DIGITAL IMAGE PROCESSING-1

Unit 4: Lecture 44-45

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Unit 4: Image Filtering and Restoration

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Last Session

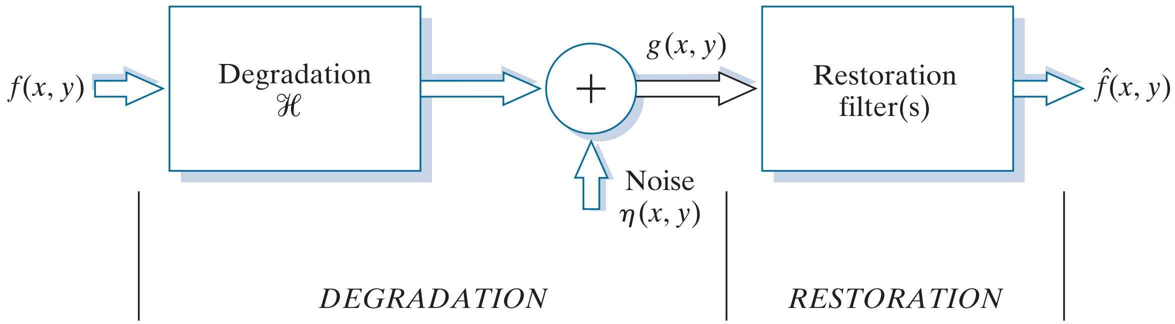
- Restoration in the presence of Noise only
  - Spatial domain
  - Frequency Domain
- Introduction to restoration in the presence of degradation

This Session

- Image Restoration in presence of degradation only
- Image Restoration in presence of degradation and noise

Restoration in the Presence of Degradation Only

Degradation / Restoration Model



- Spatial Domain: $g(x,y) = h(x,y)*f(x,y) + \eta(x,y)$
- Frequency Domain: $G(u,v) = H(u,v).F(u,v) + N(u,v)$

Linear Position Invariant Degradation

- Now

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

Taking Fourier Transform on both sides

- G(u,v) =F(u,v)H(u,v) + N(u,v)
- If we assume N(u,v)=0 then

  G(u,v) = F(u,v).H(u,v)

  Or g(x,y) = f(x,y)*h(x,y)

Linear Position Invariant Degradation

- Degradation function 'H' satisfies following two properties:
  - Linearity
    - Superposition and homogeneity property
  - Shift Invariant (Position Invariant)
    - If g(x,y) = H[f(x,y)]

    Then g(x-a,y-b)=H [f(x-a,y-b)] for any a , b & f(x,y)

- Any type of degradation can be approximated by LPI/LSI process, **since degradations are modeled as result of convolution.**

- **Restoration seeks the filters performing reverse procedure: Deconvolution filters**

Linear Position Invariant Degradation

- The term *image* ***deconvolution*** is used frequently to signify linear image restoration.
- Similarly, the filters used in the restoration process often are called ***deconvolution filters*****.**

Estimation of Degradation Function

- Suppose that we are given a degraded image without any knowledge about the degradation function H.
- To restore image we need to estimate degradation function first
- There are three methods:
  - By Observation
  - By Experimentation
  - By Mathematical modeling
- Once degradation function has been estimated, then **restoration is achieved by deconvolution**

$g(x,y) = f(x,y)*h(x,y)$

Estimation of Degradation Function: By Observation

- Assume that an image f(x,y) is degraded with an unknown degradation function H
- Then we try to estimate H from the information gathered from the image itself
- To reduce the effect of noise, we look for an area in image in which signal content is strong(an area of high contrast)
- Next step is to process the subimage to arrive at a result that is as unblurred as possible (improve quality by known enhancement methods)

Estimation of Degradation Function: By Observation

- Let $g_s(x,y)$: Observed subimage with noise

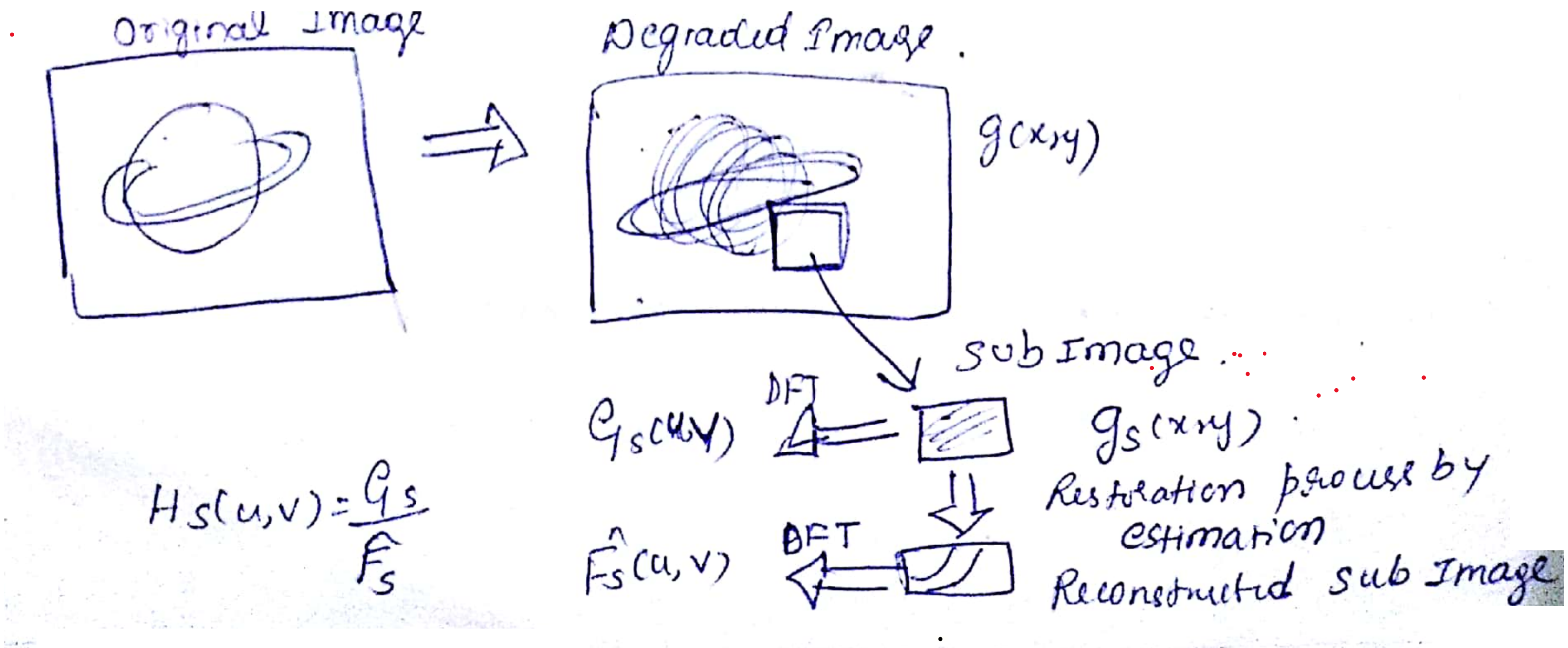
$\hat{f}_s(x,y)$: Processed subimage with low noise content(estimate of original image in that area)

- Assume that the effect of noise is negligible here (because of choosing strong signal area)

$$G_s(u,v) = H_s(u,v).F_s^{\wedge}(u,v)$$

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Estimation of Degradation Function: By Observation



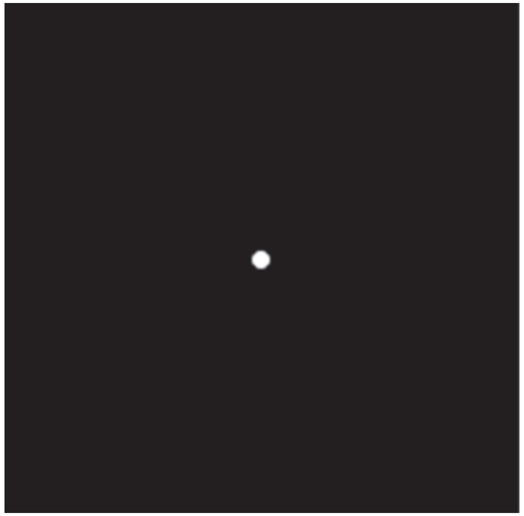
Estimation of Degradation Function: By Observation

- From the characteristics of $H_s(u,v)$ we deduce the complete degradation function H(u,v) based on our **assumption of position invariance.**
- This is a laborious process and used only for specific applications like restoring old photographs

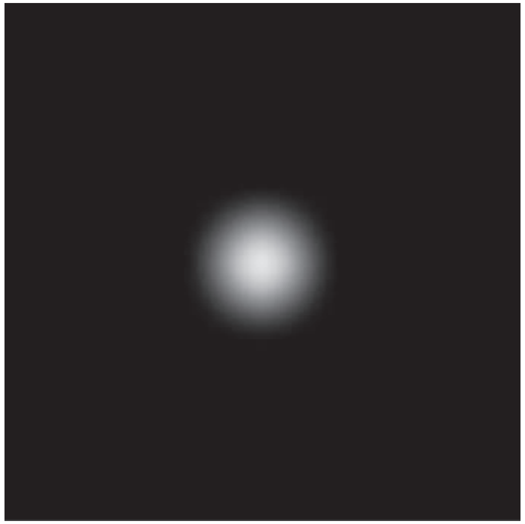
Estimation of Degradation Function: By Experimentation

- It is possible to estimate the degradation function if the equipment used to acquire the degraded image is available.
- **Step 1:** Adjust the equipment by varying the system setting such that the image obtained is similar to the degraded image that needs to be restored
- **Step 2:** Obtain the impulse response (simulated by a maximally bright dot of light)of the degradation by imaging an impulse using the same system setting(as LSI system is completely characterized by its impulse response)

Estimation of Degradation Function: Experimentation

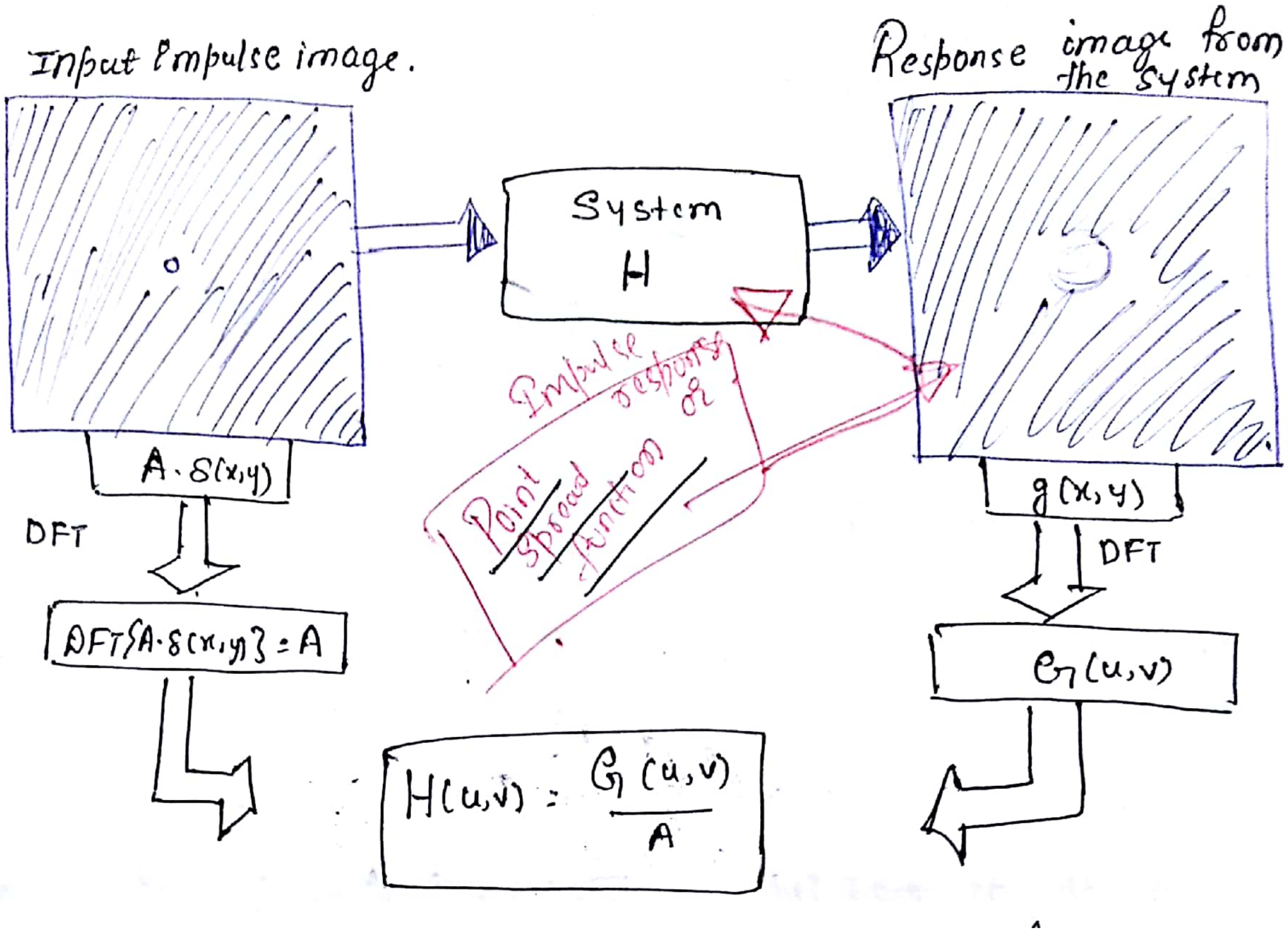


An impulse of light
(shown magnified)



Imaged (degraded) impulse
(Impulse response)

Estimation of Degradation Function: By Experimentation



Estimation of Degradation Function: By Experimentation

- Impulse response is given by

$$H(u,v) = G(u,v)/A$$

Where G(u,v) is DFT[degraded image]

A= constant describing strength of impulse

Estimation of Degradation Function: By Modeling

- Used to estimate degradation function/model
- There are several fundamental models for degradation function

Method 1:

- Degradation model based on atmospheric turbulence blur is given by (proposed by Hufnagel and Stanley [1964])

$$H(u,v) = e^{-k(u^2 + v^2)^{5/6}}$$

where k is a constant and depends on nature of turbulence.

k = 0.0025 for severe turbulence

k = 0.001 for mild turbulence

k = 0.00025 for low turbulence

- Commonly used in remote sensing

- **Except 5/6 power in exponent this function has same form as Gaussian LPF transfer function**
- **Gaussian LPF is used to model mild, uniform blurring**

Estimation of Degradation Function: Modeling

Method 2:

- Another method is to derive a mathematical model starting from basic principles
- Example: Image is blurred by uniform linear motion between image and the sensor during acquisition
- Suppose that an image f(x,y) undergoes planar motion and that $x_0(t)$ and $y_0(t)$ are time varying components of motion in x and y directions respectively

Derive the Degradation Transfer Function

$x(t-\tau) \longleftrightarrow e^{-j\tau\omega} X(\omega)$

- Blurred image g(x,y) is given by

$$g(x,y)=\int_0^{\tau} f(x-x_0(t),y-y_0(t))\,dt$$

$$where\ \tau\ is\ the\ \exp osure\ time$$

- Taking Fourier transform of g(x,y), using shifting property of Fourier transform and simplifying gives the degradation transfer function

Degradation Transfer Function

- CTFT of g(x,y) gives $$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}\,dx\,dy$$
- Substituting g(x,y) into this gives

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{0}^{T} f[x - x_0(t),\, y - y_0(t)]\,dt\right]e^{-j2\pi(ux+vy)}\,dx\,dy$$

- Reversing the order of integration results in the following expression

$$G(u,v) = \int_{0}^{T}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x - x_0(t),\, y - y_0(t)]e^{-j2\pi(ux+vy)}\,dx\,dy\right]dt$$

- The term inside the outer brackets is the Fourier transform of the displaced function $f\,[x - x_0\,(t),\, y - y_0\,(t)]$

Derive the Degradation Transfer Function

$G(u,v) = H(u,v) \cdot F(u,v)$

- Using shifting property of CTFT:
- We get $G(u,v) = \int_0^T F(u,v) e^{-j2\pi[ux_0(t) + vy_0(t)]} dt \quad = F(u,v) \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]} dt$

$x(t-\tau) \longleftrightarrow e^{-j\tau\omega} X(\omega)$

- Defining

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]} dt$$

- We get

$$G(u,v) = H(u,v)F(u,v)$$

Derive the Degradation Transfer Function

- Hence the degradation transfer function is

$$H(u,v) = \int_0^{\tau} e^{-j2\pi[ux_o(t)+vy_0(t)]}dt$$

- If motion variables $x_0(t)$ and $y_0(t)$ are known, transfer function can be obtained directly from H(u,v)

Image Restoration Techniques

1. **By Inverse Filtering**
2. Minimum Mean Square Error Filtering (Wiener Filtering)
3. Constrained Least Square Filtering

Restoration by Inverse Filtering

- Inverse filtering is a deterministic and direct method for image restoration.
- Blurred image is generated by convolving blurring filter & original image
  $g(x,y) = f(x,y) * h(x,y)$
- Taking Fourier transform

$$G(u,v) = F(u,v).H(u,v)$$

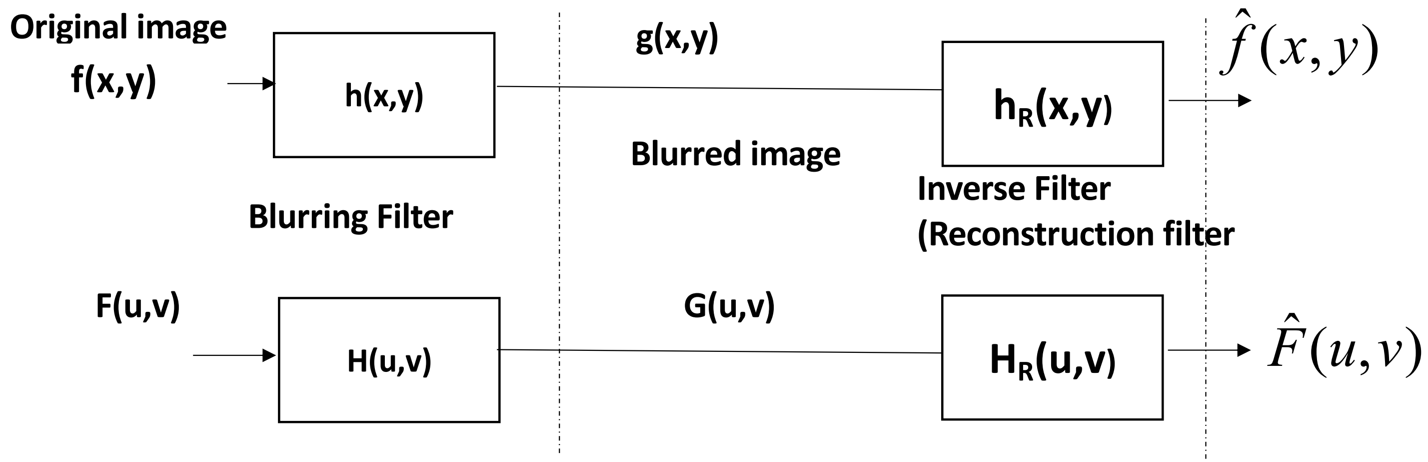
- If $H(u,v) = 0$ it will vanish $F(u,v)$, since

$$G(u,v) = F(u,v).0 = 0$$

- Irrespective of $F(u,v)$, the degraded image, $G(u,v) = 0$
- If $H(u,v) = 1$, then there is no degradation.

$$G(u,v) = F(u,v)$$

Inverse Filter in Time and Frequency Domain



$$G(u,v) = H(u,v).F(u,v)$$

And $$\hat{F}(u,v) = H_R(u,v).G(u,v)$$

$$\therefore \hat{F}(u,v) = \underbrace{H_R(u,v).H(u,v)}_{1}.F(u,v)$$

Inverse Filter in Time and Frequency Domain

- Reconstructed image can be obtained by inverse filter

$$H_R(u,v) = \frac{1}{H(u,v)}$$

- Consider restoration of images degraded by degradation filter H (which is given or obtained by known methods)

- Simplest approach is direct inverse filtering

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Problems with Inverse Filter

- If H(u,v)=0

  $H_R(u,v)=\infty$

$$H_R(u,v) = \frac{1}{H(u,v)}$$

The inverse filter becomes unstable

- Also G(u,v)=F(u,v).H(u,v) is valid if noise is absent.

- Generally degradation and noise occur together

Image with Degradation and Noise

- Consider image with degradation and Noise

$$G(u,v) = F(u,v).H(u,v) + N(u,v)$$

$$Or\ \hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

- We can’t recover f(x,y) exactly even if we know H(u,v) as N(u,v) is not known
- Also if H(u,v) has zero or small values, the ratio could dominate F(u,v), making it more noisy

Image with Degradation and Noise

- Problems:
  - We don't know $N(u,v)$
  - $H(u,v)$ often has zero values or small values.

If $H(u,v) = 0$, $N(u,v)/H(u,v) \rightarrow \infty$

If $H(u,v) \approx 0$, $N(u,v)/H(u,v) \rightarrow max$

Thus noise is amplified & dominates output.

Image with Degradation and Noise

Solution:

- One way to resolve small value problem is to limit the filter frequencies to values near origin. We know that H(0,0) is usually the highest value of H(u,v) spectrum. **(Zero Filtering)**
- Hence by limiting analysis near origin we avoid small values

Image with Degradation and Noise

Quickfix Solution:

- Limit the filter frequencies to values near the origin.

$F\text{^}(u,v)$= G(u,v)/$H(u,v)$

Eg. Atmospheric turbulence

$$H(u,v) = e^{-k\left[\left(u+M/2\right)^2+\left(v+N/2\right)^2\right]^{5/6}}$$

$$k = 0.0025, \qquad M = N = 480$$

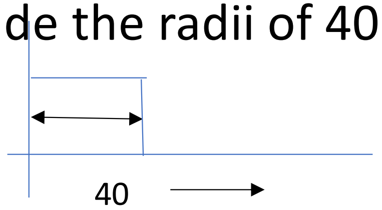
Find $F\text{^}(u,v)$= G(u,v)/$H(u,v)$

Analysis

- We get $\hat{f}(x,y) = IDFT\{\hat{F}(u,v)\}$

a) Full inverse filtering is useless due to small values of $H(u,v)$

b) Attenuate outside the radii of 40



Using Butterworth low pass filter

Result: Blurred image

c) Attenuate $\hat{F}(u,v)$ outside that radii of 70

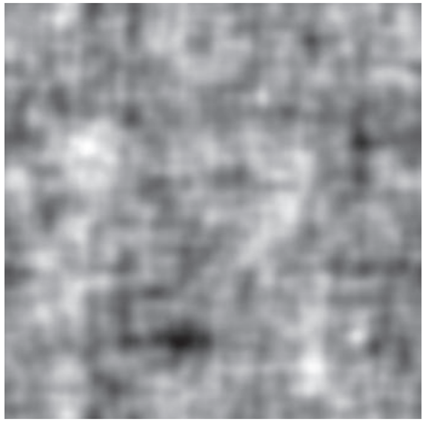
Result: Optimal image

d) Attenuate $\hat{F}(u,v)$ outside that radii of 85

Result: Noise emphasized image

Image with Degradation and Noise

Severe turbulence,
$k = 0.0025$.



Result of using the full filter.

Result with H cut off outside a radius of 40.

Best result

Result with H cut off outside a radius of 70

Result with H cut off outside a radius of 85

Analysis

- Gaussian function has no zeros so that is not a concern.
- However H(u,v) values become so small that full inverse filtering gives very noisy image and noise gets enhanced
- If the values of G(u,v)/H(u,v) are cut off outside radius of 40, 70 , 85 we see the effect of attenuation of high frequency values
- Radius near 70 gives best results

Limitations of Inverse Filtering

- It is an unstable filter
- It is sensitive to noise

- In practice inverse filter is not popularly used

1. By Inverse Filtering
2. **Minimum Mean Square Error Filtering (Wiener Filtering)**
3. Constrained Least Square Filtering

Next Session

- Restoration in the presence of degradation and noise cont..
  - Minimum Mean Square Error Filtering (Wiener Filtering)
  - Constrained Least Square Filtering

THANK YOU

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