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Abstract

We examine how inequality and openness interact in shaping the long-run growth prospects of developing countries. To this end, we develop a Schumpeterian growth model with heterogeneous households and non-homothetic preferences for quality. We show that inequality affects growth very differently in an open economy as opposed to a closed economy: If the economy is close to the technological frontier, the positive demand effect of inequality on growth found in closed-economy models may be amplified by international competition. In countries with a larger distance to the technology frontier, however, rich households satisfy their demand for high quality via importing, and the effect of inequality on growth is smaller than in a closed economy and may even be negative. In such case trade can give rise to the endogenous emergence of a 'dual economy' where some domestic sectors are highly innovative while others are not.

Keywords distance to frontier · dual economy · Dutch disease · growth · inequality · infant industry protection · non-homothetic preferences · small open economy · trade openness

JEL Classification D30 · F43 · O30 · O40

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1 Introduction

In recent years, few topics have been debated more than the rise in income inequality and countries' openness to trade. Many academic and political debates are around winners and losers from international trade in terms of income. With respect to economic growth, there is a well-developed literature on how income inequality affects growth,[1] and a large body of work examining the relation between openness and economic growth.[2] However, much less is known about how income inequality and trade openness interact in shaping a country's long-run prosperity. This is the focus of the present paper. This question is of particular importance for developing countries which can exhibit large income inequality and have often been under pressure by industrialized countries or international organizations to open their economies for international trade.

To examine how inequality and openness interact in shaping long-run economic growth, we consider a Schumpeterian growth model with heterogeneous households and non-homothetic preferences for quality. So far, the literature has used this type of framework to analyze the effects of inequality on growth in closed economies. The innovation of this paper is that we consider an open economy and show why and how the effects change when allowing for international trade. In particular, we show how the positive effect of inequality on growth found for closed economies can turn negative in an open economy that is not at the technological frontier. The key reason behind this negative relationship is that rich households can satisfy their demand for high quality via importing.

Our theoretical model considers two types of households: rich and poor. Households spend their income on a homogeneous good and a continuum of differentiated goods. Richer households demand more of the homogeneous good and higher qualities of the differentiated goods. Production of quality of a given differentiated good is constrained by the set of available blueprints for quality versions of that good. Firms can earn a patent on higher quality versions by investing in R&D and increasing the upper bound on quality for a specific differentiated good. The decision problem of an innovating firm is key to characterizing the equilibrium in the economy.[3] Depending on parameter values,

[1] Several channels of how income inequality affects growth have been put forward in the literature: Inequality might affect growth via differential propensities to save between income groups (Kaldor, 1955), via credit constraints that limit the ability of poor households to invest in the built-up of their human capital (Galor and Zeira, 1993; Galor and Moav, 2004), or via their impact on the political process and hence institutions (Alesina and Rodrik, 1994; Persson and Tabellini, 1994; Gersbach et al., 2019). Our focus will be on a demand channel as in e.g. Matsuyama (2002); Foellmi and Zweimüller (2006); Foellmi et al. (2014); Latzer (2018).

[2] See the literature discussion in section 2.

[3] We show that this decision problem boils down to a problem of optimal non-linear monopoly pricing over quality, but with three key differences when compared to the textbook case (e.g. Bolton and Dewatripont (2005)): First, there is an endogenous upper bound on quality. Second, the shape of a consumer's payoff function from one specific differentiated good depends on the full equilibrium. Third, in the open economy, foreign competition introduces a second set of individual rationality constraints. The costly quality upgrading implies that firms may find it optimal to pool rich and poor households if differences in income and / or the population share of the rich are small. The dependence of the payoff

all monopolists either pool households or separate rich from poor households. In the latter case, non-homothetic preferences over quality give rise to multi-quality firms, analogous to Latzer (2018).

A higher variance of the income distribution, keeping the skewness and average income constant, implies making the rich richer and the poor poorer while keeping their respective shares in the population constant. The increase in the income of the rich increases their willingness to pay for quality, making it more lucrative for firms to innovate to serve this demand. This leads to the robust comparative statics result that in a separating equilibrium, an increase in the variance of the income distribution has an unambiguously positive effect on growth in closed economies.

The key point of our paper is that this relationship between inequality and growth may be very different when allowing for international trade. For this purpose, we develop a small open economy (SOE) variant of our model by adding trade subject to an iceberg trade cost with a technologically advanced rest of the world (ROW).[4] In essence, this implies that if domestic firms in the SOE want to sell innovative high qualities to rich households, they need to outbid import competition for high qualities. We then identify three scenarios with respect to the effects of inequality and foreign competition on an SOE that is technologically lagging:

First, if inequality is low, the high quality demand by the rich part of the population is only slightly above the domestic technological frontier, and for these quality levels, the trade costs effectively shield the domestic firms from international competition, leading to the same equilibrium as in the closed economy.

Second, for higher but still moderate inequality, the high quality demand is further above the domestic technological frontier. Innovating domestic firms now have to face up to the international competition, but are still competitive. In this case, outside competition leads to higher domestic innovation and higher growth. The reason for this positive growth effect of foreign competition is that innovating domestic firms find it optimal to match the offer for rich households of foreign competitors by lowering their price. This triggers interesting equilibrium effects on innovation: The lower price for the high quality

function on the full equilibrium allows for interesting feedback effects on the innovation decision by firms. These effects have to be taken into account throughout. They are of particular interest in our small open economy analysis. In contemporaneous work, Bornstein and Peter (2023) also introduce non-linear monopoly pricing into a general equilibrium framework, but with a focus very different from ours: They analyze what non-linear pricing implies for misallocation.

[4] That is, ROW has already developed blueprints for higher quality versions of the differentiated goods when compared to the SOE. Therefore, while most of the existing literature on inequality and R&D deals (implicitly) with a country at the technology frontier, we are explicitly interested in an economy that is not operating at the technology frontier. We will at times use the term 'developing economy' to refer to the SOE, but to focus on the innovation channel, we consider an SOE that is technologically lagging but otherwise perfectly symmetric to the ROW, i.e. we shut down all other frictions that might impact growth in developing countries, and focus on its ability to import high qualities. This is in line with the well-known fact that richer countries export (and import) higher qualities (Schott, 2004; Hallak, 2006; Khandelwal, 2010; Bastos and Silva, 2010; Feenstra and Romalis, 2014; Flach, 2016; Schetter, 2020).

versions of *all* differentiated goods allows rich households to economize on their spending for the differentiated goods. The associated income effect induces them to increase their consumption of the homogeneous good and, in turn, this boosts their demand for quality due to the complementarity between the homogeneous good and the quality of the differentiated goods. This positive *demand effect* lifts innovation incentives above the respective level in the closed economy.

Third, if inequality is large, the quality demanded by high income earners is substantially above the domestic technological frontier, and it is no longer profitable for all domestic firms to compete with the technologically advanced foreign firms to serve the rich. Consequently, rich households start satisfying their demand for high quality by importing some of the differentiated goods, and the SOE exports the homogeneous good and/or low qualities of the differentiated goods in turn. The key observation is that this has a direct negative *business stealing effect* on innovation and growth. This effect becomes larger as inequality increases further, and domestic firms in fewer and fewer differentiated good sectors innovate to serve high qualities to the rich. Interestingly, our work thus also shows how a '*dual economy*'—with some innovative and some lagging sectors—endogenously arises in developing countries even in an ex-ante perfectly symmetric set-up. The basic intuition is that the domestic population is not rich enough to satisfy all of its demand for quality by importing pricey high qualities from abroad.

Hence, our work shows how inequality, trade openness, and distance to frontier interact in shaping a country's growth prospects, and this has important implications for the dynamic gains from trade and for policy. We show that firms in countries sufficiently close to the frontier are better positioned to profitably outbid foreign competitors, increasing the scope for stimulating growth via trade liberalizations. On the contrary, in an unequal SOE behind the world's technological frontier, rich households satisfy their demand for high quality via importing and this negative business stealing effect implies that, ceteris paribus, growth is lower than it would be in a closed economy. While a full econometric analysis is beyond the scope of the paper, appendix D nevertheless shows that this theoretical prediction can speak to basic correlations in the data.[5]

Interestingly, our set-up thus also gives rise to a novel '*Dutch disease*'-type negative effect of windfall gains on growth in developing countries: Windfall gains increase demand for quality by rich households vis-à-vis the domestic level of technology, and they therefore make it harder for domestic innovators to compete with foreign high-quality providers.

The remainder of this paper is organized as follows. Section 2 situates our paper in the

[5]This appendix presents (1) standard growth regressions using growth in GDP per capita as the dependent variable and (2) industry-level growth regressions using growth in export quality taken from Feenstra and Romalis (2014) as the dependent variable. These regressions control for an interaction of inequality and openness. Across a range of specifications, using either a large number of country controls or country fixed effects, we find that for developing countries this interaction term is typically significantly negative in the industry-level regressions, and still negative—albeit not in all cases significant—in the country-level regressions, in line with our theoretical predictions.

relevant literature. Section 3 introduces the model. Section 4 considers the closed economy and section 5 the small open economy. Section 6 provides extensions and discussions. Finally, section 7 concludes. All proofs and further details are provided in the appendix.

2 Relation to the Literature

Our paper is related to two main strands of literature.

Our main focus is on understanding how inequality impacts the growth prospects of a country via the demand for product innovation. These effects are subject to a large and growing literature. Matsuyama (2002) shows in a model of learning by doing that the effect of inequality on growth may be non-monotonic and that conventional measures of inequality such as the Gini-coefficient are not a sufficient statistic for these effects. Foellmi and Zweimüller (2006) consider a model of expanding varieties where new varieties address consumers' needs following their preference hierarchy. Inequality stimulates growth via an associated higher demand for luxury goods. Foellmi et al. (2014) consider product and process innovation, where process innovation prepares 'luxury goods' for mass production, in line with product cycles observed from the data. Latzer (2018) presents a Schumpeterian growth model featuring agents with non-homothetic preferences over quality. She shows how the desire to better discriminate between consumers of different incomes ('surplus appropriation effect') induces incumbents to invest in R&D and can give rise to multi-quality firms in equilibrium.[6] All of these models share in common that they are considering closed economies. And while a change in the income distribution may have non-trivial overall effects on growth, it is the case that a ceteris paribus increase in the income of the rich is beneficial for innovation because it increases their willingness to pay for innovated goods. We show that this channel may be very different and may, in fact, be reversed in a small open economy.[7]

Our paper is thus also related to the large literature analyzing the growth-effects of international trade (e.g. Grossman and Helpman (1991a), Acemoglu (2003), Galor and Mountford (2008), Nunn and Trefler (2010), Acemoglu et al. (2015), Gersbach and Schneider (2015), Schaefer and Schneider (2015), Arkolakis et al. (2018), Buera and Oberfield (2020), Diodato et al. (2022), Gersbach et al. (2023), Sampson (2023)). Openness to trade leads to higher competition as foreign firms enter the market. This might reduce R&D incentives for domestic firms and therefore lead to lower growth (Aghion and Howitt, 1992,

[6]This is in contrast to canonical Schumpeterian models (Aghion and Howitt, 1992) where the replacement effect (Arrow, 1962) outweighs the efficiency effect (Gilbert and Newbery, 1982; Reinganum, 1983).

[7]Hence, at a general level, our work is also related to Matsuyama (2019), who provides a thorough account of Engel's law in a global economy, and its implications for endogenous comparative advantage, structural change, and product cycles, among others. In his case, however, preferences are non-homothetic across sectors, while we consider non-homotheticities within sectors to study Schumpeterian growth. Moreover, he does not consider the effect of inequality within countries on growth, which is our main focus.

1996). At the same time, however, technology spillovers might arise as externalities from international trade (Grossman and Helpman, 1991b; Eaton and Kortum, 1999; Buera and Oberfield, 2020). In line with this, empirical studies tend to find a positive relationship between competition and growth (Nickell, 1996; Blundell et al., 1999; Schmitz, 2005), while a more recent literature suggest a U-shaped relationship between competition and growth (Aghion et al., 2005, 2009; Hashmi, 2013).[8] The key novelty of our work is that we analyze how the effects of international trade openness interact with inequality in shaping a country's growth prospects.[9,10]

3 Model

To study the growth-effects of inequality, we develop a model with non-homothetic preferences for quality and Schumpeterian growth through quality upgrading. We begin by developing the closed-economy model, which will serve as the benchmark for our open-economy analysis. We extend our framework to a small open economy model in section 5.

3.1 Households

The economy is populated by a continuum of infinitely-lived households of measure 1, $j \in [0,1]$. Households derive utility from consumption of a continuum of differentiated goods, $i \in [0,1]$, and a homogeneous good, z. Each differentiated good i represents a distinct consumption need of households that can be satisfied by consumption of one unit of exactly one of the quality versions of the good available at time t (time is discrete) $q_i \in \mathcal{Q}_i(t)$. That is, if $x_i^j(q_i;t)$ denotes the quantity of quality q_i of good i consumed by household j, we have $x_i^j(q_i;t) \in \{0,1\}$, and it takes on the value of 1 for exactly one quality version and zero for all other quality levels. This means the choice of quality is a discrete choice and there is an infinite degree of substitution between different quality versions of the same good. In what follows, we use $q_i^j(t)$ to denote the quality $q_i \in \mathcal{Q}_i(t)$ for which $x_i^j(q_i;t) = 1$. If at time t household $j \in [0,1]$ consumes a bundle $\left\{q_i^j(t)\right\}_{i\in[0,1]}$ of differentiated goods and $z^j(t)$ units of the homogeneous good, then their instantaneous

[8] Amiti and Khandelwal (2013) show that lower import tariffs (i.e. more competition) lead to quality upgrading for products close to the frontier, but discourages upgrading for products further away from the frontier.

[9] Our work is thus also, but less closely, related to the literature that incorporates non-homothetic preferences into models of international trade (e.g. Flam and Helpman (1987), Stokey (1991), Matsuyama (2000), Fajgelbaum et al. (2011), Fieler (2011), Jaimovich and Merella (2012), or Foellmi et al. (2018a)).

[10] Our work also adds a novel perspective to the discussions on infant industry protection. The theoretical literature on infant industry protection emphasizes the importance of learning-externalities either within or across industries (Krugman, 1987; Lucas Jr., 1988; Young, 1991; Matsuyama, 1992; Krugman and Elizondo, 1996; Puga and Venables, 1999; Hausmann and Rodrik, 2003; Rodrik, 2004; Melitz, 2005; Greenwald and Stiglitz, 2006). Our model also features an externality from innovation on aggregate productivity, and a potentially detrimental effect of trade on growth, which is the basis for infant industry protection. We argue, however, that this effect critically depends on the income distribution in developing countries.

utility is given by

$$u\left(\left\{q_i^j(t)\right\}_{i\in[0,1]}, z^j(t)\right) = \int_0^1 \left(q_i^j(t)\right)^{1-\beta} di \ \left(z^j(t)\right)^{\beta} \ . \tag{1}$$

Equation (1) implies that there is a complementarity between the quantity of the homogeneous good and the qualities of the differentiated goods, i.e. richer households have a higher willingness to pay for quality. This will play a central role in our subsequent analysis.[11]

Households observe the price schedules $p_i(\cdot;t) : \mathcal{Q}_i(t) \to \mathcal{R}^+$ indicating for each differentiated good i the prices for the different available quality levels. Consequently, $p_i(q_i;t)$ denotes the date t price of quality q_i of differentiated good i. Further, we denote the price of the homogeneous good at time t by $p_z(t)$. Households differ in their endowment with effective labor, ω, which they inelastically supply to the labor market. We consider two types of households, a 'high type' with high labour endowment, ω^H, and a 'low type' with low labor endowment, ω^L, and use superscript $h \in \{H, L\}$ to refer to the two different types of households. The share of high types is λ implying a corresponding share $1-\lambda$ of low types. Households earn wage rate w per unit of effective labor, which we choose to be the numéraire, i.e. we have $w(t) = 1$ at all times.

Household maximization problem

We assume that households are hand-to-mouth and consume their income in each period. Thus, the household decision problem is a static problem and so is the firm problem considered below.[12] As a consequence, we simplify notation by ignoring the dependence of all variables on time t unless explicitly stated otherwise. Further, households of the same type h face the same utility maximization problem, and we will henceforth identify households by their types. In every period, a household of type h chooses q_i^h and z^h to maximize

$$\begin{aligned} \max_{\left\{q_i^h\right\}_{i\in[0,1]}, z^h} \quad & \left[\int_0^1 {q_i^h}^{1-\beta}\, di\right] {z^h}^{\beta} \\ \text{s.t.} \quad & \int_0^1 p_i(q_i^h)\, di + p_z z^h \le I^h , \end{aligned} \tag{2}$$

where I^h denotes per-period income of household type h which equals their total expenditure in the current period. The separability of the instantaneous utility function in combination with the fact that each differentiated good has measure 0 imply that the household chooses q_i^h to maximize

$$\max_{q_i^h} \quad {q_i^h}^{1-\beta} {z^h}^{\beta} - \mu^h p_i(q_i^h)\,, \tag{3}$$

[11] A unit requirement for consumption has previously been used by e.g. Jaskold Gabszewicz and Thisse (1980); Shaked and Sutton (1982); Latzer (2018) to model non-homothetic preferences for quality.

[12] In appendix C, we specify the dynamic household problem when households are allowed to save and borrow and argue that with symmetric initial asset endowment of zero and without any aggregate investment opportunities, households will not save and consume their income in each period along the balanced growth path.

where μ^h is the shadow value of income which, by the envelope theorem, is equal to

$$\mu^h := \frac{du^h(\cdot)}{dI^h} = \frac{\frac{\partial u^h}{\partial z^h}}{p_z} = \frac{\beta Q^h {z^h}^{\beta-1}}{p_z} \ . \tag{4}$$

Here and below we use $Q^h := \int_0^1 {q_i^h}^{1-\beta}\, di$ to denote the quality aggregator of household h. Substituting equation (4) for μ^h in decision problem (3), we get

$$\max_{q_i^h} \quad {q_i^h}^{1-\beta} {z^h}^{\beta} - \frac{\beta Q^h {z^h}^{\beta-1}}{p_z} p_i(q_i^h) \, ,$$

which is equivalent to

$$\max_{q_i^h} \quad {q_i^h}^{1-\beta} \underbrace{\frac{z^h p_z}{\beta Q^h}}_{:=\theta^h} - p_i(q_i^h) \, . \tag{5}$$

From the household's maximisation problem we obtain the demand for the homogenous good z^h as well as the demands for the differentiated goods $\{x_i^h(\cdot)\}_{i\in[0,1]}$ with $x_i^h(q_i^h) = 1$ and zero for all other qualities. Importantly, from the perspective of innovating firm i, $\theta^h := \frac{z^h p_z}{Q^h \beta}$ is a sufficient statistic for household characteristics, determining their willingness to pay for quality. θ^h is exogenous to the firm and observed only by the household. It depends on the full general equilibrium in the economy. We turn to production, innovation, and the firm problem next.

3.2 Homogeneous good production

The production technology for the homogeneous good is given by

$$z = a_z A L_z \ ,$$

where A denotes the aggregate state of technology, as detailed below, L_z denotes effective labor input, and a_z is a time-invariant productivity parameter. There is perfect competition in the market for the homogeneous good, implying that its equilibrium price is

$$p_z = \frac{1}{a_z A} \ . \tag{6}$$

3.3 Differentiated good production and innovation

The quantity $x_i(q)$ of variety i at quality q can be produced according to

$$x_i(q) = \frac{1}{q} a_q A L_i(q) \ ,$$

where $L_i(q)$ denotes effective labor input and a_q is a time-invariant productivity parameter.

Blueprints for quality versions of each differentiated good i are inherited from the previous period up to the threshold quality level $\bar{q}_{i,-1}$. These blueprints are publicly available and there is a competitive fringe of firms that might enter the market.

Blueprints for new, higher-quality versions of each differentiated good can be developed through innovation.[13] Innovation entails research costs

$$h\left(\frac{\bar{q}_i}{\bar{q}_{i,-1}}\right) \tag{7}$$

to push the technological frontier for product i from $\bar{q}_{i,-1}$ to $\bar{q}_i$, where $h(\cdot)$ is C^2 and satisfies: $h(1) = 0$, $h'(1) = 0$, and $h''(\cdot) > 0$. These costs are paid using the homogeneous good.[14] Successful innovation results in a one-period patent for all qualities $q_i \in (\bar{q}_{i,-1}, \bar{q}_i]$. There is free entry into innovation and firms engage in a patent race. To enter the patent race, firms pay a fixed cost f, and the probability of winning is $f/(M_i f)$, where M_i is the mass of firms that entered the patent race for i. In equilibrium, firms then enter until profits net of fixed cost are zero in expectation, analogous to zero-profit conditions in the literature, e.g. Krugman (1980); Hopenhayn (1992); Melitz (2003). We assume that all households hold a perfectly diversified portfolio of entering firms, implying that profit income is zero for all households.[15] Taken together, the costs of R&D in sector i are $M_i \cdot f + h(\frac{\bar{q}_i}{\bar{q}_{i,-1}})$ and, consequently, the total costs of R&D in the economy add up to $\int_{i=0}^{1} M_i \cdot f + h(\frac{\bar{q}_i}{\bar{q}_{i,-1}})di$.

With the expiration of patents, production knowledge accumulated in the R&D process and in the production of new, high-quality varieties spills over to the entire economy. Such spillovers give rise to the following aggregate technology $A(t + 1)$

$$A(t+1) = \int_0^1 \bar{q}_i(t)\, di \ .$$

[13] For concreteness, we refer to the process of quality upgrading as innovation. But considering our subsequent focus on countries not at the frontier, we can alternatively interpret quality upgrading as imitation of advanced technologies.

[14] Note that we take a lab equipment approach to R&D and that the costs are denoted in the numéraire which is the wage rate. In this way, while the costs measured in terms of the numéraire stay the same over time, they would increase over time with the economy's technological level A if measured in units of the homogeneous good z as the numéraire. This can be inferred from (6). This increase in R&D costs expressed in units of the homogeneous good reflects the standard assumption of lab equipment specifications that it is harder and hence more costly to achieve the same relative increase in quality when the technological level A is higher (see e.g. Aghion and Howitt (2005)). This assumption allows for a balanced growth path where R&D investments are a constant share of GDP over time. In the context of our open economy model it implies that—given the same R&D cost function $h(\bar{q}/\bar{q}_{-1})$ and identical parameters a_z and a_q across countries (which we assume)—no country has a comparative advantage in production vs innovation in the sense that a given proportional increase of $\bar{q}$ involves the same R&D costs relative to GDP everywhere. See the beginning of section 5 and footnote 28 for further details on innovation in the SOE.

[15] Allowing for positive profits by innovating firms would not directly affect the optimal choice of $\bar{q}_i$ and, hence, aggregate growth. Profits would, however, have a general equilibrium feedback effect on innovation via their implications for the income distribution. While it is possible to incorporate such feedback loops, it would complicate the analysis without adding anything of substance to our main insights. We therefore consider the analytically more tractable case with zero profits in equilibrium. A free entry condition and, hence, zero profits in equilibrium is a common assumption in endogenous growth models. In these models, higher investment costs in R&D typically result in higher innovation probabilities and, hence, growth.

In what follows, we will consider the case of a common inherited quality level $\bar{q}_i(t-1) = \bar{q}(t-1)\ \forall\, i \in [0,1]$ as an initial condition at $t=0$, i.e. $\bar{q}_i(-1) = \bar{q}(-1)\ \forall\, i \in [0,1]$. As all differentiated good sectors are identical in terms of the firms' decision problems, we can show in section 4.2 that in the closed economy, the unique equilibrium is then symmetric across the differentiated goods sectors. This implies that the innovation levels are the same in each sector i and consequently $A(t) = \bar{q}(t-1)$ for all $t \geq 0$ resulting in the following law of motion for aggregate technology[16]

$$A(t+1) = \frac{\bar{q}(t)}{\bar{q}(t-1)} A(t) \ .$$

Whenever possible, we simplify notation again by using $A = \bar{q}_{-1}$ and correspondingly $A_{+1} = \bar{q}$.

4 Closed Economy

We are now ready to analyze the closed economy. We begin by characterizing the firms' decision problem which will be central for our results. Then we analyze the equilibrium and discuss the growth effects of inequality. We will contrast these results with the open economy in section 5.

4.1 Firms' decision problem

An innovating firm decides how much to invest in R&D in order to expand the set of blueprints for qualities from its current level $\bar{q}_{i,-1}$ to some new level $\bar{q}_i > \bar{q}_{i,-1}$. This decision is driven by the profit potential associated with these new blueprints. The competitive fringe for all pre-existing qualities $q_i \leq \bar{q}_{i,-1}$ pushes down their price to marginal cost, i.e.

$$p_i(q) = \frac{1}{a_q A} q \ , \quad \forall\, q \leq \bar{q}_{i,-1} \ ,$$

implying zero profits on these qualities. By contrast, an innovating firm can freely set the price $p_i(q)$ for all qualities $q \in (\bar{q}_{i,-1}, \bar{q}_i]$. The innovating firm then chooses $\bar{q}_i$ and $\{p_i(q)\}_{\bar{q}_{i,-1}}^{\bar{q}_i}$ to maximize its total profits. With two types of households, the decision problem of the innovating firm for good i can be written as follows:

[16] The law of motion for aggregate technology implies that process innovation is a byproduct of product innovation. In section A.2, we discuss an extension with deliberate process innovation. Foreshadowing one of our key results, we show that despite the initial condition of a common inherited quality level, in the SOE innovated qualities are typically no longer the same across firms in an equilibrium with trade, i.e. the opportunity to import high qualities from abroad gives rise to a '*dual economy*'.

Lemma 1 (Firms' decision problem)
The decision problem of innovating firm i is equivalent to:

$$\max_{q_i^H, p_i^H, q_i^L, p_i^L, \bar{q}_i} \quad \lambda\left(p_i^H - \frac{1}{a_q A} q_i^H\right) + (1-\lambda)\left(p_i^L - \frac{1}{a_q A} q_i^L\right) - h\left(\frac{\bar{q}_i}{\bar{q}_{i,-1}}\right) \tag{8}$$

$$\text{s.t.} \quad \theta^h v(q_i^h) - p_i^h \geq \max_{q \in [0, \bar{q}_{i,-1}]} \left\{\theta^h v(q) - \frac{1}{a_q A} q\right\}, \; h \in \{L, H\}, \tag{IR$^{\mathrm{h}}$}$$

$$\theta^H v(q_i^H) - p_i^H \geq \theta^H v(q_i^L) - p_i^L, \tag{IC$^{\mathrm{H}}$}$$

$$\theta^L v(q_i^L) - p_i^L \geq \theta^L v(q_i^H) - p_i^H, \tag{IC$^{\mathrm{L}}$}$$

$$q_i^h \leq \bar{q}_i, \quad h \in \{L, H\}.$$

where $v(q) := q^{1-\beta}$ and where the firm considers type $\theta^h := \frac{p_z z^h}{\beta Q^h}$ of household $h \in \{L, H\}$ as exogenously given. The value θ^h is private knowledge to the household.

The proof of lemma 1 is given in appendix B.1. The optimization problem reflects the standard assumption in monopolistic competition models according to which the individual firm has no impact on aggregate outcomes.

The firm's decision problem is one of optimal non-linear monopoly pricing over qualities, but with an endogenous choice of the upper bound on qualities and where the distribution of household types is given by the endogenous distribution of θ.[17] The set of constraints in the firm's optimization problem is a reflection of the revelation principle, according to which the optimal set of contracts is one contract for each type of households such that each household has an incentive to truthfully reveal their type. Accordingly, the first set of constraints requires that contracts are *individually rational* (IR), that is, each household must prefer their contract over their best outside option, which is in our case to consume the best option from the set of qualities $q_i \leq \bar{q}_{i,-1}$ that are available at marginal cost. The second set of constraints requires that contracts are *incentive compatible* (IC), i.e. every household must prefer their contract over the contract designed for the other household type in the economy.[18] Finally, the last constraint dictates that all qualities must be feasible, i.e. they cannot exceed the current technological frontier $\bar{q}_i$ for the respective good.

As we show in lemma 2, θ as defined in section 3.1 is an increasing function of household income and, hence, of their endowment with effective labor. In turn, this implies that households with higher income demand products of higher quality levels.

Lemma 2 (Monotonicity of household types)
θ^h is strictly increasing in I^h.

[17] Hence, in the closed economy the decision problem differs in two important ways from the textbook case of non-linear monopoly pricing over qualities (e.g. Bolton and Dewatripont (2005)). Moreover, in the open economy, foreign competition introduces a second set of individual rationality constraints—see footnote 3 for further discussions on these differences, and section 5 for the firm problem in the open economy.

[18] This constraint arises because θ^h is private knowledge. If θ^h was known to the firm, it could extract each household's full willingness to pay for quality, taking into consideration only the individual rationality constraints.

The proof of lemma 2 is given in appendix B.2.

Solution to the firm's maximization problem

Solving the firm's decision problem, we first note that the competitive fringe forces the firms to offer all qualities $q_i \leq \bar{q}_{-1}$ at their marginal costs $q_i/(a_q A)$. From the previous discussion we know that richer households have a higher willingness to pay for quality. In turn, this implies that there exists a threshold income $\hat{I}$ such that households with income $\hat{I}$ would choose $\bar{q}_{-1}$ if all qualities were priced at marginal cost. As it is never optimal for firms to charge a price below marginal costs, this in turn implies that households with income $I \leq \hat{I}$ always demand quality $q_i \leq \bar{q}_{-1}$ in equilibrium and only households with income $I > \hat{I}$ may consume innovative goods.[19] This threshold is therefore central to characterizing the different types of equilibria—and growth in these equilibria—that may exist in our economy. Lemma 3 derives this threshold income level $\hat{I}$.

Lemma 3 (Threshold income for demand for innovative qualities)
Household with income $I \leq \hat{I} := \frac{1}{a_q(1-\beta)}$ *always consume quality* $q_i \leq \bar{q}_{-1}$ *in equilibrium.*

The proof of lemma 3 is given in appendix B.3. With respect to the firm's profit maximization problem, it is now clear that if $I^h \leq \hat{I}$, $h = L, H$, the firm will not innovate and offer the households' demanded qualities $q_i^h \leq \bar{q}_{-1}$ at marginal costs.

If $I^H > \hat{I} \geq I^L$, only the rich households may demand a quality level $q_i^H > \bar{q}_{i,-1}$, while $q_i^L \leq \bar{q}_{-1}$ will be supplied at marginal costs. In this case, the firm's supply of q_i^H at price $p_i^H := p_i(q_i^H)$ is determined by solving the firm's problem with (IR^H) binding and $q_i^H = \bar{q}_i$. That is, (q_i^H, p_i^H) is the unique solution to

$$(IR^H) \qquad \theta^H v(q_i^H) - p_i^H \;=\; \theta^H v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q\ A}\ , \tag{9a}$$

$$\lambda[\theta^H v'(q_i^H) - \frac{1}{a_q A}] \;=\; h'\left(\frac{q_i^H}{\bar{q}_{i,-1}}\right)/\bar{q}_{i,-1}\ . \tag{9b}$$

Equation (9a) dictates that the high types must be indifferent between consuming q_i^H and their best outside option, viz. consuming $\bar{q}_{-1}$ at marginal costs. Equation (9b) dictates that in optimum, the willingness to pay of the high-types for a marginally higher quality—net of the extra cost of producing that higher quality—must just offset the extra cost of innovation.

Lastly, if $I^H > I^L > \hat{I}$, both household types may demand quality levels above the current technological frontier $\bar{q}_{-1}$. In this case, the firm needs to decide whether to offer separate quality-price packages to the rich and the poor households or whether instead to pool demand with one quality-price offer to both household types. In the case with separating offers for qualities $q_i^H > q_i^L > \bar{q}_{i,-1}$, conditions (IR^L) and (IC^H) are binding. As a consequence (IR^H) and (IC^L) are slack and the profit maximizing offers (q_i^h, p_i^h) are

[19] This is also the case in the SOE—see section 5.

the unique solution to

$$(IR^L) \qquad \theta^L v(q_i^L) - p_i^L = \theta^L v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q A} \ , \tag{10a}$$

$$(IC^H) \qquad \theta^H v(q_i^H) - p_i^H = \theta^H v(q_i^L) - p_i^L \ , \tag{10b}$$

$$\theta^L v'(q_i^L) - \frac{1}{a_q A} = \lambda \left(\theta^H v'(q_i^L) - \frac{1}{a_q A} \right) \ , \tag{10c}$$

$$\lambda[\theta^H v'(q_i^H) - \frac{1}{a_q A}] = h' \left(\frac{q_i^H}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1} \ . \tag{10d}$$

Intuitively, equations (10a) and (10b) dictate that (IR) is binding for the low types while (IC) is binding for the high types. Condition (10c) weighs the gains from marginally increasing the quality for the low types—and, hence, their willingness-to-pay—against the marginal cost of producing that quality and of marginally tightening (IC) for the high types. (10d) weighs the gains from marginally increasing the quality for the high types against the marginal cost of production and of innovation, reflecting the fact that q_i^H is always at the frontier.

Let $(\tilde{q}_i^h, \tilde{p}_i^h)$ be the solution to equation system (10a) - (10d). We can now distinguish three cases: First, if $\tilde{q}_i^L \leq \bar{q}_{i,-1}$ the initial premise that $q_i^L > \bar{q}_{i,-1}$ is violated and the firm will offer $\bar{q}_{i,-1}$ to the low income households at marginal costs $\bar{q}_{i,-1}/(a_q A)$, while q_i^H and p_i^H are determined by solving (9a) - (9b).

Second, if $\tilde{q}_i^H > \tilde{q}_i^L \geq \bar{q}_{i,-1}$, the firm offers $(q_i^H, p_i^H) = (\tilde{q}_i^H, \tilde{p}_i^H)$ to the high income households and $(q_i^L, p_i^L) = (\tilde{q}_i^L, \tilde{p}_i^L)$ to the low income households.

Third, if $\tilde{q}_i^L \geq \tilde{q}_i^H$, the premise that $q_i^H > q_i^L$ is violated and the firm will pool the demand of the high and low income households by offering only one quality-price combination. The profit maximizing combination is determined by solving the firm's problem with $q_i^L = q_i^H = q_i$ and $p_i^L = p_i^H = p_i$ and condition (IR^L) binding for (q_i, p_i). This yields the necessary conditions

$$(IR^L) \qquad \theta^L v(q_i) - p_i = \theta^L v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q \ A} \ , \tag{11a}$$

$$\theta^L v'(q_i) - \frac{1}{a_q A} = h' \left(\frac{q_i}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1} \ . \tag{11b}$$

The solution to the firm's maximization problem involves R&D investments f and $h(\bar{q}_i/\bar{q}_{-1})$ to increase the highest quality of variety i to $\bar{q}_i$. Given the latter, the monopolistic firm then specifies the price vector $p_i : \mathcal{Q}_i \to \mathcal{R}^+$, indicating the prices at which available qualities are offered. According to our discussion, we have $p_i(q_i) = q_i/(a_q A)$ for all $q_i \leq \bar{q}_{i,-1}$ and $p_i(q_i^h) = p_i^h$, $h = L, H$, depending on the cases as specified above. All other quality levels $\hat{q}_i > \bar{q}_{i,-1}$, $\hat{q}_i \neq q_i^h$, $h = L, H$, are not offered by the firm or, alternatively, at a price $p_i(\hat{q}_i) = \hat{p}_i$ sufficiently high but finite such that households do not demand qualities $\hat{q}_i$. Given this price schedule, the supply of variety i at quality level q_i^H is λ and that of quality level q_i^L is $(1-\lambda)$, with zero supply of any other quality levels. According to the production function for differentiated goods, the firm's labor demand amounts to $L_i(q_i^H) + L_i(q_i^L) = \frac{\lambda q_i^H + (1-\lambda) q_i^L}{a_q A}$.

4.2 Equilibrium

We define an equilibrium in the closed economy as a static equilibrium in each period while noting that our model has an infinite time horizon. As noted in sections 3.1 and 4.1, the maximization problems of the households and firms are static. Therefore the equilibrium of our model is an infinite sequence of static one-period equilibria.

Definition 1 (Equilibrium in the closed economy)
An equilibrium is a set of prices $\{\{p_i^e(q)\}_{(i,q)\in[0,1]\times\mathcal{Q}_i}, p_z^e, w^e = 1\}$, *quantities* $\{\{x_i^{H,e}(q), x_i^{L,e}(q)\}_{(i,q)\in[0,1]\times\mathcal{Q}_i},\ z^{H,e},\ z^{L,e}\}$, *labor demand* $\{\{L_i^e(q)\}_{(i,q)\in[0,1]\times\mathcal{Q}_i},\ L_z^e\}$, *and a mass of firms entering the patent race* M_i^e *and R&D-investments* $h(\bar{q}_i^e/\bar{q}_{i,-1})$ *of the winning firm for all* i, *such that the zero-profit condition in the patent race holds, profits of the firms producing the differentiated goods and the homogeneous good are maximized, the households' utilities are maximized and all markets clear, i.e.*

$$\lambda\ x_i^{H,e}(q) + (1-\lambda)\ x_i^{L,e}(q) = \frac{1}{q} a_q A L_i^e(q)\,,\ \forall (i,q) \in [0,1]\times\mathcal{Q}_i \tag{12a}$$

(differentiated good markets) ,

$$\lambda\ z^{H,e} + (1-\lambda)\ z^{L,e} = a_z A L_z^e - a_z A \int_0^1 f M_i^e + h(\bar{q}_i^e/\bar{q}_{i,-1})\ di \tag{12b}$$

(homogeneous good market) ,

$$\int_0^1 L_i^e(q_i^{H,e}) + L_i^e(q_i^{L,e})\ di + L_z^e = \lambda\ \omega^H + (1-\lambda)\ \omega^L \text{ (labor market)}\,, \tag{12c}$$

where $q_i^{H,e}$ *and* $q_i^{L,e}$ *in (12c) are the quality levels for which demand* $x_i^{H,e}(q) > 0$ *and* $x_i^{L,e}(q) > 0$, *respectively.*

The different cases in the solution to the firms' decision problem play the central role for characterizing the equilibrium in our economy. Using again $(\tilde{q}^h, \tilde{p}^h)$ to refer to the solution to the system of equations (10a) - (10d), we can formulate:

Proposition 1 (Equilibrium in the closed economy)
The equilibrium in the closed economy is unique and satisfies for $h = \{H, L\}$: $I^h = \omega^h$ *and* $(q_i^{h,e}, p_i^{h,e}) = (q^{h,e}, p^{h,e})$. *The equilibrium in the differentiated good markets can be characterized as follows:*

(A) If $\hat{I} \geq I^H \geq I^L$, *the firms do not innovate and offer qualities* $q^{h,e} = (1-\beta) I^h a_q A \leq \bar{q}_{-1}$ *at prices equal to marginal costs* $p^{h,e} = q^{h,e}/(a_q A)$.

(B) If $I^H > \hat{I} \geq I^L$, *or* $I^H > I^L > \hat{I}$ *and* $\tilde{q}^L \leq \bar{q}_{-1}$, *there is a separating equilibrium where the firms innovate to provide quality* $q^{H,e} > \bar{q}_{-1}$ *for the high types, which*

is offered at $p^{H,e}$. The price $p^{H,e}$ includes a mark-up above marginal costs. The bundle $(q^{H,e}, p^{H,e})$ is the unique solution to (9a) and (9b) where $\theta^{H,e} = (I^H - p^{H,e})/(\beta(q^{H,e})^{(1-\beta)})$. The low types consume quality $q^{L,e} \leq \bar{q}_{-1}$ priced at marginal costs $p^{L,e} = q^{L,e}/(a_q A)$.

(C) If $I^H > I^L > \hat{I}$ and $\tilde{q}^H > \tilde{q}^L > \bar{q}_{-1}$, there is a separating equilibrium with $(q^{h,e}, p^{h,e}) = (\tilde{q}^h, \tilde{p}^h)$. That is, the firms innovate to quality $q^{H,e} > \bar{q}_{-1}$ and offer qualities $\bar{q}_{-1} < q^{L,e} < q^{H,e}$ with mark-ups of different sizes at $p^{L,e} < p^{H,e}$.

(D) If $I^H \geq I^L > \hat{I}$ and $\tilde{q}^L \geq \tilde{q}^H$, there is a pooling equilibrium with $(q^{H,e}, p^{H,e}) = (q^{L,e}, p^{L,e}) = (q^e, p^e)$. The firms innovate to provide the same quality $q^e > \bar{q}_{-1}$ to both types priced at p^e including a mark-up above marginal costs. (q^e, p^e) are the unique solution to equations (11a) and (11b) where $\theta^{L,e} = (I^L - p^e)/(\beta(q^e)^{(1-\beta)})$.

The proof of proposition 1 is provided in appendix B.4.[20]

The intuition for the equilibrium is as follows. With $I^H > \hat{I}$, the existing technological level is such that at least the high income households are not able to consume their most desired quality level at marginal cost. As a consequence, firms have an incentive to innovate to satisfy the quality demands of these households. In maximizing its profits, a firm has to decide how much to invest in quality innovation and whether to offer two different quality levels for the different household types. In case (B), the firm innovates to cater to the quality demand of the rich households, while the poor households consume an already existing quality at marginal costs. The reason is as follows: On the one hand, if $I^L \leq \hat{I}$, the poor households' optimal quality level is lower than $\bar{q}_{-1}$ and hence provided at marginal costs. On the other hand, when $I^L > \hat{I}$ in case (B), the low income households consume the best already available quality $\bar{q}_{-1}$ at marginal costs because if the firm offered another quality level above $\bar{q}_{-1}$ for the low income households, it would be favorable for the rich households to consume at this lower quality level as well or the firm would have to reduce the price for q_i^H to keep the rich households consuming the higher quality. The corresponding gains in profits from the poorer households do not compensate for the losses in profits from the rich customer base. In (C), $I^L > \hat{I}$ and the share of the poor, $1 - \lambda$, is sufficiently large that it is not optimal to ignore them while income differences between the poor and the rich are sufficiently high such that it is beneficial to push out the technological level further to offer high qualities for the rich and

[20] Note that case (A) of proposition 1 summarizes the cases (ii) and (iii)(A) of the technical version of the proposition in appendix B.4, while the cases (B) and (C) of proposition 1 correspond to the cases (iii)(B) and (iii)(C) in the technical formulation. As we show in the appendix, the equilibrium allocation of labor, the price p_z^e, and the price quality bundles $(q^{h,e}, p^{h,e})$ are unique. However, we note that there is an infinite number of price vectors satisfying the equilibrium conditions, differing in the prices of qualities that are not be supplied in equilibrium. Given prices and quantities for the differentiated goods, the other equilibrium values follow. In particular, homogeneous good consumption of household h is $z^{h,e} = (I^h - p^{h,e})a_z A$, which they buy at the equilibrium price $p_z^e = 1/(a_z A)$ reflecting marginal costs. Moreover, labor demand is given by $\int_0^1 L_i^e(q^{H,e}) + L_i^e(q^{L,e})di + L_z^e = \lambda \frac{q^{H,e}}{a_q A} + (1-\lambda)\frac{q^{L,e}}{a_q A} + \lambda(I^H - p^{H,e}) + (1-\lambda)(I^L - p^{L,e})$ $+ \int_0^1 f M_i^e + h(\bar{q}_i^e/\bar{q}_{-1})di$, which is remunerated at the normalized equilibrium wage rate $w^e = 1$.

lower qualities, but still higher than $\bar{q}_{-1}$, for the poor. This is a separating equilibrium where both types of households are served by innovating firms, that is, we observe multi-quality firms in equilibrium.[21] Finally, if $I^L > \hat{I}$ and the share of the rich, λ, and income differences between rich and poor households are small enough, it is optimal for firms to pool households to economize on costs of innovation (case (D)).

We note that $q^{L,e}$, $q^{H,e}$, $p^{L,e}$, and $p^{H,e}$ are jointly pinned down by systems of equations that depend only on these variables and time-invariant parameters. Moreover, $q^{L,e}$ and $q^{H,e}$ enter these equations only via $\frac{q^{L,e}}{\bar{q}_{-1}}$ and $\frac{q^{H,e}}{\bar{q}_{-1}}$—see the detailed version of proposition 1 in appendix B.4. Consequently, the aggregate growth rate $g^e = \frac{q^{H,e}}{\bar{q}_{-1}}$ is constant over time, which constitutes the following corollary.[22]

Corollary 1 (Balanced growth path)
There is a unique balanced growth path (BGP) which is reached instantaneously. Along the BGP, the growth rate is $g^e = \frac{q^{H,e}}{\bar{q}_{-1}}$.

We will now examine how a change in the income distribution impacts equilibrium outcomes and, in particular, $q^{H,e}$, which governs growth. To simplify notation, we will throughout dispose of the superscript ^{e} to indicate equilibrium outcomes.

4.3 Inequality and growth: closed economy

Without loss of generality, we normalize endowments with effective labor such that

$$E[\omega] = \lambda\omega^H + (1-\lambda)\omega^L = 1 \equiv \bar{w} \ .$$

We further choose

$$\omega^H = 1 + \sigma\sqrt{(1-\lambda)/\lambda}, \qquad \omega^L = 1 - \sigma\sqrt{\lambda/(1-\lambda)} \ ,$$

where $\sigma \geq 0$, as this specification allows to separate changes in the variance of the income distribution from changes in its skewness[23]

$$VAR(\omega) = \sigma^2, \qquad SK(\omega) = (1-2\lambda)/(\sqrt{\lambda(1-\lambda)}) \ .$$

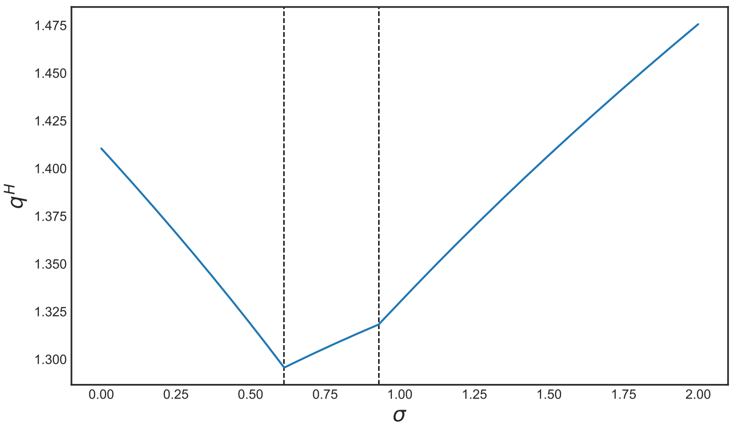
To analyze the impact of inequality on growth in our economy, we then focus on changes in the variance (i.e. in σ). This corresponds to a Lorenz-dominated shift of the income

[21] See Latzer (2018) for a detailed account of the endogenous emergence of multi-quality firms in such an environment.

[22] We note that the above also implies that a household's position vis-à-vis $\hat{I}$ remains constant over time.

[23] Our specification of endowments with effective labor relates to Foellmi et al. (2014) and Latzer (2018) as follows. As in Foellmi et al. (2014) and Latzer (2018), an increase in σ increases the income gap and leaves the share of poor households unchanged. Therefore, an increase in σ always increases inequality and a policy reducing σ leads to a Lorenz-dominating shift. On the other hand, an increase in income concentration (i.e. a decrease in λ) in our setting also increases the income gap. Hence, a change in λ leads to a Lorenz-crossing shift, as we cannot disentangle changes in income concentration and the income gap when varying λ. Unlike the specification in Foellmi et al. (2014) and Latzer (2018), λ is therefore not monotonously related to measures of inequality.

Figure 1: q^H for different values of σ $[\hat{I} < \bar{\omega}]$



Note: The figure shows the equilibrium values of q^H for different values of σ and where $\hat{I} < \bar{\omega}$. The dashed lines indicate changes in the type of equilibrium, first from a pooling to a separating equilibrium and then to a separating equilibrium where only rich households are served by innovating firms. The remaining parameter values are $a_q = 12$, $\beta = 0.5$, $\lambda = 0.2$, and $\bar{q}_{-1} = 1$. Furthermore, $h'(x) = x - 1$.

distribution and allows isolating our main mechanism of interest—an inequality-induced higher willingness to pay for innovation—and its implications in a closed vs open economy.[24] We further concentrate on the case where $\hat{I} < \bar{w}$ in the remainder of the paper. We do so because with $\hat{I} \geq \bar{w}$ low types always consume pre-existing qualities, i.e. $q^{L,e} \leq \bar{q}_{-1}$, and innovation incentives are purely driven by the rich households. The more comprehensive case where $\hat{I} < \bar{w}$ includes such a situation as well, but can also speak to other realistic scenarios where the low income households also consume quality innovations.[25] The following proposition characterizes the growth effects of a change in inequality in the closed economy for this case.

Proposition 2 (Inequality and growth in the closed economy)
If $\hat{I} < \bar{w}$, there is a U-shaped relationship between the variance of the income distribution and economic growth. The lowest growth rate is at the level of σ where the equilibrium type changes from a pooling to a separating equilibrium.

The proof of proposition 2 is given in appendix B.5.[26]

The intuition of proposition 2 can be summarized as follows. As $\hat{I} < \bar{w}$, at an equal distribution of incomes we are trivially in a pooling equilibrium, and all households would like to consume higher quality levels than $\bar{q}_{-1}$, implying positive gains from innovation. Hence, economic growth is positive in such case. As we increase σ starting from an equal income distribution, we now have to consider the different types of equilibria described in proposition 1. Initially, as we increase the variance, a pooling equilibrium will persist, but with the low income households showing a lower willingness to pay for quality and thereby

[24] We also have results for the effects of skewness on economic growth in the closed economy, which we are glad to share upon request.

[25] A discussion of the case where $\hat{I} \geq \bar{w}$ can be found in the working paper version Jaeggi et al. (2021).

[26] The statements for a separating equilibrium where both types are still served are partially based on numerical solutions for a broad range of parameter specifications, see appendix B.5.

leading to lower equilibrium quality and prices. As we increase the variance further, the willingness to pay for quality of high income households is eventually large enough to justify additional investments in quality upgrading on the side of innovating firms despite the fact that only a fraction $\lambda < 1$ of households are rich. This means that there will be a separating equilibrium. In the separating equilibrium, innovation incentives are centrally driven by the willingness-to-pay of the rich. This is where innovation incentives and growth are increasing in σ. Taken together, there is a U-shaped relationship between σ and growth, where the minimum is reached at the point where the economy switches from a pooling to a separating equilibrium, as illustrated in figure 1.

These growth effects of inequality are similar to what has previously been found in the literature and, in particular, the positive 'willingness-to-pay' effect in a separating equilibrium. In what follows, we show how this result hinges on the closed-economy assumption and how international trade can have profound consequences for the inequality-growth nexus in the developing world.

5 Small Open Economy

We now consider a small open economy (SOE) variant of our model to examine how the opportunity to trade impacts the identified link between inequality and growth. In this variant, households can satisfy their demand for any of the goods by importing it from a rest of the world (ROW) that is technologically more advanced, but perfectly symmetric to the SOE otherwise. Specifically, we assume that $a_z^{ROW} = a_z$, $a_q^{ROW} = a_q$, and that $\bar{q}_{-1} < \bar{q}_{-1}^{ROW}$, where as before, we consider the case of a common inherited quality in the first period, i.e. $\bar{q}_{i,-1} = \bar{q}_{-1} \, \forall i \in [0,1]$, and analogously for the ROW.[27] Trade between the SOE and the ROW is subject to an iceberg trade cost $\tau > 1$ that is the same across all sectors, and domestically or foreign produced versions of any given quality of a good are perfect substitutes to one another. We are interested in how the ability to import high-quality goods impacts innovation and growth in developing countries. To that end, we consider an SOE that is sufficiently far from the frontier such that (i) it is never optimal for firms in the SOE to innovate in order to serve the rich in the ROW;[28] and (ii) households

[27] In our set-up, comparative advantage thus only arises from the extensive quality margin, i.e. from the fact that the ROW can produce a broader range of qualities when compared to the SOE. This provides a tractable framework that allows studying how import competition from high-quality providers impacts the growth effects of inequality in a Schumpeterian model set-up, which is our main focus of interest. We note that we could easily allow for the SOE to have a comparative advantage in homogeneous good production. In such case, it would be more difficult for domestic innovators to compete with foreign high-quality providers, i.e. foreign competition becomes binding at lower levels of inequality, but our subsequent discussions of the inequality-growth nexus in the SOE would otherwise remain the same, as long as comparative advantage is not too extreme (or trade costs not too low) such that firms from the ROW would have lower marginal costs of serving differentiated goods to consumers in the SOE than firms from the SOE.

[28] This is the case if the SOE is sufficiently far from the frontier and large innovation steps are prohibitively expensive, e.g. in the limiting case where $h(x)$ becomes vertical at some level $\hat{x} > 1$.

can import their preferred quality from the ROW at the prevailing equilibrium prices, without being bound by the highest available quality level in the ROW. In section 6.1 below, we discuss how our main insights change if the preferred import option of the rich in the SOE is bound by the maximum quality in the ROW.

We begin with some preliminary considerations on international trade and the households' import options in the SOE, before analyzing the firms' decision problem in the next subsection.

Preliminary considerations

To analyze the equilibrium in the SOE, note first that the set-up immediately implies that there cannot be two-way trade of any given quality of a good. Hence, balanced trade is possible only if the SOE imports some high qualities $q_i > \bar{q}_i$ from abroad and exports the homogeneous good z and / or qualities $q_i \leq \bar{q}_i$ of the differentiated goods. In turn, this requires that the SOE can price the homogeneous good competitively in the world market. Thus, with positive trade it holds that[29]

$$p_z^{ROW} = \tau \frac{w}{a_z A} = \frac{w^{ROW}}{a_z A^{ROW}} \ . \tag{13}$$

To compete with firms in the SOE, foreign firms are willing to serve consumers in the SOE at their marginal costs scaled by the iceberg trade costs $\tau \geq 1$.[30] The marginal costs for firms from the ROW of providing quality q to consumers in the SOE are: $\tau \frac{q w^{ROW}}{a_q A^{ROW}}$. Using $w = 1$ again and noting that

$$w^{ROW} = \frac{\tau A^{ROW}}{A} w$$

by equation (13), these marginal costs—and, hence, the price at which foreign firms are willing to serve consumers in the SOE—can be restated as

$$p^f(q) = \tau^2 \frac{1}{a_q A} q \ .$$

[29]Note that equation (13) implies that firms in the SOE have strictly lower marginal production costs for the homogeneous good and for all qualities $q_i \leq \bar{q}_i$ of all differentiated goods than the marginal cost of firms from the ROW of serving customers in the SOE. It follows that, indeed, the only equilibrium with positive and balanced trade is one where the SOE imports high qualities and exports low qualities and / or the homogeneous good.

[30]If $q_i \leq \bar{q}_{i,-1}^{ROW}$, imported goods are offered at marginal costs because of the competitive fringe in the ROW. Regarding $\bar{q}_i^{ROW} > q_i > \bar{q}_{i,-1}^{ROW}$, the foreign firms are willing to offer these qualities at marginal costs when under competitive pressure from the domestic firms because for every price above marginal costs they make strictly positive profits from serving households in the SOE without impacting their profit potential in the ROW. Foreign qualities priced at marginal costs is thus the relevant competitive benchmark that innovating firms in the SOE must reach to sell their innovative goods domestically—see decision problem 15. Two remarks are in order: First, foreign firms might serve rich households in the SOE at a price above marginal costs if inequality is so high that the rich households satisfy *all* their demand for differentiated goods from foreign competition, and firms in the SOE are no longer competitive in any of the differentiated goods—see sections 5.2 and 5.3 for further discussions. Nevertheless, the competitive benchmark that domestic innovating firms would need to reach to serve rich households remains the same. Second, we note that if trade costs are sufficiently small, pricing in the SOE of monopolistic firms from the ROW may be constrained by a threat of re-importing to the ROW (see e.g. Foellmi et al. (2018b)). We leave such considerations out of account here. Note that we can always rule out a threat of re-importing if the SOE is sufficiently far from the technological frontier such that the imported qualities satisfy $q \leq \bar{q}_{-1}^{ROW}$.

Here and in the following, we use a superscript ^{f} to denote an offer from foreign firms to consumers in the SOE. We summarize these insights in the following lemma, and relegate a formal proof to appendix B.6.

Lemma 4 (Import prices in the SOE)
To compete against innovating firms in the SOE, foreign firms are willing to serve consumers in the SOE at a price

$$p^f(q) = \tau^2 \frac{1}{a_q A} q \; . \tag{14}$$

5.1 The domestic firms' decision problem in the SOE

The previous discussions imply that firms in the SOE cannot make profits from selling differentiated goods to the ROW—see also the proof of lemma 4. Yet, the availability of imported qualities impacts innovation incentives in the SOE, because imported qualities introduce a second set of individual rationality constraints for households: In the SOE, a contract offered by a domestic monopolist must not only be preferable to a household's best choice among the domestic competitive fringe, but also to its best import option.[31] This gives rise to the following augmented decision problem for innovating firms in the SOE:

$$\max_{q_i^H, p_i^H, q_i^L, p_i^L, \bar{q}_i} \quad \lambda \left(p_i^H - \frac{1}{a_q A} q_i^H \right) + (1-\lambda) \left(p_i^L - \frac{1}{a_q A} q_i^L \right) - h\left(\frac{\bar{q}_i}{\bar{q}_{i,-1}} \right) \tag{15}$$

$$\text{s.t.} \quad \theta^h v(q_i^h) - p_i^h \geq \max\nolimits_{q \in [0, \bar{q}_{i,-1}]} \left\{ \theta^h v(q) - \frac{1}{a_q A} q \right\}, \tag{IR$^\text{h}$}$$

$$\theta^h v(q_i^h) - p_i^h \geq \max\nolimits_{q \in [0, \bar{q}_i^{ROW}]} \left\{ \theta^h v(q) - \tau^2 \frac{1}{a_q A} q \right\}, \tag{IRf$^\text{h}$}$$

$$\theta^H v(q_i^H) - p_i^H \geq \theta^H v(q_i^L) - p_i^L \; , \tag{IC$^\text{H}$}$$

$$\theta^L v(q_i^L) - p_i^L \geq \theta^L v(q_i^H) - p_i^H \; , \tag{IC$^\text{L}$}$$

$$q_i^h \leq \bar{q}_i, \quad h \in \{L, H\} \, .$$

Decision problem (15) is the same as decision problem (8) in the closed economy, but with the additional (IRf$^\text{h}$) constraints. It is useful to simplify this decision problem by solving for the value of the best import option of a household of type θ^h. In particular, household h's best import option is to choose quality $q^{h,f}$ such that $\theta^h v'(q^{h,f}) = \tau^2/(a_q A)$.[32] It follows immediately that its optimal import quality is

$$q^{h,f} = \left[\frac{(1-\beta)\theta^h a_q A}{\tau^2} \right]^{\frac{1}{\beta}} \; , \tag{16}$$

[31] Condition (IRf) stipulates that any offer by the domestic firm must provide (weakly) higher value to the consumer than their best import option at marginal costs. This is the relevant benchmark—see footnote 30 for further details.

[32] Recall that we have assumed that the households are not bound by $\bar{q}^{ROW}$, implying $q^{h,f} < \bar{q}^{ROW}$.

which implies for the value of the best import option

$$\theta^h v(q^{h,f}) - \frac{q^{h,f}\tau^2}{a_q A} = \left[\theta^h\right]^{\frac{1}{\beta}} \left[\bar{q}_{-1}\right]^{\frac{1-\beta}{\beta}} \underbrace{\left[\frac{a_q(1-\beta)}{\tau^2}\right]^{\frac{1-\beta}{\beta}} \beta}_{:=\chi(\tau)} . \tag{17}$$

Observe from equation (17), that the value of the best import option is convex in θ^h, reflecting the fact that higher types not only value quality more, but that they also consume higher quality. As we will see, this convexity implies that for sufficiently high levels of inequality, (IRf) is binding for the high types.

In what follows, we analyze how innovation and growth in the SOE depend on inequality. To this end, we henceforth assume that:

Assumption 1

$$\tau \geq \underline{\tau} := \left[\frac{\beta^{\frac{2\beta-1}{\beta}} \left[1 - \frac{1}{a_q}\right]^{1/\beta} \left[a_q(1-\beta)\right]^{\frac{1-\beta}{\beta}}}{1 - \frac{1}{a_q}(1+\beta)}\right]^{\frac{\beta}{2(1-\beta)}}$$

As the following lemma 5 shows, assumption 1 precludes that low types find it optimal to import their differentiated products. This is attractive for our purposes for two reasons: First, it allows centering the discussions on how the possibility of rich households to import high quality from abroad impacts the growth effects of inequality, which is our main focus of interest. Second, it implies that initially, at $\sigma = 0$, (IRf) is strictly non-binding for both types, which will allow to identify the different effects of inequality on growth in the SOE as discussed in section 5.3.

Lemma 5 (Constraint (IRf$^{\mathrm{L}}$) non-binding)
Let assumption 1 be satisfied. Then constraint (IRf) is either redundant or binding for the high types.

The proof of lemma 5 is given in appendix B.7. We note that assumption 1 is sufficient but not necessary for our analysis to apply.

Solution to the firms' profit maximization problem in the SOE

Analogous to the closed economy, the domestic firms' decision problem is key to understanding the growth effects of inequality in the SOE. As in the closed economy, firms take θ^H and θ^L as given in their profit maximization.

First we recall that all households with income $I^h \leq \hat{I}$ find their optimal quality among the domestically available quality levels $q_i \leq \bar{q}_{i,-1}$ at marginal costs. This implies that potential innovation requires $I^h > \hat{I}$ for at least $h = H$.

As discussed above, the key difference between the firm's profit maximization problem in the closed and the small open economy is the inclusion of the (IRf$^{\mathrm{h}}$) constraints, where according to lemma 5 only (IRf$^{\mathrm{H}}$) may be binding. Therefore, if for any given levels of I^h

and θ^h the solution to the firm's problem in the closed economy as derived in subsection 4.1 does not violate ($\mathrm{IRf^H}$), then it is also the solution to the domestic firm's maximization problem in the SOE. Consequently, we focus on the solution to the firm's maximization problem when ($\mathrm{IRf^H}$) is binding. We organize our discussion as in subsection 4.1 by first considering the case $I^H > \hat{I} \geq I^L$, where there is no demand for innovation from the low income households, and then turn to the case $I^H > I^L > \hat{I}$, where the low income households may consume innovative quality levels as well. We refer to these cases by (B-SOE) and (C-SOE) to indicate that they relate to the cases (B) and (C) in proposition 1 with the difference that now ($\mathrm{IRf^H}$) is binding.[33]

(B-SOE) For $I^H > \hat{I} \geq I^L$, the domestic firm supplies quality level $q_i^L \leq \bar{q}_{i,-1}$ at marginal costs to the low types as in the closed economy.[34] For the high income households, the firm's offer in the closed economy was determined by the constraint ($\mathrm{IR^H}$) stipulating that any offer (q_i^H, p_i^H) yields at least the value of their best outside option, which was quality $\bar{q}_{i,-1}$ at marginal costs $1/a_q$. In the SOE with ($\mathrm{IRf^H}$) binding, the best import option provides a more attractive outside option than $\bar{q}_{i,-1}$. Consequently, the offer (q_i^H, p_i^H) is determined by solving

$$(IRf^H) \qquad \theta^H v(q_i^H) - p_i^H = \left[\theta^H\right]^{\frac{1}{\beta}} \left[\bar{q}_{i,-1}\right]^{\frac{1-\beta}{\beta}} \chi(\tau) \ , \tag{18a}$$

$$\lambda[\theta^H v'(q_i^H) - \frac{1}{a_q A}] = h'\left(\frac{q_i^H}{\bar{q}_{i,-1}}\right)/\bar{q}_{i,-1} \ . \tag{18b}$$

Note that (18b) is the same as (9b) in the closed economy. As opposed to the SOE however, the candidate solution $(q_i^{H,B}, p_i^{H,B})$ to optimality conditions (18a) and (18b) may imply negative profits if the value of the best import option is too high. Therefore, the domestic firm offers $(q_i^H, p_i^H) = (q_i^{H,B}, p_i^{H,B})$ to the high income households if

$$\pi^B(q_i^{H,B}, p_i^{H,B}) = \lambda\left(p_i^{H,B} - \frac{1}{a_q A} q_i^{H,B}\right) - h\left(\frac{q_i^{H,B}}{\bar{q}_{i,-1}}\right) \geq 0 \ , \tag{19}$$

and abstains from making an offer to the rich households otherwise.[35]

[33] With ($\mathrm{IRf^H}$) binding for the rich households while the poor households prefer domestic differentiated goods, firms in the SOE will typically find it most profitable to offer separate contracts to the rich and the poor households (provided that they still serve the rich as discussed below). However, in some special cases where inequality is not too high and τ small, the firms may want to pool contracts for the two types of households with ($\mathrm{IRf^H}$) binding. These cases are not the main focus of our analysis, and for brevity we therefore abstract from these cases in the subsequent discussions.

[34] In the closed economy, in some special cases it may be optimal for innovating firms not to serve the poor even if $I^L > \hat{I}$. This is because serving the poor $q^L > \bar{q}_{-1}$ tightens constraint $(\mathrm{IC})^{\mathrm{H}}$. In the SOE with ($\mathrm{IRf^H}$) strictly binding, this cannot happen because in such case rich households strictly prefer their best import option over consuming $\bar{q}_{-1}$ at marginal cost domestically, which gives room for marginally raising q^L above $\bar{q}_{-1}$ while $(\mathrm{IC})^{\mathrm{H}}$ remains weakly non-binding.

[35] In the closed economy it is always optimal to serve the rich households in case B as $I^H > \hat{I}$ and $h'(1) = 0$. The former implies that at $q_i = \bar{q}_{-1}$, the rich households' willingness to pay for a marginally higher q_i exceeds the marginal production cost of $1/a_q$. The latter implies that marginally increasing $\bar{q}$ is costless. By contrast, in case (B-SOE) it may be optimal not to serve the rich as their best import option is strictly better than consuming $\bar{q}_{i,-1}$ at marginal costs, i.e. simply matching this outside option is costly for the innovating firm in the SOE.

(C-SOE) Considering the case $I^H > I^L > \hat{I}$ next, we first note that according to lemma 5, (IRf$^{\mathrm{h}}$) is binding for the rich but never for the poor households which consume domestic products. In the separating equilibrium in the closed economy, conditions (IR$^{\mathrm{L}}$) and (IC$^{\mathrm{H}}$) are binding, and the profit maximizing offers (q_i^h, p_i^h) are the solution to (10a) to (10d). With (IRf$^{\mathrm{H}}$) binding, however, this solution is no longer feasible and the innovating firms' optimal contracts instead solve

$$(IR^L) \qquad \theta^L \left(v(q_i^L) - v(\bar{q}_{i,-1})\right) + \frac{1}{a_q} = p_i^L \ , \tag{20a}$$

$$(IRf^H) \qquad \theta^H v(q_i^H) - p_i^H = \left[\theta^H\right]^{\frac{1}{\beta}} \left[\bar{q}_{i,-1}\right]^{\frac{1-\beta}{\beta}} \chi(\tau) \ , \tag{20b}$$

$$(IC^H) \qquad \theta^H v(q_i^H) - p_i^H \geq \theta^H v(q_i^L) - p_i^L \ , \tag{20c}$$

$$\lambda \theta^H v'(q_i^H) - \lambda \frac{1}{a_q A} - h'\left(\frac{q_i^H}{\bar{q}_{i,-1}}\right) \frac{1}{\bar{q}_{i,-1}} = 0 \ . \tag{20d}$$

System of equations (20a) to (20d) differs in two key ways from its counterpart in the closed economy, (10a) to (10d). First, to compete with foreign high-quality providers, innovating firms in the SOE need to improve the value of their offer to the rich households, constraint (IRf$^{\mathrm{H}}$). Second, this in turn relaxes constraint (IC$^{\mathrm{H}}$), which allows innovating firms to earn higher profits by increasing q^L and p^L while holding constant $\theta^L v(q^L) - p^L$ (i.e. guaranteeing that (IR$^{\mathrm{L}}$) remains binding), up to the point where (IC$^{\mathrm{H}}$) is again binding, or up to $q_i^L = (\theta^L(1-\beta)a_q A)^{(1/\beta)}$ if (IC$^{\mathrm{H}}$) remains slack. The latter yields the highest possible profits from serving the poor. In either case, (10c) no longer applies and the optimal solution must satisfy (20c) instead.[36]

Consider the candidate solution $(q_i^{h,C}, p_i^{h,C})$ solving (20a)-(20d) as described. Again, we need to check whether it is actually profit-maximizing to serve the high income households under pressure from foreign competition. Alternatively, the firm could choose to only innovate to serve the poor households. In this case, their optimal contract is determined by optimality conditions (9a) and (9b), but adapted to the low types,[37] i.e.

$$(IR^L) \qquad \theta^L v(q_i^L) - p^L \;=\; \theta^L v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{a_q\ A} \ , \tag{21a}$$

$$(1-\lambda)[\theta^L v'(q_i^L) - \frac{1}{a_q A}] \;=\; h'\left(\frac{q_i^L}{\bar{q}_{i,-1}}\right) / \bar{q}_{i,-1} \ . \tag{21b}$$

We refer to this solution by $(q_i^{L,Cf}, p_i^{L,Cf})$. Hence, the domestic firm serves the rich if

$$\begin{aligned}\pi^C(\{q_i^{h,C}, p_i^{h,C}\}_h) := \lambda\left(p_i^{H,C} - \frac{1}{a_q A} q_i^{H,C}\right) + (1-\lambda)\left(p_i^{L,C} - \frac{1}{a_q A} q_i^{L,C}\right) - h\left(\frac{q_i^{H,C}}{\bar{q}_{i,-1}}\right) \\ \geq (1-\lambda)\left(p_i^{L,Cf} - \frac{1}{a_q A} q_i^{L,Cf}\right) - h\left(\frac{q_i^{L,Cf}}{\bar{q}_{i,-1}}\right) =: \pi^{Cf}(q_i^{L,Cf}, p_i^{L,Cf})\end{aligned} \tag{22}$$

and does not serve the rich otherwise. When Condition (22)—or Condition (19), for that matter—holds with equality, domestic firms are indifferent between innovating or not to

[36] Condition (20c) holds with equality whenever (IC$^{\mathrm{H}}$) is binding for $q_i^L < (\theta^L(1-\beta)a_q A)^{(1/\beta)}$.
[37] This is because by lemma 5, (IRf$^{\mathrm{L}}$) is never binding for the poor.

serve the rich. As the following lemma shows, in such case they would offer a strictly lower quality to rich households than their best import option.

Lemma 6 (Import quality higher than domestic quality)
When a firm in the SOE is just indifferent between innovating or not to serve rich households, the quality that it would offer to the rich households is strictly lower than the quality of the rich households' best import option.

The proof of lemma 6 is given in appendix B.8. Lemma 6 implies that in equilibrium, firms in the SOE may innovate to serve rich households in some but not all differentiated goods sectors. We discuss this further in the following sections. To that end, it will be useful to define $\Delta\pi^C(\{q_i^{h,C}, p_i^{h,C}\}_h, q_i^{L,Cf}, p_i^{L,Cf}) := \pi^C(\{q_i^{h,C}, p_i^{h,C}\}_h) - \pi^{Cf}(q_i^{L,Cf}, p_i^{L,Cf})$. Moreover, to reduce notation, we note that the solutions $(q_i^{h,C}, p_i^{h,C}), (q_i^{L,Cf}, p_i^{L,Cf})$ only depend on the model's primitives, $\bar{q}_{i,-1}$ which is given in each period, and the levels of θ^h, which are endogenously determined in equilibrium, but which the firms take as given. By inserting the firms' optimal offers, we can then write $\Delta\pi_i^C(\theta^H, \theta^L)$ and accordingly $\pi_i^B(\theta^H)$ for the maximum profits in the previously discussed case (B-SOE).

5.2 Equilibrium in the small open economy

With some adjustments, we can define the equilibrium in the small open economy analogously to the one in the closed economy. Again, we define a static equilibrium, noting that the dynamic equilibrium is an infinite sequence of static one-period equilibria. As before, we refer to the set of qualities available for consumption as $\mathcal{Q}_i(t) = [0, \bar{q}^{ROW}(t)]$, where we have made use of the fact that the SOE is technologically lagging.[38] In contrast to the closed economy, the equilibrium in the SOE takes into account the foreign supply of differentiated goods, which we refer to by $x^f(\cdot)$, in exchange for the domestically produced homogeneous good (or low quality versions of the differentiated goods). As the detailed composition of the SOE's exports is a matter of indifference, we assume for expositional reasons that only the homogeneous good is exported and denote the exported quantity by z^{exp}.[39]

Definition 2 (Equilibrium in SOE)
An equilibrium in the SOE is a set of prices $\{\{p_i^e(q), p_i^{f,e}(q)\}_{(i,q)\in[0,1]\times\mathcal{Q}_i}, p_z^e, p_z^{ROW,e}, w^e = 1\}$, *quantities* $\{\{x_i^{H,e}(q), x_i^{L,e}(q), x_i^{f,e}(q)\}_{(i,q)\in[0,1]\times\mathcal{Q}_i}, z^{H,e}, z^{L,e}, z^{exp,e}\}$, *labor demand* $\{\{L_i^e(q)\}_{i\in[0,1]\times\mathcal{Q}_i}, L_z^e\}$, *and a mass of firms entering the patent race* M_i *and R&D-investments* $h(\bar{q}_i^e/\bar{q}_{i,-1})$ *of the winning firm for all* i, *such that* $p_z^{ROW,e}$ *and* $\{p_i^{f,e}(q)\}_{(i,q)\in[0,1]\times\mathcal{Q}_i}$

[38] We note that firms in the SOE can only offer qualities up to $\bar{q}_i < \bar{q}_i^{ROW}$ and, hence, a subset of $\mathcal{Q}_i$. Accordingly, $p_i(q) = \infty$ for all $q > \bar{q}_i, q \in \mathcal{Q}_i$.

[39] To simplify the exposition, the following definition considers the case where $p_i^f(\cdot)$ is given by (14). We note, however, that in the special case where inequality is so high that the rich households import all the differentiated goods—see section 5.3—the price can be higher than marginal costs. The pricing of differentiated goods in the latter case is not of particular interest for the main mechanism and results of our model, and we therefore ignore this case in the following definition.

are given by (13) and (14), respectively, and in the SOE the profits of the firms producing the differentiated goods and the homogeneous good are maximized, the households' utilities are maximized, the zero-profit condition in the patent race holds, and all markets clear, i.e.,

$$\lambda\ x_i^{H,e}(q) + (1-\lambda)\ x_i^{L,e}(q) \;=\; \frac{1}{q} a_q A L_i^e(q) + x_i^{f,e}(q)\,, \tag{23a}$$
$$\forall (i,q) \in [0,1] \times \mathcal{Q}_i \ \textit{(differentiated good markets)}\,,$$

$$\lambda\ z^{H,e} + (1-\lambda)\ z^{L,e} + z^{exp,e} \;=\; a_z A L_z^e - a_z A \int_0^1 f M_i^e + h(\bar{q}_i^e/\bar{q}_{i,-1})\ di \tag{23b}$$
$$\textit{(homogeneous good market)}\,, \tag{23c}$$

$$\int_0^1 L_i^e(q_i^{H,e}) + L_i^e(q_i^{L,e})\ di + L_z^e \;=\; \lambda\ \omega^H + (1-\lambda)\ \omega^L\ \textit{(labor market)}\,, \tag{23d}$$

$$\int_0^1 p_i^{f,e}(q_i^{H,f,e})\ x_i^{f,e}(q_i^{H,f,e})\ di \;=\; p_z^{ROW,e}\ z^{exp,e}\ \textit{(trade balance)}\,, \tag{23e}$$

where $q_i^{H,e}$, $q_i^{L,e}$ in (23d) are the quality levels for which domestic production is positive, i.e. $L_i^e(q_i^{H,e}) > 0$ and $L_i^e(q_i^{L,e}) > 0$, respectively. Analogously, $q_i^{H,f,e}$ in (23e) refers to the quality level where foreign supply is positive, i.e. $x_i^{f,e}(q_i^{H,f,e}) > 0$.

To analyze the equilibrium, we again consider the case of a common initially inherited quality level $\bar{q}_{i,-1} = \bar{q}_{-1}$ for all i, as in the closed economy. In contrast to the closed economy, however, we now need to account for the fact that firms in the SOE may innovate to serve rich households in some but not all differentiated goods sectors and the rich households import a share of the differentiated goods from the ROW. Indeed, this is typically the case in an equilibrium with trade as we will discuss further in the next section. For now it suffices to note that in such case firms must be indifferent between innovating or not to serve the rich, i.e. it must hold that $\Delta\pi_i^C(\theta^H, \theta^L) = 0$ and $\pi_i^B(\theta^H) = 0$, respectively, depending on whether $I^L > \hat{I}$ or $I^L \le \hat{I}$. Let N_f be the share of sectors in which rich households import the differentiated goods. Without loss of generality, we assume that these are sectors $i \in [0, N_f]$, and can then write θ^H as follows

$$\theta^H = \frac{I^H - \int_0^{N_f} p_i^{H,f} di - \int_{N_f}^1 p_i^H di}{\beta(\int_0^{N_f} (q_i^{H,f})^{1-\beta} di + \int_{N_f}^1 (q_i^H)^{1-\beta} di)}\ . \tag{24}$$

Our previous considerations on the households' best import option and the solution to the firms' maximization problem as specified in subsection 5.1 indicate that and how the optimal bundles $(q_i^{H,f}, p_i^{H,f})$, (q_i^H, p_i^H) depend on the level of θ^H, which in turn depends on these prices and quantities as shown in Equation (24). The equilibrium level of θ^H is therefore a fixed point of (24). This fixed point determines the equilibrium level of θ^H that is consistent with the firms' optimal contracts, and it is therefore key to characterizing the equilibrium in the SOE. For a given N_f, we denote a solution to this fixed point problem (24) by $\theta^{H,e}(N_f)$ and analogously for $\theta^{L,e}(N_f)$.[40] We can then characterize the equilibrium in the SOE as follows:

[40]We show existence of these fixed points in appendix B.9.

Proposition 3 (Equilibrium in the SOE)
If the equilibrium in the closed economy as characterized in proposition 1 does not violate condition (IRf^H), then the equilibrium in the SOE is identical to the one in the closed economy.

If the equilibrium in the closed economy violates condition (IRf^H), then there exists an equilibrium in the SOE where the differentiated goods market equilibrium can be characterized as follows:

(B-SOE) Let $I^H > \hat{I} \geq I^L$. Then the poor households consume $q^{L,e} = (1-\beta)a_q A I^L \leq \bar{q}_{-1}$ at marginal costs $p^{L,e} = q^{L,e}/(a_q A)$ and

(i) domestic firms serve the rich households in all sectors i, i.e. $N_f^e = 0$, if $\pi^B(\theta^{H,e}(0)) \geq 0$. The firms' offer $(q^{H,e}, p^{H,e})$ is the solution to (18a) and (18b) where $\theta^{H,e}(0) = (I^H - p^{H,e})/(\beta(q^{H,e})^{(1-\beta)})$;

(ii) there is a dual economy where the rich households import a share $1 > N_f^e > 0$ of the differentiated goods with the remaining share $(1 - N_f^e)$ produced by the domestic firms, if $\pi^B(\theta^{H,e}(N_f^e)) = 0$. In sectors $i \leq N_f^e$, the rich households import quality $q^{H,f,e}$ at price $p^{H,f,e}$ as given by (14) and (16). In sectors $i > N_f^e$, the domestic firms provide the bundle $(q^{H,e}, p^{H,e})$ which is the solution to (18a) and (18b);

(iii) the rich households import all differentiated goods, i.e. $N_f^e = 1$, if $\pi^B(\theta^{H,e}(1)) < 0$. If $q^{H,f,e} \leq \bar{q}_{ROW,-1}$ according to (16), the rich households import quality $q^{H,f,e}$ at price $p^{H,f,e}$ as given by (14) and (16) with $\theta^H = \theta^{H,e}(1)$. Otherwise, innovating firms from the ROW will charge a mark-up when serving rich households in the SOE.[41]

(C-SOE) Let $I^H > I^L > \hat{I}$. Then

(i) domestic firms serve the rich households in all sectors i, i.e. $N_f^e = 0$, if $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) \geq 0$. Then the firms' offer $(q^{h,e}, p^{h,e})$ is the solution to (20a) - (20d) where $\theta^{h,e}(0) = (I^h - p^{h,e})/(\beta(q^{h,e})^{(1-\beta)})$;

(ii) there is a dual economy where the rich households import a share $1 > N_f^e > 0$ of the differentiated goods with the remaining share $(1 - N_f^e)$ produced by the domestic firms, if $\Delta\pi^C(\{\theta^{h,e}(N_f^e)\}_h) = 0$. In sectors $i \leq N_f^e$, the rich households import quality $q^{H,f,e}$ at price $p^{H,f,e}$ as given by (14) and (16) with $\theta^H = \theta^{H,e}(N_f^e)$ and the domestic firms innovate for the poor households offering $(q^{L,e}, p^{L,e}) = (q^{L,Cf,e}, p^{L,Cf,e})$ which is the solution to (21a) and (21b) with $\theta^L = \theta^{L,e}(N_f^e)$. In sectors $i > N_f^e$, the domestic firms offer the bundles $(q^{h,e}, p^{h,e}) = (q^{h,C^e}, p^{h,C^e})$ which is the solution to (20a) - (20d) with $\theta^h = \theta^{h,e}(N_f^e)$;

[41] This is because firms in the SOE no longer find it profitable to compete with the ROW for serving the rich households and, hence, foreign high-quality providers are subjected to less competitive pressure when selling their products to the SOE. The optimal contract by firms from the ROW in such case is not material for the main messages of the paper, and the characterization has therefore been omitted.

(iii) the rich households import all differentiated goods, i.e. $N_f^e = 1$ *if* $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) < 0$. *If* $q^{H,f,e} \leq \bar{q}_{ROW,-1}$ *according to (16), the rich households import quality* $q^{H,f,e}$ *at price* $p^{H,f,e}$ *as given by (14) and (16) with* $\theta^H = \theta^{H,e}(1)$. *Otherwise, innovating firms from the ROW will charge a mark-up when serving rich households in the SOE.*[42] *The domestic firm innovates to offer to the low income households* $(q^{L,e}, p^{L,e}) = (q^{L,Cf,e}, p^{L,Cf,e})$ *which is the solution to (21a) and (21b) with* $\theta^L = \theta^{L,e}(1)$.

The proof of proposition 3 is provided in appendix B.9. As in the equilibrium in the closed economy, given prices and quantities for the differentiated goods, the other equilibrium values follow. They are also provided in appendix B.9.[43]

Proposition 3 characterizes the different types of equilibria in the SOE. Which of the different cases materializes depends on primitive parameters of our model. We discuss this further in the next section, where we consider how inequality impacts growth in the SOE. These discussions also provide further economic intuition for the different cases in proposition 3.

5.3 Inequality and growth: small open economy

We are now ready to characterize the growth effects of inequality in the small open economy. We analyze equilibrium innovation and, hence, growth for sequentially increasing variances of the income distribution, σ. Analogous to the closed economy, our discussion revolves around the differentiated good firms' decision problem. We again consider the most comprehensive case where $\bar{\omega} > \hat{I}$.

Low levels of inequality

– *(IRf^H) non-binding, equilibrium in SOE identical to the one in the closed economy*

As discussed in section 5.1, for given θ's, the only difference between the firms' decision problem in the SOE and the one in the closed economy is the additional set of constraints (IRf$^{\text{h}}$), where lemma 5 implies that (IRf$^{\text{h}}$) is never binding for the low types. Hence, starting from $\sigma = 0$, we have that (IRf$^{\text{h}}$) is non-binding for both household types, and the equilibrium is identical to the equilibrium in the closed economy (see proposition 3). This

[42] See footnote 41.

[43] In proposition 3, we have shown existence of an equilibrium in the SOE for a commonly inherited quality level $\bar{q}_{-1}$ across differentiated good sectors to be able to compare it to the closed economy and to emphasize the result that in a perfectly symmetric set-up we can obtain a dual economy, i.e. non-symmetric innovation investments across the differentiated good sectors, and where the rich households import a share N_f^e of differentiated goods from ROW, while they buy the share $(1 - N_f^e)$ of differentiated goods domestically. Note that in case of a dual economy, the emerging asymmetry across sectors will persist into future periods, unless one uses the assumption of strong spillovers across the differentiated good sectors, such that in every period, the sectors start again with the same quality level inherited from the previous period. In the cases other than dual economy, all domestic differentiated goods firms make identical choices and thus in any future periods they are symmetric with respect to the quality level inherited from the previous period.

is illustrated in the top-left graph of figure 2. This graph shows a household's payoff from three different consumption choices for the differentiated good as a function of its type θ: The payoff when consuming $\bar{q}_{-1}$ from the domestic competitive fringe, $\theta v(\bar{q}_{-1}) - \frac{1}{a_q}$ (orange dashed line); the payoff when consuming the optimal pooling contract offered by innovating domestic firms, $\theta v(q^P) - p^P$ (blue solid line); and the payoff from the respective best import option, $[\theta]^{\frac{1}{\beta}} [\bar{q}_{-1}]^{\frac{1-\beta}{\beta}} \chi(\tau)$ (red dotted line). Individual rationality for the low types implies that the orange dashed and the blue solid lines intersect at θ^L which is equal to θ^H in this case. Clearly, this intersection lies above the red dotted line, i.e. both types strictly prefer contract $(q^P,\, p^P)$ over their best import option.

As we increase σ, this does not affect the orange dashed line or the red dotted line in the top-left graph of figure 2. It does, however, decrease θ^L, q^P, and p^P (see proposition 2). That is, it shifts the blue solid line upwards and makes it less steep such that its intersection with the orange dotted line moves to the left. Most importantly, however, the increase in σ also increases θ^H. As long as $(\mathrm{IRf^H})$ is non-binding, a change in σ trivially has the same effect on growth as in the closed economy: Growth initially declines while still in a pooling equilibrium and eventually increases when σ and, hence, income differences are large enough such that innovating firms find it optimal to separate high types from low types. This separating equilibrium is illustrated in the top-right graph of figure 2. The green dash-dotted line shows a household's payoff from consuming quality q^H, $\theta v(q^H) - p^H$, as a function of θ. As before, $(\mathrm{IR^L})$ implies that the orange dashed and the blue solid lines intersect at θ^L. In addition, $(\mathrm{IC^H})$ implies that the blue solid and the green dash-dotted lines intersect at θ^H. As we can observe, both types still prefer their respective contract over their best import option.

Intermediate levels of inequality

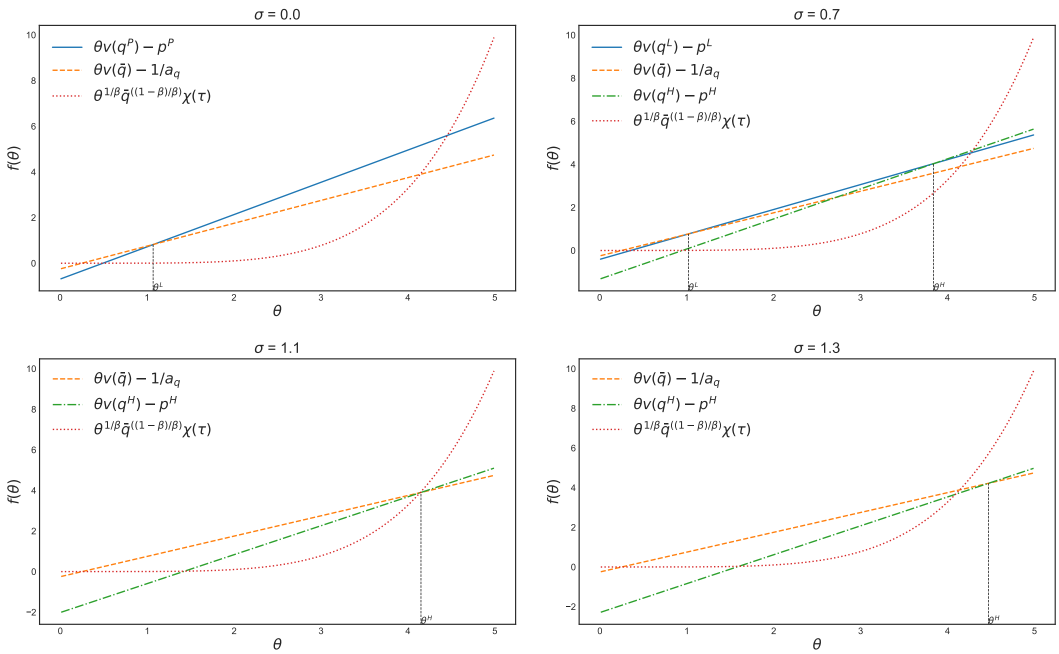
– $(\mathrm{IRf^H})$ binding, foreign competition leads to higher growth in SOE than in the closed economy

As we continue to increase σ, θ^H increases further and eventually is high enough such that high types are indifferent between consuming q^H—as determined by the optimal solution in the closed economy—and their best import option.[44] This is illustrated in the bottom-left graph of figure 2, where to clarify the exposition we show a scenario where this indifference occurs only after innovating domestic firms stopped serving the poor, i.e. it shows case (B-SOE). At this point, if we increase σ further, constraint $(\mathrm{IRf^H})$ becomes strictly binding for the high types.[45]

[44] To see that $(\mathrm{IRf^H})$ must eventually be binding, note that the value of the best import option is convex in θ, while in the closed economy the value of contract (q^H, p^H) for the rich is bounded from above by a straight line. In particular, if firms already stopped serving the poor, firms optimally set prices such that $(\mathrm{IR^H})$ holds with equality, which implies that the value of the contract is just on the orange dashed line. If they are still serving the poor, the value of the contract is determined by the blue solid line which changes as we change σ. Note, however, that it is bounded from above by a line with intercept $-\frac{1}{a_q}$ and slope $v(\hat{q}^P)$, where we use $\hat{q}^P$ to denote the optimal quality in the pooling equilibrium with $\sigma = 0$.

[45] Of course, for τ and λ high enough, $(\mathrm{IRf^H})$ is never binding while $I^L \geq 0$. We focus on the economically interesting case where $(\mathrm{IRf^H})$ is eventually binding while $I^L \geq 0$ and trade may occur. Note that

Figure 2: Illustration of the effect of inequality on innovation in the SOE



Note: The figures illustrate the optimal contracts for different values of σ. The remaining parameter values are $a_q = 4.0$, $\beta = 0.2$, $\lambda = 0.2$, $\bar{q}_{-1} = 1$, and $\tau = 3.0$. Furthermore, $h'(x) = x - 1.0$.

How will innovating firms—and, hence, the economy—respond if their optimal contract for the rich is no longer feasible due to import competition? Then domestic firms may, in principle, find it optimal to stop serving the rich. In fact, this will eventually be the case for σ high enough, as we will see below. Initially, however, when (IRf$^{\text{H}}$) is marginally binding, it is profitable for innovating firms to marginally improve the value of the contract for the rich such that they are again indifferent between consuming q^H or their best import option. This means, there is an equilibrium according to proposition 3 (B-SOE)(i) or (C-SOE)(i), depending on whether or not the domestic firms still serve the poor households with innovative qualities $q^L > \bar{q}_{-1}$. As explained in section 5.1, in the former case the firm's offer satisfies the optimality conditions (20a) to (20d). In the latter case, it satisfies optimality conditions (18a) and (18b).

The marginal increase in σ has no direct effect on q^H and, hence, growth, whether or not the domestic firms still serve the poor households with innovative qualities $q^L > \bar{q}_{-1}$. This is because for a given θ^H, q^H is pinned down by (18b) and (20d), respectively (which are identical).[46] Instead, firms improve the value of their offer to the rich by lowering the price p^H. Importantly, however, this triggers a positive general equilibrium demand effect on growth, as the lower prices p^H for all differentiated goods allow the rich to economize

for any $\tau \geq \underline{\tau}$ we can find a λ small enough such that this is indeed the case.

[46] In particular, condition (18b) and (20d), respectively, defines q^H as a function of θ^H and equates the total marginal utility from increasing q^H, $\lambda\theta^H v'(q^H)$, to the total marginal cost, $\lambda\frac{1}{a_q A} + h'\left(\frac{q_i^H}{\bar{q}_{-1}}\right)\frac{1}{\bar{q}_{-1}}$. Hence, ceteris paribus a change in q^H cannot be optimal: It would increase (decrease) the willingness-to-pay of the rich by less (more) then it would increase (decrease) the marginal cost of delivering quality to the rich.

on their spending on the differentiated goods. The associated income effect induces them to consume more of the homogeneous good and θ^H increases, which in turn increases their demand for high quality. To satisfy this demand, firms increase q^H—see condition (18b) and (20d), respectively.[47]

High levels of inequality

– *(IRf^H) binding, households import some or all of the differentiated goods*

As we keep increasing σ and, hence, θ^H, (IRf$^{\mathrm{H}}$) tightens further,[48] and this has the same qualitative effect on q^H and p^H as described previously. Eventually, however, θ^H is so high and, therefore, foreign competition fierce enough such that it is no longer profitable for firms in all domestic good sectors to serve the rich.[49] This leads to equilibrium (B-SOE)(ii), respectively (C-SOE)(ii) of proposition 3. In this equilibrium, the rich start importing some varieties from abroad, and they import strictly higher quality than what alternatively they would have been able to buy from domestic innovators as lemma 6 shows. In turn, this implies that rich households do not immediately switch to importing all differentiated goods, but instead they gradually increase the share of differentiated goods they import, N_f, as σ increases further.[50]

As noted in proposition 3, with $N_f \in (0, 1)$, firms must be indifferent between serving or not the rich. Let $\tilde{\sigma}$ be the level of inequality for which this indifference condition is just satisfied if all firms are still serving the rich (i.e. if $N_f = 0$). To shed further light on this indifference condition and how it pins down the share of differentiated goods imported, let us consider case (B-SOE), for simplicity, such that for $\tilde{\sigma}$ it holds that $I^L \leq \hat{I}$ and innovative domestic firms only serve the rich households. The firms' indifference then implies that—with $\tilde{\sigma}$—they make exactly zero profits, and these profits depend only on

[47] If innovating firms still serve the poor, we know from the solution to the firms' maximization problem in subsection 5.1 that if (IC$^{\mathrm{H}}$) is slack, innovating firms can earn higher profits by increasing q^L and p^L, holding constant $\theta^L v(q^L) - p^L$ (i.e. guaranteeing that (IR$^{\mathrm{L}}$) remains binding), up to the point where (IC$^{\mathrm{H}}$) is again binding unless the profit maximizing level of q^L can be chosen without violating (IC$^{\mathrm{H}}$).

[48] In the case where firms stopped serving the poor this follows immediately from the fact that the value of the contract for the rich in the closed economy is determined by the orange dashed line in the bottom-right graph of figure 2. The distance between this straight line and the value of the best import option as given by the strictly convex red dotted line is increasing as we increase θ beyond their intersection point.

[49] (IRf$^{\mathrm{H}}$) must eventually be strictly binding because (i) the value of the import option scales with $(\theta^H)^{1/\beta}$—see equation (17); (ii) derivations analogous to the ones for equation (17) imply that the total domestic surplus (consumer + producer surplus) from serving the rich would also scale with $(\theta^H)^{1/\beta}$ if there were no cost of innovation and the rich would consume their preferred quality at marginal cost; (iii) the cost of innovation and, consequently, the smaller increase in q^H imply that the total domestic surplus from serving the rich scales less than with $(\theta^H)^{1/\beta}$.

[50] This can be shown by contradiction. Given $\tilde{\sigma}$ as defined in the main text before (25), firms' profits are just equal whether or not they innovate to serve the rich. Hence, if σ marginally increases further and (IRf$^{\mathrm{H}}$) tightens more, firms in the SOE would, ceteris paribus, be better off not to serve the rich. Now, suppose by way of contradiction, that all domestic firms—or, for that matter, any set of positive measure of firms—would stop serving the rich in response to the marginal increase in σ. From lemma 6 we know that in such case households import strictly higher quality than what they would otherwise buy from innovating domestic firms and, hence, by (IRf$^{\mathrm{H}}$), at a strictly higher price. In turn, this would trigger a drop in θ^H implying that $\theta^H < \tilde{\theta}^H$. But with $\theta^H < \tilde{\theta}^H$, domestic firms would again be strictly better off innovating and serving the rich—due to less fierce international competition at lower levels of θ^H—, a contradiction.

θ^H.[51] Hence, as we increase σ, firms remain indifferent between serving or not the rich if θ^H remains constant at $\tilde{\theta}^H$, where

$$\tilde{\theta}^H := \frac{\tilde{I}^H - \int_0^1 \tilde{p}^H di}{\beta \int_0^1 (\tilde{q}^H)^{1-\beta} di} \tag{25}$$

is the rich households' type given $\tilde{\sigma}$ and where we used $\tilde{x}$ to indicate the value of variable x for the case of $\tilde{\sigma}$. It follows that for $\sigma > \tilde{\sigma}$ and, hence, $I^H > \tilde{I}^H$, N_f is implicitly defined by

$$\tilde{\theta}^H = \frac{I^H - \int_0^{N_f} \tilde{p}^{H,f} di - \int_{N_f}^1 \tilde{p}^H di}{\beta(\int_0^{N_f} (\tilde{q}^{H,f})^{1-\beta} di + \int_{N_f}^1 (\tilde{q}^H)^{1-\beta} di)} \ . \tag{26}$$

In other words, when the high income households start consuming imported qualities, θ^H no longer increases with σ—and, hence, neither do q_i^H and $q_i^{H,f}$. Instead, an increase in income translates into a higher share of imported qualities and, as a consequence, less growth compared to the closed economy.

It is now also clear, that if I^H increases beyond a critical level $\bar{I}^H$ implicitly defined by

$$\tilde{\theta}^H = \frac{\bar{I}^H - \int_0^1 \tilde{p}^{H,f} di}{\beta(\int_0^1 (\tilde{q}^{H,f})^{1-\beta} di)} \ , \tag{27}$$

then the rich households will consume imported qualities of all i, and θ^H, and the corresponding best imported quality level $q^{H,f}$ increase again with income I^H or σ, for that matter.[52] This case corresponds to equilibrium (B-SOE)(iii), respectively (C-SOE)(iii) in proposition 3.

More generally, as long as σ is not extremely high, firms in some differentiated good sectors are highly innovative and still serve the rich, while firms in other differentiated good sectors either innovate less to serve the poor [in case (C-SOE)] or stop innovating altogether [in case (B-SOE)]. In either case, the quality $\bar{q}_i$ in the importing sectors is lower when compared to the closed economy and $A(t+1)$ is decreasing vis-à-vis the closed economy as we increase σ further.

We can now formulate proposition 4, which follows from the previous discussions.

Proposition 4 (Inequality and growth in the SOE)
In the small open economy:

(i) For small values of the variance in incomes σ, the only equilibrium is a no-trade equilibrium, that is, equilibrium outcomes are the same as in the closed economy.

[51] When innovative firms still serve the poor, their profits when serving or not the rich also depend on θ^L which also changes with σ. This complicates the subsequent discussions, but it does not impact the essence of the argument.

[52] We note that in such case, firms from the ROW no longer serve rich households in the SOE at marginal costs inclusive of trade costs because domestic firms are strictly non-competitive. Nevertheless, from the perspective of the innovating firms in the SOE, foreign firms serving consumers in the SOE at marginal costs still is the relevant benchmark that they would need to meet to still serve the rich—see also footnote 30.

(ii) For intermediate values of the variance in incomes σ, constraint (IRf$^{\mathrm{H}}$) is binding and innovating firms block entry from foreign competitors by lowering p^H. Quality q^H and thus $A(t+1)$ is higher than in the closed economy.

(iii) For values of σ sufficiently high, domestic firms can no longer profitably compete with foreign firms in serving the rich households in all differentiated good sectors. In some sectors, high qualities are then imported. As σ increases further, eventually the share of sectors in which high qualities are imported increases and the domestic technological level $A(t+1)$ decreases.

The proposition carries the central message of this paper that the growth effects of inequality are very different in the SOE when compared to the closed economy. In the closed economy, an increase in σ has a positive effect on growth whenever firms find it optimal to offer separate qualities for the rich and the poor households. By contrast, in an SOE with inequality high enough such that (IRf$^{\mathrm{H}}$) is binding, firms initially block entry of foreign competitors by lowering p^H, leading to a positive general equilibrium effect on q^H and higher growth. This positive *demand effect* is rooted in the fact that—due to international competition—firms charge smaller mark-ups for high qualities.[53]

As inequality increases further, it is eventually high enough such that some domestic firms no longer find it optimal to serve rich households, implying that foreign competition has a negative *business stealing effect* on innovation and, hence, economic growth. This business stealing effect gets bigger as we further increase σ. This is for two reasons: On the one hand, the higher σ, the larger the share of differentiated goods that the rich import. On the other hand, in the closed economy, an increase in σ raises the taste for quality of the rich. As discussed above, this price effect is the key driver underlying a demand-driven positive relationship between inequality and growth in the closed economy. The key observation is that this channel is no longer present in the SOE if rich households satisfy their demand for high quality via importing. Corollary 2 summarizes these two central novel effects of inequality on growth in the SOE that derive from our analysis.

Corollary 2 (Growth effects of inequality in the SOE)
Compared to the closed economy, there are two novel effects of inequality on growth in the small open economy:

(i) *a positive general equilibrium* demand effect *(+) that is triggered by increased competition from foreign firms;*

(ii) *a negative* business stealing effect *(-) , when inequality is so high that domestic firms are no longer able to compete with foreign entrants in all differentiated good sectors, and rich households start satisfying their demand for some varieties via importing*

[53]In our model, a smaller mark-up ultimately results in higher demand as mark-ups are fully absorbed by the fixed cost of innovation. Note, however, that the same would also be true with positive profits as long as a smaller mark-up on the side of the firms is not passed on one-for-one to rich households via dividend payments.

high quality.

Interestingly, the previous discussions also show how non-homothetic demand for quality along with the threat of import competition from abroad can give rise to an endogenous emergence of a dual economy in developing countries, even with an ex-ante perfectly symmetric set-up. The basic intuition is that the domestic population is not rich enough to satisfy all of its demand for quality by importing pricey high qualities from abroad.[54] We summarize this insight in the following corollary.

Corollary 3 (Dual economy in the SOE)
In contrast to the closed economy, in an equilibrium with trade there is a 'dual economy' in the ex-ante perfectly symmetric SOE, i.e. some differentiated good sectors in the SOE are highly innovative, while others are lagging behind. Only for very high levels of inequality, the high income households import all types of differentiated goods and there will be no 'dual economy' as a consequence.

6 Discussion and Further Results

This section provides further discussion, first of our theoretical set-up and results, and then of empirical patterns in the data.

6.1 Theoretical Discussion

6.1.1 A novel 'Dutch disease' effect

In developing our arguments, we have assumed that there is a one-to-one mapping between the level of technology and aggregate income. In line with that view, we also considered a ROW that is perfectly symmetric to the SOE but for its technological level. This is not necessarily the case in oil-rich countries, for example. It is therefore interesting to know how a country's growth prospects change if we increase incomes, holding constant the domestic level of technology. Interestingly, in the closed economy frameworks previously considered in the literature, this will typically boost growth as higher incomes imply higher demand for quality and therefore greater gains from innovation. In our case, this is evident from considering equation (10d), which implies that firms respond to higher incomes—and therefore a higher θ^H—by increasing q^H, reflecting the higher willingness to pay for quality on the side of the rich. In the SOE, however, the increase in income also implies that the value of the best import option for the household increases, and this

[54] Our paper is thus also speaks to the literature on dual economies that goes back at least to Lewis (1954)—see LaPorta and Shleifer (2014) for an overview. In our set-up a dual economy emerges endogenously and at a general level our work is thus related to Porzio (2017), but the underlying mechanisms are very different. Porzio (2017) considers a model with sorting and matching of heterogeneous agents into becoming managers and workers. He shows how a dual economy can arise if firms in developing countries have the opportunity to adopt state-of-the-art technologies from abroad.

may have an effect on innovation and growth similar to an increase in inequality.[55] In particular, higher windfalls may—for a given level of inequality—imply that the SOE ends up being in scenario (iii) of proposition 4 where the economy suffers from the negative business stealing effect.[56] In such case, the economy might suffer from a novel negative '*Dutch Disease*' type effect of windfall gains on growth. As opposed to the textbook case, the effect here is not centered on intersectoral reallocations,[57] but on the fact that windfall gains through e.g. oil revenues imply that households get richer vis-à-vis the domestic level of technology which may imply that domestic firms find it harder to compete with foreign high-quality providers.

6.1.2 Distance to frontier, inequality, and growth in the SOE

Our previous results critically depend on the ability of rich households to satisfy their demand for high quality via importing. In turn, this ability depends on a country's distance to frontier, and we therefore discuss next the importance of a country's distance to frontier for growth and the inequality-growth nexus.[58]

When increasing the technological level in the SOE, keeping constant the income distribution and the technological level in the ROW, the SOE's GDP increases, benefiting both low and high income types. In turn, this increases the households' demand for quality. As long as the SOE is sufficiently far from the frontier, however, such an increase in aggregate technology has no effect on innovation and growth in the SOE. This is because equilibrium qualities are a constant multiple of the technological level inherited from the previous period as shown in corollary 1. This carries over to the SOE as long as it is far from the technological frontier. Specifically, this is the case as long as $\tilde{q}^{H,f} \leq \bar{q}^{ROW}$, where as before we use $\tilde{q}^{H,f}$ to denote the optimal import quality of rich households for the level of σ such that domestic firms are just indifferent between innovating or not to serve the rich, i.e. $\tilde{\sigma}$.

This, however, is no longer true if the SOE is sufficiently close to the world's technological frontier $\bar{q}^{ROW}$. In such case, and for high enough inequality, the rich households' optimal imported quality $q^{H,f}$ as defined in equation (16) is no longer available because it is beyond the technological frontier in the ROW.[59] When this happens, the best import option for

[55] If innovating firms stopped serving the poor, a proportionate windfall gain in incomes has the exact same effect on innovation and growth as an increase in σ. In a separating equilibrium, a proportionate windfall gain also increases the incomes of the low-types, which impacts (IR^{L}) and, therefore, (IC^{H}).

[56] More generally, windfall gains impact the market size for innovative goods and households' willingness to pay for innovations, and the overall effect on economic growth depends on the relative sizes of these effects and of the 'business stealing effect' through intensified foreign competition.

[57] The basic argument is that an oil boom causes a real appreciation of the domestic currency and therefore decreases an economy's competitiveness in other tradable sectors. If the primary sector has a lower growth potential, this undermines an economy's long-run growth prospects. See e.g. Corden and Neary (1982).

[58] The ability to import high quality further depends on a country's openness to trade. We briefly discuss this below.

[59] Note that this must eventually happen at strictly positive distance from the ROW because at the point where domestic firms are just indifferent between serving or not serving the rich households, the

rich households is to demand the highest quality in the ROW $\bar{q}^{ROW}$ at marginal cost. Importantly, this implies that innovating domestic firms can compete with foreign firms for higher levels of inequality. More specifically, constraint (IRf$^{\mathrm{H}}$) is binding for higher levels of inequality only, and whenever it is binding, the outside option $\bar{q}^{ROW}$ has a lower value to the household. We provide further technical details in appendix A.1 and summarize the main insights in the following proposition.[60]

Proposition 5 (Distance to frontier, inequality, and growth in the SOE) *Let $\tilde{\sigma}(\tau, A)$ be the highest σ such that domestic firms still find it optimal to serve the rich if all other domestic firms do. Let $\tilde{q}^{H,f}$ denote the optimal import quality of rich households in this case. An increase in the level of technology in the SOE, $\bar{q}_{-1}$ increases $\tilde{\sigma}(\tau, A)$ and thus allows firms in the SOE to successfully compete against foreign high-quality providers for higher levels of inequality if the SOE is sufficiently close to the technological frontier such that $\tilde{q}^{H,f} > \bar{q}^{ROW}$.*

6.1.3 Further discussion and extensions

The model could be extended in various ways as we now briefly explain. Further details are provided in appendix A.2.

In our model, there are no knowledge spillovers from the rest of the world (ROW) to the domestic country but only knowledge spillovers within the domestic economy. This reflects that firms in most developing countries are still very far from the world's technological frontier (see e.g. Cirera et al. (2022)). Reasons can be a lack of absorptive capacity (Cohen and Levinthal, 1989), and that knowledge spillovers often have local public good character (see e.g. Zucker et al. (1998)). Of course, no international spillovers is a stark simplification. Appendix A.2 illustrates how our model can accommodate international knowledge spillovers. As long as such spillovers are not too strong, they do not impact our main mechanism. Interestingly though, knowledge spillovers might get reinforced if they bring the economy closer to the technological frontier such that it can now better compete with foreign technology leaders as discussed above.

Our model set-up focuses on innovations in the quality levels of differentiated goods and assumes that efficiency improvements in production are a by-product of quality-upgrading. Of course, process innovations are a purposeful activity by firms that can play an important role for the provision of higher quality products to the poorer part of society (e.g. see Foellmi et al. (2014)). Appendix A.2 presents a simplified set-up with process innovation

optimal import option of the rich involves strictly higher quality than their domestically offered quality, i.e. it is above the technological level in the SOE as shown in lemma 6.

[60] Of course, for any distance from the frontier, as we keep on increasing σ, we will always eventually reach a point where the best import option of the rich is to consume $\bar{q}^{ROW}$ and the red dotted curve becomes a straight line in figure 2 as well. The point is, that for countries far from the frontier as shown in figure 2, this happens only at levels of inequality where domestic firms anyways no longer find it profitable to serve rich households, and it therefore has no effect on their behavior. By contrast, if the SOE is close enough to the frontier, this allows domestic firms to profitably innovate to serve the rich for higher levels of inequality.

for qualities that are inherited from the past. Such innovation intensifies competition with the domestic competitive fringe. In turn, this implies that constraint (IRf^{h}) is binding at higher levels of inequality only, but the impact of international competition on the inequality-growth nexus is otherwise the same.

To carve out the novel causal effects of inequality on growth in the SOE, we have considered an economy with two types of households. With more than two types, the analysis would be more involved, but the effects identified in proposition 4 would still be at play and, in fact, typically simultaneously. Appendix A.2 briefly discusses this with reference to the limiting case with a continuum of types.

Lastly, the previous analyses held constant a country's openness to trade. Our set-up implies that the growth effects of trade liberalizations depend on a country's distance to frontier, echoing previous findings in the literature (Aghion et al., 2005, 2009; Amiti and Khandelwal, 2013). It adds to this literature by highlighting that such growth effects also critically depend on the level of inequality in a country. See appendix A.2 for further discussion.

6.2 Empirical patterns

In sum, we consider developing countries that are not at the frontier and analyze the growth effects of inequality. Our analysis isolates a negative business stealing effect that is particularly prevalent in unequal and open countries, implying that inequality has a smaller or even negative effect on growth in an open economy when compared to a closed economy as previously considered in the literature. While a full econometric analysis is beyond the scope of the paper, we were nevertheless interested as to whether this theoretical prediction can speak to basic correlations in the data. In appendix D, we provide an empirical illustration comprising (1) standard growth regressions using growth in GDP per capita as the dependent variable and (2) industry-level growth regressions using growth in export quality taken from Feenstra and Romalis (2014) as the dependent variable. In these regressions, we then control for an interaction of inequality and openness. Across a range of specifications, using either a large number of country controls or country fixed effects, we illustrate that for developing countries this interaction term is typically significantly negative in the industry-level regressions, and still negative—albeit not in all cases significant—in our country-level regressions, in line with our theoretical predictions.[61] While our theory can thus speak to basic correlations in the data, we emphasize that these regressions only show associations and they involve variables that are difficult to measure. These regressions must therefore be interpreted with caution. See the appendix for further details.

[61] In that regard, our paper also relates to the empirical literature analyzing the linkages from income inequality to economic growth. This literature tends to find a negative effect, but the evidence is far from being conclusive—see e.g. Bénabou (1997); Barro (2000); Voitchovsky (2005); Easterly (2007); Ostry et al. (2014); Halter et al. (2014); Brueckner and Lederman (2018).

7 Conclusion

In this paper, we analyzed how inequality impacts growth in developing countries in the context of a Schumpeterian model with growth through quality upgrading and non-homothetic demand for quality. Our key insights show that the growth effects of inequality are very different in an open when compared to a closed economy: Higher inequality boosts the willingness to pay for high quality of rich households, which stimulates innovation and growth in the closed economy.

In the open economy, however, this increased taste for quality also makes importing high qualities from abroad more attractive. For low levels of inequality this triggers a positive demand effect on innovation as innovating domestic firms deter entry from foreign competitors by lowering their price on high qualities. For sufficiently high levels of inequality, however, this is no longer profitable and rich households start satisfying their demand for quality via importing, giving rise to a negative business stealing effect of inequality on growth. The size of this effect critically depends on a country's stage of development and its openness to trade. Overall, our theory suggests that in the developing world inequality is more harmful for growth in open as opposed to closed economies.

While these observations have so far largely gone unnoticed in the literature, we believe that they are of first order importance for our understanding of the growth prospects of developing countries, and they are of immediate relevance for redistributive and trade policies: In essence, our findings show how a strong (upper-) middle class can be key for sustained growth in the developing world, and how for low levels of development tariffs can have a beneficial effect on growth. The latter point is related to previous findings in the literature (Aghion et al., 2005, 2009; Amiti and Khandelwal, 2013). Our work shows how the growth effects of such policies critically depend on inequality.

Our model makes several simplifying assumptions. In developing countries, we may find stronger entry barriers for firms, more macroeconomic volatility, and slower imitation of technological advances by a competitive fringe of firms, for example. Moreover, inequality and openness to trade impact growth through additional channels, including knowledge spillovers, investments in human capital, or political institutions. Incorporating such factors will be an interesting avenue in future work to study the robustness of our findings. It would also be interesting to analyze how inequality and openness impact growth in countries at the world's technological frontier.

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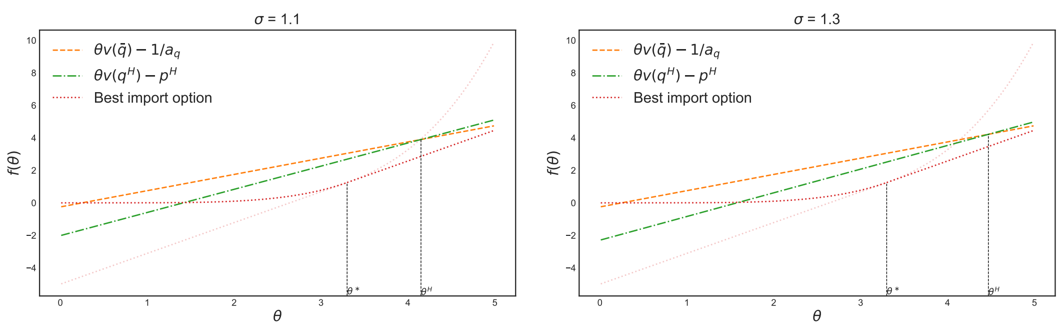
Appendix

A Details on Discussion and Extensions

A.1 Competitiveness of the SOE close to the frontier

In this appendix, we provide further details on the case where the SOE is sufficiently close to the world's technological frontier $\bar{q}^{ROW}$ such that for high enough levels of inequality, the rich households' optimal imported quality $q^{H,f}$ as defined in equation (16) is no longer available because it is beyond the current technological frontier in the ROW. When this happens, the best import option for rich households is to demand the highest quality in the ROW, $\bar{q}^{ROW}$, at marginal cost. In terms of figure 2, this implies that the red dotted line changes: It will stay the same up to the level of θ^* for which the best import option at marginal cost is exactly $\bar{q}^{ROW}$, and will become a straight line thereafter, as shown in figure 3. This straight line is tangential to the red dotted curve at θ^* and is everywhere below this line. Importantly, this implies that innovating domestic firms can compete with foreign firms for higher levels of inequality. More specifically, constraint ($\mathrm{IRf}^{\mathrm{H}}$) is binding for higher levels of inequality only, and whenever it is binding, the outside option has a lower value to the household. This is shown in figure 3 for the case where a household with $\theta^* = 3.3$ just finds it optimal to import $q^{H,f} = \bar{q}^{ROW}$. The best import option for a household as a function of θ is now indicated by the dark red dotted line. Clearly, for $\sigma = 1.1$, the level of inequality where ($\mathrm{IRf}^{\mathrm{H}}$) was just binding in figure 2, this constraint is no longer binding (left panel), because the rich can now do no better than importing $\bar{q}^{ROW} < q^{H,f}$ at marginal cost and this carries lower value. The same is still true for even higher levels of inequality (right panel).

Figure 3: Illustration of the effect of inequality on innovation in an SOE closer to the frontier



Note: The figures illustrate the values of different consumption options as a function of θ for different values of σ. The remaining parameter values are $a_q = 4.0$, $\beta = 0.2$, $\lambda = 0.2$, $\bar{q}_{-1} = 1$, and $\tau = 3.0$. Furthermore, $h'(x) = x - 1.0$.

A.2 Extensions and further discussions

In this appendix, we provide further details on the extensions and discussions in section 6.1.

Knowledge spillovers

In our model there are no knowledge spillovers from the rest of the world (ROW) to the domestic country but only knowledge spillovers within the domestic economy. This reflects that firms in most developing countries are still very far from the world's technological frontier (see e.g. Cirera et al. (2022)). Reasons can be a lack of absorptive capacity (Cohen and Levinthal, 1989), and that knowledge spillovers often have local public good character (see e.g. Zucker et al. (1998)). Of course, no international spillovers is a stark simplification and our model can accommodate international knowledge spillovers for example as follows: In period t, the domestic firm innovates up to quality $q_i^H(t) \geq \bar{q}_{i,-1}$ and at the beginning of the next period additionally obtains knowledge spillovers from ROW of size $\nu[\bar{q}_i^{ROW}(t) - q_i^H(t)]$, such that $\bar{q}_i(t) = q_i^H(t) + \nu[\bar{q}_i^{ROW}(t) - q_i^H(t)]$. Parameter $\nu \in [0, 1]$ reflects the strength of the international knowledge spillovers. This definition allows us to define $A(t+1) = \int_0^1 \bar{q}_i(t)di$ as before in subsection 3.3 and, hence, our previous analysis would directly apply to this case with spillovers. Of course, to maintain the focus of our analysis on the SOE as a developing country, we have to assume that the knowledge spillovers are not too strong, that is, the parameter ν must not be too large. Otherwise, the analysis would effectively change to technological competition between countries at the technological frontier.[62]

In the previous argument, knowledge spillovers are independent of trade and only depend on the distance to frontier. An alternative channel through which such spillovers can occur is by learning from sellers (Buera and Oberfield, 2020), and in our model such spillovers may arise because the SOE imports higher quality from abroad than available domestically—see section 5.3. It is therefore interesting to note that our analysis points to a potential amplifying mechanism of knowledge spillovers. In particular, this would be the case if the knowledge spillovers lifted the SOE to a technological level sufficiently close to the technological frontier such that domestic innovators were now better positioned to compete with foreign technology leaders—see section 6.1.

Process innovation

Our model set-up focuses on innovations in the quality levels of differentiated goods and assumes that efficiency improvements in production are a by-product of quality-upgrading. Of course, process innovations are a purposeful activity by firms that can play an important role for the provision of higher quality products to the poorer part of society (e.g. see the seminal work by Foellmi et al. (2014)).

[62] Such a model of technology competition of advanced economies can be found e.g. in Gersbach and Schneider (2015).

A detailed account of process innovation is beyond the scope of our paper. Yet, a simplified set-up can shed some light on it's impact on our growth channel in closed vs open economies. To that end, suppose that in each period, there are two sequential stages: At the first stage, the product innovator decides whether and how much to invest in improving the quality of good i and what kind of price-quality bundles to offer to the different household types, analogous to our baseline model. After that, at the second stage, potential process innovators enter a patent race analogous to the one described above for product innovation.[63] We assume that the product quality innovating monopolist does not participate in the patent race for process innovations. The winner of the patent race gets a one-period patent on a new production process for all pre-existing qualities $q_i \leq \bar{q}_{i,-1}$. This process reduces production costs of all such qualities by factor $1/\xi$, $\xi > 1$, such that the marginal production costs of any quality $q_i \leq \bar{q}_{i,-1}$ of good i reduce to

$$\frac{1}{\xi a_q A} q_i \ , \tag{A.1}$$

where A may or may not depend on aggregate quality, depending on whether or not we want to allow for knowledge spillovers from product innovation to production processes. As with quality innovation, after one period the patent expires, and the improved process can be used freely for all qualities of good i.[64]

To stay as close as possible to our baseline set-up, we assume that in the period of invention, process innovators can serve only the domestic market, i.e. process innovation does therefore not lead to any profits from exporting. The set-up then implies that there is process innovation if and only if there is domestic demand for any quality level $q_i \leq \bar{q}_{i,-1}$ at a price higher than $q_i/(\xi a_q A)$. This feeds back into the decision problem of product innovators as process innovation makes it more attractive for households to consume qualities $q_i \leq \bar{q}_{i,-1}$, i.e. it impacts households individual rationality constraints. How exactly depends on the details and, in particular, when deciding on their offers for both types of households, process innovators also need to make sure that these offers are incentive compatible. While comprehensively scrutinizing these interactions would be very interesting, here we shed some light on how process innovation impacts our main channel of interest by proceeding under the assumption that the income of the low types is sufficiently low such that (i) process innovators find it beneficial to serve the low types and (ii) low types prefer their best quality at marginal costs with no process innovation to quality $\bar{q}_{i,-1}$ even if the latter is offered at the new marginal cost after process innovation.[65]

[63] That is, firms can enter the patent race for process innovation at a fixed cost f^{PI}, and a free entry condition implies that the mass of entering firms will be such that expected profits net of fixed costs are zero for process innovators. Since firm ownership is equally distributed across households, there is again no income from firm profits.

[64] To maintain the symmetry, we further assume that the improved processes further spill-over to homogeneous-good production, but this is not essential for our main subsequent arguments.

[65] This is the case for $I \leq \hat{I}^{PI}$, where $\hat{I}^{PI}$ is the smaller solution to

$$\frac{I^\beta}{[(1-\beta)a_q]^{1-\beta}} = \frac{1}{a_q \xi} + \beta I.$$

Low types are then served by process innovators at a price equal to the marginal cost of the competitive fringe at $q_i/(a_q A)$. More importantly for our purposes here, process innovators are further willing to provide quality $\bar{q}_{i,-1}$ at a price equal to marginal costs with process innovation to the high income households. In turn, this implies for the product innovators' profit maximization problem:

$$\max_{q_i^H, p_i^H, \bar{q}_i} \lambda \left(p_i^H - \frac{1}{a_q A} q_i^H \right) - h \left(\frac{\bar{q}_i}{\bar{q}_{i,-1}} \right)$$

$$\text{s.t.} \quad \theta^H v(q_i^H) - p_i^H \geq \max_{q \in [0, \bar{q}_{i,-1}]} \left\{ \theta^H v(q) - \frac{1}{\xi a_q A} q \right\}, \tag{IR$^{\mathrm{H}}$}$$

$$\theta^H v(q_i^h) - p_i^H \geq \max_{q \in [0, \bar{q}_i^{ROW}]} \left\{ \theta^H v(q) - \tau^2 \frac{1}{a_q A} q \right\}, \tag{IRf$^{\mathrm{H}}$}$$

$$q_i^H \leq \bar{q}_i \,,$$

where, for simplicity, we consider no process innovation in the ROW. The optimality conditions in the closed economy are

$$\theta^H v(q_i^H) - p^H \;=\; \theta^H v(\bar{q}_{i,-1}) - \frac{\bar{q}_{i,-1}}{\xi \, a_q \, A} \,, \tag{A.2a}$$

$$\lambda[\theta^H v'(q_i^H) - \frac{1}{a_q A}] \;=\; h' \left(\frac{q_i^H}{\bar{q}_{i,-1}} \right) / \bar{q}_{i,-1} \,. \tag{A.2b}$$

These conditions are analogous to equations (9a) and (9b), but (IR$^{\mathrm{H}}$) is now tighter due to the process innovation. This does not directly impact the optimal choice of q_i^H, which is governed by the second condition (A.2b), but has a general equilibrium effect via θ^H. In particular, analogous to international competitive pressure, the competition from domestic process innovators induces product innovators to lower their price, which triggers a positive general equilibrium demand effect on growth.

In the open economy, the same argument implies that constraint (IRf$^{\mathrm{H}}$) is binding at higher levels of inequality only, but the impact of international competition on the inequality-growth nexus we discuss is then identical, i.e. our insights are robust to this variant with process innovation.[66]

More than two types of households

To carve out the novel causal effects of inequality on growth in the SOE, we have considered an economy with two types of households. With more than two types, the analysis would be more involved, but the effects identified in proposition 4 would still be at play

If this condition is not satisfied, the contract that process innovators offer to the high types depends on how it affects their ability to extract rents from serving the low types.

[66] Process innovation has, however, interesting implications for the long-run growth prospects of an economy. In particular, our baseline economy can be stuck in a no-growth equilibrium, either because both types of households are so poor that they demand qualities from the domestic competitive fringe or because inequality is so high that rich households import all differentiated goods from the ROW and poor households consume from the domestic competitive fringe. In either case, process innovation would gradually over time increase households' incomes and, hence, bring households that consume from the domestic competitive fringe closer to the domestic quality frontier and eventually would have them demand domestically innovated qualities.

and, in fact, typically simultaneously. Consider, for example, the limiting case with a continuum of types. In such case, innovating firms typically find it optimal to exclude a positive mass of households with income $I > \hat{I}$. That is, they offer increasing contracts in accordance with local (IC) constraints to intermediate types, and pool the highest types at the top to economize on costs of innovation.[67] In the SOE, the very rich satisfy their demand for high quality via importing. An increase in inequality implies that the marginal type who was just indifferent between importing or consuming the highest domestic quality ceteris paribus finds it now beneficial to import. To counteract this negative business stealing effect, innovating firms respond by improving the offer to the highest types, which typically triggers the positive general equilibrium demand effect on innovation and growth. In general, the overall effect of an increase in inequality on innovation incentives depends on how large these two opposing forces are. Yet, in either case the fact that parts of society satisfy their demand for quality via importing is in contrast to the beneficial willingness-to-pay effect of inequality on growth in the closed economy.

Trade openness

Our analysis points to interesting policy implications with respect to opening developing countries for trade with technologically more advanced economies. Trade costs can shield domestic innovating firms from international competition, thereby stimulating growth. In our model this is reflected in the fact that, ceteris paribus, households' best import options are less valuable the higher the trade costs—see equation (17). In turn, this immediately implies that for higher trade costs domestic firms can successfully compete against foreign high-quality providers for larger levels of inequality, i.e. $\hat{\sigma}(\tau, \bar{A})$ as defined in proposition 5 increases in τ. Our work is thus also related to a large literature on dynamic gains from trade and infant industry protection.[68] It shows how lower trade barriers may lead to more quality upgrading in industries (countries) close to the frontier, but discourage quality upgrading in industries (countries) further away from the frontier. This echoes previous findings in the literature (Aghion et al., 2005, 2009; Amiti and Khandelwal, 2013).[69] The key novelty of our set-up is that it highlights how potential gains from trade protection critically depend not only on the distance to frontier, but also

[67]If the distribution of types is without mass points, this has to be the case because it can never be optimal to bear additional innovation costs just to serve households of measure 0.

[68]See footnote 10.

[69]Aghion et al. (2005) emphasize how competition can increase or decrease innovation incentives depending on the competitive environment of firms, which is related to the firms' technological distance to the frontier. Our model similarly shows that competition can lead to higher innovation or discourage innovation. However, we emphasize how this depends on inequality and, as a consequence, on the demand for quality. Firms lagging behind the world's technological frontier can have high innovation incentives to be able to compete with foreign competition if inequality is low and consequently demand for quality is in a range where the domestic firms can successfully compete after innovating. However, with higher inequality the households' demanded level of quality is too high for a lagging domestic firm to be able to profitably cover the high costs of innovation necessary to successfully compete with foreign competition at that level. Therefore, our model shows how domestic firms' innovation incentives and competitiveness depends on the interaction between the distance to the technological frontier, openness, and the level of inequality.

on the level of inequality in a country.

B Proofs

B.1 Proof of lemma 1

We show a variant of lemma 1 with a continuous closed set of types Θ, before explaining that the case with two types then follows immediately as a special case.

Lemma 1′
The decision problem of innovating firm i is equivalent to:

$$\max_{\{q_i(\theta),p_i(\theta)\}_{\theta\in\Theta},\bar{q}_i} \quad \int_{\theta\in\Theta}\left[p_i(\theta)-\frac{1}{a_qA}q_i(\theta)\right]f_\theta(\theta)\,d\theta - h\left(\frac{\bar{q}_i}{\bar{q}_{i,-1}}\right)$$

$$\text{s.t.} \quad \theta v(q_i(\theta))-p_i(\theta)\geq \max_{q\in[0,\bar{q}_{i,-1}]}\left\{\theta v(q)-\frac{1}{a_qA}q\right\}\,, \quad \forall\,\theta\in\Theta \qquad \text{(IR)}$$

$$\theta v(q_i(\theta))-p_i(\theta) = \max_{\hat{\theta}\in\Theta}\left\{\theta v(q_i(\hat{\theta}))-p_i(\hat{\theta})\right\}\,, \quad \forall\,\theta\in\Theta \qquad \text{(IC)}$$

$$q_i(\theta)\leq\bar{q}_i, \quad \forall\,\theta\in\Theta\,,$$

where $v(q) := q^{1-\beta}$ and where the firm considers type $\theta^h := \frac{p_z z^h}{\beta Q^h}$, $Q^h := \int_0^1 {q_i^h}^{1-\beta}\,di$, of household h as exogenously given. θ^h is private knowledge to the households and is distributed according to $f_\theta(\theta)$ with support Θ, with this probability density function (pdf) being common knowledge.

Proof To show the desired result, we proceed in two steps.

Step 1. From the discussions in the main text—decision problem (5)—we know that households choose quality to maximize

$$\max_{q_i^h} \quad {q_i^h}^{1-\beta}\underbrace{\frac{z^h p_z}{\beta Q^h}}_{:=\theta^h} - p_i(q_i^h)\,.$$

From the perspective of innovating firm i, $\theta^h := \frac{z^h p_z}{Q^h\beta}$ is a sufficient statistic for household characteristics, which is exogenous to the firm and observed only by the household. θ is distributed according to $f_\theta(\theta)$, which depends on the full general equilibrium in the economy.

Let Θ denote the set of pairwise distinct elements in $\{\theta^j\}_{j\in[0,1]}$. Then, by the revelation principle (cf. e.g. Mas-Colell et al. (1995, Proposition 23.C.1)), the innovating firm can limit attention to truthful revelation mechanisms, i.e. for each $\theta\in\Theta$ a quality-price bundle $(q_i(\theta),p_i(\theta))$ such that households find it optimal to truthfully reveal their type, that is

$$\theta v(q_i(\theta))-p_i(\theta) = \max_{\hat{\theta}\in\Theta}\left\{\theta v(q_i(\hat{\theta}))-p_i(\hat{\theta})\right\}\,, \quad \forall\,\theta\in\Theta\,, \qquad \text{(IC)}$$

where $v(q_i) = {q_i}^{1-\beta}$ and $p_i(\theta) := p_i(q_i(\theta))$.

Step 2. The competitive fringe implies that all qualities $q_i \leq \bar{q}_{i,-1}$ are offered at marginal cost, which, in turn, implies that every household must weakly prefer its offered contract $(q_i(\theta^h), p_i(\theta^h))$ to its best choice among all qualities $q \leq \bar{q}_{i,-1}$

$$\theta v(q_i(\theta)) - p_i(\theta) \geq \max_{q\in[0,\bar{q}_{i,-1}]} \left\{\theta v(q) - \frac{1}{a_q A} q\right\} \ , \quad \forall\, \theta \in \Theta \ . \tag{IR}$$

Reformulating the decision problem for a finite discrete set of types Θ with population weights f_θ for any type $\theta \in \Theta$ such that $\sum_{\theta\in\Theta} f_\theta = 1$ would only change the integral in the objective function to a sum, i.e. to

$$\max_{\{q_i(\theta), p_i(\theta)\}_{\theta\in\Theta}, \bar{q}_i} \qquad \sum_{\theta\in\Theta} \left[p_i(\theta) - \frac{1}{a_q A} q_i(\theta)\right] f_\theta - h\left(\frac{\bar{q}_i}{\bar{q}_{i,-1}}\right) \ ,$$

while the conditions (IR), (IC) and feasibility $q(\theta) \leq \bar{q}_i$, $\forall\theta \in \Theta$ remain unaffected. It is now obvious that this is equivalent to the case with two types θ^h, $h = L, H$, where $f_\theta^H = \lambda$, $f_\theta^L = 1 - \lambda$, and $q_i(\theta^h) = q_i^h$. The lemma then immediately follows when also taking into account the endogenous choice of $\bar{q}_i$.

$\square$

B.2 Proof of lemma 2

Consider any two types $\theta^H, \theta^L \in \Theta$. We show that

$$\theta^H > \theta^L \quad \Rightarrow \quad I^H > I^L \tag{i}$$

$$\theta^H = \theta^L \quad \Rightarrow \quad I^H = I^L \ . \tag{ii}$$

The result then follows.

(i) The following conditions are necessary for incentive compatibility for both types:

$$\theta^H v(q_i(\theta^H)) - p_i(\theta^H) \geq \theta^H v(q_i(\theta^L)) - p_i(\theta^L) \tag{IC^H}$$

$$\theta^L v(q_i(\theta^L)) - p_i(\theta^L) \geq \theta^L v(q_i(\theta^H)) - p_i(\theta^H) \ . \tag{IC^L}$$

Rearranging terms and combining the two conditions, we get

$$\theta^H \left[v(q_i(\theta^H)) - v(q_i(\theta^L))\right] \geq \theta^L \left[v(q_i(\theta^H)) - v(q_i(\theta^L))\right] \ .$$

Using $\theta^H > \theta^L$ along with the fact that $v'(\cdot) > 0$, we get

$$q_i(\theta^H) \geq q_i(\theta^L)\ \forall\, i \in [0, 1] \ ,$$

and, hence

$$Q^H = \int_0^1 \left(q_i(\theta^H)\right)^{1-\beta} di \geq \int_0^1 \left(q_i(\theta^L)\right)^{1-\beta} di = Q^L \ . \tag{B.1}$$

Moreover, incentive compatibility requires that $p_i(\theta^H) \geq p_i(\theta^L)\ \forall\, i \in [0,1]$, implying that

$$\int_0^1 p_i(\theta^H)\, di \geq \int_0^1 p_i(\theta^L)\, di\ . \tag{B.2}$$

Finally, by the monotonicity of households' preferences, the budget constraint always holds with equality, i.e. we have

$$p_z z^h = I^h - \int_0^1 p_i(\theta^h)\, di \quad \forall\, h \in [0,1]\ . \tag{B.3}$$

Combining (B.1), (B.2), and (B.3) with the definition of θ, we conclude

$$\theta^H > \theta^L \quad \Rightarrow \quad I^H > I^L\ .$$

(ii) It remains to show that

$$\theta^H = \theta^L \quad \Rightarrow \quad I^H = I^L\ .$$

We proceed by contradiction. Suppose there exist two types of households with $I^H > I^L$ satisfying $\theta^H = \theta^L$. Then it must be that $Q^H > Q^L$ and that $\int_0^1 p_i(\theta^H)\, di > \int_0^1 p_i(\theta^L)\, di$. Hence, for some measurable subset $\hat{\mathcal{I}} \subseteq [0,1]$ we must have that

$$q_i(\theta^H) > q_i(\theta^L) \quad \forall\, i \in \hat{\mathcal{I}}\ ,$$

where incentive compatibility for both H and L requires

$$\theta^h v(q_i(\theta^H)) - p_i(\theta^H) = \theta^h v(q_i(\theta^L)) - p_i(\theta^L) \quad \forall\, i \in [0,1],\ h \in \{L, H\}\ . \tag{B.4}$$

This, however, contradicts profit maximization by innovating firms $i \in \hat{\mathcal{I}}$. To see this, note that for firm i to offer two distinct contracts to one type of households, both contracts must yield the same profit to the firm. Consider, for concreteness, the case of $q_i(\theta^H) < \bar{q}_i$.[70] Then, we must have

$$p_i(\theta^H) - p_i(\theta^L) = \frac{1}{a_q A}\left(q_i(\theta^H) - q_i(\theta^L)\right)\ . \tag{B.5}$$

(B.4), (B.5), and the concavity of $v(\cdot)$ imply that for every $\tilde{q}_i \in (q_i(\theta^L), q_i(\theta^H))$ there exists a $\tilde{p}_i \in (p_i(\theta^L), p_i(\theta^H))$ such that

$$\theta^h v(q_i(\theta^L)) - p_i(\theta^L) = \theta^h v(\tilde{q}_i) - \tilde{p}_i\ , \quad h \in \{L, H\}$$

and

$$\tilde{p}_i - p_i(\theta^L) > \frac{1}{a_q A}\left(\tilde{q}_i - q_i(\theta^L)\right)\ .$$

The contract $(\tilde{q}_i, \tilde{p}_i)$ yields higher profits for the firm than both $(q_i(\theta^H), p_i(\theta^H))$ and $(q_i(\theta^L), p_i(\theta^L))$. It satisfies (IC) and (IR) for households L, H. Moreover, it weakly relaxes (IC) to all other households because it is less preferred than $(q_i(\theta^L), p_i(\theta^L))$ by all types $\theta < \theta^L$ and less preferred than $(q_i(\theta^H), p_i(\theta^H))$ by all types $\theta > \theta^H$. Hence, offering $(q_i(\theta^H), p_i(\theta^H))$ and $(q_i(\theta^L), p_i(\theta^L))$ cannot be profit maximizing.

□

[70] It is straightforward to extend the argument to the case of $q_i(\theta^H) = \bar{q}_i$.

B.3 Proof of lemma 3

We begin with a preliminary observation and then show the desired result.

Lemma 7

The equilibrium price of quality q_i^h, $h \in \{L, H\}$, *of any differentiated good* $i \in [0,1]$ *is never below its marginal cost of production, i.e.*

$$p_i^h \geq \frac{q_i^h}{a_q A} \;, \quad h \in \{L, H\}.$$

Proof We proceed by contradiction. Suppose innovating firm i offers contracts (q_i^h, p_i^h) and $(q_i^{\hat{h}}, p_i^{\hat{h}})$, $h \neq \hat{h} \in \{L, H\}$, and where $p_i^h < \frac{q_i^h}{a_q A}$ and $p_i^{\hat{h}} \geq \frac{q_i^{\hat{h}}}{a_q A}$.[71] Contract (q_i^h, p_i^h) is loss making for firm i. Consider the following variant to these contracts:

$$\tilde{q}_i^{\hat{h}} = q_i^{\hat{h}}$$
$$\tilde{p}_i^{\hat{h}} = p_i^{\hat{h}}$$

and

$$\tilde{q}_i^h = \underset{q \in \{[0,\bar{q}_{-1}], q_i^h, q_i^{\hat{h}}\}}{argmax} \; \{\theta^h v(q) - \tilde{p}_i^h\}$$
$$s.t. \quad \tilde{p}_i^h = \begin{cases} \frac{\tilde{q}_i^h}{a_q A}, & \text{if } \tilde{q}_i^h \in \{[0, \bar{q}_{-1}], q_i^h\} \\ \tilde{p}_i^{\hat{h}}, & \text{if } \tilde{q}_i^h = \tilde{q}_i^{\hat{h}} \end{cases} .$$

By construction, contract $(\tilde{q}_i^h, \tilde{p}_i^h)$ satisfies (IR) and (IC) for households h. Moreover, as either $\tilde{q}_i^h = q_i^h$ and $\tilde{p}_i^h > p_i^h$, or $(\tilde{q}_i^h, \tilde{p}_i^h)$ is a contract that has already been available previously, contract $(\tilde{q}_i^{\hat{h}}, \tilde{p}_i^{\hat{h}})$ satisfies (IR) and (IC) for household $\hat{h}$. Yet, contracts $(\tilde{q}_i^h, \tilde{p}_i^h)$, $(\tilde{q}_i^{\hat{h}}, \tilde{p}_i^{\hat{h}})$ yield strictly larger profits to firm i when compared to contracts (q_i^h, p_i^h) and $(q_i^{\hat{h}}, p_i^{\hat{h}})$, a contradiction to the latter being profit maximizing.

□

Now, suppose all qualities were offered at marginal cost. Then household $h \in \{H, L\}$ would maximize its instantaneous utility (1) subject to

$$\int_0^1 q_i^h \frac{1}{a_q A} \, di + z^h \frac{1}{a_z A} = I^h \; .$$

Standard derivations then imply that $q_i^h = q^h \; \forall \, i \in [0,1]$ and that

$$q^h \frac{1}{a_q A} = (1 - \beta) I^h \; . \tag{B.6}$$

Now, the solution to (B.6) is household h's consumed quality unless this quality level is not available or some other quality is sold at a price below marginal cost. By lemma

[71] Note that the firm will never price both contracts below marginal cost because this would imply that it is making losses and staying out of business and making zero profits is always an option for the firm.

7, the latter will never happen in equilibrium. Moreover, the competitive fringe for pre-existing qualities implies that qualities $q_i \leq \bar{q}_{-1}$ are offered at marginal cost in equilibrium. Recalling that $A = \bar{q}_{-1}$, it follows that a household with income $\hat{I} := \frac{1}{a_q(1-\beta)}$ just finds it optimal to consume quality $\bar{q}_{-1}$. Moreover, the solution according to (B.6) is increasing in I^h. Hence, households with income $I < \hat{I}$ find it optimal to consume quality $q_i < \bar{q}_{-1}$.

□

B.4 Proof of proposition 1

We first state the formal version of proposition 1, which we prove in the following.

Proposition 1′
There is a unique equilibrium satisfying for $h = \{H, L\}$: $q_i^{h,e} = q^{h,e}$ and $p_i^{h^e} = p^{h,e}$ $\forall i \in [0,1]$. Depending on parameter values, this equilibrium can be characterized according to one of the following cases:

(i) $\boldsymbol{I^L \leq I^H \leq \hat{I}}$ **:**
$q^{L,e} = (1-\beta)a_q A I^L$, $p^{L,e} = \frac{1}{a_q A} q^{L,e}$
$q^{H,e} = (1-\beta)a_q A I^H$, $p^{H,e} = \frac{1}{a_q A} q^{H,e}$

(ii) $\boldsymbol{I^H > \hat{I} \geq I^L}$ **:**
$q^{L,e} = (1-\beta)a_q A I^L$, $p^{L,e} = \frac{1}{a_q A} q^{L,e}$
$q^{H,e} > \bar{q}_{-1}$ and $p^{H,e}$ are the unique solutions to:

$$\frac{I^H - p^{H,e}}{\beta} \left[1 - \left(\frac{\bar{q}_{-1}}{q^{H,e}} \right)^{1-\beta} \right] + \frac{1}{a_q} = p^{H,e}$$

$$\lambda \frac{1-\beta}{\beta} \left(I^H - p^{H,e} \right) - \lambda \frac{1}{a_q} \frac{q^{H,e}}{\bar{q}_{-1}} - \frac{q^{H,e}}{\bar{q}_{-1}} h' \left(\frac{q^{H,e}}{\bar{q}_{-1}} \right) = 0$$

(iii) $\boldsymbol{I^H \geq I^L > \hat{I}}$ **:**

(A) If the solution to the system of equations in part (B) involves $q^L \leq \bar{q}_{-1}$, there is a separating equilibrium with $q^{L,e} = \bar{q}_{-1}, p^{L,e} = \frac{1}{a_q}$ and where $q^{H,e}, p^{H,e}$ are the solutions to the equations as shown in (ii).

(B) There is a separating equilibrium where $q^{L,e}$, $p^{L,e}$, $q^{H,e}$, and $p^{H,e}$ are the unique solutions to:

$$\frac{I^L - p^{L,e}}{\beta}\left[1 - \left(\frac{\bar{q}_{-1}}{q^{L,e}}\right)^{1-\beta}\right] + \frac{1}{a_q} = p^{L,e}$$

$$\frac{I^H - p^{H,e}}{\beta}\left[1 - \left(\frac{q^{L,e}}{q^{H,e}}\right)^{1-\beta}\right] + p^{L,e} = p^{H,e}$$

$$I^L - \lambda\left(I^H - p^{H,e}\right)\left(\frac{q^{L,e}}{q^{H,e}}\right)^{1-\beta} - (1-\lambda)\frac{\beta}{(1-\beta)a_q}\frac{q^{L,e}}{\bar{q}_{-1}} = p^{L,e}$$

$$\lambda\frac{1-\beta}{\beta}\left(I^H - p^{H,e}\right) - \lambda\frac{1}{a_q}\frac{q^{H,e}}{\bar{q}_{-1}} - \frac{q^{H,e}}{\bar{q}_{-1}}h'\left(\frac{q^{H,e}}{\bar{q}_{-1}}\right) = 0\ .$$

(C) If the solution to the system of equations in part (B) involves $q^L \geq q^H$, there is a pooling equilibrium, i.e. $q^{L,e} = q^{H,e} = q^{Pe}$ and $p^{L,e} = p^{H,e} = p^{Pe}$ which are the unique solutions to:

$$\frac{I^L - p^{Pe}}{\beta}\left[1 - \left(\frac{\bar{q}_{-1}}{q^{Pe}}\right)^{1-\beta}\right] + \frac{1}{a_q} = p^{Pe}$$

$$\frac{1-\beta}{\beta}\left(I^L - p^{Pe}\right) - \frac{1}{a_q}\frac{q^{Pe}}{\bar{q}_{-1}} - \frac{q^{Pe}}{\bar{q}_{-1}}h'\left(\frac{q^{Pe}}{\bar{q}_{-1}}\right) = 0\ .$$

We prove each part of proposition 1′ in turn.

(i) Follows from the proof of lemma 3 and the fact that qualities $q \leq \bar{q}_{-1}$ are offered at marginal cost.

(ii) From the proof of lemma 3 we know that for all differentiated goods we have: $q^{Le} = (1-\beta)a_q A I^L$ and $p^{L,e} = \frac{1}{a_q A}q^{L,e}$ and that household H's preferred option among freely available qualities is $\bar{q}_{-1}$. Moreover, it is never optimal for the firm to upgrade quality more than what is needed to serve the high types, i.e. we have $\bar{q}_i = \max\{\bar{q}_{-1}, q_i^H\}$. Hence, firm i's decision problem simplifies to

$$\max_{q_i^H, p_i^H} \quad \lambda\left[p_i^H - \frac{1}{a_q A}q_i^H\right] - h\left(\frac{q_i^H}{\bar{q}_{-1}}\right)$$

$$\text{s.t.} \quad \theta^H v(q_i^H) - p_i^H \geq \theta^H v(\bar{q}_{-1}) - \frac{1}{a_q A}\bar{q}_{-1}\ . \qquad (\mathrm{IR^H})$$

As the firm's profits are strictly increasing in p_i^H, ($\mathrm{IR^H}$) always holds with equality in equilibrium. Rearranging ($\mathrm{IR^H}$), substituting in for p_i^H in the objective, and differentiating with respect to q_i^H, we get the following necessary conditions for profit maximization:

$$\theta^H v(q_i^H) - \theta^H v(\bar{q}_{-1}) + \frac{1}{a_q A}\bar{q}_{-1} = p_i^H \qquad (\mathrm{IR^H})$$

$$\lambda\theta^H v'(q_i^H) - \lambda\frac{1}{a_q A} - \frac{1}{\bar{q}_{-1}}h'\left(\frac{q_i^H}{\bar{q}_{-1}}\right) = 0\ . \qquad (\mathrm{B.7})$$

Note that for every $\theta^H > 0$, the first order conditions (IR^H) and (B.7) have at most one solution, implying that any equilibrium has to be symmetric across differentiated goods. Using the symmetry, $A = \bar{q}_{-1}$, the fact that $I^H - p^H = p_z z^H$, the definitions of θ and $v(\cdot)$, and rearranging terms, we get

$$\frac{I^H - p^H}{\beta}\left[1 - \left(\frac{\bar{q}_{-1}}{q^H}\right)^{1-\beta}\right] + \frac{1}{a_q} = p^H \tag{B.8}$$

$$\lambda\frac{1-\beta}{\beta}\left(I^H - p^H\right) - \lambda\frac{1}{a_q}\frac{q^H}{\bar{q}_{-1}} - \frac{q^H}{\bar{q}_{-1}}h'\left(\frac{q^H}{\bar{q}_{-1}}\right) = 0\ , \tag{B.9}$$

which are the expressions shown in proposition 1′. Finally, to see that these equations have a unique solution and that this solution involves $q^{H,e} > \bar{q}_{-1}$, observe that (B.8) describes an increasing relationship between p^H and q^H starting from $p^H = \frac{1}{a_q}$ and $q^H = \bar{q}_{-1}$ and converging to $p^H = \frac{I^H}{1+\beta} + \frac{\beta}{(1+\beta)a_q}$ as $q^H \to \infty$, while (B.9) describes a decreasing relationship between p^H and q^H starting from $p^H = I^H - \frac{\beta}{(1-\beta)a_q}$ and $q^H = \bar{q}_{-1}$, and reaching $p^H = 0$ at the solution of

$$\frac{1-\beta}{\beta}\lambda I^H = \frac{\lambda}{a_q}\frac{\hat{q}^H}{\bar{q}_{-1}} + \frac{\hat{q}^H}{\bar{q}_{-1}}h'\left(\frac{\hat{q}^H}{\bar{q}_{-1}}\right)\ .$$

The result then follows from $I^H > \frac{1}{a_q(1-\beta)}$.

(iii) We show existence and uniqueness of the equilibrium by construction. In particular, we follow the standard procedure for addressing this optimization problem, i.e. we eliminate (IR^H) as it is redundant and consider the firm's maximization problem ignoring (IC^L). Noting further that (IR^L) and (IC^H) are always binding,[72] this yields the following first order conditions for profit maximization:

$$\theta^L\left(v(q_i^L) - v(\bar{q}_{-1})\right) + \frac{1}{a_q} = p_i^L \tag{B.10}$$

$$\theta^H\left(v(q_i^H) - v(q_i^L)\right) + p_i^L = p_i^H \tag{B.11}$$

$$\theta^L v'(q_i^L) - \lambda\theta^H v'(q_i^L) - (1-\lambda)\frac{1}{a_q A} \leq 0 \tag{B.12}$$

$$\lambda\theta^H v'(q_i^H) - \lambda\frac{1}{a_q A} - h'\left(\frac{q_i^H}{\bar{q}_{-1}}\right)\frac{1}{\bar{q}_{-1}} = 0\ , \tag{B.13}$$

with the complementary slackness condition for (B.12) being

$$\left[\theta^L v'(q_i^L) - \lambda\theta^H v'(q_i^L) - (1-\lambda)\frac{1}{a_q A}\right]\left[q_i^L - \bar{q}_{-1}\right] = 0\ . \tag{B.14}$$

For θ^L and θ^H given, these equations have exactly one solution. If this solution implies $q_i^H \geq q_i^L$, it characterizes the uniquely optimal choice of firm i. If it involves $q_i^H < q_i^L$, then the uniquely optimal choice is instead to pool consumers.[73] We will get back to this point later and characterize the separating equilibrium first, if it exists.

[72]If not, the firm could increase profits by raising p^L and / or p^H.

[73]This solution may involve $q_i^H < q_i^L$ because the cost of innovation are made dependent on q_i^H in the above first-order-conditions, i.e. these conditions apply only if $q_i^H \geq q_i^L$. If $q_i^H < q_i^L$ they ignore the fact that the cost of innovation would be governed by q_i^L in such case.

Note first that the fact that for θ^L and θ^H given, equations (B.10) to (B.14) have a unique solution implies that there can only exist a symmetric separating equilibrium. This equilibrium can be derived by the following algorithm that takes into account the endogeneity of θ^h, $h \in \{L, H\}$, with respect to the equilibrium outcomes:

(1) For every $\hat{q}^L$, there is a unique $\hat{p}^L$ satisfying (B.10). For $\hat{q}^L$ and $\hat{p}^L$ given, (B.11) describes a monotonously increasing relation between p^H and q^H, starting at $\hat{q}^H = \hat{q}^L$ and $\hat{p}^H = \hat{p}^L$ and converging to $\hat{p}^H = \frac{I^H + \hat{p}^L \beta}{1+\beta}$ as $\hat{q}^H \to \infty$. (B.13), on the other hand, describes a monotonously decreasing relation between p^H and q^H, starting at $\hat{q}^H = \bar{q}_{-1}$ and $\hat{p}^H = I^H - \frac{\beta}{(1-\beta)a_q}$ and reaching $\hat{p}^H = 0$ at the solution of

$$\frac{1-\beta}{\beta}\lambda I^H = \frac{\lambda}{a_q}\frac{\hat{q}^H}{\bar{q}_{-1}} + \frac{\hat{q}^H}{\bar{q}_{-1}} h'\left(\frac{\hat{q}^H}{\bar{q}_{-1}}\right) .$$

Hence, for every $\hat{q}^L$, (B.10), (B.11), and (B.13) have at most one solution for $\hat{p}^L$, $\hat{p}^H$, $\hat{q}^H$.

(2) Start with $\hat{q}^L = \bar{q}_{-1}$ and follow the procedure as described above. Plug the derived $\hat{q}^L, \hat{q}^H, \hat{p}^L, \hat{p}^H$ into (B.12).[74] If inequality (B.12) is satisfied, $\hat{q}^L, \hat{q}^H, \hat{p}^L, \hat{p}^H$ are the unique equilibrium values (case A).

(3) If inequality (B.12) is violated, add some small $\Delta > 0$ to $\hat{q}^L$ and repeat procedure (1). Keep adding $\Delta > 0$ to $\hat{q}^L$ until (B.12) is satisfied.[75] If the inequality is strict, apply a bisection algorithm until convergence to the equilibrium values (case B).[76]

(4) The unique symmetric solution to equations (B.10)-(B.14) may imply $q^L \geq q^H$. In such case there exists no separating equilibrium, and the unique equilibrium is a symmetric pooling equilibrium which is the solution to

$$\theta^L \left(v(q^P) - v(\bar{q}_{-1})\right) + \frac{1}{a_q} = p^P \tag{B.15}$$

$$\theta^L v'(q^P) - \frac{1}{a_q A} - \frac{1}{\bar{q}_{-1}} h'\left(\frac{q^P}{\bar{q}_{-1}}\right) = 0 . \tag{B.16}$$

Using the definitions of θ and $v(\cdot)$, along with the fact that $A = \bar{q}_{-1}$ yields the expressions given in proposition $1'$ (case C).

(5) Finally, it remains to be shown that an equilibrium according to case (A) and (B), respectively, is unique if it exists. To see this, assume that a symmetric separating equilibrium exists with $\hat{q}^L < \hat{q}^H$ and note first that the arguments in steps (1) to (3) above

[74] Note that by $I^H > \frac{1}{(1-\beta)a_q}$ there is indeed a solution for (B.10), (B.11), and (B.13) with $\hat{q}^L = \bar{q}_{-1}$.

[75] Note that by (B.10) increasing $\hat{q}^L$ results in a higher $\hat{p}^L$ and a lower $\hat{\theta}^L$. This does not affect (B.13), but shifts the solutions to (B.11) in the q^H, p^H diagram down and to the right, i.e. according to (B.11) every $\hat{q}^H$ is now associated with a lower $\hat{p}^H$. Together, this implies that the unique solution to (B.11) and (B.13) has now a higher $\hat{q}^H$ and a lower $\hat{p}^H$. Moreover, by (B.13), it is also associated with a higher $\hat{\theta}^H$. Now, a higher $\hat{q}^L$ in conjunction with a lower $\hat{\theta}^L$ and a higher $\hat{\theta}^H$ imply that the left hand side of (B.12) is decreasing.

[76] Note that this is indeed an equilibrium and in particular that the above reasoning also implies that no firm has an incentive to deviate by pooling types in its sector. This follows from the fact that given θ, i.e. given the equilibrium strategy of all other firms in the economy, the solution to equations (B.10) to (B.14) is uniquely optimal.

imply that if an equilibrium according to case (A) and (B) exists, there can be no other separating equilibrium. To see that there can also be no pooling equilibrium in such case, suppose that there exists some $\tilde{q}^L$ such that equations (B.10), (B.11), and (B.13) are simultaneously satisfied if $\tilde{q}^H = \tilde{q}^L = \tilde{q}$ for all i. As by assumption there is a symmetric separating equilibrium with $\hat{q}^H > \hat{q}^L$, step (3) then implies that for these values the inequality in condition (B.12) must be strict. This, in combination with the fact that equation (B.13) holds implies that the left-hand-side of equation (B.16) would be negative for this value, i.e. in a potential pooling equilibrium it must be that $q < \tilde{q}$. But for $q^L < \tilde{q}$ we know from the reasoning above that the unique symmetric solution to equations (B.10), (B.11), and (B.13) implies $q^H > q^L$, i.e. there can be no pooling equilibrium.

□

B.5 Proof of proposition 2

For $\sigma = 0$, the unique equilibrium is a pooling equilibrium with positive growth. As σ increases, and, hence, I^L decreases, the growth rate in the pooling equilibrium declines. To see this, observe that as I^L decreases, both equilibrium conditions for the pooling equilibrium shift downwards in the q^P, p^P diagram, but that (B.16) shifts more, implying that both q^P, and p^P decline. This, in turn, implies that higher-σ pooling equilibria are associated with a higher θ^H. Hence, for some σ large enough, (B.13) holds with equality.[77]

As we increase σ further, we switch from a pooling equilibrium to a separating equilibrium. In the separating equilibrium, an increase in σ has two different effects: (i) The associated increase in I^H has a strictly positive effect on growth.[78] (ii) The associated decrease in I^L has an indirect effect on growth as its effect on q^L and p^L impacts p^H and, hence, θ^H which, in turn, pins down q^H via (B.13). This effect may initially be negative but will eventually be positive as well for σ large enough such that θ^L and q^L sufficiently small.[79]

[77] Note that (B.13) and (B.16) together imply that (B.12) will also hold with equality and that for $q^H = q^L$ and therefore $p^H = p^L$ (B.11) trivially holds.

[78] To show (i), we proceed by contradiction. In particular, note that (B.13) defines an increasing relationship between θ^H and q^H, i.e. for growth to decline it must be that θ^H declines as well. (B.10) and (B.12) then imply that q^L must increase while θ^L decreases. But then equations (B.10) and (B.11) imply that p^H must decrease as well, a contradiction to θ^H being decreasing given that I^H increases and q^H decreases.

[79] A decrease in I^L has a negative (positive) effect on growth if for the previously given q^H the price p^H increases (decreases). A decrease in I^L will, ceteris paribus, lower θ^L and, hence, lower p^L at a given q^L to satisfy individual rationality of the low types. Firms do, however, respond to the decrease in I^L by lowering quality for the low types, q^L, according to optimality condition (B.12). Now, equations (B.10) and (B.11) define marginal changes in θ^L and q^L such that—given the new equilibrium values for q^L and p^L and the previous equilibrium values for q^H and p^H—incentive compatibility for high types is just satisfied. In particular, totally differentiating (B.10), we get

$$dp^L = d\theta^L \left[v(q^L) - v(\bar{q}_{-1})\right] + \theta^L v'(q^L) dq^L \ ,$$

while totally differentiating (B.10) and using that q^H, p^H, and θ^H are constant, we get

$$dp^L = \theta^H v'(q^L) dq^L \ .$$

We show numerically that for a broad range of parameter specifications the direct effect via an increase of I^H always dominates. In particular, we numerically solve for q^H as a function of σ assuming $h'(x) = c(x-1)^\alpha$ for all possible parameter specifications from the following set:

Table 2: Parameter choices for numerical solutions

λ :	$\{0.05, 0.15, ..., 0.95\}$
β :	$\{0.05, 0.15, ..., 0.95\}$
a_q :	$\{2, 4, ..., 20\}$
c :	$\{1, 2, 4, 8, 12\}$
α :	$\{0.05, 0.2, 1, 10, 20\}$

For each possible combination of these parameter specifications, q^H is increasing as a function of σ in a separating equilibrium.[80]

Combining the previous two equations and rearranging terms, we get

$$\frac{dq^L}{d\theta^L} = \frac{v(q^L) - v(\bar{q}_{-1})}{(\theta^H - \theta^L)v'(q^L)} . \tag{B.17}$$

Equation (B.17) characterizes how q^L has to change in response to a marginal change in θ^L for (IR^L) and (IC^{H}) still to be satisfied given q^H, p^H, and θ^H. On the other hand, noting that in a separating equilibrium equation (B.12) holds with equality and totally differentiating using again that θ^H stays constant by assumption, we get

$$\frac{dq^L}{d\theta^L} = \frac{v'(q^L)}{v''(q^L)} \frac{1}{\lambda\theta^H - \theta^L} . \tag{B.18}$$

Equation (B.18) characterizes the optimal change of q^L in response to a marginal change of θ^L for a given θ^H. Now, if the right-hand-side of (B.18) is larger than the right-hand-side of (B.17), then the optimal response of q^L to a marginal decrease of θ^L is larger in absolute terms than the one needed to have (IC^{H}) satisfied at the old levels of q^H and p^H. As the high types value quality more, this decreases the attractiveness of contract (q^L, p^L) to high types which, in turn, allows firms to increase p^H. As a consequence, growth is lower via the negative general equilibrium effect of a higher p^H on θ^H. In other words, a decrease of I^L lowers growth if

$$\frac{v'(q^L)}{v''(q^L)} \frac{1}{\lambda\theta^H - \theta^L} > \frac{v(q^L) - v(\bar{q}_{-1})}{(\theta^H - \theta^L)v'(q^L)} .$$

Using the definition of $v(\cdot)$ and rearranging terms, this is equivalent to

$$\frac{\theta^H - \theta^L}{\theta^L - \lambda\theta^H} < \frac{\beta}{1-\beta} \left[1 - \left(\frac{\bar{q}_{-1}}{q^L} \right)^{1-\beta} \right] .$$

Now, the right-hand-side of the above condition approaches zero as $q^L \to \bar{q}_{-1}$ while the left-hand-side is strictly positive, which shows that, indeed a decrease in I^L eventually has a positive effect on growth.

[80] If σ is large enough such that the economy reaches the point where innovating firms find it optimal to no longer serve the low types—i.e. if the solution to the system of equations in proposition 1′(iii)(B) involves $q^L \leq \bar{q}_{-1}$—this is necessarily the case and an increase in σ has a positive effect on growth. Formally, this follows from noting that as I^H increases, both (B.8) and (B.9) shift upwards with (B.9)—which is downward sloping—shifting more.

B.6 Proof of lemma 4

The expression in lemma 4 has been derived in the main text under the condition that in equilibrium the SOE prices the homogeneous good competitively in the ROW (equation (13)).

To show this, note first that firms from the SOE cannot serve consumers in the ROW at a price that is higher than the marginal cost of firms from the ROW in their home market. For the homogeneous good and for differentiated goods with quality $q \leq \bar{q}_{-1}^{ROW}$, this follows immediately from the competitive fringe in the ROW. For differentiated goods with quality $q > \bar{q}_{-1}^{ROW}$, this would violate profit maximization of firms in the ROW. To see that, suppose by way of contradiction that the SOE sells quality $\hat{q} > \bar{q}_{-1}^{ROW}$ of some differentiated good i at a price $\hat{p}(\hat{q}) > \frac{w^{ROW}\hat{q}}{a_q A^{ROW}}$ to a positive measure $(1 - \lambda^{ROW}) > 0$ of consumers in the ROW. By assumption, the SOE does not serve rich households in the ROW, i.e. $(1 - \lambda^{ROW}) < 1$ and $\bar{q}_i^{ROW} > \hat{q}$. It follows that in such case firm i in the ROW makes variable profits $\hat{\pi}_i^{ROW} = \lambda^{ROW} \left(\hat{p}^{H;ROW} - \frac{w^{ROW}}{a_q A^{ROW}} \bar{q}_i^{ROW} \right)$. Suppose instead that the firm offered quality $\hat{q}$ at a price $\hat{p} - \epsilon$ with ϵ positive but small and quality $\bar{q}_i^{ROW}$ at a price $\hat{p}^{H;ROW} - \epsilon$, with all other prices unaffected. Clearly, in such case rich households in the ROW would still opt to consume $\bar{q}_i^{ROW}$ while poor households would now prefer to buy quality $\hat{q}$ domestically rather than importing it from the SOE. It follows that with these contracts, firm i's profits would be

$$\begin{aligned} \hat{\pi}_i^{ROW;2} &= \lambda^{ROW} \left(\hat{p}^{H;ROW} - \epsilon - \frac{w^{ROW}}{a_q A^{ROW}} \bar{q}_i^{ROW} \right) + (1 - \lambda^{ROW}) \left(\hat{p} - \epsilon - \frac{w^{ROW}\hat{q}}{a_q A^{ROW}} \right) \\ &= \hat{\pi}_i^{ROW} + (1 - \lambda^{ROW}) \left(\hat{p} - \frac{w^{ROW}\hat{q}}{a_q A^{ROW}} \right) - \epsilon \, . \end{aligned}$$

For $0 < \epsilon < (1 - \lambda^{ROW}) \left(\hat{p} - \frac{w^{ROW}\hat{q}}{a_q A^{ROW}} \right)$, this would strictly increase firm i's variable profits in the ROW while not affecting its cost of innovation. This, however, contradicts that serving only the rich at a price $\hat{p}^{H;ROW}$ is profit maximizing.

The previous arguments imply that the SOE cannot sell a product in the ROW at a price above marginal costs of producing that same product in the ROW. The symmetry of the set-up in combination with the fact that it is never optimal to serve the ROW at a price below marginal costs in the SOE inclusive of trade costs then immediately implies that in an equilibrium with positive exports it must hold

$$\tau \frac{w}{a_z A} \leq \frac{w^{ROW}}{a_z A^{ROW}} \, .$$

But then, this condition must hold with equality—equation (13)—for if not, the homogeneous good firms in the SOE would make strictly positive profits from serving the ROW, which would contradict profit maximization of homogeneous good firms in the SOE as they are price takers in the ROW.

$\square$

B.7 Proof of lemma 5

We show that constraint (IRf) cannot be binding for the low types. With $I^L \leq \hat{I}$ this is trivially the case. We thus consider the case of $I^L > \hat{I}$ and show that low types prefer quality $\bar{q}_{-1}$ over any imported quality.

As argued in the main body of the text, it is never optimal to import quality $q \leq \bar{q}_{-1}$. Hence, constraint (IRf) can only be binding if the preferred importing quality satisfies $q > \bar{q}_{-1}$. Combined with the fact that the marginal utility of quality is increasing in θ, this implies that low types will prefer quality $\bar{q}_{-1}$ over their best import option if this is the case for some $\hat{\theta} \geq \theta^L$.

Now, the income of low types is bounded from above by 1. Moreover, θ^L is decreasing in both, q^L and p^L. We conclude that θ^L is bounded from above by

$$\bar{\theta}^L := \frac{1 - \frac{1}{a_q}}{\beta \bar{q}_{-1}^{1-\beta}} .$$

A household of type $\bar{\theta}^L$ prefers quality $\bar{q}_{-1}$ over its best import option if

$$\bar{\theta}^L v(\bar{q}_{-1}) - \frac{1}{a_q} \geq \left[\bar{\theta}^L\right]^{\frac{1}{\beta}} \left[\bar{q}_{-1}\right]^{\frac{1-\beta}{\beta}} \chi(\tau) .$$

Using the definitions of $\bar{\theta}^L$, $v(\cdot)$, and $\chi(\tau)$, this can be rewritten as

$$\frac{1 - \frac{1}{a_q}}{\beta \bar{q}_{-1}^{1-\beta}} \bar{q}_{-1}^{1-\beta} - \frac{1}{a_q} \geq \left[\frac{1 - \frac{1}{a_q}}{\beta \bar{q}_{-1}^{1-\beta}}\right]^{\frac{1}{\beta}} \left[\bar{q}_{-1}\right]^{\frac{1-\beta}{\beta}} \left[\frac{a_q(1-\beta)}{\tau^2}\right]^{\frac{1-\beta}{\beta}} \beta .$$

Solving for τ and simplifying terms yields the expression given in assumption 1.

$\square$

B.8 Proof of lemma 6

To show the desired result, we consider the limiting case where domestic firms are just indifferent between innovating or not to serve the rich and then proceed by contradiction. In particular, we show that $q^{H,f} \leq q^H$ contradicts that it is optimal for domestic firms not to serve the rich households, where $q^{H,f}$ denotes the quality of the best import option and q^H denotes the optimal domestically-provided quality.

Suppose that $q^{H,f} \leq q^H$. The best importing quality satisfies the first-order condition for utility maximization of the rich

$$\theta^H v'(q^{H,f}) = \frac{\tau^2}{a_q A} ,$$

implying that

$$p^{H,f} = \frac{\tau^2}{a_q A} q^{H,f} = \theta^H v'(q^{H,f}) q^{H,f} , \tag{B.19}$$

where $p^{H,f}$ denotes the price of imported quality $q^{H,f}$. In the limiting case where domestic firms are just indifferent between serving or not the rich households, (IRf) is binding for the rich and, hence,

$$\theta^H v(q^{H,f}) - p^{H,f} = \theta^H v(q^H) - p^H$$

and therefore

$$\begin{aligned} p^H &= \theta^H \left[v(q^H) - v(q^{H,f}) + p^{H,f}\right] \\ &= \theta^H \int_{q^{H,f}}^{q^H} v'(x)dx + \theta^H v'(q^{H,f})q^{H,f} \\ &\geq \theta^H v'(q^H)q^H \ . \end{aligned}$$

The second equality follows from using the fundamental theorem of calculus and equation (B.19). The inequality follows from the fact that $v(\cdot)$ is concave and that $q^{H,f} \leq q^H$, by assumption, and from simplifying terms. The above inequality is strict whenever $q^{H,f} < q^H$.

Now, there are two possibilities for when domestic firms are indifferent between serving or not the rich households. (i) Either they make zero profits and are equally well off stopping to innovate altogether. (ii) Or they would be equally well off innovating at a lower rate to just serve the poor. We show that neither is possible.

(i) The first order condition for q^H implies

$$\lambda\theta^H v'(q^H) - \lambda\frac{1}{a_q A} - \frac{1}{\bar{q}_{-1}}h'\left(\frac{q^H}{\bar{q}_{-1}}\right) = 0 \ . \tag{B.20}$$

Clearly, the fact that $p^H \geq \theta^H v'(q^H)q^H$ and the convexity of $h(\cdot)$ imply that firms are making strictly positive profits from just serving the rich households, i.e. a solution with no innovation cannot be optimal.

(ii) The fact that firms cannot be indifferent between serving both types of households or just the poor follows from a revealed preference argument. In particular, in the separating equilibrium, it must be that ($\mathrm{IR^L}$) is binding.[81] Moreover, firms could opt to offer poor households contract ($\tilde{q}^L$, $\tilde{p}^L$), where we use this to denote the contract that firms would offer the low types in the hypothetical scenario where they just serve these types. This contract also satisfies ($\mathrm{IR^L}$) with equality, i.e. low types are indifferent between contracts ($\tilde{q}^L$, $\tilde{p}^L$) and (q^L, p^L). We now show that offering ($\tilde{q}^L$, $\tilde{p}^L$) and ($\tilde{q}^H$, $\tilde{p}^H$) would yield strictly higher profits than when just offering ($\tilde{q}^L$, $\tilde{p}^L$), where $\tilde{q}^H = q^H$ and $\tilde{p}^H$ is as defined below. In turn, this implies that the optimal contracts in the separating equilibrium yield strictly higher profits than when just offering ($\tilde{q}^L$, $\tilde{p}^L$).

[81] ($\mathrm{IRf^L}$) is never binding. Hence, the only possibility where ($\mathrm{IR^L}$) is not binding is a hypothetical case where ($\mathrm{IC^L}$) is binding, for otherwise firms could increase profits by increasing p^L which would only relax ($\mathrm{IC^H}$). ($\mathrm{IC^L}$), however, cannot be binding because, by assumption, the rich are indifferent between consuming the domestically produced quality q^H or importing a weakly lower quality. As richer households have a stronger taste for quality, poor households must then weakly prefer the best import choice of the rich households over (q^H, p^H) and, therefore, strictly prefer their own best import choice, i.e. ($\mathrm{IRf^L}$) would have to be strictly binding in such case, a contradiction.

If $\tilde{q}^L \leq q^L$, this follows immediately because the change in the contract of the poor would not affect the contract for the rich and because firms make positive profits from serving the rich.

If $\tilde{q}^L > q^L$ and ($\mathrm{IC^H}$) is not binding, the same reasoning from before applies. If ($\mathrm{IC^H}$) is binding, then the price of q^H changes to

$$\tilde{p}^H = \tilde{p}^L + \int_{\tilde{q}^L}^{q^H} \theta^H v'(x) dx$$

and we have

$$\begin{aligned}
&(1-\lambda)\left(\tilde{p}^L - \frac{1}{a_q A}\tilde{q}^L\right) - h\left(\frac{\tilde{q}^L}{\bar{q}_{-1}}\right) \\
&< \left(\tilde{p}^L - \frac{1}{a_q A}\tilde{q}^L\right) - h\left(\frac{\tilde{q}^L}{\bar{q}_{-1}}\right) \\
&< \left(\tilde{p}^L - \frac{1}{a_q A}\tilde{q}^L\right) - h\left(\frac{\tilde{q}^L}{\bar{q}_{-1}}\right) + \int_{\tilde{q}^L}^{q^H} \lambda\theta^H v'(x) - \lambda\frac{1}{a_q A} - \frac{1}{\bar{q}_{-1}} h'\left(\frac{x}{\bar{q}_{-1}}\right) dx \\
&= \left(\tilde{p}^L - \frac{1}{a_q A}\tilde{q}^L\right) + \lambda\left(\tilde{p}^H - \tilde{p}^L\right) - \lambda\frac{1}{a_q A}\left(q^H - \tilde{q}^L\right) - h\left(\frac{q^H}{\bar{q}_{-1}}\right) . \qquad \text{(B.21)}
\end{aligned}$$

The first inequality follows from $\tilde{p}^L - \frac{1}{a_q A}\tilde{q}^L > 0$ and $\lambda > 0$. The second inequality follows from using (B.20) and the fact that $v(\cdot)$ is concave and $h(\cdot)$ is convex. The equality follows from solving the integral. The result then follows from noting that the expression in the last row is equal to total profits with this alternative separating contract.

□

B.9 Proof of proposition 3

The fact that the various cases as characterized in proposition 3 represent equilibria, follows from the discussions in the main text. In particular, the solution to the firms' problem implies that firms maximize profits given households' utility maximizing consumption choice, and the definition of $\theta^{h,e}(N_f)$ implies that in turn these solutions are consistent with the ensuing $\theta^{h,e}$. It remains to show that an equilibrium does indeed exist (step 1) and to characterize the equilibrium values in the markets other than the differentiated good markets (step 2). This appendix discusses each step in turn.

Step 1. The characterization of the equilibrium in proposition 3 relies on $\theta^{h,e}(N_f^e)$, which is the fixed point of (24) for a given N_f^e. That is, it considers situations where the ensuing solution to the domestic firms' profit maximization problem (q_i^h, p_i^h) implies levels of θ^h that are in turn consistent with the firms' solution given the respective equilibrium level of N_f^e. To show existence, we therefore need to show that such fixed points exist for some $N_f^e \in [0,1]$. We begin with showing two lemmata that characterize existence of fixed points of (24) for arbitrary given N_f.

Lemma 8

There is an infimum $\underline{N}_f \geq 0$ such that for all $N_f > \underline{N}_f$, the fixed point $\theta^{H,e}(N_f)$ exists.

Proof To show that (24) possesses a fixed point $\theta^{H,e}(N_f)$, we can rewrite it as

$$I^H - \int_0^{N_f} \beta\,\theta^H \cdot (q_i^{H,f})^{1-\beta} + p_i^{H,f}\,di - \int_{N_f}^1 \beta\,\theta^H \cdot (q_i^H)^{1-\beta} + p_i^H\,di = 0\;. \tag{B.22}$$

Using equations (14) and (16), we obtain

$$\int_0^{N_f} \beta\,\theta^H \cdot (q_i^{H,f})^{1-\beta} + p_i^{H,f}\,di = N_f \cdot (\theta^H)^{\frac{1}{\beta}} \left[\frac{(1-\beta)a_q A}{\tau^2}\right]^{\frac{1-\beta}{\beta}}\;.$$

The contract offered by the domestic firm is determined by (IRf$^{\mathrm{H}}$) and (18b) (which is equivalent to (20d) as well as to the corresponding equation in the closed economy (10d)). We first note that (18b) defines a unique level q_i^H only depending on θ^H, the previous period's maximum quality level $\bar{q}_{i,-1}$, and the exogenous primitives of the model. Further note that the implicit function theorem implies that the function $q_i^H(\theta^H)$ defined by (18b) is an increasing function in θ^H.[82] The condition (IRf$^{\mathrm{H}}$) then defines the price p_i^H such that for the firms' optimal $q_i^H(\theta^H)$, (IRf$^{\mathrm{H}}$) holds with equality. With $\bar{q}_{i,-1} = \bar{q}_{-1}$ for all i, we get[83]

$$\begin{aligned}\int_{N_f}^1 \beta\,\theta^H \cdot (q_i^H)^{1-\beta} + p_i^H\,di \;&=\; (1-N_f)\cdot(\theta^H)^{\frac{1}{\beta}}[(1-\beta)a_q A]^{\frac{1-\beta}{\beta}} \cdot \\ &\left[(1+\beta)\left(1+\frac{a_q}{\lambda}h'\left(\frac{q^H(\theta^H)}{\bar{q}_{-1}}\right)\right)^{-\frac{1-\beta}{\beta}} - \beta\tau^{-2\frac{1-\beta}{\beta}}\right]\;.\end{aligned}$$

To show the desired result, we now proceed in two steps. We first show that for $N_f = 1$ a fixed point must exist and then show the desired result.

(i) $N_f = 1$: We first consider the case where the high income households import all differentiated goods varieties. Then (B.22) reads

$$I^H - (\theta^H)^{\frac{1}{\beta}}\left[\frac{(1-\beta)a_q A}{\tau^2}\right]^{\frac{1-\beta}{\beta}} = 0\;.$$

---

[82] Applying the implicit function theorem to (20d) yields

$$\frac{d\,q_i^H}{d\,\theta^H} = \frac{\lambda v'(q_i^H)}{-\lambda\theta^H v''(q_i^H) + h''(q_i^H/\bar{q}_{i,-1})/\bar{q}_{i,-1}^2} > 0\,. \tag{B.23}$$

The sign is positive as $v(\cdot)$ is concave and $h(\cdot)$ is convex.

[83] More concretely, we use

$$p_i^H = \theta^H v(q_i^H) - (\theta^H)^{\frac{1}{\beta}}(\bar{q}_{-1})^{\frac{1-\beta}{\beta}}\chi(\tau)\;,$$

from (IRf$^{\mathrm{H}}$) and

$$q_i^H = (\theta^H)^{\frac{1}{\beta}}[(1-\beta)a_q A]^{-\frac{1}{\beta}}\left[1+\frac{a_q}{\lambda}h'\left(\frac{q_i^H}{\bar{q}_{-1}}\right)\right]^{-\frac{1}{\beta}}\;,$$

from equation (20d). Some minor transformations considering that $A = \bar{q}_{-1}$ then yield the expression of interest.

As the left-hand side monotonously declines with θ^H, there is a unique root

$$\theta^{H,e}(N_f = 1) = (I^H)^\beta \left[\frac{\tau^2}{(1-\beta)a_q A}\right]^{1-\beta} .$$

Also note that $\theta^{H,e}(N_f = 1)$ increases with I^H.

(ii) $N_f \in (0,1)$: If only a share of differentiated goods are imported, (B.22) can be written as

$$\begin{aligned} I^H \quad - \quad & N_f \cdot (\theta^H)^{\frac{1}{\beta}} \left[\frac{(1-\beta)a_q A}{\tau^2}\right]^{\frac{1-\beta}{\beta}} - (1-N_f) \cdot (\theta^H)^{\frac{1}{\beta}} [(1-\beta)a_q A]^{\frac{1-\beta}{\beta}} \cdot \\ & \cdot \left[(1+\beta)\left(1 + \frac{a_q}{\lambda} h'\left(\frac{q^H(\theta^H)}{\bar{q}_{-1}}\right)\right)^{-\frac{1-\beta}{\beta}} - \beta\tau^{-2\frac{1-\beta}{\beta}}\right] = 0 . \end{aligned} \tag{B.24}$$

We know that the left-hand-side (LHS) of (B.24) declines with θ^H for $N_f = 1$. Since LHS of (B.24) is continuous in N_f, there must exist a $0 \leq \underline{N}_f < 1$ such that for all $N_f > \underline{N}_f$ the root $\theta^{H,e}(N_f)$ exists.

□

Based on lemma 8, we define $N_f^{min} = \underline{N}_f$ if $\theta^{H,e}(\underline{N}_f)$ exists, and $N_f^{min} = \underline{N}_f + \varepsilon$ for some small but positive ϵ such that $\theta^{H,e}(N_f^{min})$ exists. Lemma 8 implies that for all $N_f \geq N_f^{min}$, a fixed point for θ^H exists. Lemma 9 below shows that for any given $N_f \geq N_f^{min}$ there are corresponding fixed points $\theta^{L,e}(N_f)$ with respect to the firms' offers for the low income households.

Lemma 9

For any $N_f \in [N_f^{min}, 1]$ and corresponding fixed point $\theta^{H,e}(N_f)$, there exists a fixed point $\theta^{L,e}(N_f)$ such that the optimal contract for the low income households (q_i^L, p_i^L) by the domestic firms given $\theta^L = \theta^{L,e}(N_f)$ in turn implies that $\theta^L = \theta^{L,e}(N_f)$.

Proof To prove lemma 9, we need to distinguish the cases (i) where the domestic firm offers qualities $q_i^L \leq \bar{q}_{-1}$ and (ii) where it offers qualities $q_i^L > \bar{q}_{-1}$.

(i) This reflects case (B-SOE) in the domestic firm's problem in subsection 5.1 where $I^L \leq \hat{I}$. Then $q_i^L \leq \bar{q}_{-1}$, independently of whether the domestic firm serves the high income households or not. Consequently, the low income households consume the same quality $q_i^L = [\theta^L(1-\beta)a_q A]^{\frac{1}{\beta}} \leq \bar{q}_{-1}$ at marginal costs independent of the share N_f of imported high quality goods for the rich households. We then obtain

$$\theta^{L,e}(N_f) = (I^L)^\beta [(1-\beta)a_q A]^{\beta-1} , \ \ \forall N_f \in [0,1] .$$

(ii) This corresponds to the case (C-SOE) in the domestic firm's problem in subsection 5.1 where $I^L > \hat{I}$. Then we can write for θ^L with given N_f:

$$\theta^L = \frac{I^L - \int_0^{N_f} p_i^{L,Cf} di - \int_{N_f}^1 p_i^{L,C} di}{\beta(\int_0^{N_f} (q_i^{L,Cf})^{1-\beta} di + \int_{N_f}^1 (q_i^{L,C})^{1-\beta} di)} . \tag{B.25}$$

This can be re-written as

$$I^L - \int_0^{N_f} \beta\,\theta^L \cdot (q_i^{L,Cf})^{1-\beta} + p_i^{L,Cf}\, di - \int_{N_f}^1 \beta\,\theta^L \cdot (q_i^{L,C})^{1-\beta} + p_i^{L,C}\, di = 0 . \tag{B.26}$$

Replacing $p_i^{L,C}$ and $p_i^{L,Cf}$ via condition (IR^{L}), i.e. via equation (20a) (or equivalently (21a)), and taking into account the symmetry of offers in sectors $i \leq N_f$ and $i > N_f$, respectively, we obtain

$$\begin{aligned} I^L \quad &- \quad N_f \cdot \left[\theta^L\big((1+\beta) \cdot (q^{L,Cf})^{1-\beta} - (\bar{q}_{-1})^{1-\beta}\big) + \frac{1}{a_q}\right] \\ &- \quad (1-N_f) \cdot \left[\theta^L\big((1+\beta) \cdot (q^{L,C})^{1-\beta} - (\bar{q}_{-1})^{1-\beta}\big) + \frac{1}{a_q}\right] = 0 . \end{aligned} \tag{B.27}$$

It then remains to be shown that $q^{L,Cf}$ and $q^{L,C}$ are increasing functions of θ^L for (B.27) to possess a unique root $\theta^{L,e}(N_f)$. For $q^{L,Cf}$ this follows from (21b) via the implicit function theorem.[84]

When the domestic firm serves the high income households, we have to distinguish the cases where (IC^H), i.e. (20c), is slack or binding. Starting with the case where (20c) is slack, the domestic firm innovates up to q^H, and $q^{L,C}$ can be freely chosen. In this case we obtain $q^{L,C} = [\theta^L(1-\beta)a_q A]^{\frac{1}{\beta}} > \bar{q}_{-1}$, which increases with θ^L. If (IC^H) is binding, it determines $q^{L,C}$. Using (IRf$^{\mathrm{H}}$) and (IR$^{\mathrm{L}}$) (20a), it can be written as

$$\begin{aligned} F(\theta^L, q^{L,C}) \quad &= \quad \theta^{H,e}(N_f)\; v(q^{L,C}) - \theta^L\, v(q^{L,C}) + \theta^L\, v(\bar{q}_{-1}) \\ &\quad -\frac{1}{a_q} - [\theta^{H,e}(N_f)]^{\frac{1}{\beta}} (\bar{q}_{-1})^{\frac{1-\beta}{\beta}} \chi(\tau) = 0 . \end{aligned}$$

The implicit function theorem yields

$$\frac{d\,q^{L,C}}{d\,\theta^L} = -\frac{\frac{\partial F}{\partial \theta^L}}{\frac{\partial F}{\partial q^{L,C}}} = \frac{v(q^{L,C}) - v(\bar{q}_{-1})}{(1-\beta)(q^{L,C})^{-\beta}(\theta^{H,e}(N_f) - \theta^L)} > 0 .$$

Consequently, the left-hand side of (B.27) is strictly decreasing in θ^L for any given N_f and thus, there exists a unique root $\theta^{L,e}(N_f)$.

[84]That is, we obtain from the implicit function theorm:

$$\frac{d\,q_i^L}{d\,\theta^L} = \frac{(1-\lambda)v'(q_i^L)}{-(1-\lambda)\theta^L v''(q_i^L) + h''(q_i^L/\bar{q}_{i,-1})/\bar{q}_{i,-1}^2} > 0 . \tag{B.28}$$

The sign is positive as $v(\cdot)$ is concave and $h(\cdot)$ is convex.

In addition, we note (i) that the implicit function theorem implies that $\theta^{L,e}(N_f)$ increases with I^L for any given N_f and (ii) that $\theta^{L,e}(N_f)$ is a continuous function of N_f.

$\square$

We now build on lemmata 8 and 9 to show that an equilibrium exits. We proceed in four steps:

(i) We note that $\pi^B(\theta^{H,e}(N_f))$ and $\Delta\pi^C(\{\theta^{h,e}(N_f)\}_h)$ are continuous functions of N_f for $N_f \in [N_f^{min}, 1]$.

This is the case as lemma 8 implies that equation (B.24) defines an implicit function $\theta^{H,e}(N_f)$ that—according to the implicit function theorem—is continuous over $N_f \in [N_f^{min}, 1]$. Similarly, from lemma 9 we obtain the corresponding function $\theta^{L,e}(N_f)$ that is also continuous on $[N_f^{min}, 1]$. Recalling the definitions of $\pi^B(\theta^H)$ and $\Delta\pi^C(\{\theta^h\}_h)$ in subsection 5.1, $\pi^B(\theta^H)$ and $\Delta\pi^C(\{\theta^h\}_h)$ are continuous functions of θ^H and $\{\theta^h\}_h$, respectively. With $\theta^{h,e}(N_f)$ continuous on $[N_f^{min}, 1]$, $\pi^B(\theta^{H,e}(N_f))$ and $\Delta\pi^C(\{\theta^{h,e}(N_f)\}_h)$ are continuous on $N_f \in [N_f^{min}, 1]$.

(ii) From lemma 8 and 9 we know that $\theta^{H,e}(1)$ and $\theta^{L,e}(1)$ exist. Hence, if $\pi^B(\theta^{H,e}(1)) \leq 0$ or $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) \leq 0$, respectively, there exists an equilibrium with $N_f^e = 1$, i.e. where rich households import all differentiated goods from the ROW (B-SOE(iii) and C-SOE(iii), respectively). Similarly, if $N_f^{min} = 0$ and $\pi^B(\theta^{H,e}(0)) \geq 0$ or $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) \geq 0$, respectively, there exists an equilibrium with $N_f^e = 0$, i.e. where rich households purchase all differentiated goods from innovative local firms (B-SOE(i) and C-SOE(i), respectively). It thus remains to be shown that an equilibrium exists if (a) $\pi^B(\theta^{H,e}(1)) > 0$ and either $N_f^{min} > 0$ or $N_f^{min} = 0$ and $\pi^B(\theta^{H,e}(0)) < 0$, and (b) $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) > 0$ and either $N_f^{min} > 0$ or $N_f^{min} = 0$ and $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) < 0$, respectively.

(iii) If $N_f^{min} = 0$, $\pi^B(\theta^{H,e}(0)) < 0$ and $\pi^B(\theta^{H,e}(1)) > 0$, then by step (i), there exists an $N_f^e \in (0, 1)$ for which $\pi^B(\theta^{H,e}(N_f^e)) = 0$, which then constitutes an equilibrium as firms are indeed indifferent between serving or not the rich (B-SOE(ii)). Similarly, if $N_f^{min} = 0$, $\Delta\pi^C(\{\theta^{h,e}(0)\}_h) < 0$ and $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) > 0$, there exists an $N_f^e \in (0, 1)$ for which $\Delta\pi^C(\{\theta^{h,e}(N_f^e)\}_h) = 0$, which then constitutes an equilibrium (C-SOE(ii)).

(iv) Lastly, lemma 10 below shows that if $N_f^{min} > 0$, there must be an $\tilde{N}_f \geq N_f^{min}$ for which $\pi^B(\theta^{H,e}(\tilde{N}_f)) < 0$ (in case B-SOE), respectively $\Delta\pi^C(\{\theta^{h,e}(\tilde{N}_f)\}_h) < 0$ (in case C-SOE). Step (i) then again implies that there exists an $N_f^e \in (0, 1)$ for which $\pi^B(\theta^{H,e}(N_f^e)) = 0$ and $\Delta\pi^C(\{\theta^{h,e}(N_f^e)\}_h) = 0$, respectively, which thus constitutes an equilibrium (B-SOE(ii) and C-SOE(ii), respectively).

Lemma 10

If $N_f^{min} > 0$ and $\pi^B(\theta^{H,e}(1)) > 0$ in case (B-SOE), respectively $\Delta\pi^C(\{\theta^{h,e}(1)\}_h) > 0$ in case (C-SOE), then for each of the cases (B-SOE) and (C-SOE) there exists a $\tilde{N}_f \geq N_f^{min}$ such that $\pi^B(\theta^{H,e}(\tilde{N}_f)) < 0$ and $\Delta\pi^C(\{\theta^{h,e}(\tilde{N}_f)\}_h) < 0$, respectively.

The proof of lemma 10 uses the following steps:

(1) First, we define the right-hand side of (24) as function $G(\theta^H; N_f)$ for a given level of imports N_f. That is,

$$G(\theta^H; N_f) := \frac{I^H - N_f\, p^{H,f} - (1 - N_f)\, p^H}{\beta(N_f\, v(q^{H,f}) + (1 - N_f)\, v(q^H))}. \tag{B.29}$$

Recall that q^H is an implicitly defined function of θ^H via (18b) (resp. (20d)) and with this, p^H is an implicitly defined function of θ^H via ($\mathrm{IRf^H}$), i.e. (20b). Since $q^{H,f}$, q^H, and $p^{H,f}$ increase with θ^H, we obtain that

(a)
$$\frac{\partial G(\theta^H; N_f)}{\partial \theta^H} < 0,$$
if $\frac{\partial p^H}{\partial \theta^H} > 0$, or if $\frac{\partial p^H}{\partial \theta^H} < 0$ and N_f sufficiently large.

(b)
$$\frac{\partial G(\theta^H; N_f)}{\partial \theta^H} > 0,$$
if $\frac{\partial p^H}{\partial \theta^H} < 0$ and sufficiently strongly negative and N_f sufficiently small.

This implies that $N_f^{min} > 0$ can only occur in case (b) where $\frac{\partial p^H}{\partial \theta^H} < 0$. Thus, for the proof of lemma 10 this is the relevant case, which can be depicted qualitatively by the following graph:

(2) Consider condition ($\mathrm{IRf^H}$) which characterizes price p^H:

$$p^H(\theta^H) = \theta^H \cdot (q^H(\theta^H))^{1-\beta} - (\theta^H)^{\frac{1}{\beta}} (\bar{q}_{-1})^{\frac{1-\beta}{\beta}} \chi(\tau)$$

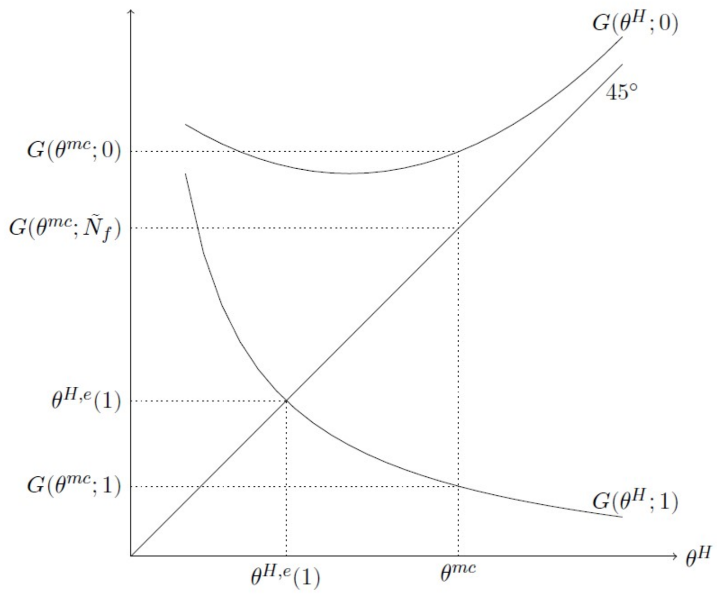
Since p^H declines with θ^H in the case of interest where $N_f^{min} > 0$, we can define the level θ^{mc}, where the price p^H equals the marginal production costs of (optimal) quality q^H as defined by (20d), i.e.

$$p^H(\theta^{mc}) = \frac{q^H(\theta^{mc})}{a_q A}\ . \tag{B.30}$$

Note that such a θ^{mc} must exist, as for $\theta = \theta^{H,e}(1)$, the firms make profits from serving the rich and hence $p^H(\theta^{H,e}(1))$ must be greater than marginal production costs. As $p^H(\theta^H)$ on the left-hand side of (B.30) declines with θ^H while $q^H(\theta^H)$ on the right-hand side increases with θ^H, there must be a θ^{mc} where they intersect. Further this implies that $\theta^{mc} > \theta^{H,e}(1)$.

(3) From (1) we know that $G(\theta^H; N_f = 1)$ is a strictly decreasing function, which possesses a fixed point at $\theta^{H,e}(1)$. It then follows from our discussions (see also graphically above) that $G(\theta^{mc}; N_f = 1) < \theta^{H,e}(1) < \theta^{mc}$.
On the other hand, when $N_f^{min} > 0$, then $G(\theta^H; N_f = 0)$ does not intersect with the

Figure 4: Illustration of $G(\theta^H;0)$ and $G(\theta^H;1)$



Note: The figure illustrates $G(\theta^H;0)$ and $G(\theta^H;1)$ as functions of θ^H and indicates the fixed point $G(\theta^{mc};\tilde{N}_f) = \theta^{mc}$. Note that the important property of $G(\theta^H;0)$ is that is does not intersect with the bisectrix and regarding $G(\theta^H;1)$ that it is strictly declining with θ^H.

bisectrix (see graph above). Consequently, $G(\theta^{mc}; N_f = 0) > \theta^{mc}$. Thus there must exist a $\tilde{N}_f \geq N_f^{min}$ such that $G(\theta^{mc}; \tilde{N}_f) = \theta^{mc}$, i.e. for $\tilde{N}_f$ it holds $\theta^{H,e}(\tilde{N}_f) = \theta^{mc}$. But with the price only covering marginal costs, the additional profits from serving the rich must be negative as innovation costs cannot be covered.

□

Step 2. It remains to provide the equilibrium values for the homogeneous good and for labor that follow from the equilibrium in the differentiated goods markets as given in proposition 3.

In case (B-SOE) homogeneous good consumption of the rich households is $z^{H,e} = (I^H - N_f^e\, p^{H,f,e} - (1 - N_f)\, p^{H,B,e}) a_z A$ and for the poor households $z^{L,e} = (I^L - p^{L,B,e}) a_z A$ which they buy at the equilibrium price $p_z^e = 1/(a_z A)$ reflecting marginal costs. The amount of exported homogeneous goods is $z^{exp,e} = N_f\, \lambda\, \tau\, \frac{a_z}{a_q}\, q^{H,f,e}$, which is traded at the equilibrium price $p_z^{ROW} = \tau/(a_z A)$ reflecting marginal costs times the iceberg trade costs. Labor demand is given by

$$
\begin{aligned}
\int_{N_f^e}^1 L_i^e(q^{H,B,e})di + \int_0^1 L_i^e(q^{L,B,e})di + L_z^e \;=\;& (1-N_f^e)\lambda\frac{q^{H,B,e}}{a_qA} + (1-\lambda)(1-\beta)I^L \\
&+\lambda(I^H - N_f^e\,p^{H,f,e} - (1-N_f^e)\,p^{H,B,e}) \\
&+(1-\lambda)\beta I^L + N_f\,\lambda\,\frac{\tau^2}{a_qA}\,q^{H,f,e} \\
&+\Big[\int_{N_f^e}^1 fM_i^e + h(q^{H,B,e}/\bar{q}_{-1})di\Big]\ ,
\end{aligned}
$$

which is remunerated at the normalized equilibrium wage rate $w^e = 1$.

In case (C-SOE) homogeneous good consumption of the rich households is $z^{H,e} = (I^H - N_f^e p^{H,f,e} - (1-N_f^e)p^{H,C,e})a_zA$ and for the poor households $z^{L,e} = (I^L - N_f^e p^{L,Cf,e} - (1-N_f^e)p^{L,C,e})a_zA$ which they buy at the equilibrium price $p_z^e = 1/(a_zA)$ reflecting marginal costs. The amount of exported homogeneous goods is $z^{exp,e} = N_f^e\lambda\,\tau\,\frac{a_z}{a_q}\,q^{H,f,e}$, which is traded at the equilibrium price $p_z^{ROW} = \tau/(a_zA)$ reflecting marginal costs times the iceberg trade costs. Labor demand is given by

$$
\begin{aligned}
&\int_{N_f^e}^1 L_i^e(q^{H,C,e})di + \int_0^{N_f^e} L_i^e(q^{L,Cf,e})di + \int_{N_f^e}^1 L_i^e(q^{L,C,e})di + L_z^e \\
=\;& \lambda(1-N_f^e)\frac{q^{H,C,e}}{a_qA} + (1-\lambda)\left[N_f^e\frac{q^{L,Cf,e}}{a_qA} + (1-N_f^e)\frac{q^{L,C,e}}{a_qA}\right] \\
&+\lambda(I^H - N_f^e\,p^{H,f,e} - (1-N_f^e)\,p^{H,C,e}) + (1-\lambda)(I^L - N_f^e\,p^{L,Cf,e} - (1-N_f^e)\,p^{L,C,e}) \\
&+\lambda\,N_f^e\,\frac{\tau^2}{a_qA}\,q^{H,f,e} + \left[\int_0^{N_f^e} fM_i^e + h(q^{L,Cf,e}/\bar{q}_{-1})di + \int_{N_f^e}^1 fM_i^e + h(q^{H,C,e}/\bar{q}_{-1})di\right]\ ,
\end{aligned}
$$

which is remunerated at the normalized equilibrium wage rate $w^e = 1$. Also note the measure of firms participating in the patent races M_i^e does not differ for sectors which import and only innovate to serve the poor, $i \in [0, N_f^e]$, and for those where domestic firms compete to serve the rich households, $i \in [N_f^e, 1]$. This is because for $N_f^e \in (0,1)$, expected profits must be the same across sectors.

□

C Specification of the dynamic household problem

In this appendix, we briefly discuss the households' dynamic utility maximization problem for the case where they are not hand-to-mouth.

In the households' dynamic utility maximization problem, total lifetime utility sums up to

$$
U\left(\left\{q_i^j(t)\right\}_{i,t\in[0,1]\times\{0,1,\ldots\}}, \left\{z^j(t)\right\}_{t\in\{0,1,\ldots\}}\right) = \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t u\left(\left\{q_i^j(t)\right\}_{i\in[0,1]}, z^j(t)\right)\ .
$$

Taking prices as given, the households maximize their utility subject to their inter-temporal budget constraint

$$a^j(t+1) = (1+r(t))a^j(t) + I^j(t) - \int_0^1 p_i(q_i^j(t);t)\,di\; - p_z(t)z^j\ ,$$

and the no-Ponzi-game condition

$$\lim_{t\to\infty} a^j(t+1)\prod_{t=0}^{\infty}\frac{1}{1+r(t)} = 0\ .$$

In the above, $a^j(t)$ are household j's total asset holdings at the beginning of period t, $I^j(t)$ denotes its total per-period income net of interest earnings, and $r(t)$ is the per-period interest rate. There are no aggregate investment opportunities in the economy, implying that total net asset holdings are zero, i.e. at any point in time we have[85]

$$\int_0^1 a^j(t)\,dj = 0\ .$$

We further assume that all households have zero initial asset holdings $a^j(0) = 0$ at $t = 0$. All households are thus perfectly symmetric except for their labor productivity ω^h which remains the same over time. As a consequence, in our economy no household has an incentive to lend or borrow, which in turn implies that asset holdings are always 0 for all households, and each household just consumes its per-period income I^j. Hence, even if households were allowed to save or borrow, the balanced growth path would continue to be a dynamic equilibrium with symmetric initial asset endowments of zero.[86]

D Empirics

In this appendix, we provide basic correlations on the inequality-growth nexus. These correlations are far from conclusive due to endogeneity concerns and potential measurement issues. Nevertheless, they provide basic patterns in the data that our theory can speak to.

In open economies, inequality impacts growth through various channels and the overall effect may not be conclusive. Yet, our theory points to an important negative 'business

[85] A free-entry condition guarantees that profits are always equal to zero—see section 3.3. The main focus of our work is on how income inequality impacts aggregate growth via demand-induced quality upgrading. While we could in principle allow for aggregate savings, e.g. by introducing capital as a second factor of production, this would not affect our main mechanism of interest over and above any potential effect on the income distribution. We therefore simplify the exposition by ignoring this possibility throughout.

[86] One might wonder whether rich households might have an incentive to save today in order to avoid being bound by the upper bound on quality as detailed in section 3.3. This is not the case in our economy because, loosely speaking, households have a constant-over-time 'distance' between their ideal qualities and the technological frontier, i.e. saving today to mitigate the problem would exacerbate it in future. Hence, with symmetric initial asset endowments of zero, the equilibrium would be the same as discussed in section 4.2 even if households were allowed to borrow or save.

stealing effect' of inequality on growth in open economies far from the frontier: Ceteris paribus, the more unequal the income distribution is and the more open the economy, the larger is this effect. Moreover, any additional increase in incomes of rich households and, hence, their willingness to pay for innovation, no longer benefits growth in the domestic economy if they satisfy their demand for quality via importing. This is in contrast to the closed economy where higher willingness to pay for quality on the side of the rich leads to more innovation. Our theory therefore suggests that in developing countries, inequality should have a smaller—or more negative—effect on growth in open as opposed to closed economies.

A thorough econometric analysis of this theoretical prediction is challenging and beyond the scope of the paper. Instead, in this appendix we run two different sets of simple regressions that nevertheless can inform us whether our theory reflects basic associations in the data or not. First, we run industry-level growth regressions using growth in export quality taken from Feenstra and Romalis (2014) as the dependent variable. These are our main regressions as growth in quality is closest to our theoretical model and as some of our variables of interest—distance to frontier and openness—vary at the industry level. Second, as quality indices might be prone to measurement error and to better compare our results to previous work in the literature, we perform standard growth regressions using growth in GDP per capita as the dependent variable.

To perform these regressions, we need data on growth at the country-industry and at the country level, respectively, as well as data on inequality, openness, and distance to frontier along with other control variables. We begin with introducing our data, before turning to the model specification and results.

D.1 Data

Export quality and GDP per capita: To measure quality upgrading at the country-industry level, we use data on export quality at the SITC4 industry classification level taken from Feenstra and Romalis (2014), i.e. we use export quality to proxy for domestic production capabilities. The quality indices in Feenstra and Romalis (2014) are the mirror image of quality-adjusted price indices in the Penn World Tables, and they are derived from a theoretical model different from ours. As such, they rely on modeling assumptions and are not readily comparable to quality in our model. Yet, to the best of our knowledge, these are the best available quality indices for analyses of "[...] the time-series or cross-country properties of these indexes" (Feenstra and Romalis, 2014, p. 480), and we therefore use this data.[87] As there might nevertheless be concerns regarding the use of these indices for

[87] We provide a brief summary of the procedure how quality is measured in Feenstra and Romalis (2014) for the reader to better understand our main variable in the empirical illustrations.
First, they set up an extended Melitz (2003) model where consumers have CES expenditure functions over quality-adjusted prices with destination-country specific taste for quality. The latter allows for non-homothetic demand for quality, similar to Fajgelbaum et al. (2011). Their general strategy is then to

our purposes here, we corroborate our industry-level analysis with country-level growth regressions. To measure growth in GDP per capita, we use data on real per capita GDP taken from the Penn World Tables (PWT), version 9.0 (Feenstra et al., 2015).

Inequality: To measure inequality, we use Gini indices in our baseline specification. Gini indices are taken from Solt (2016), as this source combines data from various other databases and makes comparable the Gini indices across countries. We use the Gini index after redistribution. We provide robustness checks using the income shares of the top 10% and top 20%, respectively, in appendix D.3. For these income shares, we rely on data stemming from the World Development Indicators (WDI) (The World Bank, 2018).

Distance: Our main theoretical prediction relies on the possibility to import high qualities from abroad, i.e. it applies to countries not at the frontier. To classify a country-industry pair and a country, respectively, as being not at the frontier, we use our data on export quality and GDP per capita from above. We then generate an indicator for whether a country's export quality in a given industry belongs to the bottom 75% within that industry across countries in the year 2000. Analogously, in our country-level regressions, we classify a country as being developing if its GDP per capita in USD belongs to the bottom 75% in the year 2000.[88] We present robustness checks using alternative specifications for distance to the frontier in appendix D.3.

Openness: To measure a country's openness in a given industry, we combine data on imports by industry taken from Feenstra and Romalis (2014) with data on nominal GDP

derive exporter-importer-product specific quality-adjusted price indices. Given their assumptions, these indices are cardinally comparable across exporters. Quality is defined as the ratio of raw price indices over quality adjusted price indices.
In their model, quality is an endogenous choice of firms and can be produced with labor under a given productivity level which is assumed to be Pareto-distributed across firms. The firm choice of quality is such that all exporters from country i charge the same price in destination j, i.e. differences in productivity are fully reflected in quality differences. Their model features a demand- and a supply-side channel of quality on trade: On the demand-side, ceteris paribus, higher quality leads to higher sales, giving rise to a negative link between trade and quality-adjusted prices. On the supply side, as the sales of country i to destination j increase, the set of firms serving the market expands and, hence, their average productivity declines. This implies a positive link between trade and quality-adjusted prices. Using the Pareto distribution of firm productivities, Feenstra and Romalis (2014) derive an expression linking relative quality-adjusted prices of two exporters to a given destination to relative c.i.f. prices, f.o.b. prices, tariffs, and exogenous market-access cost shifters—see their equation (18).
To take this equation—and an analogous equation for import qualities—to the data, they estimate their model parameters using a structural expression for relative exports and auxiliary regressions to estimate the Pareto shape parameter and the taste shifter for quality—see their section IV.B for details. Equipped with these parameter estimates, they compute relative quality-adjusted prices. These quality-adjusted prices and the raw prices are then aggregated over partner countries and products within an industry using the so-called GEKS method to derive aggregate export and import "quality-adjusted price indices" and raw "price indices", respectively. The indices of export and import quality are obtained by dividing the price indices by the corresponding quality-adjusted price indices.
We note that these measures of quality rely on the modeling assumptions, regressions involving various proxy and potentially mis-measured variables, and aggregations that may not be innocuous in unbalanced data. Moreover, they are derived from a model different from ours, and the interpretation of log changes in quality is not as clear cut in our model with non-homothetic preferences. A very good treatment of the challenges involved can be found e.g. in Deaton and Heston (2010).

[88] We use a binary indicator for distance because according to our theory, distance does not matter for countries sufficiently far from the frontier.

taken from the WDI. From this data, we then compute the share of total imports in a given industry and year over GDP and normalize this share by the average share across countries in the same industry to control for cross-industry heterogeneity in size.[89] In our country-level regressions, we use the share of total imports over GDP taken from the WDI. We present robustness checks using alternative measures for openness in appendix D.3.

Control variables: In our regressions without country fixed effects, we further include a series of country-level controls following Barro (2015). We take data for life expectancy, fertility, consumer price inflation, and the terms of trade from the WDI. From Barro and Lee (2013) we take years of schooling for males and females. The PWT provide us with data on investment shares and government consumption shares. Finally, we take a measure of political rights combining data from Freedom House (2016) and Bollen (1980) and standardize it to be between zero and one.

Merging the country level data to the industry specific data gives us a data panel tracking industry-country pairs over time. The industry level export and import data are available for the years 1985–2010. Therefore, our panel spans 25 years, and we use the same years also for the country-level regressions. To increase the variation in the data, we collapse the panel to a five year frequency, such that we have six periods in our panel. For each five year period, we keep the last value available not to lose observations with a data point in 2004 but not in 2005, for example. We exclude resource-rich countries (i.e. countries whose resource rents exceed 20% of their GDP on average) as well as micro states with a population of less than one million, averaged over all years.[90] The panel then covers 131 countries and a total of 485 industries.[91] Table 3 provides descriptive statistics for our dataset and appendix D.3 robustness checks using the subset of industries that make (durable) final goods.

D.2 Specification and results

Equipped with this data, we estimate the following industry-level regressions.

$$\begin{aligned}\ln\left(\frac{q^s_{x,c,t}}{q^s_{x,c,t-j}}\right) =& \beta_1 \ln(q^s_{x,c,t-j}) + \beta_2 Open^s_{c,t-j} + \beta_3 Ineq_{c,t-j} + \beta_4 Dist^s_c \\ &+ \beta_5[Open^s_{c,t-j} \times Ineq_{c,t-j}] + \beta_6[Open^s_{c,t-j} \times Dist^s_c] \\ &+ \beta_7[Ineq_{c,t-j} \times Dist^s_c] + \beta_8[Open^s_{c,t-j} \times Ineq_{c,t-j} \times Dist^s_c] \\ &+ controls + \epsilon^s_{c,t} \ , \end{aligned} \tag{D.1}$$

[89] We use this way to control for average industry-size because output data is not available at the disaggregated country-industry level.

[90] Data on resource rents as a share of GDP are taken from the WDI and data on total population from the PWT.

[91] Note that the total number of industries in the original data is 1646. However, for some countries and industries, there are several measures of units and, hence, some industries appear more than once. We focus on kilograms as the unit measure, since this is the most common in the data. Furthermore, we exclude industries producing homogenous goods, according to the conservative version of the Rauch (1999) index, as quality differs very little in these sectors. This reduces the number of industries to 485.

Table 3: Descriptive statistics main variables

Variable	Mean	Std. dev.	Min.	Max.	N
Part I: Macro variables:					
Import share	0.26	0.27	0.00	3.31	704
Real GDP	0.48	1.41	0.00	15.27	704
Population	45.59	148.93	0.73	1340.97	704
Real GDP per capita	12.30	13.44	0.31	81.69	704
Gini	0.39	0.09	0.20	0.62	616
Income share top 20%	0.48	0.08	0.33	0.71	360
Income share top 10%	0.32	0.08	0.19	0.62	360
Life expectancy	66.20	10.73	31.98	82.98	727
Fertility	3.42	1.93	0.96	8.18	727
Schooling (female)	6.78	3.28	0.37	13.23	655
Schooling (male)	7.60	2.83	1.11	13.36	655
Investment share	0.19	0.09	0.01	0.66	704
Government share	0.19	0.09	0.05	0.74	704
Democracy index	0.58	0.36	0.00	1.00	727
CPI inflation	0.50	5.68	-0.04	117.50	628
Terms of trade	1.08	0.51	0.15	5.62	668
Part II: Industry variables:					
Export quality	1.25	1.48	0.00	134.35	191448
Import quality	1.17	0.61	0.03	24.51	263124
Import share	0.00	0.00	0.00	0.26	250597
Import share (adjusted)	0.01	0.03	0.00	1.00	250597

Note: The export and import quality data, as well as the sectoral import shares, are taken from Feenstra and Romalis (2014). The country import shares are taken from the PWT. Real GDP is measured in trillion USD, population in millions and real GDP per capita in 1000 USD. The Gini index is after redistribution. Life expectancy is measured at birth in years, fertility is number of births per woman, schooling is measured in years. The democracy index is standardized between zero and one. Terms of trade is the ratio of the export value index and the import value index.

where $q^s_{x,c,t}$ is export quality in country c, year t, and sector (or industry) s. $Open^s_{c,t-j}$ is our measure of openness at the sectoral level, $Ineq_{c,t-j}$ is a measure of inequality, i.e. the Gini index in our baseline specification, and $Dist^s_c$ is an indicator whether sector s in country c has a large distance to the technology frontier. $controls$ is a set of control variables which includes industry times year fixed effects and either the large set of country controls following Barro (2015) or country fixed effects. Finally, $\epsilon^s_{c,t}$ is an error term. In appendix D.3, we present robustness checks using country times year and country times industry fixed effects, respectively, in our industry-level regressions. As explained above, we use a data panel with a five year frequency (i.e. $j = 5$ and the data are collapsed to a frequency of five years).

To alleviate potential concerns regarding measurement of export quality and to connect our results with previous research on growth and inequality, we also estimate the specified regression equation at the country level, replacing quality upgrading by growth in GDP per capita. In these regressions, we use the measures for openness and distance to frontier at the country level as described above. The other control variables are the same as in our

Table 4: Baseline results for growth regressions

Dependent variable in t:	Growth t to $t+1$ in export quality				Growth t to $t+1$ in GDP per capita			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log export quality	-0.73***	-0.73***	-0.74***	-0.75***				
	(0.01)	(0.01)	(0.01)	(0.01)				
Log GDP per capita					-0.09***	-0.09***	-0.48***	-0.51***
					(0.01)	(0.02)	(0.05)	(0.05)
Openness	0.00	0.01	0.01***	0.02	0.01	-0.11	0.04	-0.25
	(0.00)	(0.02)	(0.00)	(0.01)	(0.01)	(0.12)	(0.03)	(0.27)
Inequality	-0.27***	-0.48	0.58**	0.53	-0.13	0.53*	1.20**	0.46
	(0.09)	(0.38)	(0.26)	(0.38)	(0.12)	(0.30)	(0.49)	(1.05)
Distance	-0.35***	-0.30*	-0.32***	-0.27**	0.05**	0.40***		
	(0.02)	(0.16)	(0.02)	(0.12)	(0.02)	(0.15)		
Openness × Inequality		-0.01		-0.02		0.37		0.50
		(0.05)		(0.04)		(0.32)		(0.82)
Openness × Distance		0.02		0.00		0.18		0.48*
		(0.03)		(0.02)		(0.13)		(0.28)
Inequality × Distance		-0.15		-0.23		-1.00**		-0.04
		(0.44)		(0.35)		(0.40)		(1.31)
Openness × Inequality × Distance		-0.06		-0.01		-0.50		-1.01
		(0.06)		(0.05)		(0.35)		(0.85)
Control variables	Yes	Yes	No	No	Yes	Yes	No	No
Industry × Year FE	Yes	Yes	Yes	Yes	No	No	No	No
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	Yes	No	No	Yes	Yes
Key coefficient		-0.07**		-0.04*		-0.14		-0.50**
Wald test		0.02		0.07		0.44		0.02
Observations	95211	95211	121769	121769	379	379	486	486

Note: Openness is the log of the relative import share at the industry level (the log of the country-level import share in the country-level regressions). Inequality is the Gini index in levels. Distance indicates whether the export quality of an industry was amongst the lower 75% in the year 2000 for the industry level regressions and whether a country's per capita GDP was amongst the lower 75% in the year 2000 for the country level regressions. Control variables is a series of control variables at the country level, as introduced in section D.1. Key coefficient is the sum of the coefficients for the interaction between Openness and Inequality and the interaction between Openness, Inequality, and Distance. The Wald test tests whether the sum of the two coefficients is zero and reports the p-value of this test. Standard errors are clustered at the industry×year and country×year level (for industry level analysis) or at the country level (for country level analysis), respectively. Significance at the 10% level is indicated by *, at the 5% level by **, and at the 1% level by ***.

industry-level regressions with year fixed effects replacing the industry times year fixed effects.

We estimate equation (D.1) using OLS fixed effects regressions. The main results for the different specifications are reported in table 4. Our prime interest is in the sum of coefficients β_5 and β_8, which measures how in developing countries the effect of inequality depends on openness to trade. We expect the sum of the coefficients to be negative: Given a country is developing, higher inequality should have a smaller—or more negative—effect on growth in an open than in a closed economy.

The results show that, as expected, we find conditional convergence for both industry level export quality as well as aggregate GDP growth. For the variables of interest, namely openness, inequality, and level of development, the results for the individual effects are inconclusive. However, as the bottom of the table shows, inequality and openness are jointly inversely related to growth in developing countries. In all specifications, the sum of β_5 and β_8 as specified in equation (D.1) is negative, and for the industry level regressions as well as for the country regression with country fixed effects it is statistically significant.[92] The results in table 4 therefore suggest that inequality is associated with less quality upgrading (or growth in GDP per capita) in more vs. less open developing countries, as predicted by our theory. While these regressions suffer from endogeneity and measurement concerns, they show associations in the data that our theory may be able to speak to:[93] once a country is away from the technological frontier, high inequality and the possibility of the rich class to import high quality goods from abroad reduce innovation incentives for domestic producers. A series of robustness tests confirm these associations in the data. These robustness tests are documented in the next section.

D.3 Robustness checks for table 4

In this appendix, we present robustness checks for the empirical results presented in table 4.

Table 5 shows robustness checks for the distance to frontier measure. We vary the threshold level to define an industry or country as being distant from the frontier as well as the reference year. The first two columns use a threshold of 50% instead of 75% to classify a sector as not belonging to the technology frontier. In columns (3) and (4) we change the reference year from 2000 to 1985. As the growth rate in export quality has an impact on whether a sector is close to the frontier, we choose the first year of our data as the reference year to alleviate endogeneity problems stemming from potential reverse causality. The remaining columns repeat the exercise for the country-level data. Overall, the results indicate that the way how distance to frontier is defined does not crucially impact our results.

Table 6 provides results for different specifications of the openness measure. Instead of using the continuous adjusted import share, we use a binary variable indicating whether a sector's openness is amongst the 75% highest across countries (first two columns). Furthermore, instead of taking the openness measure for every year, we define openness in the

[92] The point estimates in column (4) imply that the effect on 5-year growth in export quality of increasing inequality from the 25th to the 75th percentile is 0.8% lower in developing countries with openness at the 75th percentile than in developing countries with openness at the 25th percentile.

[93] For example, a country's openness as measured here depends on domestic demand, production cost, and quality, both directly and because these factors might influence lobbying for trade liberalization / protection. Similarly, there might be omitted variables that influence both a country's current distance to frontier and its future growth prospects, and distance to frontier also impacts trade and, thus, our measurement of openness. Moreover, our dependent variable might suffer from measurement issues and is not perfectly consistent with our theory, as previously discussed.

Table 5: Robustness results: Distance

Dependent variable in t:	Growth t to $t+1$ in export quality				Growth t to $t+1$ in GDP per capita			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log export quality	-0.76***	-0.77***	-0.56***	-0.60***				
	(0.01)	(0.01)	(0.01)	(0.01)				
Log GDP per capita					-0.11***	-0.49***	-0.08***	-0.45***
					(0.02)	(0.05)	(0.01)	(0.04)
Openness	-0.01	0.01	0.01	0.03**	-0.05	0.09	0.01	-0.02
	(0.02)	(0.01)	(0.02)	(0.01)	(0.05)	(0.09)	(0.15)	(0.30)
Inequality	-0.26	0.54*	-0.90**	-0.14	0.26	0.02	-0.00	-0.40
	(0.26)	(0.30)	(0.35)	(0.35)	(0.17)	(0.62)	(0.28)	(1.06)
Distance	-0.27*	-0.30***	-0.11	-0.17**	0.21		0.40**	
	(0.15)	(0.11)	(0.12)	(0.09)	(0.23)		(0.18)	
Openness × Inequality	0.03	0.01	-0.02	-0.05	0.14	-0.32	0.04	-0.28
	(0.04)	(0.03)	(0.05)	(0.04)	(0.12)	(0.22)	(0.41)	(0.81)
Openness × Distance	0.04*	0.01	0.03	0.01	0.09	0.34**	0.12	0.36
	(0.02)	(0.02)	(0.02)	(0.01)	(0.11)	(0.16)	(0.17)	(0.31)
Inequality × Distance	-0.23	-0.13	0.07	0.17	-0.61	0.00	-0.89*	0.28
	(0.38)	(0.31)	(0.31)	(0.24)	(0.54)	(1.27)	(0.46)	(1.31)
Openness × Inequality × Distance	-0.10*	-0.04	-0.07	-0.02	-0.22	-0.49	-0.33	-0.38
	(0.06)	(0.04)	(0.05)	(0.04)	(0.27)	(0.40)	(0.45)	(0.85)
Control variables	Yes	No	Yes	No	Yes	No	Yes	No
Industry × Year FE	Yes	Yes	Yes	Yes	No	No	No	No
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Country FE	No	Yes	No	Yes	No	Yes	No	Yes
Key coefficient	-0.07**	-0.04	-0.09***	-0.06***	-0.08	-0.81**	-0.29	-0.67***
Wald test	0.05	0.16	0.00	0.00	0.76	0.02	0.14	0.01
Observations	95211	121769	59690	80615	379	486	334	425

Note: The specifications are the same as in table 4, except for the measure of Distance. The first two columns use a dummy variable indicating whether the export quality of an industry was amongst the lower 50% in the year 2000. Columns (3) and (4) again use 75% as the threshold but use 1985 as the reference year. Columns (5) to (8) repeat the exercise using GDP per capita and the country level data. Note that the industry level results are robust to defining the distance measure at the country level as well.

year 2000 and use this measure for all years (columns (3) and (4)). The results hold also if we use 1985 instead of 2000.[94] Columns (5) to (8) repeat the exercise for the country level. The table shows that our main results also do not hinge on the exact definition of openness.

Table 7 repeats the exercise using different measures of inequality. Columns (1) and (2) show the results using the share of incomes going to the top quintile as the inequality measure, while columns (3) and (4) use the share of incomes going to the top decile. Columns (5) to (8) show the results for the country level data. For the sectoral regressions, the estimated key coefficient remains negative in all specifications, while it becomes statistically insignificant once country fixed effects are included. For the country-level regressions, the results are robust as well. However, note that it is not straightforward

[94] Our result is also robust to using country-level instead of country-industry level measures for openness in our industry regressions.

Table 6: Robustness results: Openness

Dependent variable in t:	Growth t to $t+1$ in export quality				Growth t to $t+1$ in GDP per capita			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log export quality	-0.73***	-0.75***	-0.73***	-0.74***				
	(0.01)	(0.01)	(0.01)	(0.01)				
Log GDP per capita					-0.08***	-0.49***	-0.09***	-0.48***
					(0.01)	(0.05)	(0.01)	(0.05)
Openness	0.02	0.04	-0.00	0.02	0.36**	-0.10	-0.04	
	(0.07)	(0.05)	(0.02)	(0.01)	(0.14)	(0.16)	(0.14)	
Inequality	-0.43**	0.71**	-0.41	0.62	1.17***	-0.99	0.40	2.00*
	(0.21)	(0.32)	(0.35)	(0.39)	(0.35)	(0.70)	(0.35)	(1.19)
Distance	-0.52***	-0.33***	-0.24	-0.32***	0.53***		0.34*	
	(0.10)	(0.08)	(0.15)	(0.11)	(0.14)		(0.20)	
Openness ×	-0.02	-0.04	0.01	-0.02	-0.92**	0.54	0.21	2.38**
Inequality	(0.19)	(0.15)	(0.05)	(0.04)	(0.40)	(0.43)	(0.39)	(1.08)
Openness ×	0.11	0.06	0.03	-0.01	-0.29*	0.28	0.10	
Distance	(0.08)	(0.06)	(0.02)	(0.02)	(0.17)	(0.18)	(0.16)	
Inequality ×	0.44*	0.00	-0.25	-0.05	-1.33***	3.03***	-0.88*	-1.76
Distance	(0.24)	(0.21)	(0.40)	(0.33)	(0.37)	(0.81)	(0.50)	(1.53)
Openness ×	-0.28	-0.16	-0.08	0.01	0.76*	-0.96**	-0.35	-3.14***
Inequality × Distance	(0.21)	(0.18)	(0.06)	(0.05)	(0.46)	(0.46)	(0.43)	(1.12)
Control variables	Yes	No	Yes	No	Yes	No	Yes	No
Industry × Year FE	Yes	Yes	Yes	Yes	No	No	No	No
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Country FE	No	Yes	No	Yes	No	Yes	No	Yes
Key coefficient	-0.30***	-0.21***	-0.08***	-0.01	-0.16	-0.41*	-0.15	-0.76**
Wald test	0.00	0.00	0.01	0.67	0.47	0.06	0.51	0.05
Observations	95211	121769	96136	124441	379	486	379	486

Note: The specifications are the same as in table 4, except for the measure of Openness. Instead of using the continuous measure of the import share, we use a binary variable indicating whether the industry openness is amongst the higher 75% (columns (1) and (2)). In columns (3) and (4), we take the log industry import share in the year 2000 instead of the yearly import share. Columns (5) to (8) repeat the exercise using the country level data. Columns (5) and (6) show the results using a binary variable indicating whether the country's import share is amongst the highest 50%. We use the 50% threshold in order to avoid multicollinearity of the interaction terms. Using the 75% threshold and omitting openness does not substantially change the results. The last two columns use the log of the country's import share in the year 2000. Note that for all specifications using the year 2000 as the reference year, we could use 1985 (the first year in our dataset) instead and the results remain robust and become even more pronounced.

to compare the results using these definitions of inequality to the results with the Gini index, as the sample has changed due to the limited data availability for income shares.

Table 8 shows our industry-level regressions with additional fixed effects. Columns (1) to (4) repeat our main specification from table 4, for convenience. Columns (5) and (6) (columns (7) and (8)) replace the country fixed effects with country times year (country times industry) fixed effects. The estimated key coefficient remains negative in both cases, albeit no longer significant.

Finally, table 9 shows additional robustness checks for our industry-level regressions. Columns (1) to (2) repeat our main specification from table 4, but where we add a country's GDP per capita as an additional control with little effect on our key coefficient.

Table 7: Robustness results: Inequality

Dependent variable in t:	Growth t to $t+1$ in export quality				Growth t to $t+1$ in GDP per capita			
Inequality measure in t:	Top 20%		Top 10%		Top 20%		Top 10%	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log export quality	-0.75***	-0.77***	-0.75***	-0.77***				
	(0.01)	(0.01)	(0.01)	(0.01)				
Log GDP per capita					-0.09***	-0.48***	-0.09***	-0.47***
					(0.02)	(0.10)	(0.02)	(0.10)
Openness	0.04	0.02	0.03	0.02	-0.09	0.35	-0.06	0.18
	(0.04)	(0.03)	(0.03)	(0.02)	(0.14)	(0.81)	(0.11)	(0.54)
Inequality	-0.58	0.30	-0.57	0.42	0.64	-2.17	0.65	-1.79
	(0.55)	(0.54)	(0.59)	(0.58)	(0.76)	(2.00)	(0.96)	(2.30)
Distance	-0.47	-0.25	-0.45**	-0.28	0.41		0.29	
	(0.30)	(0.29)	(0.22)	(0.21)	(0.32)		(0.25)	
Openness × Inequality	-0.06	-0.00	-0.06	0.01	0.15	-0.88	0.12	-0.75
	(0.07)	(0.07)	(0.08)	(0.07)	(0.37)	(2.11)	(0.46)	(2.24)
Openness × Distance	-0.01	-0.01	-0.01	-0.01	0.13	-0.01	0.09	0.03
	(0.04)	(0.04)	(0.03)	(0.03)	(0.17)	(0.82)	(0.13)	(0.55)
Inequality × Distance	0.04	-0.37	0.00	-0.48	-0.79	0.70	-0.79	0.45
	(0.65)	(0.62)	(0.70)	(0.67)	(0.82)	(2.04)	(1.02)	(2.36)
Openness × Inequality × Distance	-0.01	-0.01	-0.01	-0.02	-0.25	0.25	-0.21	0.20
	(0.09)	(0.08)	(0.10)	(0.09)	(0.41)	(2.14)	(0.50)	(2.28)
Control variables	Yes	No	Yes	No	Yes	No	Yes	No
Industry × Year FE	Yes	Yes	Yes	Yes	No	No	No	No
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Country FE	No	Yes	No	Yes	No	Yes	No	Yes
Key coefficient	-0.07*	-0.01	-0.07*	-0.01	-0.10	-0.63**	-0.09	-0.56*
Wald test	0.08	0.69	0.10	0.74	0.53	0.02	0.58	0.06
Observations	53301	60115	53301	60115	222	234	222	234

Note: The specifications are the same as in table 4, except for the measure of Inequality. Instead of the Gini index, we use the income share earned by the top 20% (columns (1), (2), (5), (6)) and the top 10% (columns (3), (4), (7), (8)), respectively.

In columns (3) to (8), we limit attention to final-good producing industries, which are of particular interest for our theoretical arguments. We classify industries as final goods based on the Broad Economic Categories (BEC), rev. 5.[95] In columns (3) and (4), we consider only final-good industries. Columns (5) and (6) consider only final goods that are also processed. Lastly, columns (7) and (8) consider only durable final goods.[96] The estimated key coefficient remains mostly unaffected in the specifications with country fixed effects, albeit less significant due to the reduced sample size. It gets amplified in specifications without country fixed effects.

[95] We first match the BEC to HS product codes and then the HS product codes to SITC industries using concordances from eurostat (https://ec.europa.eu/eurostat/ramon/relations/index.cfm?TargetUrl=LST_REL). We treat an industry as being final-good producing if more than 50% of the matched HS product codes are final goods.

[96] We note that once we aggregate goods to the SITC classification, all durable final goods are also processed.

Table 8: Robustness results: Fixed Effects

Dependent variable in t:	Growth t to $t+1$ in export quality							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log export quality	-0.73***	-0.73***	-0.74***	-0.75***	-0.75***	-0.74***	-1.15***	-1.15***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Openness	0.00	0.01	0.01***	0.02	0.01***	0.01	0.02***	0.05
	(0.00)	(0.02)	(0.00)	(0.01)	(0.00)	(0.02)	(0.00)	(0.03)
Inequality	-0.27***	-0.48	0.58**	0.53			0.89***	-0.06
	(0.09)	(0.38)	(0.26)	(0.38)			(0.34)	(0.72)
Distance	-0.35***	-0.30*	-0.32***	-0.27**	-0.33***	-0.25**		
	(0.02)	(0.16)	(0.02)	(0.12)	(0.02)	(0.12)		
Openness × Inequality		-0.01		-0.02		-0.01		-0.08
		(0.05)		(0.04)		(0.04)		(0.07)
Openness × Distance		0.02		0.00		0.00		-0.01
		(0.03)		(0.02)		(0.02)		(0.03)
Inequality × Distance		-0.15		-0.23		-0.27		0.87
		(0.44)		(0.35)		(0.35)		(0.92)
Openness × Inequality × Distance		-0.06		-0.01		-0.02		0.04
		(0.06)		(0.05)		(0.05)		(0.09)
Control variables	Yes	Yes	No	No	No	No	No	No
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country × Year FE	No	No	No	No	Yes	Yes	No	No
Country × Industry FE	No	No	No	No	No	No	Yes	Yes
Country FE	No	No	Yes	Yes	No	No	No	No
Key coefficient		-0.07**		-0.04*		-0.03		-0.03
Wald test		0.02		0.07		0.18		0.44
Observations	95211	95211	121769	121769	126371	121769	120433	119859

Note: The specifications are the same as in table 4, except for the selection of the fixed effects as detailed in the table.

Table 9: Robustness results: Log GDP and BEC

Dependent variable in t:	Growth t to $t+1$ in export quality							
	all goods		final		final processed		final durable	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log export quality	-0.74***	-0.75***	-0.67***	-0.69***	-0.67***	-0.70***	-0.68***	-0.71***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Log GDP per capita	0.10***	0.05*						
	(0.01)	(0.03)						
Openness	0.00	0.02	0.03	0.02	0.06	0.01	0.05	0.01
	(0.02)	(0.01)	(0.03)	(0.02)	(0.04)	(0.03)	(0.04)	(0.03)
Inequality	-0.25	0.46	-0.70	0.06	-0.98	0.13	-0.93	0.20
	(0.38)	(0.39)	(0.52)	(0.50)	(0.62)	(0.55)	(0.66)	(0.58)
Distance	-0.26	-0.27**	-0.09	-0.17	-0.10	-0.17	-0.08	-0.18
	(0.16)	(0.12)	(0.21)	(0.16)	(0.25)	(0.19)	(0.26)	(0.20)
Openness × Inequality	0.00	-0.02	-0.07	-0.01	-0.13	-0.02	-0.12	-0.00
	(0.05)	(0.04)	(0.08)	(0.06)	(0.10)	(0.07)	(0.10)	(0.08)
Openness × Distance	0.02	0.00	0.04	0.01	0.04	0.01	0.05	0.01
	(0.03)	(0.02)	(0.03)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)
Inequality × Distance	-0.26	-0.23	-0.57	-0.35	-0.51	-0.28	-0.58	-0.30
	(0.44)	(0.35)	(0.57)	(0.46)	(0.68)	(0.54)	(0.73)	(0.57)
Openness × Inequality × Distance	-0.06	-0.01	-0.12	-0.04	-0.10	-0.02	-0.12	-0.04
	(0.06)	(0.05)	(0.09)	(0.07)	(0.11)	(0.08)	(0.11)	(0.09)
Control variables	Yes	No	Yes	No	Yes	No	Yes	No
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	No	Yes	No	Yes	No	Yes	No	Yes
Key coefficient	-0.05**	-0.04*	-0.18***	-0.05	-0.23***	-0.04	-0.24***	-0.04
Wald test	0.04	0.08	0.00	0.12	0.00	0.30	0.00	0.31
Observations	95211	121769	18988	24076	15905	20147	14660	18605

Note: The specifications are the same as in table 4, columns (2) and (4). Columns (1) and (2) include log GDP per capita as an additional explanatory variable, while columns (3) to (8) use only final goods, final processed goods, and final durable goods, respectively, according to the Broad Economic Categories (BEC), which have first been matched to HS product codes and then to SITC industries using concordances from eurostat (https://ec.europa.eu/eurostat/ramon/relations/index.cfm?TargetUrl=LST_REL). Industries have been classified as being e.g. final-good producing if more than 50% of the matched HS product codes are final goods.

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