CHAPTER 8

Strategic voting, nonseparability, and probabilistic voting

Discretion is the better part of virtue,
Commitments the voters don't know about can't hurt you.
(Ogden Nash, *The Old Dog Barks Backward,* 1972, "Political Reflections")

In earlier chapters, we have presented a model of how people vote in a "committee" setting. As the reader will recall, the assumptions of committee voting are: (a) Each participant has full information about all possible alternatives and has well-defined *preferences* over those alternatives; (b) those preferences are *separable across issues,* so that even if the policy space is multidimensional, we can act as if issues are voted on one at a time; (c) each participant knows the preferences of all other participants; (d) all participants have *free, equal power to propose alternatives;* and (e) votes are assumed to be *sincere,* in the sense that if a member prefers alternative A to alternative B, he or she votes for A over B in a pairwise majority rule comparison.

We have also noted that this model is only a starting point. Real committee voting decisions are far more complicated. One complication is that the location of members' "ideal points" is the result of a variety of outside pressures and use of information, so that the ideal point is induced rather than being a primitive "preference."[1] Still, since we have taken member ideal points as exogenous, accounting for the origin or process of formation of the preference is not too difficult. All we need is that an ideal point exists.

A more difficult problem is the "agenda," or actual sequence of votes that leads to a decision. As John Kingdon points out:

> It is important to keep in mind that in the process of setting the agenda and specifying the alternatives, a good many policy options are eliminated from consideration. There is a myriad of subjects that could conceivably be decided by Congress or by any other authoritative decision body. Governmental

decision-makers, particularly congressmen, cannot attend to them all or even to a very large fraction of them. The subjects that do become part of the decisional agenda, therefore, represent only a part of the population of subjects that are potential agenda items. This selection of which subjects to address and which ones to overlook is a kind of structural "decision" of major consequence. . . . [When] a matter does reach the decisional agenda, *the process by which alternatives are evoked and seriously considered is also crucial.* (1981, pp. 282–3; emphasis added)

In this chapter we consider three advanced topics in the committee voting model that help us account for the complexity of both the voting decision and the process of agenda formation.[2]

Strategic voting

We have assumed that participants in political decision-making processes have reacted to each choice situation by casting a vote that reflects their "true," or sincere, preference. More specifically, given a vote between alternatives A and B, voter *i* votes for the alternative he most prefers. Of course, this makes sense if the present vote is the last one and no further choices will be presented.

Such a stopping rule is rarely in force, however. More often, a vote is really a choice between a set of future votes, the sequence of which depends on the "agenda" rule adopted by the organization. If political actors are sophisticated, they can recognize that a vote leads not to an outcome, but to another vote. Their preferences over outcomes can plausibly induce preferences over branches of an agenda "tree," as the following example makes clear.

Suppose the only three foods in the world were apples, broccoli, and carrots. Each type of food is sold only in large crates. Consider three people who, if they cooperate, will have just enough money to buy one, but only one, crate of food. The preference profiles of the three people, Mr. 1 (who loves apples), Ms. 2 (who loves carrots), and Mr. 3 (who loves broccoli), are listed in Table 8.1.

The premise of the example is that choice is collective: If they cannot agree on a food, all will go hungry, because no one has enough money to buy a crate alone. But the three disagree (as the reader can see from Table 8.1) about what to buy. After discussing the choices endlessly, they realize they will never reach a consensus, and no one is going to

Table 8.1. *Preference "lists" of three voters over apples, broccoli, and carrots*

	Person		
Ranking	Mr. 1	Ms. 2	Mr. 3
Best	Apples	Carrots	Broccoli
Middle	Broccoli	Apples	Carrots
Worst	Carrots	Broccoli	Apples

change his or her mind (if you don't like broccoli, you just don't like it; there is little room for persuasion). So, our three people decide to vote. They learned in ninth grade civics that voting is the only fair way to make collective decisions. Besides, they are all getting hungry.

The alert reader may feel sympathy at the naive hope that voting will solve the problem. The reason faith in voting is naive is that the preferences profiled in Table 8.1 do not admit of a Condorcet winner. By majority rule, apples are preferred to broccoli is preferred to carrots are preferred to apples, always by 2 to 1 margins. Say, for example, apples are voted against broccoli first. Mr. 1 and Ms. 2 both prefer apples to broccoli, Mr. 3 vainly dissents, and apples are selected. Apples are then compared with the remaining choice, carrots; Mr. 1 votes for apples, Ms. 2 votes for carrots, and Mr. 3 votes for carrots. It is quite possible that the three stop there, if they don't think things through, since nobody covered agenda manipulation in ninth grade civics. On the face of it, the "carrots preferred to apples preferred to broccoli" group preference seems fair enough.

If the first vote were between apples and carrots, then broccoli would be the collective choice (carrots beat apples, but lose to broccoli, in pairwise majority voting). If carrots and broccoli are the first comparison, then apples emerge as the favored food. The point is that there exists *some* agenda leading to *any* given outcome. Interestingly, this result is surprisingly general, as McKelvey (1976a, 1976b, 1986), Schofield (1978a, 1984), and McKelvey and Schofield (1986) showed. We can state the result in the form of a very much simplified theorem, which is more restrictive (and easier to understand) than the actual result.

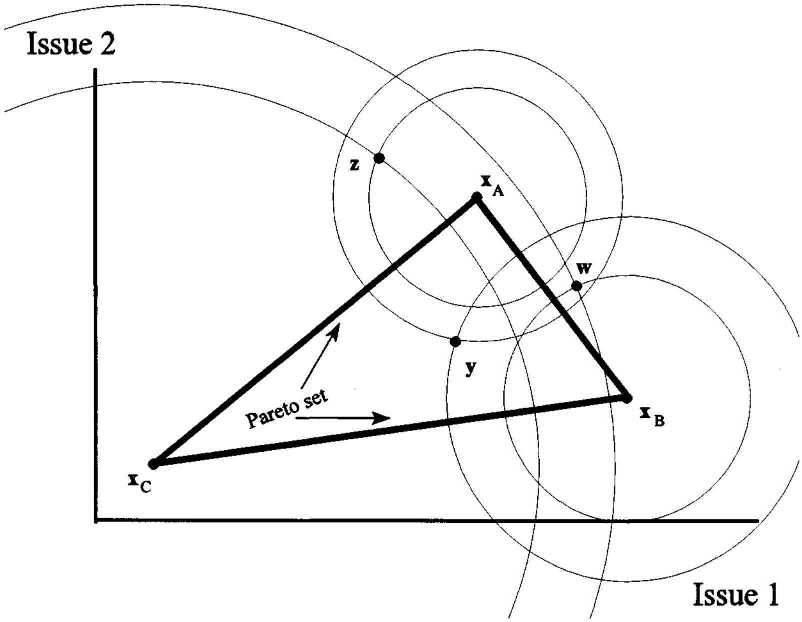

Figure 8.1. The existence of an agenda from **y** to **z**.

Chaos theorem. *Imagine a continuous policy space $\mathcal{P}$. Let the number of dimensions in $\mathcal{P}$ be at least three, assume decisions are by simple majority rule, and let there be at least three voters with proposal power. Let voting be sincere. If there is not at least one "median in all directions," it is possible to construct an agenda, or sequence of comparisons of pairs of alternatives, that leads to any alternative in the space.*

Intuitively, this theorem simply means that given a status quo **y** and any other alternative **z**, there exists an agenda (sequence of pairwise votes) that leads from **y** to **z**. Under some circumstances, in fact, this agenda has only one intermediate step **w**, so that a sophisticated agenda maker can go from **y** to **w** to **z**, no matter what values of **y** and **z** are chosen. We can illustrate such a "trajectory" in Figure 8.1: Beginning with **y**, a majority (B and A) prefers **w**. But then a majority (B and C) prefers **z** to **w**. What the chaos theorem says is that for any **y** and **z** there exists a **w** (where neither **z** nor **w** is necessarily contained in the Pareto set) that allows the agenda setter to go from **y** to **z**.

But wait just a minute. This result requires that the voters be completely ignorant of politics! Let us go back to our apples, broccoli, and carrots choosers. It is true that if no one knows social choice and an agenda is constructed at random by our hungry heroes, the choice could be *any one* of the three foods. In a way, this is "fair" because if the *agenda* is chosen at random, the implied outcome is (in effect) also randomly selected. If there is no median in all directions, a *naive choice over agendas* is tantamount to a *lottery over outcomes.*

If this kind of choice is routine, however, it seems unlikely that people could really fail to understand the underlying correspondence: Choosing an agenda implies a choice of an outcome. As Riker (1980) pointed out, if people disagree about outcomes and understand politics, their disagreement will take the form of a disagreement over the agenda, or institutions of choice more generally.

More fundamentally, what can we say from an ethical perspective about the "fairness" of the outcome if some people understand agendas, but others don't? The answer is disturbing: If only one person gets to pick the agenda and enforce a stopping rule, and if participants vote sincerely, there is no important difference between majority rule and dictatorship. Figure 8.2 contains the agendas selected by Mr. 1, Ms. 2, and Mr. 3. In each case the outcome is the same as if that person were dictator and had sole decision power. There is one sinister difference between dictatorship and a "monopoly agenda setter," however. Dictatorship is openly and definitionally undemocratic. By contrast, people naively voting on alternatives whose order is decided by an agenda setter may be unaware that the choice of outcome is, in effect, dictatorial.

Each alternative in the agenda is "considered," it is true, but the order of consideration dictates the outcome.[3] Getting to vote doesn't mean choice is democratic, though letting people go through the motions may imbue the choices of the agenda setter with an aura of legitimacy. To put it another way, there is nothing inherently "fair" about a sequenced majority rule decision. The trappings of democracy (people get to vote) may simply be the mechanism of manipulation.

There is some evidence that control of the agenda does confer just this sort of power on the "setter." Agenda control can be imbedded either in the rules of parliamentary procedure (see, e.g., Denzau and Mackay, 1983) or in proposals made by bureaucrats to elected officials

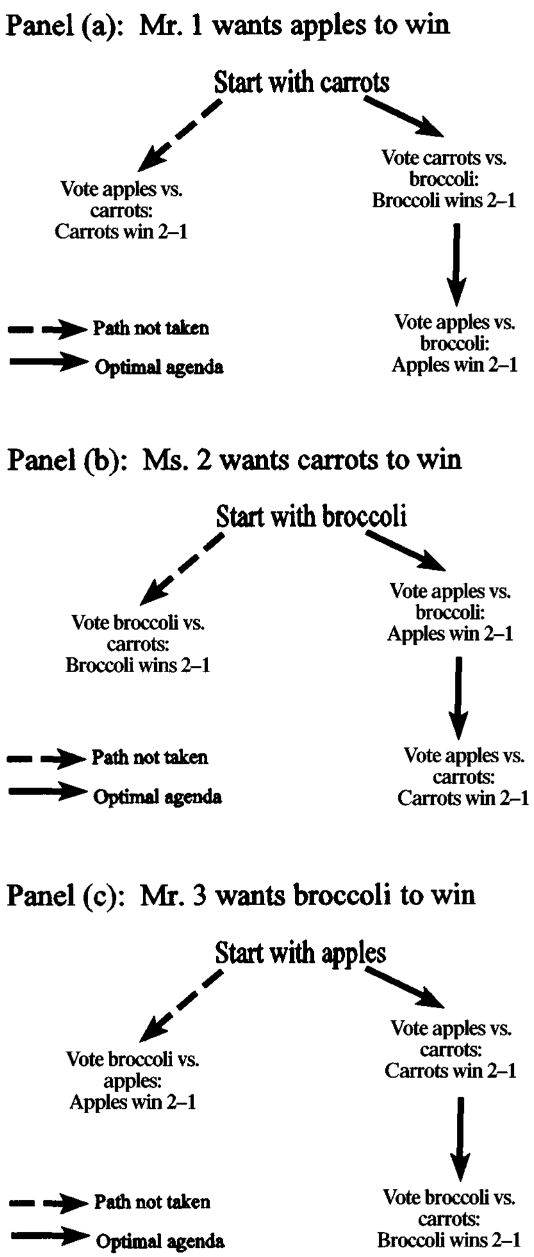

Figure 8.2. My agenda means my outcome.

(see, e.g., Niskanen, 1971; Romer and Rosenthal, 1978, 1979; Rosenthal, 1990). On the other hand, there have been claims that elected officials would not allow the sorts of rules that really give bureaucrats proposal power (Weingast and Moran, 1983; McCubbins and Schwartz, 1984; Shepsle and Weingast, 1987). More fundamentally, if "institutions" give real power to an agenda setter, there may be a contest over choice of institutions themselves (Riker, 1980); cycles over outcomes should be transformed into cycles over institutions. Still, if there is a monopoly agenda setter, that person can act as something close to a dictator if other participants vote their true preferences.

But wait just another minute; why assume people vote their sincere preferences in the face of agenda manipulation? What if the other participants know the power of the agenda setter? What if they recognize that voting over alternatives in the first stage is really voting over future agendas? Can't the *voters manipulate their votes,* in the same way the *setter can manipulate the alternatives*? The answer is obviously yes.

For that matter, why preclude strategic or sophisticated voting in any circumstance, not just when the agenda is set by someone else? Until we allow that voting might not be sincere, we *still* won't have captured the reality of political choice in committees. One of the first scholars to treat the difference between "sincere" voting (i.e., vote your true preferences) and "sophisticated" voting (vote strategically, in effect choosing over agendas rather than outcomes) was Farquharson (1969). There is considerable theoretical work (e.g., McKelvey and Niemi, 1978; Denzau and Mackay, 1981; Enelow, 1981) and empirical evidence (Enelow and Koehler, 1980; Denzau, Riker, and Shepsle, 1985) that claim sophisticated voting may be part of real-world political processes.

How would sophisticated voting work in our simple example of collective food purchase? We will suppose that Mr. 1 is the setter, for simplicity, since the exposition is the same for each participant. Mr. 1, of course, wants an outcome of apples, so he has the process begin with broccoli against carrots. Since he knows broccoli beats carrots, but loses to apples, this ensures the "right" outcome if the other participants cooperate by voting sincerely.

But suppose Mr. 3 has read Farquharson (1969) and recognizes that a first stage vote for broccoli over carrots is not the end of the process. Mr. 3 mentally looks ahead on the agenda tree in Figure 8.2 and notes that a first-stage vote for *broccoli* is really just a vote for *apples*! Mr. 3

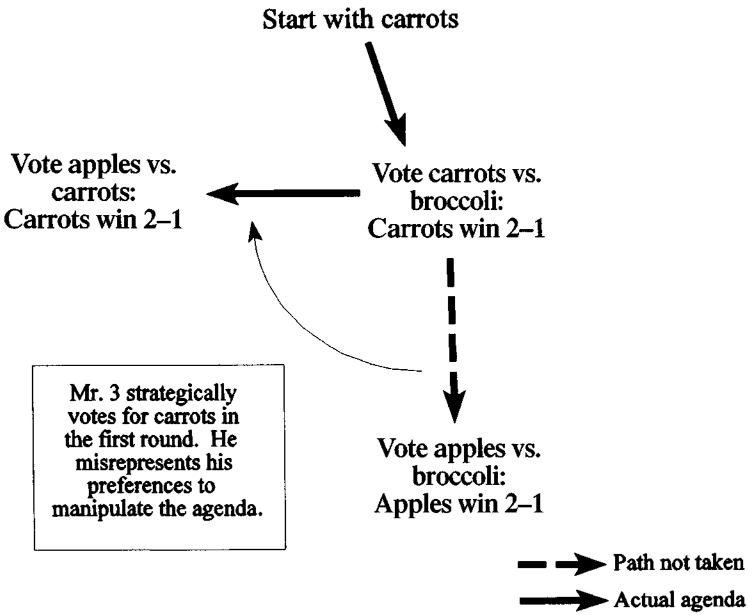

Figure 8.3. Mr. 3 votes strategically, and Mr. 1's agenda leads to carrots, Mr. 1's least-preferred alternative.

detests apples. He decides to vote strategically for carrots. Ms. 2 is happy to vote for carrots (her sincere preference is for carrots over broccoli, anyway). Consequently, carrots win the first round. Since Mr. 1's agenda specifies that the first-round winner is then voted against apples, the result is that carrots win again, this time on (sincere) votes by both Ms. 2 and Mr. 3. The agenda and new outcome are shown in Figure 8.3.

The point of this example is that the power of the agenda setter to dictate the sequence of votes may not be enough to ensure the setter his or her most-preferred outcome. If there are specific rules that force the elected body to accept the take-it-or-leave-it proposal, as the Romer and Rosenthal (1978, 1979) work assumes, then the setter has broad power. If the other participants aren't clever enough to recognize the need to choose sophisticated votes, then again the setter has his or her own way. But isn't it a little silly to base a theory on the presumption that people systematically make mistakes? Successful politicians understand politics. They recognize chances to vote for a particular sequence of future votes instead of the apparently simple issue at hand.[4]

The example in this section has dealt with simple majority rule. The logic of the example can be generalized to any voting procedure in an important theorem called the "Gibbard–Satterthwaite theorem." First, however, we will give a definition of a technical term, "strategy proofness," adapted from Mueller (1989, p. 395).

Strategy proofness. Suppose voter i is trying to decide how to vote. Let M_i be the "message" i gives the voting procedure when he states his true and sincere preference (so M_i could be a vote, a list of votes, or whatever is required by the voting procedure to register a preference). Let $\mathcal{M}_i$ be any distortion or strategic misrepresentation in M_i, so that $M_i \neq \mathcal{M}_i$. Now consider two outcomes of the actual application of the voting procedure, whatever it is:

> *x is the outcome when voter i states M_i, and all other voters state their true preferences.*
> *y is the outcome when voter i states $\mathcal{M}_i$, and all other voters state their true preferences.*

Then a voting procedure is strategy-proof, or immune to strategic voting, if and only if there is no achievable alternative y that voter i prefers to x, regardless of the distorted message $\mathcal{M}_i$ that i chooses. More simply, a voting procedure is strategy-proof if and only if the voter's best strategy is *always* to be sincere, regardless of what other voters do.

There is an important logical connection between voting procedures that are strategy-proof and social choice rules that obey the IIA axiom, which we discussed in Chapter 5.[5] Vickrey (1960), in his proof of Arrow's theorem, offered two conjectures. First, social choice rules satisfying IIA are also strategy-proof. Second, strategy proofness implies satisfaction of IIA.[6] This conjecture was correct, as was shown by Gibbard (1973) and Satterthwaite (1975).[7] A simple summary of the theorem is:

Gibbard–Satterthwaite theorem. *No voting rule that can predictably choose one outcome from many alternatives is strategy-proof, unless it is dictatorial.*

"Predictably" here means that the voting rule is not random, so that the voter can see some correspondence between the message or vote she reveals and the outcome. The Gibbard–Satterthwaite theorem is a classic social choice good news, bad news result. On the one hand, it means that all voting procedures (at least, all those that are not trivial or useless) are manipulable under many circumstances. Further, we cannot trust the particular votes or messages that voters deliver to represent their true preferences. This is bad news, because it means that voters often don't vote honestly, and the reasons have to do with the voting procedure itself, not the character of voters.

The good news is that "manipulation" (i.e., strategic voting) may be the mechanism that gives voters countervailing power over agenda setters. The apparent dictatorial power of those who manipulate the agenda is ameliorated by the ability of voters to manipulate their votes. Consequently, while the *means* of manipulation seems dishonest, strategic voting may actually ensure the "right" *end.* As we saw in our apples–broccoli–carrots example, simple agenda manipulation did not determine the outcome.

Multiple issues and nonseparable preferences: Order matters

The order in which alternatives are considered may determine the outcome, holding participants' preferences fixed. We have called the "order of consideration" the agenda and claimed that under some circumstances agenda control may give one participant disproportionate power. The agenda also matters in a multidimensional setting: The agenda determines the order in which policies or projects are considered. If preferences are not separable, the order of consideration may actually change issue-by-issue preferences, as we saw in Chapters 3 and 4.

What was shown earlier was that voting on each issue separately can solve the problem of multidimensional instability of majority rule. However, separating the issues in the *agenda* works only if *preferences* are also separable. The reason is that issue-by-issue voting reduces multidimensional choice to an unrelated series of unidimensional choices. Unfortunately (for the researcher, at least), separability of preferences among all issues is a very narrow and limiting assumption. What can we say if preferences are nonseparable and if (as in the preceding section) members recognize the implications of agendas?

Our answer needs to account for two obvious problems: First, nonseparability makes the outcome appear indeterminate, because preferences on each issue are conditional on what else has been decided. Second, voters have induced preferences over institutions (in this case, the agenda, or order of consideration of issues). The reason is that different agendas yield multidimensional outcomes given other participants' expected votes. These outcomes are closer to, or farther from, each voter's ideal point. More simply, if the order of consideration determines outcomes, then participants who care about outcomes will also care about the order of consideration.

Consequently, if there is a cycle in voting outcomes, there may be an induced cycle in attempts to choose the institutions that lead to choices. To put it another way, if the choice of agenda leads to a particular outcome, then disagreement over ultimate outcomes simply transfers to a disagreement over agendas, provided all participants are sophisticated.

As an illustration, let us consider again the legislative subcommittee from Chapter 3, faced with a choice of different appropriations for two projects. As the reader will recall, in the earlier discussion we argued that nonseparability forces consideration of conditional preferences, because the amount members expect to be spent on Project 1 affects their ideal preference for spending on Project 2. The reverse is also true: Project 1 preferences depend on Project 2 spending. In the preceding section we saw that the agenda may determine the outcome, but nonseparability makes the agenda even more important, though for a very different reason. The order of consideration can change conditional preferences, leading participants to favor different alternatives.

One theory that seeks to explain the stability of outcomes in legislatures (or, more accurately, in the U.S. Congress) is "structure-induced equilibrium" (SIE). Originally argued in two papers (Shepsle, 1979; Shepsle and Weingast, 1981), SIE theory notes the high potential costs of cycling and conflict in legislatures evokes an institutional response. Rules and practices will evolve to mitigate the problem of instability in a multidimensional policy space. The particular institution that SIE theorists have argued for is the committee system in the U.S. Congress, with a set of (arguably) disjoint jurisdictions over policy areas.[8]

The story SIE theorists use is that multidimensional instability is prevented, or at least ameliorated, by dramatically reducing the space any committee has jurisdiction over. Further, the status quo has a privileged position: It is usually voted last against the "perfected" amendment, or proposed change to policy, in parliamentary procedure. Consequently, the possibilities for cycling are diminished, and the power of committee members is enhanced.[9]

According to SIE theorists, committee proposals have a privileged status, within their recognized jurisdictions, because of "reciprocity," or the norm that floor members defer to committee members.[10] The basis of this agreement is that if I defer to you in your jurisdiction, you will defer to me in mine. Since each of us has a seat on a committee whose issue jurisdiction matters to our constituents, this reciprocity is self-enforcing and mutually advantageous. The proposals that come out of committees will either be that of the median member of the committee (if one believes Shepsle and Weingast, 1981; 1987) or the committee's expectation of the median of the floor (if one believes Krehbiel, 1991).

SIE theory holds a justifiably important place in theories purporting to explain legislative institutions and the importance of institutions in shaping outcomes. There is a largely unspoken premise to the argument, though, that is unrealistic. That premise is that preferences are separable. If we allow that preferences might be nonseparable, the prediction that policy will be the vector of medians of committee members in each respective jurisdiction breaks down.

That is not to say that the committee system has no role in creating or maintaining stability. It is just that the simple prediction that the outcome is the vector of medians has to be adjusted for the complexity of expectations in real legislatures. The role of expectations about the sequence of consideration is complexity was raised (though only implicitly) by Kramer (1972). The explicit criticism offered by Denzau and Mackay (1981) and Enelow and Hinich (1983b, 1983c) is both detailed and telling: One must be very careful not to interpret the SIE approach as a general answer, unless the question of forecasts by voters is also taken into account.

To see how the vector of medians prediction breaks down, recall the example of five members in an appropriations committee, members A, B, C, D, and E, from Chapter 3. Their ideal points (from Chapter 3)

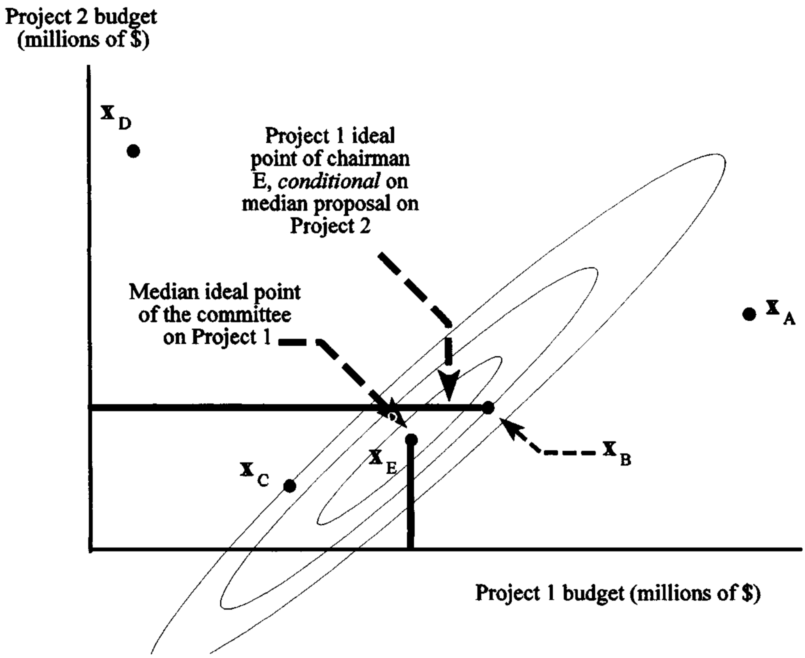

Figure 8.4. Medians of the committees are not the observed proposals, if preferences are nonseparable.

are depicted in Figure 8.4. Let us suppose that there are two subcommittees of this committee, each charged with making a recommendation for one project budget. The subcommittee for Project 2 is chaired by Ms. B (who happens to have the unidimensional median ideal point for Project 2), and the subcommittee for Project 1 is chaired by Mr. E (who likewise has the unidimensional median ideal for Project 1). Finally, let us suppose that all members, *except Mr. E*, have circular indifference curves (not depicted in Figure 8.4), but that Mr. E has sharply nonseparable preferences, as shown.

It is hard to predict the Project 2 subcommittee budget, because it may depend on expectations, logrolls, or other strategic considerations of real legislatures. For simplicity, however, let us stipulate that the Project 2 subcommittee recommends the unidimensional median favored by the subcommittee chair, Ms. B. The prediction of the naive SIE model is that the same approach (i.e., pick the unidimensional median) will then predict the budget for Project 1, *given* the Project 2 budget.

Yet this prediction is clearly wrong if preferences are nonseparable. The Project 1 budget will be larger than the simple unidimensional Project 1 median. Since the Project 2 budget ($80 million) exceeds Mr. E's ideal budget for Project 2 ($60 million), the conditional ideal of Mr. E for Project 1 also increases, because of complementarities in his preferences for the two projects.

The simplest version of SIE focuses on issue-by-issue voting. This focus is apparently made possible by the fact that committee jurisdictions in the U.S. Congress are disjoint, or nearly so. Still, this crucial simplification does not work unless preferences of committee members are also nonseparable.[11] More simply, what Kadane (1972) called "division of the question" (i.e., separating issues in the *agenda*) is one assumption about the process. Separability of *preferences* is another thing entirely. We could find that agendas separate issues and/or that preferences are separable across issues.

If preferences are nonseparable, then the median committee ideal point on any issue is conditional on either (1) other issues that have already been decided or (2) the *expected* decisions on other issues (Denzau and Mackay, 1981; Enelow and Hinich, 1983b, 1983c). As an example, imagine there is a change in the membership of one committee (call it Committee One) after an election, resulting perhaps from some new conservative members replacing liberal senior members. The new median position on the committee is implemented by the floor, if the norm of reciprocity (Weingast, 1979) operates. But this new policy in one dimension changes the conditional ideal points of all the other committees where members' preferences are nonseparable with respect to the newly changed policy. Consequently, if preferences are nonseparable, there is the potential that *all* the nation's policies change because of the electoral defeat of a *few members* on one committee.[12]

What happens if the preferences of members on Committee One are nonseparable? When all the other policies are adjusted, the location of the conditional median ideal point within One's policy jurisdiction is changed, too. The sequence of events can be summarized thus: (1) There is a change in the membership of Committee One, changing the median preference on the issue within its jurisdiction. (2) Other committees adjust policies in response. (3) Committee One readjusts, in response to the changes in other policies, or even changes in policies that members of One anticipate. There may be an equilibrium to this process of mutual readjustment, but there need not be: *The adjustment process can go on indefinitely.* (See Exercise 8.3.) Of course, the adjustments might be very large or so small as to be indiscernible. In any case, the smoothly functioning, "issue-by-issue" logic of the SIE model is greatly complicated when preferences are not separable.

Probabilistic voting

In Chapter 7, we claimed that the most promising form of statements about turnout may be "probabilistic," taking the form of an increased or decreased probability of voting. In this section, we will introduce what has been called "probabilistic voting" (Hinich, Ledyard, Ordeshook, 1972; Hinich, 1977; Coughlin, 1992), a theory that can generate the predictions of the preceding section.

Probabilistic voting has been widely misunderstood, partly because of the computational complexity of moving from deterministic decision rules to decisions described by a probability distribution. However, we have already seen (in Chapter 6) at least one example of a "rational" decision rule very much like probabilistic voting in the mixed-strategy equilibrium suggested by Ledyard (1984). Ledyard's voters randomized over the two strategies (vote or abstain) with only a small probability of voting in any given election. Nonetheless, as Ledyard showed, this strategy generated an equilibrium where turnout is large enough that the probability that one more voter will affect the outcome is negligible.

Probabilistic voting is, in general, simply a way of building in uncertainty about the behavior of the population of citizens. That doesn't mean individual voters use probabilities to decide how to vote, of course. On the morning of the election, the citizen may know for certain if he or she intends to vote, and for whom that vote will be cast. But there may be an automobile accident, snowstorm, or sprained ankle that prevents travel to the polls. The voter may even have a last minute change of heart on whom to support. The problem is that the researcher cannot hope to know the idiosyncratic decision rule used by each voter. In statistical terms, the key features of the voter's decision are "unobservable."

It is worth elaborating the sources of uncertainty about voter information, and choice rules, from the perspective of the *researcher.* A complete statement of the logic of probabilistic voting is given by Coughlin (1992). Paraphrasing Downs (1957, p. 212), Hinich and Munger (1994, p. 167) give a simplified summary of explanations of differences in the information that voters use, and the consequent differences in the way voters choose. The researcher's ability to model information use by voters accurately is limited, because:

- No externally generalizable rule guides voters' information search. Instead, each voter uses his or her unique, idiosyncratic experiences and values to guide the voting decision.
- Even if rules were general, voters would inevitably also have different samples from the complete set of information available. Since rules do differ, voters' opinions about candidates may differ strikingly.
- If information and perceptions of credibility were identical, opinions would be identical. Since information and beliefs differ, ex ante opinions about candidate behavior once in office may differ widely.
- As researchers, we can divide the set of determinants for a given individual's vote into those that *researchers* can identify and measure in some consistent fashion, and those that are *specific to each voter* and are unobservable to the researcher.

Probabilistic voting focuses on general tendencies of information use in the electorate, but allows that there may be idiosyncratic characteristics. For example, if $|x_i - x_A| < |x_i - x_B|$, we can say that voter i is *more likely* to vote for candidate A than for candidate B. More accurately, of the members of a *population* of voters who share ideal point x_i most will vote for A. Some voters will have different information or beliefs, but overall closeness is determinative.

Probabilistic voting builds in the researcher's uncertainty about atomistic voter decisions from the outset. The approach is a means of aggregating propensities to make choices based on observable characteristics of voters, candidates, and the features of the election. Thus, it is possible to compare the likelihood of certain outcomes with the deterministic predictions of the classical model. An illustrative comparison is given in Table 8.2.

Research on vote choice, including much of the work specifically based on the spatial model (e.g., Enelow and Hinich, 1982a, 1982b, 1984a, 1984b, 1989, 1994; Grofman, 1985; Enelow, Endersby, and Munger, 1993; Hinich and Munger, 1994), suggests that more than just spatial position "matters." Other important factors include the charac-

Table 8.2. *Comparison of the predictions of the classical and probabilistic voting models*

Feature of election	Prediction of classical spatial model	Prediction of probabilistic voting model
$\|x_A - x_i\| < \|x_B - x_i\|$	*i* votes for A	Voter *i* is *more likely* to vote for A. Probability rises with difference in closeness. If difference is small, *i* may vote for B, though A is "closer."
$\|x_A - x_i\| > \delta$ $\|x_B - x_i\| > \delta$	*i* alienated, abstains	*i* *more likely* to abstain; probability of abstention increases with distance to closest candidate position.
$\|x_A - x_i\| \approx \|x_B - x_i\|$	*i* indifferent, abstains	*i* *more likely* to abstain; probability of abstention increases as a function of the difference between candidate positions.

Note: A is the first candidate, whose position is x_A; B is the second candidate, whose position is x_B; *i* is the voter, whose ideal point is x_i.

ter of the candidate, perception of competence and probity, and loyalty to party or influence by campaign advertising. Probabilistic voting takes account of the multivariate aspects of political choice, but allows the observable factors in the spatial model to have predictable impacts. Monocausal explanations of human behavior are usually incomplete. Probabilistic voting assumes that the observer knows only *some* of the reasons why voters vote as they do.

It is useful to compare the predictions of the classical model and the probabilistic model graphically, in terms of the probability of voting for a particular candidate. The classical model is a "step function": If (i) candidate A is closer to the voter's ideal, in WED, and (ii) the voter is not alienated, then the voter votes for A with a probability equal to one. If condition (i) is not met, the voter votes for B or is indifferent. If condition (ii) is not met, the voter abstains, again with probability one.

For the sake of example, let us assume that the voter is not alienated ($|x_A - x_i| < \delta$, and $|x_B - x_i| < \delta$). Thus, we can analyze vote choice as follows: The voter will either vote for A, abstain out of indifference, or vote for B, depending on the value of the net candidate differential. Let us allow that the area of indifference may be "thick," so that the voter may abstain out of indifference if the candidates' positions are *almost* identical in terms of WED from x_i. (This situation was depicted in Figure 7.2.) Then we can illustrate the decision calculus of voters in panel (a) of Figure 8.5. As the figure shows, the voter votes for A, votes for B, or abstains, based on distance alone. No other factors, such as judgments about character, competence, or party are allowed to matter.

In panel (b), the predictions of the model are profoundly different. Voters with ideal points far from one of the candidate positions choose the other candidate with certainty, as before. (In principle, of course, the threshold where $0 < \text{Prob}(B) < 1$ might extend far enough to include all voters.) However, inside this threshold difference, any voter may pick any one of the three alternatives (vote for A, vote for B, abstain) with some positive probability. We claim the probability of choosing a candidate gets larger as the difference in perceived difference is bigger, but the voter may still choose one of the other two actions. In particular, a voter closer to A may vote for B, because factors other than simple distance matter.

The probabilistic voting approach is also useful because it affords a more accurate way of adding up expected votes. In providing advice to politicians, campaign consultants don't focus much on individual voters; they focus on groups. Some groups are likely to suppport one candidate, other groups are likely to support someone else. That doesn't mean every member of a group behaves in the same way, but overall different proportions of the groups are likely to support or oppose a given candidate. Further, support from a group may be "hard" (that group's members can be counted on to turn out, and not to change their minds) or "soft" (a high proportion of the group's members *say* they will vote for the candidate, but they may abstain or vote for someone else on a whim). Under the probabilistic approach, votes are counted as expected values, so that "soft" support counts less, just as it does in real politics. The following example makes this clearer.

Suppose there are five voters (1, 2, . . . , 5) and two candidates (A and B). Imagine that 1 and 2 are closer to A, and 3, 4, and 5 are closer to B. The classical model would predict that B wins the election, 3 to 2, and that would be the end of it. The probabilistic voting model, however, allows that voters may differ in the intensity of their support for candidates, based on other factors. Suppose that the breakdown of possible actions, by voter, is like that in Table 8.3. Since all we can predict is *expected* votes, it is clear that this race may be "too close to call," in

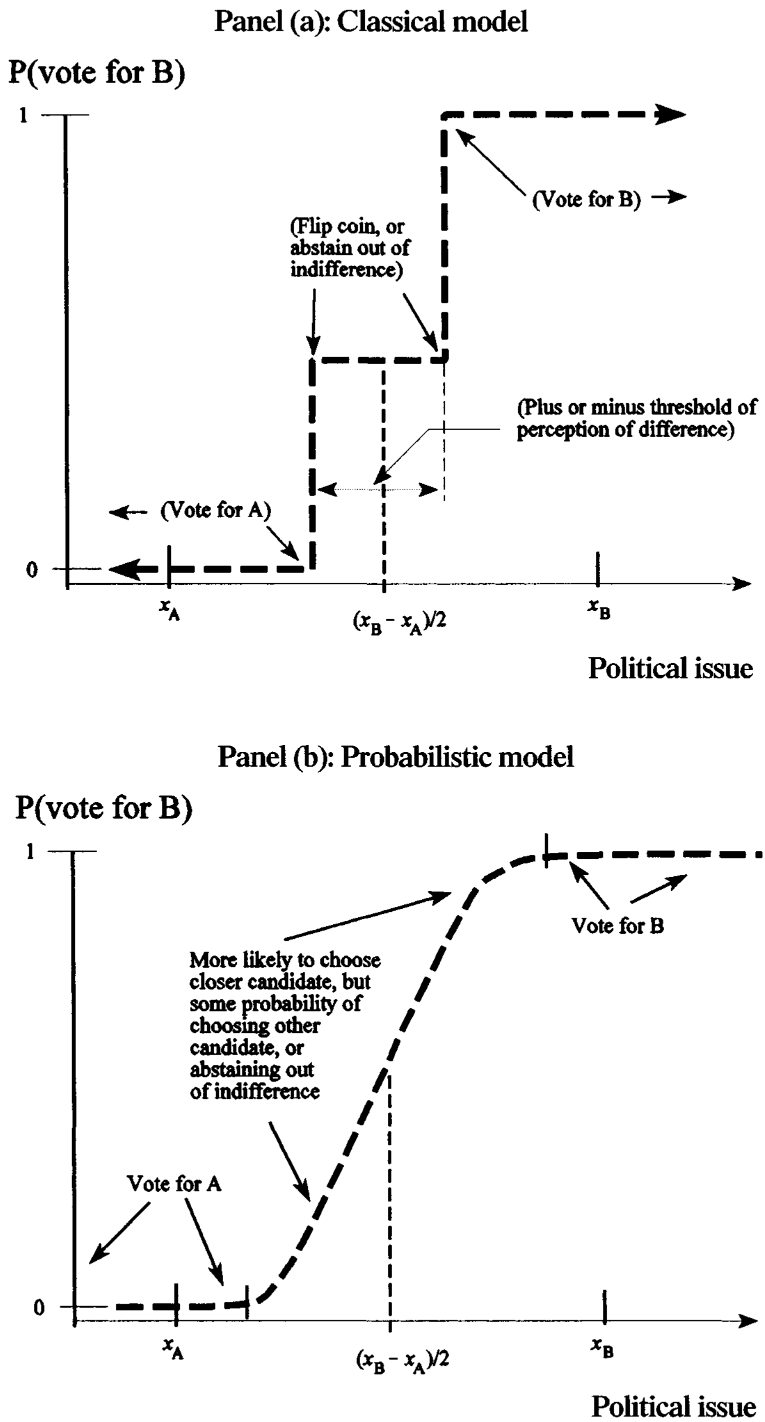

Figure 8.5. Comparing the classical and probabilistic models of vote choice.

Table 8.3. *Predicting outcomes as sums of expected values*

Voter	Probability of vote for A	Probability of vote for B	Probability of abstention (indifference)
1	1.00	0.00	0.00
2	0.80	0.05	0.15
3	0.10	0.45	0.45
4	0.05	0.60	0.35
5	0.00	0.95	0.05
Expected vote totals	1.95	2.05	1.00

the parlance of the electioneers. Notice that the "supporters" of B, whose vote was unquestioned in the classical model, are actually more equivocal than A's likely supporters. Candidate B's support is soft: There is a 1 in 20 chance that even voter 4 will cast his vote for A!

Real politicians must consider not just what people say they will do, but the likelihood they will go to the polls and do it. In our "early poll," based on the classical model, the prediction would have been 60% support for B, 40% for A (people rarely admit to pollsters they aren't going to vote). Assuming the probabilities in the table are independent, we can calculate some examples of the likelihood of other possible (though unlikely) outcomes:

A gets four votes, B gets none, one abstention: 0.0002
A gets one vote, B gets one, 4 abstentions: 0.0012
A gets three votes, B gets one, one abstention: 0.029
A and B each get one vote, three abstentions: 0.026

We don't have space to consider all the possible outcomes (there are fifty-four different outcomes with nonzero probabilities). The point is simple: Political contests can have surprising results, not wholly explained by the positions of the candidates and the ideal points of the voters.

The classical mode makes a useful simplification for the sake of pedagogy, but real politics is highly uncertain. Outcomes are the aggregation of many hard to predict choices by voters with myriad, possibly highly idiosyncratic decision rules. Fortunately, these idiosyncracies and many of the uncertainties that pervade real political choice can be incorporated into the model using probabilistic voting theory.

Conclusions

In this chapter we have examined some weaknesses of the classical model of analytical politics. The three topics we covered were (1) the implausibility of naive, or sincere, voting, (2) the implausibility of purely separable preferences in a multidimensional policy space, and (3) the implausibility of shared deterministic voting rules for all participants.

As we have shown, for each of these implausibilities there is an extension that makes the analytical model more realistic and general. These extensions are accomplished, however, only with a significant increase in the complexity of the model. We can summarize the differences in the results of the model as follows:

- Accounting for strategic voting allows committee members to have a sophisticated understanding of both political choice and the power of the agenda, but we are no longer able to make clear predictions about the outcome of majority rule processes with an agenda setter.
- Accounting for nonseparability makes the representation of preferences much more realistic, but makes the study of institutions whose apparent function is to "separate" issues much more complicated. We can no longer analyze one choice in isolation.
- Accounting for the diversity of voter decision rules and information in mass elections affords the researcher much greater flexibility in predicting vote choice. Probabilistic voting, however, is a self-consciously incomplete model of voter choice, because it allows that choices might be (a) idiosyncratic (with some shared elements), and (b) based on factors that have little to do with the classical model. These factors might include the perceived character of the candidate or other "nonspatial" attributes of the choice.

This is not meant to belittle the value of the extensions covered in this chapter. In each case, analytical political theorists have made good use of the perspectives we have covered here, and the value of the increased realism these extensions allow is obvious. It is important for the reader to recognize that the stylized model of earlier chapters is the one often criticized by opponents of rational choice. In fact, it is common to see current research question the assumptions made by analytical political theorists, and then quote some text from Downs (1957). Such criticisms may be correct, but they are anachronistic. The criticism of the original Downsian model of mass elections is blunted by understanding the extensions that this chapter has laid out.

EXERCISES

8.1 Assume that preferences for three foods (antipasto – A, brussels sprouts – B, and cauliflower – C) are given in the following table. Suppose all members vote sincerely and that Person 1 is the (politically knowledgeable) agenda setter. By prior agreement the agenda will consist of only two votes: One alternative against another, with the winner against the remaining alternative. The winner of the second contest becomes the group's choice. Which of the three alternatives is the group's choice? How can you be sure?

	Person		
Ranking	1	2	3
Best	A	B	C
Middle	C	A	B
Worst	B	C	A

8.2 Let all the facts of Exercise 8.1 remain the same, and use the same agenda as Person 1 used when all voting was sincere. However, allow all members to vote strategically (i.e., choosing over final outcomes rather than just the two alternatives being voted on). What is the outcome? Draw the sequence of votes in an "agenda tree," like those in Figure 8.2.

8.3* (Adapted from Enelow and Hinich, 1984b, Section 8.8). Suppose there are three voters, A, B, and C, with the following ideal points on two projects (i.e., there is a two-dimensional policy space).

$$\mathbf{x}_A = \begin{bmatrix} 25 \\ 55 \end{bmatrix} \quad \mathbf{x}_B = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \quad \mathbf{x}_C = \begin{bmatrix} 120 \\ 40 \end{bmatrix}$$

Note: Exercises marked * are advanced material.

Let the corresponding matrix of salience and interaction weights be:

$$\text{(Person A)}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\quad \text{(Person B)}\begin{bmatrix}1 & -9\\-9 & 1\end{bmatrix}\quad \text{(Person C)}\begin{bmatrix}7 & 0\\0 & 1\end{bmatrix}$$

(a) Suppose that (for some reason) all three voters expect that the Project 1 budget will be the median on that dimension: $x_{B1} = 100$. Given this expectation, and assuming free proposal power, what will be the Project 2 budget if Project 2 is considered alone?

(b) Taking the new Project 2 budget you found in part (a), determine the Project 1 budget if the committee now votes, assuming free proposal power and a fixed Project 2 budget.

(c) Now start over: Taking the Project 1 budget from part (b) and assuming that all members believe that is the final budget, determine the Project 2 budget the committee will choose, assuming free proposal power. Why is it different from the budget you found in part (a)? Is there an equilibrium to this process, or will the committee cycle back and forth forever?