9

Basic Laws of Party Seats and Votes – and Application to Deviation from Proportionality

What we have reached at this point in the book is a set of *basic laws of party seats and votes*. Each of these laws has been derived in preceding chapters. In this chapter we summarize them and explain how they qualify as laws in the strongest scientific sense of the term. Then we explore how they can be applied to a further quantity of interest, deviation from proportionality. We do not (yet) have a law of how electoral systems connect to deviation, but we have a mathematical expression that draws on the basic laws.

BASIC LAWS OF PARTY SEATS AND VOTES

1 Law of Largest Party Seats

The most likely seat share of the largest party in an elected assembly is $s_1=(MS)^{-1/8}$, where S is the number of seats in the assembly and M is the number of seats in the average district.

2 Law of Number of Assembly Parties

The most likely effective number of parties in an elected assembly is $N_S=(MS)^{1/6}$, where S is the number of seats in the assembly and M is the number of seats in the average district.[1]

In social sciences, the term "law" has been at times conferred rather loosely to empirical quantitative regularities the cause of which remains unknown and even to directional tendencies lacking any quantitative content. The laws above are stronger than that. *First*, they have a logical underpinning. Indeed, they can be derived from probabilistic considerations without any data input. *Second*,

[1] What about the number of seat-winning parties, $N_{S0}=(MS)^{1/4}$? While the largest seat share and the effective number of parties can be determined fairly unambiguously, the operational determination of the number of parties winning at least one seat is at the mercy of local parties and independent candidates. Thus we would not claim $N_{S0}=(MS)^{1/4}$ as a law, even while it is an indispensable link in deriving the two laws.

they are confirmed empirically, as being as close to the world median outcomes as one could hope.

These laws are deterministic regarding the average outcome: if you carry out sufficiently many measurements on reasonably fair elections worldwide, the averages will approach $s_1=(MS)^{-1/8}$ and $N_S=(MS)^{1/6}$, respectively. These laws are probabilistic for individual cases, with about one half of outcomes falling short of the average expectation values and about one half exceeding them; however, outcomes far from expectation values are rarer than those close to them.

So far we have applied these laws only to a select group of countries using "simple" electoral rules. There is nothing inherently problematic with that: after all, the law of ideal gases, so basic to physics and chemistry, applies only to a nonexisting ideal entity! In Chapters 15 and 16 we will probe their applicability to other systems that are more complex, but in this chapter the focus remains on simple systems.

These laws enable us to calculate the most likely outcome in a country, *in the absence of any further information* beyond M and S. If we do have further information, we may make further inferences on the direction in which this country would tend to deviate from worldwide expectations.[2]

With more caution, we also put forward two further potential laws.

3 Law of Largest Party Votes

The most likely vote share of the largest party in assembly elections is $v_1=[(MS)^{1/4}+1]^{-1/2}$, where S is the number of seats in the assembly and M is the number of seats in the average district.

4 Law of Number of Electoral Parties

The most likely effective number of parties in assembly elections is $N_V=[(MS)^{1/4}+1]^{2/3}$, where S is the number of seats in the assembly and M is the number of seats in the average district.[3]

The reasons for caution are wider scatter in data and less thorough testing, especially trying to extend these laws to complex systems. Note, however, that this testing has already been more stringent than is the case for many a tendency

[2] How do these laws relate to the so-called Duverger's law? Suppose we have S=256 and M=1. We would expect a largest seat share of 50.0 percent and an effective number of seat-winning parties 2.52. Is the largest party sufficiently large and the number of parties sufficiently low to confirm Duvergerian expectation? This would be a matter of taste, given that the so-called Duverger's law lacks quantitative specificity. Note that an expectation of s_1=0.50, as an average, would imply, if the executive were parliamentary, about one half of the time there would be either a minority or coalition government.

[3] Here we have even more reason not to propose $N_{V0}=(MS)^{1/4}+1$ as a law, given the phantom nature of the "pertinent" number of electoral parties, even while $N_{V0}=(MS)^{1/4}+1$ is an indispensable link in deriving the two laws.

loosely called law in social sciences. We prefer to be more demanding in what we call a law, starting with the requirement that it be based on logical deduction and estimation of a specific quantitative relationship, before determining whether it also might be an empirical regularity.

Now, having summarized these basic laws, let us see if we can extend in further directions. The first extension is to another nationwide outcome, deviation from proportionality. Then there will be several further extensions in subsequent chapters.

ELECTORAL RULES AND DEVIATION FROM PROPORTIONALITY

A central concept in the analysis or design of electoral systems is proportionality. In this chapter, we turn our attention to how nationwide electoral system characteristics affect this important outcome of elections. *Deviation from proportionality* means a summary index of the differences between the seat shares and vote shares of parties. Recall from Chapter 4 that the two most widely used measures are:

Loosemore and Hanby's (1971)$D_1 = 0.5\sum|s_i - v_i|$, and
Gallagher's (1991)$D_2 = [0.5\sum(s_i - v_i)^2]^{1/2}$.

Deviation, by either measure, tends to be largest in elections using one-seat districts, especially when assemblies are small, so that Seat Product MS is less than 100. Deviation tends to be smallest for nationwide seat allocation, where S is more than 100 and MS thus more than 10,000. Table 9.1 shows a clear impact of the Seat Product.

Developing a model of just what that effect looks like, however, turns out to be somewhat challenging. In this chapter, we show an approximation that does reasonably well, even though it is not as complete a logical prediction as we have been aiming for with other quantities.

How might we make the relationship between Deviation from PR and MS more specific and quantitative? In Chapter 4, we showed a graph (Figure 4.1) of D_2 against a country's mean district magnitude, M. We suggested that the best fit was approximately

$$D_2 = 0.10M^{-1/3}.$$

TABLE 9.1 *Deviation from PR tends to decrease with increasing Seat Product* MS

MS	Less than 100	100 to 10,000	Over 10,000
Average D_2	0.153	0.069	0.0157
No. of elections	102	155	35

This expression, of course, leaves out the assembly size, *S*. Moreover, it is purely empirical. Logically a connection to *MS* should exist, but what would it be? At this stage, we are stretching the limits of our basic approach. We are now several logical steps removed from where we began, going from the number of seat-winning parties to the vote share of the largest. Each step piles up more random scatter. On top of that, taking differences of almost equal values such as s_1 and v_1 makes relative error explode.[4] Such scatter could blur out any impact of Seat Product *MS*. For successive individual elections, deviation from PR is notoriously fickle, even while the effective number of parties remains practically the same.

Toward Logical Model of Deviation from Proportionality

So far, we are unable to provide a precise logical model for either measure of deviation from proportionality. However, we can approximate one for Gallagher's D_2, as explained in this section.[5] We base it on (s_1-v_1), the difference between the seat and vote shares of the largest party nationwide. The largest party typically has the biggest impact of all parties on D_2. When only one party has a greater share of seats than of votes and only one other party has a lesser share of seats than of votes, then $D_2=[0.5\Sigma(s_i-v_i)^2]^{1/2}$ boils down to this difference, $|s_1-v_1|$. Most often the party with the most votes gets a bonus in seats, making the difference a positive value. Then $D_2=s_1-v_1$. This is our base line.[6] If several other parties have losses ($s_i<v_i$), then D_2 is less than s_1-v_1. But it also may happen that several large parties have bonuses ($s_i>v_i$,) at the expense of a single major loser; then D_2 is more than s_1-v_1. So we settle on

4 Assume M=10 and S=100, so that MS=1000. The expected largest seat share is s_1=0.42, and the largest vote share is v_1=0.39, so that s_1-v_1=0.03. Assume a relative error of just ±2 percent on s_1 and v_1: they can range from 0.412 to 0.429, and from 0.382 to 0.398, respectively. Then s_1-v_1 could range from as little as 0.014 to as much as 0.047. Indeed, an error range of ±4 percent on s_1 and v_1 could turn s_1-v_1 negative.

5 While we consider Loosemore-Hanby's index to be at least as good a measure as D_2, we have found no way to start a logical model for D_1. Empirically, Taagepera (2007: 79) observes that $D_1 \approx D_2^{.863}$, on the average, but we have no logical model to account for such relationship. Taagepera and Shugart (1989a: 270–273) show a good fit between D_1 and the ratio $r=(N_V-N_S)/N_V$; namely $r=(D_1-.02)^{0.7}$. We still do not know the logical reason behind it.

6 Lijphart (1994: 160–162) listed stable period averages in various countries for D_2 and also for an even simpler measure of deviation from PR: the largest single gap, $D_\infty=\max|s_i-v_i|$. Taagepera (2007: 67) observed that the values of D_2 and D_∞ are fairly close. Most often D_2 falls slightly short of D_∞, but sometimes even exceeds it. So we take a leap of faith and assume that $D_2 \approx D_\infty$. Often the largest party supplies this maximum gap: $D_\infty=s_1-v_1$. Yet it also happens that the losses of a third party exceed the gains of the largest party. It can even happen that s_1-v_1 is negative in an individual election. As we strive for general average relationships, we'll ignore these very rare cases.

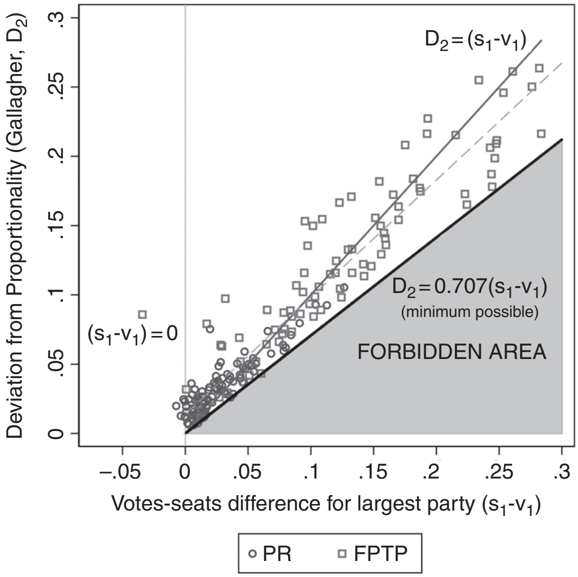

FIGURE 9.1 Deviation from proportionality (D_2) versus the difference between the largest party's seat and vote shares (s_1–v_1)

$$D_2 \approx s_1 - v_1,$$

even though this assumption is quite questionable.[7]

Despite being questionable, $D_2 \approx s_1 - v_1$ is clearly close to being accurate, as seen in Figure 9.1. This figure also shows the minimum possible D_2, for given s_1–v_1, as the thick black line. This line represents the equation,

$$D_2 = 0.707(s_1 - v_1).$$

No election result could be below this line.[8] The statistical best fit, D_2=0.0138 + 0.845(s_1–v_1) [R^2=0.913]. This best fit is depicted in Figure 9.1 as the dashed line. It deviates from $D_2=s_1-v_1$ in an expected way. Indeed, when (s_1–v_1) happens to be a perfect 0, it makes sense for D_2 to be slightly positive (hence intercept 0.0138), because s_i–v_i is still likely to be off perfect 0 for other parties. On the other hand, a very large s_1–v_1 materializes most easily when a large party faces split opposition. Evenly split opposition would lead to D_2=0.867(s_1–v_1); the observed slope, 0.845, is close.

[7] When relationships cannot be firmly modeled, one has the choice: give up or do one's best. Giving up leaves us with nothing. Trying may lead to partial advance, because the disparities between the tentative model and the actual average pattern may offer hints of how to improve the model.

[8] Mathematically, this lower limit is approached when the large party faces a profusion of tiny parties that win no seats, so that $(s_i–v_i)^2$ approaches 0 for each of them. Then $D_2=[0.5(s_1–v_1)^2]^{1/2}=0.707(s_1–v_1)$.

It is, of course, possible for s_1–$v_1$to be negative. Figure 9.1 shows a light vertical line at s_1–v_1=0 in order to allow us to see at a glance that most cases of s_1–v_1<0 are only barely negative. The one case that is most visibly into the negative range is India 1989.[9] The graph also shows, with the dashed line, the best fit, reported previously.

In general, Figure 9.1 confirms, it is not as ridiculous as it might have seemed initially to suggest that $D_2 \approx s_1 - v_1$ would be a good building block. Thus we might be able to connect the average expectation for D_2 to Seat Product *MS*. This should be possible because we have seen (from Chapters 7 and 8) that, on the average,

$$s_1 = (MS)^{-1/8} \text{and}$$
$$v_1 = (N_{S0} + 1)^{-1/2} = [(MS)^{1/4} + 1]^{-1/2}.$$

The resulting complex expression $s_1 - v_1 = (MS)^{-1/8} - [(MS)^{1/4} + 1]^{-1/2}$ can be approximated by

$$s_1 - v_1 = (1/3)(MS)^{-1/3}. \tag{9.1}$$

This approximation holds within ±6 percent of the value of $(MS)^{-1/8} - [(MS)^{1/4} + 1]^{-1/2}$, throughout the observed range of *MS*. The coefficient and exponent (which just happen to have the same numerical value) are not empirical (data based) but result from a purely mathematical transformation.[10] To the extent D_2=s_1–v_1 holds, we would then also expect

$$D_2 = (1/3)(MS)^{-1/3}. \tag{9.2}$$

We can graph both the *MS*-derived expectation for largest party seat–vote difference (s_1–v_1) and the complete value of deviation from proportionality (D_2) against the Seat Product, *MS*. Figure 9.2 shows the direct graphing of s_1–v_1 against *MS*. The actual values of s_1–v_1 tend to exceed slightly the logical expectation at low *MS* and to fall slightly below it at very high *MS*.[11]

Figure 9.2 also differentiates by electoral formula. The FPTP systems are shown with squares. PR systems are with circles; the circles are filled in if the formula is D'Hondt, but open otherwise.[12] The impact of PR formula is not

[9] In this election, the Indian National Congress party won only 36.15 percent of the seats despite 39.5 percent of the votes. The other cases, all with –0.004<(s_1–v_1)<0, are Finland, 1951, and the Swiss elections of 1959 and 1963.

[10] A slightly better fit to $(MS)^{-1/8}$–$[(MS)^{1/4}+1]^{-1/2}$ is possible, but keeping to simple values of coefficients has its advantages.

[11] It is worth calling attention to the significant outlier at *MS*=36, s_1–v_1=0.001, as this is Trinidad and Tobago, 2001, one of the elections discussed in Chapter 5.

[12] The non-D'Hondt PR formulas are all either Hare quota with largest remainders or Modified Ste.-Laguë, with the exception of Brazil and Finland, which use hybrid D'Hondt/SNTV as explained in the appendix to Chapter 7.

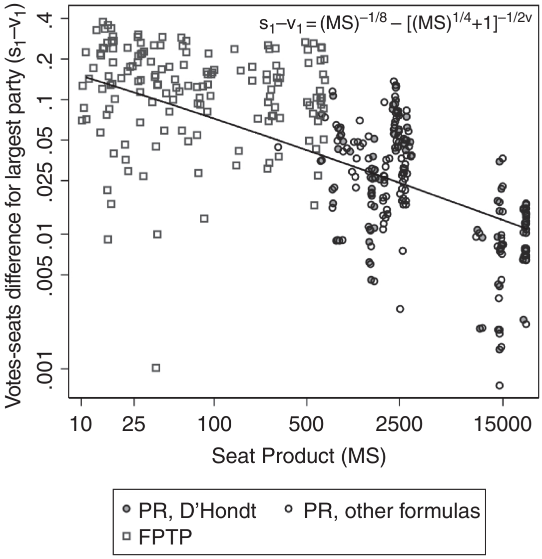

FIGURE 9.2 Difference between the largest party's seat and vote shares ($s_1–v_1$) and the seat product (*MS*)

obvious, although there is some tendency of non-D'Hondt cases to be lower, for a given *MS*, than are the D'Hondt elections.

In Figure 9.3, we graph D_2 against *MS*. Equation 9.2, plotted as the dashed line, is visibly too low. However, the important thing is that the slope, –1/3, that we showed in Figure 9.2 still clearly works. What we find is that the fit is better fit with approximately D_2=1.5($s_1–v_1$), hence

$$D_2 = 0.50(MS)^{-1/3}, \tag{9.3}$$

shown as the solid line in Figure 9.3. (See regression in the chapter appendix.)

Figure 9.3 again differentiates FPTP (squares) from PR (circles), and uses filled-in circles for D'Hondt.[13] It is again apparent that PR formula is less important than the Seat Product. Nonetheless, note that the data points for Israel (labeled in the graph), which is among the few countries to have used D'Hondt and Hare quota and largest remainders in different times periods with the same Seat Product (Hazan et al., 2018), shows a clear differentiation by formula, with the D'Hondt elections exhibiting higher D_2.

It is from the connection to *MS* in Equation 9.3 that we derived the expression, D_2=0.10 $M^{-1/3}$, reported way back in Chapter 4, before we had

[13] Only parliamentary systems are shown; a regression in the appendix includes presidential systems and provides support for Equation 9.3.

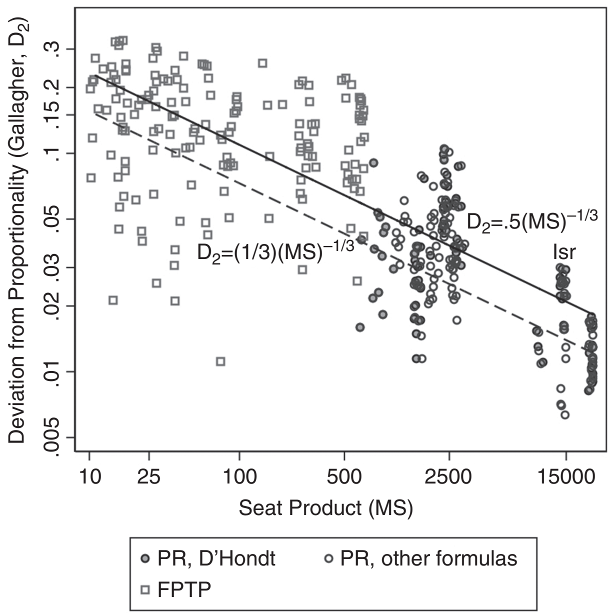

FIGURE 9.3 Deviation from proportionality (D_2) and the seat product (MS)

introduced the concept of the Seat Product. Thus the same slope works to connect D_2 to a country's mean magnitude or to its Seat Product.[14] The slope itself is, within the range of actual assembly sizes, an almost precise match to a logically derived difference of seats and votes for the largest party – that is, derived from MS, the Seat Product (Equation 9.2).

In both Figures 9.2 and 9.3 we see one noteworthy cluster of data points under FPTP that are especially far off the graph's solid line. The set of points with MS>450 and both s_1–v_1 and D_2 greater than 0.125 consists entirely of elections from India and the UK. These are the FPTP parliamentary systems with the largest assemblies, although other elections in these countries are not so far off the line. Among the PR systems, a few elections in Spain have unusually high s_1–v_1 and D_2. In Spain, as in India and the UK, low magnitude and a large number of districts both tend allow individual district-level deviations to accumulate if the leading party performs unusually well at converting votes into seats.

Recall that N_S=$(MS)^{1/6}$, as our best average estimate (Chapter 7). It then would follow from D_2=$0.50(MS)^{-1/3}$ that

$$D_2 = 0.50/{N_S}^2 \tag{9.4}$$

[14] More specifically, for this sample, mean S is around 200. Applying Equation 10.3 with this value yields D_2 =$0.09M^{-1/3}$; we round the constant to 0.10 for purposes of the discussion in Chapter 4.

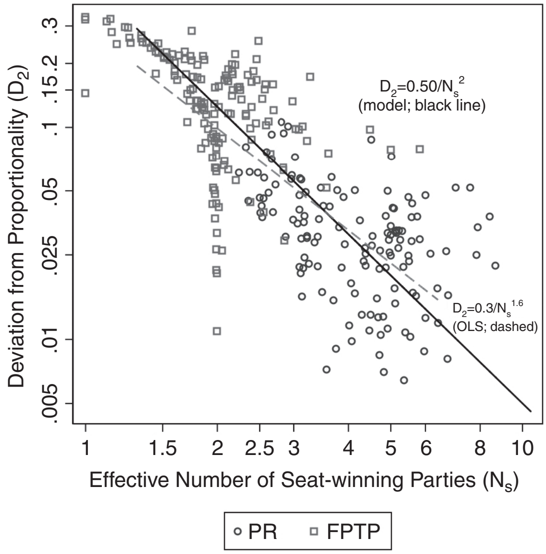

FIGURE 9.4 Deviation from proportionality (D_2) and effective number of seat-winning parties (N_S)

The causal linkage between D_2 and N_S is as indirect as it could be. Indeed, the closest common "ancestor" that enters their derivation from *MS* is N_{S0} (and partly, s_1). Therefore, one might expect enormous scatter when graphing D_2 against N_S. However, the fit is quite good, even if a best fit from an OLS regression (the output of which is in the chapter appendix) diverges somewhat from Equation 9.4, as we see in Figure 9.4. The solid line in the figure is Equation 9.4.

Thus we have a systematic relationship between D_2 and N_S, although we hesitate to call it a law, because the relationship of D_2 to *MS* is itself somewhat short of a law (as shown earlier in this chapter). In retrospect, some inverse relationship between D_2 and N_S makes sense. A very low number of assembly parties – which is most likely to result from FPTP (see square symbols in Figure 9.4) – is likely to result from heavy disproportionality, while a very high number of parties can make it into the assembly only when disproportionality is low.[15]

CONCLUSION: CONNECTIONS AMONG CONNECTIONS

Starting with the institutional Seat Product, we have been able to identify four basic laws of party votes and seats. These have been derived and tested on

[15] Previous graphing of disproportionality against the number of parties (e.g., Taagepera and Shugart 1989a:107 and Taagepera 2007: 68) missed this relationship because they graphed against the number of electoral parties, N_V, rather than N_S.

nationwide party systems in the preceding two chapters, and were summarized at the beginning of the present chapter. Then, in this chapter, we apply them to an index that indicates a core question for assessing electoral-system performance: proportionality.

When we apply the laws to the index of Deviation from PR (D_2, introduced by Gallaher, 1991), we step on shakier ground. Seat structure follows directly from the Seat Product (Chapter 7). Vote structure follows from seat structure and from the bold and risky "plus one" hypothesis (Chapter 8). Whatever discrepancies occur at the seats level will be transmitted to the votes level, possibly magnified. When we reach deviation from PR, the subtraction of vote shares from seat shares can have two effects.

First, it can cause random error to explode. We find that scatter increases indeed, but remains manageable. Second, tiny but systematic deviations from the "plus one" hypothesis regarding votes can systematically shift even the average of the pattern for deviation from PR, especially in conjunction with deviations from the assumption that D_2 equals $s_1 - v_1$. We do observe such a shift: in Figure 9.3, D_2 tends to be systematically larger than expected, by a factor of 1.5.

Further fine-tuning of the model for D_2 would be potentially useful.[16] Yet this should not obscure the fact that the broad dependence of deviation from PR on the number of seats that are available in the assembly and districts has become predictable. If a country with given *M* and *S* considers altering either of them, we can estimate the extent to which it would change the deviation from PR. Note that much of the scatter in Figure 9.3 is due to country-specific factors that would remain the same for the given country. When its *MS* is altered, its D_2 can be expected to move on a curve parallel to the worldwide average curve.

We thus stand at a point at which a panoply of nationwide party system effects can be linked together, all back to the most basic of institutional inputs, assembly size (*S*) and mean district magnitude (*M*) – at least for simple systems. In Chapter 15 we will offer an extension with one more parameter, which allows us to estimate values in two-tier compensatory PR systems. In Chapter 16 we consider other complex types of electoral systems, and find that at least some of them show patterns that are predictable as if they were simple after all.

In Figure 9.5, we offer a schematic of how the various quantities derive ultimately from the Seat Product, *MS*. Table 9.2 summarizes the key equations linking the quantities together. Table 9.3 gives examples of what the equations would produce as expected outputs for given values of *MS*. We already have seen that these generally conform to real-world averages (in previous chapters, except for D_2, which was shown in this chapter).

[16] Plus we lack a model to predict Loosemore-Hanby's D_1. This is not too severe a deficiency, given that D_2 has become the more prevalent measure.

TABLE 9.2 *Nationwide equations for the Seat Product Model*

Nationwide seats		
$\mathbf{N_{S0} = (MS)^{1/4}}$		
$\mathbf{s_1 = (MS)^{-1/8}}$ BASIC LAW 1	$s_1 = N_{S0}^{-1/2}$	
$\mathbf{N_S = (MS)^{1/6}}$ BASIC LAW 2	$N_S = N_{S0}^{2/3}$	$N_S = s_1^{-4/3}$
Nationwide vote-seat interaction		
$\mathbf{N_{V0}=N_{S0}+1}$		
$v_1 = (N_{S0}+1)^{-1/2}$	$v_1 = (s_1^{-2}+1)^{-1/2}$	
$N_V = (N_{S0}+1)^{2/3}$	$N_V = (s_1^{-2}+1)^{2/3}$	$N_V=[N_S^{3/2}+1]^{2/3}$
Nationwide votes		
$N_{V0}=(MS)^{1/4}+1$		
$\mathbf{v_1 = [(MS)^{1/4}+1]^{-1/2}}$ BASIC LAW 3	$v_1 = N_{V0}^{-1/2}$	
$\mathbf{N_V=[(MS)^{1/4}+1]^{2/3}}$ BASIC LAW 4	$N_V = N_{V0}^{2/3}$	$N_V = v_1^{-4/3}$
Nationwide deviation from proportional representation		
$s_1-v_1=(1/3)(MS)^{-1/3}$ [theoretical]		
$D_2=0.50(MS)^{-1/3}$ [empirically adjusted coefficient]		

The four basic laws and their antecedents are shown in bold; the rest follows from algebra.

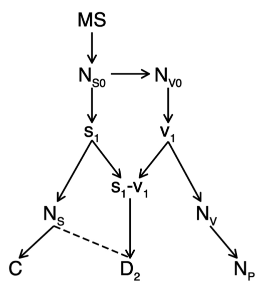

FIGURE 9.5 Schematic of quantities deriving from the Seat Product

The basic sequence shown on the left side of Figure 9.5, $MS \rightarrow N_{S0} \rightarrow s_1 \rightarrow N_S$, was tested in Chapter 7 (see also Taagepera 2007). Once "winners plus one" is introduced (Chapter 8), we repeat the basic sequence $N_{S0} \rightarrow s_1 \rightarrow N_S$ on the right side of the schematic, for measures based on votes: $N_{V0} \rightarrow v_1 \rightarrow N_V$. Finally, s_1-v_1 joins the two strands, and D_2 needs further and fuzzier juggling. The pattern is symmetrical, apart from *MS* directly impacting seats, not votes.

Each step involves noise (data scatter). We are estimating predictions from predictions. Thus, as one graphs and regresses against each other factors several steps away, one might expect increasing scatter, as the number of

TABLE 9.3 *Average expectations at various levels of* **MS**

MS	**1**	**10**	**100**	**1000**	**10,000**	**100,000**
$s_1 = (MS)^{-1/8}$	1	0.750	0.562	0.422	0.316	0.237
$v_1 = [(MS)^{1/4}+1]^{-1/2}$	0.707	0.600	0.490	0.389	0.302	0.231
$N_S=(MS)^{1/6}$	1	1.468	2.154	3.162	4.642	6.813
$N_V=[(MS)^{1/4}+1]^{2/3}$	1.587	1.976	2.588	3.527	4.946	7.066
$D_2=0.5(MS)^{-1/3}$	0.5	0.232	0.108	0.050	0.023	0.011

steps increases. But this is not quite so. Since N_S and N_V are five steps removed, huge scatter might be expected on this account. Yet it stands to reason that they cannot diverge widely and just randomly. The same applies to s_1 and v_1, three steps removed. The parallel nature of $N_{S0} \rightarrow s_1 \rightarrow N_S$ and $N_{V0} \rightarrow v_1 \rightarrow N_V$ may keep their fluctuations in line, even though this is hard to demonstrate.

In contrast, $s_1 - v_1$, so close to both s_1 and v_1, would offer much more scatter, when graphed against either s_1 or v_1, compared to s_1 graphed against v_1. This is the effect of subtracting rather similar quantities. Indeed, the closer the systematic parts of s_1 and v_1 are, the more random their relative difference becomes.

The eagle-eyed reader might notice an entirely new quantity connected to N_V out on the right of Figure 9.5: "N_P." This refers to the effective number of presidential candidates. We have noted in both Chapters 7 and 8 that quantities such as N_S and N_V in presidential systems are somewhat more scattered than is the case for parliamentary systems. Nonetheless, we have stressed, there is no need for a systematically different approach to presidential systems. The existence of a politically significant, elected office of the presidency has consequences for the party system, but they are less distinct in their patterns than may be expected. In fact, in Chapter 11 we are able to support the very bold claim that the Seat Product can predict the values of N_P, too. This is remarkable, given that the Seat Product is a feature of the assembly, not of the presidency. Yet what we will show in Chapter 11 is that for presidential democracies the Seat Product actually predicts trends in N_P better than it predicts assembly party-system quantities such as N_S and N_V! The two core institutional parameters, M and S, really are fundamental.

The last feature on the lower left in Figure 9.5 is cabinet duration (C), derived from the effective number of assembly parties, as indicated in Chapter 7. While it is outside the direct focus of the present book, cabinet duration extends the predictive power of the Seat Product from electoral to governmental realm, if the executive type is parliamentary. Note also the dashed line joining

N_S to D_2: while there is no direct causal link between the two, the previous logical relationships imply a simple equation joining the two (as in Figure 9.4).

We have now completed our set of chapters on the basic models for nationwide party systems, although we will come back to them in Part III (on presidential systems) and Part V (where we analyze complex systems). While understanding the nationwide level is critical, it is nonetheless the case that most electoral systems consist of numerous districts, in each one of which a contest among several parties takes place. To account for the district level, Chapter 10 shows how to model the impact of these nationwide factors on those multiple districts that comprise what we think of as "the electoral system".

What we will demonstrate in Chapter 10 is that same schematic depicted for the nationwide party system in Figure 9.5 also recurs at the district level. It is perhaps obvious that one of the core institutional parameters, M, would be critical at the district level. What is less obvious is that S – a nationwide feature by definition – also would be. Yet we will see that the impact of M depends on how much of a share of the country's total S any given district represents.

Appendix to Chapter 9

In this chapter we observe that

$$s_1 - v_1 = (MS)^{-1/8} - [(MS)^{1/4} + 1]^{-1/2} \approx (1/3)(MS)^{-1/3} \qquad (9.1)$$

as a close approximation.[17] We further suggest

$$D_2 = (1/3)(MS)^{-1/3} \qquad (9.2)$$

Equation 9.2 is tested by Regression One in **Table 9.A1**. The estimated coefficient is very near the expected, –0.333. The constant term should be –0.477 (the log of 1/3). While this is within the confidence interval, a clearly better fit is obtained by D_2=0.50$(MS)^{-1/3}$ (Equation 9.3), as we saw in Figure 9.3. This revised intercept would be –0.301 (the log of 0.5), and is closer to the regression-estimated constant.

Regression Two in Table 9.A1 tests Equation 9.4, which states that D_2=0.50/N_S^2. The regression output only partially supports the result, as neither expected coefficient is within the confidence intervals. However, as shown in Figure 9.4, the best fit derived from Regression Three is not greatly divergent from Equation 9.4. More importantly, we are on firm logical ground with both the slope of –1/3 in Equation 9.2 and, of course, the slope of 1/6 in the

17 If graphed, the quantities derived from $(MS)^{-1/8}$–$[(MS)^{1/4}+1]^{-1/2}$ and $(1/3)(MS)^{-1/3}$ map almost perfectly on to an equality line; they are correlated at 0.994.

TABLE 9.A1 *Regressions for Deviation from Proportionality* (**D_2**)

	(1)	**(2)**
log*MS*	–0.322	
Expected: –0.333	(0.0250)	
	[–0.372 – –0.271]	
logN_S		–1.551
Expected: –2.0		(0.131)
		[–1.816 – –1.286]
Constant	–0.362	–0.514
Expected: see text	(0.0622)	(0.0474)
	[–0.488 – –0.236]	[–0.609 – –0.418]
Observations	295	295
R-squared	0.584	0.594

SPM, deriving N_S from *MS*. Thus we must have –2 in Equation 9.4. If we recalculate the intercept with the coefficient on logN_S forced to be –2, we obtain an intercept of –0.328, which (unlogged) is 0.470. This is so close to 0.50 that we would not allow the regression's estimated confidence intervals to override Equation 9.4.