# Unit-5

## TRANSMISSION LINES

Electromagnetic waves can propagate through

(i) unbounded media
Eg. TV broad casting

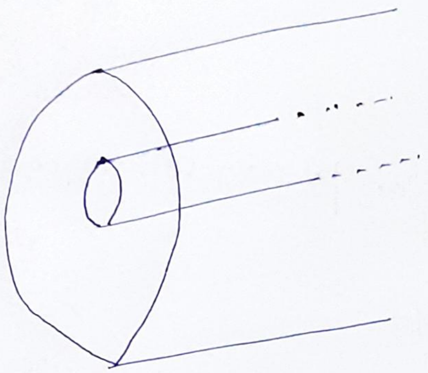
(ii) Guided structures
Eg: Transmission lines, Wave guides

In this unit we will consider wave propagation through transmission lines. Transmission lines can be analyzed using (i) EM Field theory
(ii) Electric circuit theory

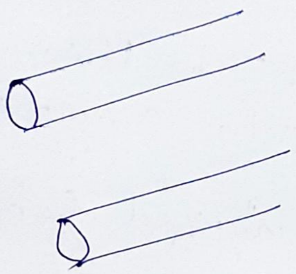
Transmission lines analyzed from Electro-Magnetic field theory is complex, because it involves variables $(x, y, z, t)$. There fore it is not preferred.

Transmission lines analyzed from Electric circuit theory involves transmission line parameters R = Resistance per unit length L = Inductance per unit length, G = ~~Capacitance~~ conductance per unit length and C = Capacitance per unit length. It is simple and convenient for analysis.

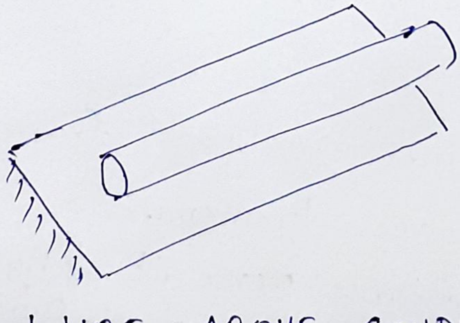
# Different types of transmission lines



COAXIAL LINE



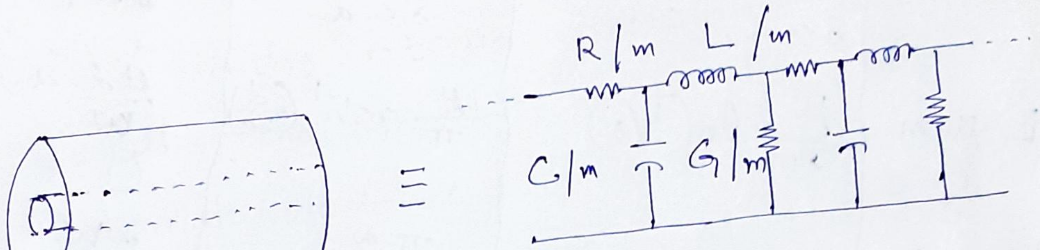
TWO WIRE LINE

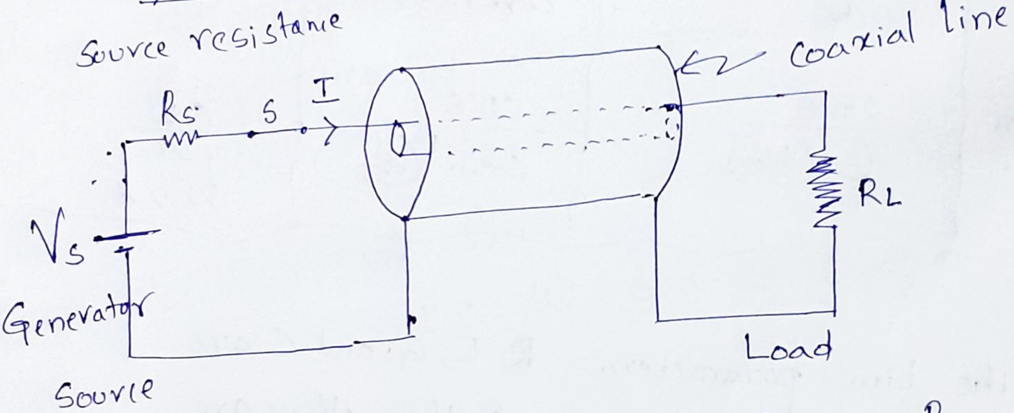


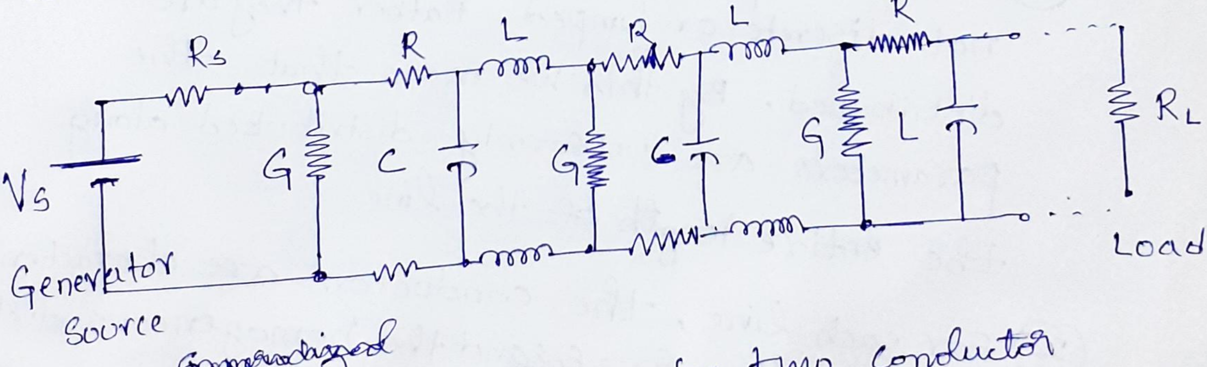
WIRE ABOVE CONDUCTING PLANE

# TRANSMISSION LINE PARAMETERS

Consider a coaxial cable which is a transmission line. This can be represented by distributed parameters as follows




Generalized
Distributed Parameters of a two conductor transmission Line

# Distributed Line Parameters at High Frequencies

| Parameter | Coaxial line | Two-wire line | Planar Line |
|---|---|---|---|
| R Ω/m | $\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$ <br> $\delta \ll a,\ c-b$ | $\frac{1}{\pi a\delta\sigma_c}$ <br> $\delta \ll a$ | $\frac{2}{w\delta\sigma_c}$ <br> $\delta \ll t$ |
| L H/m | $\frac{\mu}{2\pi}\ln(b/a)$ | $\frac{\mu}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right)$ | $\frac{\mu d}{w}$ |
| G S/m | $\frac{2\pi\sigma}{b}$ | $\frac{\pi\sigma}{\cosh^{-1}(d/2a)}$ | $\frac{\sigma w}{d}$ |
| C F/m | $\frac{2\pi\varepsilon}{\ln(b/a)}$ | $\frac{\pi\varepsilon}{\cosh^{-1}\left(\frac{d}{2a}\right)}$ | $\frac{\varepsilon w}{d}$ <br> $w \gg d$ |

1. The Line parameters R, L, G and C are not discrete or lumped. Rather, they are distributed. By this we mean that the parameters are uniformly distributed along the entire length of the line.

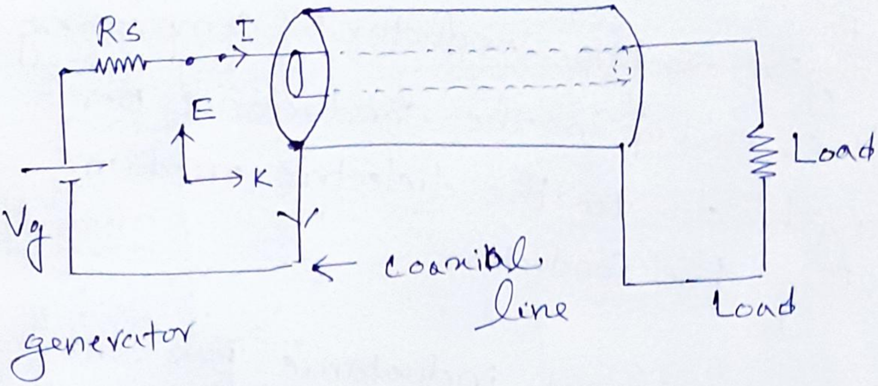
2. For each line, the conductors are characterized by $\sigma_c$, $\mu_c$, $\varepsilon_c = \varepsilon_0\varepsilon_r$ and the homogeneous dielectric seperating the conductors is characterized by $\sigma$, $\mu$, $\varepsilon$

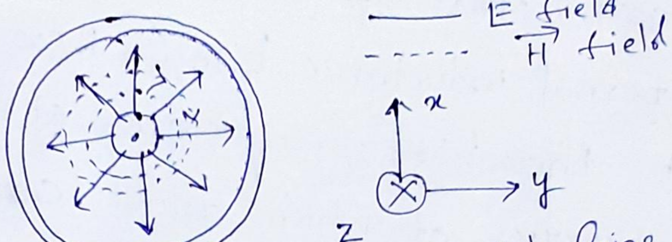
3) $G \neq \frac{1}{R}$. R is the ac resistance per unit length of the conductors comprising the line, and G is the conductance per unit length due to the dielectric medium Seperating the conductors.

4) 'L' is the external inductance per unit length, that is $L = L_{ext}$. The effects of internal inductance $L_{in}$ $(= R/\omega)$ are negligable at the high frequencies at the high frequencies at which most communication System operate

5) For each line,

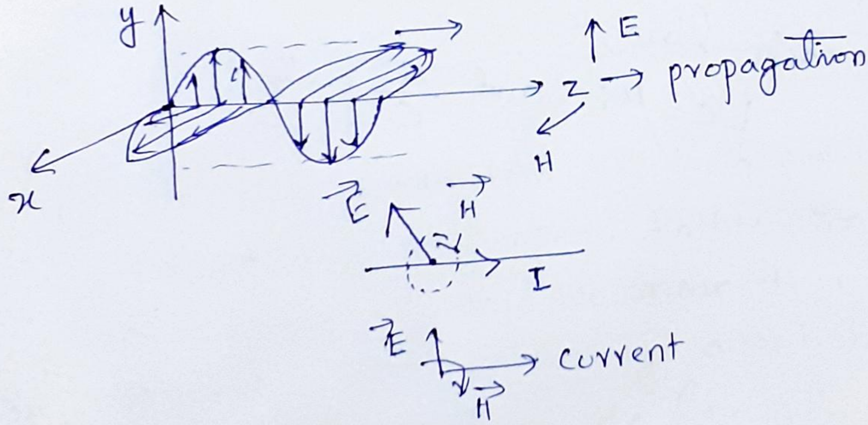
$$LC = \mu\varepsilon \quad \text{and} \quad \frac{G}{C} = \frac{\sigma}{\varepsilon}$$



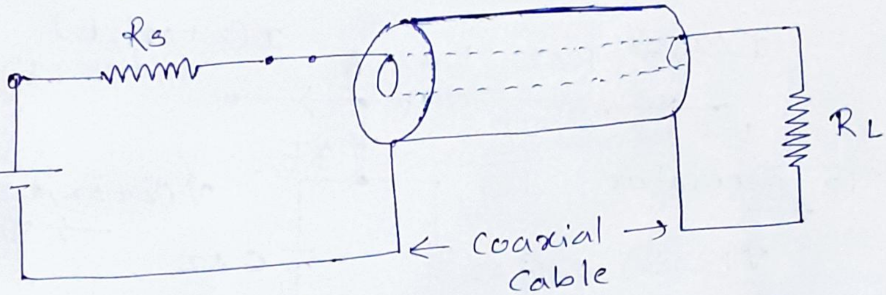




the coaxial line

$\vec{E}$ and $\vec{H}$ fields on



# TRANSMISSION LINE EQUATIONS




A two conductor transmission line supports a TEM wave, that is, the electric and magnetic fields on the line are perpendicular to each other and transverse to the direction of wave propagation.

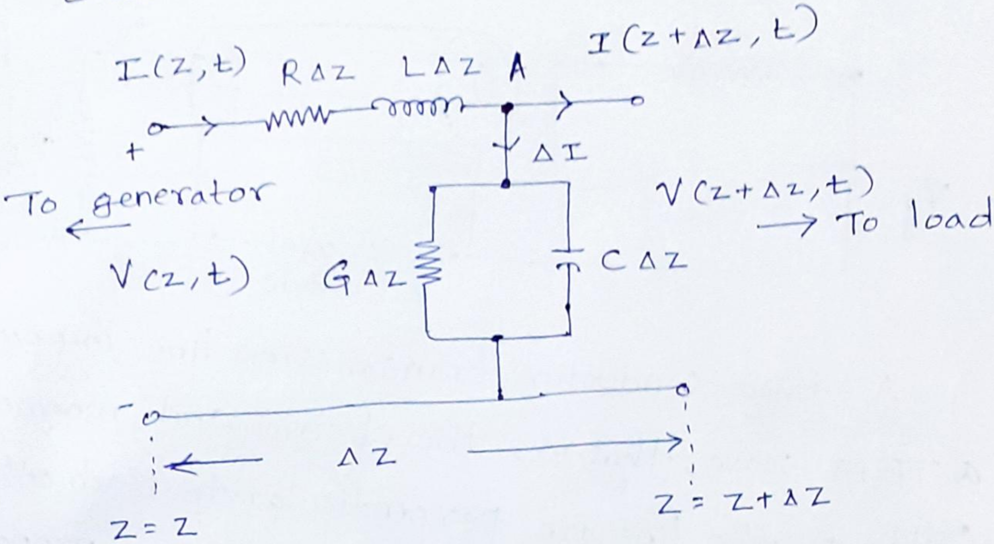
An important property of TEM wave, ~~waves~~ is that the fields $\vec{E}$ and $\vec{H}$ are uniquely related to voltage "V" and current "I" respectively

$$V = -\int_L \vec{E}\cdot\vec{dl}$$

$$I = \oint_L \vec{H}\cdot\vec{dl}$$

In view of this, we will use circuit quantities V and I in solving the transmission line.

The equivalent circuit of a portion of the transmission line can be represented as




This type of equivalent circuit model is called an L-type equivalent circuit model.

By applying Kirchoff's Voltage Law to the outer loop of the circuit we get

$$V(z,t) - R\Delta z\, I(z,t) - L\Delta z \frac{\partial I}{\partial t}(z,t) - V(z+\Delta z,t) = 0$$

$$-\frac{[V(z+\Delta z,t) - V(z,t)]}{\Delta z} = R\, I(z,t) + L\frac{\partial I}{\partial t}(z,t)$$

Taking the limit $\Delta z \to 0$

$$-\frac{\partial V(z,t)}{\partial z} = R\, I(z,t) + L\frac{\partial I(z,t)}{\partial t} \rightarrow (1)$$

Applying KCL at node A

$i_c = C\frac{dv}{dt}$ current through the capacitor

$$I(z,t) = I(z+\Delta z,t) + \Delta I$$

$$I(z,t) = I(z+\Delta z,t) + \left[G\Delta z\, V(z+\Delta z,t) + C\Delta z \frac{\partial}{\partial t}(V(z+\Delta z,t))\right]$$

$$-\frac{\left[I(z+\Delta z, t) - I(z,t)\right]}{\Delta z} = G V(z+\Delta z, t) + C\frac{\partial V(z+\Delta z, t)}{\partial t}$$

As $\Delta z \to 0$

$$-\frac{\partial I(z,t)}{\partial z} = G V(z,t) + C\frac{\partial V(z,t)}{\partial t} \rightarrow (2)$$

If we Assume harmonic time dependence so that

$$V(z,t) = Re\left[V_s(z) e^{j\omega t}\right] \rightarrow (3)$$

$$I(z,t) = Re\left[I_s(z) e^{j\omega t}\right] \rightarrow (4)$$

In equation (3) and (4) $V_s(z)$ and $I_s(z)$ are the phasor forms of $V(z,t)$ and $I(z,t)$

From (1) we have

$$-\frac{\partial V(z,t)}{\partial z} = R I(z,t) + L\frac{\partial I}{\partial t}(z,t)$$

$$-\frac{\partial}{\partial z}\left[Re\left[V_s(z) e^{j\omega t}\right]\right] = R.Re\left[I_s(z) e^{j\omega t}\right] + L \frac{\partial}{\partial t}\left[Re\, I_s(z) e^{j\omega t}\right]$$

$$-Re\left[\frac{\partial V_s(z)}{\partial z} e^{j\omega t}\right] = R.Re\left[I_s(z) e^{j\omega t}\right] + L\left[Re\, I_s(z) j\omega\, e^{j\omega t}\right]$$

cancelling $Re\{\ \}$ and $e^{j\omega t}$ on both sides

$$-\frac{dV_s}{dz} = R I_s + j\omega L I_s = (R + j\omega L) I_s \rightarrow (5)$$

From (2)

$$-\frac{\partial}{\partial z} I(z,t) = G V(z,t) + C \frac{\partial V}{\partial t}(z,t)$$

$$-\frac{\partial}{\partial z} Re\left[I_s(z) e^{j\omega t}\right] = G\, Re\left[V_s(z) e^{j\omega t}\right] + C \frac{\partial}{\partial t}\left[Re\left[V_s(z) e^{j\omega t}\right]\right]$$

$$-Re\left[\frac{dI_s(z)}{dz} e^{j\omega t}\right] = Re\left[G V_s(z) e^{j\omega t}\right] + Re\left[j\omega C V_s(z) e^{j\omega t}\right]$$

Cancelling Re{ } and $e^{j\omega t}$ on both sides

$$-\frac{dI_s(z)}{dz} = (G + j\omega C) V_s(z)$$

$$-\frac{dI_s}{dz} = (G + j\omega C) V_s \rightarrow (6)$$

Consider $-\frac{dV_s}{dz} = (R + j\omega L) I_s$

differentiating on both sides w.r.to. z

$$-\frac{d^2 V_s}{dz^2} = (R + j\omega L) \frac{dI_s}{dz} \rightarrow (7)$$

$$-\frac{d^2 V_s}{dz^2} = (R + j\omega L)\left(-(G + j\omega C)\right) V_s$$

$$-\frac{d^2 V_s}{dz^2} = -(R + j\omega L)(G + j\omega C) V_s$$

$$\cancel{-}\frac{d^2 V_s}{dz^2} = \cancel{-}\gamma^2 V_s$$

$$\boxed{\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0} \rightarrow (8)$$

where
$$\boxed{\begin{aligned} \gamma &= \alpha + j\beta \\ \gamma &= \sqrt{(R+j\omega L)(G+j\omega C)} \end{aligned}} \rightarrow (9)$$

where $\alpha$ = Attenuation constant in Np/m or decibel/m

$\beta$ = Phase Constant in radians/meter

$\gamma$ = Propagation constant

$$\boxed{\lambda = \frac{2\pi}{\beta}} \rightarrow (10)$$

Multiply 'f' on both sides

$$\lambda f = \frac{2\pi f}{\beta}$$

$$\boxed{u' = \frac{\omega}{\beta} = f\lambda = \text{Velocity}} \rightarrow (11)$$

Consider $\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0 \rightarrow (12)$

The solution to equation (12) is given by

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z} \rightarrow (13)$$

where ~~[illegible]~~ $V_o^+$, $V_o^-$, $I_o^+$ and $I_o^-$ are ~~[illegible]~~ denote

Wave amplitudes,

$e^{-\gamma z}$ denote the wave travelling in $+z$-direction $\longrightarrow$

$e^{+\gamma z}$ denote the wave travelling in $-z$-direction $\longleftarrow$

$$V(z,t) = Re\left[V_s(z)e^{j\omega t}\right]$$

$$= Re\left[V_o^+ e^{-\gamma z} e^{j\omega t} + V_o^- e^{+\gamma z} e^{j\omega t}\right]$$

$$= Re\left[V_o^+ e^{-\alpha z} \cdot e^{-j\beta z} \cdot e^{j\omega t} + V_o^- e^{+\alpha z} e^{+j\beta z} e^{j\omega t}\right]$$

$$= V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z)$$

lll'y $I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$

## Characteristic Impedance "$Z_o$"

The characteristic Impedance "$Z_o$" of the line is the ratio of the positively travelling voltage wave to the current wave at any point on the line.

Consider $V = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$

$I = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$

Consider $V = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$

differentiating with respect z

$$\frac{dV}{dz} = (-\gamma) V_o^+ e^{-\gamma z} + (\gamma) V_o^- e^{+\gamma z} \rightarrow (1)$$

But $\frac{dV}{dz} = -(R+j\omega L) I_s$

$$\frac{dV}{dz} = -(R+j\omega L)\left(I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}\right) \rightarrow (2)$$

1 = 2

$$-(R+j\omega L)I_0^+ e^{-\gamma z} - (R+j\omega L)I_0^- e^{+\gamma z} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} \quad (3)$$

Equaling the coefficients $e^{-\gamma z}$ on both sides

$$\cancel{-}(R+j\omega L)I_0^+ = \cancel{-}\gamma V_0^+$$

$$\frac{V_0^+}{I_0^+} = \frac{(R+j\omega L)}{\gamma}$$

$$\frac{V_0^+}{I_0^+} = \frac{(R+j\omega L)}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$\boxed{\frac{V_0^+}{I_0^+} = Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}} = R_0 + jX_0$$

Similarly if we equate the coefficients of $e^{+\gamma z}$ we get

$$-(R+j\omega L)I_0^- = \gamma V_0^-$$

$$-\frac{V_0^-}{I_0^-} = Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega L}}$$

$R_0$ is the real part of $Z_0$

$X_0$ is the imaginary part of $Z_0$

$\therefore Z_0 = \frac{1}{Y_0}$ $Y_0$ = characteristic Admittance

There are ~~two~~ three types of transmission line

they are (i) Lossless Transmission Line

(ii) Distortionless Transmission Line

(iii) General transmission line

## 1) Lossless Line ($R=0$, $G=0$)

A transmission line is said to be lossless, if the conductors of the line are perfect conductors i.e $\sigma_c = \infty$ and the dielectric medium seperating them is lossless, i.e whose conductivity $\sigma = 0$

$\therefore$ Since $\sigma_c = \infty$, $\sigma = 0$

$$R = 0$$
$$G = 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(0 + j\omega L)(0 + j\omega C)}$$

$$\gamma = j\omega\sqrt{LC}$$

$$\gamma = \alpha + j\beta$$

$\therefore$ $\alpha = 0$ $\quad$ $\beta = \omega\sqrt{LC}$

Velocity $\quad u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

$$u = f\lambda$$

$$Z_0 = \sqrt{\left(\frac{\cancel{R}^{0} + j\omega L}{\cancel{G}^{0} + j\omega C}\right)}$$

$$Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

## 2) Distortionless Line $\left(\frac{R}{L} = \frac{G}{C}\right)$

A signal normally consists of a band of frequencies, wave amplitudes of different frequency components will be attenuated differently in a lossy line because '$\alpha$' is frequency dependent. Since, in general, the phase velocity of each frequency component is also frequency dependent, this will result in distortion.

A distortionless line is one in which the attenuation constant '$\alpha$' is frequency independent while the phase constant '$\beta$' is linearly dependent on frequency.

For a distortionless line

$$\frac{R}{L} = \frac{G}{C} \rightarrow (1)$$

We know $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\gamma = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)} \rightarrow (2)$$

$$\gamma = \sqrt{RG}\left(1 + j\omega \times \frac{C}{G}\right)^{1/2}\left(1 + j\omega\frac{C}{G}\right)^{1/2} \quad \text{using (1) in (2)}$$

$$\gamma = \sqrt{RG}\left(1 + j\omega\frac{C}{G}\right) = \alpha + j\beta \rightarrow (3)$$

$$\therefore \alpha = \sqrt{RG} \qquad \beta = \omega\sqrt{RG}\,\frac{C}{G} = \frac{\omega\sqrt{R}}{\sqrt{G}} \times C$$

$$\beta = \omega\sqrt{\frac{L}{C}} \times C = \omega\sqrt{LC}$$

Since $\alpha = \sqrt{RG}$ does not depend on frequency, whereas '$\beta$' is a linear function of frequency, because

$\beta = \omega\sqrt{LC}$ 16

$$Z_0 = \sqrt{\left(\frac{R + j\omega L}{G + j\omega C}\right)}$$

$$Z_0 = \sqrt{\frac{R\,(1 + j\omega L/R)}{G\,(1 + j\omega C/G)}}$$

$$Z_0 = \sqrt{\frac{R}{G} \times \frac{\cancel{(1 + j\omega L/R)}}{\cancel{(1 + j\omega L/R)}}}$$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_0 + jx_0$$

$$\therefore\ R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}\ ,\ X_0 = 0$$

Velocity $u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = f\lambda$

— The phase velocity is independent of frequency because the phase constant $\beta$ linearly depends on frequency.

# Transmission Line Characteristics

| Case | Propagation Constant $\gamma = \alpha + j\beta$ | characteristic Impedance $Z_0 = R_0 + jX_0$ |
|---|---|---|
| General | $\sqrt{(R + j\omega L)(G + j\omega C)}$ | $\sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$ |
| Lossless | $0 + j\omega\sqrt{LC}$ | $\sqrt{\dfrac{L}{C}} + j0$ |
| Distortionless line | $\sqrt{RG} + j\omega\sqrt{LC}$ | $\sqrt{\dfrac{L}{C}} + j0$ |

*) A lossless line is a also distortionless, but a distortionless is not necessarily lossless

*) ~~A~~ lossless lines are desirable in power transmission

*) Telephone lines are required to be ~~lossless~~ distortionless.

## Problems

1) An air line has a characteristic impedance of $70\,\Omega$ and a phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

### Solution

An air line can be regarded as a lossless line because $\sigma = 0$ and $\sigma_c = \infty$

For a lossless line $R = 0$, $G = 0$

$$\therefore \alpha = 0$$

$$Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

$$\beta = \omega\sqrt{LC}$$

$$\frac{(Z_0 = R_0)}{\beta} = \frac{1}{\omega C}$$

where $R_0 = Z_0$

$$C = \frac{\beta}{(\omega R_0)} = \frac{3}{2\times\pi\times 100\times 10^{6}\times 70}$$

$Z_0$

$$\boxed{C = 68.2\times 10^{-12}\ \text{F/m}}$$

$$L = R_0^2 C = (70)^2(68.2\times 10^{-12})$$

$$\boxed{L = 334.2\times 10^{-9}\ \text{H/m}}$$

2) A distortionless line has $Z_0 = 60\,\Omega$, $\alpha = 20\,m\,Np/m$, $u = 0.6c$, where $c$ is the speed of light in vacuum. Find $R$, $L$, $G$, $C$ and $\lambda$ at 100 MHz.

Solution:

For a distortionless line

$$RC = GL$$

$$G = \frac{RC}{L}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

$$R = \alpha Z_0$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$R = \alpha Z_0 = (20 \times 10^{-3})\,60 = 1.2\,\Omega/m$$

$$u = \frac{1}{\sqrt{LC}} \Rightarrow u^2 = \frac{1}{LC} \qquad L = \frac{1}{u^2 C}$$

$$L = \frac{1}{u^2 C} \times \frac{Z_0}{Z_0}$$

$$L = \frac{Z_0}{\frac{1}{L\cancel{C}} \times \cancel{C} \times \sqrt{\frac{L}{C}}} = \frac{Z_0}{\frac{1}{\sqrt{LC}}} = \frac{Z_0}{u}$$

$$L = \frac{Z_0}{u} = 333 \times 10^{-9}\ H/m$$

$$G = \frac{\alpha^2}{R} = \frac{400\times10^{-6}}{1.2} = 333\times10^{-6}\ S/m$$

$$u Z_0 = \frac{1}{C}$$

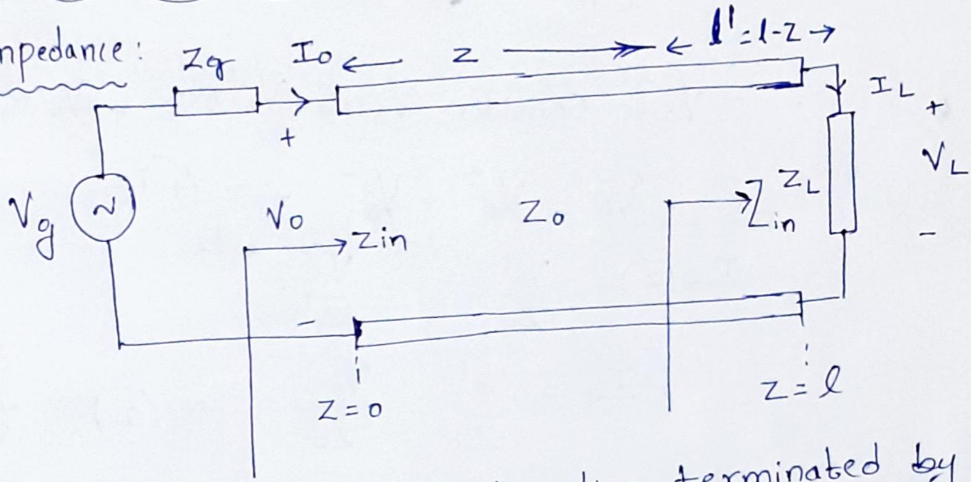
$$C = \frac{1}{u Z_0} = \frac{1}{0.6\times 3\times10^{8}\times 60}$$

$$C = 92.59\times10^{-12}\ F/m$$

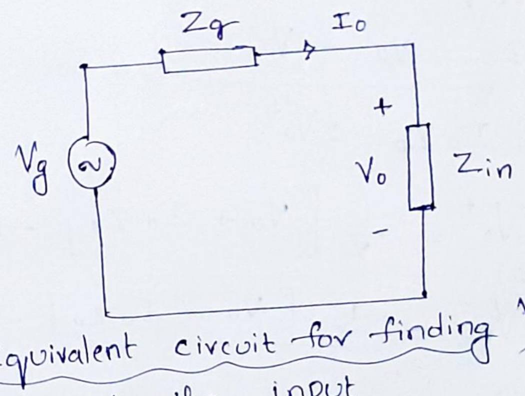
$$\lambda = \frac{u}{f} = \frac{0.6\times3\times10^{8}}{10^{8}} = 1.8\,m$$

# Input Impedance, Standing Wave Ratio, And Power

**Input Impedance:**




Input Impedance due to a line terminated by a load.




Equivalent circuit for finding $V_o$ and $I_o$ in terms of $Z_{in}$ at the input

Consider a transmission line of length "$l$", characteristic impedance $Z_o$, propagation constant "$\gamma$"

Let Voltage equation as a function 'z' is given by

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

The current equation in terms of 'z' is given by

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{+\gamma z}$$

$$V_s(z)\Big|_{z=0} = V_0 = V_0^+ + V_0^-$$

$$I_s(z)\Big|_{z=0} = I_0 = \cancel{I_0}\,\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

~~Var~~ Consider $V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$

$$V_0 = V_0^+ + V_0^- \rightarrow (1)$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$I_0 = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} \rightarrow (2)$$

$$V_0 = V_0^+ + V_0^-$$
$$I_0 Z_0 = V_0^+ - V_0^-$$

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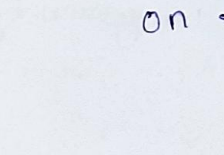
$$V_0 + I_0 Z_0 = 2V_0^+$$

$$V_0^+ = \frac{1}{2}\left[V_0 + I_0 Z_0\right] \rightarrow (3)$$

$|||^{ly}$ $$V_0^- = \frac{1}{2}\left[V_0 - I_0 Z_0\right] \rightarrow (4)$$




$\therefore$ $$V_0 = \frac{Z_{in}}{Z_{in} + Z_g} V_g \qquad I_0 = \frac{V_g}{Z_{in} + Z_g}$$

on the other hand if we are given the conditions

$$V_L = V_s(z)\Big|_{z=l} \qquad I_L = I_s(z)\Big|_{z=l}$$

$$V_L = V_0^+ e^{-\gamma l} + V_0^- e^{+\gamma l}$$

$$I_L = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{+\gamma l}$$

$$V_L = V_0^+ e^{-\gamma l} + V_0^- e^{+\gamma l} \rightarrow (5)$$

$$Z_0 I_L = V_0^+ e^{-\gamma l} - V_0^- e^{+\gamma l} \rightarrow (6)$$

(5) + (6) $\quad V_L + Z_0 I_L = 2 V_0^+ e^{-\gamma l}$

$$V_0^+ = \frac{1}{2}\left[V_L + Z_0 I_L\right] e^{+\gamma l} \rightarrow (7)$$

(5) − (6) $\quad V_L - Z_0 I_L = 2 V_0^- e^{+\gamma l}$

$$V_0^- = \frac{1}{2}\left[V_L - Z_0 I_L\right] e^{-\gamma l} \rightarrow (8)$$

$$Z_{in} = \left.\frac{V_s(z)}{I_s(z)}\right|_{z=0} = \frac{V_0^+ \cdot \cancel{e^{-\gamma z}}^{\,0} + V_0^- \cancel{e^{+\gamma z}}^{\,0}}{\frac{V_0^+}{Z_0} \cancel{e^{-\gamma z}}^{\,0} - \frac{V_0^-}{Z_0} \cancel{e^{+\gamma z}}^{\,0}}$$

$$Z_{in} = \frac{Z_0 (V_0^+ + V_0^-)}{(V_0^+ - V_0^-)} \rightarrow \cancel{(8)} \rightarrow (9)$$

$$Z_{in} = \left.\frac{V_s(z)}{I_s(z)}\right|_{z=\gamma l} = \frac{V_0^+ e^{-\gamma l} + V_0^- e^{+\gamma l}}{\frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{+\gamma l}}$$

$$= \frac{Z_0 \left[V_0^+ e^{-\gamma l} + V_0^- e^{+\gamma l}\right]}{\left[V_0^+ e^{-\gamma l} - V_0^- e^{+\gamma l}\right]}$$

using (7) and (8)

From (7) $\quad V_0^+ = \frac{1}{2}\left[V_L + Z_0 I_L\right]e^{+\gamma l}$

From (8) $\quad V_0^- = \frac{1}{2}\left[V_L - Z_0 I_L\right]e^{-\gamma l}$

Since $V_L = I_L Z_L$

(7) becomes $V_0^+ = \frac{1}{2}\left[I_L Z_L + Z_0 I_L\right]e^{+\gamma l} \rightarrow$ (7a)

(8) becomes $V_0^- = \frac{1}{2}\left[I_L Z_L - Z_0 I_L\right]e^{-\gamma l} \rightarrow$ (8a)

using (7a) and (8a) in (9)

$$Z_{in} = Z_0\left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right]$$

$$Z_{in} = Z_0\left[\frac{\frac{1}{2}(I_L Z_L + I_L Z_0)e^{+\gamma l} + \frac{1}{2}(I_L Z_L - I_L Z_0)e^{-\gamma l}}{\frac{1}{2}(I_L Z_L + I_L Z_0)e^{+\gamma l} - \frac{1}{2}(I_L Z_L - I_L Z_0)e^{-\gamma l}}\right]$$

$$Z_{in} = Z_0\left[\frac{(Z_L + Z_0)e^{\gamma l} + (Z_L - Z_0)e^{-\gamma l}}{(Z_L + Z_0)e^{\gamma l} - (Z_L - Z_0)e^{-\gamma l}}\right]$$

$$Z_{in} = Z_0\left[\frac{Z_L(e^{\gamma l} + e^{-\gamma l}) + Z_0(e^{\gamma l} - e^{-\gamma l})}{Z_L(e^{\gamma l} - e^{-\gamma l}) + Z_0(e^{\gamma l} + e^{-\gamma l})}\right]$$

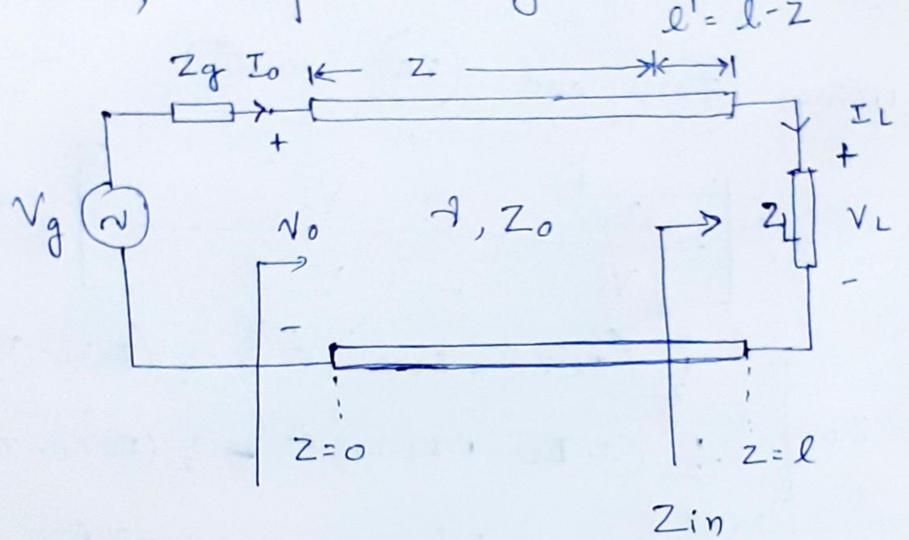
$$\boxed{Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right]}$$

$$\because \quad \tanh \gamma l = \frac{(e^{\gamma l} - e^{-\gamma l})/2}{e^{\gamma l} + e^{-\gamma l}/2}$$

$$Z_{in} = Z_0 \left[ \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]$$ corresponds

to lossy transmission line. It is a general expression for finding "$Z_{in}$" at any point on the line.

To find "$Z_{in}$" at a distance "$l'$" from the load, we replace $l$ by $l'$. where $l' = l - z$




For a lossless line $\alpha = 0$, $\gamma = \alpha + j\beta = j\beta$
$\tanh j\beta l = j \tan \beta l$ and $Z_0 = R_0$

$$\therefore \quad Z_{in} = Z_0 \left[ \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right]$$ (lossless lines)

"$\beta l$" is usually referred to as electrical length of the line. and can be expressed in degress or radians.

For a lossless line, the input impedance varies periodically with distance "$l$" from the load.

# Reflection Coefficient or Voltage Reflection Coefficient

Voltage Reflection coefficient $\Gamma_L$ is the ratio of the reflected wave Voltage to the incident wave Voltage at the load.

$$\Gamma_L = \frac{\text{Voltage of the reflected wave}}{\text{Voltage of the incident wave}}$$

$$\Gamma_L = \frac{V_0^- e^{+\gamma l}}{V_0^+ e^{-\gamma l}}$$

But we know, at the load

$$V_0^- = \frac{1}{2}\left[I_L Z_L - I_L Z_0\right] e^{-\gamma l}$$

$$V_0^+ = \frac{1}{2}\left[I_L Z_L + I_L Z_0\right] e^{+\gamma l}$$

$$\Gamma_L = \frac{V_0^- e^{+\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{\cancel{\frac{1}{2}}\left[I_L Z_L - I_L Z_0\right] \cancel{e^{+\gamma l}} \cdot \cancel{e^{-\gamma l}}}{\cancel{\frac{1}{2}}\left[I_L Z_L + I_L Z_0\right] \cancel{e^{+\gamma l}} \cdot \cancel{e^{-\gamma l}}}$$

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

- The voltage reflection coefficient at any point on the line is the ratio of the reflected voltage wave to that of the incident wave

In general $\Gamma(z) = \dfrac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \dfrac{V_0^-}{V_0^+} e^{2\gamma z}$

If $z = l - l'$

$$\Gamma(z) = \frac{V_0^-}{V_0^+} e^{2\gamma(l-l')}$$

$$\Gamma(z) = \underbrace{\frac{V_0^-}{V_0^+} e^{2\gamma l}}\, e^{-2\gamma l'}$$

$$\boxed{\Gamma(z) = \Gamma_L e^{-2\gamma l'}}$$

- The current reflection coefficient at any point on the line is the negative of the voltage reflection coefficient at that point.

$\therefore$ Current reflection Coefficient at the load $= \dfrac{I_0^- e^{\gamma l}}{I_0^+ e^{-\gamma l}} = -\Gamma_L$

## Standing Wave Ratio (S)

Standing Wave Ratio is denoted by "S"

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{\cancel{1 + |\Gamma_L|}}{\cancel{1 - |\Gamma_L|}}$$

$$= \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

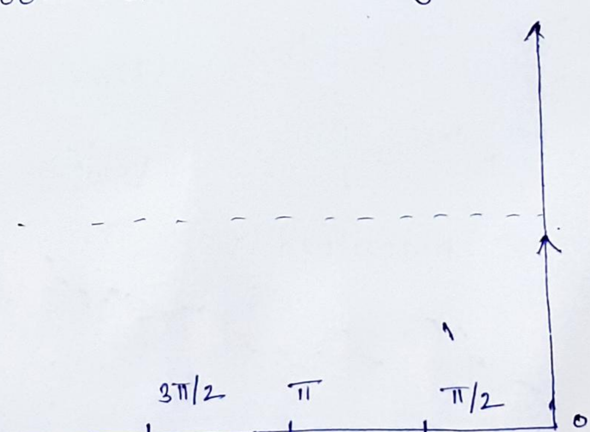
$$I_{max} = \frac{V_{max}}{Z_0}$$

$$I_{min} = \frac{V_{min}}{Z_0}$$

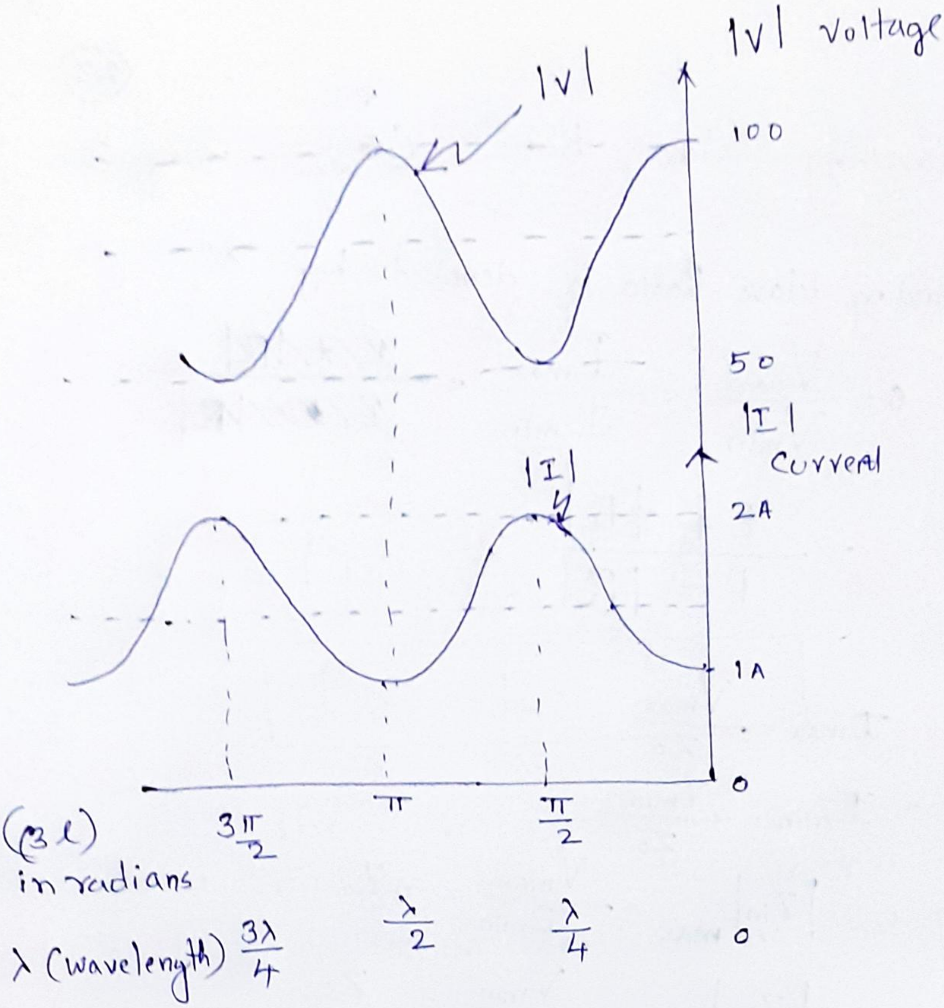
$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = S Z_0$$

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{S}$$

Consider a lossless line with characteristic impedance of $Z_0 = 50\Omega$. For the sake of simplicity, we assume that the line is terminated in a pure resistive load $Z_L = 100\Omega$ and the voltage at the load is 100V (rms)




Voltage and current standing wave pattern on a lossless line terminated by a resistive load.




Voltage and current standing wave patterns on a lossless line terminated by a resistive load.