15

Extending the Seat Product Model: Upper Tiers and Ethnic Diversity

In this final part of the book, we ask what we can expect from models of electoral systems. The topics include whether institutions alone are sufficient to predict the average trend in party systems, or whether many existing works on the topic are correct in their claims that electoral-system effects are operational only under certain conditions of social cleavages that produce demands for party fragmentation. Another key topic is the applicability of our models to complex electoral systems. This chapter takes up both of these topics, and the next takes the latter topic a step further by considering a wide range of features that introduce complexity, including second rounds, ranked-choice ballots, and thresholds. Then the concluding chapter wraps up with wider themes of electoral systems, and models of them, in the context of countries' politics, and the role of logical models in social science.

Our first task in this chapter is to extend the Seat Product Model (SPM). In Chapters 7–10, we developed and tested the SPM for simple electoral systems. This chapter introduces two main extensions: (1) the consideration of ethnic diversity; (2) the role of upper tiers of composite systems. We find that the added predictive power of including ethnicity is minimal (with few exceptions), and that our extended model can offer quantitative understanding of one major class of complex electoral system, two-tier compensatory PR (including mixed-member proportional; see Chapter 3 for definitions).

Both of these extensions are good news for both the science of electoral systems and practical, real-world, system design. Two-tier compensatory systems are common, and thus covering them under the SPM is an important advance. Moreover, while practitioners can, in principle, manipulate a country's electoral system when the current system is inadequate, a country's ethnic diversity is relatively fixed and is not subject to re-engineering, at least not absent redrawing national boundaries or engaging in unacceptable antidemocratic practices.

DEVELOPING AND TESTING AN EXTENDED SEAT PRODUCT MODEL

In the present section we introduce and test our extended version of the SPM. By introducing ethnic fragmentation and the upper tiers of compensatory PR systems, we can further compare the SPM with conventional approaches. Most such approaches include these variables (as well as variables specific to presidentialism – see Chapters 11 and 12). As was the case in Chapter 7, here our first modeling is concerned with predicting the effective number of seat-winning parties (N_S). In a subsequent section of the present chapter, we will extend the logic of Chapter 8 to test the ability of our extended SPM to predict the effective number of vote-earning parties (N_V).

Following common practice of standard regression analysis of party systems, such as Clark and Golder (2006), we will test whether our simple model for nationwide party systems can be improved if we account for social diversity. As with prior works, we use the *effective number of ethnic groups* (N_E).[1] This will allow us to determine whether the standard approaches are correct in claiming that features of the electoral system that generate permissiveness – that is, higher magnitude and upper tiers, as well as assembly size – are associated with a high effective number of parties only in the presence of social demand for many parties.

The other new factor added to the model in this chapter is the upper tier of composite systems. The largest set of such systems has an upper tier that is *compensatory* (Chapter 3). Taagepera (2007) devotes an entire chapter to complex systems, but ultimately concludes that it is impossible to understand how upper tiers affect seat distributions. This is an unfortunate limitation, standing in contrast to the conventional approaches. Since Cox (1997) and including Clark and Golder (2006) and Hicken and Stoll (2011), many scholars have estimated regressions in which there is a linear additive term for the percentage of seats in any upper tiers, typically along with an interaction for social cleavages.

In order to apply the SPM to two-tier compensatory systems, we first model how the basic tier (i.e., the component of the system excluding the upper tier) shapes party-system fragmentation. Logically, this should be through the product of the mean magnitude and the total size of the basic tier. We call this product the *basic-tier seat product*, which we will designate as MS_B, with the subscript B reminding us that it is the seat product of the basic tier alone. In this way, a simple (one-tier) system is one where the basic tier comprises the entire seat product ($MS=MS_B$).

How does the effect of an upper tier enter? If our hunch is right that the basic-tier seat product has the same relationship to the effective number of parties in

[1] Derived from Fearon (2003), which is the same measure used by Clark and Golder (2006) and by Li and Shugart (2016) in their attempt to replicate Clark and Golder's results on parliamentary systems.

systems with or without an upper tier, then the equation for a multitier system requires some adjustment to the basic-tier seat product that varies with the size of upper tiers. A compensatory upper tier could only increase the effective number of parties, relative to the N_S of simple systems with the same MS_B (i.e., $(MS_B)^{1/6}$). Thus the adjustment factor must be at least one. This gives us the following formula,

$$N_S = J^t(MS_B)^{1/6} \tag{15.1}$$

where J is the base of the upper-tier adjustment factor,[2] which is raised to the power t, the upper-tier share; MS_B is the product of the average magnitude (M) of the basic tier and the total number of seats in that tier (S_B). Note that t and S_B are not two separate inputs; they are connected as S_B=S(1-t). However, it is more convenient to enter them separately. The base, J, must be greater than 1.0, but its precise value is determined empirically. In the nationwide dataset we have, we can determine that we must have J=2.5.[3] The resulting formula is:

$$N_S = 2.5^t(MS_B)^{1/6} = 2.5^t[MS(1-t)]^{1/6} \tag{15.2}$$

This model holds for $t\leq 0.5$, the usual range for upper-tier seat shares.[4] We emphasize that the logic behind Equations 15.1 and 15.2 applies only to complex systems that are compensatory – the "simplest" form of "complex" system. In Chapter 16, we will consider whether it also works for systems that have upper tiers that are not designed as compensatory.

Testing the Extended Model

We are ready now to perform regression tests of our extended model:

$$\log N_S = \alpha + \beta_1 \log(MS_B) + \beta_2 t + \beta_3 \log(N_E) + \beta_4[\log(MS_B) * \log(N_E)].$$

[2] We use J simply because it is the first consonant sound one hears in the word, *adjustment*.

[3] This value of the parameter is based on the sample of multitier compensatory systems for which Bormann and Golder (2011) report a known number of seats allocated in one or more upper tiers. For these cases, we ask, what would be the expected N_S (from the SPM) if we ignored the upper tier? We find that, for these systems, actual N_S is, on average, 1.27 times as large as it would be if we calculated the Seat Product from the basic tier alone. The mean upper tier in these systems represents 25 percent of the total assembly size. In Equation 15.1, therefore, we need a value of J such that J^t=1.27 when t=0.25. This can be calculated via the following steps:

$$t\log J = log 1.27;$$
$$\log J = 0.10/0.25 = 0.40.$$

Therefore, we should expect a coefficient on t that is approximately 0.4, which is indeed what we find in the regressions (reported in Table 15.1). The inverse log of 0.4 is 2.51, hence the value of J in Equation 15.2. We test the underlying logic behind this complex set of steps later in this chapter.

[4] Purely empirically, Equation 15.2 could be well approximated by N_S=(1+0.8t)$(MS)^{1/6}$.

TABLE 15.1 *The extended Seat Product Model, including upper tiers, ethnic fragmentation, and systems with presidential executives*

	1	2	3
	All execs., established	**All execs., any age since 1990**	**Full pooled sample**
Seat Product (MS_B), logged	**0.171**	**0.173**	**0.166**
*Expected coeff.: **0.167***	(0.0251)	(0.0328)	(0.0237)
F test that coefficient on MS_B =1/6	0.866	0.852	0.981
Upper-tier ratio (t)	0.376	0.411	0.40006
*Expected coeff: **0.40***	(0.103)	(0.120)	(0.103)
F test that coefficient on t=0.4	0.825	0.926	0.9995
Eff. No. Ethnic Groups (N_E), logged	–0.0445	–0.0706	–0.189
	(0.269)	(0.318)	(0.275)
MS_B X N_E	0.0671	0.192	0.148
	(0.116)	(0.118)	(0.112)
Constant	–0.0661	–0.114	–0.0664
	(0.0749)	(0.108)	(0.0728)
Observations	376	197	432
R-squared	0.462	0.520	0.473

Of course, we expect β_1=1/6 (Chapter 7), while β_2=log*J*. From Equation 15.2, we should expect β_2=0.4, the decimal logarithm of 2.5. We have no specific expectations for β_3, but if the standard works in the field are correct, both should be positive and significant. Given the industry-standard expectation that the effect of a permissive system is felt only in the presence of demand via social heterogeneity, we will also seek to determine via analysis of β_4 whether there is a multiplicative effect of institutions and ethnicity.

In Table 15.1, we present three regressions. In Regression One, we consider only established democracies, defined as those that had their first democratic election before 1989. Regression Two includes elections since 1990, regardless of date of first election. Finally, Regression Three is our fully pooled model, including all postwar democratic elections in simple or two-tier compensatory systems for which the ethnic-diversity data were available.

Having models with and without newer democracies is important to test the breadth of applicability of the Seat Product Model. Clark and Golder (2006) report substantively different results for samples with and without the inclusion of post-1989 democracies. Thus the inclusion of newer democracies is an important check on the idea that the "context" of new democracies might temper or override electoral-system effects (Moser and Scheiner 2011; Ferree et al. 2013).

We see that in all of three regressions the coefficient on the basic-tier Seat Product (MS_B) is always near the expected one-sixth. The F tests reveal that in no case can we reject the null hypothesis that coefficient is equal to 1/6. Further, the coefficient on t (upper-tier ratio) is also near 0.4, as expected.[5] In addition, the constant terms are always statistically indistinguishable from zero, also as expected.

The coefficients reported in Table 15.1 confirm our expectation that the impact of the seat product is consistent across systems with and without compensatory upper tiers. We do not see any notable difference when we include or exclude newer democracies. Moreover, consistent with what Li and Shugart (2016) discovered in attempting to replicate Clark and Golder's (2006) findings, our results are not sensitive to whether or not we include India.[6] Unlike the conventional regression coefficients in the literature, those testing the Seat Product Model, as extended in this chapter, are remarkably stable.

As for the effect of ethnic diversity, β_3, the coefficient on N_E, is not significant in any of the three regressions, and the coefficient on $\log N_E$ is actually negative. Moreover, β_4, the coefficient on the interaction term, is also insignificant, although it is positive in each model – especially in the samples that include the newer democracies (Regressions Two and Three), precisely the set where context aside from rules might have the greatest impact. For these reasons, and given the difficulty of analyzing interactions from just coefficients and standard errors (Brambor, Clark, and Golder 2006), we examined the marginal effects. We determined that the interaction is indeed significant – albeit barely – when $200 > MS_B > 15{,}850$.[7] Given that this range encompasses over 85 percent of all the elections included in Regression Three the finding appears to support the conventional expectation of permissive electoral systems having an effect on increasing N_S only under conditions of high heterogeneity, and perhaps especially so when we include younger democracies. We now explore this notion further.

In Figure 15.1 we compare the estimates of Regression Three against the institutions-only SPM. In the figure, we compare how different the predicted values are when we include ethnic fragmentation versus leaving the latter out. We see, plotted with the x symbol, the predicted value from the SPM

[5] Likewise, F tests show that we cannot reject the null hypotheses that these coefficients are 0.400.

[6] For reasons of space, we do not report models without India; in each sample reported in Table 15.1 any difference was trivial. We discuss the Indian case in some detail later in this chapter.

[7] A plot of the marginal effect is available at our online appendix www.cambridge.org/votes_from_seats. A further question is whether there is also an interaction of N_E with the upper-tier share, t. Other authors find such an interaction to be significant. Regressions of the extended SPM testing this show an extremely weak effect of this interaction. Thus the (small) impact of N_E on N_S apparently works through the basic-tier Seat Product more than through the compensation mechanism.

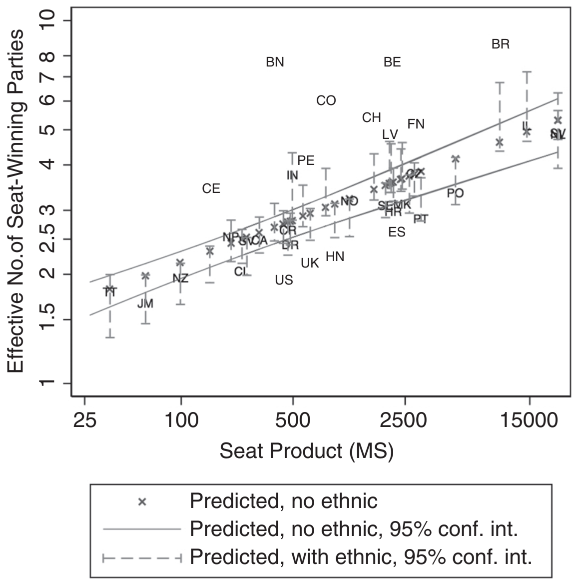

FIGURE 15.1 Comparing predictions from the institutions-only model to the predictions when considering ethnic fragmentation

(Equation 7.1), using only institutional variables.[8] The 95 percent confidence intervals, calculated from Regression Three in Table 7.2 (where we did not include ethnicity), are indicated with gray lines. Then, using capped bar plots, we see the 95 percent confidence interval on the predictions of Regression Three from Table 15.1, incorporating ethnicity. The country abbreviation is located at the actual value for that country, averaged over the elections in the sample (for visual clarity, although the regression estimates are derived from individual elections).

What we see is that inclusion of ethnicity does not substantially improve the fit of predicted to actual values in most cases. Only a few cases have actual values that are outside the 95 percent confidence interval of the institutions-only model while also being within the comparable interval for the model that includes ethnicity. One of these cases is India, a long-term democracy with very high N_E, which we discuss further below. In a few other cases, inclusion of the ethnic factor improves the prediction only because the country's low ethnic fragmentation helps account for a value of N_S that is even lower than its institutions would predict. This more accurately predicted lower N_S when accounting for low ethnic diversity is observed in one highly restrictive

[8] The figure shows only simple systems, due to the difficulty of graphing both MS_B and *Upper-tier ratio* on one axis. We return to analysis of two-tier systems later in this chapter.

electoral system, Jamaica, but also in some permissive systems, such as Poland and Portugal. Thus, from Figure 15.1, we can conclude that the Seat Product Model, based only on institutions, predicts the effective number of seat-winning parties at least as accurately for the large majority of democracies as does the combination of institutions and a widely used indicator of social diversity.

Extended Seat Product Model for Votes

Having dealt with seat-winning parties, we are now ready to test the connection between the Seat Product, MS, and the effective number of vote-earning parties for the extended sample. As in the preceding section, the analysis includes two-tier compensatory systems and a country's ethnic diversity. We already saw in Chapter 8, specifically in Figure 8.4, that the relationship between seat and vote fragmentation is not fundamentally different in two-tier complex systems. Thus it should be possible to derive votes from seats for the wider sample that includes the complex systems in the same manner as we did for simple (in Chapter 8). Combining our extended SPM (Equation 15.2) and our model for deriving N_V from N_S (Equation 8.3), we can test:

$$N_V = \{[2.5^t(MS_B)^{1/6}]^{3/2} + 1\}^{2/3} = [4^t(MS_B)^{1/4} + 1]^{2/3} \tag{15.3}$$

(The "4" is rounded from $2.5^{3/2}$=3.95.) In Table 15.2, we report three regressions to test Equation 15.3 by pooling simple and two-tier systems. The first two are "institutions only," while the third brings in the ethnic factor. Because ethnic fragmentation data are missing on several countries, we show the institutions model both on the wider sample and on a sample restricted to those for which the ethnic data are available.

The dependent variable in each is the logged effective number of vote-earning parties (logN_V) and the key independent variable in each is the *log of the number of pertinent vote-earning parties* (logN_{V0}). Because this latter quantity is not directly measurable, as explained in Chapter 8, we estimate it from the Seat Product. That is, what is entered into the regression is the part of Equation 15.3 that is in the square brackets. We expect the coefficient on this input variable to be two-thirds (0.667). We see in Table 15.2 that the actual estimate ranges from 0.625 to 0.712 (thus –0.042 to +0.045, relative to the expected 0.667), depending on the sample. In all cases it is not statistically distinguishable from the expectation, as reported by the F test statistic in the table. Thus we can consider Equation 15.3 to be strongly supported on our pooled sample that includes two-tier proportional systems and presidential democracies.

Regression Three introduces the "industry standard" interactive effect of institutions and the *log of the effective number of ethnic groups* (logN_E). We have no specific expectation for the coefficients on either logN_E or its

TABLE 15.2 *Three regressions for the effective number of vote-earning parties (N_V), including two-tier systems and the effective number of ethnic groups (N_E)*

	(1) Institutions only; incl. 2-tier and presidential	(2) Institutions only; all systems with non-missing N_E	(3) With N_E (interactive; all systems)
$\log N_{V0}=\log[4^t(MS_B)^{1/4}+1]$	**0.625**	**0.675**	**0.712**
expected: ***0.667***	(0.0614)	(0.102)	(0.1071)
F test that coeff = 2/3	0.500	0.935	0.673
$\log N_E$			–0.110
			(0.352)
$\log[(MS)^{1/4}+1] \text{ X } \log N_E$			0.566
			(0.475)
Constant	0.0291	–0.0158	–0.116
	(0.0508)	(0.0941)	(0.0965)
Observations	553	433	433
R-squared	0.395	0.317	0.403

interaction with $\log N_{V0}$, but if the standard works in the field are correct, both should be positive and significant. However, we find that neither is close to significant, and the coefficient on $\log N_E$ is actually negative. Nonetheless, when we examine the marginal effects, we find that the interaction of institutionally derived $\log N_{V0}$ and $\log N_E$ is significant as long as $\log N_{V0}$ is greater than about 0.60, meaning N_{V0} greater than 4.0.[9] This corresponds to a significant effect as long as MS>80, for a simple system. Thus we find some support for the "industry-standard" interactive effect, which we should explore further.

What the regression output cannot directly tell us is how much of an improvement the inclusion of ethnic diversity offers over our institutions-only model. For all elections covered by the sample of Regression Three (Table 15.2), the mean ratio of actual to N_V predicted by our logical model is 1.042 (median 0.950, standard deviation 0.412). For the same sample, the mean ratio of actual N_V to the estimated derived from Regression Three is 1.053 (median 0.983, standard deviation 0.363). The improvement from including N_E is thus slight.

Of particular interest is whether the inclusion of ethnic diversity helps us understand individual countries that are relatively extreme on N_E. To probe the fit of our model (Equation 15.3) relative to the regression that includes the

[9] A plot of the marginal effect is available at our online appendix www.cambridge.org/votes_from_seats.

TABLE 15.3 *Comparing the logical model and the regression that includes ethnic effects*

Country	**No. of elections**	**Mean N_V**	**N_E**	**Mean ratio of actual N_V to Equation 15.3**	**Mean ratio of actual N_V to Regression Three (Table 15.2)**
		Lowest 25% of N_E			
Portugal	13	3.42	1.04	0.845	0.906
Poland	4	4.46	1.05	**0.991**	1.143
Netherlands	20	5.17	1.08	0.941	0.943
Germany	17	3.78	1.1	0.792	0.886
Norway	16	4.28	1.11	**1.054**	1.133
Austria	19	2.93	1.14	0.645	0.717
Denmark	25	4.85	1.15	**1.098**	1.22
		Highest 25% of N_E			
Israel	18	5.68	2.11	**1.087**	0.895
Macedonia	5	4.38	2.15	1.145	**1.022**
Brazil	6	9.62	2.22	1.954	1.564
Belgium	3	9.52	2.31	2.437	2.087
Switzerland	17	5.82	2.35	1.549	1.336
Latvia	4	6.67	2.41	1.5	1.444
Canada	21	3.27	2.48	1.101	1.026
Peru	5	5.36	2.76	1.586	1.351
Trinidad & Tobago	12	2.29	2.83	1.0004	1.007
Nepal	2	3.91	3.1	1.376	1.226
India	10	5.27	5.29	1.629	**1.146**

The ratio indicating the better-performing model is in bold if one model is within the range, 0.80–1.25, but the other is not, or if one is very close to 1.000 while the other is not.

ethnic effect (Regression Three from Table 15.2), Table 15.3 shows these ratios. The ratios start with the observed values of N_V for a given election and divide it by the predicted value from either Equation 15.3 or Regression Three (from Table 15.2). We then take a mean value of each ratio for each country and report these means in the table.

The cases included in Table 15.3 are those countries that have unusually low or high effective number of ethnic groups, relative to our full sample. A country is included in Table 15.3 if its value of N_E is in the lowest 25 percent of countries or the highest 25 percent, the ranges in which the effect of ethnicity is most likely to be felt. We see from the ratios for the set of relatively homogeneous or heterogeneous countries that the inclusion of N_E improves the prediction substantially only in a few cases. Among the low-N_E cases, the predictions of the two approaches tend not to differ greatly. However, the institutions-only model does better for some cases than does the incorporation of these countries' (low) ethnic diversity. Denmark, Norway, and Poland stand out as cases substantially better explained by our model (Equation 15.3).

In the high-diversity cases, again few are predicted better by inclusion of N_E. A case where the model including ethnicity performs a good deal better is India. This is striking, given how highly fragmented the system is, in spite of FPTP. Canada's N_V is predicted almost perfectly by the inclusion of N_E; however, the institutions-only model does only slightly worse. Several of our highest-diversity cases do indeed have unexpectedly high N_V, but are not predicted much better by inclusion of N_E than without it; examples include Benin, Brazil, Nepal, Peru, and Switzerland.

The case of Israel, our example of nationwide PR in Chapter 6, is predicted markedly better by institutions only, despite its very high N_V. We do not need to invoke the ethnic dimension, at least as measured by the proxy available to us for the wider sample, in order to understand how fragmented the vote is in Israel, on average. The very high Seat Product is sufficient to understand the country's high mean value of N_V.[10]

One case of high N_E is actually accounted for equally well by the institutions-only model (Equation 15.3) and by the model that includes ethnicity: Trinidad and Tobago, our example for FPTP in Chapter 5. Its ratios of almost exactly 1.0 from either model are consistent with the idea, common on the study of party systems, that even high diversity is not reflected if the electoral system is highly restrictive. Note, however, that *most of the mainstream literature would consider India's and Trinidad's electoral systems as equally restrictive*, as both have M=1. However, the Seat Product identifies India's larger assembly as making its system considerably more permissive; combined with its exceptional degree of ethnic diversity, we are able to account for its party-system fragmentation. We discuss the case of India in more detail below.

What we have shown here is that ethnic diversity of a country is a much less important factor in predicting its electoral fragmentation, as measured by either N_S or N_V, than is widely believed. While there is a statistically significant marginal effect, there are not many countries whose unusual degrees of ethnic diversity can be invoked as an explanation for why their mean values of N_V diverge from our Seat Product Model prediction. Contrary to much scholarship, then, *there simply is not much evidence that the effect of electoral systems is conditional on social factors* – at least not when these factors are measured by the effective number of ethnic groups, as has been the preference of many scholars. This does not mean that there is no effect of social diversity on party

[10] For a more nuanced account of the impact on its party system of changing social divisions over time in Israel, see Stoll (2013).

systems (Moser et al. 2018). It means only that this measure, N_E, widely used in the scholarly literature for exactly this purpose, is not able to detect such an effect in most democracies and – more importantly – that most countries are quite close to the prediction of our logical models, which are based solely on institutional factors.

The Seat Product Model and the Effective Number of Alliances: India

The Indian case is fundamental to checking on the accuracy of theories of the impact of electoral systems. India is the world's largest democracy. Perhaps, if a model of key democratic processes leaves India as an outlier, the model might need to be reconsidered. The case has long vexed scholars working on such topics. For instance, Riker (1982) resorted to a highly ad hoc approach to explaining why India was an "exception" to "Duverger's law."[11] More recent work also has had trouble with India, though the authors may not have been aware.

In Chapter 7, we reviewed the attempt by Li and Shugart (2016) to replicate the Clark and Golder (2006) regressions on the parliamentary subsample. Li and Shugart discovered that the Clark-Golder approach was not robust to the exclusion of the Indian case. Such lack of robustness is important because their approach assumes that ethnic diversity is an essential input variable, and that its effect is interactive with district magnitude. This is particularly a problem for proponents of the conventional method, given that India has by far the highest ethnic fragmentation of any of the countries that they (or we) have included. Fortunately, the coefficient of the Seat Product Model itself is robust to including or dropping the Indian case (as reported in Chapter 7). However, we should explore this case further, because it offers important lessons for Duverger's tendencies and the question of how parties adapt to the constraints of the first-past-the-post (FPTP) electoral system.

Many treatments of the so-called Duverger's law take the Indian case as an exception that must be explained. At one time the puzzle was the existence of a single dominant party, rather than the expected "Duvergerian" two-party system (Riker 1982). More recently, the puzzle has been the existence of an extremely fragmented party system, despite the continued use of FPTP.

Figure 15.2 shows Indian N_S over time, along with the mean prediction from the Seat Product Model. Focusing on the dashed trend line, we see that until the late 1980s, N_S was consistently below the Seat Product prediction. Then in the

[11] Riker argued that the then-dominant Congress Party was the Condorcet winner (i.e., would beat any single contender in a pairwise competition), but offered no evidence for the assertion, which is particularly debatable given that on the occasions when it has faced a grand alliance of most of the opposition, it sometimes has been defeated – including in 1979, before Riker's article was published, and again in some recent elections discussed in this section. See Ziegfeld (2018) for details.

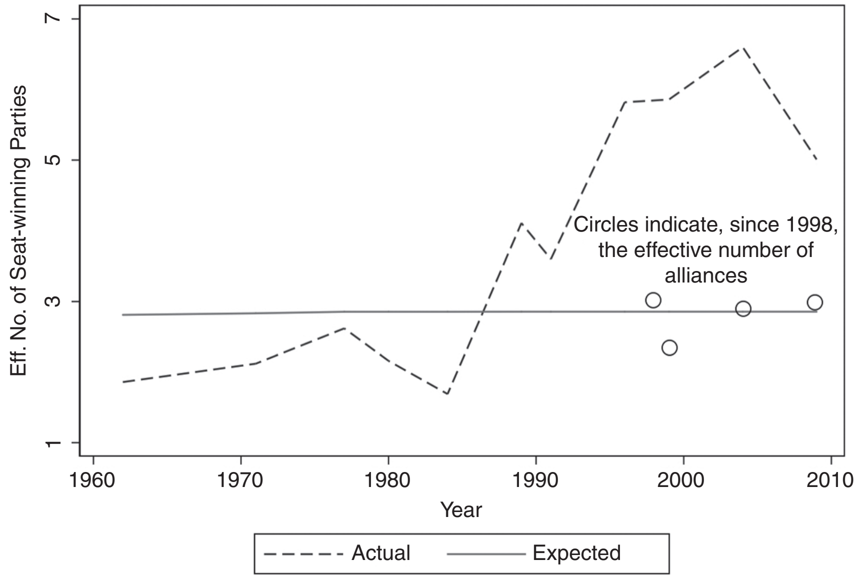

FIGURE 15.2 Effective number of seat-winning parties and (since 1998) alliances in India over time

more recent period, N_S has surged, remaining above 5.0 from 1996 through 2009. While India's assembly size has increased over time, the line for expected N_S remains flat visually because the increases have been small.

The period of Indian politics since 1998 has been marked by two major and some smaller *alliances*, each of which presents a single candidate in any given district (see the discussion near the end of Chapter 5). When we consider the individual parties that comprise these alliances to be distinct components for the calculation of the effective number, we obtain the strikingly high values tracked by the dashed line in Figure 15.2. However, one could be justified in starting from a different premise when calculating N_S. Indian voters are faced with a single candidate from each of the various alliances, because each presents a single candidate from just one of the alliance's component parties in each district. Thus we might want to know what the effective number would be if calculated it based on alliances.

The circles in Figure 15.2 represent the values of *effective number of seat-winning alliances* since 1998, the election at which the National Democratic Alliance first appeared. It is striking how closely the distribution of seats among these alliances approximates the predictions of the Seat Product Model. After all, the model is "blind" to the politics behind the effective number of components in the national legislature – whether those components are called representatives of distinct subcomponents (parties) or national alliances.

Looked at in this light, *India actually appears somewhat less exceptional than it is generally taken to be.*

As for the votes, we saw above in Table 15.3 that the effective number of vote-earning parties – here meaning the component parties, many of which are ethno-specific – is rather well accounted for by including the effect of N_E in addition to *MS* (Regression Three in Table 15.2). In other words, recent Indian elections have featured a very large number of parties, many of them representing specific ethnic groups, and a regression that takes into account the country's high ethnic diversity accurately accounts for N_V. These parties operate within large pre-electoral alliances, and the effective number of these alliances winning seats is well accounted for by the Seat Product Model (if we replace "parties" with "alliances"). Which is the better indicator – the alliances or the parties comprising them? We can only answer with, "better at what?" Both measures capture something important about the country's politics. Nonetheless, the management of business in parliament and the formation (or dissolution) of governing cabinets is done through the alliances (Heath et al. 2005: 152–154). It is thus striking that the SPM is quite accurate at predicting the effect of alliances, as if they were the parties in Indian parliamentary politics.

India is not the only case where this issue of whether to count alliances instead of their component parties arises. As we first noted in Chapter 6, electoral lists in proportional systems also sometimes consist of more than one party. If lists are open, voters are able to cast their vote for a candidate of one of the component parties rather than for the alliance as a whole. Should analysts count the parties or the composite alliances? When, as in the Indian FPTP case, alliances are consistent across districts and present only a candidate from one partner, it is possible to recalculate the effective number of alliances, rather than the component parties (as we saw in Figure 15.2). It is also possible to do the same in Chile, where a multiparty system is tempered by the existence of two major nationwide alliances. However, in some proportional-representation countries the set of parties that may be in alliance with one another differs from district to district, even in the same election. Chapter 14 offered a detailed discussion of pre-election alliances in PR systems.

DEVIATION FROM PROPORTIONALITY: INCORPORATING TWO-TIER SYSTEMS

In Chapter 9, we developed a formula that related deviation from proportionality to the Seat Product for simple systems. It is axiomatic that two-tier compensatory systems are "more proportional" than otherwise similar simple systems. That is precisely what a compensatory upper tier does, after all: reduce disproportionality! Let us see if we can specify how much.

We will extend Equation 9.3 for D_2 by including a parameter in a regression test for the upper tier, such that our extended equation will be of the form:

$$D_2 = 0.50j^t(MS_B)^{-1/3},$$

where j is the base of the adjustment factor for the impact of the compensatory tier. As with its counterpart in Equation 15.1, we must estimate it empirically. It must be the case that $0<j<1$, because a compensatory tier can only deflate D_2 relative to what results from the basic-tier. When we run a regression to derive j, we find its value to be approximately 0.06, and thus the formula becomes:

$$D_2 = 0.50(0.06^t)(MS_B)^{-1/3} \qquad (15.4)$$

Recall that t and S_B are connected as $S_B=S(1-t)$.

The output of this regression is shown and explained further in the chapter appendix. In Figure 15.3, we plot the deviation from proportionality, D_2, for all of the two-tier compensatory PR systems. The main trend line plotted in black is Equation 15.4 for the mean observed value of t in our nationwide data sample, which is around 0.25. When an upper tier consists of a quarter of all assembly seats, Equation 15.4 becomes (with rounding):

$$D_2 = 0.25/(MS_B)^{1/3} \qquad (15.5)$$

We must emphasize that this is not a logical model! It contains multiple empirical steps. Only the 1/3 has a logical basis (see Chapter 9). Nonetheless, it suggests an elegant, albeit tentative, conclusion: it says that the "average" two-tier compensatory system reduces deviation from proportionality to half the predicted value for a system that has an identical basic tier but no compensation. The gray line near the top of Figure 15.3 represents values twice the predictions of Equation 15.4, which happens to be identical to Equation 9.3. No election is observed above this line, and thus the average prediction for simple systems may be an upper limit for two-tier systems.

A lighter gray line in Figure 15.3 shows values half the prediction of Equation 15.4. We see several elections are considerably below this line. From a design standpoint, this is not troubling. Electoral system designers would choose a two-tier PR system when they want to minimize deviation from proportionality; getting less disproportionality than they bargained for should not, therefore, be a problem. The useful finding here for practitioners is that the *upper* extent of disproportionality rarely is very much greater than we estimate its average to be in simple systems with the same (basic-tier) number of seats and mean district magnitude.

A further implication of this section is that the precise size of the upper tier may not matter greatly for proportionality, as long as it is not much below about 20 percent. The very large upper tiers of Germany, South Africa, and some other two-tier systems, are overkill. We had already suggested in Taagepera and Shugart (1989a: 131) that a compensatory upper tier need be

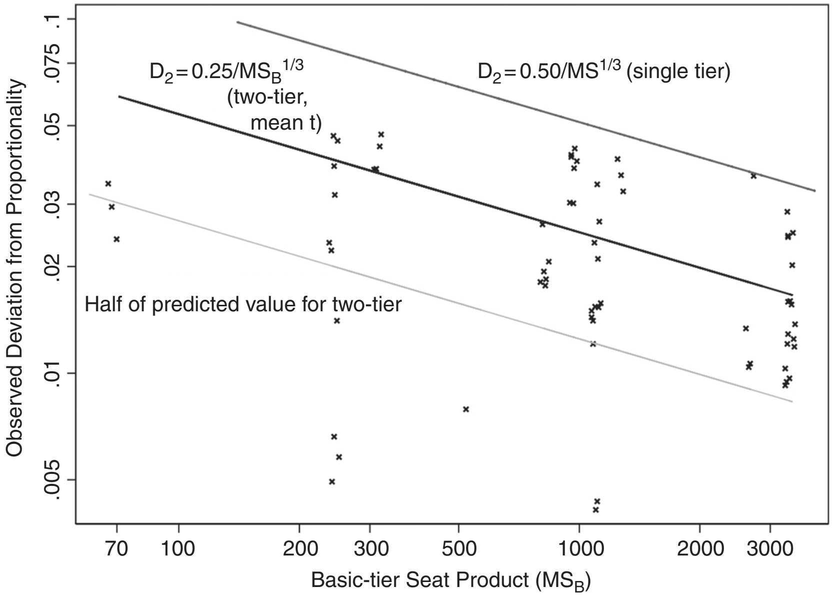

FIGURE 15.3 Deviation from proportionality (D_2) in two-tier compensatory PR systems

no bigger than the percentage Deviation from PR expected to result from the basic tier. We can test more directly the impact of the relative size of the compensation tier by turning to our district-level dataset, and disaggregating nationwide party-system outcomes into their basic-tier and upper-tier components. This is the task of the next section.

THE BASIC TIER IN COMPENSATORY SYSTEMS

In preceding sections of this chapter, we tested some models of party system outcomes in pooled samples that included simple and two-tier systems. Their results suggest the conclusion that the basic tier of the two-tier system works much like a simple system. Then the outcome (e.g., N_S or D_2) is adjusted by the application of upper-tier compensation. Up to now, however, the conclusion contains a bit of a leap of faith: we have not looked at the actual outcomes in the basic tier. We have only tested regressions in which the outcome variable was a nationwide indicator.

In this section, we return to our district-level dataset and perform tests on the basic-tier of complex systems directly. This procedure allows us to probe the process underlying the functioning of two-tier systems. We do so first by looking at the individual districts themselves. It might be expected that districts in two-tier systems would be fundamentally different from their counterparts in simple systems. After all, by definition, these districts are not the only way that parties can win seats. They can also win via the upper tier. After the district-level analysis we aggregate the districts within the basic tier, to see the extent to which upper tiers adjust the basic-tier outcome in a systematic way, as predicted by our extended Seat Product Model (Equation 15.2).

The Effective Number of Seat-Winning Parties in Basic-Tier Districts

It might seem as if the models of district-level party fragmentation that we developed in Chapter 10 for simple systems could not possibly extend to two-tier systems. By definition, the basic tier districts in compensatory PR systems is not decisive in determining the overall makeup of the assembly – in total contrast to the situation in simple, single-tier systems. Therefore, perhaps it follows that there would be little or no relationship between features of basic-tier districts and the representation of parties in that tier.

To give an idea of how models derived from single-tier systems need only one small adjustment to account for the pattern in basic tiers, we offer Figure 15.4.[12] It shows the districts of our two-tier systems, and the relationship of the effective number of seat-winning parties (N'_S) to district magnitude. Partially replicating Figure 10.1, this graph also plots the simple (single-tier) systems.[13] We can see quite clearly that the general trend is for the data points for two-tier systems to be somewhat higher than the main trend for simple systems.

The thick dotted curve takes into account the embeddedness of districts in the wider system, much as did Figure 10.1 and several other district-level graphs that we have seen. The remarkable aspect of this curve is that it captures the same relationship as Equation 14.2:

$$N'_S = M^{4kk/3}. \tag{15.6}$$

In the case of Equation 14.2, the quantity explained was the suballiance parties in the hybrid D'Hondt/SNTV countries – those open-list PR systems that often feature candidates of more than one party on the same list. Here it is parties, per se, but in systems where the district might be said to be "doubly embedded" – first it is embedded in a basic tier consisting of various other districts, and then it is embedded in a two-tier design in which some percentage of the assembly is elected via compensatory PR in national (or regional) districts.

Equations 14.2 and 15.6 both contain a double k, our embeddedness factor. This is because, *unlike with simple systems*, the number of seat-winning parties,

12 Readers seeking more detail on these processes than we offer here are referred to our online appendix www.cambridge.org/votes_from_seats.

13 Excluding the M=S cases, in order to focus on the range that is relevant to the comparison to the two-tier systems.

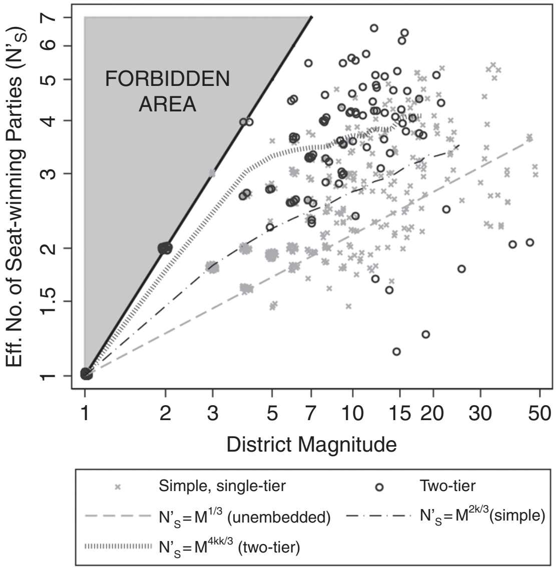

FIGURE 15.4 How the magnitude of a district shapes the effective number of seat-winning parties in the basic tier of two-tier systems, compared to simple systems

of any size, turns out to be $N'_{S0}=M^k$, instead of $N'_{S0}=M^{1/2}$, as it is with simple systems (Equation 1.1, tested in Chapter 7).[14] The implication is that parties in two-tier systems enter even in districts where they do not expect to win. They do so because winning the basic-tier district is typically not necessary to gaining representation, but entering is necessary to earning votes that may help it win seats in the upper tier.[15] By campaigning for and earning such votes, these parties are more likely to pick up a seat in a given basic-tier district even where they are relatively weak. Their ability to win seats is still constrained by the district magnitude, but it is boosted via the district's double-embeddedness in a more complex electoral-system design. We can see just how much it is boosted, on average, by comparing the thick dotted curve representing Equation 15.6 with the curve for simple systems (equivalent to Equation 10.5, and plotted with the dot-dash pattern), and the equation for a hypothetical unembedded district, plotted with the light gray dashed line.[16]

[14] The regressions supporting these results are included in our online appendix. www.cambridge.org/votes_from_seats

[15] The statement about entry is true for all cases in which the basic tier $M>1$. It is not strictly true for cases of MMP in which districts are $M=1$ and there is a separate party-list vote. Nonetheless, in most cases, even small parties enter the basic-tier districts with candidates whose presence helps the party "show the flag" for earning list votes.

[16] We see eight data points from two-tier systems below this line for the unembedded relationship. Two are from one Danish district in different elections, one is from Estonia, and all the others are

Both the D'Hondt/SNTV hybrid and the presence of an upper tier are forms of complexity. However, they lend themselves to modeling with the same theoretical approach that we applied to simple systems (Chapter 10). While such systems may seem as if they could not be modeled, in fact they can be.

The Basic Tier and the Upper Tier: Fitting the Components Together

The preceding section concerned individual districts of two-tier systems. This is one piece of the puzzle, but in order to understand the process of compensatory PR, we need to consider how the basic and upper tiers fit together as components of a two-tier system. The equations for two-tier systems overall – the Extended Seat Product Model (Equation 15.2) and the formula for Deviation from PR (Equation 15.4) – both make the following claim: the output (N_S or D_2) is a product of the basic-tier aggregate output, times the adjustment term. The adjustment term itself is a base raised to the tier ratio (the share of the total assembly elected from the upper tier): J^t for N_S and j^t for D_2. We can check the logic now in two steps.

The first step is to ask whether the output (N_S or D_2) in the basic tier conforms to the same relationship as the models for simple systems. In other words, we want to know whether the aggregate outputs of just the basic tiers of the two-tier cases conform to:

$$N_{SB} = (MS_B)^{1/6} \tag{15.7}$$

$$D_{2B} = 0.5(MS_B)^{-1/3}. \tag{15.8}$$

Once again, the addition of the B to our subscripts reminds us that these are quantities for the basic tier only. We run regressions (shown in the chapter appendix) to test both of these. We also graph the data and the lines formed by these equations in Figure 15.5, which consists of two panels. In the left we see the basic-tier effective number of seat-winning parties, N_{SB}, while in the right we see the basic-tier deviation from PR (D_{2B}).

We find that Equation 15.7 is confirmed precisely. So is the slope in Equation 15.8, although the intercept is somewhat higher than it was for the wider set of cases, including simple systems. The 0.5 constant term in our earlier equations for D_2 was empirically determined anyway; what is most impressive is the repeat of the expected exponent.

Having confirmed that the basic tier of compensatory systems works essentially equivalent to a simple system, we now ask whether the adjustment effect is as implied by Equations 15.2 and 15.4. In other words, does raising

from South Africa (which is noteworthy for its dominant party, notwithstanding its extremely proportional electoral system – see Ferree 2018).

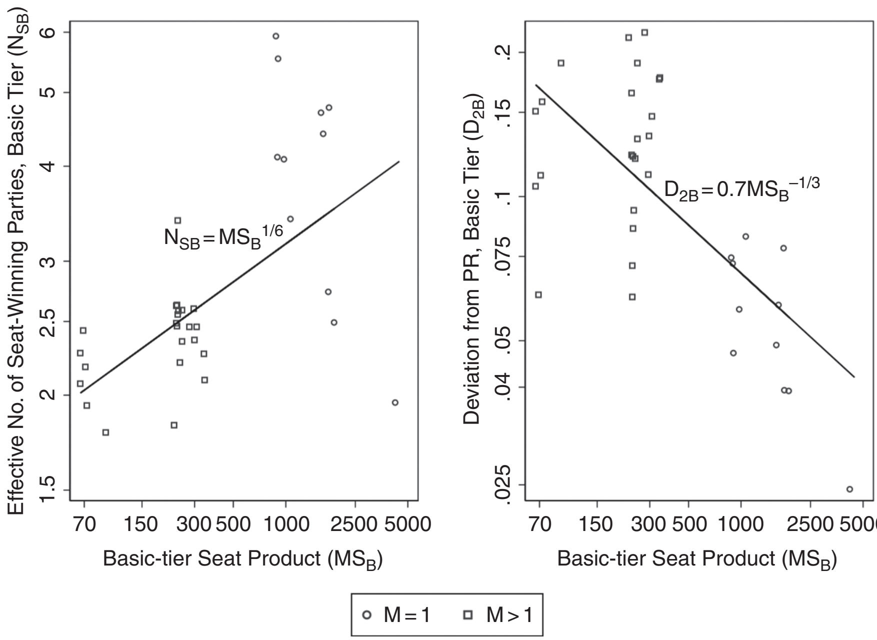

FIGURE 15.5 The impact of basic tier seat product (MS_B) on the effective number of seat-winning parties (N_{SB}) and deviation from proportionality (D_2)

a base coefficient to the upper tier ratio, t, accurately capture the tendency of the compensatory tier to adjust the basic-tier result? We can answer this by determining the mean adjustment factor in the two-tier systems that we have in our district-level dataset, the one we use in this section because it allows us to measure the impact of the two tiers separately. They are:

$$N_S/N_{SB} = 1.38;$$
$$D_2/D_{2B} = 0.36.$$

That is, after the application of compensation via the upper tier, N_S on average is 1.38 times what it was from the basic tier alone, whereas D_2 is 0.36 times as high. Given the previously estimated values of our adjustment factors, we ask whether the mean upper-tier ratio, which happens to be 0.364 in this dataset, yields the correct result:

$$J^{0.364} = 2.5^{0.364} = 1.396;$$
$$j^{0.364} = 0.06^{0.364} = 0.359.$$

The results are almost spot on. Therefore, by using the basic-tier district-level data and comparing to the nationwide, we have confirmed that the process is as we posited in developing Equations 15.2 and 15.4 earlier in this chapter.

This finding is highly relevant for the design of two-tier systems: the implication is that the number of parties (actual and effective) can be constrained by employing low or moderate district magnitude in the basic tier. The degree to which the upper tier will inflate this is broadly predictable through the share of all assembly seats that are available for upper-tier compensation. The degree to which the disproportionality arising from the basic tier will be reduced through the compensation seats is likewise broadly predicable, through the same parameter.

With this information, it is possible to understand two-tier compensation processes systematically. It is therefore also possible to estimate how the design of the components of two-tier systems can be crafted towards producing the relative degree of district-level and nationwide party system patterns that are desired.[17] Of course, there remains ample room for political variation away from the model estimates. This should be even more so for two-tier systems than for simple ones, for the reason that more complexity of the system means more uncertainty of the result. Yet the models shown in this chapter offer an advance, in that previously the best that political scientists could say was the directional statement: more compensatory seats means a higher effective number of parties and lower deviation from proportionality. Now, from the analysis carried out in this chapter, we can offer something closer to a quantitative prediction[18] if we know three parameters: the size of the assembly, the size of the basic tier,[19] and the mean magnitude of basic-tier districts.

CONCLUSION

This chapter extended the Seat Product Model (SPM) by considering two additional variables, ethnic fragmentation and upper tier share. Both variables are included in many standard regression-based accounts of electoral systems and party systems, and thus the SPM would be much less generalizable were it to ignore these factors. Fortunately for the applicability of the SPM, we have seen that accounting for ethnic fragmentation has much less impact than commonly believed, and that a compensatory upper tier can be included in the SPM with just one additional parameter.

What we have found is that even though the ethnic structure matters statistically for some of the outputs, its substantive effects are most often

[17] We have not considered the impact of relatively rare cases in which the upper tier is itself districted. These may not have different effects from cases of nationwide compensation if the basic tier is itself not too restrictive. However, for cases like MMP (basic tier districts of M=1) it is potentially consequential. Analysis of the Scottish Parliament system shows that our extended SPM predicts its N_S well, despite assuming erroneously that the upper tier is one district, but understates D_2. (See our on line appendix at www.cambridge.org/votes_from_seats).

[18] See Chapters 1 and 17 for discussion of direction hypotheses versus quantitative prediction.

[19] Recall that the tier ratio, t, is $S- S_B/S$.

negligible. This is a valuable insight, considering that the measurement of ethnicity is subject to serious controversy: see Stoll's (2008) discussion of the impact of measurement selection, Lublin's (2014) criticism of counting subthreshold groups, Potter's (2014) arguments regarding the mismatch of theory and data in the measurement of (district-level) diversity, and the extensive words of caution by Fearon himself (2003: 197–198, 200). Thus a parsimonious model that has good predictive power without the inclusion of ethnic variables should be preferred when the purpose is to predict nationwide party-system fragmentation under a given electoral system.

For two-tier systems, a small extension of the Seat Product Model allows us to include two-tier compensatory systems within its coverage. Through analysis of both the nationwide and district-level data, we find that two-tier systems affect N_S first through the seat product of the basic tier alone, and then through an inflation of this basic-tier value according to the share of the assembly that is elected in the upper tier. Larger upper tiers thus can be expected to inflate N_S more, relative to the impact of the basic tier. The relationship of N_V to N_S is equivalent across two-tier and simple systems, as already shown in Figure 8.4.

A key implication of our analysis is that *the basic tier of two-tier compensatory systems remains fundamental* to our expectations of how fragmented the party system will be. This need not have been the case; the existence of upper-tier compensation could have overridden the impact of the size of the basic tier and the average magnitude of its districts. Yet, for estimating what the effective number of parties tends to be, on average, the basic tier is indeed basic to the outcome.

The basic tier also remains fundamental to the impact of a two-tier system on deviation from PR, D_2. Compensation surely means that D_2 can be only reduced by the upper tier. As long as the upper tier represents at least a quarter of the total assembly, we can expect D_2 to be about half what would be expected if the basic tier were the entire assembly. However, it is quite likely to be even lower than that expectation. In other words, very large upper tiers appear to be unnecessary to produce a substantial reduction in disproportionality.

An informal iron law has been long observed in electoral system design: one can obtain more proportionality (which may be desirable) only at the cost of more fractured party landscape (which may be undesirable). Our Figure 9.4 confirms and quantifies this tradeoff, for simple systems. Two-tier compensatory systems relax this iron grip. For instance, a basic tier that does not encourage fragmentation, combined with a modest-sized compensatory tier (perhaps 25 percent of total seats), favors both a moderate effective number of parties and low disproportionality. We can have our cake and eat half of it too. In principle, simplicity is desirable, but the complexity of having two tiers pays off in practice – and to the extent expected in theory.

This chapter thus clarifies an important design principle, as there is often resistance to making districts large geographically or expanding assembly size in order to accommodate a large compensatory tier. If the compensatory tier can be considerably smaller than 50 percent of the total assembly and still provide adequate PR, then the basic tier can accommodate more compact districts within a reasonably sized assembly. These findings, it must be emphasized, refer only to compensatory upper tiers. Thus they may not apply to mixed-member majoritarian systems (as defined in Chapter 3) or other systems with noncompensatory upper tiers. In Chapter 16, we consider these and other complex systems.

Appendix to Chapter 15

This appendix reports results of regressions discussed in Chapter 15. In **Table 15.A1** we see a regression in which the output variable is Deviation from Proportionality (D_2) and the inputs are the basic-tier seat product (MS_B) and the upper tier ratio (t). Based on findings in Chapter 9, we expect a coefficient on MS_B of –0.333. We do not have a specific expectation for the coefficient on t, other than that it should be negative and significant. The constant term should be –0.301, so that when unlogged the equation derived from the regression matches Equation 9.3 for a simple system, i.e., $D_2=0.5/(MS_B)^{1/3}$. The results closely match the expectations, and lead us to Equations 15.4 and 15.5, reported in the main text of the chapter.

In Table 15.A2, we see the regressions on district-level outcomes in the basic tiers of our two-tier systems. All variables in each regression are entered as their decimal logarithms. Regression One confirms that the embeddedness

TABLE 15.A1 *The effect of two-tier systems on Deviation from Proportionality*

	(1)
VARIABLES	**log(D_2)**
(M_{SB}), logged	–0.342
Expected: –0.333	(0.0371)
	[–0.416 – –0.268]
tier ratio (t)	–1.209
Expect neg. sign	(0.205)
	[–1.620 – –0.799]
Constant	–0.317
Expected: –0.301	(0.120)
	[–0.558 – –0.0771]
Observations	342
R-squared	0.455

Robust standard errors in parentheses.
95 percent confidence intervals in brackets.

TABLE 15.A2 *Regressions for basic tier of two-tier systems: district level*

VARIABLES	1 No. of seat-winning parties $\log(N'_{S0})$	2 Size of the largest party $\log(s'_1)$	3 Size of the largest party $\log(s'_1)$	4 Effective No. of seat-winning parties $\log(N'_S)$
$k*\log(M)$	0.960***			
	(0.0508)			
	[0.857 – 1.063]			
$k*\log(N'_{S0})$		–0.986***		
		(0.0574)		
		[–1.104 – –0.868]		
$k^2*\log(M)$			–0.969***	1.271***
			(0.143)	(0.162)
			[–1.262 – –0.676]	[0.939 – 1.603]
Constant	–7.41e–06	2.73e–05	–0.000215	0.000147
Expected: 0	(0.000172)	(6.21e–05)	(0.000131)	(0.000127)
	[–0.000358 – 0.000343]	[–0.000100 – 0.000155]	[–0.000484 – 5.33e–05]	[–0.000113 – 0.000407]
Expected coeff.:	1.000	–1.000	–1.000	1.333
Observations	5,110	5,110	5,110	5,110
R-squared	0.963	0.947	0.860	0.902
rmse	0.0139	0.0107	0.0173	0.0185

Robust standard errors in parentheses.
95 percent confidence intervals in brackets.
*** p<0.01, ** p<0.05, * p<0.1

factor, k, is needed to understand the relationship of the number of seat-winning parties of any size (N'_{S0}) to district magnitude (M) – unlike in the case of simple systems (Chapter 9). The regression supports the expectation, $N'_{S0}= M^k$. Regressions Two and Three are for the output variable, size of the largest party (s'_1); in Regression Two the input is N'_{S0}. Following the same logic as in Chapter 9, we expect $s'_1 = N'^{-k}_{S0}$, which is supported by the result (because the expected –1 is within the 95 percent confidence interval of the estimated coefficient on $k*\log(N'_{S0})$.

When we take the next step, which is to connect s'_1 to M, we expect $s'_1 = (M^k)^{-k}=M^{-kk}$, which is confirmed (within the confidence interval) by Regression Three. Finally, because we have $N'_S = s'^{-4/3}_1$ (Table 9.2), we expect $N'_S = (M^{-kk})^{-4/3}=M^{-4kk/3}$; this is supported by Regression Four, where the

TABLE 15.A3 *Regressions on basic-tier effective number of seat-winning parties and deviation from PR*

	(1) Testing SPM in basic tier $\log(N_{SB})$	(2) Deviation from PR in basic tier $\log(D_{2B})$
log_MB	0.166	−0.334
	(0.0700)	(0.104)
Expected:	**0.167**	**−0.333**
Constant	0.0124	−0.157
	(0.161)	(0.300)
Expected:	**0**	**(empirical)**
Observations	34	34
R-squared	0.337	0.463

Robust standard errors in parentheses.

estimated 1.271 is close to the expected 1.333 (and the latter is within the confidence interval).

In Table 15.A3 we see regression results to test whether our formulas for nationwide simple systems also work in the basic tiers of two-tier systems. Regression One is a test of the main claim of the extended SPM (Equation 15.1): that the effective number of seat-winning parties in the basic tier (N_{SB}) – i.e., prior to the application of nationwide compensation – is in fact equivalent to $(MS_B)^{1/6}$. We see that the expectation is confirmed almost precisely.[20]

Regression Two tests the effect of MS_B on deviation from PR (D_2). In line with our prior equations for nationwide deviation form PR (Equations 9.1, 9.2, 15.4, and 15.5), we expect the coefficient to be –0.333. Again, it is confirmed almost precisely. The constant term is estimated by the regression to be –0.0157, which unlogged is 0.697. This differs from the estimated 0.5 of Equations 9.2 (simple systems) and 15.5 (which pooled simple and two-tier). Moreover, the constant in Regression Two is not statistically significant. This is not a troublesome finding, as the constant in our equations for D_2 is empirically determined in all cases. What is important – and remarkable – is that we continue to get the expected –0.333 coefficient.

Both regressions confirm the logic of how two-tier systems work: first they shape party-system outcomes in the basic tier. Then the compensatory upper tier inflates N_S and deflates D_2.

[20] It narrowly misses the p=.05 standard of significance; it is actually p=0.055. This is trivial, especially in light of a virtual point confirmation of a logically determined parameter.