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Note Title 14-09-2021

Performance of the System

Electrical | Mechanical | Electromechanical System

$v_i(t) = Ri(t) + L\frac{di(t)}{dt}$

$a_1\frac{dy(t)}{dt} + a_0 = b_0 x(t)$

Simplifying assumptions / Idealizing assumption

LTI & lumped parameter

ODE with constant coefficients

State-space Representation

Electrical
→ current
→ voltage

Mechanical
→ position
→ velocity

$\downarrow$ LT

Algebraic equations (s-domain / freq domain)

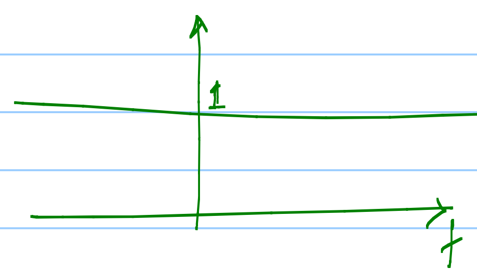
BD SFG

Transfer function (Zero IC)

Input Signals / Basic signals

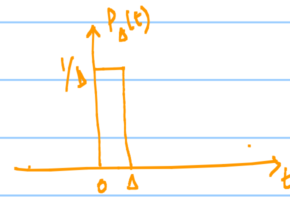
1. Impulse Signal / Dirac Delta function.

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$



$$\boxed{\delta(t) = 0 \quad t \neq 0 \ \& \int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

$$\delta(t) = \lim_{\Delta \to 0} P_\Delta(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}\left(1(t) - 1(t-\Delta)\right)$$



$$\lim_{\Delta \to 0} \mathcal{L}T\{P_\Delta(t)\} = \lim_{\Delta \to 0} \mathcal{L}T\left[\frac{1}{\Delta}\left(1(t) - 1(t-\Delta)\right)\right]$$

$u(t) = 1(t)$

$$1(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$= \lim_{\Delta \to 0} \frac{1}{\Delta}\left[\frac{1}{s} - \frac{e^{-\Delta s}}{s}\right]$$

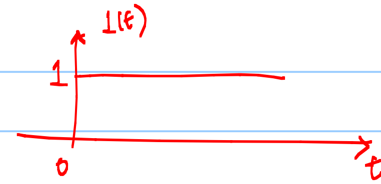
$$= \frac{1}{s}\lim_{\Delta \to 0} \frac{1}{\Delta}\left(1 - e^{-\Delta s}\right)$$

$$\lim_{\Delta \to 0} \frac{\frac{d}{d\Delta}(1 - e^{-\Delta s})}{\frac{d}{d\Delta}\Delta}$$

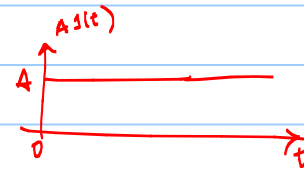
$$\boxed{\mathcal{L}\{\delta(t)\} = \frac{1}{s} \times s = 1}$$

2. Step Signal

$$u(t) = 1(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

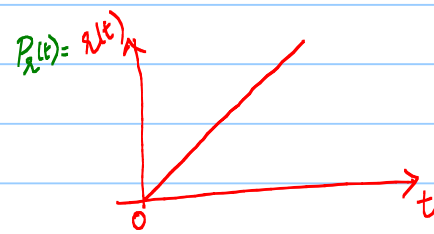


$$LT\{1(t)\} = \frac{1}{s}$$



$$LT\{A\,1(t)\} = \frac{A}{s}$$

3. Ramp Signal

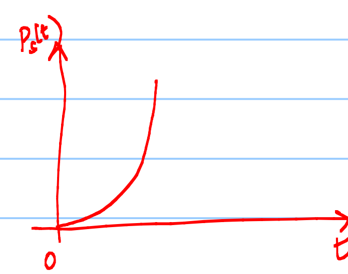


$$r(t) = P_r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$LT\{r(t)\} = 1/s^2$$

$$LT\{A\,r(t)\} = A/s^2$$

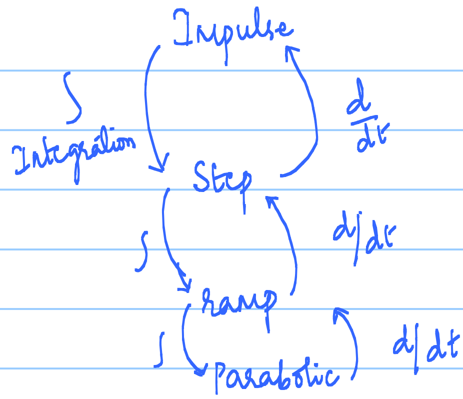
4. Parabolic Signal / input

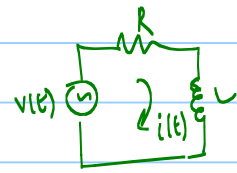


$$P_s(t) = \begin{cases} t^2/2 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

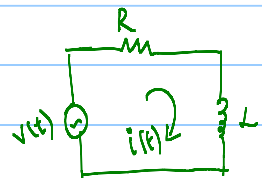
$$LT\{P_s(t)\} = \cancel{2}!/s^3 \times \cancel{2} = 1/s^3$$

$$LT\{A P_s(t)\} = A/s^3$$





First Order System:



$$v(t) = Ri(t) + L\frac{di(t)}{dt}$$

$LT \rightarrow$

$$V(s) = RI(s) + LsI(s)$$

$$\frac{I(s)}{V(s)} = \frac{1}{Ls+R} = \frac{1/R}{\boxed{L/R}\,s+1}$$

$$V_o(t) = Ri(t)$$

$$V_o(s) = RI(s)$$

$$\frac{V_o(s)}{V(s)} = \frac{R}{Ls+R} = \frac{\cancel{R}}{\cancel{R}(L/R\,s+1)} = \frac{1}{L/R\,s+1}$$

$$G(s) = \frac{V_o(s)}{V(s)} = \frac{1}{L/R\,s+1} = \frac{1}{\tau s + 1 \times s^0} \qquad \tau = L/R$$

Time constant form.

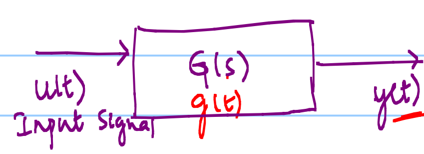
$$\boxed{G(s) = \frac{K}{\tau s + 1}}$$

K – DC gain $= \lim_{s \to 0} G(s)$

τ – Time constant

$$\boxed{G(s) = a/s+a}$$

Impulse response of the first order system



$$y(t) = g(t) * u(t)$$

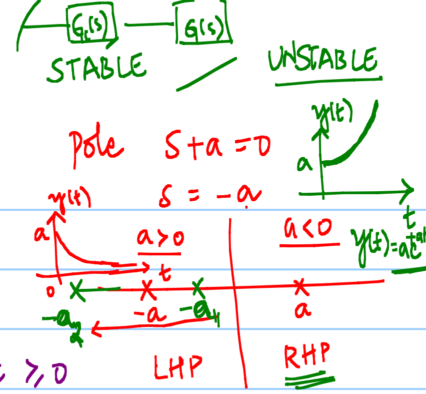
$$Y(s) = G(s)\,U(s)$$

Let $u(t) = \delta(t)$

$$\boxed{\frac{Y(s)}{U(s)} = G(s)} \Rightarrow Y(s) = G(s)\,U(s)$$

$U(s) = \mathcal{L}\{\delta(t)\} = 1$

$Y(s) = \frac{a}{s+a} U(s) \qquad a > 0$



$Y(s) = \frac{a}{s+a}$

ILT $\rightarrow$

$y(t) = a e^{-at} 1(t) \quad = a e^{-at}, \; t \geq 0$

$= \frac{K}{\tau} e^{-t/\tau} \qquad = a e^{-t/\tau}, \; t \geq 0$

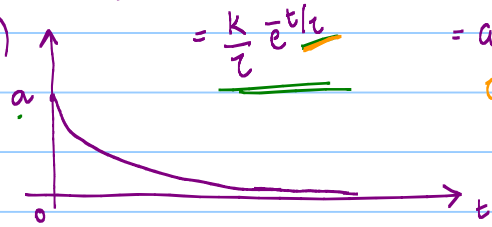
$\tau = 1/a$

$G(s) = \frac{K}{\tau s+1}$

$Y(s) = G(s)U(s)$

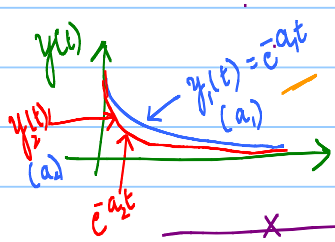
$= \frac{K}{\tau s+1} \times 1$

$y(t) = \frac{K}{\tau} e^{-t/\tau} 1(t)$

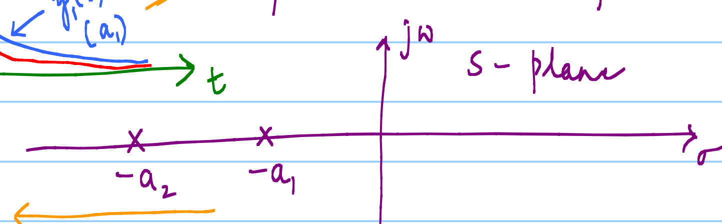


$a_1 < a_2 \qquad G_1(s) = \frac{a_1}{s+a_1} \qquad G_2(s) = \frac{a_2}{s+a_2} \qquad \tau_1 = 1/a_1 \quad \tau_2 = 1/a_2$

$\boxed{\tau_1 > \tau_2}$



pole $= -a_1$ $\qquad$ pole $= -a_2$



The response of the first order system dies down / decays faster as the pole is placed further away from $j\omega$ axis in LHP

Step response of the first order system

$u(t) = 1(t)$ unit step. $\quad U(s) = 1/s$

$Y(s) = G(s)U(s) \qquad\qquad G(s) = \frac{K}{\tau s+1}$

$Y(s) = \frac{a}{s+a} \times \frac{1}{s}$

$Y(s) = \frac{A}{s} + \frac{B}{s+a}$

$A = s\,Y(s)\Big|_{s=0} = \frac{a}{a} = 1$

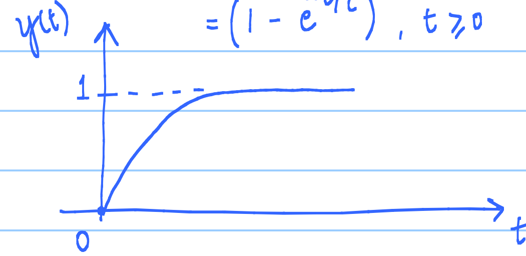
$B = (s+a)\,Y(s)\Big|_{s=-a} = -1$

$Y(s) = \frac{1}{s} - \frac{1}{s+a}$

ILT $\longrightarrow$

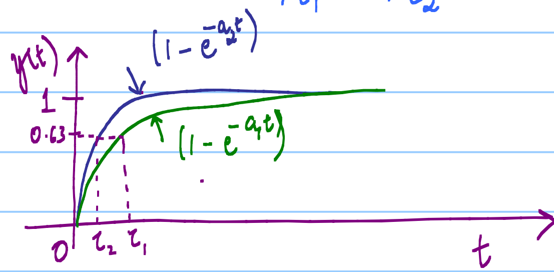
$$y(t) = 1(t) - e^{-at}\,1(t) = (1 - e^{-at}), \; t \geq 0$$

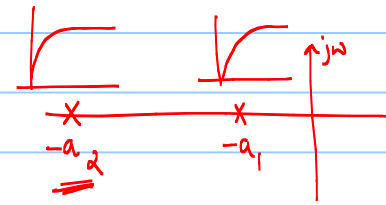
$$= (1 - e^{-t/\tau}), \; t \geq 0 \qquad = K(1 - e^{-t/\tau}), \; t \geq 0$$



STEP RESPONSE

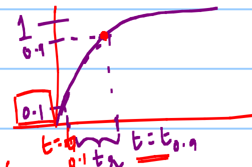
$a_1 < a_2$ or $\tau_1 > \tau_2$ $\quad 1/\tau_1 < 1/\tau_2$





The pole near to the origin makes the system sluggish.

Time constant is the time it takes for the step response y(t) to rise to 63% of its final value



Rise time: t_r

The time takes by the step response to go from 0.1 to 0.9 of its final value
10% to 90%

$$t_r = t_{0.9} - t_{0.1}$$

$t = t_{0.9} \quad y(t_{0.9}) = 0.9 \times 1$

$t = t_{0.1} \quad y(t_{0.1}) = 0.1 \times 1$

$$y(t) = 1 - e^{-at}$$

$y(t) = 0.9$

$$y(t_{0.9}) = 0.9 = 1 - e^{-at_{0.9}}$$

$$\Rightarrow e^{-at_{0.9}} = 0.1$$

taking ln

$$-at_{0.9} = \ln 0.1 = -2.3$$

$$\Rightarrow \boxed{t_{0.9} = \frac{2.3}{a}}$$

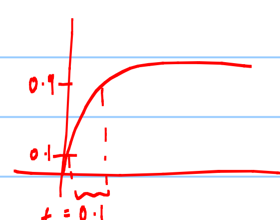
$y(t_{0.1}) = 0.1$

$$y(t_{0.1}) = 0.1 = 1 - e^{-at_{0.1}}$$

$$\Rightarrow \boxed{t_{0.1} = \frac{0.11}{a}}$$

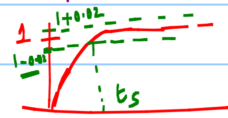
$$t_r = \frac{2.3}{a} - \frac{0.11}{a}$$

Rise time $\boxed{t_r = \frac{2.197}{a}}$



Settling time : t_s

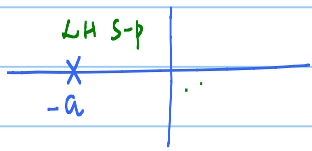
Time required for the system response to settle within certain Percentage ($\boxed{2\%}$ or 5%) of the input signal amplitude (1) / final value of the output
(step)



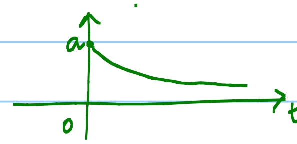
$$y(t_s) = 0.98 = 1 - e^{-at_s}$$
$$\Rightarrow e^{-at_s} = 0.02$$
$$\ln \longrightarrow$$
$$-at_s = \ln 0.02$$
$$\boxed{t_s = 4/a}$$

Location of first order system

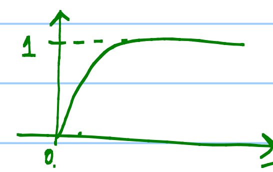
$G(s) = \frac{a}{s+a}$



$IR = a e^{-at}, \; t \geq 0$

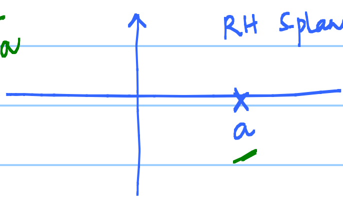


$SR = (1 - e^{-at}), \; t \geq 0$

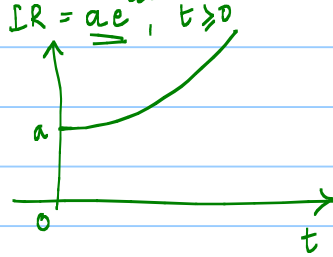


→ System Stable

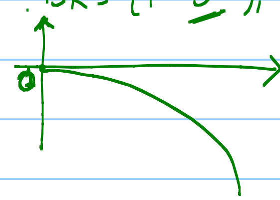
$G(s) = \frac{a}{s-a}$



$IR = a e^{at}, \; t \geq 0$

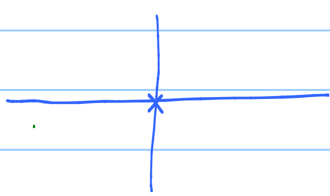


$SR = (1 - e^{at}), \; t \geq 0$



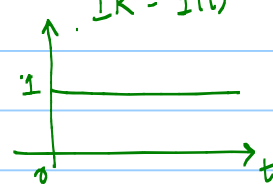
→ System unstable

$G(s) = \frac{1}{s}$



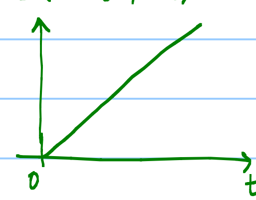
$Y(s) = G(s)R(s)$
$= \frac{1}{s} \times 1$

$IR = 1(t)$



$Y(s) = G(s)R(s) = \frac{1}{s} \times \frac{1}{s}$

$SR = t, \; t \geq 0$



→ System is Marginally stable

Ramp response of the first order system:

$$G(s) = \frac{a}{s+a}$$

$u(t) = t \qquad U(s) = 1/s^2$

$$Y(s) = G(s)\,U(s) = \frac{a}{s^2(s+a)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+a}$$

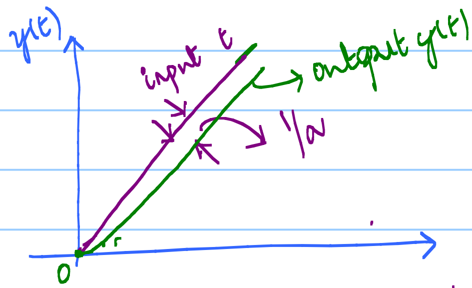
$A = -\frac{1}{a}$

$B = 1$

$C = 1/a$

$$Y(s) = -\frac{1}{a}\frac{1}{s} + \frac{1}{s^2} + \frac{1}{a(s+a)}$$

$$y(t) = \boxed{-\frac{1}{a}} + t + \frac{1}{a}e^{-at}, \quad t \geq 0$$



$\frac{0}{0}$ - growth order

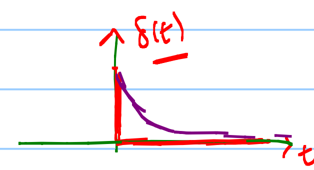
→ Output signal always grows at a lesser rate compared to input signal

Total response = Natural Response + Forced response.

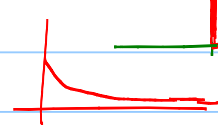
Natural Response ↓ due to initial conditions and dies down as $t \to \infty$ is called " TRANSIENT RESPONSE "

Since input is zero " ZERO INPUT RESPONSE "

Forced response ↓ due to input & stay forever " STEADY STATE RESPONSE " " ZERO STATE RESPONSE "



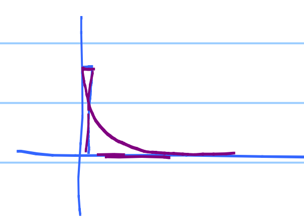
Steady state error :



Impulse response : $y(t) = a e^{-at}, \; t \geq 0$

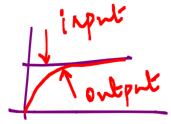
Error $\quad e(t) \triangleq u(t) - y(t)$

$e(t) = \delta(t) - a e^{-at}$



Steady state error $e_{ss} = \lim_{t\to\infty} e(t) = 0$

⇒ The first order system tracks the impulse input



$$\boxed{G(s) = \frac{k}{\tau s+1}}$$

Step response:

error $\quad e(t) = \underset{u(t)}{1(t)} - \underset{y(t)}{(1-e^{-at})1(t)}$

$$e(t) = 1 - (1-e^{-at})$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} e^{-at} = 0$$

$$e(t) = 1 - K(1-e^{-at})$$

$$e_{ss} = \lim_{t\to\infty}(1-K-Ke^{-at})$$

$$e_{ss} = 1-K$$

⇒ The first order system tracks the step input with error $(1-K)$
if $K=1$, then error is zero

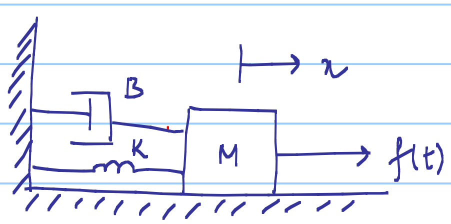
Ramp response

$$e(t) = t - \left(\frac{-1}{a} + t + \frac{1}{a}e^{-at}\right)$$

$$e_{ss} = 1/a$$

→ The first order system tracks the ramp (input) with constant error $1/a$.

Second Order System



$$f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2+Bs+K}}$$

as^2+bs+c

roots $s_1, s_2 = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

case (i): $b^2 = 4ac$, roots are repeated & real

case (ii) $b^2 < 4ac$, roots are complex

case (iii) $b^2 > 4ac$, roots are real & unique

case (iv): $b=0$, $\quad as^2+c=0 \Rightarrow s = \pm j\sqrt{\frac{c}{a}}$ $\quad$ Imaginary roots

Translate the above case to Mechanical system

$$\frac{-B \pm \sqrt{B^2 - 4MK}}{2M}$$

case (i) : $B^2 = 4MK$, $s_1, s_2 = -B/2M$

case (ii) : $B^2 < 4MK$, complex roots

case (iii) : $B^2 > 4MK$, real & unique

case (iv) : $B = 0 \Rightarrow$ System has no damping. $s_1, s_2 = \pm j\sqrt{\frac{K}{M}}$

→ If damping is zero ($B=0$), there will be oscillation for the impulse input & the frequency of oscillation is $\sqrt{K/M}$

Natural frequency : (Undamped frequency)

$$\omega_n^2 = K/M = c/a \Rightarrow \omega_n = \sqrt{c/a}$$

Damping ratio :

zeta $\zeta \triangleq \frac{\text{actual damping constant}}{\text{Critical damping constant}}$

$B^2 = 4MK$

$B = \sqrt{4MK}$

$$= \frac{B}{\sqrt{4MK}} = \frac{B}{2\sqrt{MK}} = \frac{b}{2\sqrt{ac}}$$

$$\zeta = \frac{b}{2\sqrt{ac}} \times \frac{a}{a} =$$

$$= \frac{b/a \times \cancel{\sqrt{a}}\sqrt{a}}{2\cancel{\sqrt{a}}\sqrt{c}}$$

$as^2 + bs + c$

$a(s^2 + b/a\, s + c/a)$

$a(s^2 + 2\zeta\omega_n s + \omega_n^2)$

$$\zeta = \frac{b/a}{2\sqrt{c/a}} = \frac{b/a}{2\omega_n}$$

$$\zeta = \frac{b/a}{2\omega_n}$$

$$\boxed{2\zeta\omega_n = b/a \quad , \quad c/a = \omega_n^2}$$

$$as^2 + bs + c = a(s^2 + b/a\, s + c/a) = a(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

General Second Order system

$$\boxed{G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

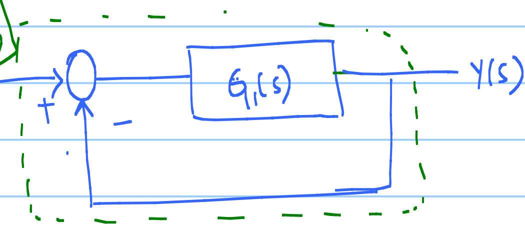
DC Gain

$K = \lim_{s \to 0} G(s)$

$K = 1$

$$G_1(s) = \frac{n_1}{d_1} = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{n_1(s)}{d_1(s)}}{1 + \frac{n_1(s)}{d_1(s)}} = \frac{n_1(s)}{d_1(s) + n_1(s)}$$



$$G(s) = \frac{G_1(s)}{1 + G_1(s)}$$

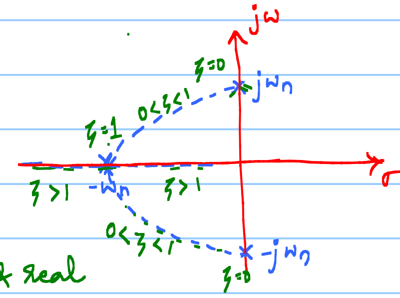
$$G_1(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2/s(s+2\zeta\omega_n)}{1 + \omega_n^2/s(s+2\zeta\omega_n)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

roots $\quad S_1, S_2 = \dfrac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4\omega_n^2}}{2}$ $\qquad a = 1,\ b = 2\xi\omega_n,\ c = \omega_n^2$

$= -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$

$= -\xi\omega_n \pm \omega_n \underbrace{\sqrt{\xi^2 - 1}}_{d}$



case (i): $\xi = 1$ $\quad$ roots $S_1, S_2 = -\omega_n$

Critical damping. $\quad$ roots are repeated & real

case (ii): $\xi = 0$, $\quad S^2 + \omega_n^2 = 0 \quad s_1, s_2 = \pm j\omega_n$ $\quad$ roots are imaginary

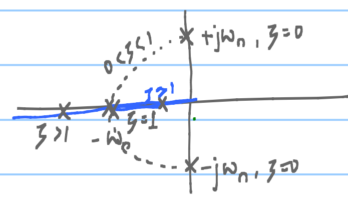
case (iii): $\quad 0 < \xi < 1 \qquad \xi^2 - 1 < 0$

$\Rightarrow S_1, S_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ $\qquad$ roots are complex

case (iv): $\quad \xi > 1 \qquad \xi^2 - 1 > 0$

$\Rightarrow S_1, S_2 = -\xi\omega_n \pm \omega_n d = -\omega_n(\xi \pm d)$ $\qquad$ roots are real & unique

when $\xi < 0$ roots will lie in right half s-plane



IMPULSE RESPONSE: $\quad u(t) = \delta(t)$

$G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad U(s) = 1 \qquad Y(s) = G(s)U(s)$

$Y(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \times 1$

$\sin at \longleftrightarrow \dfrac{a}{s^2 + a^2}$

$= \dfrac{\omega_n^2}{\underbrace{\boxed{s^2 + 2\xi\omega_n s + \xi^2\omega_n^2}}_{+b^2} \underbrace{- \xi^2\omega_n^2 + \omega_n^2}_{-b^2}}$ $\qquad s^2 + 2\xi\omega_n s + \omega_n^2 + b^2 - b^2$ (a^2, 2, b, a)

$= \dfrac{\omega_n^2}{(s + \xi\omega_n)^2 + \underbrace{\omega_n^2(1 - \xi^2)}_{\omega_d^2}} = \dfrac{\omega_n^2 \, \omega_d}{\omega_d\left((s + \xi\omega_n)^2 + \omega_d^2\right)}$

Damped frequency: ω_d $\qquad \boxed{\omega_d = \omega_n\sqrt{1 - \xi^2}}$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_d^2}\right\}$$

Applying freq shift property

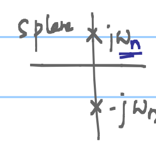
$$= e^{-\zeta\omega_n t}\,\mathcal{L}^{-1}\left\{\frac{\omega_n^2\,\omega_d}{\omega_d(s^2+\omega_d^2)}\right\}$$

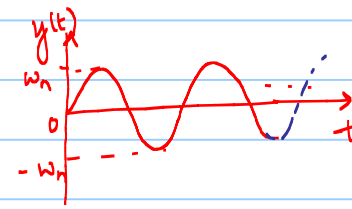
$$= \frac{\omega_n^2 e^{-\zeta\omega_n t}}{\omega_d}\,\mathcal{L}^{-1}\left\{\frac{\omega_d}{s^2+\omega_d^2}\right\}$$

$$y(t) = \frac{\omega_n^2}{\omega_d}\, e^{-\zeta\omega_n t}\sin(\omega_d t)\ ,\quad t \geq 0$$

$0<\zeta<1$ $$y(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t}\sin(\omega_d t)\ ,\ t\geq 0$$

When $\zeta=0$, roots are imaginary $$G(s) = \frac{\omega_n^2}{s^2+\omega_n^2}$$

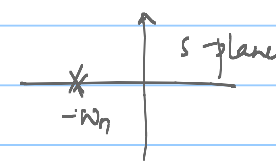




$$\boxed{y(t) = \omega_n \sin(\omega_n t)\ ,\ t\geq 0}$$

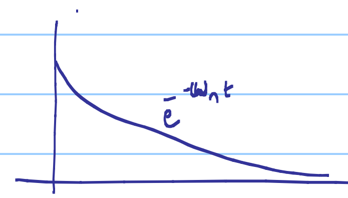
Response is oscillatory in nature

When $\zeta = 1$, roots are real & repeated $$G(s) = \frac{\omega_n^2}{s^2+2\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$



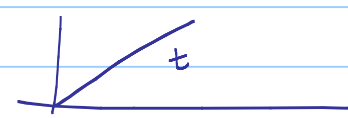
$$Y(s) = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{\omega_n^2}{(s+\omega_n)^2}\right\}$$



$$= e^{-\zeta\omega_n t}\,\mathcal{L}^{-1}\left\{\frac{\omega_n^2}{s^2}\right\}$$

$$\boxed{y(t) = \omega_n^2 e^{-\omega_n t}\, t}\ ,\ t\geq 0$$



$$\frac{dy(t)}{dt} = -\omega_n^3 e^{-\omega_n t}\,t + \omega_n^2 e^{-\omega_n t} = 0$$

$ab=0$

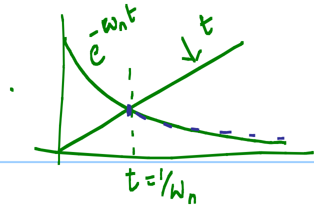
$$\boxed{\omega_n^2 e^{-\omega_n t}(-t\omega_n+1) = 0}$$

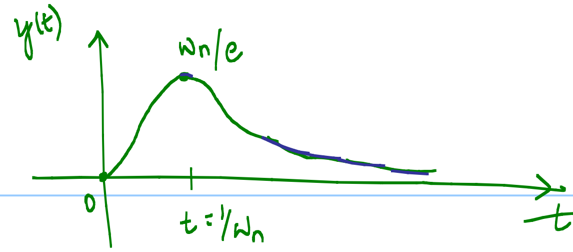
$$-t\omega_n + 1 = 0$$

$$\boxed{t = 1/\omega_n}$$

$$\frac{d^2y(t)}{dt^2} = +\omega_n^4 e^{-\omega_n t}\,t - \omega_n^3 e^{-\omega_n t} - \omega_n^3 e^{-\omega_n t}\Big|_{t=1/\omega_n} = -\omega_n^3/e < 0$$

$\Rightarrow y(t)\big|_{t=1/\omega_n}$ results in maximum value of $y(t)$

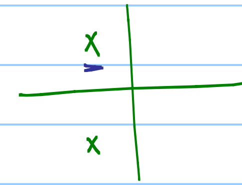




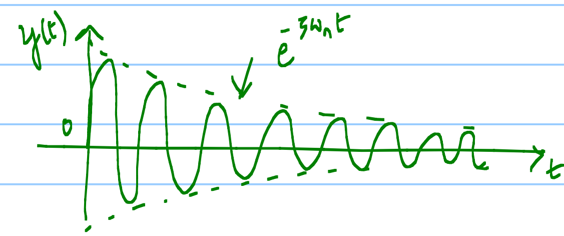
when $0 < \zeta < 1$

roots are complex $\quad y(t) = \dfrac{\omega_n^2}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$

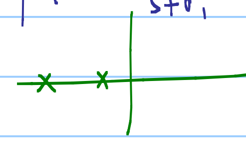
roots



Damped Sinusoid

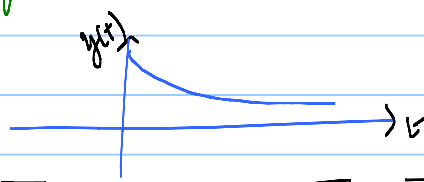


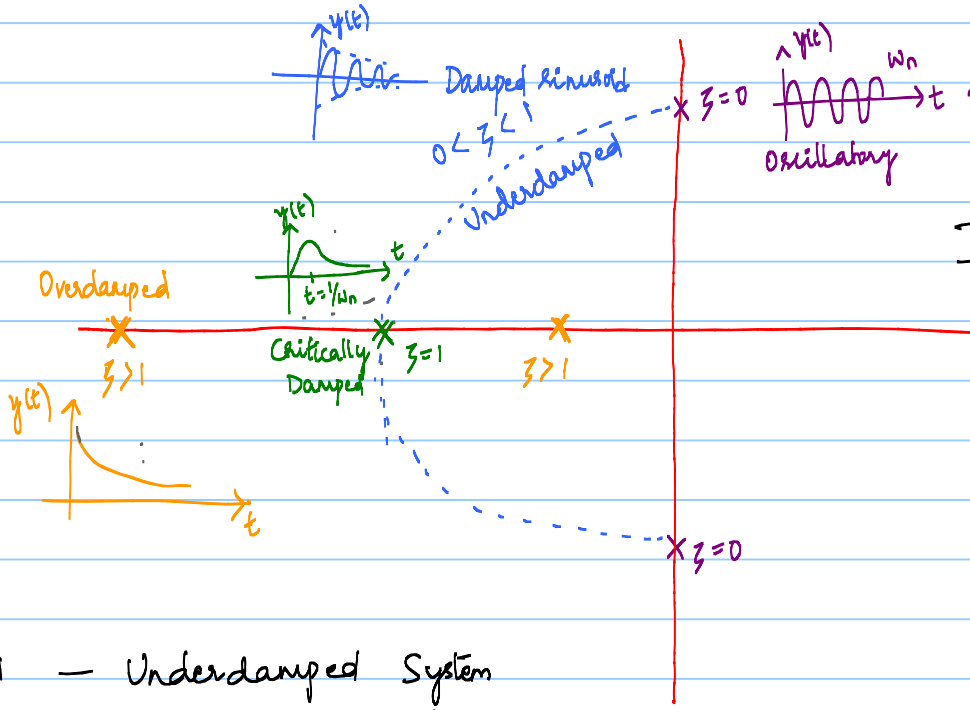
$$\boxed{G(s) = \frac{A}{s+\sigma_1} + \frac{B}{s+\sigma_2}}$$



When $\zeta > 1$,

$$y(t) = C_1 e^{-\sigma_1 t} + C_2 e^{-\sigma_2 t}$$





IMPULSE RESPONSE PLOTS

$0 < \zeta < 1$ — Underdamped System

$\zeta = 1$ — Critically damped system

$\zeta = 0$ — Undamped System

$\zeta > 1$ — Overdamped System

Step Response of Second Order System $\quad u(t) = 1(t)$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$U(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s}\,\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$Y(s) = \boxed{\frac{A^{1}}{s} + \frac{Bs + C}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \qquad \frac{(s^2 + 2\zeta\omega_n s + \omega_n^2 + (Bs + C)s}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$A = Y(s) \times s \Big|_{s=0} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\Bigg|_{s=0} = 1 \qquad \cancel{s^2} + 2\zeta\omega_n s + \omega_n^2 + B\cancel{s^2} + Cs$$

$$B = -1 \quad C = -2\zeta\omega_n$$

$$Y(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2 + (\zeta\omega_n)^2 - (\zeta\omega_n)^2}$$

Completion of squares

$$Y(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

denominator is obtained after completing the square

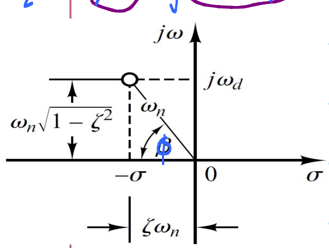
$$\mathcal{L}^{-1}\left\{\frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}\right\} = e^{-\zeta\omega_n t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2 + \omega_d^2}\right\} = e^{-\zeta\omega_n t}\cos(\omega_d t),\ t \geq 0$$

$$\mathcal{L}^{-1}\left\{\frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}\right\} = e^{-\zeta\omega_n t}\,\mathcal{L}^{-1}\left\{\frac{\zeta\omega_n}{s^2 + \omega_d^2}\right\} = e^{-\zeta\omega_n t}\,\frac{\zeta\omega_n}{\omega_d}\sin(\omega_d t),\ t \geq 0$$

$$\omega_d \rightarrow \omega_n\sqrt{1-\zeta^2}$$

$$s_1, s_2 = \overbrace{-\zeta\omega_n}^{\sigma} \pm j\overbrace{\omega_n\sqrt{1-\zeta^2}}^{\omega_d}$$

$$y(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta\cancel{\omega_n}}{\cancel{\omega_n}\sqrt{1-\zeta^2}}\sin\omega_d t\right),\ t \geq 0$$



$$\cos\phi = \zeta$$

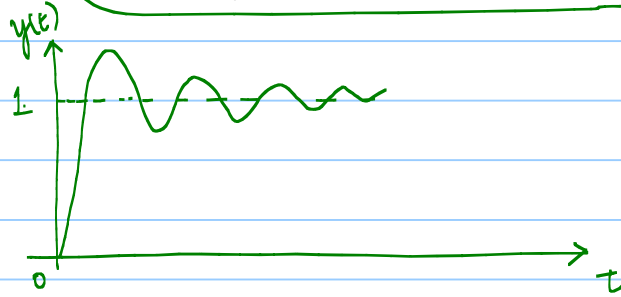
$$\sin\phi = \sqrt{1-\zeta^2}$$

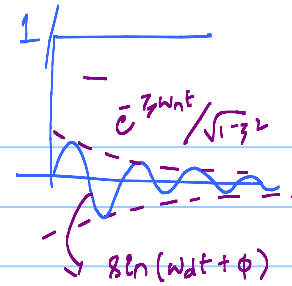
$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\underbrace{\sqrt{1-\zeta^2}}_{\sin\phi}\cos\omega_d t + \underbrace{\zeta}_{\cos\phi}\sin\omega_d t\right),\ t \geq 0$$

$$\boxed{y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi),\ t \geq 0} \qquad 0 < \zeta < 1$$

Case(i): $0 < \zeta < 1$ — underdamped system

$$\boxed{y(t) = 1 - \frac{\cancelto{1}{e^{-\zeta\omega_n t}}}{\cancelto{>1}{\sqrt{1-\zeta^2}}} \sin(\cancelto{\omega_n}{\omega_d} t + \cancelto{\pi/2}{\phi}), \quad t \geq 0}$$





Case ii: $\zeta = 0$

Undamped System

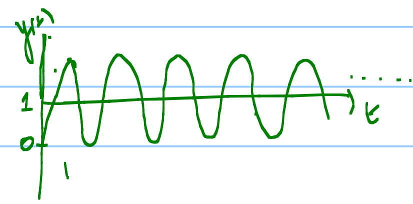
$$\omega_d = \omega_n\sqrt{1-\zeta^2} = \omega_n$$

$$y(t) = 1 - \cos(\omega_n t), \quad t \geq 0$$

$$G(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$Y(s) = \frac{1}{s} \frac{\omega_n^2}{(s^2+\omega_n^2)}$$

$Y(s) =$

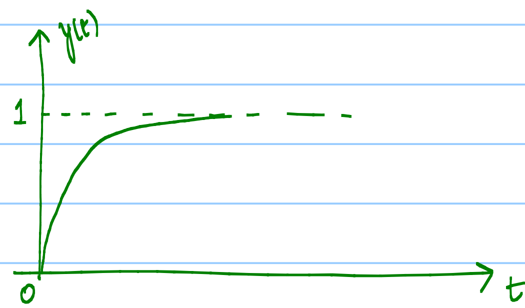


Case (iii): $\zeta = 1$,

Critically damped system

$$y(t) = \overset{1}{a} + \overset{-1}{b} e^{-\omega_n t} + \overset{\omega_n}{c} t e^{-\omega_n t}, \quad t \geq 0$$

$$Y(s) = \frac{1}{s} \frac{\omega_n^2}{(s+\omega_n)^2}$$

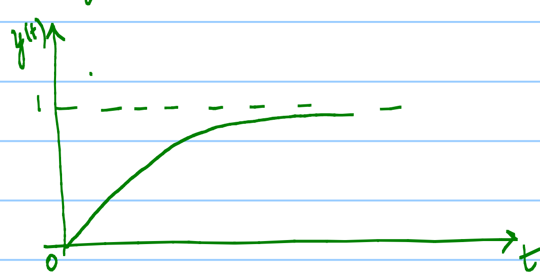


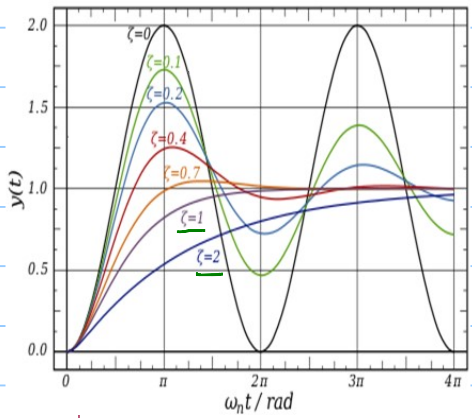
Case (iv): $\zeta > 1$,

Overdamped System

$$y(t) = 1 - a e^{-\sigma_1 t} - b e^{-\sigma_2 t}$$

$$G(s) = \frac{\omega_n^2}{(s+\sigma_1)(s+\sigma_2)}$$





- The degree of damping will indicate the nature of transients.
- For the ratio equal to Zero, the system will have no damping at all and continue to oscillate indefinitely.
- The ratio when increased from 0 to 1 (0 to 100%), will reduce the oscillations, with exactly no oscillations and best response at damping ratio equal to 1.
- On further increasing the damping ratio, the degree of damping has been overdone, this will cause sluggish performance/longer transients in the system.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$R(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

$$G(s) = \frac{400}{s^2 + 12s + 400}$$

$\zeta = 0.3, \quad \omega_n = 20$

Step response $y(t) = 1 - \frac{e^{-6t}}{\sqrt{1-0.3^2}} \sin(19.08t + \phi), \quad t \geq 0$ — Underdamped System

$\phi = 1.266$ rad

$$G(s) = \frac{900}{s^2 + 90s + 900}$$

$2\zeta\omega_n = 90$

$\zeta = \frac{90}{2 \times 30} = 1.5$

$\zeta = 1.5 \quad \omega_n = 30$

step response $y(t) = \cancel{a}^{\,1} + \cancel{b}^{\,+0.17} e^{-78.54t} + \cancel{c}^{\,-1.17} e^{-11.46t}, \quad t \geq 0$ — Overdamped system

$$G(s) = \frac{225}{s^2 + 30s + 225}$$

$\zeta = 1, \quad \omega_n = 15$

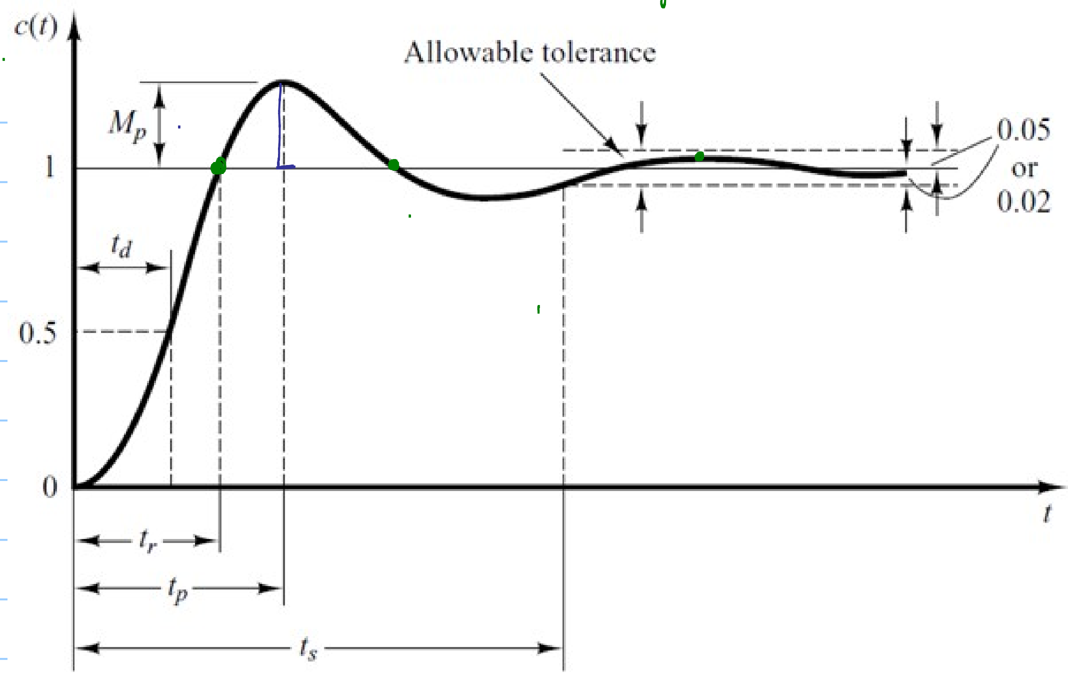
$y(t) = a + b e^{-15t} + c t e^{-15t}, \quad t \geq 0$ — Critically damped system

$$G(s) = \frac{625}{s^2 + 625}$$

$\zeta = 0, \quad \omega_n = 25$

$y(t) = 1 - \cos(25t), \quad t \geq 0$ — Undamped system

Time domain specifications: Underdamped second order system $0 < \zeta < 1$



Undershoot and

Time domain Specification:

1. Rise time t_r:

Underdamped system $(0 < \zeta < 1)$ – Time taken for the step response to go from 0 % to 100% of its final value

Critically damped $(\zeta = 1)$ and Overdamped systems $(\zeta > 1)$ – Time taken for the step response to go from 0 to 90 % of its final value (10% to 90%)

Underdamped systems: t_r

$$y(t_0) = 0$$

$$y(t_{100}) = 1$$

$$t_r = t_{100} - t_0 = t_{100}$$

$$y(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

$$y(t_r) = 1 = 1 - \frac{e^{-\zeta \omega_n t_r}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_r + \phi)$$

$$\frac{e^{-\omega_n \zeta t_r}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_r + \phi) = 0$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = 0$$ ($e^{-\zeta \omega_n t} \to 0$ only $t \to \infty$, but we are looking at transient)

$$t_r = \frac{n\pi - \phi}{\omega_d}$$

$n=1$, first peak

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$ See ϕ - rad $\quad \sin\phi = \sqrt{1-\zeta^2}$, $\cos\phi = \zeta$

2. Peak time t_p:

The time required for the step response to reach the first peak of the overshoot.

Differentiate $y(t)$ and solve for t_p.

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t_p}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_p + \phi), \quad t \geq 0$$

$$\frac{dy(t)}{dt} = \frac{\zeta\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_p + \phi) - \frac{e^{-\zeta\omega_n t}\, \omega_d}{\cancel{\sqrt{1-\zeta^2}}} \cos(\omega_d t_p + \phi)$$

$\omega_d = \omega_n \cancel{\sqrt{1-\zeta^2}}$

$$\Rightarrow \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\left[\underbrace{\zeta}_{\cos\phi} \sin(\omega_d t_p + \phi) - \underbrace{\sqrt{1-\zeta^2}}_{\sin\phi} \cos(\omega_d t_p + \phi)\right] = 0$$

$$\Rightarrow \cos\phi \sin(\omega_d t_p + \phi) - \sin\phi \cos(\omega_d t_p + \phi) = 0$$

$$\Rightarrow \sin(\omega_d t_p) = 0$$

$$\Rightarrow \omega_d t_p = n\pi$$

$$\therefore t_p = \frac{n\pi}{\omega_d}$$

Since we are looking first peak, $n=1$.

$$\boxed{t_p = \frac{\pi}{\omega_d}} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Maximum Overshoot: M_p

It is the maximum peak value of the step response measured from unity

$$\text{Maximum \% overshoot} = \frac{y(t_p) - \overset{y(\infty)}{1}}{\underset{(y(\infty))}{1}} \times 100\%$$

$$y(t)\Big|_{t_p = \pi/\omega_d} = 1 - \frac{e^{-\zeta\omega_n \pi/\omega_d}}{\sqrt{1-\zeta^2}} \sin(\omega_d \pi/\omega_d + \phi)$$

$$= 1 - \frac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}} \sin(\pi + \phi)$$

$$= 1 + \frac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}} \sin\phi$$

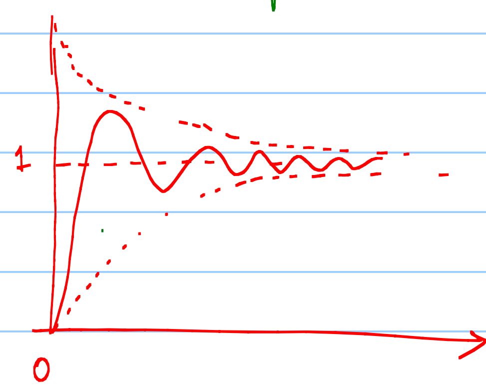
$$y(t)\Big|_{t=t_p} = 1 + \frac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\cancel{\sqrt{1-\zeta^2}}} \cancel{\sqrt{1-\zeta^2}}$$

Max % overshoot $= \cancel{1} + e^{-\zeta\pi/\sqrt{1-\zeta^2}} - \cancel{1}$

$$\boxed{M_p(\%) = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%}$$

Settling time : t_s

It is the time required for the step response curve to reach & stay within the range about the final value (2% or 5%)



The envelope decides the amplitude of sin

2% $\quad e^{-\zeta\omega_n t_s} = 0.02$

$$\boxed{t_s = \frac{3.91}{\zeta\omega_n} \approx \frac{4}{\zeta\omega_n}}$$

$$1 + \frac{e^{-\zeta\omega_n t_s}}{\sqrt{1-\zeta^2}} = 1.02$$

or

$$1 - \frac{e^{-\zeta\omega_n t_s}}{\sqrt{1-\zeta^2}} = 0.98$$

5% $\quad e^{-\zeta\omega_n t_s} = 0.05$

$$\boxed{t_s = \frac{2.99}{\zeta\omega_n} \approx \frac{3}{\zeta\omega_n}}$$

Given $\zeta = 0.6$, $\omega_n = 5$ rad/sec, find t_p, t_r, t_s, M_p

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.78 \text{ sec}$$

$$t_r = \frac{\pi - \phi}{\omega_d}$$

$\cos\phi = \zeta$

$\phi = \cos^{-1}(0.6)$

$= 0.93$ rad

$$t_r = \frac{3.14 - 0.93}{5\sqrt{1-0.6^2}}$$

$$t_r = 0.55 \text{ sec}$$

2% $\quad t_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{0.6\times 5} = 1.33$ sec

5% $\quad t_s = \dfrac{3}{\zeta\omega_n} = 1$ sec

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = e^{-0.6\times 3.14/\sqrt{1-0.6^2}}$$

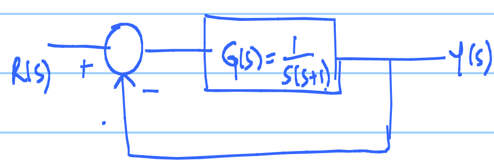
$$= 0.093$$

$$M_p(\%) = 9.3\%$$

- **For a unity negative feedback system having open loop transfer function**

$G(s) = \dfrac{1}{s(s+1)}$ — OLTF

- **Damping ratio**
- **Damped frequency and natural frequency**
- **Peak time**
- **Peak overshoot for step input**



CLTF, $\quad T(s) = \dfrac{G(s)}{1+G(s)}$

$$= \frac{1/s(s+1)}{1 + 1/s^2+s}$$

$$T(s) = \frac{1}{s^2 + s + 1} \qquad \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n^2 = 1 \Rightarrow \omega_n = 1$ rad/sec — Natural frequency

$2\zeta\omega_n = 1$

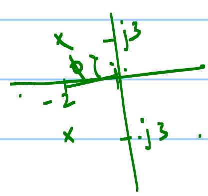
damping ratio $\zeta = 1/2 = 0.5$

Damped freq $\omega_d = \omega_n\sqrt{1-\zeta^2} = 0.866$ rad/sec

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{0.866} = 3.62 \text{ sec}$$

$$\%\ \text{overshoot} = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\%$$

$$= 16.3\%$$



$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$= \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = \underbrace{-\zeta\omega_n} \pm j\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}$$

$\zeta = \cos\phi$

The closed loop poles of a system are given as $-2 \pm j3$. Find $\zeta, \omega_d, \omega_n, t_p, M_p$ for step input.
t_r, t_s

$S_1 = -2 + j3 \qquad S_2 = -2 - j3$

denominator $\rightarrow (S - S_1)(S - S_2)$

$(\underbrace{S+2}_{a} - \underbrace{j3}_{b})(S + 2 + j3)$

$(S+2)^2 - (j3)^2 = S^2 + 4S + 4 + 9$

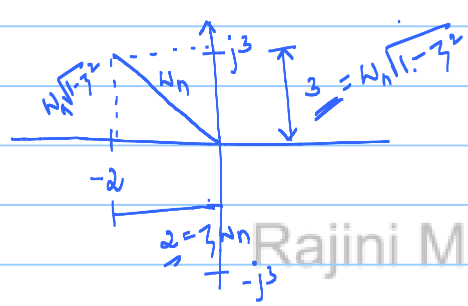
denominator $\rightarrow S^2 + 4S + 13$

$\omega_n^2 = 13$

$\omega_n = \sqrt{13}$ rad/sec

$2\zeta\omega_n = 4$

$\zeta = 2/\sqrt{13} = 0.55$



$s^2 + 2\zeta\omega_n s + \omega_n^2$

$s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

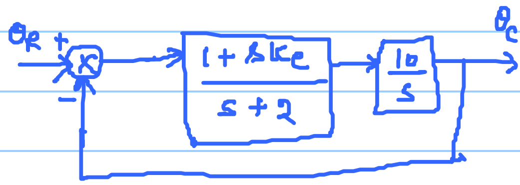
$\omega_n = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\zeta\omega_n = 2 \Rightarrow \zeta = 2/\sqrt{13}$

$t_r = \frac{3.14 - 0.98}{3} = 0.717$ sec $\quad t_p = \frac{\pi}{3} = 1.04$ sec $\quad t_s(2\%) = \frac{4}{\zeta\omega_n} = 2$ sec

$\% M_p = 12.63\%$

Determine the values of t_s, M_p & e_{ss} (unit ramp input) with and without error rate control k_e. Given $\zeta = 0.6$ (with k_e)



$$\frac{\theta_c(s)}{\theta_R(s)} = \frac{\frac{10(1+sK_e)}{s(s+2)}}{1 + \frac{10(1+sK_e)}{s(s+2)}}$$

$$= \frac{10 + 10sK_e}{s^2 + 2s + 10sK_e + 10}$$

$\omega_n = \sqrt{10} \quad \zeta = 0.6$

$2\zeta\omega_n = 2 + 10K_e$

$K_e = 0.179$

$K_e = 0.18$

$s^2 + 2\zeta\omega_n s + \omega_n^2 \Rightarrow$

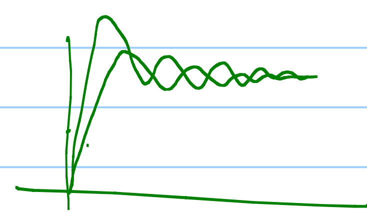
$\omega_n^2 = 10 \Rightarrow \omega_n = \sqrt{10}$

$2\zeta\omega_n = 2 + 10K_e$

$\Rightarrow K_e = \frac{2\zeta\omega_n - 2}{10}$

$= \frac{2 \times 0.6 \times 3.16 - 2}{10}$

$= 0.18$



$\omega_d = \sqrt{10}\sqrt{1-0.6^2} = 2.52$ rad/sec

$t_p = \pi/\omega_d = 1.24$ sec

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.6 \times \sqrt{10}} = 2.11 \text{ sec}$$

$$\% M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100 = 9.47\%$$

$$t_r = \frac{\pi - \cos^{-1}(0.6)}{2.52} = 0.87 \text{ sec}$$

Without Ke :

$K_e = 0$

$$G(s) = \frac{10}{s(s+2)}$$

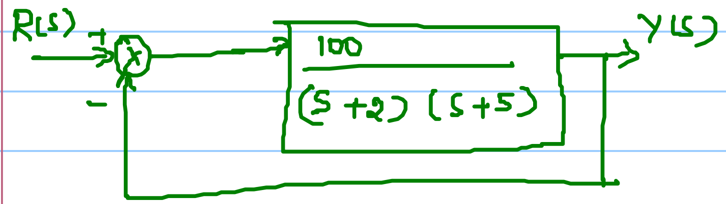
$$, \frac{\theta_c}{\theta_R} = \frac{10}{s^2 + 2s + 10}$$

$\omega_n = \sqrt{10}$, $2\zeta\omega_n = 2 \Rightarrow \zeta = \frac{1}{\sqrt{10}} = 0.32$

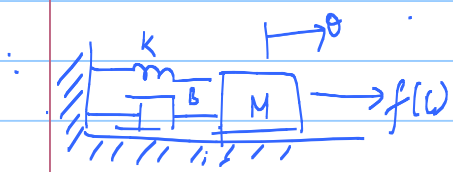
$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.32 \times \sqrt{10}} = 4 \text{ secs}$$

$$\% M_p = e^{-\pi \times 0.33/\sqrt{1-0.33^2}} \times 100 = \underline{35\%}$$

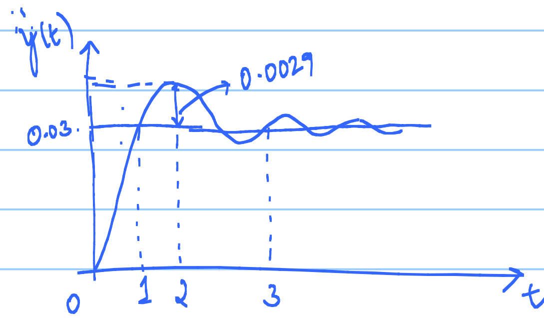
$$t_p = \frac{\pi}{\omega_d} = 1.04 \text{ sec}$$



Determine % overshoot, ξ & ω_n.



force = 8.9 N



$t_p = 2$ sec

$t_r = 1$ sec

$y(\infty) = 0.03$

peak $= 0.0029 + 0.03$

$$f(t) = M\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} + K\theta(t)$$

$$\frac{\theta(s)}{F(s)} = \frac{1}{\underline{Ms^2 + Bs + K}}$$

$s^2 + 2\xi\omega_n s + \omega_n^2$

$\omega_n = \sqrt{K/M}$ $\quad 2\xi\omega_n = B/M$

$$\xi = \frac{B}{2M\sqrt{K/M}} = \frac{B}{2\sqrt{KM}}$$

$$\theta(s) = \frac{1}{Ms^2 + Bs + K} F(s)$$

$$F(s) = \frac{8.9}{s}$$

$$\theta(s) = \frac{8.9}{s(Ms^2 + Bs + K)}$$

Final Value Theorem

function $f(t)$

$$f_{ss} = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

$$\lim_{t \to \infty} \theta(t) = \lim_{s \to 0} s\theta(s) \Rightarrow \theta(\infty) = \frac{8.9}{K} = 0.03 \text{ (from the graph)}$$

$$\Rightarrow K = 296.67 \text{ N/m}$$

$$t_p = \frac{\pi}{\omega_d} = 2 \Rightarrow \omega_d = \pi/2 = 1.57 \text{ rad/sec}$$

$$M_p = \frac{\theta(t_p) - \theta(\infty)}{\theta(\infty)} = \frac{0.03 + 0.0029 - 0.03}{0.03} = \frac{0.0029}{0.03} = 0.096$$

$$e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.096$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} = \ln(0.096)$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{2.34}{\pi}$$

$$\frac{\zeta^2}{1-\zeta^2} = \left(\frac{2.34}{\pi}\right)^2$$

$$\zeta = 0.591$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{1.571}{\sqrt{1-0.591^2}} = 1.958 \text{ rad/sec}$$

$$\underbrace{s^2+2\zeta\omega_n s+\omega_n^2}_{} \qquad Ms^2+Bs+K \rightarrow M\underbrace{(s^2+B/M\, s+K/M)}_{}$$

(1) $\omega_n^2 = K/M \Rightarrow M = K/\omega_n^2 = \dfrac{296.7}{(1.958)^2} = 77.38$ kg

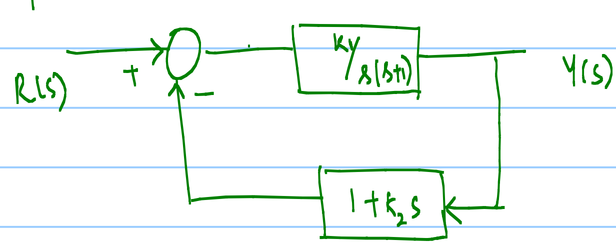
$\omega_n = \sqrt{K/M}$

(2) $2\zeta\omega_n = B/M$

$B = 2\zeta\omega_n \times M = 2\times 0.591\times 1.958\times 77.38$

$B = 179.08$ N/m/sec

For the following block diagram, the given specification to be satisfied are $M_p = 0.2$, $t_p = 1$ sec.



Design K_1 & K_2 and also find t_r, t_s(2%).

Given $M_p = 0.2$ $\quad t_p = 1$

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.2$$

$$\frac{\zeta^2\pi^2}{1-\zeta^2} = (1.699)^2$$

$$\left(\pi^2 + (1.609)^2\right)\zeta^2 = (1.609)^2$$

$$\zeta = \sqrt{\frac{(1.609)^2}{\pi^2 + (1.609)^2}}$$

$$\zeta = 0.45$$

$$t_p = \frac{\pi}{\omega_d} = 1$$

$$\Rightarrow \omega_d = \pi = 3.14 \text{ rad/sec}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{\pi}{\sqrt{1-(0.45)^2}} = 3.53 \text{ rad/sec}$$

$$T(s) = \frac{K_1}{s^2 + (K_1K_2+1)s + K_1}$$

Comparing with $s^2 + \underbrace{2\zeta\omega_n}\, s + \omega_n^2$

$$\Rightarrow \omega_n^2 = K_1$$

$$\boxed{K_1 = (3.53)^2 = 12.46}$$

$$\Rightarrow 2\zeta\omega_n = K_1K_2 + 1$$

$$K_2 = \frac{2\zeta\omega_n - 1}{K_1} = 0.175$$

$$\boxed{K_2 = 0.175}$$

$$t_r = \frac{\pi - \cos^{-1}(\zeta)}{3.14} = \frac{\pi - \cos^{-1}(0.45)}{3.14}$$

$$t_r = 0.65 \text{ sec}$$

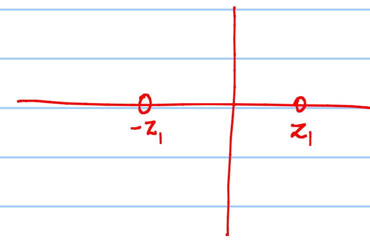
$$(2\%)\ t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.45 \times 3.53} = 2.52 \text{ sec}$$

$$(5\%)\ t_s = \frac{3}{\zeta\omega_n} = \frac{3}{0.45 \times 3.53} = 1.89 \text{ sec}$$

Effect of addition zero to the second order system

Let a zero at $s = -z$ be added to the second order transfer function

$$\frac{Y_1(s)}{R(s)} = \frac{\omega_n^2/z\,(s+z)}{s^2+2\zeta\omega_n s+\omega_n^2}$$



$$Y_1(s) = \frac{\frac{\omega_n^2 s}{z}}{s^2+2\zeta\omega_n s+\omega_n^2} \times \frac{1}{s} + \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

$$= \frac{s}{z}\underbrace{\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}} + \underbrace{\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}}$$

I.LT → Step response.

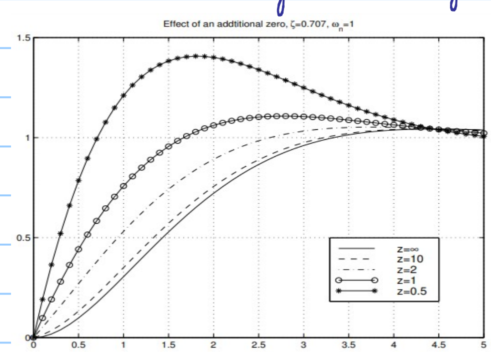
$$Y_1(s) = \frac{s}{z}Y(s) + Y(s)$$

ILT →

$$y_1(t) = \frac{1}{z}\frac{d}{dt}y(t) + y(t)$$

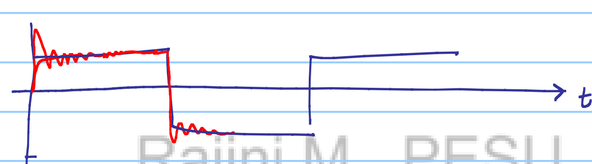
→ When zero is placed near to the origin the effect of the additional zero on step response would be more pronounced & will have less effect on the step response as it moves (placed) away from the origin

→ Smaller value of z i.e close to the origin, more is peak overshoot (generally avoided in the design unless system is sluggish)



Hence, the additional zero in the left half-plane speeds up transients, making rises and falls sharper. Smaller values of z make this effect more prominent

→ The effect of a zero on the system response is it makes the system respond faster.

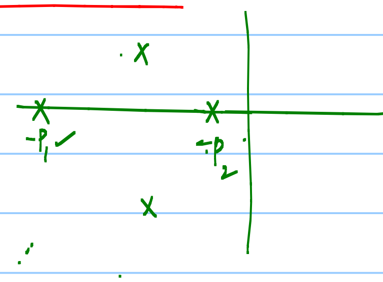


→ The response starts from negative value when the zero is placed in RHS

→ Additional zero doesnot affect stability.

Effect of additional pole on the second order system.

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



$$\frac{Y_1(s)}{R(s)} = \frac{p\,\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(s+p)}$$

Step response:

$$Y_1(s) = \frac{p\,\omega_n^2}{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)(s+p)}$$

$$Y_1(s) = \frac{A}{s+p} + \underbrace{\frac{B}{s} + \frac{Cs+D}{(s^2+2\zeta\omega_n s+\omega_n^2)}}_{\text{ILT} \,-\, y(t) \,-\, \text{actual step response}}$$

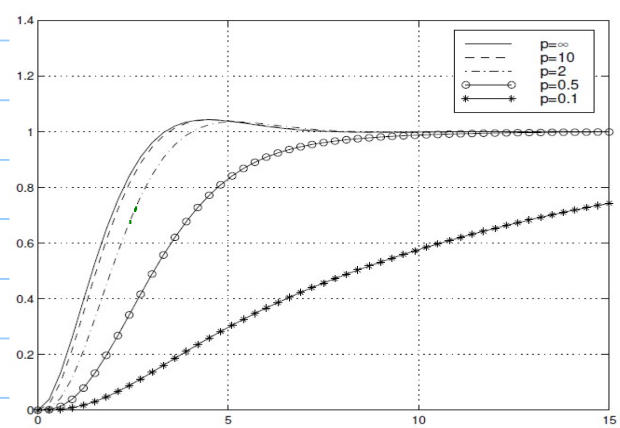
$$Y_1(s) = \frac{A}{s+p} + Y(s)$$

ILT →

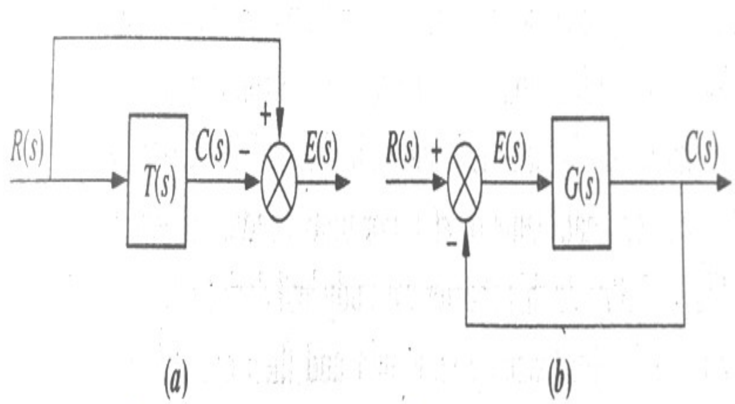
$$y_1(t) = A e^{-pt} + y(t)$$

→ with the additional pole the system becomes slow

→ pole near to origin will make system sluggish & the poles becomes "dominant pole"



Steady state error: e_{ss}



a) Closed loop control system error

b) Representation for UFB

Def 1: $E(s) = R(s) - Y(s)$

from Block diagram reduction: $Y(s) = \frac{G(s)}{1+G(s)H(s)} R(s)$

$$E(s) = R(s)\left(1 - \frac{G(s)}{1+G(s)H(s)}\right)$$

$$\boxed{E(s) = \left(\frac{1+G(s)H(s) - G(s)}{1+G(s)H(s)}\right) R(s)}$$

Def 2: $E(s) = R(s) - H(s)Y(s)$

$Y(s) = G(s)E(s)$

$E(s) = R(s) - H(s)G(s)E(s)$

$$E(s)\left(1 + H(s)G(s)\right) = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1+G(s)H(s)}}$$

Steady state error

$$\boxed{e_{ss} = \lim_{s \to 0} s\,E(s)}$$