8

Winners Plus One: How We Get Votes from Seats

Now we literally deduce votes from seats, in line with the book's title. We take a completely novel step and extend the Seat Product Model to votes distribution. This transition was hard to come by. For instance, the chapter on "The institutional impact on votes" in Taagepera (2007) is in retrospect a total flop, and the approach to estimating votes fragmentation taken by Li and Shugart (2016) lacked a theoretical foundation. Yet the additional assumption needed will be seen to be of utmost simplicity.

When a given number of parties wins seats in a representative assembly, how many more are likely to try their luck? How are these two numbers connected logically? If a logical model can be proposed, does it agree with the empirical average pattern? We can expect, of course, that the actual worldwide data will show variation over a wide range. In fact, we can expect any data on vote-earning parties to show more variation than comparable data on seat-winners, for the same reason that we argue for starting the analysis with the seats and only then extending votes: the seats are ultimately constrained by the institutions, whereas the votes actually are not. Whatever total number of seats comprise the assembly, we can be certain that there are no more parties than this number that gain representation. But how many others may run, and earn votes? The only constraint is that obviously there could not be more parties earning votes than there are voters, but that observation is hardly helpful. The extent to which the number of parties earning votes is greater than that winning seats is softly constrained, and only by the willingness of voters to tolerate "wasted votes" and the elites who form parties to keep running and losing.

Despite the expectation of wide variation in the number of electoral parties, the average value for a given country or a given institutional setup can offer a useful benchmark. When establishing a logically expected number of parties competing, we implicitly also ask: *Given the number of parties that won seats, was the number of parties competing for seats in a particular country unusually high or low, or as expected*? If there is marked deviation from expectation, then one may wish to look for the reason. In the absence of such a benchmark, anything goes and nothing gets explained.

 https://doi.org/10.1017/9781108261128.009

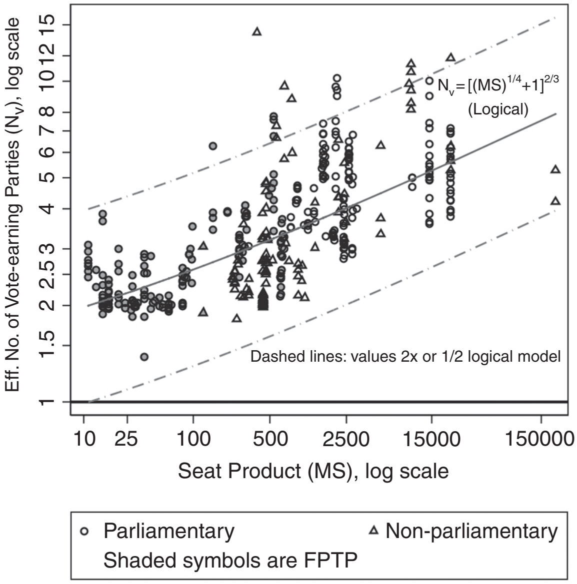

FIGURE 8.1 The effective number of vote-earning parties and the Seat Product

As a preview to what this chapter is about, we offer Figure 8.1. This graph shows how Seat Product, *MS*, affects the effective number of *vote-earning* parties (N_V). The data points represent 285 individual elections in thirty-two countries. The solid curve shows the logically based model we are going to develop:

$$N_V = [(MS)^{1/4} + 1]^{2/3}. \tag{8.1}$$

This equation is a good deal more complex than the one for seats, N_S=$(MS)^{1/6}$. This is the nature of cumulative science: we start with simple building blocks like N'_{S0}=$M^{1/2}$ (Equation 1.1, tested in Chapter 7) and build up complex structures. The solid curve in Figure 8.1 that follows Equation 8.1 is visibly a good fit, even though it is not a regression estimate.

The dashed curves represent observed values of N_V twice or one-half the prediction of Equation 8.1. Most cases are well within the range delimited by the dashed curves, but there are a few elections that are outside the range, on the high side. Several of these are presidential systems (depicted with triangles). We will discuss the reasons for why presidential systems are sometimes more fragmented than parliamentary systems in Chapter 11.

Votes are only indirectly and incompletely constrained by a purely institutional input like the Seat Product (*MS*). Therefore, we would expect more scatter for vote-based N_V than for seat-based N_S in Figure 7.1. Surprisingly, scatter is only slightly heavier here.

The amount of scatter should not detract from the bigger picture, which is that the overall fit is good. The degree of fit in Figure 8.1 shows that we can make progress in predicting average outcomes from institutional inputs. We discuss in the next section how we build the model just previewed; a later section of this chapter subjects it to a regression test on simple systems. Complex systems will be discussed in passing in this chapter, but not systematically analyzed till Chapter 15.

"PLUS ONE," BUT NOT M+1

The now fairly well-established idea of an "*M*+1 rule" for the number of "serious" or "viable" *candidates* offers a useful starting point. This rule was elaborated by Reed (1990, 2003) and Cox (1997). A recent review summarizes it as follows: "The *M*+1 rule, whereby the number of parties or candidates in a district is capped by the district magnitude (*M*) plus one" (Ferree, et al., 2013: 812). This "parties *or* candidates" becomes problematic when *M* is large and the number of parties is thus sure to be smaller than *M*.

The original notion behind the *M*+1 rule was that it generalized the so-called Duverger's law. When *M*=1, it hardly matters whether we think of the competing agents as parties or candidates – each party generally presents just one candidate, so the concepts merge. "Duverger's law" predicts two parties (two candidates) when *M*=1. Reed's contribution was to say that under single nontransferable vote (SNTV) rules (see Chapter 3) formerly used in Japan, the number of "serious" candidates also was near *M*+1. Reed operationalized "serious" via the effective number, and he explicitly meant candidates, not parties. Under SNTV, larger parties typically nominate more than one candidate per district (but fewer than *M* – see Chapter 13). Because there are *M* winners, and they are those with the highest individual vote totals regardless of party, there are *M*+1 viable candidates,[1] according to Reed's argument. In other words, the district has *M* winners, and one close loser. The rest tend to fall farther behind, at least under certain conditions specified by Reed and extended by Cox.

Cox himself recognizes the limits of applicability of the *M*+1 rule to larger magnitudes and to the (effective) number of parties, saying it specifies an "upper bound" rather than a prediction. It is obvious that it would be a poor prediction of the number of parties. For instance, in the single nationwide district of *M*=150 seats in The Netherlands, 151 parties is an overkill.[2] Above some moderate level of *M* – Cox (1997: 100) suggests "about five" seats – the *M*+1

[1] Cox (1997: 99) defines viable as "proof against strategic voting."

[2] By the same token, 151 viable candidates presumably would be too low, as many a party that wins seats also likely has one or more viable losing candidates. (The Netherlands uses a "flexible" list – see Chapter 6 for definition – and thus individual candidates can enhance their own viability if they successfully campaign for preference votes.)

rule is not the principal factor limiting proliferation of parties (Cox 1997: 122). Further, Cox (1997: 102n) states that the application of the M+1 rule to lists in PR systems is “substantially less compelling.” Thus, he concludes, “something else” other than the strategic voting that leads to M+1 viable candidates under FPTP and SNTV must be at work when we are concerned with the number of vote-earning parties in PR systems (Cox 1997: 110).[3]

For making sense of the number of viable vote-earning parties, we agree that the “plus one” is an important logical building block. However, we suggest the “plus one” should be added to the *number of seat-winning parties*, not the total number of seats in a district. The objective of this chapter is to understand how electoral systems shape the effective number of vote-earning parties for elections to national assemblies. The Reed and Cox notion of viability being conditioned by competition for seats is logically correct. We take this key insight a step further. As already sketched in Figures 1.4 and 7.5 the total numbers of seats available in a district and nationwide impose physical constraints on the number of winning parties. No district can have more than M winning parties (or fewer than one), nor can the assembly have more than S winners (or fewer than one). This was the logic behind the Seat Product, MS, explained and successfully tested in Chapter 7.

Unlike seat-winning parties, however, the number of vote-earners is not strictly constrained. It is challenging to specify *ex ante* how tolerant of “wasted votes” the country’s politicians, election financiers, and voters will be. While we know the actual upper bound on the number of seat-winning parties (M in any district, S nationwide), the upper bound on parties that might be viable vote-earners is not knowable. However, it is surely constrained by the number of parties with seats. The number of vote-earning parties clearly is not lower than the number with seats. It likely tends not to be greatly higher, as most voters and other actors will not long tolerate voting for a hopeless party.

With this logical foundation, we can specify a new concept that will be crucial to this chapter and again in Chapter 10 (which deals with district-level effects): *the number of pertinent vote-earning parties*. We will use the term, “pertinent,” rather than “viable,” in order to differentiate our concept from those of preceding authors. We mean something subtly, but importantly, different. We are not asking whether some number of parties, beyond the winners, is perceived as sufficiently viable so as not to suffer from strategic defection. We are asking how many are sufficiently important to contribute to our prediction of the effective number of vote-earning parties (N_V). They are “pertinent” if counting them helps us estimate N_V. Thus our logical basis for model building is:

The number of pertinent vote-earning parties (N_{V0}) is the number of seat-winning parties (N_{S0}), plus one.* More briefly: *strivers are winners plus one.

[3] Cox goes on to suggest the answer is “economies of scale” that lead actors to “coordinate” around a smaller number of *party lists*.

The statement defines N_{V0} as deriving from a quantity that we already know can be logically predicted: N_{S0} (as we saw in Chapter 7). The rest of the logical model building, leading to Equation 8.1, then follows, as we shall see in the next sections of this chapter. While the M+1 rule is restricted to a single electoral district, this "winners plus one" model can also apply to electoral parties nationwide. In this chapter we will test the plausibility of this notion at the national level; we extend it to the district level in Chapter 10.

We will test "winners plus one" using the known number of assembly parties (of any size), which we have already shown in Chapter 7 to be a quantity about which we can build systematic models. Perhaps surprisingly, given how blunt the notion of winning parties, plus one, might appear, it works well. Given that it does, we can also connect the concept later in this chapter to the purely institutional input, the Seat Product, MS, thereby providing the logical basis behind Equation 8.1.

Connecting Vote-Based and Seat-Based Effective Numbers

We show the hypothesis "successful parties plus one" in bold, because this is a major advance first published in this book. Several later chapters hinge on this simple relationship. It applies to N_0, the *actual* number of parties:

$$N_{V0}=N_{S0}+\mathbf{1}. \tag{8.2}$$

It does *not* apply to the *effective* numbers. What we add is one real party that barely fails to win a seat, not an abstract "effective" quantity. This equation is the crucial link where the average patterns for **votes** are deduced **from** the average patterns for **seats**, and hence from the purely institutional Seat Product MS.

The reasoning that took us in Chapter 7 from the actual number of seat-winning parties to the largest seat share and then on to the effective number of assembly seats remains valid when we substitute votes for seats. So, in parallel with $s_1 = N_{S0}{}^{-1/2}$ and $N_S = s_1{}^{-4/3}$ (Table 7.3), we get

$$v_1 = N_{V0}{}^{-1/2} \quad \text{and} \quad N_V = v_1{}^{-4/3}$$

for the largest vote share, and the effective number of vote-earning parties.

Figure 8.2 compares the fit of $N_S = s_1{}^{-4/3}$ for seats in the left panel (as reproduced from Figure 7.4) to the fit of $N_V = v_1{}^{-4/3}$ for votes in the right panel. Forbidden areas are shown in both panels, corresponding to those depicted in Figure 7.4. Votes show slightly more scatter, but the agreement with the logical model is still very good.

From $v_1 = N_{V0}{}^{-1/2}$ and N_{V0}=N_{S0}+1 it follows that

$$v_1 = (N_{S0} + 1)^{-1/2}.$$

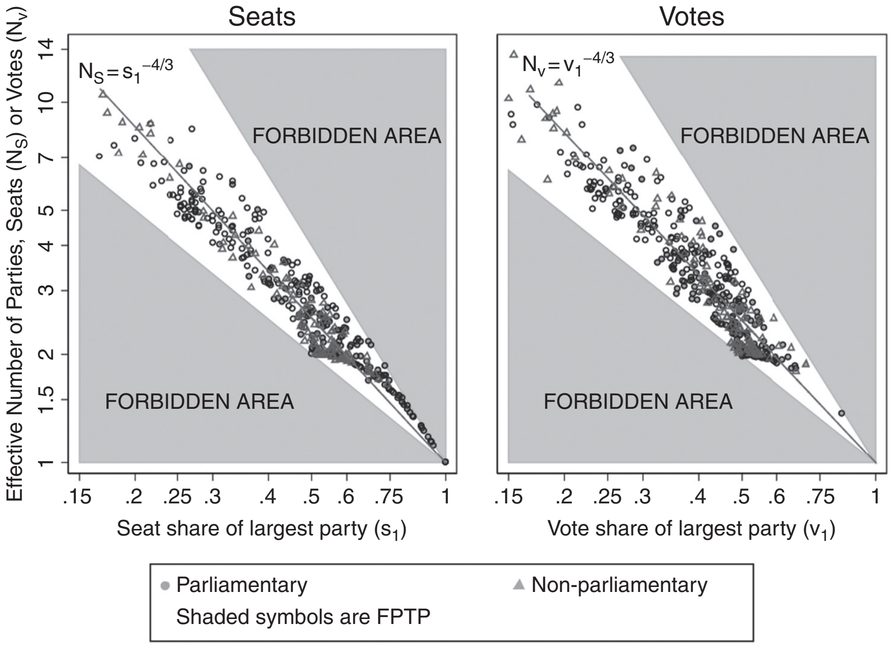

FIGURE 8.2 How the effective numbers are related to the largest shares of seats and votes

Figure 8.3 compares the impact of the number of seat-winning parties (N_{S0}) on the largest seat and vote shares, respectively. The left panel reproduces the left panel of Figure 7.3 for the largest seat share. The right panel shows the impact of N_{S0} on the largest vote share. Due to the "+1" the relationship is curved. The degree of fit is comparable for seats and votes, confirming that the model $v_1=(N_{S0}+1)^{-1/2}$ fits the data.

This is an important connection, as we are unable to measure "actual vote-earning parties" directly in a meaningful way. We surely do not mean any party that earns even a single vote the same way that for "actual seat-winning parties" we do indeed mean to include even a party that wins only one seat. Thus we need a "phantom" quantity for how to measure how many parties are serious – or, as we call them, *pertinent*. This is what Equation 8.2 represents – the number of pertinent vote-earning parties, represented by the number of winners with at least one seat (N_{S0}) plus one serious striver that came up just short.

If the logic is on the right track, it means we can also connect the effective numbers for votes and seats, using basic algebra. Inserting $v_1=(N_{S0}+1)^{-1/2}$ into $N_V=v_1{}^{-4/3}$, we get $N_V=(N_{S0}+1)^{2/3}$. Reversing $N_S=N_{S0}{}^{2/3}$ (Table 7.3) leads to $N_{S0}=N_S{}^{3/2}$. Inserting this into

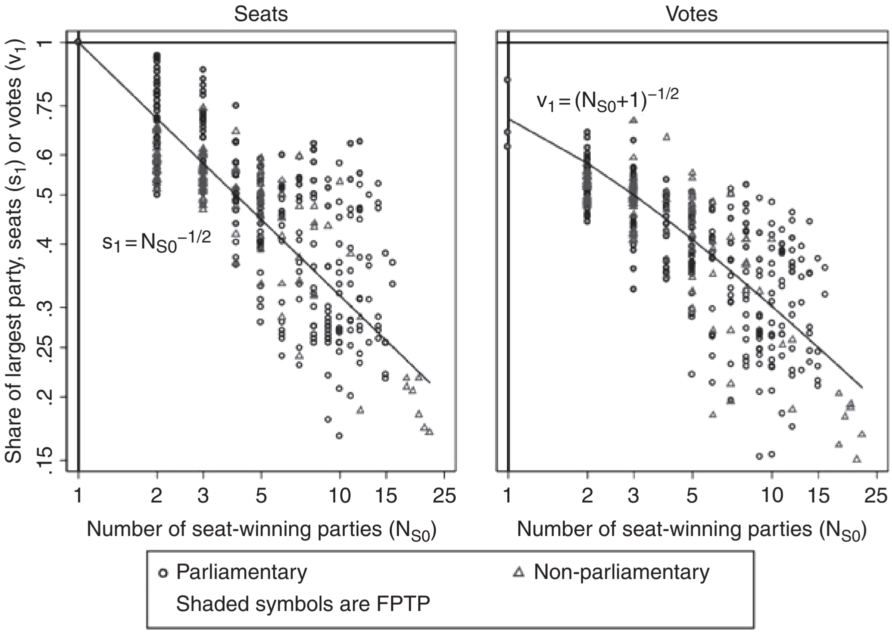

FIGURE 8.3 How the largest seat and vote shares relate to the number of seat-winning parties

$$N_V = (N_{S0} + 1)^{2/3}$$

yields

$$N_V = [N_S{}^{3/2} + 1]^{2/3} \tag{8.3}$$

Equation 8.3 offers a powerful test of the underlying logic of how we get votes from seats, bypassing the major uncertainties surrounding the "real" value of N_{V0} in Equation 8.1. The effective numbers, N_V and N_S, can be measured unambiguously – at least in principle.

In Figure 8.4 we plot the data for a visual test of Equation 8.3. We plot several institutional variations with different symbols, in order to see if the relationship between N_V and N_S is somehow fundamentally different when the rules are different. The symbols differentiate FPTP (M=1), nationwide PR (M=S), two-tier PR systems, as well as presidential democracies.[4]

The graph has the effective number of seat-winning parties (N_S) on the x-axis and the effective number of vote-earning parties (N_V) on the y-axis, both on logarithmic scales. The light gray line represents N_V= N_S. Visibly, very few data points fall below this equality line, although many come close when the number of parties is large.

[4] We do not further differentiate the presidential cases by electoral system.

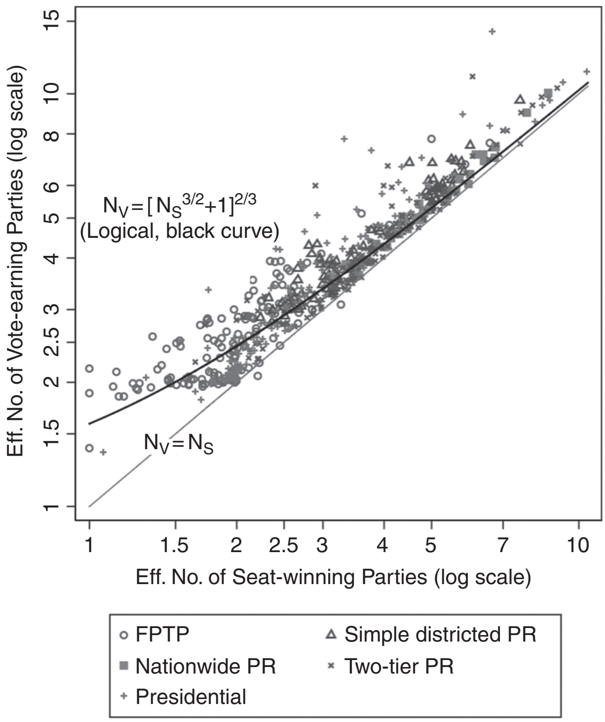

FIGURE 8.4 The effective numbers of parties, votes versus seats

The curve plotted in black corresponds to the logical model, $N_V = [N_S{}^{3/2} + 1]^{2/3}$. We see that the fit is strong. In fact, it is so strong that few cases, *regardless of electoral system or executive type*, are outside of a narrow band that follows the logical model. The logical model, Equation 8.3, clearly fits the pattern well, thereby offering support for our theory of "strivers are winners, plus one."

Among the relatively few cases that are outside the main data cloud, it is noteworthy that many are presidential systems and a few have two-tier rules. (We will discuss the additional considerations that affect presidential democracies in Chapters 11 and 12.) Moreover, there is some tendency for N_V to be higher than expected in those M=1 systems in which N_S is less than about 1.5. More noteworthy, however, is that M=1, districted PR, two-tier, and M=S (nationwide PR) systems are intermixed. It may seem surprising that less proportional electoral systems do not have a tendency to have N_V substantially greater than their typically low N_S, but in fact, Equation 8.3 accurately assesses the relationship of votes to seats independent of electoral system. (We show regression results later.) Thus the regression and graph both support our logical model – "strivers are winners plus one."

So far we have considered the fit of Equation 8.3, which has the effective number of seat-winning parties on the right hand side: $N_V = [{N_S}^{3/2} + 1]^{2/3}$. However, what we really should ask is, can we connect N_V to the Seat Product, *MS*? Of course, we have already spoiled the suspense! We showed in Figure 8.1 at the beginning of this chapter that such a connection works. In the following section we discuss this finding in more detail.

Predicting the Effective Number of Vote-Earning Parties: Are Institutions Sufficient?

If we are able to connect N_V to the Seat Product (*MS*), as we previewed as a promising task at the start of this chapter, it would permit an estimation of the fragmentation of the vote in a country based solely on its institutions. This is advantageous because such a prediction would be based on inputs that are, in principle, subject to design and reform: the average district magnitude and the assembly size. Given that we have previously established $N_V = [{N_S}^{3/2} + 1]^{2/3}$ and, even earlier (Chapter 7), $N_S = (MS)^{1/6}$, we expect Equation 8.1 to result for simple systems[5]:

$$N_V = [(MS)^{1/4} + 1]^{2/3}.$$

This was the curve plotted in Figure 8.1. Similarly, it follows from $v_1 = (N_{S0} + 1)^{-1/2}$ that the largest vote share can be estimated from the Seat Product:

$$v_1 = [(MS)^{1/4} + 1]^{-1/2}.$$

However, data plot in Figure 8.1 showed some nontrivial scatter. In particular, there were several countries with a tendency to have elections with $N_V >> [(MS)^{1/4} + 1]^{2/3}$. Such cases raise the question of whether we should consider inclusion of the factor of ethnic diversity. After all, it is generally agreed by other authors (reviewed in Chapter 7), that countries with greater social diversity have a systematically higher effective number of parties, at least if they combine such diversity with "permissive" electoral institutions (i.e., high *MS*). We will present tests that include ethnic diversity in Chapter 15; we find that the impact of this additional factor is less than commonly believed, but it does help understand some countries' party systems. For now, we stick to the institutional variables, to uncover the generalizable patterns that form a baseline against which country-level deviation can be compared.

[5] Extension to two-tier systems will be done in Chapter 15.

Statistical Testing of the Connection Between the Effective Numbers of Parties for Votes and Seats

In this section, we begin undertaking empirical testing of the model that ultimately leads to Equation 8.1, introduced at the beginning of this chapter. In the preceding subsection, we proposed Equation 8.3, $N_V = [{N_S}^{3/2} + 1]^{2/3}$. We can test it by running the following OLS regression:

$$\text{Log}N_V = \alpha + \beta\log({N_S}^{3/2} + 1).$$

We should obtain a coefficient of β=2/3, and a constant, α=0. What we actually find is remarkably close. First, if we run it on just the parliamentary systems with the simple electoral systems – the set to which the model is designed to apply – we obtain:

$$\text{Log}N_V = 0.00093 + \mathbf{0.687}\ \log({N_S}^{3/2} + 1)\ \ [R^2 = 0.920,\ 288\ \text{obs.}].$$

This is a remarkably good fit, confirming the logic behind Equation 8.3. We can then double the number of observations by including presidential democracies and two-tier proportional electoral systems (as defined in Chapter 3). The result is:

$$\text{Log}N_V = -0.0322 + \mathbf{0.722}\ \log({N_S}^{3/2} + 1)\ \ [R^2 = 0.904,\ 598\ \text{obs.}].$$

Now we have added a large number of observations where we could expect the distribution of votes to parties to be less adjusted to the seats – either because the assembly election is not also the source of executive authority (due to direct presidential election) or because nationwide compensation from a second tier may make voters less aware of institutional constraints. Yet the inclusion of these cases only slightly alters the fit of the empirical regression to our logical model.[6]

Statistical Testing of the Connection Between the Effective Numbers of Vote-Earning Parties and the Seat Product

Following our usual procedure, we can test Equation 8.1 as follows:

$$\log N_V = \alpha + \beta\log[(MS)^{1/4} + 1].$$

We should find α=0 and β=2/3. The right-hand side of this equation is our proxy for the *number of pertinent vote-earning parties* (N_{V0}), logged. As explained earlier in the chapter, N_{V0} cannot be measured directly, hence we estimate it from the Seat Product.

[6] If we include the two-tier systems, but discard the presidential systems, the coefficient is 0.689. Thus the complexity of two-tier allocation does not affect the accuracy of the result, even though some are visible outliers in Figure 8.4. Rather, it is presidential systems that perturb it somewhat – as we would already expect from these systems' greater visible scatter in Figure 8.1, at the start of this chapter.

TABLE 8.1 *Regression for the effective number of vote-earning parties* ($\mathbf{N_V}$)

	(1) Institutions only parl., simple	(2) Institutions only all execs., simple
log[(MS)¼ +1]	0.679***	0.706***
Expected: 0.667	(0.0722)	(0.0747)
	[0.533–0.825]	[0.557–0.855]
Constant	0.0148	–0.0275
Expected: 0.000	(0.0481)	(0.0550)
	[–0.0826–0.112]	[–0.137–0.0823]
Observations	285	389
R-squared	0.618	0.486
rmse	0.108	0.132

Robust standard errors in parentheses.
95 percent confidence intervals in brackets.

In Table 8.1 we see output of two regression tests of Equation 8.1. In Regression One, the sample consists of parliamentary democracies with simple electoral systems. This is the subset for which we have the strongest expectations, because we derived the Seat Product Model, including the extension to nationwide electoral party systems in this chapter, with simple parliamentary systems in mind. The result confirms Equation 8.1, as 0.679 is hardly different from the expected 0.667 and constant of 0.0148 is not statistically distinguishable from zero.

Regression Two adds in the presidential systems. The result does not change greatly; the expected coefficient of 2/3 remains within the 95 percent confidence interval of the regression's coefficient, even though the latter increases to 0.706. The constant remains effectively zero. The somewhat reduced fit reminds us again that presidential systems are more variable than parliamentary. We will discuss this matter in some detail in Chapter 11. Nonetheless, both regressions show that for simple electoral systems, if we know average district magnitude (M) and assembly size (S), we can estimate its effective number of vote-earning parties (N_V). The degree of accuracy of the fit to worldwide averages of many systems is remarkable.

So far then, we have achieved much, as existing approaches to explaining the effective number of vote-earning parties in national elections offer only quite vague answers to the question of how high we should expect N_V to be. Moreover, they typically include some index of social heterogeneity in their models, which has various problems for generalizability of results: it is difficult to measure accurately, is only a proxy for the various social factors that might lead to demand for additional parties, and is not always available for all countries. If we can have success predicting electoral party systems from just a country's assembly size and its mean district magnitude, we should regard that as a significant advance. In Chapter 15, we will offer extended tests that include two-tier systems as well as ethnic diversity.

HOW ELECTORAL SYSTEM IS CONGRUENT WITH CURRENT POLITICS

In Chapter 7, we showed a schematic, Figure 7.1, which suggested that the electoral system has a direct impact on seats and only an indirect impact on votes. The latter effect is indirect because votes are more directly affected by current politics and culture. Why then does the purely institutional model $N_V=[(MS)^{1/4}+1]^{2/3}$ (Equation 8.1) almost agree with the statistical best-fit (as reported in Table 8.1)? Does the psychological effect of the electoral system reach even beyond votes, into current politics and culture? Partly this is so indeed: politicians, voters, and commentators are very much aware of the limitations imposed by electoral rules. But there is also another reason.

Rather than put history into the same box with current politics, maybe we should place a separate box, "Original political culture," underneath the scheme in Figure 7.1, as we do now in Figure 8.5. This means political culture at the time the values of *S* and *M* were chosen, which includes many factors that are difficult to measure. We will attempt one such measure in Chapter 15, through the inclusion of the effective number of ethnic groups in our regression tests. While assembly size is rather tightly constrained by size of population (Taagepera and Shugart, 1989a: 173–183; see also Figure 2.1 of this book), original political culture very much determined the choice of district magnitude. Here we mean more than a country's ethnic diversity, which is the noninstitutional factor most often included in cross-national regressions by other authors. We mean something more difficult to theorize about systematically, such as prior political legacies of the country or enduring attitudes about how politics should work.

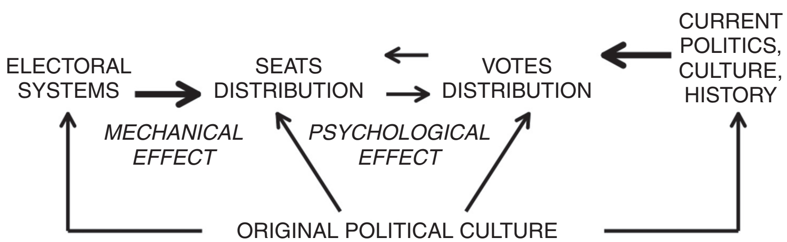

FIGURE 8.5 How current politics and the electoral system are shaped by political culture

For example, countries that were once colonies of the UK tend to retain a majoritarian culture and opt for M=1, FPTP. Notably, this is so whether ethnic diversity is unusually low (as in Jamaica) or the highest of any long-term democracy (India). One key exception is New Zealand, which changed to a proportional system in the 1990s; there are various indications that the country's norms about how politics should work were shifting in a more consensual direction around the time that the process of electoral reform was underway (Blais and Shugart, 2008; Vowles, 2008).

On the other hand, countries with a deeper tradition of political and social accommodation typically opted for large-M or two-tier systems, often early in their process of democratization. The precise reasons for why some countries adopted proportional representation remain controversial, and are not of direct concern to us (see Boix 2003, Colomer 2005, Ahmed 2013). The political culture that stands behind the adoption of majoritarian or proportional rules becomes the basis of the current culture as well. Thus, the point that is of more immediate concern for us is that part of the success of the Seat Product Model, for both seats and votes, is due to this congruence between M and present culture.

CONCLUSIONS

We have seen that the rule, "strivers are winners plus one," a modified form of Reed's and Cox's well-known M+1 rule, provides a reasonable starting point for estimating the effective number of vote-earning parties at the national level (N_V). The model that we derive from this starting point leads to a new concept that we will use again in future chapters: *the number of pertinent vote-earning parties* (N_{V0}). We showed that a model proxies this "phantom" quantity by the actual number of seat-winning parties (N_{S0}), plus one. When we replaced N_{S0} with $(MS)^{1/4}$, which we already had confirmed in Chapter 7, we were able to derive an institutions-only logical model for predicting N_V. Thus we are able to predict worldwide average N_V quite accurately from solely the assembly size and the mean district magnitude. It may seem that it could not be that straightforward – after all, many factors might motivate the formation of political parties and voter response to them. Yet, as shown in this chapter and the preceding one, the seat-winning parties are conditioned by the electoral institutions, and the vote-earning parties tend to follow from the seat-winners in a predictable fashion. Some individual countries deviate from the model appreciably. In these cases the model supplies a benchmark and flags the need to look for factors that cause the deviation.

So far we have modeled the number of parties at the nationwide level, primarily focused on parliamentary democracies with simple electoral systems. Presidential systems will be treated systematically in Chapters 11 and 12, while extension to two-tier systems and consideration of the impact of ethnic diversity is the topic of Chapter 15. Chapter 9 ties the nationwide effects all together into a series of four "basic laws" and extends the logic to another key output variable: deviation from PR.