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Principles of Digital Signal Processing

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DSP

Discrete Fourier Transform

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Frequency domain sampling
DFT

DFT and IDFT expressed as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad k = 0, 1, \ldots, N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \qquad n = 0, 1, \ldots, N-1$$

Where, $W_N = e^{-j2\pi/N}$

The *Nth* root of unity

Frequency domain sampling

DFT

Linear transformation equations

DFT → $X_N = W_N x_N$

IDFT → $x(n)$

$$x_N = \frac{1}{N} W_N^{*} X_N$$

X_N → Vector → DFT coefficients

x_N → i/p vector

W_N → N X N transformation matrix.

Frequency domain sampling

DFT

$x(n) = [0, 1, 2, 3]$ → 4-pt DFT.

$X_4 = W_4 \, x_4$

$= \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} x_4$

$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$

$W_4^0 = 1$

$W_4^1 = -j$

$W_4^2 = -1$

$W_4^3 = +j$

$W_N^n = \left(e^{-j2\pi/N}\right)^n$

$X_4 = [6, \; -2+2j, \; -2, \; -2-2j]$

Frequency domain sampling

DFT

IDFT

$X(k) = [6, -2+2j, -2, -2-2j]$

find $x(n)$ → 4 pt sequence.

Linear transformation eqn

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}$$

$n = 0 \ldots N-1$

$$x_N = \frac{1}{N} W_N^* X_N$$

$$x_N = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix} \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$x_N = [0, 1, 2, 3]$$

Frequency domain sampling

DFT

a) find the # of Complex Multiplications and Complex Additions for $N=8$.

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad k = 0 \cdots N-1.$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \quad n = 0 \cdots N-1$$

Complex Multiplications — $N^2 = 8^2 = \underline{64}$

Complex Additions — $N(N-1) = 8 \times 7 = \underline{56}$

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THANK YOU

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