CHAPTER 5

Social choice and other voting models

She pours, to her spirit's content, a nightingale's woeful lament
That e'en though the voting be equal, his ruin will soon be the sequel.
(Aristophanes, *The Frogs,* Part III, 405 B.C.)

The previous chapters have assumed that society's decisions are made by simple majority rule. In the real world, many decisions really are made that way. Many other decisions, however, are made using other means of adding up or counting people's political preferences. In this chapter we consider public decisions more broadly and look at other ways of choosing. Because we are going to cover many topics quickly, it is well to begin with an intuitive overview.

One of the conventions of social choice theory is to describe each of several important results as a "paradox," a term deriving from *paradoxon,* a Greek word meaning "beyond opinion or belief." Thomas Schwartz described the role of paradox in analytical politics: "Deduce a contradiction from reasonable-looking, widely held assumptions, and you have a paradox; the better entrenched the assumptions, the more paradoxical the paradox" (1986, p. 116).

We have argued that the location of a middle within public opinion, given the voting rules of the society, is the key feature in determining both the nature of policy and its stability in a nation. But we have had trouble *finding* a middle in complicated policy spaces under majority rule. This difficulty suggests that majority rule decisions are susceptible to manipulation, because there is no determinate outcome in many circumstances. At worst, democratic decision processes may be quite incoherent, as William Riker argued.

In Chapter 2 we considered a problem with the stability of majority rule outcomes when preference profiles are not single-peaked. The example itself dates back to the Marquis de Condorcet, who published a lengthy discussion of the mathematical properties of sequences of pairwise majority rule contests in his *Essai sur l'application de l'analyse à la* *probabilité des décisions rendues à la pluralité des voix* (1785). Condorcet himself considered the result to be merely a feature of majority rule, rather than a paradox. Nonetheless, "Condorcet's paradox" has become part of the language of modern social choice theory. What is the deduced implication that is "beyond belief"?

Before we give the answer, it is useful to define a technical term: "transitivity." Suppose a person is asked to rank three alternatives, A, B, and C. Imagine she responds with a list that has two comparisons:

C is better than B.
B is better than A.

Now, it *seems* obvious that anyone who likes C better than B and likes B better than A would also like C better than A. Still, we have no direct evidence because our subject did not tell us about her preferences in a comparison of C versus A.

From a technical perspective, the conclusion "C is better than A" is not obvious, and it is not trivial. In fact, C is *deducibly* preferred to A in our example if and only if the preferences of the person in question are transitive.

Transitivity. Preferences are *transitive* if, for three alternatives, C preferred to B preferred to A necessarily implies that A is not preferred to C (weak transitivity) or that C is preferred to A (strong transitivity).

Note that transitivity is a concept that might be applied to a preference ordering of an individual or a society choosing among alternative policies.

We are now in a position to state Condorcet's paradox.

Condorcet's paradox. Suppose all individual preferences are transitive, but not necessarily single-peaked. Then the social preference ordering under majority rule may be intransitive.

The paradox is that the aggregation of *individually* transitive preferences leads to an *aggregate* intransitivity.[1] The society finds itself in an endless cycle of "best" alternatives, none of which commands a majority against all other alternatives.

Our Hun–Gats, from Chapters 2 and 3, found themselves in such a cycle: A majority preferred staying home to going south. Then a majority preferred going north to staying home. Then a majority preferred going south to going north! To a casual observer, it may look as though this is a group of people who don't know what they want, but that's the paradox: Each person individually knows exactly what he or she wants. It is the society that can't make a decision, because majority rule is intransitive in this case.

In this chapter, we discover that Condorcet's paradox is an example in a general class of paradoxes arising from using *any* social choice mechanism, except dictatorship. The results are called "paradoxes" because the general problem seems so simple, yet turns out to be insoluble.

The meaning of collective choice. Suppose all citizens are perfectly informed about all policies. We then solicit citizens' ordered "lists" of policies, ranking all feasible alternatives from best to worst, assuming that the information on the list is accurate and not strategically misrepresented to manipulate the outcome. Then, given sincere, accurate information in the lists about individual relative valuation of policies, compute the aggregate list for the society. The aggregate list looks like an individual's list, again ranking polices from best to worst. The difference is that "best" now is from the collective perspective.

Social choice theorists have repeatedly demonstrated that there is no sure way of choosing a transitive aggregate list if all citizens' preferences count. This is true even under the best circumstances (perfect information, no manipulation), if no restrictions are placed on the form of individual lists. The objections made by some – that these assumptions are unrealistic – miss the point. If social choice is not tractable under idealized circumstances, then adding imperfect information and manipulation makes the problem harder, not easier. What is beyond belief about paradoxes of social choice is that even the simplest kinds of voting schemes yield surprises.

We will briefly examine the best known of these paradoxical results, Arrow's (1963) impossibility theorem. Arrow laid out a set of properties or conditions that (arguably) are desirable features of a social choice mechanism. The impossibility theorem is a deduction that no social choice mechanism can possess all these features. In particular, all voting procedures must violate at least one of the conditions that Arrow demonstrated to be mutually inconsistent. Since Arrow's paradox applies to *any* nondictatorial aggregation mechanism, it encompasses all of what we might consider "democratic" decisions by societies. We will review briefly a variety of ways of choosing collectively and discuss some of their advantages and disadvantages.

Choosing how to choose

Decisions may be made by one person, by some people, or by everyone. The set of citizens required to make a decision or choice is called the "decisive set."

Decisive set. A set C of citizens is "decisive" if for two alternatives **y** and **z** the fact that all members of C like **y** better than **z** is sufficient to ensure that **y** is selected over **z** by the society, regardless of the opinions of citizens not members of C. We will call $K(C)$ the "size" of C, or the minimum number of people required to be decisive.

We have seen an example of a decisive set under majority rule: C is any group of $(N/2) + 1$ citizens. Thus, there are many different potential Cs, each of which must have $K(C) \geq (N/2) + 1$ citizens.

Both the inclusion of a given person's preferences in a decision (enfranchisement) and the decision to use a particular means of summarizing these preferences (aggregation mechanism) can affect the decision. Generally we think of "private" decisions (What will I have for lunch? What shoes will I wear with this outfit?) as being very different from "group" decisions (What is the appropriate budget for education? What is the right speed limit on limited access highways?).

But distinguishing private and group decisions begs the question. Apparent differences among "kinds" of choices are caused by differences in enfranchisement and aggregation mechanisms, not necessarily by inherent properties of the choices themselves. Lunches and shoes could plausibly be collectively chosen (as in the military or in a school with a dress code). Similarly, levels of spending and speeding might be picked by individuals, at least in principle. Government vouchers could be applied toward the bill at a private school of a family's choice. Drivers might drive as fast as they like on highways, with government road crews occasionally clearing the mangled remains.

Thus, "social choice" has two elements:

- *Public decision:* The choice will have a significant public impact, affecting more than one individual. This might be because the choice affects others; this effect is called an "externality." The effect is called a "positive externality" if the effect is a benefit, or a "negative externality" if the effect is harmful. Alternatively, the choice might involve the level of provision of a "public good." Public goods (e.g., national defense) are characterized by zero marginal cost of production and high cost of exclusion from consumption.
- *Collective decision:* It is mandated, by rule or practice, that the choice will be made by more than one person. Technically, all this means is that $C > 1$. If collective decisions are made by majority rule, $K(C) \geq (N/2) + 1$. If the rule is unanimity, the decisive set may be all enfranchised citizens: $K(C) = N$.

The two aspects of choice may appear to go together, but they are distinct. Choices might be public but not collective: Suppose I build a factory that produces sooty smoke and my downwind neighbors suffer. There may be no institution that *enfranchises* them in the decision to build the factory.[2] Their preferences receive zero weight in the public (but not collective) decision, because $C = 1$. In the extreme, *all* decisions might be made by one person (a dictator), with no collective enfranchisement whatsoever, though all the dictator's choices might be "public."

Conversely, choices may be collective but not public: A society may decide to outlaw some consensual sexual practice involving two mentally competent adults. Another society might require that cyclists wear helmets. In both cases peoples' choices are being regulated, even though these activities affect no one else.[3] In this case, other people are enfranchised to decide individual behavior: The sexual partners and the bareheaded biker get a voice in the collective decision, but their preference counts only as a few among many. It is no longer true that $C = 2$ (for the consensual sexual activity) or that $C = 1$ (for the cyclist).

Our primary concern in this chapter will be with the implications of different *aggregation mechanisms,* not alternative *enfranchisement rules.*[4] The following section lays out the limits of the abilities of aggregation mechanisms to solve collective choice problems. The rest of the chapter considers several specific collective choice rules and their implications.

Arrow's paradox and the limits of social choice

Condorcet showed that majority rule may be intransitive, even if each individual has transitive preferences over the alternatives. Intransitivity is a kind of breakdown, assuming that some choice is required. That choice may be to preserve the status quo, and do nothing, but the choice itself must be clear and determinate. Intransitivity means the society is incapable of choosing among several mutually exclusive outcomes, without resort to random or imposed "choice." One might ask whether this potential for incoherence extends to other aggregation mechanisms. The definitive answer is less than fifty years old and dates to Arrow (1952; revised 1963).

That answer is disturbing for defenders of democracy or for advocates of any particular form of collective choice. We will present a simple overview of Arrow's technical result, but it is worth beginning by summarizing the intuition of the result first:[5]

Arrow's paradox. The only collective choice mechanism that is always transitive, allowing for any possible fixed set of pairwise preferences over alternatives, is dictatorship.

The "paradox" is that the only transitive collective decision rule that obeys the technical criteria Arrow sets out is dictatorship, or rule by one. Such a decision rule is not "collective" at all! Dictatorship resolves disagreements by restricting the decisive set to contain only one person. How did Arrow arrive at this conclusion?

The "impossibility" result

We will consider only a simplified paraphrasing of Arrow's theorem and will not consider the technical aspects of the proof of the theorem at all. The reader interested in pursuing this subject more deeply can find an introduction to the literature in Mueller (1989, especially chapters 19–20) and a treatment in depth by Schwartz (1986) and Kelly (1988). Our exposition of the logic of Arrow's result can be summarized this way:

(1) Specify a set of desirable characteristics for an aggregation mechanism, or way of "counting" preferences registered by enfranchised citizens.
(2) Determine the set of collective choice mechanisms that have these desirable characteristics.
(3) Ask how many of these choice rules are *not* dictatorial. The answer is: Not one! Any social choice mechanism exhibiting all the characteristics Arrow listed as desirable *must* be dictatorial.

Some scholars have questioned the merit of Arrow's list. Others have suggested substitute axioms that are weaker or quite different, but Arrow's original set of desirable characteristics is not implausible. The version of these characteristics we will use is adapted from Mueller (1989), itself adapted from Vickrey (1960). To describe the list of desirable characteristics for social choice mechanisms, we will need to define some terms and concepts.

Consider three different states of the world, S_1, S_2, S_3, representing discrepant policy vectors 1, 2, and 3, respectively. We can then describe the set of desirable characteristics as follows:

1. *Unanimity (also, the Pareto criterion):* If all enfranchised citizens agree (for example) that S_1 is better than S_2, then S_1 is selected by the collective choice rule over S_2.
2. *Transitivity:* The collective choice mechanism is transitive, so that if S_1 is selected over S_2, and S_2 to S_3, then S_1 is selected over S_3.
3. *Unrestricted domain:* For any individual and for any pair of alternatives S_1 and S_2, any of the following six preference orderings (from best to worst) is possible:

	1	2	3	4	5	6
Best	S_1	S_1	S_3	S_2	S_2	S_3
Middle	S_2	S_3	S_1	S_1	S_3	S_2
Worst	S_3	S_2	S_2	S_3	S_1	S_1

4. *Independence of irrelevant alternatives (IIA):* The social choice between any two alternatives must depend only on the individual rankings of the alternatives in question in the preference profile of the group. Thus, if S_1 is socially preferred to S_2, then it will still be socially preferred if we rearrange the orderings of the other alternatives

while leaving the paired rankings of S_1 and S_2 the same. For example, the following two sets of preference profiles of three citizens must yield the same social ordering for S_1 and S_2, if the social choice rule is IIA:

Preference profile set I (for persons 1, 2, and 3)

	1	2	3
Best	S_1	S_2	S_1
Middle	S_2	S_1	S_3
Worst	S_3	S_3	S_2

Preference profile set II

	1	2	3
Best	S_1	S_3	S_1
Middle	S_3	S_2	S_2
Worst	S_2	S_1	S_3

Notice that the relative rankings of S_1 and S_2 are the same in profile sets I and II. All that is different is the position of S_3 in the rankings. For example, in set I, person 1 ranks the alternatives S_1, S_2, S_3. In set II, person 1 ranks them S_1, S_3, S_2. In both cases, 1 likes S_1 better than S_2. Independence of irrelevant alternatives requires that this pairwise comparison of rankings does not depend on the position of other, "irrelevant" alternatives (such as S_3 in our example).

The final "good" characteristic of mechanisms for the democratic aggregation of preferences is probably the most obvious: No one person possesses all power to decide.

5. *Nondictatorship:* There is no dictator. If person 2 (for example) is a dictator, then if person 2 ranks S_1 above than S_2, then "S_1 better than S_2" is the social ranking, regardless of how anyone else, or even everyone else, ranks S_1 compared with S_2.

With these conditions established, we can state a version of the "impossibility" theorem more precisely.

Impossibility theorem. *Consider the set of all collective choice rules that satisfy requirements 1–4 (unanimity, transitivity, unrestricted domain,* *and IIA). Every element of the set of collective choice mechanisms satisfying these requirements violates requirement 5, implying the existence of a dictator.*

What does the impossibility result leave us? A menu of choice: Any mechanism for aggregating individual preferences must lack at least one of the desirable properties 1–4. We have listed nondictatorship separately because tyranny is incommensurable with democracy. To put it another way, nondictatorship is a starting point if the goal is to compare ideal forms of government.

Clearly, this outright exclusion of dictatorship is a normative choice and could be quarreled with. Plato might well have disagreed, arguing that it is democracy that causes tyranny:

> Democracy is precisely the constitution out of which tyranny comes; from extreme liberty, it seems, comes a slavery most complete and most cruel. . . . When a democratic city gets worthless butlers presiding over its wine, and has drunk too deep of liberty's heady draught, then, I think, if the rulers are not very obliging and won't provide plenty of liberty, it calls them blackguards and oligarchs and chastises them . . . and any who obey the rulers they trample in the dust as willing slaves and not worth a jot. (Plato, *The Republic,* Book VI, 560A–564B)

The only way to have justice is, in Plato's view, to have a just dictator:

> It is no wonder that the multitude do not believe what we are saying. For they have never seen in existence the project now being discussed . . . a man, balanced and equated with virtue as nearly as possible to perfection in word and deed, and, moreover, holding sovereignty in a city perfectly equated with him. . . . Let there be one man who has a city obedient to will, and he might bring into existence the ideal polity about which the world is so incredulous. (Plato, *The Republic,* Book VI, 498D–502B).

Still, Arrow disallows dictatorship. A similar argument (for restricting the menu of choice for ideal forms of government) applies to the Pareto criterion, though on more practical grounds: It is hard to imagine adopting a rule that would prevent change if literally *everyone* favored the change. If nothing else, society could unanimously change the rules![6]

If we insist on nondictatorship and the Pareto criterion in our social choice rules, we are left with three options: decision rules that allow *intransitivity,* rules that allow *independent alternatives* to affect pairwise choices of other alternatives, and rules that restrict the set of *preferences* that will be allowed (i.e., violate universal domain). A complete discussion of the implications of relaxing the postulates of the impossibility theorem are beyond the scope of this book; the interested reader should consult Schwartz (1986) or Kelly (1988).

What we will do instead is look at some alternatives to simple majority rule. People have come up with many different ways to choose. Each of these methods of aggregating preferences has some advantages and some drawbacks. The search for an "optimal" system is very difficult, however. All systems have potential problems with fairness, and all systems can be manipulated.

Alternative decision rules

There is a difficulty with majority rule as a normative prescription for all members of society. That difficulty is that the majority's will must serve all, though it is only selected by most. Such a process must rely for its legitimacy on the majority's forbearance: "To be governed by appetite alone is slavery, while obedience to a law one prescribes to oneself is freedom" (Rousseau, 1973, Book I, chapter 8). If the majority does not act on "appetites," but rather enacts only good laws, *the same policy would be chosen* by one person, by a group, or by the whole society, provided the choosers are wise, well informed, and well intentioned. Such an approach begs the question of collective choice by assuming the problem away: The collective is organic, not fractious.

This moral force of unanimity can be achieved for majorities by artifice, as in Rawls's (1971) "veil of ignorance." Rawls posits a thought experiment: Suppose you didn't know what your position would be in the society. Then what laws, rules, and policies would you select? The answer, Rawls claims is that "each is forced to choose for everyone" (p. 140). Since you don't know what your self-interest is, you must choose for society, rather than for your own "appetites." If each chooses for everyone, the distinction between private and collective choice vanishes.

But what if the majority decides to act on its appetites? Suppose, for example, most of the society wants simply to enslave the rest. Even on a smaller scale, it is perfectly possible that the majority may want to enrich itself at the cost of the minority, and ultimately at the cost of virtue and order for the society. The reader may recall that such a pessimistic prediction was Aristotle's view of unfettered majoritarianism, or what Riker (1982) called "populism." Abraham Lincoln, harking back to Jefferson's belief that revolution, even in a democracy, could be just, said: "If by the mere force of numbers a majority should deprive a minority of any clearly written constitutional right, it might, in a moral point of view, justify revolution – certainly would if such a right were a vital one" (Inaugural Address, March 4, 1861).

How can we choose rules that determine how we choose policies and have any faith in the justice of the choice? The choice of aggregation mechanism will determine, in part, the nature of the society by advantaging certain alternatives. We will consider three major sets of alternatives to simple majority rule: (1) optimal majority rule, (2) the Borda count and approval voting, and (3) proportional representation.

Optimal majority rule

The first variation on majority rule is some form of majority rule itself, allowing the size of the required "majority" (i.e., decisive set) for an affirmative decision to be different from the simple majority $C = (N/2) + 1$. After all, what is so special about 50% (plus one voter) as the minimum group in favor? In theory, as we have discussed above, the size of the group making a decision can vary from one person to the whole society.

In practice, lots of normal collective business is done by majority rule. But even within the context of real-world collective decisions, the size of the proportion of enfranchised voters required to make a decision varies widely. In many legislative assemblies, unanimous consent is required to amend or waive temporarily the rules of procedure. To change the U.S. Constitution, two different supermajorities (two-thirds of a national assembly to propose, three-quarters of state assemblies to ratify) are required. There are examples of decisive sets smaller than even simple majority, including the U.S. Supreme Court's practice of affirming a writ of *certiorari* based on a vote of less than one-half of the seats on the court (four out of nine). What is the "best" decisive set as a proportion of the polity? The answer to almost all important questions, of course, is that "it depends." But what does the answer depend on?

The classic analytical treatment of optimal majority is Buchanan and Tullock's *The Calculus of Consent.* Taking an economic approach,

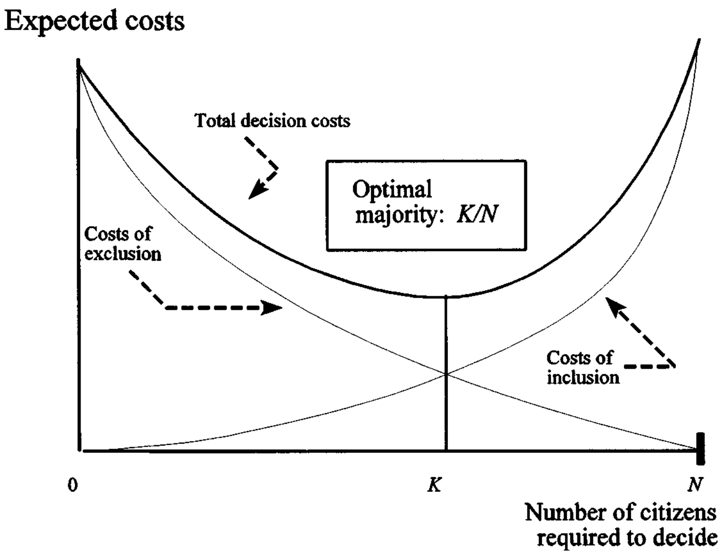

Figure 5.1. Buchanan and Tullock's "optimal majority" analysis.

Buchanan and Tullock note that there are *costs* of widely shared decision power as well as *benefits.* The costs of *including* more people in the required majority entail defining and amending the proposal, explaining it to the voters, providing payoffs to solve strategic maneuvering of swing voters, and so on. These are called "decision costs," because the costs fall on those whose preferences count in the decision. The costs of *excluding* members of society from the required majority can be thought of as the costs of being forced to obey a policy that one opposes. We will call these costs "external costs."

We can depict the problem of optimal majority graphically, as in Figure 5.1. Decision costs rise dramatically as we near a rule of unanimity, because each voter becomes a potential swing voter. Anyone can threaten to withhold approval unless certain concessions or payoffs are made. Similarly, external costs fall as we near unanimity, because by definition there is less chance a policy can be enacted without approval of everyone affected.

The optimal majority is *K/N,* because it minimizes the sum of the costs of inclusion and the costs of exclusion. These costs, though hard to quantify, clearly figure in how we choose how to choose, as public decisions fall into three categories:

(a) *Access decisions:* $0 < K < (N/2) + 1$. One member of the U.S. House or Senate is required to introduce a bill. If no one introduces the bill, it is completely blocked. At least four members of the U.S. Supreme Court are required to grant a writ of certiorari, or petition for a case to be heard. Neither of these decisions by any means ensures success; all that is granted is access.

(b) *Routine decisions:* $K = (N/2) + 1$. The smallest strict majority is a very common value for the decisive set in democracies, from tiny private clubs to the U.S. Congress and Supreme Court. This value for K is the smallest value that ensures no simultaneous passage of two directly contradictory measures. Thus, both inclusion and exclusion costs are moderate. For simple majority rule to have such wide real-world application, it must minimize the (perceived) costs of making routine collective decisions.

(c) *Changing the Rules:* $(N/2) + 1 < K \leq N$. The rules of choice govern the kinds of outcomes that the choices represent. A decision to change the rules has more far-reaching and unpredictable effects than a decision made under a fixed set of rules. Consequently, the costs of excluding enfranchised members is higher for rule change decisions, and the optimal majority for rule changes is more than 50% + 1.

In most cases, there is no single decision rule used for all choices; it depends. For example, in the U.S. Congress the introduction of a bill requires just one member. The passage of a bill requires 50% + 1 of the members present and voting. A resolution to propose an amendment to the Constitution requires a two-thirds majority. A motion to suspend the normal rules requires unanimous consent. Business in the Senate can be held up almost indefinitely by dilatory tactics or "filibuster," unless three-fifths of the membership vote for a resolution of "cloture," closing off further debate.

If there is a general rule, it does appear that we require larger majorities for larger questions, just as Buchanan and Tullock suggested. This conclusion is also quite consistent with some of Rousseau's thought, though his justifications are very different from those given by Buchanan and Tullock:

A difference of one vote destroys equality; a single opponent destroys unanimity; but between equality and unanimity, there are several grades of unequal division, at each of which this proportion may be fixed in accordance with the condition and needs of the body politic.

There are two general rules that may serve to regulate this relation. First, the more grave and important the questions discussed, the nearer should the opinion that is to prevail approach unanimity. Secondly, the more the matter in hand calls for speed, the smaller the prescribed difference in the numbers of votes may be allowed to become: where an instant decision has to be reached, a majority of one vote should be enough. (Rousseau, 1973, Book II, chapter 2, p. 278)

Equilibrium for larger majorities

It appears that different decisive sets are appropriate for different choice situations. But now we must ask what effects these differences may have on the existence and nature of equilibrium. We will still face the generic failing of majority rule systems, of course: The society may cycle among alternatives within a subset of the overall space of feasible political choices. But given this caveat, is there a generalization of the MVT that applies to optimal majorities K/N, where $K > N/2$?

The answer is yes.[7] As in Chapter 3, define N_1 as the number of points on one side of a hyperplane $\mathcal{H}$ (including points on $\mathcal{H}$), and N_2 as the number of points on the other side (again, including points on $\mathcal{H}$). The generalized K-majority voter theorem (KMVT) is then just an obvious generalization of the GMVT.[8] The KMVT can be stated as follows:

KMVT. *An alternative* **y** *is a K-majority equilibrium for the society if and only if* $N_1 \geq N - K + 1$ *and* $N_2 \geq N - K + 1$ *for every* $\mathcal{H}$ *containing* **y**.

Interestingly, the KMVT reduces to the MVT for majority rule, since for majority rule $N - K + 1 = N/2$, where $K = N/2 + 1$. Thus, a more general statement of the GMVT in the preceding chapter might have been to require the numbers of voters on each side of the separating hyperplane to be at least $N - (N/2 + 1) + 1$.

The practical difficulty with supermajority rules is the dramatic increase in the number of equilibria for the society. There is no way of choosing one over another at the outset. Further, there is no way of

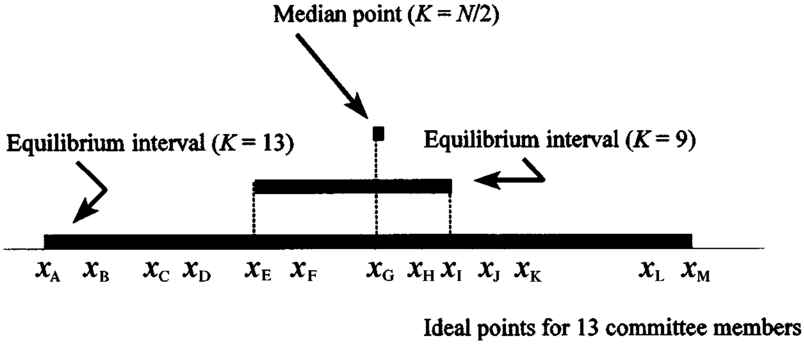

Figure 5.2. An example of supermajority rule: many equilibria, little change.

changing the choice once any equilibrium position is established as the status quo. Another way to think of the KMVT, as Figure 5.2 shows, is to realize that supermajority rules profoundly advantage the status quo position. The larger the required majority, the more difficult is any kind of change, *once a status quo is established.*

As the figure shows, larger majorities imply broader equilibrium intervals, illustrating why the KMVT is a generalization of the GMVT. If $K = N$, the set of equilibria is the Pareto set. For $N > K > N/2$, the set of equilibria is an interval. As K shrinks to $N/2$, the set of equilibria is the median ideal point (as in our example), or a median interval containing two ideal points (if the median is not unique, under the conditions outlined in Chapter 2).

If any point in the set of possible equilibria is established as the status quo by accident, practice, or strategic action, it is protected from change under supermajority rules. Consequently, supermajority rules ensure stability, but at the expense of flexibility. The status quo is preserved like an insect in amber, even if most citizens support some other alternative.

Runoffs and pluralities. One other consideration remains in our discussion of majorities. We have required that votes be conducted in a series of pairwise comparisons. That is, though the set of alternatives may be quite large, our consideration of majority rule has required that each new proposal be voted against the status quo, with the winner becoming the new status quo. Majority rule can also be applied to more than two alternatives, but analysis becomes much more complex. Further, the existence of equilibria (particularly "centrist" equilibria) becomes problematic. One solution is to have a modified form of majority rule over three or more alternatives: If any alternative receives more than half the vote, that alternative wins and becomes the status quo. Otherwise, the top two alternatives (in terms of votes received) are selected for a *runoff* election.

An apparently similar (but very different) procedure is *plurality rule.* In plurality rule systems, whichever party or candidate receives the most votes wins, regardless of whether a majority of the vote is received. To see the difference, consider the following vote shares, for a four-candidate election: candidate 1, 27%; candidate 2, 26%; candidate 3, 24%; candidate 4, 23%.

In a majority rule with runoff election, candidates 1 and 2 would have to stand for election again. The outcome is very much in doubt, since we have no idea how the 47% who cast ballots for candidates 3 or 4 compare candidates 1 and 2. Importantly, the candidate who wins will be the one who appeals to more of the 47% of the voters who did not vote for candidates 1 or 2 in the first round. Under plurality rule, the leftover 47% are irrelevant: Candidate 1 wins the election outright, and the other three candidates get nothing.

While majority rule with runoff and plurality rule are fairly widely used in the real world of politics, it is interesting to note that neither has quite the direct centralizing tendency so apparent in the two-alternative, majority rule world of the median voter result. Plurality rule, in particular, may lead to the nonexistence of equilibrium if candidate entry or movement is free and unrestricted. Plurality rule may also lead to equilibria that are "decidedly noncentrist" (Cox, 1987). Cox shows that some candidates (if there are several) will take positions outside the two central quartiles of the distribution of voter ideal points even in equilibrium. This point is elaborated, for a variety of voting systems, in Cox (1990).

Borda count and approval voting

Majority rule and its variations are based on the premise that each person gets one, and only one, vote. It is simple, easy to understand,

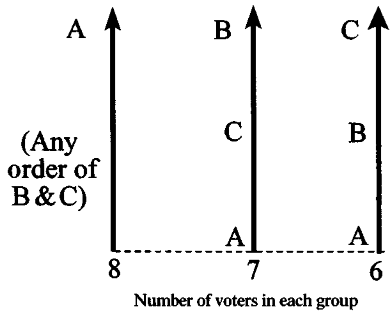

Figure 5.3. Borda's example of how majority rule picks the "wrong" alternative: A wins, but either B or C is better. (From Borda, 1781; reprinted in Black, 1958, p. 157, Fig. 156)

and technically defensible when the polity is seeking a choice of a single "best" outcome from the set of alternatives. The blunt and decisive quality of majority rule is a disadvantage, however, if the goal is to select a set of alternatives, or if the polity is trying to rank alternatives in a way that reflects voters' preferences from best to worst, rather than just choosing a single outcome or candidate.

In this section, we will consider some aspects of two types of voting over "lists" of alternatives (or candidates), with the presumption that the number of alternatives may be much larger than the two we have generally assumed so far. These two voting rules are the *Borda count,* and *approval voting.*

Borda count. Jean-Charles de Borda (1733–99) anticipated some of the observations of Condorcet, whose achievements we outlined earlier. His objection to majority rule was different from Condorcet's, however (see Black, 1958, for discussion and background). Condorcet's criterion for value in elections was that if a majority preferred an alternative, that alternative should be selected. Borda was concerned that majorities might pick the *wrong* alternative, even if Condorcet's criterion was satisfied.

Suppose there are twenty-one voters, who must choose over three alternatives A, B, and C. Suppose further that the preference rankings of voters fall into three categories, as in Figure 5.3. If we use a majority (actually, plurality) choice rule, only first-place votes count. We don't know the second- and third-place preferences of voters in the first group. All we know for sure is that they like alternative A best, and they are the most numerous (eight first-place votes for A). Since B receives seven votes and C gets only six, A is the chosen policy.

Borda's point was that thirteen voters, a clear majority, may actually like alternative A *least.* The problem with plurality rule, he felt, was that it counted only first-place votes. Borda suggested several possible alternative decision rules, but the one most often associated with him is the "Borda count": Let each voter assign to each alternative a number corresponding to his or her ranking of that alternative.

Thus, each voter in the middle group in Figure 5.3 gives alternative B a rank of 1, C a rank of 2, and A a rank of 3. (If there were more alternatives, say, q of them, then the ranks would go down to the worst, or qth, alternative on each voter's ballot.) The authority conducting the election then adds up the scores, or "marks," for each alternative, and that alternative with the *smallest* number (for Borda, the most "merit") wins.

Borda left the exact distribution of preferences for C and B in the first group unspecified (again, see Black, 1958). For the sake of example, let us suppose that four of the eight voters like B better than C, and the others prefer C over B. What would be the result of using the Borda method? Alternative A has eight first-place votes, zero second-place marks, and thirteen third-place marks, for a Borda count of forty-seven. Alternative B has seven first-place votes, ten second-place, and four third-place, for a count of thirty-nine. Alternative C has six first-place marks, eleven second-place, and four third-place, for a count of forty. Using the Borda count, then, B wins over C in a close race, with A well back in the voting.

We might change the preferences of the first group for B over C in the example, but the basic result would be the same: No matter what preference for B over C is used, A always comes in *last,* rather than first as under majority rule. When the preferences of voters beyond their first-place rankings are considered, A is eliminated from serious consideration in this example.

Approval voting. The Borda count has been criticized as requiring of citizens too much character and information. Borda requires that voters rank all alternatives. If some do not, the outcome will depend on the way abstention is counted. Further, though Borda noted that "my scheme is intended only for honest men,"[9] the Borda count provides opportunities for voting strategically, misrepresenting one's preference ordering to change the outcome.[10]

An alternative that preserves some of the qualities of the Borda count is *approval voting:* Each voter votes for as many of the candidates as he or she likes, and the candidate with the most votes wins.[11] Another way to think of approval voting is to imagine that each voter makes a list of candidates, ranked from best to worst. The voter then draws a line between the worst acceptable candidate and the best unacceptable candidate. Every candidate the voter approves gets a vote, but those below the line get nothing.

Returning to the example in Figure 5.3, suppose that voters in the first group think that only A is acceptable. Imagine that voters in the middle and last groups consider both B and C acceptable. What will be the result under approval voting? A will receive eight votes, just as under plurality rule. Candidates B and C will each receive thirteen votes, so that the specific outcome will depend on how we assume ties are broken. The point is that A will not have a chance as long as A is judged not acceptable by a majority of voters, which is the spirit of Borda's example.[12]

Both the Borda count and approval voting must violate one of Arrow's axioms, but which one? The answer is independence of irrelevant alternatives (IIA). The social choice under the Borda rule or approval voting may depend on the relative positions of two alternatives compared with other (irrelevant) alternatives. Violation of IIA is the basis of the manipulability of the Borda count. This may be more of a disadvantage in small-group settings than for mass elections, of course. In any case, IIA is almost always violated for any choice rule that requires scoring an entire list of alternatives rather than making a single best choice.

A simple example (adapted from Arrow, 1963, p. 27) illustrates another property of the Borda count: It is not *independent of path.*[13] This is actually a different problem from independence of irrelevant alternatives, since it involves the presence or absence of particular alternatives, not their relative locations in the preference rankings of voters. The interesting thing about Arrow's example is that it reveals an access point for strategy: For the Borda count and related social choice rules, outcomes can be sensitive to the set of alternatives that appear on the ballot, even in a single-stage decision.

Imagine there are three voters (A, B, and C) and four alternatives (*x*, *y*, *z*, and *w*). Further, suppose that all voters use the Borda count and vote sincerely. Consider two different ballots, one with four alternatives and another with only three.

Vote 1: A comparison of x, z, and w, with y also considered

	Voter			
Rank of alternative	A	B	C	Borda count
x	1	2	2	5
z	4	1	1	6
w	2	4	4	10
y	3	3	3	9

Clearly, *x* wins, because it has the smallest Borda count. Now, however, suppose that one alternative, *y*, is eliminated from the contest.

Vote 2: A comparison of x, z, and w, without y

	Voter			
Rank of alternative	A	B	C	Borda count
x	1	2	2	5
z	3	1	1	5
w	2	3	3	8

The outcome is now different: *x* and *z* tie, though nothing about their *relative* value has changed. All that is different is the decision context: For vote 1, *y* was on the ballot, and for vote 2, *y* did not appear. Furthermore, if there is now a run-off between *x* and *z, z* actually wins. Consequently, dropping *y* from consideration changes the outcome. This sensitivity to the inclusion or absence of other alternatives is different from the relative rankings of irrelevant alternatives, having more to do with how alternatives are retained or dropped in the early stages of elections or amendment procedures. That is the reason the "path" is important in this example.

Table 5.1. *Plurality rule and proportional representation compared*

Party	% Vote received	Pluralities won	Seats (plurality)	Seats (PR)
Green	45	55	55	45
Red	35	45	45	35
Blue	20	0	0	20
Totals	100	100 Elections	100 Seats	100 Seats

Proportional representation

A wide variety of collective decisions, particularly choices of political representatives in national assemblies for geographic districts, are made using "proportional representation" (PR) rules. There are many PR rules, but they share the characteristic that *each party's share of seats in the assembly is approximately that party's share of votes in the last election.* The ideal for a pure PR system, then, would be:

$$\frac{\textit{Party's seats}}{\textit{Total seats}} \approx \frac{\textit{Party's votes}}{\textit{Total votes}}$$

In practice, this ideal is violated in many ways and for very practical reasons. One of the most common modifications made to pure PR systems is the *threshold,* or minimum vote required for a party to seat members in the assembly. For example, Greece requires that a party receive at least 15% of the votes. Israel has an "exclusion threshold" of only 1.5% (Sartori, 1994). Such rules have two effects: (1) There is a departure from the ideal of pure proportionality, since a party can receive up to the vote threshold, minus one vote, and receive no seats in the legislature. (2) Consequently, people may vote strategically, eschewing sincere votes for small parties that have no chance and concentrating on one of the larger parties.

To contrast a PR system with a plurality rule system, consider Table 5.1.[14] In the table, we see a contrast between 1-seat allocations under plurality rule and a pure PR rule for three parties. The implications of plurality and PR rules, even given identical vote totals in either case, for the distribution of power in a 100-seat assembly are strikingly different.

Notice that the table does not give individual election results, but only nationwide totals. The number of pluralities won was chosen arbitrarily, but represents a possible outcome: The Green party comes in first in fifty-five races, the Reds come in first forty-five times, and Blues don't win any races outright. Under a plurality rule, these are exactly the proportions of seats the two parties hold: Greens 55%, Reds 45%. (The Blue party is presumably demonstrating in favor of electoral reform.)

Under a pure PR system, the Blue party would get fully 20% of the seats, corresponding to its 20% of votes in the election. There is no clear majority party, since the largest number of seats is held by the Greens, who have only 45% of the assembly under their control. The Greens will be obliged to form a coalition if they want to govern, or face the possibility that a Blue–Red coalition government (the Purple) will form and control 55% of the votes, a working majority.

The process of coalition government formation is complex and quite beyond the scope of this book; the interested reader should consult Laver and Schofield (1990). Our point is simply that once a PR election has been held, it is by no means clear that a government has been selected; in fact, the process may have barely begun. Any two of the three parties in our example are capable of joining and forming a government. On the other hand, no one party can govern. Consequently, the nature of the government is very much in doubt. Cycling over coalition partners simply moves the incoherence of the democratic process from voter choice to bargaining among elites.

A summary of the difference between plurality and PR rules is given by Dennis Mueller:

> If the purpose of the election is to select a government, a chief executive, a single party to rule the country, then the plurality rule for electing representatives, or for electing the chief executive, should be employed. This rule will tend to produce two parties or candidates. . . . If the purpose of the election is to select a body of representatives that mirrors as closely as possible the preferences of the citizenry, then PR is the appropriate electoral rule. . . . PR is a system for choosing *representatives,* not for picking between final packages of outcomes. (1989, pp. 222, 226; emphasis in original)

In some ways, the consideration of whether plurality/majority rule (sometimes called "first past the post" elections) or PR is better mimics our earlier discussion on the optimal size of majorities. If a single decision among several mutually exclusive alternatives is required, then plurality rule, or majority rule with runoff, has clear advantages. The polity may simply need to decide something *right now,* so the costs of delay from deliberation and negotiation in a PR system may not be worth paying.

On the other hand, questions of far-reaching consequence for all citizens may require a representation of many points of view. In such decisions, representation may even be an end in itself, since not just the decision but the legitimacy of the decision is crucial to the survival of the society. For such decisions, voters may want to be sure they have a representative of their own choosing. PR systems are appealing in such circumstances because the deliberative process of the legislative assembly mimics, in proportions of perspectives, the population as a whole.

Summary and conclusions

In this chapter, we have outlined some intuition behind alternative voting institutions. The use of the word "alternative" means "ways of choosing other than simple majority rule," for majority rule holds a privileged status as a benchmark in social choice. Gary Cox (1987), in discussing alternative voting institutions, notes:

> Another way to organize the findings herein is along a "degree of centrism" axis. Holding down one end of this axis would be the Condorcet procedures, under which candidates have a dominant strategy to adopt the position of the median voter. Other procedures (such as Borda's method and approval voting) under which the unique convergent Nash equilibrium is the situation in which all candidates adopt the median position would come next, followed by systems (such as negative voting) under which there are multiple convergent equilibria. Finally, holding down the other end of the centrism axis would be procedures such as plurality rule, under which candidates will not converge at any point, instead spreading themselves out more or less evenly along the policy dimension. (p. 99)

Dictatorship, for most of us, is unacceptable as a way of organizing government. In the weighing of order versus liberty on the scales of choosing a "good" society, dictatorship provides only order. But liberty without order may lead to chaos. The signal contribution of Arrow and the social choice theorists that have followed him has been to demonstrate that there is no perfect alternative to dictatorship. Ultimately, dictatorship may always be with us, because order without liberty may be better than liberty without order. The design of government institutions that can preserve order and still protect liberty is perhaps the most difficult, yet most rewarding of all human enterprises.

EXERCISES

5.1. Suppose that there are seventeen members of a committee, with ideal points on a policy line that correspond to their identities (Mr. 1 most prefers a policy of 1, Mr. 11 likes a policy of 11, and so on). Suppose that the current policy is 7. According to the KMVT, what is the *smallest* K that preserves the status quo (7) as an equilibrium?

5.2. Suppose five voters (Messrs 1–5) have the following preference rankings for three candidates (A, B, and C).

	Voter				
	1	2	3	4	5
Best	A	B	C	B	A
Middle	C	C	A	C	C
Worst	B	A	B	A	B

Which candidate will win under the following decision rule:

a. *Majority rule,* each voter gets one vote for first preference, *with runoff* in the event of no majority on first ballot.

b. *Borda count,* each voter ranks candidates from 1 (best) to 3 (worst). Smallest total "score" wins.

5.3.* Give a formal proof of the KMVT.

Note: Exercises marked * are advanced material.