Solutions to the Assignment for Week 1

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Exercise 1

Formal Derivation of the Compliance Condition with Recurring Costs

In the standard model of self-enforcing democracy, the expected lifetime payoff from *compliance* is given by:

$$V_{\text{comply}} = \frac{\pi(R-L)+L-k}{1-\delta}$$

where:

- R: payoff from being in power,
- L: payoff from being out of power,
- π: probability of winning the next election,
- k: cost of regular political engagement,
- $\delta \in (0,1)$: discount factor.

When the cost of fighting c is a *one-time* cost, the expected lifetime payoff from fighting is:

$$V_{\text{fight}} = \frac{pR}{1-\delta} - c$$

Now suppose instead that the cost of fighting is *recurring* — that is, it is incurred in each period the conflict persists. The present discounted value of the cost is:

$$\sum_{t=0}^{\infty} \delta^t c = \frac{c}{1-\delta}$$

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Hence, the expected payoff from fighting becomes:

$$V_{\text{fight}}^{(\text{recurring})} = \frac{pR - c}{1 - \delta}$$

The compliance condition requires:

$$V_{\text{comply}} \geq V_{\text{fight}}^{(\text{recurring})}$$

Substituting in:

$$\frac{\pi(R - L) + L - k}{1 - \delta} \geq \frac{pR - c}{1 - \delta}$$

Multiplying both sides by $1 - \delta$, we obtain the compliance condition:

$$\pi(R - L) + L - k \geq pR - c$$

Intuitive Justification for Recurring Costs

In many real-world scenarios, the costs of political violence or conflict are not one-off expenditures. Civil wars, insurgencies, or authoritarian crackdowns often impose sustained economic damage, human suffering, and institutional decay over time. Modelling these as *recurring costs* might better capture the long-term nature of such strife.

Comparison with the One-Time Cost Case

Under a one-time cost assumption, fighting is relatively more attractive, since the cost is paid only once and not discounted. By contrast, recurring costs reduce the attractiveness of fighting over time. Thus:

- Compliance is *more likely* with recurring costs.
- Peaceful democratic competition is *more stable* under this assumption.

Implications for Political Stability

The recurring cost assumption highlights the stabilising role of sustained conflict costs. When political violence imposes long-term, compounding hardship, actors are more likely to abide by democratic rules. This implies that structural conditions or institutions that increase the long-term costs of conflict can help support democratic stability and make peaceful power transfers more robust.

Setup and Assumptions

We focus on the compliance condition in the model of self-enforcing democracy, under the assumption that the cost of fighting c is *recurring*. It is completely fine if you assumed one-time cost; the derivations are in the appendix of the slides. The compliance condition is then given by:

$$\frac{\pi(R-L)+L-k}{1-\delta} \geq \frac{pR-c}{1-\delta}$$

Multiplying through by $1-\delta$ yields:

$$\pi(R-L)+L-k \geq pR-c$$

Letting $S=R-L$, the condition becomes:

$$\pi(S)\cdot S+L-k \geq p(R)-c$$

Rearranged, the function defining the threshold level of electoral stakes S^* can be written as:

$$F(S;\delta,c,k,p)=\pi(S)\cdot S-pS-(1-p)L+k+c$$

We assume that S^* is implicitly defined by the condition $F(S^*;\delta,c,k,p)=0$. We now study how S^* changes with respect to each parameter using the Implicit Function Theorem.

Implicit Differentiation

By the Implicit Function Theorem:

$$\frac{dS^*}{d\theta}=-\frac{\frac{\partial F}{\partial \theta}}{\frac{\partial F}{\partial S}}$$

where $\theta \in \{\delta,c,k,p\}$. We compute these derivatives case by case.

1. Discount Factor δ

Although δ appears in the original compliance condition, it cancels when both sides are multiplied by $1-\delta$, so:

$$\frac{dS^*}{d\delta}=0$$

2. Cost of Fighting c

$$\frac{\partial F}{\partial c} = 1, \quad \Rightarrow \quad \frac{dS^*}{dc} = -\frac{1}{\pi'(S) \cdot S + \pi(S) - p}$$

Sign: If the denominator is positive (as in many realistic cases), then:

$$\frac{dS^*}{dc} > 0$$

Interpretation: Higher fighting costs make conflict less attractive, pushing actors toward compliance and allowing democracy to be sustained at lower stakes.

3. Cost of Engagement k

$$\frac{\partial F}{\partial k} = 1, \quad \Rightarrow \quad \frac{dS^*}{dk} = -\frac{1}{\pi'(S) \cdot S + \pi(S) - p}$$

Sign: Again assuming the denominator is positive,

$$\frac{dS^*}{dk} > 0$$

Interpretation: Higher participation costs make compliance less attractive, necessitating higher electoral stakes to sustain it.

4. Probability of Winning Conflict p

$$\frac{\partial F}{\partial p} = -S, \quad \Rightarrow \quad \frac{dS^*}{dp} = \frac{S}{\pi'(S) \cdot S + \pi(S) - p}$$

Sign: Provided the denominator is positive,

$$\frac{dS^*}{dp} > 0$$

Interpretation: A higher chance of winning a violent conflict makes defection more attractive, requiring higher stakes from electoral competition to preserve compliance.

Summary Table of Effects

We summarise the direction of comparative statics depending on the sign of $\pi(S^*) - p$ and the slope $\pi'(S^*)$:

	$\pi(S^*) > p$		$\pi(S^*) < p$	
	$\pi'(S^*) > 0$	$\pi'(S^*) < 0$	$\pi'(S^*) > 0$	$\pi'(S^*) < 0$
$\frac{dS^*}{dc}$	> 0	< 0	> 0	< 0
$\frac{dS^*}{dk}$	> 0	< 0	> 0	< 0
$\frac{dS^*}{dp}$	> 0 (always)			

Interpretation

- **Cost of fighting** c: When elections are competitive ($\pi > p$) and electoral stakes increase electoral probability ($\pi' > 0$), raising c deters conflict and increases compliance — so S^* rises.
- **Cost of political engagement** k: Higher k makes compliance costlier. When democratic competition is credible ($\pi > p$), it requires higher stakes to sustain compliance.
- **Probability of winning conflict** p: As fighting becomes more attractive, compliance requires higher stakes — S^* rises.

1. Model Setup

We model electoral competition as a contest between an incumbent I and an opposition challenger O. Each actor chooses a non-negative effort level $e_i \in \mathbb{R}_{\geq 0}$, where $i \in \{I, O\}$. Electoral outcomes are probabilistic and determined by relative effort levels.

Contest Success Function

Let π_O denote the probability that the opposition wins the election. We adopt a Tullock-type contest success function (CSF):

$$\pi_O = \frac{e_O}{e_O + e_I}, \qquad \pi_I = 1 - \pi_O = \frac{e_I}{e_O + e_I}$$

with the convention that if $e_O = e_I = 0$, then $\pi_O = \pi_I = \frac{1}{2}$.

Payoffs

Let the stakes of the election be denoted by $S > 0$, interpreted as the net benefit of holding office. Players value winning at S and receive zero otherwise. The cost of effort is quadratic and increasing:

$$c_i(e_i) = \frac{1}{2}e_i^2$$

The utility for each player is:

$$U_I(e_I, e_O) = \frac{e_I}{e_I + e_O} \cdot S - \frac{1}{2}e_I^2$$
$$U_O(e_I, e_O) = \frac{e_O}{e_I + e_O} \cdot S - \frac{1}{2}e_O^2$$

2. Solving for Nash Equilibrium

We seek symmetric pure strategy Nash equilibria where both players choose positive effort. Each player maximises their utility with respect to their own effort, taking the other's effort as given.

Opposition's First Order Condition (FOC)

$$\frac{\partial U_O}{\partial e_O} = \frac{S(e_I + e_O) - Se_O}{(e_I + e_O)^2} - e_O = 0 \Rightarrow \frac{Se_I}{(e_I + e_O)^2} = e_O$$

Incumbent's FOC

$$\frac{\partial U_I}{\partial e_I} = \frac{Se_O}{(e_I + e_O)^2} - e_I = 0 \Rightarrow \frac{Se_O}{(e_I + e_O)^2} = e_I$$

Symmetric Equilibrium Assume $e_I = e_O = e^*$. Then:

$$\frac{Se^*}{(2e^*)^2} = e^* \Rightarrow \frac{S}{4e^*} = e^* \Rightarrow e^{*2} = \frac{S}{4} \Rightarrow e^* = \frac{\sqrt{S}}{2}$$

Equilibrium efforts:

$$e_I^* = e_O^* = \frac{\sqrt{S}}{2} \quad \Rightarrow \quad \pi_O = \pi_I = \frac{1}{2}$$

3. Existence and Uniqueness

- The cost functions are strictly convex and differentiable.
- The contest success function is smooth and strictly increasing in own effort.

- Best response functions are strictly concave and intersect only once.

Conclusion: A unique symmetric pure strategy Nash equilibrium exists.

4. Toward a Hump-Shaped $\pi(S)$

In the symmetric baseline, $\pi(S) = \frac{1}{2}$ for all S. To generate a hump shape, we relax symmetry or introduce strategic constraints on the opposition.

Example Extension

Suppose the opposition faces an additional marginal cost that increases with S, e.g.:

$$c_O(e_O) = \frac{1}{2}e_O^2 + \alpha S e_O \quad \text{with } \alpha > 0$$

This models scenarios where:

- The opposition fears repression when contesting high-stakes elections.
- Higher stakes make it harder to coordinate or fund mobilisation.

In this case:

- At low S: $e_O \uparrow \Rightarrow \pi_O \uparrow$
- At high S: marginal cost becomes prohibitive $\Rightarrow e_O \downarrow \Rightarrow \pi_O \downarrow$

Result: $\pi(S)$ becomes hump-shaped.

5. Interpretation

This contest model provides a strategic microfoundation for the assumption that $\pi(S)$ is non-monotonic. The opposition's willingness to contest elections varies endogenously with stakes — and may decline if risks, repression, or coordination costs outweigh potential gains.

Exercise 2

2(a) First-Order Condition with Asymmetric Uniform Noise

We consider the simplified version of the Ferejohn (1986) model, in which the performance outcome is given by:

$$o = e + \varepsilon,$$

where $\varepsilon \sim \text{Uniform}(-a, b)$, with $a, b > 0$ and $a \neq b$. Thus, the noise term is drawn from a uniform distribution on the asymmetric interval $[-a, b]$, and has a constant density:

$$f(\varepsilon) = \frac{1}{a+b} \quad \text{for } \varepsilon \in [-a, b].$$

Voters apply a cutoff rule, re-electing the incumbent if and only if:

$$o \geq o^*.$$

The incumbent chooses effort e to maximise the following objective:

$$\max_e \left[R \cdot \Pr(o \geq o^*) - c(e)\right],$$

where we assume that the cost of effort is quadratic: $c(e) = \frac{1}{2}e^2$.

Since $o = e + \varepsilon$, we have:

$$\Pr(o \geq o^*) = \Pr(\varepsilon \geq o^* - e).$$

Given the cumulative distribution function (CDF) of ε, for $x \in [-a, b]$,

$$F(x) = \frac{x+a}{a+b},$$

the probability of re-election becomes:

$$\Pr(o \geq o^*) = 1 - F(o^* - e) = \frac{e - o^* + a}{a+b}.$$

Substituting into the incumbent's objective:

$$\max_e \left[R \cdot \frac{e - o^* + a}{a+b} - \frac{1}{2}e^2\right].$$

To find the optimal level of effort, take the first-order condition with respect to e:

$$\frac{d}{de}\left(\frac{R(e - o^* + a)}{a+b} - \frac{1}{2}e^2\right) = \frac{R}{a+b} - e = 0.$$

Solving yields:

$$e^* = \frac{R}{a+b}.$$

Conclusion: The optimal level of effort by the incumbent is decreasing in the total width of the noise distribution, $a + b$, and increasing in the value of holding office, R.

2(b) and (c) Effect of Asymmetric Support on Optimal Effort (with Interpretation)

From part (a), the incumbent's optimal level of effort is:

$$e^* = \frac{R}{a + b},$$

which depends inversely on the total width of the support of the noise distribution. As the support widens — either due to more extreme negative or positive shocks — the marginal benefit of exerting effort declines, since outcomes become more dominated by noise.

Link to Political Luck: Changes in the support of the noise distribution can be interpreted in terms of how *lucky* or *unlucky* politicians are likely to be:

- **Increased negative support:** If a increases while b remains constant, the lower bound of the distribution becomes more negative. This means that the incumbent faces a greater risk of experiencing a *bad shock* — a poor observed outcome — even if they exert high effort. In this case, bad luck is more common. Because performance is more likely to look bad regardless of effort, the incentive to work hard diminishes. Thus, optimal effort *decreases*.
- **Increased positive support:** If b increases while a is fixed, good outcomes become more likely due to *positive shocks*. The incumbent may appear competent simply due to good luck, even with low effort. Again, the marginal return to effort declines, leading to lower optimal effort.

Real-World Examples: In crisis-prone or volatile political environments — such as those facing frequent natural disasters, wars, or economic shocks — negative shocks are common. Voters may struggle to distinguish between poor outcomes due to bad luck and genuine underperformance. In such contexts, politicians might reduce effort strategically, knowing that even hard work may not yield visible results.

Conversely, in boom periods (e.g., during an oil windfall), favourable outcomes may be attributed to incumbents even when driven by external factors. Politicians facing such environments may similarly underperform, relying on good luck to carry them to re-election.

Conclusion: As the noise distribution becomes more asymmetric in support — whether toward bad or good outcomes — the probability of re-election becomes less sensitive to the incumbent's effort. This reduces the incentive to exert effort, leading to strategic shirking in both cases.

2(d) Expected Voter Loss as a Function of the Cutoff o^*

We consider a setting in which voters observe a noisy outcome:

$$o = e + \varepsilon, \quad \text{where } \varepsilon \sim \mathcal{N}(0, \sigma^2),$$

but cannot directly observe the incumbent's effort e. Voters apply a cutoff rule: re-elect the incumbent if and only if $o \geq o^*$.

Suppose voters incur asymmetric losses based on the type of decision error:

- Loss $L_1 > 0$ if they mistakenly **re-elect** a low-effort incumbent,
- Loss $L_2 > 0$ if they mistakenly **replace** a high-effort incumbent,
- No loss if the re-election decision is correct.

Assume:

- The incumbent may exert either high effort e_H or low effort e_L, with $e_H > e_L$,
- Voters believe there is a 50% chance of high effort and 50% chance of low effort.

Let:

- $P_{R|L} = \Pr(o \geq o^* \mid e = e_L)$: the probability of re-electing a low-effort incumbent (a costly mistake),
- $P_{R^c|H} = \Pr(o < o^* \mid e = e_H)$: the probability of replacing a high-effort incumbent (also a mistake).

Then the expected voter loss is:

$$\mathbb{E}[\text{Loss}] = \frac{1}{2} L_1 \cdot P_{R|L} + \frac{1}{2} L_2 \cdot P_{R^c|H}.$$

This expression captures the total expected loss from both types of re-election errors, as a function of the chosen cutoff o^*.

Probabilities of Mistaken Decisions Using the Standard Normal CDF

Assume the noise term $\varepsilon \sim \mathcal{N}(0, \sigma^2)$, so the observed outcome $o = e + \varepsilon$ is normally distributed:

$$o \sim \mathcal{N}(e, \sigma^2).$$

We express the two types of mistaken voter decisions in terms of the standard normal cumulative distribution function, denoted by $\Phi(\cdot)$.

(i) Mistakenly re-electing a low-effort incumbent: Let the incumbent exert low effort e_L. The outcome o is then distributed as $\mathcal{N}(e_L, \sigma^2)$. The probability of mistakenly re-electing the incumbent is:

$$P_{R|L} = \Pr(o \geq o^* \mid e = e_L) = 1 - \Phi\left(\frac{o^* - e_L}{\sigma}\right).$$

(ii) Mistakenly replacing a high-effort incumbent: Let the incumbent exert high effort e_H. The outcome o follows $\mathcal{N}(e_H, \sigma^2)$. The probability of mistakenly replacing the incumbent is:

$$P_{R^c|H} = \Pr(o < o^* \mid e = e_H) = \Phi\left(\frac{o^* - e_H}{\sigma}\right).$$

These expressions can be substituted directly into the expected voter loss function derived in part (a).

2(e) Should Voters Set a Higher or Lower Cutoff When $L_1 > L_2$?

From above, the expected voter loss as a function of the cutoff o^* is:

$$\mathbb{E}[\text{Loss}](o^*) = \frac{1}{2}L_1 \cdot \left[1 - \Phi\left(\frac{o^* - e_L}{\sigma}\right)\right] + \frac{1}{2}L_2 \cdot \Phi\left(\frac{o^* - e_H}{\sigma}\right),$$

where:

- L_1 is the loss from mistakenly re-electing a low-effort incumbent,
- L_2 is the loss from mistakenly replacing a high-effort incumbent,
- $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

Suppose $L_1 > L_2$, so that false positives (Type I errors: re-electing a bad incumbent) are considered more costly than false negatives (Type II errors: replacing a good incumbent).

To minimise expected loss, voters should choose the cutoff o^* that optimally trades off these two types of mistakes. Note the following:

- The term $1 - \Phi\left(\frac{o^*-e_L}{\sigma}\right)$ *decreases* as o^* increases: a higher cutoff makes it less likely to re-elect a low-effort incumbent.
- The term $\Phi\left(\frac{o^*-e_H}{\sigma}\right)$ *increases* with o^*: a higher cutoff makes it more likely to mistakenly reject a high-effort incumbent.

Given $L_1 > L_2$, the loss function is more sensitive to the first term. To reduce the weighted contribution of the first term to expected loss, voters should set a relatively *higher* cutoff o^*, thereby lowering the probability of re-electing low-effort incumbents.

Conclusion: When the loss from re-electing low-effort incumbents is greater than the loss from mistakenly removing high-effort ones ($L_1 > L_2$), the optimal cutoff o^* should be set *higher*. This policy reflects a more conservative or risk-averse re-election standard, consistent with voters' preference to avoid costly false positives.

2(f) Political Interpretation of Voter Loss Asymmetry

The model assumes that voters incur asymmetric losses depending on the type of error made:

- A **false positive** (loss L_1): re-electing a low-effort or incompetent incumbent,
- A **false negative** (loss L_2): replacing a high-effort or competent incumbent.

The assumption that $L_1 \neq L_2$ allows us to model the idea that voters may view these two types of mistakes differently. In practice, whether voters fear one kind of error more than the other is highly context-dependent and shaped by political, institutional, and historical factors.

Situations where $L_1 > L_2$: Voters are more concerned about re-electing bad leaders. This might be especially plausible in contexts where incompetence, corruption, or inaction can cause significant harm:

- In fragile or post-conflict states, such as South Sudan or Haiti, poor leadership can exacerbate instability or humanitarian crises.
- During emergencies (e.g., pandemics, wars), voters may demand a higher level of competence and become less tolerant of underperformance.
- Where civic institutions and watchdogs are strong, public intolerance of waste or corruption may encourage voters to "vote out" questionable incumbents, even at the risk of losing capable ones.

Example: In Italy during the early 1990s, the *Mani Pulite* (Clean Hands) investigation uncovered widespread political corruption, leading to the collapse of several major parties. The public's reaction was one of outrage, resulting in a significant political realignment. Voters became highly intolerant of corruption, prioritising the removal of (tainted) incumbents, even at the risk of political instability (Daniele et al., 2020; Foresta, 2020).

Situations where $L_2 > L_1$: Voters are more concerned about removing good leaders. This may occur in contexts where:

- Competent leadership is rare, and voters wish to retain individuals who have shown capacity.
- The incumbent has developed strong personal legitimacy, reformist credentials, or is rather charismatic.
- Political stability and continuity are prioritised over accountability, especially in transitional regimes or post-conflict settings.

Example: In post-genocide Rwanda, many voters supported President Kagame despite concerns about democratic backsliding or autocratisation, motivated by perceptions of competence and national reconstruction, suggesting higher L_2 (Reyntjens, 2024, 2018; Booth and Golooba-Mutebi, 2012).

Conclusion: The asymmetry in voter loss preferences reflects deeper political realities. Whether voters choose a high or low performance threshold depends not only on informational constraints, but also on their *relative aversion to false positives and false negatives*. This asymmetry can help explain electoral behaviour across diverse institutional settings.

Exercise 3

3(a): Why Stationary Cutoff Rules May Be Suboptimal

I. Core claim from the literature: Stationary cutoffs are normatively suboptimal

- Stationary rules re-elect incumbents if performance exceeds a fixed threshold, regardless of career stage or history.
- Both Acharya et al. (2025) and Gieczewski and Li (2024) show these rules often fail to provide optimal incentives, especially under moral hazard (which arises from hidden action).

- Instead, optimal accountability requires *dynamic, history-dependent* rules that adjust cutoffs over time.

II. Key intuitions from Acharya et al. (2025)

- Effort is most incentivised when re-election is uncertain; therefore, **first-term politicians work hardest**.
- Voters should reward early effort with future leniency — leveraging career incentives to extract public service.
- Optimal rules generate **increasing job security** over time conditional on success; politicians become gradually entrenched.
- Stationary rules ignore this intertemporal trade-off and fail to backload rewards.

III. Contributions from Gieczewski and Li (2024)

- They correct a technical error in Ferejohn (1986), showing that non-stationary rules can be implemented in equilibrium without requiring commitment.
- Voters can condition future retention on past performance in a way that is incentive-compatible.
- This enhances the feasibility of *declining* or *adaptive* cutoff paths.

IV. Intuition: Why declining cutoffs might improve accountability

- Strict early cutoffs screen out low-effort incumbents, especially when shocks are not too frequent or severe.
- Later leniency allows:
  - Time for long-term policies to yield results,
  - Forgiveness for accumulated shocks, or stochastic misfortune.
- This mirrors probationary employment models or tenure systems: early hurdles, later stability.

V. Critical reflections on this logic

- If shocks are noisy or skewed, early performance may not reveal true effort; strict cutoffs risk **false negatives**.

- Without persistent types, high effort early on does not guarantee continued performance — ongoing incentives still matter.
- Declining cutoffs may invite complacency unless tied to history-dependent rules (as in Acharya et al.).

VI. Conclusion

- Both theoretical models and intuitive reasoning suggest that **adaptive re-election standards**, not fixed thresholds, better align voter incentives with long-run political accountability.
- Dynamic rules allow voters to condition on performance histories, shape political careers, and tailor expectations to changing circumstances.

3(b): Adaptive Cutoff Trajectories and Accountability

I. Why adaptive cutoffs improve accountability

- Adaptive (time-varying or history-dependent) cutoffs allow voters to tailor incentives across a politician's career.
- Unlike stationary rules, adaptive thresholds can reflect:
  - The stage of a political career (e.g., early probation, later stability),
  - Accumulated performance history,
  - Shifts in political uncertainty or observability.

II. Linking adaptive cutoffs to voter loss asymmetries

- The optimal trajectory depends on which type of error voters are more concerned about:
  - *Type I error*: Re-electing a low-effort politician (loss L_1),
  - *Type II error*: Replacing a high-effort politician (loss L_2).

III. Declining cutoff trajectories (strict early, lenient later)

- Best when voters are:
  - Highly averse to Type I errors ($L_1 \gg L_2$),

  - Willing to risk removing some good-but-unlucky politicians early on,
  - Confident in their ability to adjust standards based on observed performance.
- Also justified when:
  - Performance signals improve with tenure,
  - Policy benefits are delayed (e.g., investments or reforms take time to yield visible results),
  - Later periods may be more shock-prone or voters more forgiving.

IV. Increasing cutoff trajectories (lenient early, strict later)

- Best when voters are:
  - More concerned about Type II errors ($L_2 \gg L_1$),
  - Willing to temporarily retain low-effort politicians who got lucky,
  - Expecting to apply more stringent evaluation as information improves over time.
- May arise when:
  - Early performance is noisy or hard to attribute,
  - Voters anticipate improved observability (e.g., institutional strengthening, more media scrutiny).

V. Responsive or performance-contingent cutoffs

- Cutoffs adjust based on the incumbent's observed performance path, not just time.
- Capture the benefits of both declining and increasing rules.
- Require mechanisms for credible conditionality and monitoring.

VI. Conclusion

- Adaptive cutoffs outperform fixed rules because they can embed screening, forgiveness, and incentive logic dynamically.
- The ideal path depends on political context, voter preferences, institutional maturity, and information quality.

3(c) Do the Models Rationalise Incumbency Advantage?

I. Empirical fact: Incumbents are more likely to be re-elected

- Across many democracies, re-election rates for incumbents exceed those of challengers.
- This pattern is known as the *incumbency advantage*.

II. Do the models explain this phenomenon? Yes — partially.

(i) Rational entrenchment via performance history (Acharya et al., 2024)

- Politicians who perform well in early terms accumulate *continuation value*.
- Voters optimally relax re-election standards over time, making these incumbents *less electorally vulnerable*.
- High performers become entrenched not due to bias, but as a rational response to good past behaviour.

(ii) Selection through dynamic screening

- Strict early thresholds filter out low-effort types.
- Those who survive are likely to be higher effort or higher quality.
- This endogenous selection increases the average re-election probability of incumbents over time.

III. Comparison to standard explanations

- Common empirical explanations for incumbency advantage include:
  - Voter bias or name recognition,
  - Partisan loyalty or safe seats,
  - Information asymmetry (more is known about incumbents),
  - Resource advantages (campaign funding, media access).
- These are typically behavioural, institutional, or informational in nature.

IV. Contribution of the model-based explanation

- The Acharya et al. model provides a *rational, incentive-compatible* justification for incumbency advantage.

- It arises even with fully rational voters and no bias, purely from optimal career incentives and dynamic accountability.
- However, this does not rule out the coexistence of behavioural or structural factors in real-world settings.

3(d) Risks and Limitations of Dynamic Cutoff Rules

I. Dynamic cutoffs improve flexibility — but are not risk-free

- While adaptive rules often enhance accountability, poor implementation or overreaction can undermine their effectiveness.
- Dynamic cutoffs can introduce volatility, ambiguity, and manipulation if not designed carefully.

II. Key risks of over-adjusting cutoffs
(i) Volatility and unpredictability

- If voters adjust the cutoff too frequently or erratically, politicians may be unable to anticipate the standards they will face.
- This reduces the clarity of incentives and may discourage consistent or long-term effort.

(ii) Perverse signalling and complacency

- Rapidly lowering the cutoff may signal to incumbents that voters will tolerate declining performance.
- This can induce strategic shirking: politicians reduce effort rationally in response to perceived leniency.

(iii) Manipulability and politicisation

- In practice, adaptive rules may be harder to institutionalise and more vulnerable to manipulation.
- Cutoffs that appear responsive may instead be perceived as arbitrary, biased, or driven by political interests.

(iv) Misattribution of noise

- Voters may mistakenly adjust cutoffs in response to stochastic shocks rather than true performance.
- This risks rewarding low-effort but lucky incumbents or punishing high-effort but unlucky ones.

III. When might stationary cutoffs be preferable?

- In environments with high noise or limited observability of effort.
- When institutional capacity for monitoring and enforcement is low.
- When commitment to time-consistent rules is infeasible or public trust in dynamic mechanisms is weak.

Conclusion: Dynamic cutoff rules hold significant promise for improving accountability, but only if voters can adjust them in a predictable, credible, and history-contingent manner. Poorly implemented dynamics may reduce, rather than enhance, political discipline.

Exercise 4

Function for computing optimal effort

```
# Load required package
library(stats)

# Define the function to compute optimal effort
optimal_effort <- function(k_t, sigma = 0.5, R = 1.0) {
  # Define the objective function (to minimise)
  objective <- function(e) {
    prob <- 1 - pnorm((k_t - e) / sigma)
    return(-R * prob + 0.5 * e^2)
  }

  # Optimise over the range [0, 5]
  result <- optimize(objective, interval = c(0, 5))
  return(result$minimum)
```

```
}

# Example: compute optimal effort when k_t = 1.0
e_star <- optimal_effort(k_t = 1.0)
print(e_star)
```

20-period simulation

```
 # Load required library
library(tidyverse)
library(stats)

# --- Parameters ---
T <- 20              # number of periods
k0 <- 1.0            # initial cutoff
alpha <- 0.02        # cutoff decline rate
sigma <- 0.5         # standard deviation of noise
R <- 1               # reward from reelection

# --- Storage vectors ---
cutoffs <- numeric(T)
efforts <- numeric(T)
reelect_probs <- numeric(T)

# --- Simulation loop ---
for (t in 1:T) {
  k_t <- k0 - alpha * (t - 1)
  cutoffs[t] <- k_t

  # Define payoff function
  incumbent_payoff <- function(e) {
    prob <- 1 - pnorm((k_t - e) / sigma)
    cost <- 0.5 * e^2
    return(-(R * prob - cost))
  }
```

```
  opt <- optimize(incumbent_payoff, interval = c(0, 5))
  e_star <- opt$minimum
  efforts[t] <- e_star
  reelect_probs[t] <- 1 - pnorm((k_t - e_star) / sigma)
}

# --- Combine into long-format dataframe for clean plotting ---
df <- data.frame(
  Time = 1:T,
  Cutoff = cutoffs,
  Effort = efforts,
  Reelection_Prob = reelect_probs
) %>%
  rename(‘Reelection Probability‘ = Reelection_Prob) %>%
  pivot_longer(cols = c(Cutoff, Effort, ‘Reelection Probability‘),
               names_to = "Variable", values_to = "Value")

# --- Plot ---
ggplot(df, aes(x = Time, y = Value, colour = Variable)) +
  geom_line(size = 1.2) +
  geom_point(size = 2, shape = 21, fill = "white", stroke = 0.8) +
  scale_colour_manual(values = c("Cutoff" = "#1b9e77",
                                 "Effort" = "#d95f02",
                                 "Reelection Probability" = "#7570b3")) +
  labs(
    title = "Effort, Cutoffs, and Reelection Probability Over Time",
    subtitle = "Ferejohn-style dynamic accountability model with declining voter standar
    x = "Period",
    y = "Value",
    colour = "Metric"
  ) +
  expand_limits(y = c(0, 1)) +
  theme_bw(base_size = 14) +
  theme(
    plot.title = element_text(face = "bold", size = 16),
```

```
    plot.subtitle = element_text(size = 12, colour = "grey40"),
    legend.position = "bottom",
    legend.title = element_text(face = "bold"),
    panel.grid.minor = element_blank()
  )
```

Interpreting the Simulation Results

The simulation models a dynamic version of the Ferejohn accountability model in which voters apply a declining performance threshold over time:

$$k_t = k_0 - \alpha t.$$

In each period, the incumbent chooses effort e_t to maximise the probability of re-election minus effort costs. The resulting trends in optimal effort and re-election probability offer the following insights:

1. Declining cutoffs reduce effort and accountability

- As voter standards k_t fall over time, the probability of re-election becomes less sensitive to effort.
- Incumbents rationally exert less effort: the marginal return to effort declines.
- This is a form of *shirking*, which is different from Acharya et al. (2025) because in our case the declining cutoff is not performance-dependent..

2. Re-election becomes more likely despite falling effort

- The simulation shows an increase in the probability of re-election over time.
- This occurs because the cutoff k_t declines faster than the fall in effort.
- Politicians thus become more likely to be re-elected even as their performance declines.

3. Shirking can be rational

- The decline in effort is not due to incompetence or laziness, but reflects optimal behaviour under weaker re-election incentives.
- Since effort is costly and the probability of re-election increases over time, the incumbent has less need to work hard.

Exercise 5

5(a): Simulation of the simplified Fearon (1999) model

```
# --- Parameters ---
n_politicians <- 1000
sigma <- 1
x_good <- 0
x_bad <- 0.5
cutoff_k <- -0.5

# --- Choose case: low or high share of bad types ---
share_bad <- 0.3  # Change to 0.7 for the second case

# --- Step 1: Generate types ---
set.seed(123)  # for reproducibility
types <- sample(c("good", "bad"),
                size = n_politicians,
                replace = TRUE,
                prob = c(1 - share_bad, share_bad))

# --- Step 2: Assign effort (x) based on type ---
x_vals <- ifelse(types == "good", x_good, x_bad)

# --- Step 3: Simulate signal z = -x^2 +  ---
eps <- rnorm(n_politicians, mean = 0, sd = sigma)
z_vals <- -x_vals^2 + eps

# --- Step 4: Apply reelection rule ---
reelected <- z_vals > cutoff_k
n_reelected <- sum(reelected)

# --- Step 5: Analyse reelected politicians ---
reelected_types <- types[reelected]
reelected_x <- x_vals[reelected]
```

```
# --- Compute statistics ---
total_reelected <- n_reelected / n_politicians
bad_among_reelected <- mean(reelected_types == "bad")
voter_utility <- -mean(reelected_x^2)

# --- Output results ---
cat("=== Case:", share_bad * 100, "% bad types, cutoff =",
    cutoff_k, "===\n") === Case: 30 % bad types, cutoff = -0.5 ===
cat("Reelection rate:", round(total_reelected, 2), "\n") Reelection rate: 0.67
cat("Share of bad types among reelected:", round(bad_among_reelected, 2), "\n")
Share of bad types among reelected: 0.27
cat("Voter welfare (U = -E[x^2]):", round(voter_utility, 3), "\n")
Voter welfare (U = -E[x^2]): -0.067
```

5(b) Re-election Outcomes and Type Composition

Re-election rule: Politicians are re-elected if their observed performance signal $z > k$, where $k = -0.5$. For each simulation case, we compute:

- The fraction of all politicians who are re-elected,
- Among those re-elected, the share who are actually bad types.

Results:

Share of Bad Types	Fraction Re-elected	Bad Types Among Re-elected
30%	69.2%	29.5%
70%	62.7%	68.6%

Interpretation:

- When only 30% of the pool are bad types, voters are able to re-elect mostly good politicians. The fraction of bad types among re-elected incumbents remains close to their population share.
- When 70% of politicians are bad types, a much larger share of those re-elected are also bad. In this case, voter selection fails not because voters behave irrationally, but because the pool of available politicians is heavily skewed toward bad types.

- Despite the re-election rule, noise in the signal $z = -x^2 + \varepsilon$ allows many bad types to pass as good types — especially when they are numerous.

These results highlight a central challenge in selection-based models: even rational voting rules can struggle to filter out bad types when noise is high and the candidate pool is of low quality.

5(c) Voter Welfare

Voter utility is defined as:

$$U = -\mathbb{E}[x^2 \mid \text{reelected}]$$

In this setup:

- Good types choose $x = 0$, so $x^2 = 0$,
- Bad types choose $x = 0.5$, so $x^2 = 0.25$.

Results:

Share of Bad Types	**Bad Among Re-elected**	**Voter Welfare**
30%	29.5%	−0.074
70%	68.6%	−0.171

Interpretation:

- When only 30% of politicians are bad types, the majority of re-elected incumbents are good, and the average policy distortion x^2 is low. Voter welfare is correspondingly higher.
- When 70% are bad types, many of those re-elected have high x^2, and voter welfare falls sharply.
- This demonstrates a central insight of Fearon's selection model: voter welfare is highly sensitive to the underlying type distribution. Even if voters apply a consistent and rational re-election rule, it is much harder to achieve good outcomes if the political pool is dominated by bad types.

Exercise 6

6(a)-(c)

```
        # Load necessary library
library(dplyr)

# Set seed for reproducibility
set.seed(2024)

# Parameters
N <- 1000
type_probs <- c(0.5, 0.5)  # 50% good, 50% bad
mus <- list(good = 1.0, bad = -1.0)
sigmas <- c(0.5, 1.0, 2.0)
cutoff <- 0

# Store results
results <- list()

for (sigma in sigmas) {
  # Assign types
  is_good <- runif(N) < type_probs[1]

  # Generate z based on type
  z <- ifelse(is_good,
              rnorm(N, mean = mus$good, sd = sigma),
              rnorm(N, mean = mus$bad, sd = sigma))

  # Apply decision rule
  reelected <- z > cutoff

  # Compute metrics
  reelection_rate <- mean(reelected)
  true_positive_rate <- sum(is_good & reelected) / sum(is_good)
  false_positive_rate <- sum(!is_good & reelected) / sum(!is_good)
```

```
  # Store results
  results[[length(results) + 1]] <- data.frame(
    sigma = sigma,
    reelection_rate = reelection_rate,
    true_positives = true_positive_rate,
    false_positives = false_positive_rate
  )
}

# Combine results into a data frame
df_voter_noise <- bind_rows(results)
print(df_voter_noise)
```

6(d)

Simulation Summary:

What happens to voter accuracy when signals become noisier?

- As σ increases, voter accuracy deteriorates. Specifically:
  - **True positives** fall — good politicians are less likely to be re-elected.
  - **False positives** rise — bad politicians are more likely to be mistakenly retained.
- At low noise ($\sigma = 0.5$), nearly all good types are correctly re-elected and very few bad types pass.
- At high noise ($\sigma = 2.0$), almost one-third of bad types are mistakenly re-elected.

Do voters make more mistakes when σ is large? Why?

- Yes. Higher σ means the observed signal $z = \mu + \varepsilon$ is more influenced by random noise than by the politician's underlying type.
- This blurs the distinction between good and bad types — making it harder for voters to condition decisions on true quality.
- Even though the voter rule ($z > 0$) is simple and consistent, its effectiveness declines as noise increases.

What might this imply for real-world voting decisions when information is limited?

- When performance information is noisy — due to poor transparency, media bias, or economic volatility — voters are more likely to:
  - Re-elect unfit incumbents (false positives),
  - Fail to reward competent politicians (false negatives).
- This weakens democratic accountability. Elections remain formally competitive, but the feedback mechanism between performance and electoral outcomes becomes less reliable.
- The results underscore the importance of institutional mechanisms that reduce noise — e.g., audits, independent media, and reliable policy metrics — to improve voter decision-making.

Exercise 7

Note: This outline identifies core arguments from James Fearon's chapter, but students are encouraged to develop their own interpretations and extensions.

(a) What is the difference between selection and sanctioning?

- **Sanctioning model:**
  - Elections are mechanisms for *incentivising good behaviour*.
  - Politicians are self-interested and strategic; they act well only when threatened with non-reelection.
  - Voters monitor outcomes and punish poor performers.
- **Selection model:**
  - Elections are used to *choose good types* — politicians who are intrinsically motivated to act in the public interest.
  - Voters aim to sort between good and bad types based on signals.
  - Good types behave well regardless of future electoral incentives.
- **Key differences:**

  - Sanctioning assumes strategic behaviour; selection assumes fixed traits.
  - Sanctioning focuses on *incentives*; selection focuses on *screening*.
  - Voters rely on retrospective evaluations (sanctioning) or broader trait inference (selection).

(b) Why does Fearon argue that real-world voter behaviour fits the selection model better?

- Fearon cites four types of evidence suggesting voters act as selectors rather than sanctioners:

  1. **Disdain for ambition:** Voters dislike candidates seen as merely seeking reelection — inconsistent with the sanctioning logic that reelection incentives should be good.
  2. **Support for term limits:** Popular despite their elimination of future incentives to behave — a puzzle for sanctioning logic.
  3. **Valuing principles over responsiveness:** Voters reward candidates who stick to principles, even when unpopular, and punish ”waffling” — inconsistent with a purely retrospective, sanctioning model.
  4. **Lack of last-term shirking:** Politicians do not significantly reduce effort in their final term, contrary to what the sanctioning model would predict.

(c) Why can trying to select good types undermine incentives to behave well (mixed model)?

- In the mixed model, voters use performance to infer type.
- This raises the threshold for what counts as good — since voters expect high performance from good types, bad types must perform even better to “pass”.
- But this makes it less attractive for bad types to exert effort: the benefits of mimicry shrink as expectations rise.
- **Result:** Effort falls, and accountability may worsen — a paradox where selection logic undermines sanctioning incentives.

(d) Do democratic elections function better as selection or sanctioning devices in practice?

- Fearon's view: empirically, voter behaviour resembles *selection* logic more closely.
- Normatively, sanctioning may not work well in practice due to:
  - Limited monitoring of politicians,
  - Voter inattention or lack of information,
  - Crudeness of reelection as a punishment/reward mechanism.
- Students may argue for one model over the other or suggest that elections combine both functions.
- Institutional context likely matters: in low-information settings, selection may dominate; in high-accountability settings, sanctioning could be more effective.

Exercise 8

Note: This outline identifies key conceptual trade-offs, using the logic of electoral accountability models. Students are encouraged to reflect on these points and develop their own arguments or examples.

(a) Why does the prospect of reelection incentivise effort in two-period models like Ferejohn's? What happens in the final term under term limits?

- In Ferejohn-style models, politicians exert costly effort in the first period because:
  - Voters condition re-election on observed outcomes.
  - Effort increases the likelihood of favourable outcomes and therefore re-election.
- In the **final term**, when re-election is not possible, politicians have no electoral incentive to exert effort.
- This creates a classic "lame duck" problem: effort collapses in the final period because there is no future electoral reward.

(b) When voter signals are noisy, why might term limits help?

- In noisy environments (e.g., shocks, media distortion), performance signals are unreliable.
- Voters may struggle to distinguish bad types from unlucky good ones — potentially leading to the re-election of underperformers.
- Term limits can act as a **hard cap** on tenure, ensuring periodic turnover even when voters cannot confidently sanction poor performers.
- In this sense, term limits partially substitute for sanctioning when monitoring fails.

(c) Can term limits improve accountability by "flushing out" low-quality incumbents? When is this logic persuasive?

- Yes, in some contexts. If voters cannot reliably detect bad types, mandatory turnover ensures they don't stay indefinitely.
- This is particularly persuasive in:
  - Systems with weak or captured media,
  - Environments with high electoral manipulation or clientelism.
- Term limits may help **reduce entrenchment**, especially when incumbents benefit from resource or institutional advantages.

(d) What are the costs of term limits?

- **Weakened incentives in final term:** Without re-election pressure, incumbents may shirk or extract rents.
- **Loss of good performers:** Term limits remove both good and bad types indiscriminately.
- **Short-termism:** Politicians may avoid long-term investments with deferred payoffs.
- **Power shift to non-elected actors:** Bureaucrats or party elites may gain influence over policy due to incumbent turnover.

(e) When are term limits more helpful or harmful for accountability?

- **Helpful** when:
  - Voter information is limited or distorted,
  - Institutional checks are weak,
  - Incumbents enjoy unfair advantages (e.g., media capture, patronage).
- **Harmful** when:
  - Voters can monitor performance reliably,
  - Elections effectively reward effort,
  - Political experience and continuity are valuable.
- The optimal rule may be context-dependent — some countries may benefit from term limits, others not.

(f) Optional: If signal noise were reduced, would the case for term limits be stronger or weaker?

- **Weaker:** With better monitoring (e.g., through audits, independent media, transparent metrics), voters can use elections to reward good performance and remove poor performers.
- The justification for term limits declines if voters can exercise effective electoral control.
- In low-noise contexts, term limits may unnecessarily disrupt accountability by removing effective leaders.

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