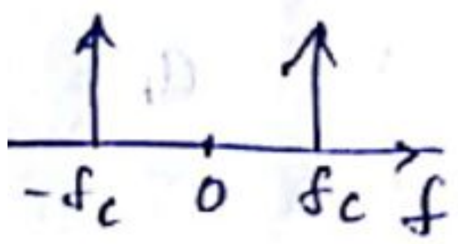
2. The phase spectrum is modified.

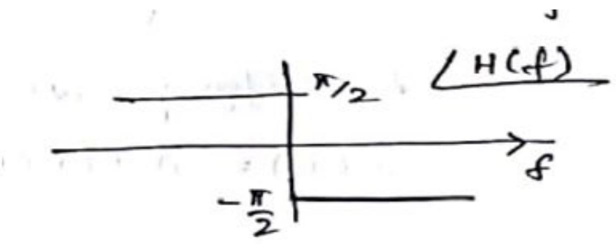
$\hat{M}(f)$ has a phase shift of $\pm\frac{\pi}{2}$ wrt m(f)

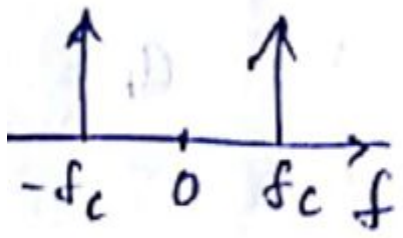


3. $H\{(\hat{m}(t)\}$ = - $m(t)$

4. $m(t)$ and $\hat{m}(t)$ are orthogonal to each other





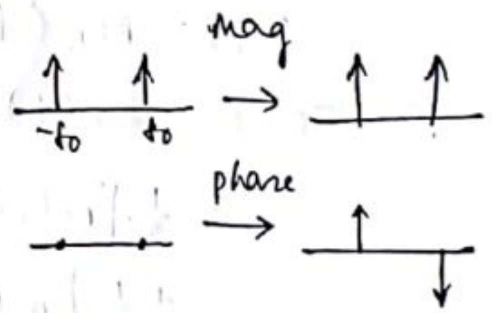


HILBERT TRANFORM

Properties

$$m(t) = A\cos 2\pi f_0 t$$

$$\hat{m}(t) = A\cos\left[2\pi f_0 t - \frac{\pi}{2}\right] = A\sin 2\pi f_0 t$$



$$m(t) = A\sin 2\pi f_0 t$$

$$\hat{m}(t) = A\sin\left[2\pi f_0 t - \pi/2\right] = -A\cos 2\pi f_0 t$$

$$H\{\hat{m}(t)\} = -m(t)$$

proof: $\hat{M}(f) = -j\,\mathrm{sgn}(f)\,M(f)$

$$\hat{\hat{M}}(f) = \left[-j\,\mathrm{sgn}(f)\right]^2 M(f)$$

$$= -M(f)$$

Properties

4. $m(t)$ and $\hat{m}(t)$ are orthogonal to each other.

proof: $$\int_{-\infty}^{\infty} m(t)\,\hat{m}^*(t)\cdot dt = \int_{-\infty}^{\infty} M(f)\,\hat{M}^*(f)\cdot df$$

[By parseval's theorem]

$$= \int_{-\infty}^{\infty} M(f)\,[-j\,\mathrm{sgn}(f)\cdot M(f)]^*\cdot df$$

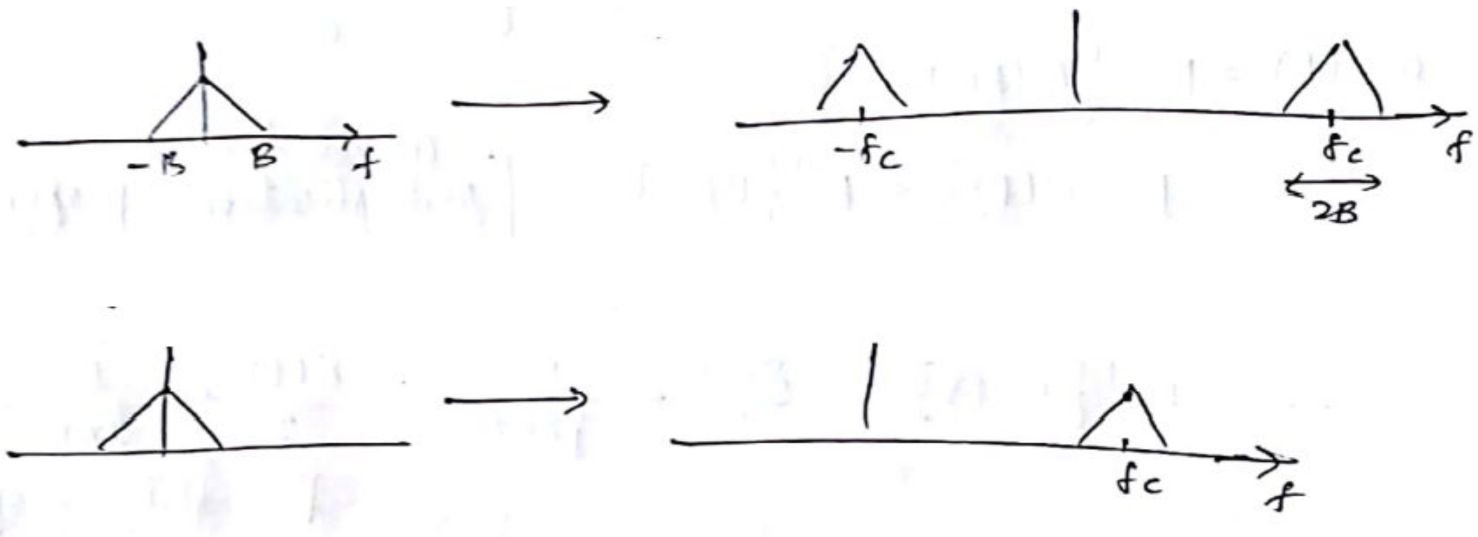
$$= j\int_{-\infty}^{\infty} \underbrace{|M(f)|^2}_{\text{even}}\cdot \underbrace{\mathrm{sgn}(f)}_{\text{odd}}\cdot df = 0.$$

$$\left[\langle x(t), y(t)\rangle = \int_{-\infty}^{\infty} x(t)\,y^*(t)\cdot dt\right].$$

Motivation

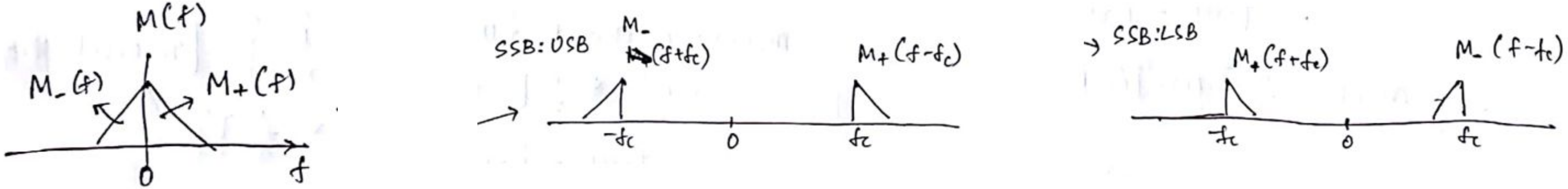
- Directly evaluating the convolution of a bandpass signal with a bandpass system is problematic.
- For sampling bandpass signals
- A single signal processing unit can deal with multiple sources

Motivation



Will B.W improve?

Spectrum is one-sided □ signal in time is complex □ Two channels to transmit (no saving of B.W)



CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Motivation

$$M_+(f) = M(f) \cdot U(f)$$

$$U(f) = 0, \quad f < 0$$
$$= 1, \quad f > 0$$

$$\therefore \; m_+(t) = F^{-1}\{M(f) \cdot U(f)\}$$

$$= F^{-1}\{M(f)\} * F^{-1}\{U(f)\}$$

$$= m(t) * F^{-1}\{U(f)\}.$$

0.5

0.5

+

$$u(t) = \frac{1}{2}[1 + \mathrm{sgn}(t)] \qquad U(f) = \frac{1}{2}[\delta(f) + \frac{1}{j\pi f}]$$

$$F^{-1}\{U(f)\} = \frac{\delta(t)}{2} - \frac{1}{j2\pi t} = \frac{\delta(t)}{2} + \frac{j}{2\pi t}$$

Duality property

CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Motivation

$$\therefore\ m_+(t) = m(t) * \frac{1}{2}\left[\delta(t) + \frac{j}{\pi t}\right]$$

$$= \frac{1}{2}\left[m(t) + j\left\{m(t) * \frac{1}{\pi t}\right\}\right]$$

$$\therefore\ m_+(t) = \frac{1}{2}\left[m(t) + j\hat{m}(t)\right] \longrightarrow (1)$$

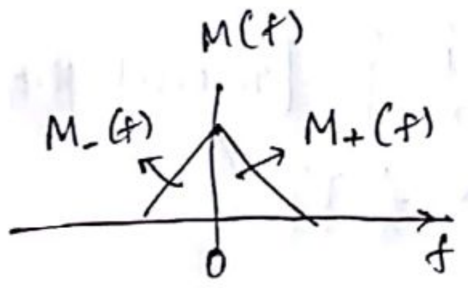
$$M_-(f) = M(f)\,U(-f) \qquad m_-(t) = m(t) * F^{-1}\{U(-f)\}$$

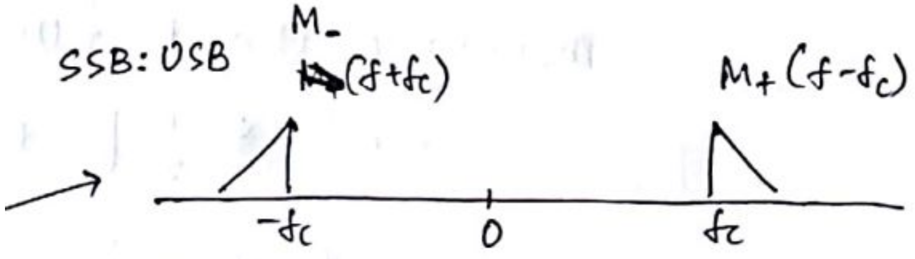
$$= m(t) * \frac{1}{2}\left[\delta(t) - \frac{j}{\pi t}\right] \quad \text{[reversal property]}$$

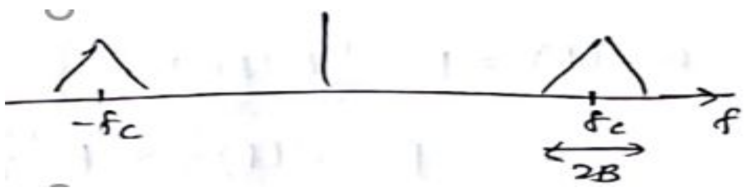
$$= \frac{1}{2}\left[m(t) - j\left\{m(t) * \frac{1}{\pi t}\right\}\right] \qquad m_-(t) = \frac{1}{2}\left[m(t) - j\hat{m}(t)\right]$$

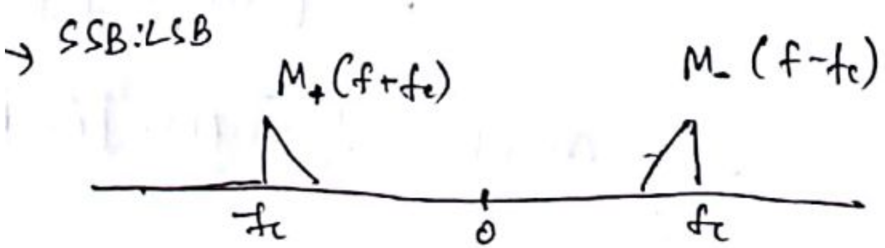
CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Motivation









For USB

$$S(f) = M_+(f-f_c) + M_-(f+f_c)$$

$$\therefore \quad s(t) = m_+(t)\, e^{j2\pi f_c t} + m_-(t)\, e^{-j2\pi f_c t}$$ [freq. shift property]

CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Motivation

$$= \frac{1}{2}\left[m(t) + j\hat{m}(t)\right]\left[\cos 2\pi f_c t + j \sin 2\pi f_c t\right]$$
$$+ \frac{1}{2}\left[m(t) - j\hat{m}(t)\right]\left[\cos 2\pi f_c t - j \sin 2\pi f_c t\right]$$

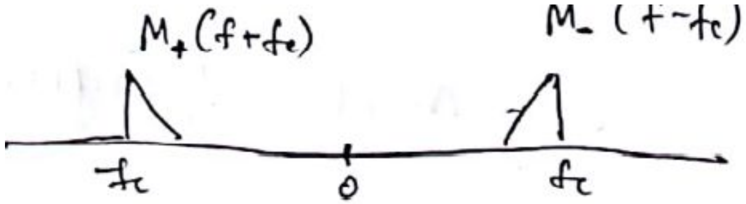
Simplifying, we get

$$s(t) = m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t$$

<u>For LSB</u>

$$S(f) = M_+(f + f_c) + M_-(f - f_c)$$

$$\therefore s(t) = m_+(t)e^{-j2\pi f_c t} + m_-(t)e^{j2\pi f_c t}$$
$$= \frac{1}{2}\left[m(t) + j\hat{m}(t)\right]\left[\cos 2\pi f_c t - j \sin 2\pi f_c t\right]$$
$$+ \frac{1}{2}\left[m(t) - j\hat{m}(t)\right]\left[\cos 2\pi f_c t + j \sin 2\pi f_c t\right]$$



$$\therefore s(t) = m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t.$$

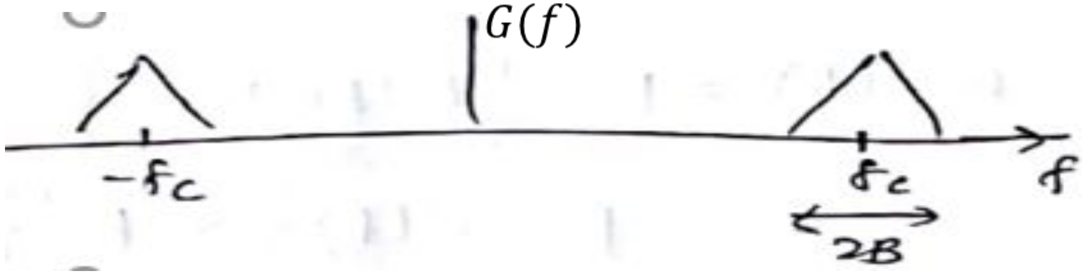
CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Pre-Envelope

- Let m(t) be a real signal. Its pre-envelope is defined as
- $m_+(t) = m(t) + j\hat{m}(t)$

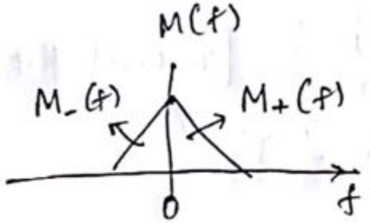
$M_+(t) = \frac{1}{2}\left[m(t) + j\hat{m}(t)\right]$

- $M_+(f) = \begin{cases} 2M(f) & f > 0 \\ M(0) & f = 0 \\ 0 & f < 0 \end{cases}$



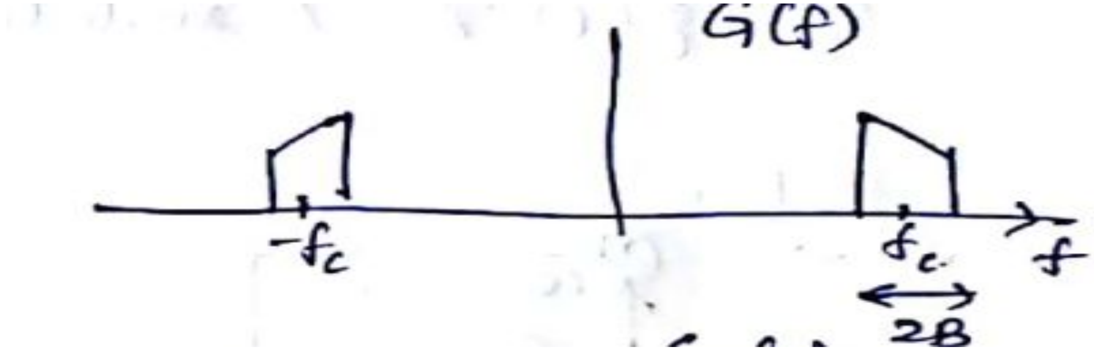
$g(t) \Longrightarrow G(f)$

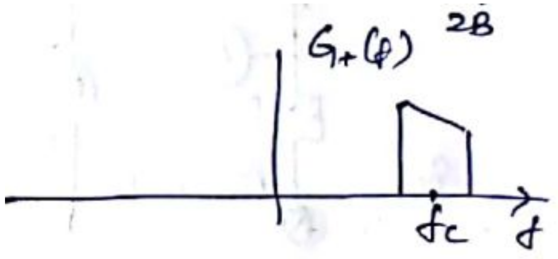
$g(t) = m(t)cos2\pi f_c t$



CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Pre-Envelope





- Its Pre-envelope is

$$g_+(t) = g(t) + j\hat{g}(t) \dashrightarrow (1)$$

- We can express as $g_+(t)$ as

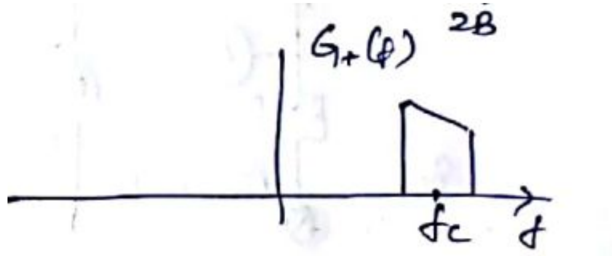
$$g_+(t) = \tilde{g}(t)e^{j2\pi f_c t} \dashrightarrow (2)$$

CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Pre-Envelope

- Let g(t) be a real signal whose spectrum is restricted to $\pm f_c \pm B\ Hz$
- g(t) is a Bandpass signal.
- Its Pre-envelope is

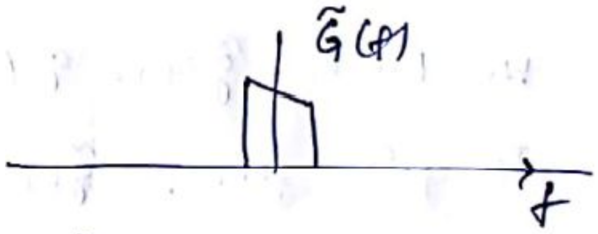
$$g_+(t) = g(t) + j\hat{g}(t) ----\rightarrow (1)$$



where $\hat{g}(t)$ is the Hilbert transform of g(t)

- We can express as $g_+(t)$ as

$$g_+(t) = \tilde{g}(t)e^{j2\pi f_c t} ----\rightarrow (2)$$



where $\tilde{g}(t)$ is a lowpass signal. It is called complex envelope of g(t)

- $\tilde{g}(t) = g_I(t) + jg_Q(t)$, where $g_I(t)$ and $g_Q(t)$ are baseband signals

CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Pre-Envelope

- From equation (1), we have

$g(t) = R_e\{g_+(t)\} = R_e\left\{\tilde{g}(t)e^{j2\pi f_c t}\right\}$; from equation (2)

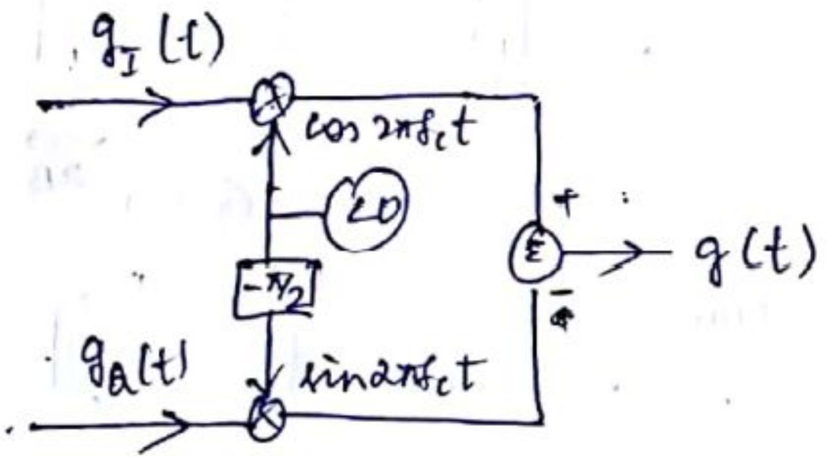
- Substitute for $\tilde{g}(t)$ we have

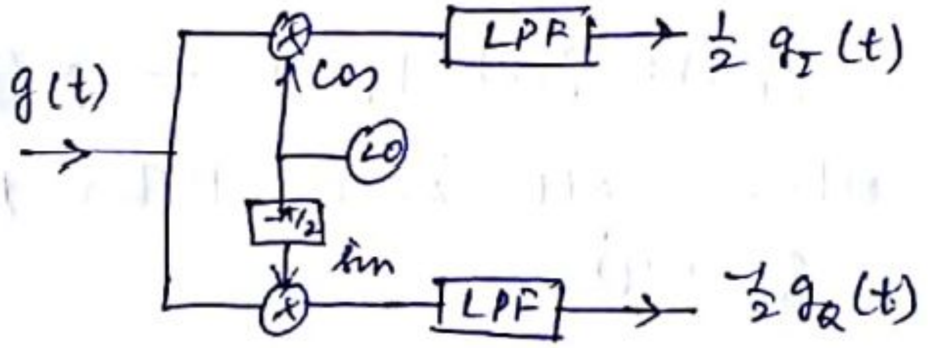
$$g(t) = g_I(t)cos(2\pi f_c t) - g_Q(t)sin(2\pi f_c t)$$

- This is the representation of a bandpass signal in terms of its baseband In-phase and Quadrature phase components.
- $\tilde{g}(t) = g_I(t) + jg_Q(t)$ is baseband "equivalent" representation of $g(t)$.

CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Pre-Envelope





$$\tilde{g}(t) = g_I(t) + j g_Q(t).$$

$$g_I(t) = a(t) \cos \phi(t)$$

$$g_Q(t) = a(t) \sin \phi(t)$$

$$\tilde{g}(t) = a(t) e^{j\phi(t)}$$

$$a(t) = \sqrt{g_I^2(t) + g_Q^2(t)} \qquad \phi(t) = \tan^{-1}\left[\frac{g_Q(t)}{g_I(t)}\right].$$

$$g(t) = a(t)\cos\phi(t)\cdot\cos 2\pi f_c t - a(t)\sin\phi(t)\sin 2\pi f_c t$$

$$= a(t)\cos\left[2\pi f_c t + \phi(t)\right]$$

Pre-Envelope

If $G_+(f)$ *is symmetric about* f_c, *then*

- $G(f)$ is even symmetric.
- $\tilde{g}(t)$ is real ➔ $\tilde{g}(t) = g_I(t)$
- $a(t) = g_I(t)$
  $\phi(t) = 0$
- $g(t) = g_I(t)\cos 2\pi f_c t = a(t) \cos 2\pi f_c t$

CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Problem 1

$g(t) = \cos 2\pi f_c t$. Find $g_+(t)$, $\tilde{g}(t)$ & $a(t)$

$$g_+(t) = g(t) + j\hat{g}(t) = \cos 2\pi f_c t + j \sin 2\pi f_c t = e^{j 2\pi f_c t}.$$

$$\tilde{g}(t) = g_+(t)\, e^{-j 2\pi f_c t} = e^{j 2\pi f_c t} \cdot e^{-j 2\pi f_c t} = 1$$

$$\therefore \tilde{g}(t) = a(t) = 1$$

CANONICAL REPRESENTATION OF BANDPASS SIGNALS

Problem 2

$g(t) = \sin 2\pi f_c t$. Find $g_+(t)$, $\tilde{g}(t)$ & $a(t)$

$$g_+(t) = g(t) + j\hat{g}(t) = \sin 2\pi f_c t - j\cos 2\pi f_c t$$

$$= -j\left[\cos 2\pi f_c t + j\sin 2\pi f_c t\right]$$

$$= -j \cdot e^{j2\pi f_c t} = e^{j[2\pi f_c t - \pi/2]}$$

$$\tilde{g}(t) = g_+(t)\,e^{-j2\pi f_c t} = e^{j2\pi f_c t} \cdot e^{-j\pi/2} \cdot e^{-j2\pi f_c t} = e^{-j\pi/2}$$

$$= -j$$

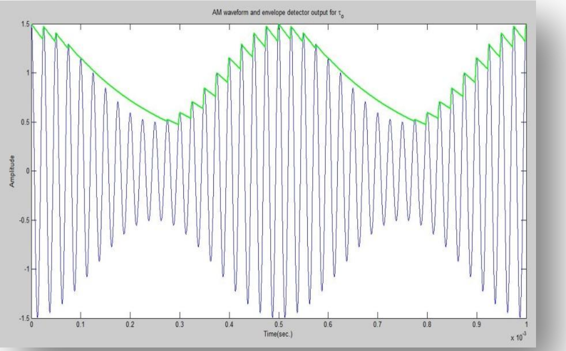
$\Rightarrow g_I(t) = 0$ & $g_Q(t) = -1$

$a(t) = 1$, $\phi(t) = -\pi/2$.

ANGLE MODULATION

Types and Details

- There are two types of angle modulation techniques
  - Frequency modulation
  - Phase modulation



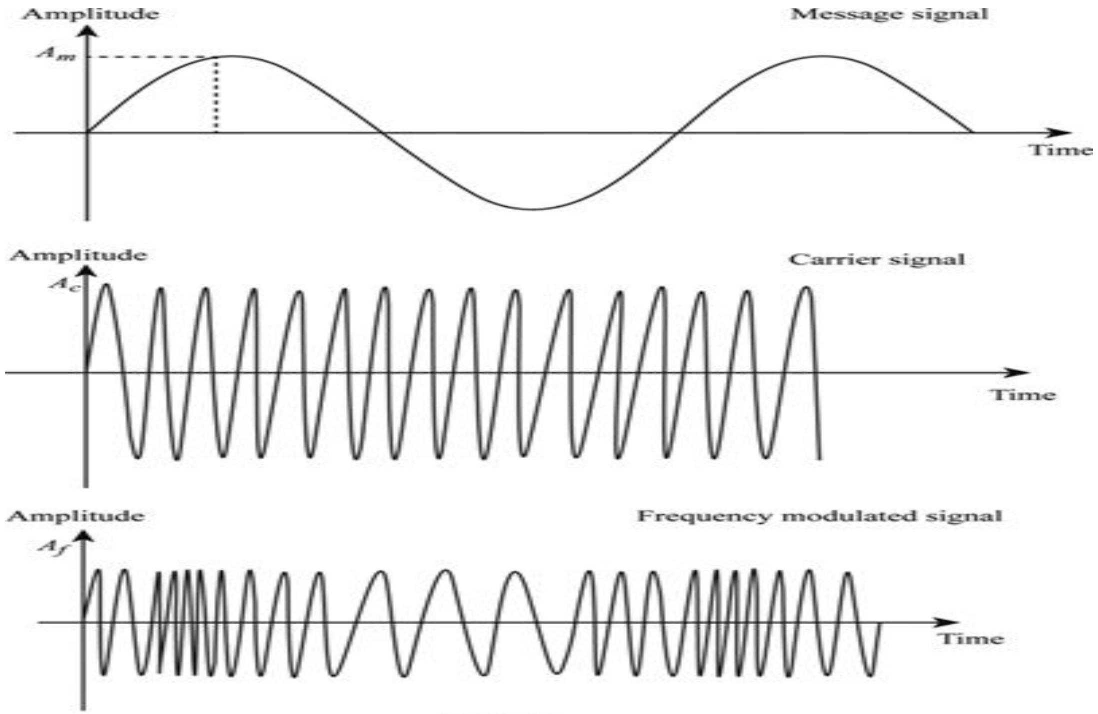


Figure 1

ANGLE MODULATION

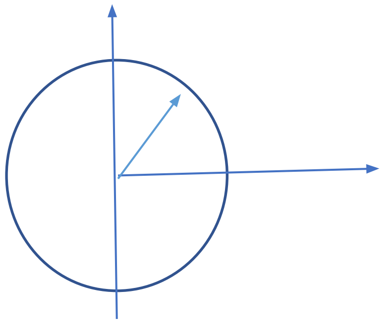
Types and Details

- Transmission BW is a concern in AM
- FM was (wrongly) thought as a technique to reduce BW arbitrarily
- Added noise at the receiver affects the amplitude
- Effect of noise can be mitigated by using angle modulation

- Consider signal $s(t) = A_c \cos \theta(t)$ -- angle is varied according to $m(t)$
- Typically, we choose $\theta(t) = 2\pi f_c t = \omega_c t$

$\theta(t)$

ω_c



$\theta(t) = \int_0^t \omega_c \, d\tau = 2\pi f_c t$

Phasor representation

T= $1/f_c$

ANGLE MODULATION

Types and Details

$$\theta_i(t) = \int_0^t \omega_i(\tau)d\tau = \int_0^t 2\pi f_i(\tau)d\tau = 2\pi \int_0^t f_i(\tau)d\tau$$

$$f_i(t) = f_c + k_f m(t)$$

If m(t)=0 □ $f_i(t) = f_c$

If m(t) = +ve □ $f_i(t)\ will\ increase\ over\ f_c$

$$\theta(t) = 2\pi \int_0^t f_i(\tau)d\tau = 2\pi\, f_c\mathrm{t} + 2\pi k_f \int_0^t m(\tau)d\tau$$

Phase Modulation

- The expression for a phase modulated signal (“similar” to AM) is given by

$$s(t) = A_c \cos[\underbrace{2\pi f_c t + k_p m(t)}_{\theta(t)}]$$

- Here, k_p is called as **phase sensitivity** (rad/V)
- If there are discontinuities in $m(t)$ ⇒ Discontinuities in phase of modulated signal ⇒ Unwanted high frequency components

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- Instantaneous frequency of carrier is modulated by the message signal
- The instantaneous frequency is modelled as $f_i(t) = f_c + k_f m(t)$
- Here, k_f is the **frequency sensitivity** (Hz/V)
- Recall that the frequency is the rate of change of phase

$$\theta(t) = 2\pi \int_0^t f_i(\tau)\, d\tau = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

- Therefore, the modulated signal is given as

$$s(t) = A_c \cos\left\{\overbrace{2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau}^{\theta(t)}\right\}$$

PHASE MODULATION

Single Tone Modulation

- We consider $m(t) = A_m \cos(2\pi f_m t)$ -- a sinusoidal signal
- It can be seen that $s(t) = A_c \cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)]$
- For illustration, plotting phase variation is difficult; hence we work with instantaneous frequency, since it is easier to plot/analyze
- The instantaneous frequency can be derived to be

$$f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt} = f_c - k_p A_m f_m \sin(2\pi f_m t)$$

- From above, note that the maximum frequency shift

$$\Delta f = k_p A_m f_m = \beta f_m$$

FREQUENCY MODULATION

Single Tone Modulation

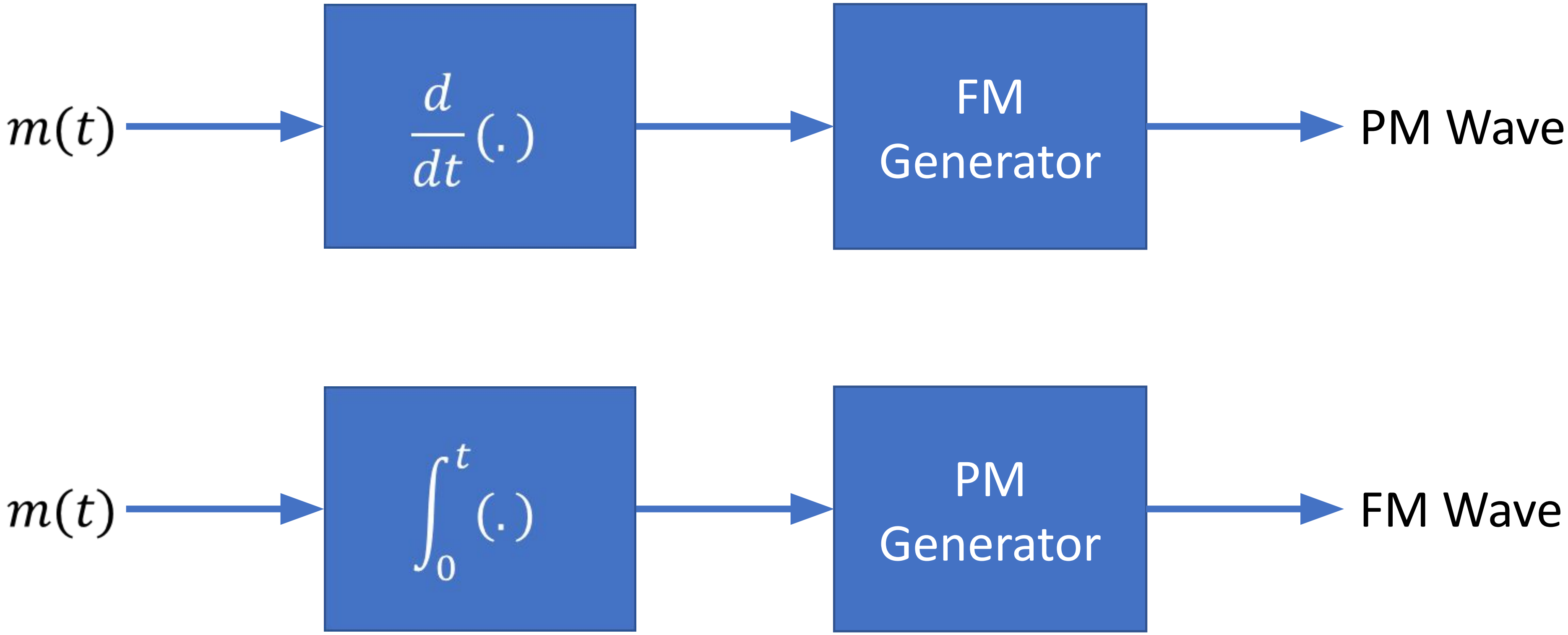
- We consider $m(t) = A_m \cos(2\pi f_m t)$ -- a sinusoidal signal
- It can be seen that

$$s(t) = A_c \cos\left\{2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m \tau) d\tau\right\}$$

- This can be further simplified as $s(t) = A_c \cos\left\{2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)\right\}$
- Therefore, maximum frequency shift $\Delta f = k_f A_m$
- Parameter $\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m}$ is defined as the modulation index for FM
- Also, β is the maximum phase deviation for FM

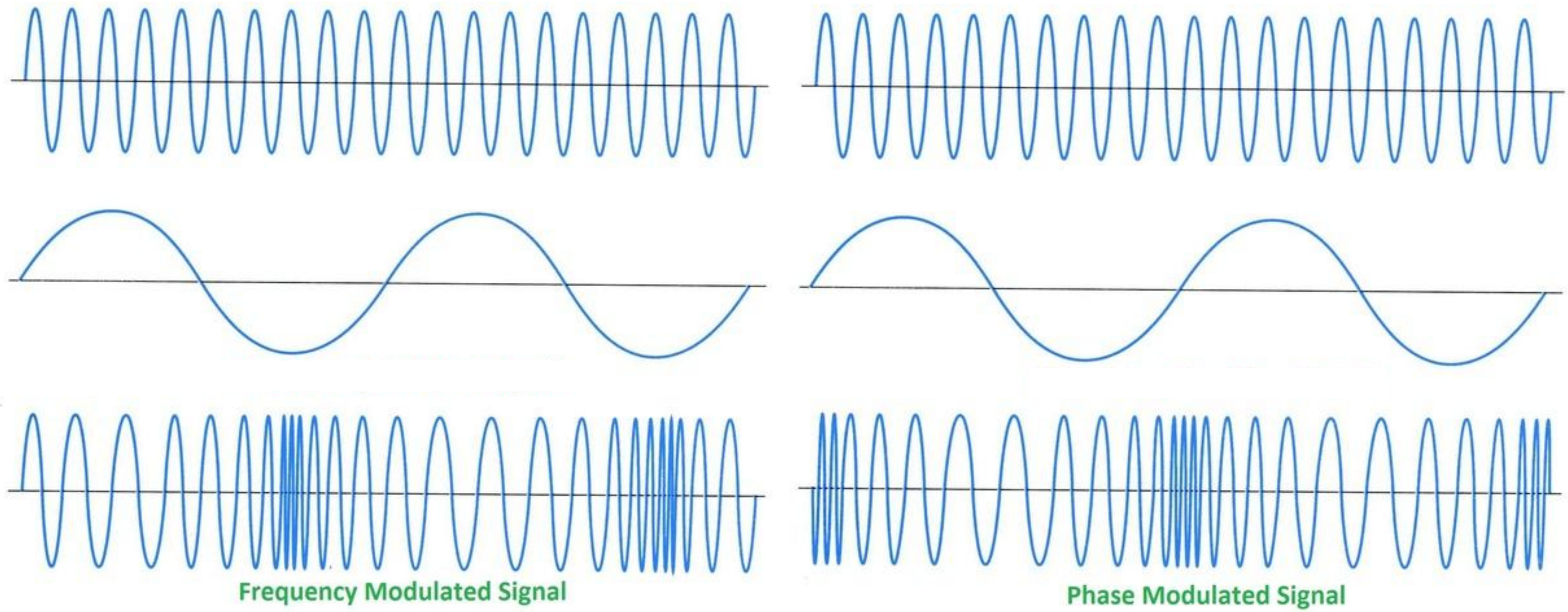
RELATION BETWEEN FM AND PM

Following the Definitions of $s(t)$ and $f_i(t)$



RELATION BETWEEN FM AND PM

Time-Domain Waveforms



Narrowband FM

- Once again, we consider $m(t) = A_m \cos(2\pi f_m t)$ -- a sinusoidal signal
- Recall that $s(t) = A_c \cos\left\{2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m \tau) d\tau\right\}$
- Since $\beta \ll 1$ for narrowband FM, we use the following approximations

$$\cos[\beta \sin(2\pi f_c t)] \approx 1 , \qquad \sin[\beta \sin(2\pi f_c t)] \approx \beta \sin(2\pi f_c t)$$

- To simplify $s(t)$ as

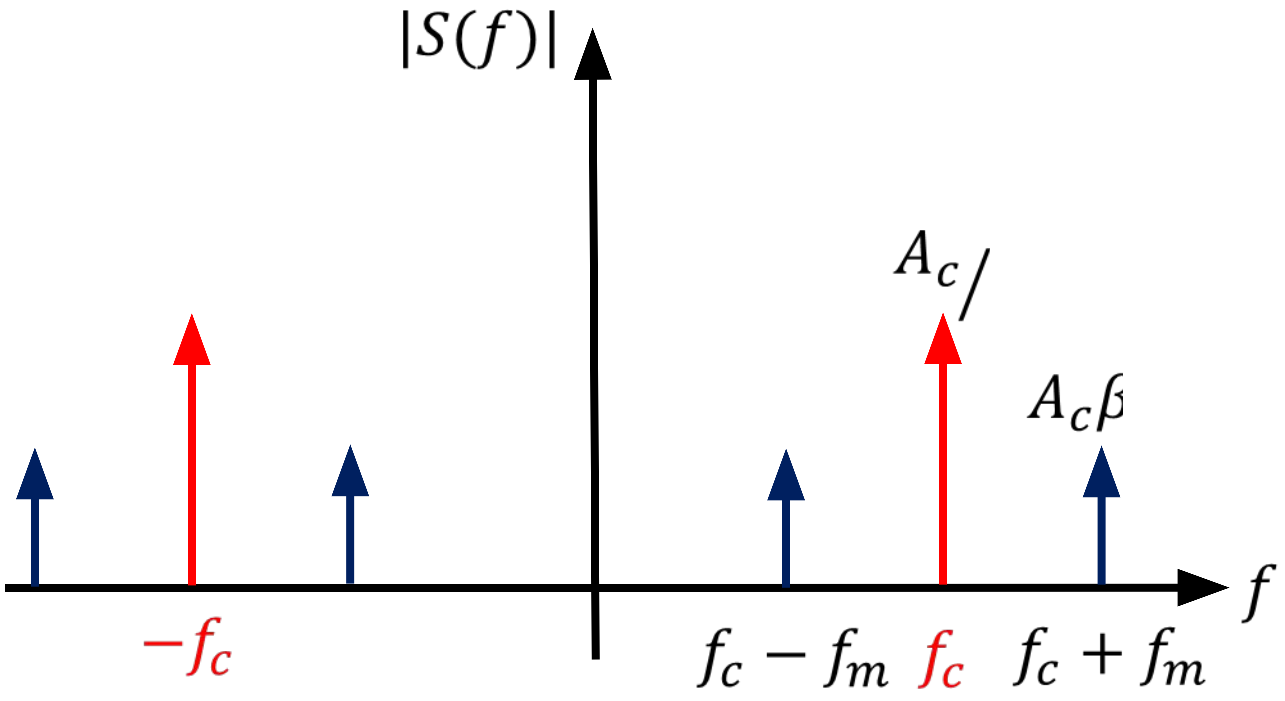
$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \beta}{2} \cos(2\pi(f_c + f_m)t) - \frac{A_c \beta}{2} \cos(2\pi(f_c - f_m)t)$$

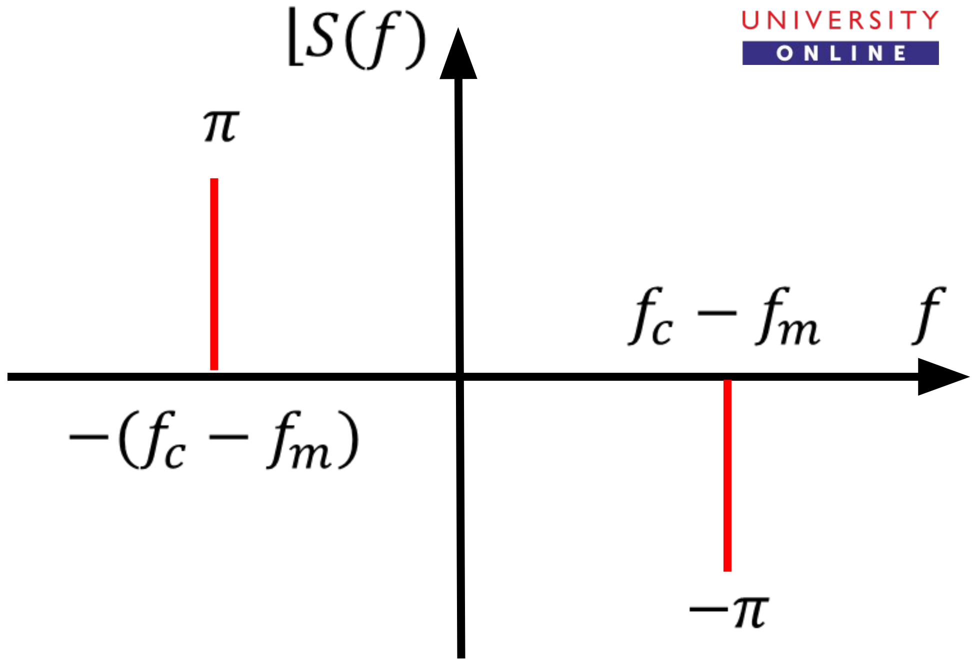
- Comparing with single tone AM equation, which was

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2} \cos(2\pi(f_c + f_m)t) + \frac{A_c \mu}{2} \cos(2\pi(f_c - f_m)t)$$

NARROWBAND FM

Spectrum and Comparison with AM





- In AM, sideband are in phase with the carrier
- In NBFM, the LSB has an offset of π radians with respect to carrier
- BW for both AM and NBFM are $2f_m$

THANK YOU

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