Solutions to selected exercises

CHAPTER 2

2.1 Arrange the positions in order $\{-2, 2, 6, 9, 9\}$ to see that the median is 6 (there are two points smaller than 6 and two points greater). The mean, or average, is the sum of the above positions divided by the number of positions (five): 24/5.

2.2 By the definition of symmetry, changes in either direction will yield similar changes in a voter's utility. The two voters with positions less than 3 will clearly vote for it, and the two with positions greater than 8 will choose it. The middle voter at position 6 has symmetric preferences, so equal changes in either direction yield equal changes in utility; since 3 is further away than 8, the median voter will choose 8 (closer is better) and 8 will win 3 to 2.

2.3 If 7 is added to the committee, then no unique median exists; 6 and 7 are both medians. If this new committee is asked to choose between 8 and 3, then 8 will again be chosen.

2.4 Order the positions to find the median $\{-4, -3, 6, 7, 21\}$. By definition, 6 must be the median. If one replaces 7 with 25 the new ordering is $\{-4, -3, 6, 21, 25\}$ and the median stays the same. The mean, however, changes (increases).

2.5 Probably not. One can imagine a society in which a few outliers change the mean from the true middle of society. Instead, one might want to argue that the mode is most important, insofar as Aristotle is arguing for a large, vigorous middle class.

2.6 The three measures of central tendency are in general not the same for a given distribution. One can easily imagine distributions where they happen to be the same: for example, the set {5, 5, 5, 5, 5, 5} or {1, 2, 3, 4, 5}.

2.7 The median income is likely a better indicator of where the middle class is due to the effect that outliers have upon the mean. A small group of very rich people can move the mean higher than it would be naturally; alternatively, a small group of very poor people can move it lower. In general, if the mean is higher than the median it means the distribution is weighted to the right (i.e., the rich in this example); if the mean is lower than the median, the distribution is weighted to the left (i.e., the poor).

2.8 *Proof by Contradiction:* Assume n is odd, and that two medians exist (or there is an interval bounded by $\{x, y\}$). Suppose $x < y$. By definition, the number of points to the left of x must be the same as the number to the right of y (if the number to the left of x if less than the number to the right of y, then x is not a median; the opposite case also holds for y). Thus, n_l for x + n_r for y is some even number (since $n_l = n_r$ we are in effect doubling one of them, which proves the result is even). The only other numbers that have been excluded from the set are x and y; adding them makes n even (since all we have done is add 2 to an already even number). This results in a contradiction (n is odd), proving the result.

CHAPTER 3

3.1 **z** is not a Condorcet winner, as its win set is not empty.

3.2 If the issues are separable, then the median for issue 1 is 5 and the median for issue 2 is 10. The ordered pair is thus (5,10), but this is no surprise given the result that voting on separable issues one at a time results in the same outcome regardless of order.

3.3 There is a Condorcet winner at (5,10), which is also a median in all directions.

CHAPTER 4

4.1 For such a problem, SED is a linear transformation of WED. If the SED is $((x_1 - y_1)^2 + (x_2 - y_2)^2))^{1/2}$, then the WED is $[\lambda(x_1 - y_1)^2 + \lambda(x_2 - y_2)^2)]^{1/2}$.

4.2 Since a_{12} is < 0, we are left with a negative interaction term. If x_1 (equal to 20) is $> x_{i1}$, then the conditional ideal on x^*_{i2} is larger than x_{i1}; if $x_1 < x_{i1}$, the opposite will hold.

4.3 Assume one of the two points is the origin. Then, the key to the proof is to realize that if there are three dimensions (x, y, z), one can apply the Pythagorean theorem not only to the plane described by (x, y), but also to the right triangle created by the vector reaching from the origin to the point (x, y) and the side (0, z). Applying the Pythagorean theorem twice yields:

$$\sqrt{[(x^2 + y^2)^{1/2}]^2 + z^2} = \text{the equation for SED}$$

One can easily broaden this result to any two points, rather than the origin and another point, by transformation of the origin.

CHAPTER 5

5.1 What we want to find is a K such that no other position could receive a K-majority. Take for example, $K = 9$ (which is $n/2 + 1$). The position 9 would thus be a KMVT, supplanting 7 in an election. The smallest K, which would maintain 7 as the status quo, is thus $K = 10$.

5.2 First round $x_A = 2$, $x_B = 2$, $x_C = 1$; second round $x_A = 3$, $x_B = 2$. Then x_A gets a score of 10, $x_B = 11$, and $x_C = 9$. Member C would win the election.

CHAPTER 6

6.1 The general formula for voter preference is WED from Chapter 4. Given this, candidate B wins 3 to 0 based upon the position $[2\ 3]^T$.

6.2 From Equation 6.5, the candidate with the least distance and variance will win the election. If the variance for candidate R is 2, then R will win the election. In fact, R's variance could be as high as 14 to be insured of winning (at 14, there would be a coin toss to determine the victor).

6.3 Given the discussion of Nash equilibrium in the chapter, the candidates will converge upon the median in all cases. Thus, all the elections will result in a tie, giving them an expected utility of at least ½ (representing the chance that they win); if a candidate doesn't move (e.g., because the position is far from the ideal), the candidate's expected utility is less, because he or she will certainly lose the election (W in the utility function) and receive little utility from the ending policy position (x in the utility function).

CHAPTER 7

7.1 Given Equation 7.1, any of the governor's initiatives would increase a factor in the equation, thus increasing the number of probable voters. And as these initiatives focus on the first two factors, they will have a larger impact than factors 3 and 4 (once you get a person to the voting booth, it is likely he or she will actually vote).

7.2 Using the formula for WED, the citizen prefers candidate A to B ($50^{1/2}$ to $68^{1/2}$) and will vote since the distance to the candidate is less than 10.

7.3 Again, using the formula for WED, the voter is a distance of $5^{1/2}$ from candidate A and $37^{1/2}$ from B. Subtracting these terms yields a difference greater than 2, indicating the voter is not alienated.

CHAPTER 8

8.1 If person 1 is strategic and can set the agenda, the first pairwise contest will be B against C. C loses to B 2 to 1. The second contest would be B against A. B loses to A 2 to 1. The cycle could continue, but the malevolent person A has the ability to stop after these two votes.

8.2 Consult Figure 8.1 – one can make a 1 to 1 substitution of vegetables and find the answer (i.e., the two problems are isomorphic).