CHAPTER 2

The spatial model of Downs and Black: One policy dimension

You will be safest in the middle.
("Publius Ovidius Naso," or
"Ovid," *Metamorphoses,*
Book II, no. 137)

The idea of spatial competition comes from Hotelling (1929) and Smithies (1941), who used "space" to describe firms' need to be near markets. Spatial theory was adapted for analytical politics by two pioneers: Anthony Downs, in *An Economic Theory of Democracy* (1957), and Duncan Black, in *The Theory of Committees and Elections* (1958). They uncovered two of the most important theoretical contributions of analytical political theory:

- Political power lies at the "middle" of the distribution of citizens *effectively* enfranchised by the society's political institutions.
- The stability of political systems is a variable, or subject of analysis. Stability depends on the distribution and nature of citizens' preferences, as well as the rules used to add up these preferences for social choices.

The contribution of the spatial theorists who have built on the work of Downs and Black has been to state these two principles of political power very precisely. Most importantly, analytical politics works to distinguish situations where the principles are true, false, or conditionally true based on other variables outside the model. The two principles themselves, however, have been recognized by political theorists for more than two thousand years.

The first spatial theorist: Aristotle

Aristotle (384–322 B.C.) was a student of Plato, a tutor to Alexander the Great, and a great thinker. One of his accomplishments has gone unnoted, however, until now: Aristotle was the first spatial theorist. More accurately, he was the first to realize that political conflict has a *spatial* organization. The argument, of which we will give only excerpts, has three distinct elements:

- *Assumption:* The world view of citizens can be described by their position along a single (ordered) dimension. For Aristotle, the dimension is wealth or "class." Further, a citizen's position on this dimension determines the government action he or she likes best.
- *Causal conclusion:* In *any* government, the middle rules. The reason is that power lies at the center of those who can vote.
- *Policy conclusion:* The decision to enfranchise different groups changes the distribution of preferences in the polis. Changing the distribution of preferences alters the political "constitution" of the society.[1]

See if you can pick out these three elements in Aristotle's argument in the following passage:

> Now in all states there are three elements: one class is very rich, another very poor, and a third in a mean. It is admitted that moderation and the mean are best . . . for in that condition of life men are most ready to follow rational principle. . . . Thus it is manifest that the best political community is formed by citizens of the middle class, and that those states are likely to be well-administered, in which the middle class is large, and stronger if possible than both the other classes, or at any rate than either singly; for the addition of the middle class turns the scale, and prevents either of the extremes from being dominant. . . . These considerations will help us to understand why most governments are either democratical or oligarchical. The reason is that the middle class is seldom numerous in them, and whichever party . . . transgresses the mean and predominates, draws the constitution its own way, and thus arises either oligarchy or democracy. (Aristotle, 1979, Book 4, Chapter 11)

For Aristotle "democracy" is a government where the poor are numerous and are allowed to vote. A government where only the wealthy can vote is an "oligarchy." Neither of these societies, Aristotle claims, are really consistent with virtue, because such societies are neither just nor stable. He goes on, in chapter 12 of Book 4:

> The legislator should always include the middle class in his government. If he makes his laws oligarchical, to the middle class let him look; if he makes them democratical, he should equally by his laws try to attach this class to the state. There only can the government ever be stable where the middle class exceeds one or both of the others, and in that case there will be no fear that the rich will unite with the poor against the rulers. For neither of them will ever be willing to serve the other, and if they look for some form of government more suitable to both, they will find none better than this.

The nature of institutions, and the good society

The best government is stable and favors neither the poor nor the wealthy too heavily, in Aristotle's view. Such a government is possible only if there is a large group in the center, so that neither extreme can dominate. Consider Figure 2.1, depicting two distributions of citizens by wealth. Panel (a) depicts a society wracked by revolution. If the poor are many and enfranchised, they will favor policies of redistribution and high taxes on the rich. If only the wealthy can vote, they will defend their oligarchy by denying freedoms to the poor, for fear the poor will rise against them.

The society portrayed in panel (b) is more symmetric. Further, the center of the distribution of wealth describes many citizens. It is unlikely that such a society will see wild swings in government form. The middle class gets the government it wants, and the poor and the wealthy have diametrically opposed goals (more democracy versus more oligarchy, respectively).

Aristotle may be right that the first society has an inferior government. We would also have to concede, however, that *it is not the first government's fault.* The flaw lies with the distribution of preferences, which prevent *any* form of government in the first society from being stable. The character of government and the stability of policy can be affected by the distribution of voter preferences. Consequently, the choice of whose votes "count" is tantamount to choosing a specific policy outcome.

In the next section, we begin to analyze the claims about the importance of the center and the stability of outcomes. To do the analysis, we must introduce spatial theory.

The importance of the middle: An example

An appropriations subcommittee meets to decide the budget for construction of river and harbor projects for the coming year. Suppose the government spent $80 million on such projects last year. The subcommittee has already held hearings, so each member knows what he or she wants. Because the debate was public, each member also knows the preferences of other members.

The rules of the committee are as follows:

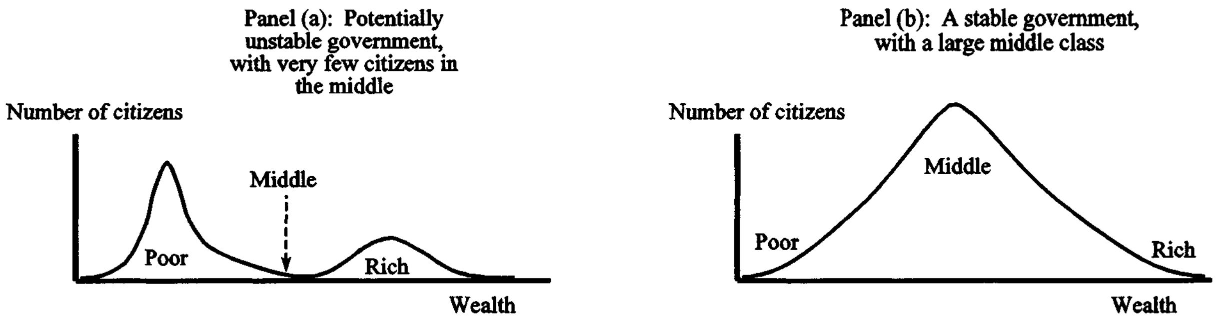

Figure 2.1. Two distributions of citizens, by wealth.

Table 2.1. *Ideal budgets for committee members A–E*

	Member	Most preferred budget for rivers and harbors (millions of $)	
	A	150	
	B	100	
→	***E***	***80***	← middle, or ***median,*** voter
	C	50	
	D	10	

- Each member can make any proposal on the rivers and harbors budget, ranging from zero to an arbitrarily large (but still finite) number.
- New proposals are compared against the *status quo,* using majority rule. *All members must vote,* either for or against, each proposal.
- The initial status quo is last year's budget ($80 million). If a new proposal loses, it is discarded. If it wins, the proposal becomes the new status quo.
- The subcommittee has five members: A, B, C, D, and E. These members all vote based only on their preferences for the proposals then under consideration. This kind of voting is called *sincere.*

Mr. A thinks the subcommittee should recommend almost doubling the budget to $150 million. Ms. B also believes that $80 million was too little, but thinks that $100 million is the right amount. Mr. C believes that the projects built last year are enough and that the committee should recommend just enough money for maintenance, or $50 million.

Conservative Mr. D thinks rivers and harbors legislation is pure "pork barrel" spending, so just $10 million is needed for emergency repairs to existing dams. Moderate Mr. E believes that last year's budget was fine and that $80 million should be allotted again this year. The ideal budgets of all five committee members (in decreasing order) are given in Table 2.1. Two members want budgets larger than Mr. E's ideal. Two other members want to spend less than Mr. E's ideal. Consequently, the "middle" voter is member E, at $80 million.

Now we know who wants what. How can we use spatial theory to predict what will happen when the committee votes? One thing we must know is how unhappy members feel about budgets different from their ideal. Remember, there can be only one budget, and there are five different people, with five different ideals, in our example. Obviously, at least four members are not going to get exactly what they want. Now we will invoke the "ordered dimension" assumption, which allows us to conceive of preferences spatially. The simplest assumption about judgments by members is *distance.* Changes *away from* the ideal make members less happy. Members are more satisfied with changes *toward* their ideal budget.

How members choose: Preferences and representation

The meaning of the words "away from" and "toward" seem self-evident, and the use of distance as a description of preferences seems clear. That is always a bad sign. The fact is that very little is self-evident in analytical politics (or any other kind of political theory), because we have not yet carefully identified our assumptions. It is important to step back and think about what the words mean.

The general problem is to assign a *preference function* to each member. Preference functions describe how people feel about different government policies. In this case, the preference function tells us the level of satisfaction, or acceptance, of each level the budget might take. This is important, because preference functions allow us to rank members' reactions to every possible budget proposal. One way this ranking might take place is based on the difference between the proposal and ideal budget for each member.

There is an "as if" aspect to the assignment of a preference function, of course, since members may not do the calculations implied by a particular function. Preference functions simulate, or *represent,* each member's preferences. To see this, choose two proposals (y and z) arbitrarily, allowing y and z to take any values in the feasible range of budgets. Then we can give two important definitions.

Preference Function. Define a preference function U as follows:

$$\text{Utility of proposal } y = U(y) = (\text{Level of satisfaction}) \qquad (2.1)$$

A function *U represents* a member's preferences if it accurately reflects that member's reactions to proposals. To summarize these reactions, we use two key relations: *preference* and *indifference.*

Preference. The member likes one alternative more than the other. Then both of the following must be true:

1. If the member (strictly) prefers y to z, then $U(y) > U(z)$.
2. If $U(y) > U(z)$, then the member prefers y to z.

Indifference. The member likes the two alternatives equally well. Then both of the following must be true:

1. If $U(y) = U(z)$, then the member is indifferent between z and y.
2. If the member is indifferent between z and y, then $U(z) = U(y)$.

If we knew a member's preference function, we could guess how he or she reacts to any proposed budget. But no one, including the member, needs to know the exact preference function to represent preference. In analytical politics, we generally are concerned with a particular kind of function for the representation of preference. These are called "spatial" preference functions, where *satisfaction is higher the closer a proposal is to the member's ideal.* The only properties of spatial preference functions that we will assume for now are the following:

(a) *Unidimensionality:* There is only one issue (here, the budget), and members like their own ideal budgets best. This is the "ordered dimension" assumption mentioned above.
(b) *Preferences are single-peaked:* Each member likes new proposed budgets more the "closer" they are to the member's ideal.
(c) *Voting is sincere:* If $U(y) > U(z)$, the member votes for y, regardless of what other proposals might later be raised if y becomes the new status quo. More intuitively, sincere voting means that the member considers only the two alternatives being voted on and disregards the agenda of future votes.

There is an additional property of spatial preference functions that we will find useful to invoke:

(d) *Symmetry:* A preference function is symmetric if equal departures from the ideal in opposite directions yield equal declines in satisfaction.

Properties (a)–(c) are the assumptions of the spatial model of political preferences, with (d) used only sometimes. These assumptions re-

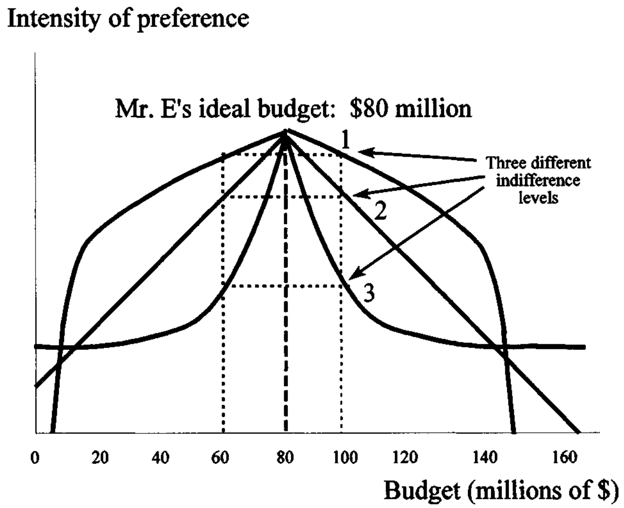

Figure 2.2. Three possible preference functions for committee member E.

strict the set of possible preferences we have to consider. That is not to say that (a)–(d) are always accurate! For example, if members can forecast the sequence of future votes (the *agenda*), they may not vote sincerely, as we will see in Chapter 8.

Likewise, symmetry is a stronger assumption than may be required to say important things about political choice. Let's just consider the preferences of one member, Mr. E. If two alternatives are the same distance from E's ideal point, $80 million (say, y = $60 million and z = $100 million) and the preference function is symmetric, then E won't care which proposal is chosen (U($60m) = U($100m)). We say that E is *indifferent* between the two proposals. If one alternative (say, $40 million) is farther away than another (such as $100 million), E prefers the closer alternative. Remember, there are but two ways E can react to any pair of budget proposals: He can prefer one, or he can be indifferent between them. Both relations are consistent with *many* different spatial preference functions. The graph of a preference function is called the *preference curve.*

Three very different preference curves are depicted in Figure 2.2. The *intensity* of preference (the height of the curve, or the vertical axis in Figure 2.2) is arbitrary. Each curve depicts a preference function that

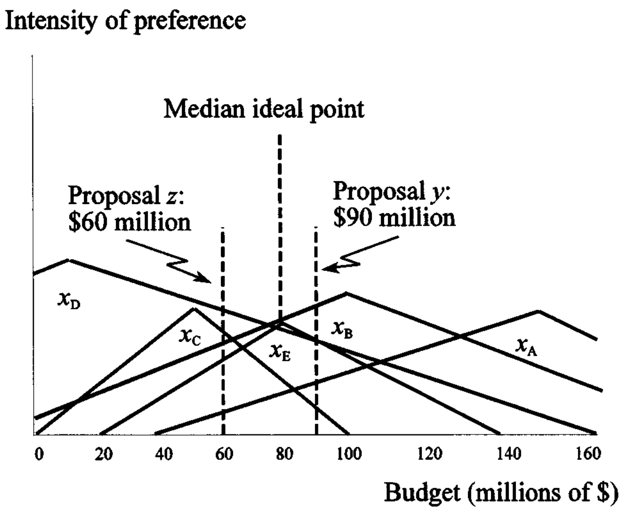

Figure 2.3. Two proposals, z and y, and preference functions for committee members.

satisfies assumptions (a)–(d), so each of the three curves is equally valid as a representation of the preferences of member E in our example. The fact that they are strikingly different reflects the fact that functions that represent preferences are not unique.

Other committee members have preferences, too. One of many possible forms is depicted in Figure 2.3. Note that each of these (linear) preference curves satisfies assumptions (a)–(d), so that they are single-peaked and symmetric. For example, B wants \$100 million, prefers \$90 million to \$80 million, and is indifferent between \$85 million and \$115 million.

What if we *drop* the restrictive assumption of symmetry? The preference curves wouldn't (necessarily) look like those in Figures 2.2 and 2.3, because increases may look different from decreases, compared with the ideal budget, for each member. We say symmetry is "restrictive" because it rules out plausible forms of political preference, such as that depicted in Figure 2.4. Here, a committee member (G) has an ideal budget of \$5,400 per student.

Mr. G feels strongly that spending should be no less than this ideal, because his main concern is that the school budget be *at least* \$5,400

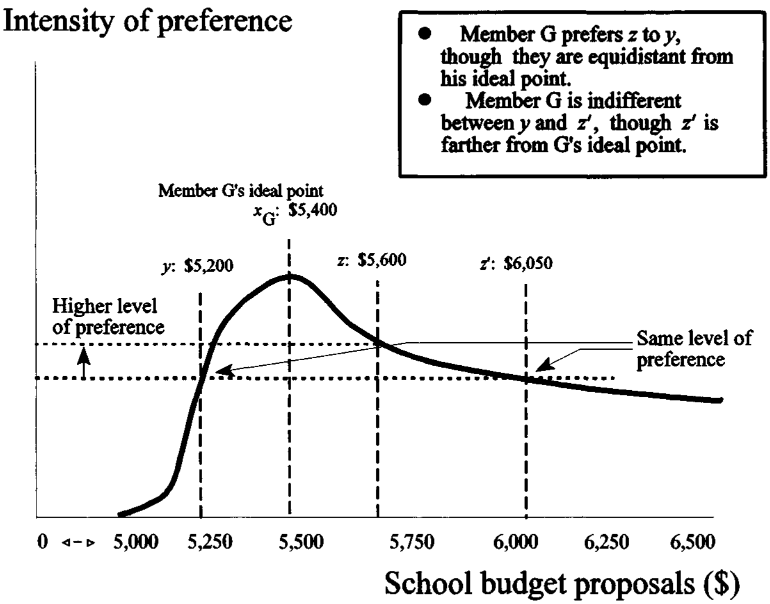

Figure 2.4. Example of asymmetric preferences: Member G likes increases in school spending more than cuts.

per student. As the figure shows, increases in spending beyond his ideal don't bother him much. If member G compares proposal z with y, he likes z better ($U(z) > U(y)$), though y is closer to his ideal point. We would have to overspend dramatically to find a budget (z') that G values as little as y. Mr. G's preferences are asymmetric because he values deviations from his ideal point differently depending on whether the proposed spending level is (from his perspective) too much or too little.

Can we say anything about what the committee is likely to decide, given only assumptions (a)–(c) (leaving out symmetry)? Suppose A proposes to increase spending over last year's level, to y million dollars ($y >$ \$80 million). How many members will prefer y to the last year's budget? Two members (A and B), *at most,* will prefer any $y >$ \$80 million. In the example in Figure 2.3, we set $y =$ \$90 million. As expected (since $y >$ \$80 million, and A and B want increases), A and B prefer y to \$80 million.

Imagine C proposes to decrease spending to z, where \$0 $< z <$ \$80 million. One possibility is the proposal in Figure 2.3, where z = \$60 million. Both C and D prefer z to the status quo, but A, B, and E prefer \$80 million. What we have shown (informally) is that any budget larger than \$80 million will lose by a vote of 3 to 2. We have also shown that any budget less than \$80 million will lose by a 3 to 2 vote.

This phenomenon is general, because the same reasoning applies to any status quo in the middle. All that is required is that each member like budgets less the more they differ from that member's ideal budget. Symmetric preference is not required, for any member. The "middle" ideal budget (\$80 million) will win a majority against any other alternatives, such as $z <$ \$80 million or $y >$ \$80 million. Even if groups of members try to form coalitions, the middle member's ideal point will win if each member votes sincerely. What are we to make of this?

Political power rests in the middle of the distribution of voters, that's what. The example we have just seen is a key theoretical result for majority rule decisions on one issue. This result is commonly called the *median voter theorem* (from now on, MVT). Now that we have explained just what is meant by "distance" for representing preference, we can address the technical details that make the MVT result true or conditionally true.

More careful definitions

Let's go back to the beginning. The general problem is that a society of N people must choose *one* position on an issue. This situation is common in public life: There cannot be two different defense budgets in a given year, for example, or two different minimum drinking ages. Assume (perhaps less realistically) that the issue in question can be separated from decisions on other issues. Further, let all members vote sincerely. What will happen? Can we predict what they will decide, as a group, given information only about their individual goals and the decision rule?

The first step is to specify how members rank alternatives. For simplicity, consider a *representative citizen i,* where i can be anyone in the society from 1 to N. Assume i has an *ideal point* x_i, or a unique[2] position strictly preferred to any other alternative. (Notice that for ideal points, and for most other purposes in this book, *subscripts* are used to identify *people,* so x_1 is person 1's ideal point, and x_i is i's ideal point.)

Finally, let $|x|$ denote the absolute value of a number x. For example,

$|-7| = 7$; $|56| = 56$. Consider two alternatives y and z, where both y and z are on the *same side* of x_i (this means symmetry is irrelevant). We can now define two relations, called "preference" and "indifference."

Preference

$$i \textit{ prefers } y \text{ to } z \text{ if and only if } |y - x_i| < |z - x_i| \qquad (2.2)$$

Indifference

$$i \text{ is } \textit{indifferent} \text{ between } y \text{ and } z \text{ if and only if } |y - x_i| = |z - x_i| \qquad (2.3)$$

The only innovation in these definitions is that now preference intensity varies with distance. Unlike more abstract preference functions, both these definitions clearly require that the issue being considered be continuous and unambiguously ordered along a single dimension (for more detail, see Appendix 2A). For the example we have been considering, the dimension is budget size, but the model is equally applicable to many other spatial settings.

Symmetry is *not* assumed in these formal definitions. Without assuming symmetry, we can define preference and indifference only for the case where y and z are on the same side of the ideal point. If preferences are symmetric, then (2.2) and (2.3) also describe i's evaluation of y and z when y and z are on *different* sides of x_i. This seems a powerful statement, but the power of the claim derives from the restrictiveness of the assumption of symmetry.

Now that indifference and preference have been defined for individual *i,* we can consider these same relations, and the distribution of ideal points $\{x_1, x_2, \ldots, x_N\}$ for many citizens, and see what happens when they have to vote.

The middle is the "median"

At the beginning of this chapter, we claimed that power resides in the middle of the distribution of voters. But just what does "middle" mean in large groups of voters? Social scientists use at least three different measures of the central tendency of a distribution:

Mean: The sum of the values divided by the number of values. In our subcommittee example, this is:

$$(10 + 50 + 80 + 100 + 150) / 5 = \$78 \text{ million}$$

Mode: The most frequently occurring value. Each of the five ideal points occurs just once in the subcommittee example, so the mode provides little guidance: The "distribution" has five modes.

Median: The middle value. The middle value, as we discussed, is $80 million, because there are two ideal points larger and two smaller than this value.

In *pairwise* majority rule elections (elections comparing exactly two alternatives), a proposal preferred by one half ($N/2$) of the voters is guaranteed to be at least a tie. If an alternative receives more than $N/2$ votes, it wins outright. This property suggests that the median captures an important part of what is meant by the "middle" in politics.[3] Simply put, *the median value beats or ties all other alternatives in pairwise majority rule elections.*

To see this, consider the definition of a median position:

- Choose an arbitrary x_i
- Let N_L be the number of citizens for whom $x_L \leq x_i$
- Let N_R be the number of citizens for whom $x_i \leq x_R$.

Then a median position can be defined as follows.

Median position. A position x_i is a *median position,* or x_{med}, if and only if:

(1) $N_R \geq N/2$

and

(2) $N_L \geq N/2$.

This definition, adapted only slightly from Duncan Black's formulation,[4] is cumbersome. But it is also general, because it encompasses odd and even N and situations where several voters share an ideal point. A more intuitive definition is given by Thomas Schwartz: "x is a median position if 'no majority of individuals have ideal points (peaks) to the left or to the right of x'" (1986, p. 87).

The difference between the two definitions has to do with the treatment of the median position itself. In our definition, N_L and N_R *both* encompass the ideal point(s) at the median position, so $N_L \geq N/2$ and $N_R \geq N/2$. In Schwartz's definition, no majority can have ideal points *strictly* to the left or right of the median position. The advantage of

Black's definition is that it measures directly the vote that the median position will receive in a majority rule election. For any proposal to the left of x_i, the median will receive at least N_R votes. For any proposal to the right of x_i, the median position will receive at least N_L votes.

Odd N, even N, and uniqueness of the median position

In some cases, such as for the congressional subcommittee example (Figure 2.3) the median ideal point was easy to identify. There were *three* members (D, C, and E) who had ideal points less than or equal to x_{med}. There were also *three* (E, B, and A) with ideal points at least as big as x_{med}. E is a median position, since $N/2 = 2.5$, $N_L = 3 > N/2 = 2.5$, and $N_R = 3 > N/2 = 2.5$.

The median is not always so easy to identify. For one thing, the median may not be unique. Suppose that E, whose ideal point is depicted in Figure 2.3, was absent on the day of the vote. Now $N = 4$, and *all positions between 50 and 100* (including $x_C = 50$ and $x_B = 100$) are "medians." Consider, for example, $x = 70$. There are two ideal points above 70 and two below 70. The same is true for 69, or 71, or 98.

If N is odd, and assumptions (a)–(c) are met, there must be a unique median. However, several voters may share this ideal point, so while there is a unique median there may be no unique "median voter." For example, suppose that $x_1 = 10$, $x_2 = 4$, $x_3 = -2$, $x_4 = 0$, and $x_5 = 7$ in a five-person society. Then the unique median position is 4, and x_2 is the median voter.

Now consider a seven-person society, where $x_1 = 10$, $x_2 = 4$, $x_3 = -2$, $x_4 = 0$, $x_5 = 7$, $x_6 = 2$, and $x_7 = 2$. N is odd, so the median position must be unique. A moment's thought shows this is true: There are five citizens with ideals larger than or equal to 2 and four with ideals smaller than or equal to 2. But there is no unique median voter, because the median position is held by two citizens, 6 and 7.

Once we allow citizens to share ideal points, it turns out that a unique median can exist even if N is even. Suppose $x_1 = 9$, $x_2 = 3$, $x_3 = 3$, and $x_4 = -2$. Clearly, $x_{med} = 3$. There is only one median position (though there are two "median voters"), even though the number of voters is even.

It is worth summarizing what we know about the existence and nature of median points, given the distribution of voter ideal points:

- If N is odd, then the *median is always unique.* This is true even if ideal points are shared. (See Exercise 2.6.)
- If N is even and no ideal points are shared, there is a *closed interval*[5] of medians.
- If N is even and some ideal points are shared, then there may be either a unique median or a closed interval of medians.

The median voter theorem

We are now in a position to state and prove the median voter theorem (MVT) and a corollary. The MVT result is based on the two assumptions discussed earlier:

(1) There is a single issue. This means that all the alternatives can be represented as points on one line, and each voter has an ideal point x_i on this line.
(2) Preferences are such that given two alternatives y and z *on the same side* of x_i, individual i votes for y if and only if $|y - x_i| < |z - x_i|$. This type of preference is called "single-peaked."

Median voter theorem. Suppose x_{med} is a median position for the society. Then the number of votes for x_{med} is greater than or equal to the number of votes for any other alternative z.

Proof. Suppose $z < x_{\text{med}}$. All ideal points to the right of x_{med} are closer to x_{med} than to z. But since x_{med} and z are both to the left of x_i, all voters with ideal points to the right of x_{med} vote for x_{med} versus z, by assumption (b). Likewise, all voters with ideal points exactly equal to x_{med} vote for x_{med}. Since x_{med} is a median position, there are at least $N/2$ voters such that $x_i \geq x_{\text{med}}$. So x_{med} receives at least $N/2$ votes, ensuring victory over any alternative to the left. The same reasoning applies if $z > x_{\text{med}}$.

To paraphrase: A median position can *never* lose in a majority rule contest. It can *tie* other alternatives if there is a closed interval of medi-

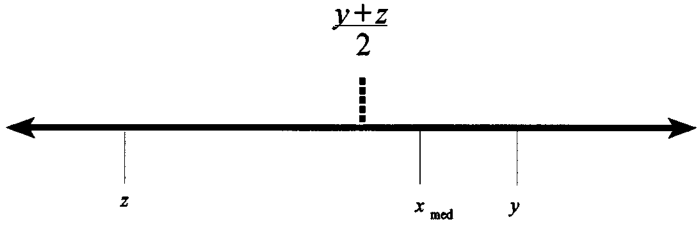

Figure 2.5. Midpoint between z and y is less than the median position.

ans, since any two median points will by definition each receive $N/2$ votes, but it cannot lose. The statement and proof of the MVT is probably best attributed to Duncan Black.[6]

Corollary to the median voter result

The corollary requires two extra assumptions:

(3) The society has exactly one median position x_{med}.
(4) Voter preferences around x_i are symmetric, so that every voter i prefers y to z when $|y - x_i| < |z - x_i|$, even if y and z are on opposite sides of x_i.

Corollary. If y is closer to x_{med} than z, then y beats z in a majority rule election.

Proof. Again, let $z < y$. Since y is closer to x_{med} than z is, it must be true that $(y + z)/2 < x_{med}$, whether y and z are on the same side of x_{med} or not (for an example, see Figure 2.5). The position $(y + z)/2$ is the point that is equidistant from y and z. Thus, all ideal points to the right of $(y + z)/2$ (i.e., the set of x_i such that $x_i > (y + z)/2$) are closer to y than to z, whereas all ideal points to the left of $(y + z)/2$ are closer to z than to y. Since x_{med} is unique, we know that the number of ideal points equal to or to the right of x_{med} is greater than $N/2$. These ideal points are closer to y than to z, because $(y + z)/2 < x_{med}$. Since preferences are symmetric, y receives a majority of the votes.

The MVT says that if the median position itself is one of the alternatives being compared, the median always wins. The corollary implies that for any other pair of proposals, the alternative *closer* to the median wins if preferences are symmetric.

What if preferences aren't single-peaked?

In the passage quoted at the beginning of this chapter, Aristotle noted that one source of instability of governments is wide variations in the distribution of wealth. He assumed that such differences caused disagreement over the basic structure of society: Poor people favor democracy, while the wealthy favor oligarchy. If Aristotle is right, such conflict could cause instability on a grand scale. There is a more common kind of instability in political processes, however, caused by the form of preferences people have over policies. This type of instability is observed when some people's preferences are not single-peaked.

If a significant proportion of voters have preferences that are not single-peaked, there may be no median voter, even if the other assumptions of the MVT are met. Black (1958) showed that single-peaked preference is akin to the assumption of *transitivity,* generally innocuous in economic choice but a crucial and restrictive assumption in the study of politics. Riker (1982) argues persuasively that the possibilities for genuine "popular rule" may be sharply circumscribed by the structure of preferences among voters. It is still not clear whether this incoherence and unpredictability extend to the aggregation of judgments, as opposed to preferences.[7]

It is hard to say who first discovered the difficulties that non-single-peaked preferences cause for the coherence of democratic processes.[8] There is no doubt, however, about the importance of the result: *If preferences are not single-peaked, it may be strictly impossible to arrive at a nonarbitrary outcome, or collective judgment, by majority rule.* This is not some minor mathematical curiosity, because it means that claims about the existence of the middle are only conditionally true.[9]

To see how preferences that are not single-peaked can affect democratic choice, we will consider two very different examples: (1) a simplified characterization of U.S. public attitudes toward United Nations (U.N.) involvement in Bosnia, and (2) the "what do we do now" problem of the Hun–Gats, from Chapter 1.

The U.N. in Bosnia

Imagine it is September 1995. Bosnian government forces are trying to break the siege of the Bosnian capital, Sarajevo. Bosnian Serb rebel forces, with the tacit support of the government of the Serbian Republic (which controlled most of the Yugoslav federal army forces and heavy weapons such as tanks and artillery), are shelling the city of Sarajevo. The rebels are blocking the delivery of truckloads of humanitarian aid by international relief agencies. Suppose (simplifying greatly) that attitudes of U.S. citizens about U.N. involvement in the Bosnian war can be organized into three groups.

Stay Outites. This group thinks that the fighting in the former Yugoslavia is simply a civil war. There is no international threat, unless one or more world powers become involved. They oppose any direct foreign military involvement. This group might allow the use of a limited number of troops to secure routes for humanitarian aid, but would prefer to stay out completely and supply only food and medicine to refugees.

New World Orderists. People with this preference profile consider the conflict to be a threat to the stability of international trade and stability. They are concerned that the war itself will spread beyond the Bosnian borders or that tensions over resolution of the conflict will raise tensions between Russia and the West. Their ideal solution is to insert U.N. peacemakers between combatants, along existing lines of control by both sides. They want to pressure Bosnian Serbs to give back captured territory, but do not advocate actual fighting by U.N. troops. The worst alternative would be to do nothing, because of the potential for unrestricted conflict to spread.

Serb Blamers. This group sees the rebels (and their Serbian sponsors) as the aggressors in the conflict. The Serb Blamers want to punish the Serbs for their attempts at "ethnic cleansing" and force them to relinquish all captured territory, by force if necessary. Second best would be to withdraw all U.N. troops and foreign citizens and end the arms embargo to allow the Bosnians to fight on a more equal basis. The worst solution would be to use U.N. troops to enforce the current (illegiti-

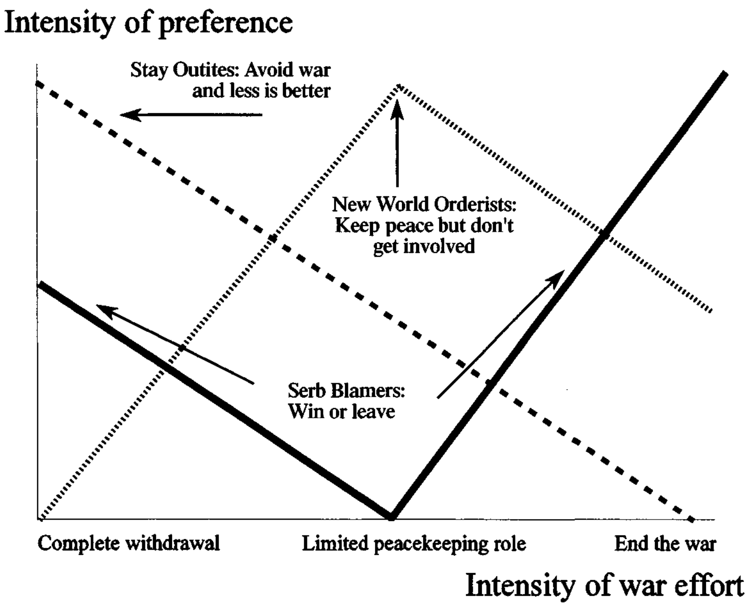

Figure 2.6. Preference profiles for New World Orderists, Stay Outites, and Serb Blamers for intensity of U.N. involvement in the former Yugoslav Republic.

mate) front lines as borders and prevent the Bosnians from lifting the siege of Sarajevo.

Figure 2.6 depicts the preference profiles of the three groups. The reader will note that while the preferences of Stay Outites and New World Orderists are single-peaked, the preferences of Serb Blamers are not. Serb Blamers prefer either of the extremes of the policy dimension to alternatives in the middle. Suppose that the U.N. Security Council is a committee composed of three people, a Serb Blamer, a New World Orderist, and a Stay Outite. The Security Council uses simple majority rule to decide the extent of U.N. troop involvement.[10] Though "intensity of involvement," the horizontal axis in Figure 2.6, is continuous (i.e., it is possible to make small changes in policy in either direction), we will consider only three policy alternatives: complete withdrawal, limited peacekeeping, and assault the Serbian rebel troops. The voters, alternatives, and their rankings are depicted in Table 2.2.

We can choose a starting point (status quo) arbitrarily. Suppose that

Table 2.2. *Rankings of alternatives by three-person committee*

	Member		
Ranking	New World Orderist	Stay Outite	Serb Blamer
Best	Limited action	Complete withdrawal	All-out war
Middle	All-out war	Limited action	Complete withdrawal
Worst	Complete withdrawal	All-out war	Limited action

the U.N. Secretary General has already started a "peacekeeping action," so the committee is meeting to decide if they want to change from peacekeeping to some other policy. The Stay Outite then proposes withdrawal. The Stay Outite and the Serb Blamer prefer withdrawal to the status quo (police action). The New World Orderist prefers the status quo. The withdrawal alternative wins, 2 to 1.

Now suppose the Serb Blamer proposes aggressive threats and air strikes to punish the rebels and take back their ill-gotten gains. Serb Blamer and New World Orderist both vote to stop the war by assaulting the rebels, so aggressive action beats withdrawal, 2 to 1. But the New World Orderist is upset at this escalation of tension and proposes going back to passive peacekeeping. The Stay Outite likes that idea, and of course the New World Orderist does too. Peacekeeping, the original status quo, beats aggressive intervention, 2 to 1, and is reimplemented, while all the world wonders.

What happened here? The sequence of majority rule votes moved us from the original status quo, passive peacekeeping, through each of the other alternatives, right back to where we started! As Figure 2.7 shows, this "cycle" has no end. Majority rule votes on pairs of alternatives will never settle on one outcome in this scenario because preferences aren't single-peaked.

The world was repulsed by its inability to choose a policy in dealing with the war among the former republics of Yugoslavia in the mid-1990s. Ultimately, the international political system was strained almost to the breaking point, and many questioned whether it is possible for an international agency to act effectively. No doubt some of these criticisms were justified. Yet it is hard to avoid noticing that the U.N. ap-

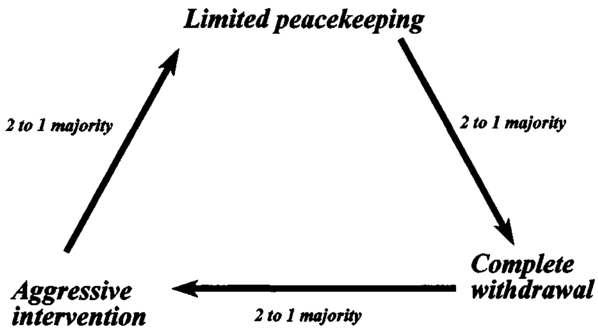

Figure 2.7. The majority rule voting cycle over intensity of U.N. involvement in Bosnia.

peared incompetent because the "middle course" was untenable. A sizable proportion of the world's nations appeared to believe that the U.N. should either go in strong or get out, yet very few of those nations showed much willingness to commit troops to such an intervention.

This is a common problem in majority rule cycles. There is never a majority clearly in favor of any policy for very long. While the political conflict was much more complex than our depiction here, there is some evidence (see, e.g., Russett and Shye, 1993) that *precisely* the sorts of preferences that would lead to a cycle in majority rule voting outcomes can be found in public opinion surveys. In the 1960s, Barry Goldwater and his supporters made very explicit arguments that the United States should either fight an all-out war in Vietnam or withdraw all troops immediately. The "middle" course, a limited action where the United States provided a steadily escalating number of advisors and war materiel to the Army of the Republic of Vietnam, was (for the Goldwater faction) the worst alternative. The preferences implied by this perspective are not single-peaked; the outcome implied by a significant faction with non-single-peaked preferences is a majority rule cycle.

The Hun–Gats

What of our undecided Hun–Gats? The reader will recall we left this tribe of hunter–gatherers sitting around their campfire, trying to decide

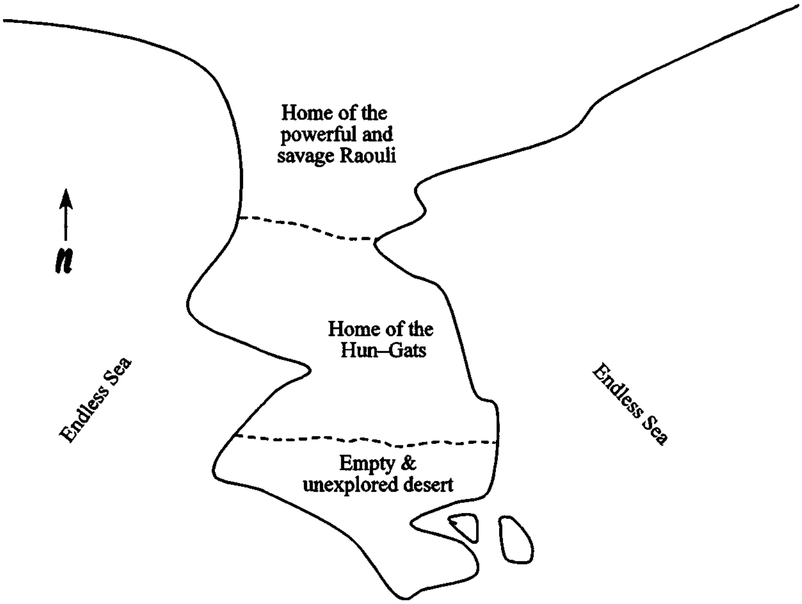

Figure 2.8. The spatial decision problem of the Hun–Gats.

what to do. A map of the Hun–Gats' situation appears in Figure 2.8. Assuming they don't split up, the Hun–Gats must choose among three mutually exclusive courses of action:

- Go north and face the warlike and cannibalistic Raouli, who consider trespassers a chance for a bodacious barbecue. The land of the Raouli is fertile and filled with game and good water.
- Stay at home in the land of the Hun–Gats near Muddy River. There is very little game, fish, or edible plant life left in this "home," but they have water and know the area. Also, nobody else wants it, so they don't have to fight.
- Go south and explore the unknown desert at the end of the peninsula. There may be sources of water far to the south, and there may be game and edible plants around these oases. There also may be nothing to drink or eat.

The Hun–Gats all have identical "preferences," in the sense that they all (1) want to live in a place where they can find food and water and

Table 2.3. *Ranking of alternatives by Hun–Gat factions*

	Faction		
Ranking	Raouli will kill us, stay far away	Anywhere but here	No place like home
Best	Go south	Go north	Stay here
Middle	Stay here	Go south	Go north
Worst	Go north	Stay here	Go south

(2) want to avoid being brained by heavy clubs and then eaten. However, they disagree (i.e., have differing judgments) about the best way to achieve this goal.

There are three distinct factions in the tribe. Each faction has a preference profile induced by a particular judgment about the implications of different courses of action. The three groups and their preferences are described in Table 2.3. The timid "Raouli will kill us!" faction fears the other tribe and believes the farther away they live the better. This group wants to try going south, with staying here second best, and moving north the worst alternative.

The "Anywhere but here!" faction is made up of the younger males of the tribe. They are tired of subsisting near the Muddy River. They are also convinced the Raouli can't be as fierce as the timid old people claim. For this faction, the best action is to move north. Next best is to move south and explore the desert. Worst is staying here and starving by inches.

The "No place like home!" faction thinks the land of the Hun–Gats is not so bad, because they are terrified at the prospect of dying of thirst in the desert. Their best alternative is to remain at home. Second best is to move North, where at least the land is familiar and fertile.

If these groups are of approximately equal size, the result will once again be a cycle. The reason is that the preference profile of the "Anywhere but here!" faction is not single-peaked.[11] They think that the middle alternative is the worst. If the status quo is remaining where they are, then a majority of the tribe (those afraid of the Raouli and those who prefer anything to staying home) will vote to move south. But then a majority (the "Anywhere but here!" and the "No place like home!"

factions) will vote to move north. Finally, a majority will prefer to stay home rather than go north, bringing us back to the original status quo.

There is no general will. There is no way out for the Hun–Gats, no matter how earnestly they try to implement Rousseau's optimistic instructions.[12] For a coherent middle to exist, the preferences of participants must be single-peaked. To put it another way, if a significant proportion of citizens think the "middle course" is wrong, democracy can be unstable and even incoherent.

The discussion in this section represents a slightly different statement of the thesis advanced in William Riker's (1982) *Liberalism against Populism.* In this famous book, Riker argues that too much faith has been placed by political theorists in the power of participatory, or populist, democracy:

> What is different between the liberal and populist views is that, in the populist interpretation of voting, the opinions of the majority *must* be right and *must* be respected because the will of the people is the liberty of the people. In the liberal interpretation, there is no such magical identification. The outcome of voting is just a decision and has no special moral character. (p. 14; emphasis in original)

Riker's interpretation of voting is simple: Elections are a way to control officials and nothing more. Some observers have called this conclusion unduly pessimistic or have objected that the normative conclusions make citizens skeptical about government.

This critique of Riker's conclusions, however, ignores the *positive* character of his argument. The nonexistence of a single best alternative is not of itself good or bad. Rather, cycles are a generic property of majority rule decision processes, under some circumstances. As long as the "middle" either does not exist or exists conditionally and ephemerally, a faith in the ability of majorities *always* to discover objective truth is dangerous and misleading.

The best counterargument for Riker's indictment of populism is probably a focus on arriving at judgments rather than choosing based on judgments already made. Coleman and Ferejohn (1986) point out that voting may be the only way of making good judgments. Their defense of "populism" rests on its ability to identify the right thing to do, just as Rousseau argued. "The desirability of a voting rule will then depend on its reliability – the extent to which the collective judgments it generates converge with what is in fact the correct judgment" (Coleman and Ferejohn, 1986, pp. 16–17). This argument is extended and qualified by Ladha (1996) and Ladha and Miller (1996). A key difficulty with the argument is that it requires choices among exactly two mutually exclusive alternatives. If there are three or more alternatives, then voting on judgments is subject to the problem of cycling majorities, as the Hun–Gats example shows.

Conclusion

In this chapter, we saw both *definitions* and *results.* It is important to keep these two building blocks distinct, as definitions are in effect the premises of arguments, and results are conclusions based on those premises. One key definition was that of *preferences* over a *space of alternatives.* The idea is that voting choices can be predicted by comparing *distances* from an ideal point to an alternative. The two basic results of this chapter can be summarized by restating the MVT and its corollary requiring the additional assumption of symmetric preferences.

> Median voter theorem: *A median position cannot lose to any other alternative in a majority rule election.*

The MVT implies that the middle of the distribution of citizen preferences in a society holds a privileged position in political competition. If the median position is unique (identified with just one voter), we call this very important person the "median voter."

> Corollary: *In a comparison of alternatives that are not median positions, the alternative closer to the median wins.*

The corollary shows that under certain conditions (unique median and symmetric preferences), closeness to the middle is the basis of political power. Consequently, even if the status quo is different from the median, there are pressures for new alternatives to move toward the center until a median position is reached.

Together, these two conditions establish an important benchmark in what we can say about the likely outcomes of political conflict. More important, we have given some formal support for the intuitive claim that the "center" of the society is where political power lies. As we have pointed out repeatedly, this notion of the center as the source of political power is widely held and is taken for granted by the media and political operatives alike.

In the preceding section, we showed that an additional restriction on preferences, called "single-peakedness," is paramount. One of the most famous and influential arguments about the possibility of "populism," or determinate democratic choice in the face of disagreement, was Riker's (1982). Riker's work brought together the results of many other social choice theorists. The primary conclusion of this body of research is that unless preferences are single-peaked, there may not be a viable "center" that leads to stable policy choices. In this view, fear of tyranny by the majority is perfectly legitimate. In the will of the majority there does not, of necessity, inhere the moral force attributed to the "general will" by Rousseau and others.

In the next chapter, we will generalize the MVT to allow for more than one dimension. It will turn out that the simple intuition about distance and voting that led us this far is not reliable. In two or more dimensions, the "center" may not exist at all, even if preferences of individuals are single-peaked.

Appendix 2A: "Space" means ordered dimensions and continuity

We have been using a spatial representation of preferences, but have not properly defined just what it means for issues to be represented as dimensions and for preferences to be points in a "space." The use of a space to represent preferences is usually claimed to have two requirements:

- *Alternatives are ordered:* It must be possible to arrange policy alternatives along a dimension, from "less" to "more." Further, this perception of order must be shared, so that all voters are choosing from the same dimension.
- *Policy space is continuous:* Technically, a space is continuous if between any two feasible alternatives there lies another feasible alternative.

What if dimensions are not ordered?

The assumption that an issue can be thought of as an ordered space may be difficult to sustain for some issues, where there are multiple alternatives distinguished as categories rather than as measured levels of a property. In the Hun–Gats example, the three alternatives (north, stay, south) are *not* ordered. Nothing important would be changed if the choices were north, stay, east, for example. But this means that "preference functions" can handle nonordered alternatives perfectly well. If a Hun–Gat prefers to go north rather than staying home, and wants least of all to go south, all we need is a (transitive) preference function such that:

$$U(\text{north}) > U(\text{stay}) > U(\text{south})$$

Such a preference function is not strictly "spatial," of course, so we cannot use distance between alternatives in defining which alternative is preferred. However, as we have pointed out earlier, there are many ways of representing preference other than using distance. If alternatives are not ordered, it is perfectly possible to define preference functions of a more abstract sort. This more general type of analysis is well developed in the advanced formal theory literature, including Schofield (1985) and Schwartz (1986).

Importantly, the reader should note that defining "single-peakedness" does *not* require ordered dimensions. If one arrays three or more alternatives randomly, all that is required is that a substantial proportion of the voters consider the middle alternative to be worse than at least one of the alternatives on both sides, *no matter what the order of the alternatives.* The question of which preference profile violates single-peakedness then becomes arbitrary, of course. Nonetheless, single-peakedness in the set of preferences is violated, and the result is a cycle in majority rule outcomes.

What if alternatives are not continuous?

The second assumption, *continuity,* is met (at least approximately) for most policy debates on taxes or spending, since between (for example) proposals of \$1,000 and \$1,500 lies another alternative: \$1,250; between \$1,000 and \$1,250 lies \$1,125; and so on. However, for other policies (such as abortion rights and capital punishment), continuity may not be a useful assumption.

Critics of the spatial model have pointed out that continuity may not hold, particularly in elections involving the general public. In some ways this is true, as we shall see when we discuss alternative theories later in this book. However, the criticism may be overblown, because critics have confused two types of assumptions: *necessary* assumptions and *sufficient* assumptions. It is certainly true that spatial theorists often assume a shared, continuous ordering over policy alternatives, because this assumption is sufficient to simplify exposition of the model. But these assumptions (particularly the continuity assumption) are not necessary for the logic of the model to be applied.

EXERCISES

2.1 For a given issue, let $x_1 = 2$, $x_2 = 6$, $x_3 = 9$, $x_4 = 9$, and $x_5 = -2$. What is the median position of the five voters? What is the mean, or average, position?

2.2 Suppose this five-member committee (from Exercise 2.1) is asked to vote (sincerely) on two alternatives, $y = 8$ and $z = 3$. If preferences are symmetric, which alternative will win?

2.3 Suppose $x_6 = 7$ is added to the committee. What is (are) the median position(s) now? If the new, six-member committee is asked to choose between $y = 8$ and $z = 3$, what happens?

2.4 For another issue, let $x_1 = -3$, $x_2 = 21$, $x_3 = 7$, $x_4 = 6$, and $x_5 = -4$. What is the median position and mean ideal point of these five voters? Suppose x_3 is replaced by $x_3' = 25$. Which changes more, the mean or the median position?

2.5 Given Aristotle's discussion of the distribution of citizens that makes for the best society, is "mean" an accurate description of what he meant?

2.6 Three different measures of the central tendency of a distribution were discussed in the chapter: mean, mode, and median. For what sorts of distributions are these three concepts identical?

2.7 The median income for a family of four in the United States is about $28,000. The mean income for a family of four is about $34,000. Which is a better measure of "middle class," as you understand that term? Draw a distribution of income in which the mean is larger than the median. Do you think this type of distribution also describes the patterns of incomes in other countries?

2.8* Prove the following statement: "If N is odd, the median is always

Note: Exercises marked * are advanced material.

a single point, and not an interval." (*Hint: Assume an interval of ideal points exists, and then use the median voter result proved in this chapter to show that an interval of medians implies N is even. It may help to remember that any integer multiplied by two is even.*)