**System of linear equations**

A system of equations in which all the unknowns appear in 1stdegree only is called ***system of linear equations.***

Consider equations with unknowns

. . . .

. . . .

**Matrix form of system of linear equations**

The above system can be written as

Here, called coefficient matrix

called unknown matrix

called constant matrix

**Geometry of linear equations**

We have 2 geometric interpretations of equations and their solution. We call the interpretation the

(i) row picture

(ii) column picture

**(i) Row picture**

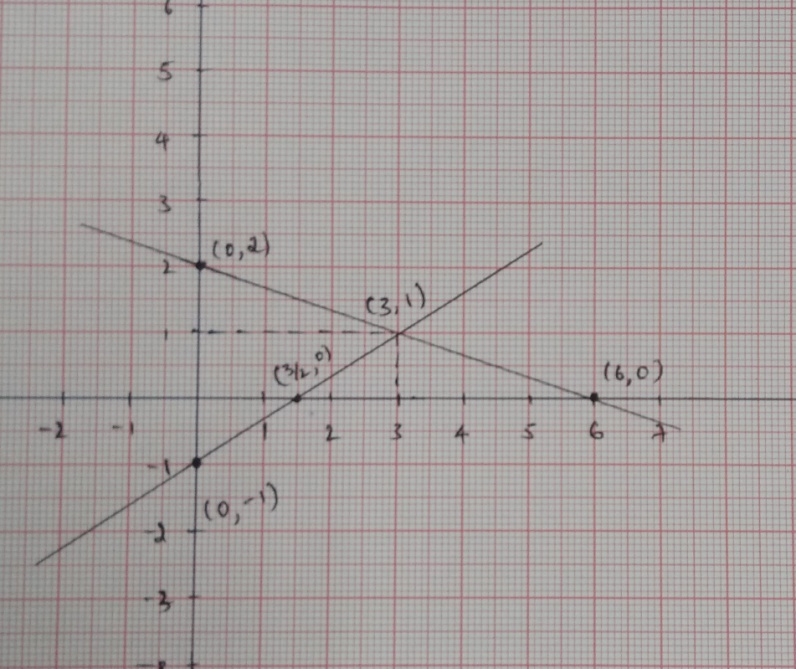
In the row picture, we draw a line for each equation . The solution of the system of linear equation is the unique intersection of all the lines.

**(ii) Column picture**

In column picture, we draw a vector for each column of the matrix and draw the vector . Column picture is the linear combination of the columns of the matrix which equal to vector .

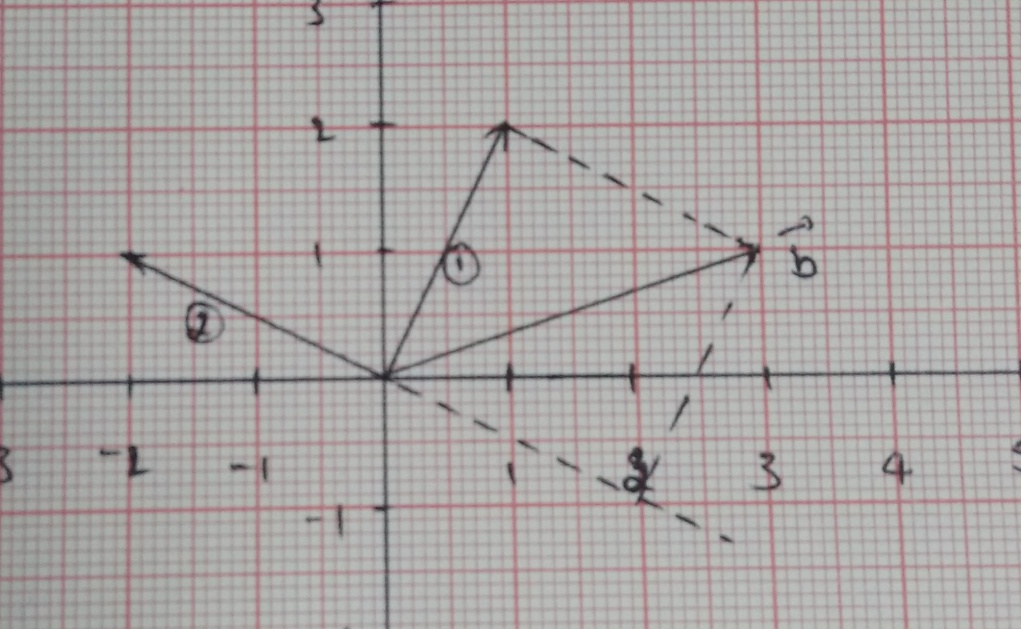
**Problems**

1. Explain the row approach to solve the system with neat diagram.



The two lines intersect at (3,1). . This is non-singular case.

2. Explain the column approach to solve the system with a neat diagram.



From the graph

is the solution.

System is non-singular.

**The method of Gaussian elimination**

In this method unknowns are eliminated successively and the system is reduced to an upper triangular form using which unknowns are found by back substitution.

Consider

The augmented matrix for the above given system is given by

by back substitution, we get the solution of given system of linear equation.

**Problems**

1.Solve the following system by the method of Gaussian elimination

**Solution:**

The augmented matrix is given by

The pivot elements are

From the above matrix:

From the above equations one can get.

2. Solve the following system by the method of Gaussian elimination

**Solution:**

The augmented matrix is given by

This is a case of temporary break down

Exchange

The pivot elements are

From the above matrix

From the above equations one can get .

3. Solve the following system by the method of Gaussian elimination

**Solution:**

The augmented matrix is given by

This is a case of permanent break down.

4. Investigate the values of such that

has (i) unique solution (ii) infinitely many solution (iii) no solution

**Solution**

The augmented matrix is given by

**Unique solution:**

From the Echelon form it is seeing that when

hence we get a unique solution when.

**No Solution**

For the system to have no solution. Thus must be 2 and must be 3.

For this to happen should be equal to 7 and i.e.,

).

**Many solutions**

For the system to have infinitely many solution we should have

. Thus.

For this to happen

5. Use Gaussian elimination to test for consistency of the system of linear equations Find the solution of the system is consistent. In the case of inconsistency change the coefficient of suitably in the third equation (viz 2) so that the system yields a unique solution with (Do not substitute for in the given set of equations). Solve also for .

**Solution**

The augmented matrix is given by

Therefore the system is inconsistent.

Given system yields unique solution

i.e.,

(Do the operation in reverse way)

6. Do the three planes have atleast one common point of the intersection? Explain; is the system consistent if the last equation is changed to ? If so solve the system completely.

**Solution**: The augmented matrix is given by

Hence the system is inconsistent.

Now, replace last equation by

Therefore, unknowns can be chosen arbitrarily.

**Elementary Matrices**

An elementary matrix is a matrix obtained from the identity matrix by performing one single elementary row operation.

It is a square matrix with 1’s on the main diagonal and almost one non-zero entry off the main diagonal.

1. Which put into triangular form? Multiply those’s to get such that. Also express as, where is the coefficient matrix of the following system:

**Solution**: The augmented matrix is given by

undoing the transformation to get back to to see that

is a lower triangular matrix. It can be easily written as

**Triangular Factorization**

(i) (with no exchange of rows)

Here is the lower triangular matrix with 1’s on the diagonal. is the upper triangular matrix which appear after forward elimination. Diagonal entries of are the pivots.

(ii) (with no exchange of rows)

wherehave 1’s on the diagonal &is the diagonal matrix of pivots.

**Permutation Matarix**

If is an identity matrix of order then permutation matrix is obtained by interchanging only two rows of .

* When a matrix has to be factorized into at same point, row exachange might be needed, then is multiplied by corresponding permutation matrix and then factorized into. Thus
* is always same as
* When using Gauss elimination method to find, all the row operations involved. These row operations will help to find using the identity matrix.
* To write start with the identity matrix & use the following rule

→ Any row that involves adding a multiple of one row to other

Eg: put the value in row and column of the identity matrix i.e., position.

* Order of = no. of rows of given matrix.

1. Find factorization for

**Solution:**

2.

**Solution:**

Square matrix of order = no. of rows

3. Find and factorization for

**Solution:**

is the diagonal matrix of pivots, Here and have 1’s in the diagonal.

Therefore divide each row of by its pivot.

4. Find factorization for

A cannot be factorized in the given form.

Introduce permutation matrix

5. Factorize into

Dummy matrix

**Invertible Matrices**

A matrix of order is said to be is said to be invertible matrix if there (exist) is a matrix of order such that where is the identity matrix of order A matrix is called as inverse of and it is denoted by

* Inverse matrix is unique
* If is invertible then is also invertible
* =
* Invertible matrix is also called as non-singular matrix
* Let & if then
* If is invertible then only one solution to the system is

**Inverse by row reduce to to** [Gauss-Jordan Method]

Let be a matrix of order and be an identity matrix of some order. Then is called as augmented matrix. Transform the matrix into identity matrix by eliminating row operations. Then identity matrix in augmented matrix is transform into

* The inverse of a matrix exist if and only if Gauss elimination produces

[Permanent break down inverse does not exists]

or

A square matrix is invertible if and only if elimination yields the same pivots as rows

1. Obtain the inverse of (or) use Gauss-Jordan method to solve

**Solution:**