DIGITAL COMMUNICATION

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QUANTIZATION

Delta Modulation

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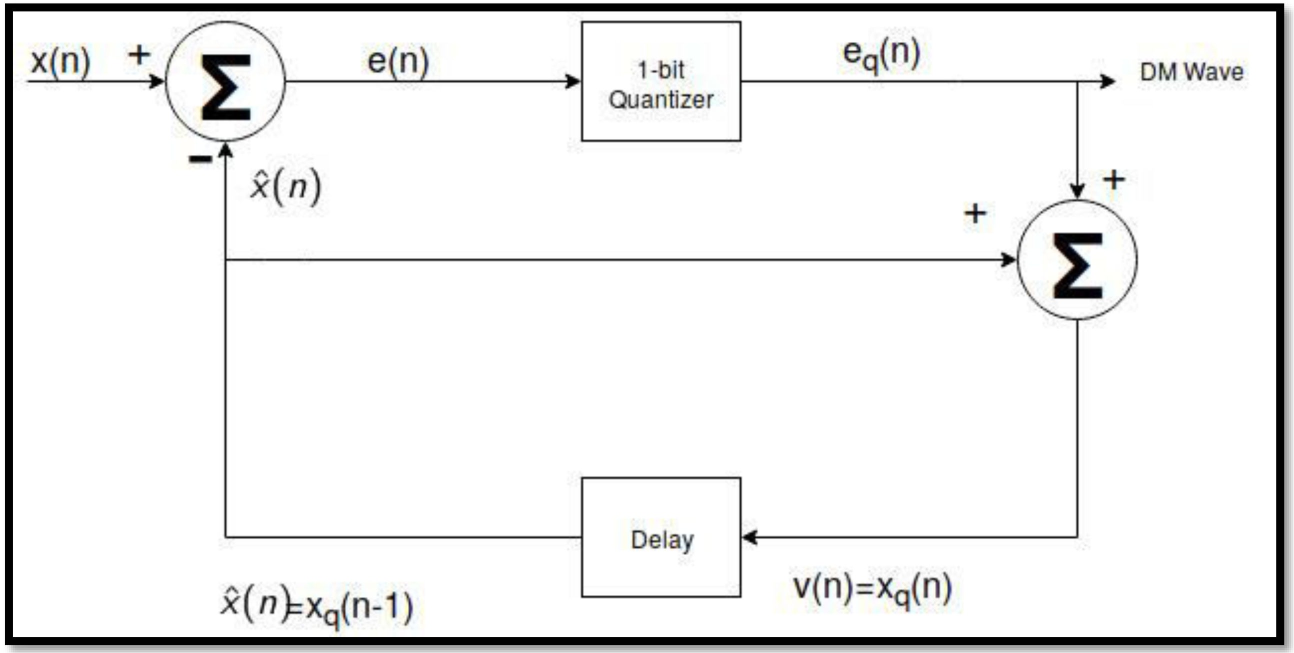
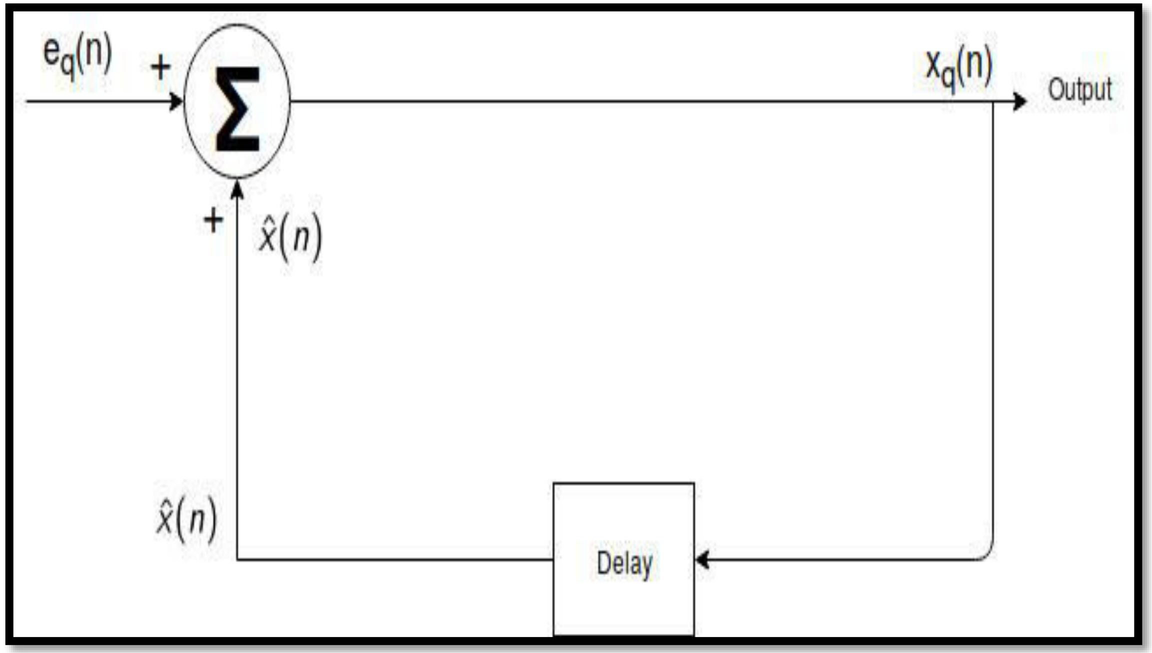
DELTA MODULATION

One-Bit DPCM

- Staircase approximation to a continuous signal
- Delta modulation (DM) is a special case of DPCM (quantize the error)
- DM quantizes the difference between a sample and its approximation by 1 bit
- With one bit of quantization, the differences are coded into two levels
- Let $+\delta$ and $-\delta$ denote positive or negative difference levels





DELTA MODULATION

Transfer Characteristics

- Step size of the DM quantizer is $\Delta = 2\delta$

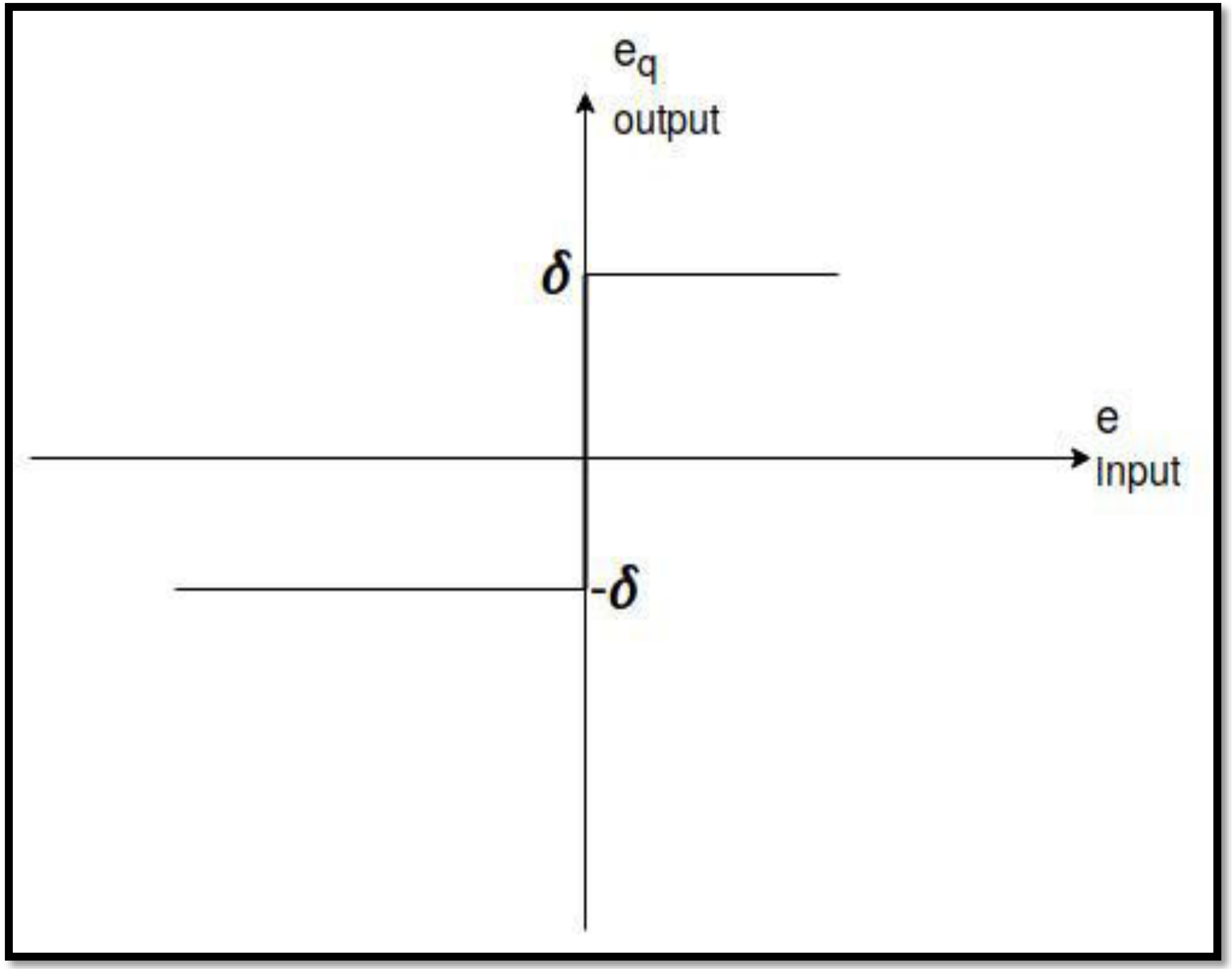
$$e_q(n) = \begin{cases} +\delta, & e(n) > 0 \\ -\delta, & e(n) \leq 0 \end{cases}$$

- Only one bit is required for quantization

- Example sequence:

$\{-\delta, \delta, \delta, -\delta, \delta\} \Rightarrow \{0,1,1,0,1\}$

- Used for compression (speech, image)



DELTA MODULATION

Equations

$$\mathrm{v}(n) = x_q(n) = e_q(n) + \hat{x}(n)$$

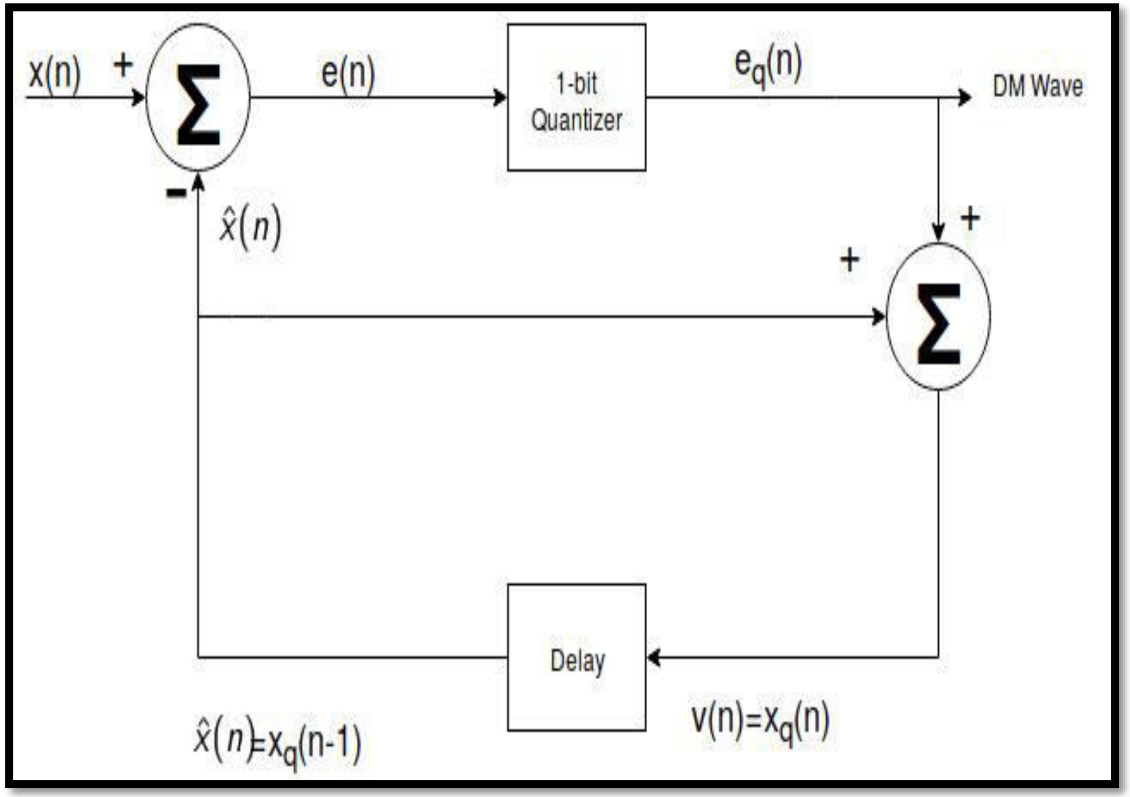
$$= e_q(n) + x_q(n-1)$$

$$= e_q(n) + e_q(n-1) + \hat{x}(n-1)$$

$$= e_q(n) + e_q(n-1) + x_q(n-2)$$

$$= e_q(n) + e_q(n-1) + e_q(n-2) + \hat{x}(n-2)$$

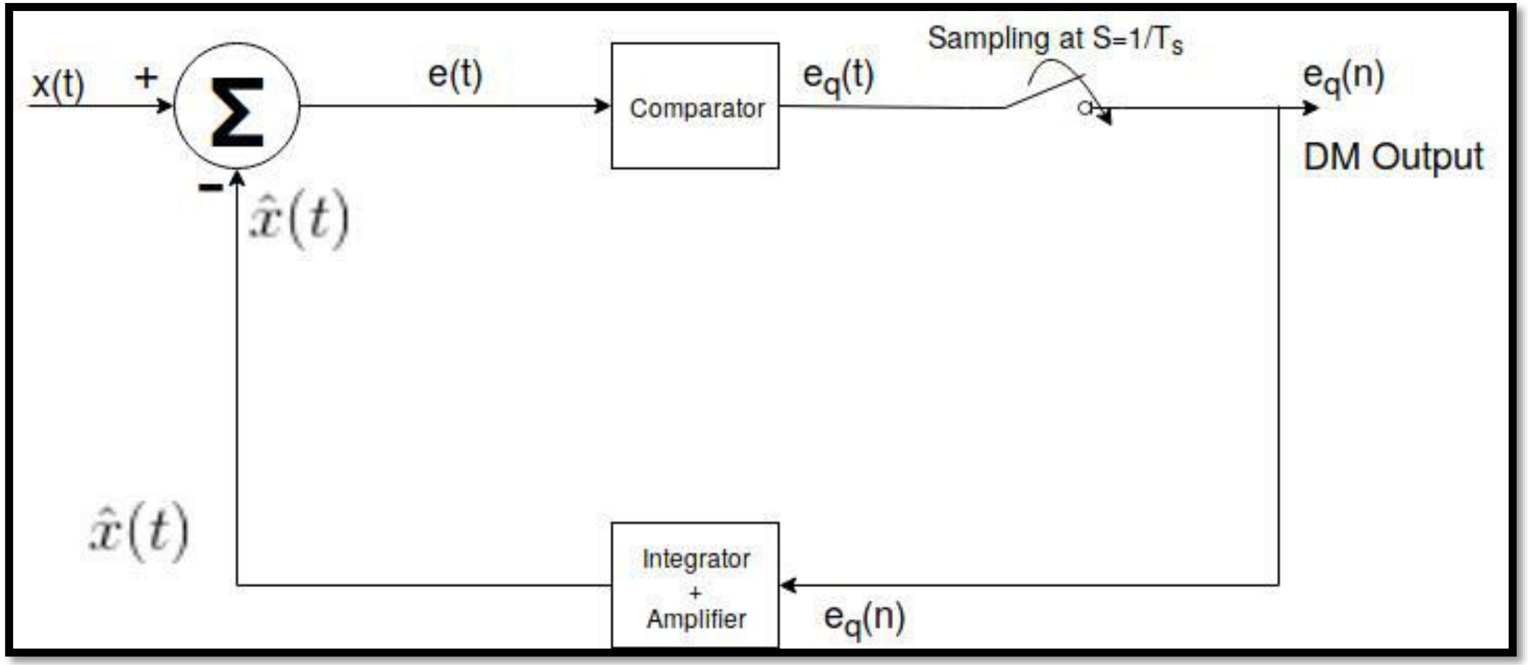
$$= \sum_{k=1}^{n} e_q(n) + \hat{x}(0)$$



If $\hat{x}(0)$=0, $\mathrm{v}(n) = x_q(n) = \sum_{k=1}^{n} e_q(n)$

DELTA MODULATION

Practical Implementation

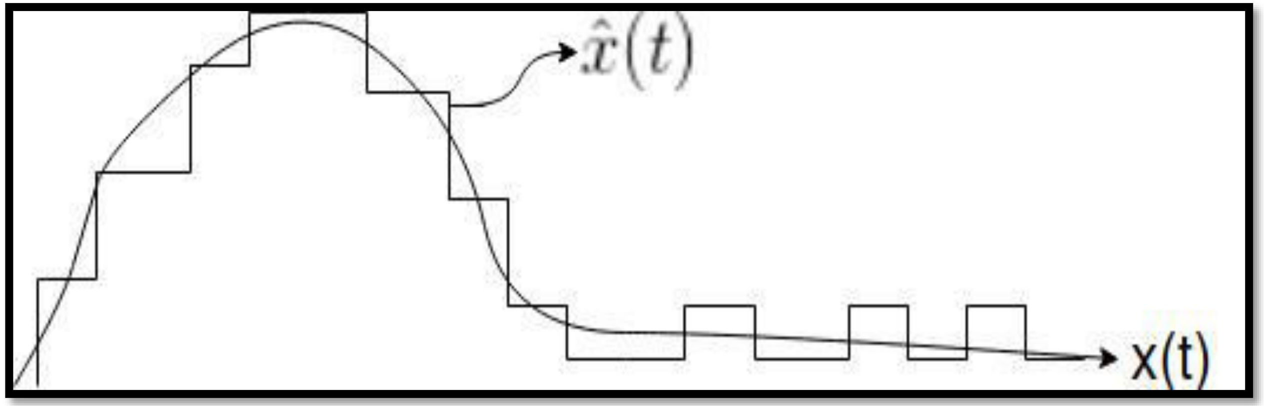


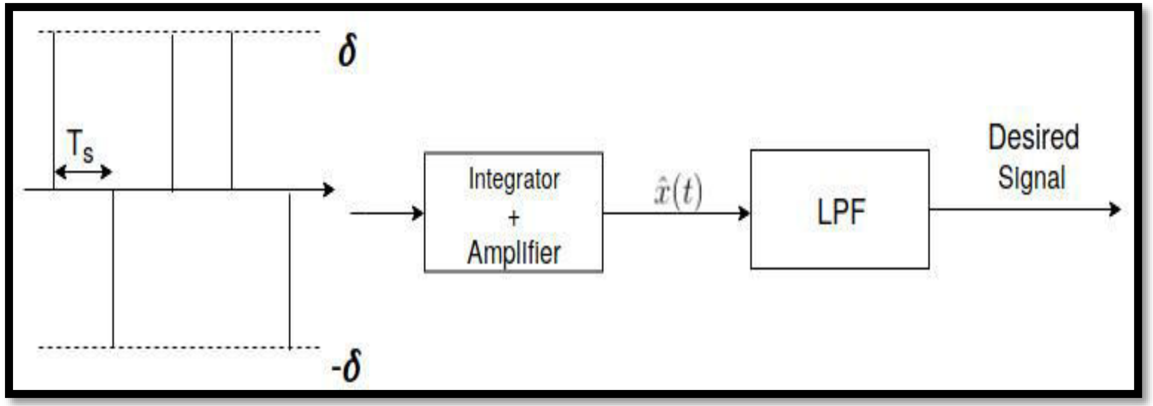
- The quantized sample is obtained as an accumulation of the quantized error samples
- At each sampling instant, the accumulator increments the approximation to the input signal by $\pm\delta$, depending on the binary output of the modulator

DELTA MODULATION

Smoothing

- An LPF is used to get a smooth reconstructed message signal $x(t)$
- It also rejects noise in the high frequency staircase approximation
- Naturally, the BW of the LPF is chosen to be the BW of $x(t)$
- DM is simple to construct, and eliminates the need for word framing in bit sequence
- Key choice of parameters: δ and f_s (quantization noise Vs. bandwidth)
- To account for rapid and slow variations in signal – should not be too high/low





DELTA MODULATION

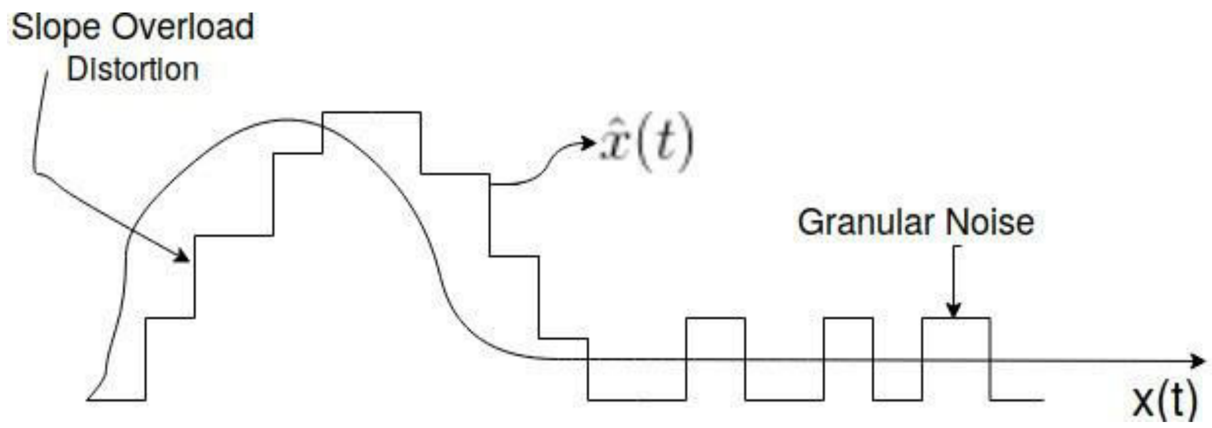
Types of Errors/Distortions

- Slope overload distortion
  - Occurs due to rapid rise or fall of the signal – staircase approximation does not catch up
  - Results in high error between staircase approximation and reconstructed signal
  - Typically due to small step size $\Delta = 2\delta$
- Granular noise
  - Occurs when the signal is relatively flat
  - Results in high error again
  - Typically due to larget step size $\Delta = 2\delta$
- Step size has to be intelligently chosen
- To match the nature of signal,

  adaptive delta modulation is used

To avoid slope overload distortion

$$\frac{\delta}{T_s} \geq \max \frac{dx(t)}{dt}$$



DELTA MODULATION

Problem 1

Let $x(t) = \; A\cos 2\pi f_o t$. For a given δ and T_s, what is the maximum value of A that to avoid slope overload distortion? Also find the maximum signal power.

Solution

Given that $x(t) = A\cos 2\pi f_0 t$, ${}^{dx(t)}\!/_{dt} = 2\pi f_0 A \sin 2\pi f_0 t$, and $\max\left|{}^{dx(t)}\!/_{dt}\right| = 2\pi f_0 A$

Recall that to avoid slope overload distortion, we need

$$\frac{\delta}{T_s} \geq \max\left|{}^{dx(t)}\!/_{dt}\right|$$

This implies that $\delta f_s \geq 2\pi f_0 A$, which simplifies to $A \leq \frac{\delta}{2\pi} \times \frac{f_s}{f_0}$

Recall that for the given $x(t)$, $P = {}^{A^2}\!/_{2}$. Therefore,

$$P \leq \frac{\delta^2}{8\pi^2} \times \frac{f_s^2}{f_0^2}$$

Problem 2

A message signal has a bandwidth of W Hz, and is sampled at Nyquist rate. The samples are quantized with 8-bit PCM. If the resultant bit rate is 40 Mbps, find W.

Solution

Given that the bit rate is 40 Mbps, the number of words/samples per second = $\frac{40}{8} = 5$ Mbps.
Therefore, the sampling frequency $f_s = 5$ MHz.

Given that it was sampled at the Nyquist rate, $f_s = 2W$, and therefore, $W = 2.5$ MHz.

DELTA MODULATION

Problem 3

A signal $x(t) = 5\cos 10^5 \pi t$ is sampled at twice the Nyquist rate. It has to be quantized such that SNR is atleast 43 dB. Find the minimum possible bitrate.

Solution

Given that $x(t) = A\cos 2\pi f_0 t$, recall that $SNR_{dB} = 6N + 1.76 \geq 43\ dB$ (given). Simplification on N yields $N \geq 6.87$, which becomes $N \geq 7$.

Note that $f_s = 4f_0 = 4 \times 5 \times 10^4 = 200$ kHz.
Therefore, minimum possible bit rate is $7 \times 200 = 1.4$ Mbps.

Problem 4

A signal $x(t) = 5\cos 10^5\pi t + 10\cos 10^4\pi t$ is sampled at twice the Nyquist rate. It has to be quantized such that SNR is atleast 43 dB. Find the minimum bitrate and Δ.

Solution

Given that both cos signals in $x(t)$ are orthogonal, and total power $P = \frac{5^2}{2} + \frac{10^2}{2} = \frac{125}{2}$.

Now, $\Delta = \frac{2\times 15}{2^N} = \frac{30}{2^N}$, and $\sigma_Q^2 = \frac{\Delta^2}{12} = \frac{900}{12\times 2^N}$

Therefore, $SNR = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{5}{6}\times 2^{2N}$, which reduces to $SNR_{dB} = 6N - 0.79$.

DELTA MODULATION

Problem 4

A signal $x(t) = 5\cos 10^5\pi t + 10\cos 10^4\pi t$ is sampled at twice the Nyquist rate. It has to be quantized such that SNR is atleast 43 dB. Find the minimum bitrate and Δ.

Solution

It is given that $SNR_{dB} = 6N - 0.79 \geq 43\ dB$ (given).
Simplification on N yields $N \geq 7.29$, which becomes $N \geq 8$.

Note that $f_s = 4f_m = 4 \times 5 \times 10^4 = 200$ kHz.
Therefore, minimum possible bit rate is $8 \times 200 = 1.6$ Mbps.

THANK YOU

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