DIGITAL COMMUNICATION

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QUANTIZATION AND PULSE SHAPING

Quantization Error/ Noise

SNR for transmission with Quantization Noise

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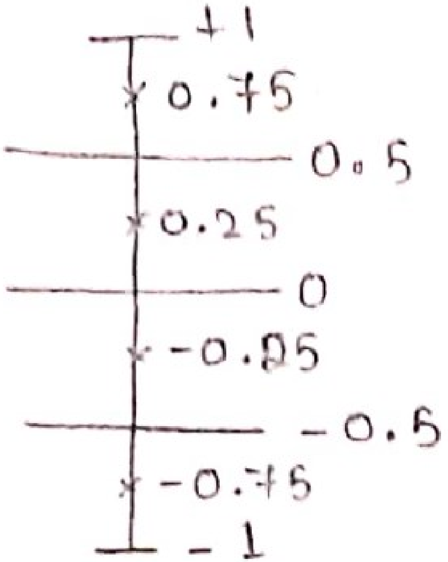
Quantization Error/ Noise

Consider the previous example:

$x(n)$	$y(n)$	$q(n) = y(n) - x(n)$
0.38	0.25	−0.13
0.51	0.75	0.24
⋮	⋮	⋮



Quantization Error/ Noise

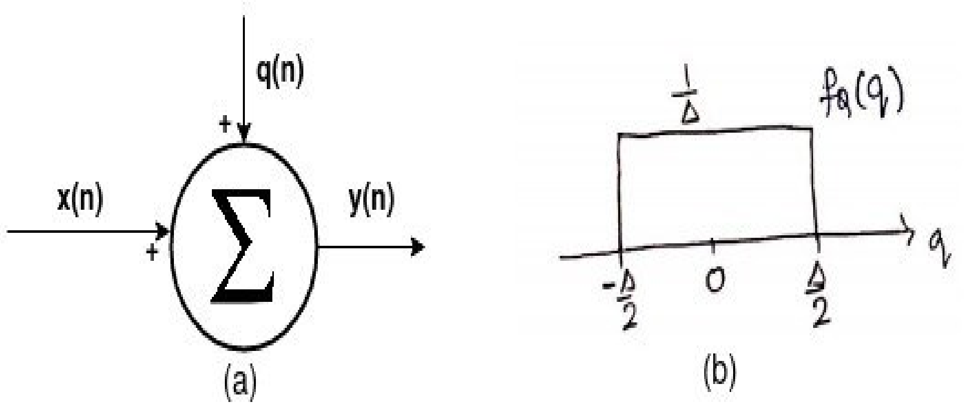


Figure 5 :Quantization based figures

We can represent the quantization noise as an addition of the real value of the signal with a fixed quantization value as shown in figure(5 a).

$\therefore y(n) = x(n) + q(n). \Rightarrow q(n) = y(n) - x(n).$

W.k.t. $y(n) = a_k \; if \; x(n) \in \Delta_k \Rightarrow q(n) = a_k - x(n) \; if \; x(n) \in \Delta_k$

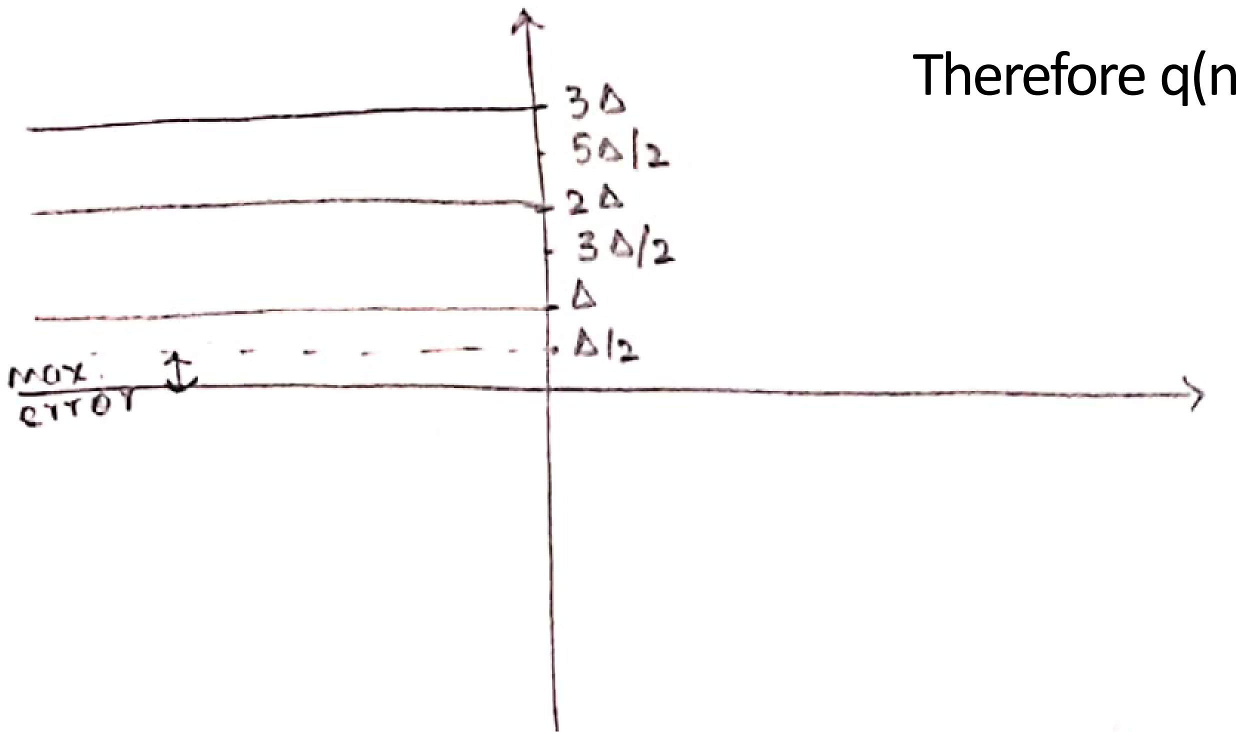
The pdf of a quantization scheme of uniform quantization is as shown in figure(5 b). Its step width is: $\Delta = 2A/2^N$ (1)

Quantization Error/ Noise

q(n) is called the quantization error/ noise [q(n) = approx. – actual value]

Quantization error cannot be removed

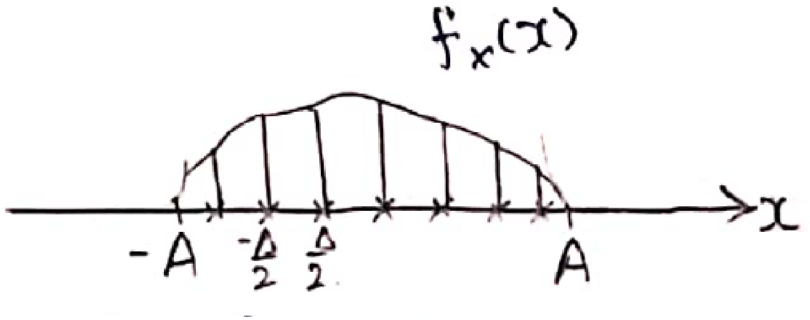
Since x(n) is random, q(n) is also random



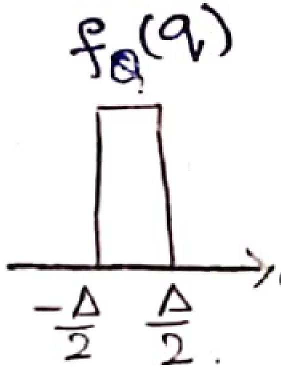
Therefore q(n) takes values in the range $-\frac{\Delta}{2}$ to $\frac{\Delta}{2}$.

SNR for transmission with Quantization Noise

PDF of x(n) & q(n):



$\Rightarrow$

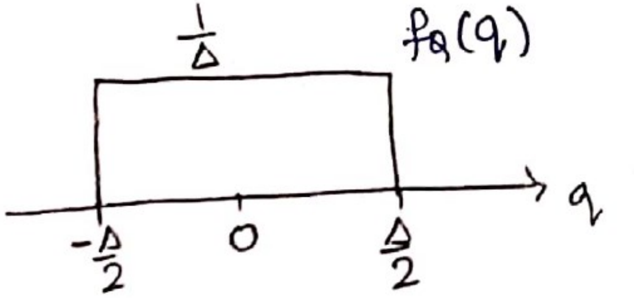


pdf of q(n) for large values of N.

PDF of x(n)

If N is large then $\boxed{\Delta = \frac{2A}{2^N}}$ is small hence we can assume that the pdf of the quantization error is uniform over the range $\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$

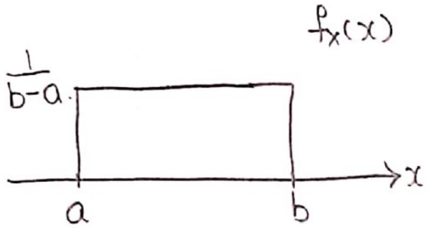
SNR for transmission with Quantization Noise



pdf of q(n) for large values of N

E[Q] = 0 (i.e., mean of the random variable Q = 0)

x(n) & q(n) are the realizations of the random variable X & Q respectively



$$\boxed{\begin{aligned} E[X] &= \frac{a+b}{2} = \mu_X \\ \sigma_X^2 &= \frac{(b-a)^2}{12} \end{aligned}}$$

SNR for transmission with Quantization Noise

Show that the variance of x is $\frac{(b-a)^2}{12}$. for the given uniform pdf.

Sln w.k.t. $\sigma_x^2 = E[X^2] - \mu_X^2$.

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx.$$

$$= \int_a^b x^2 \cdot \frac{1}{(b-a)}\, dx.$$

$$= \left.\frac{x^3}{3(b-a)}\right|_a^b.$$

SNR for transmission with Quantization Noise

$$E[x^2] = \frac{b^3 - a^3}{3(b-a)}$$

$$\sigma_x^2 = \frac{b^2 + ab + a^2}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{4b^2 + 4ab + 4a^2 - 3a^2 - 6ab - 3b^2}{12}$$

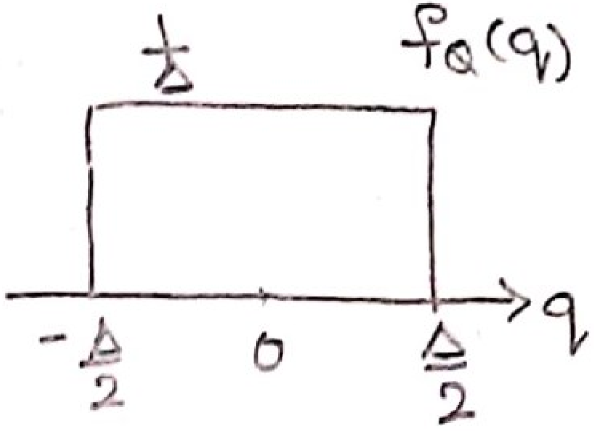
$$= \frac{b^2 - 2ab + a^2}{12}$$

$$\therefore \boxed{\sigma_x^2 = \frac{(b-a)^2}{12}}$$

SNR for transmission with Quantization

Hence, we get $\boxed{\sigma_Q^2 = \frac{\Delta^2}{12}}$

as $(b-a) = \Delta$.



$\therefore$ The Average Power of the Quantization is. $E[Q^2] = \sigma_Q^2 + \mu_Q^2$.

as $\mu_Q^2 = 0$

$\therefore E[Q^2] = \sigma_Q^2$

$\boxed{E[Q^2] = \frac{\Delta^2}{12}} \rightarrow$ Average power for the zero mean Random variable.

SNR for transmission with Quantization Noise

Hence we can estimate the performance of a quantizer by defining a term called Signal to Quantization Noise Ratio as follows:

$$SNR = \frac{\text{Average Signal Power}}{\text{Average Power of the Quantization Noise}}$$

Hence;

$$SNR = \frac{E[X^2]}{\sigma_Q{}^2} = \frac{\sigma_X{}^2}{\sigma_Q{}^2} = \frac{\sigma_X{}^2}{\Delta^2/12}$$

The signal is usually assumed to have zero mean hence we have average signal power as $\boxed{E[X^2] = \sigma_X{}^2}$

$$\therefore \boxed{SNR = \frac{\sigma_X{}^2}{\sigma_Q{}^2} = \frac{\sigma_X{}^2}{\Delta^2/12}}$$

THANK YOU

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