10

All Politics Is National? How "Embeddedness" in a National Assembly System Shapes Votes and Seats in a District

In the 2015 Canadian election, the Green Party received 3.5 percent of the vote nationwide. Yet it won only one seat. The result is perhaps not a surprise. After all, the country uses the FPTP electoral system, and normally we do not expect small parties to gain many seats in such systems. In fact, what is perhaps most surprising is that the party ran some semblance of a national campaign despite being plausibly competitive in only a very few districts.[1] Why would a party run candidates and campaign where it can't win? One plausible reason, we suggest, is that it needs to show viability as a national party. It is not enough to emphasize local focus, or for a party to concentrate on a few places where it can win, because the local district is part of the national assembly election and most voters will be thinking of national politics.[2] That is the theme of this chapter.

The puzzle of national campaigning despite viability in only some districts is not only a feature of FPTP politics. In fact, most PR systems also use districts (although, by definition, districts in such systems will be multiseat[3]). In these systems, too, we may see parties entering in districts all around the country despite having a realistic chance at winning a seat in only a few places.

Consider the case of another Green Party, this time in Czechia (Czech Republic). In the 2006 election, this party won just 6.3 percent of the vote nationwide. The Czech electoral system is a simple PR system, with fourteen districts of highly varying magnitude, ranging from eight to twenty-five. The Green Party obtained its six seats (out of 200) in just five of these

[1] For instance, it ran a candidate in every district, and according to the CBC's *Tracking the Leaders* interactive map, the Green Party leader visited constituencies in several provinces across the country, in few of which it came anywhere near victory in either 2015 or the prior (2011) election (www.cbc.ca/news/multimedia/map-tracking-the-leaders-1.3081740).

[2] We do not claim that all parties act this way; there are, of course, strictly regional parties like the Bloc Quebecois in Canada or the Scottish National Party in the UK. We are referring to parties that claim to speak to national issues.

[3] A few otherwise PR systems have one or two districts that elect just one member (examples include the Åland Islands district in Finland, Madre de Dios in Peru, and the districts of Ceuta and Melilla in Spain).

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districts. Nonetheless, there were only two districts in which the party failed to collect at least 5 percent of the vote. One of these (Moravia) was a district in which it won a seat, thanks to the very high magnitude, M=23. Yet the party won 6.7 percent of the vote in the lowest-magnitude district (Krlovy, M=4) despite having no realistic chance of a seat, and 6 percent or more in districts where magnitudes of ten to thirteen made a seat unlikely. This party was recognized from opinion polls as being assured of having sufficient nationwide support to enter parliament. Yet clearly many voters were willing to vote for this national party even in districts where a vote for it would be objectively "wasted," in that it would be unlikely to help the party win *in the voter's own district*.

The examples just given were of Green parties, a type of party found in many countries and advocating socially liberal and proenvironmental policies. They make good examples for the relationship between electoral systems and voting behavior because of their similarity in approach to politics across a wide range of countries (Spoon 2011, Belden n.d.). Perhaps one might object, however, that their voters are just idealistic and not motivated by winning seats. We doubt this – the Canadian Greens campaign around how they can make a difference in parliament, and the Czech party even entered into a coalition government following the 2006 election – but we can offer one more example from a very different type of party.

In Albania, the Republican Party, a conservative party emulating its American namesake, was a fairly minor party in the 2013 election. It won just 3.1 percent of the vote and three of the 140 seats. Albania also uses a simple PR system (first adopted in 2009, after a period of "mixed-member" rules). As in Czechia, district magnitude is highly variable: the twelve districts elect from four to thirty-two seats. The Republicans won one seat in each of three districts (with magnitudes of thirteen, fourteen, and thirty-two). Yet they obtained vote shares greater than their nationwide share in four other districts in which their prospects for seats would have been very bleak, including Kukis (M=4) and Gjirokastir (M=5).

In each of these cases, we see parties running and winning substantial vote shares even where they could not have expected to win. Each of these has seats elsewhere in the country, and therefore a reasonably secure hold on parliamentary representation. The conclusion we can draw from these examples, and many others we could have mentioned, is that voters are often motivated by national politics even when they vote in electoral systems where votes are used exclusively for allocating seats within self-contained electoral districts. The implication is that *all politics is national*, at least in the sense of electoral actors – party leaders, candidates, and voters – cueing on nationally viable parties even when they surely will not win locally. This chapter is about how the national affects the local. We analyze outcomes at the district level, but with a systematic focus on how the national electoral system shapes the local competition for seats and votes.

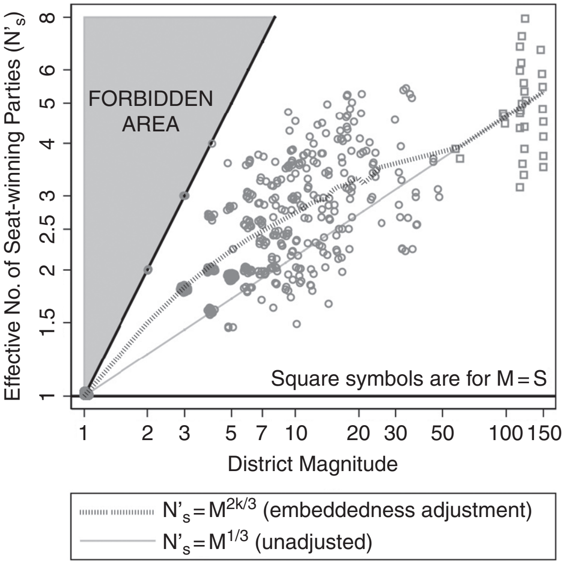

FIGURE 10.1 How the magnitude of a district shapes the effective number of seat-winning parties

Before turning to the logic, let us first see why the district level can be understood only with reference to the national. The following two graphs serve to illustrate the broader point. Figure 10.1 shows how the effective number of seat-winning parties in a district is related to magnitude. This graph is comparable to Figure 7.2, which showed the nationwide effective number of seat-winning parties (N_S) graphed against the Seat Product, *MS*. Indeed, inside a single district, the reasoning that led to $N_S=(MS)^{1/6}$ would lead to $N'_S=M^{1/3}$. But in Figure 10.1 this equation, plotted via the light solid line, fits only for nationwide single districts, where $M=S$ (plotted with square symbols). When districts are embedded in a larger country, they tend to a higher effective number of parties than the simple model would predict. Why is that? Because nationwide politics enters the game.

One might expect the district-level model to be simpler than the nationwide. But this is not so when the district is *embedded* in a nationwide context which affects the district. Indeed, the model here shows, via the thick dotted curve, a twisted path produced by the equation, $\boldsymbol{N'_S=M^{2k/3}}$, where the parameter k depends on the relative weight of this district within the country's entire assembly. There will be many graphs in this chapter that have a thick dotted curve like this one. While the derivation of this specific curve will be explained later, we must emphasize at the outset that curves of this sort that we plot in this chapter (and in Chapter 14) are not post hoc best fits. Rather, they represent a logical model, plotting the y-axis variable at a given value of the x-axis

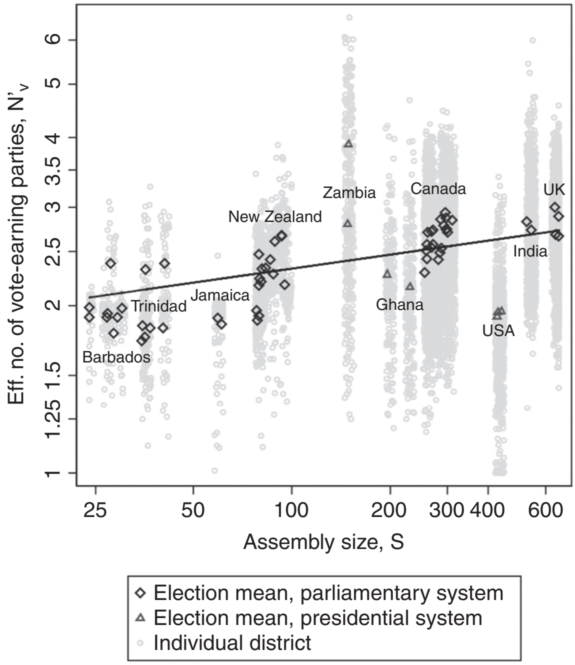

FIGURE 10.2 How the assembly size shapes the effective number of vote-winning parties in single-seat districts

variable and this new parameter, *k*, that will be central to the mission of this chapter and depends both on *M* and *S*.

The logical model curve shifts towards being identical to the straight line near the right side of the graph. Where the curve and line converge marks the shift from relatively small districts where $M<<S$ to nationwide single district where $M=S$. The more complete logically derived model (which accounts for nationwide impact) can be seen to fit the average trend in both regions. Thus, Figure 10.1 reveals that *we need a somewhat complex model to make sense of the district level* effective number of seat-winning parties – even in a "simple" electoral system in which all seats are allocated in districts (i.e., without a second tier – a yet more complex matter that we return to later).

Our second preliminary graph is Figure 10.2. It shows something perhaps very surprising: the effective number of *vote*-earning parties at the district level in FPTP systems as a function of assembly size. It is generally held that the number of parties in a single-seat district tends to be around two; that is the claim of "Duverger's law." Let us focus initially on the diamond symbols in the graph, which show the mean value for each election in a parliamentary system. We can see that this mean tends to be around two, *as long as the assembly is small*. However, as assembly size increases, mean N'_V rises such that it tends to be 2.5 or greater in the largest countries.

The line in Figure 10.2 will be explained later. For now suffice to say that it again derives from the parameter k in order to reflect the impact of other districts on the votes constellation of any given district. We will explain the relationship in more detail later, after the logic of how this connection of the district and national features works. The broad point is that, even in – no, especially in – FPTP systems, the average degree of fragmentation of the vote is predicted not by the district magnitude so much as by the assembly size, which is obviously a nationwide factor. The average trend represented by the diamonds in Figure 10.2 could be possible only if many voters did not vote based on the district outcome – as standard coordination-based arguments (e.g., Cox 1997) would have it – but on national politics.[4] That is why this chapter is called "All politics is national?"; how districts are embedded in a national system predicts much of the variance in district-level N'_V.

Figure 10.2 shows each district in our FPTP elections with a light gray circle. The more a given area of the data plot looks like a gray smear instead of a series of discernible circles, the more clustered the observations are in that region. What is clear is that individual districts in any given election can vary wildly around the mean. No broad model could possibly claim to predict what a specific district's effective number of vote-earning parties will be. Nonetheless, there is a systematic relationship of district-level mean N'_V to assembly size, a point never before noted in the vast literature on whether "Duverger's law" accurately captures what goes on at the district level. Notably, the trend in India's two depicted elections (2004 and 2009) also follows the same pattern, despite the fact that India is generally considered as somehow "exceptional" to the general rule of how FPTP behaves. Clearly it is not, at the district level, once we take into account its large assembly size. In fact, if India is exceptional, then so is the UK, a country most political scientists probably think of as a fairly typical representative of its electoral system.

Far more deviant than the UK or India is the United States. The remaining symbols in Figure 10.2 are the triangles that mark the election means for presidential democracies. There are only three presidential countries included. Of all countries in the graph, the US is the most significant outlier. Its mean N'_V, lower than most elections in even the small country of Barbados, is far lower than we should expect for its large assembly. On the other hand, Zambia has one election with an unexpectedly high mean value, while Ghana is closer to fitting the parliamentary pattern. As we have seen before, presidential systems tend to be more variable, but with the exception of the US (and Zambia 2001),

4 For such an argument on the Canadian case, see Johnston (2017).

those that use FPTP are not markedly different from the parliamentary cases. We will now turn to the logic of the national impact on the district level.

THE CONCEPT OF EMBEDDEDNESS

Scholars have claimed over many years that key processes leading to outcomes expected from Duverger's propositions actually occur at the district level. In recent years, there have been significant advances in analyzing district-level electoral data (Moser and Scheiner 2012; Singer 2013). The previous chapters have focused on *national* level measures of effective numbers of parties (seats and votes). Here we aim for a deeper understanding of the factors that impact the adaptation of party systems to the institutional context by focusing on the *district* level. The ultimate aim of this chapter is to tie the national and district levels together, to arrive at an understanding of how the districts in which votes are cast and seats allocated is embedded in the larger electoral system that elects the national legislature.

Everything we have done in the two previous chapters on the national level could be repeated in each district, as if it were a separate state with a nationwide single district. This approach *almost* works, but not quite: the number of parties consistently tends to be somewhat larger than expected, as we saw in Figure 10.1. This is so because nationwide politics interferes with the district-level politics.

The main conclusion of this chapter is that, indeed, at least in reasonably well-established parliamentary democracies with simple electoral systems, it may be true that the districts in which seats actually are allocated have their own local issues and personalities, yet *all politics is national*. By that we mean that a logical model of district-level vote fragmentation can be developed, and confirmed with regression analysis, based on the interplay between national electoral systems (the Seat Product, or its components, assembly size and average district magnitude) and the magnitude of the individual district. In other words, it is not quite accurate to say that the district level is decisive in Duvergerian dynamics. We can model the average trend in a district's effective number of parties, both seats and votes, by knowing its magnitude and the size of the national assembly in which it is embedded.

A key point is that two districts of a given magnitude will display different party-system competitive patterns according to the national systems of which they are a part. This is the sense in which politics can be national even when seat-allocation occurs entirely within local or regional districts. However, this does not mean that there is no room for local politics to enter as well; rather, it means that attempts to model how local electoral systems "project" to the national may be misguided. Rather, we find that we can project in the opposite direction: first model the national system, as we did in earlier chapters, and then model how it projects onto the districts.

We will undertake our analysis of district-level dynamics, and how the national electoral system shapes them, by reference to a dataset consisting of over 11,500 district-level results in 102 elections in sixteen countries. Our analysis will mainly focus on parliamentary (or parliamentary-leaning) executive formats. However, a key point is that – as we already saw at the national level – the district-level dynamics are not fundamentally different in presidential democracies. This is surprising, given how much presidencies are thought to shape the overall political process, but we are able to show that the same basic electoral-system factors shape parliamentary and presidential systems. The latter are, however, more variable, a point we shall return to at several points in this book – especially Chapters 11 and 12.

We use the Belden and Shugart (n.d.) dataset to test and refine several models of how institutions shape party-system fragmentation. Some of the models date back to the late 1980s or early 1990s (e.g., Taagepera and Shugart 1989a, 1993), others are of more recent vintage (Taagepera 2007), and several are original to our current joint work.

Taagepera and Shugart (1993) suggested that the actual number of parties that win at least one seat in each district (N'_{S0}, with the prime mark denoting the district-level measure), should be approximately *the square root of the district's magnitude*.[5] We already tested and confirmed this logical model graphically in Chapter 1 (Figure 1.2) and via regression in Chapter 7, because it is a key building block of the Seat Product Model for predicting nationwide effective number of seat-winning parties (N_S). The relationship of the actual number of seat-winning parties at the district level is our fundamental building block here as well. First, we must consider analytically the notion with which we opened this chapter: that appeals to national viability may affect vote-earning performance at the district level. This is important to consider because if it does so systematically, then we need to take it into account when attempting to predict the district-level effective number of vote-earning parties.

A Logical Model of a District's Embeddedness

In our earlier work (Taagepera and Shugart 1993) we argued that the simple model identified already in Chapter 1 as Equation 1.1,

$$N'_{S0} = M^{0.5},$$

would likely understate the relationship, because it tacitly presumes an isolated district not subject to nationwide, i.e., extra-district, pressures. We proposed an adjustment, based on how a given district is "embedded" in the nationwide electoral system. Here we explain the theoretical notion behind this concept of embeddedness.

[5] This is in full agreement with the nationwide $N_{S0}=(MS)^{1/4}$, shown in Chapter 7. Indeed, within the district S is the same as M, so that $(MS)^{1/4}=(M^2)^{1/4}=M^{1/2}$.

In practice we observe an "isolated", or nonembedded, district only in cases where $M=S$. In such cases there is a single nationwide district encompassing the entire assembly electoral system. Examples include Israel, the Netherlands, and San Marino. All other districts are embedded in a national system that consists of some number of other districts.

The notion of embeddedness is that parties that are smaller players in national politics, such as the examples we provided at the start of this chapter, may enter candidates and run campaigns even in low-magnitude districts where they are relatively weak. They do this perhaps to "show the flag" – i.e., to prove that they are in fact *national* parties. Once a party has been set up and is participating in the nationwide debate, and perhaps has won some seats somewhere, the costs to putting up at least minimal effort in other districts are lowered.[6] The theoretical notion of embeddedness is that these efforts and costs to entry are lower in a district of a given magnitude that is one of many districts in a wider assembly electoral system than they would be if the district were isolated.

Imagine a single nationwide district of just five seats; probably few small parties would bother. But now imagine a five-seat district in a 300-seat assembly where the party can win elsewhere, perhaps especially if some other districts have large magnitude. Now the party will tend to allocate some resources away from its strong areas to others, in order to make the point that it is a serious player on the national scene. If this resource-shifting happens, the votes and potentially the seats for other, larger parties, may be suppressed, relative to the "isolated" district of the same magnitude. Some parties other than the largest may win an additional seat in some districts (where $M>1$) beyond what they might have been expected to have won were the district isolated.

In parallel with such flag showing by small parties, nationwide competition by the top-two parties may also spill into districts. Consider two districts with lopsided support, so that seats are likely to divide up as four to one and one to four, respectively; hence $s'_1=0.8$ and $N'_S=1.47$. To appear to be making nationwide effort, it is in both party's interest to shift resources to winning a second seat where they are weak, even at potential cost of sacrificing a seat in their respective strongholds. Then these districts may shift to three to two and two to three, respectively, so that $s'_1=0.6$ and $N'_S=1.92$. Note that in such a case the number of seat-winning parties (N'_{S0}) does not increase. In contrast, small party flag showing might increase N'_{S0}, if it helps some party to win its first seat.

Drawing on this notion of embeddedness, Taagepera and Shugart (1993) argued, a parameter capturing the impact of national politics on the district must reduce to Equation 1.1 when $M/S=1$, but the exponent, 0.5, in that

[6] Our logic has affinities to Cox's (1997: 110–111) "economies of scale" argument for the formation of parties.

equation must increase as the *M/S* ratio becomes smaller. In this way the embeddedness parameter reflects the theoretical point that, the smaller the share of all seats that a given district represents, the more room there is for impact from other districts to affect this district's politics.

We have already seen in Chapter 7 that for the number of parties winning at least one seat, the model without adjustment for embeddedness, i.e., $N'_{S0}=M^{0.5}$, is a good fit. But this lack of necessary adjustment applies only to N'_{S0}. We show in this chapter that adjustment is absolutely needed to make sense of other outcomes of interest, including the *effective* numbers of both seats and votes (as Figures 10.1 and 10.2 at the start of this chapter showed). The details of the derivation of this embeddedness parameter, *k*, are complex. The resulting formula is:

$$k = 0.5 + 0.2076\log(S/M)/M^{0.25} \qquad (10.1)$$

The formula is baffling,[7] and those readers wanting to see our logic and many steps of calculation should refer to the appendix, where we spell it out in detail. Our next steps here are to show how the parameter, *k*, improves our prediction of the size of the largest party. From there we move on to the effective number of seat-winning parties, and then on to votes.

FROM THE NUMBER OF PARTIES OF ANY SIZE TO THE SIZE OF THE LARGEST

In order to provide the logical foundation behind the link shown in Figure 10.1, between N'_S and *M*, we must estimate the seat share of the largest party, s'_1. The *average* fractional seat share of the seat-winning parties must be $1/N'_{S0}$. That is, if there are four parties winning seats, each one averages one quarter of the seats. The fractional share going to the party with the most seats in the given district, s'_1, must be at least this average. The upper limit on s'_1 is 1, when that party wins all the seats.[8] In the absence of other knowledge, the expected average value is again the geometric mean:

$$s'_1 = {N'_{S0}}^{-.5}. \qquad (10.2)$$

That is, the seat share of the largest party in the district should be, on average, the inverse square root of the actual number of seat-winning parties. However, this expression ignores embeddedness. As a result, it would prove to be a poor fit

[7] The formula is different from (and, we believe, an improvement on) that in Taagepera and Shugart (1993), but the intuition behind it is the same. This *k* boils down to 0.5 when *M*=*S*. It reaches its maximum when *M*=1; then *k*=0.5+0.2076log*S*.

[8] More precisely (as mentioned in Chapter 7), it is one, minus one seat for each of $N'_{S0}-1$ remaining parties. This additional complication adds no extra precision to our modeling enterprise, and is thus set aside. For discussion, see Taagepera and Shugart (1993) and Taagepera (2007:135).

to the data.[9] Thus we should run the regression instead with the embeddedness parameter included. When we do so, we are testing whether, instead of $s'_1 = N'_{S0}{}^{-.5}$, we might have $s'_1 = N'_{S0}{}^{-k}$.

In order to test the combined impact of N'_{S0} and the district's embeddedness on s'_1, we run the OLS regression on the logarithms with k included in the input variable:

$$\log s'_1 = \alpha + \beta(k \log N'_{S0}).$$

Because of the laws of logarithms and exponents, and if we find the expected α=0, this is equivalent to:

$$s'_1 = N'_{S0}{}^{k\beta}.$$

If a given district were isolated (i.e., nationwide PR), we already know from the way the national embeddness factor, k, is constructed that k=0.5. Thus if our expectations about the relationship between the number of seat-winning parties and the seat share of the largest are correct, we expect $\beta = -1$ to apply to the full sample (and, of course, we expect α=0), such that for any given district, the equation becomes:

$$s'_1 = N'_{S0}{}^{-k}. \tag{10.3}$$

When we run OLS regression, the result is:

$$\log s'_1 = -0.00016\ \mathbf{-0.990} k \log N'_{S0} [R^2 = .954].$$

The result[10] offers clear validation of Equation 10.3, which states that the simpler proposed model ($s'_1 = N'_{S0}{}^{-.5}$) is valid for an isolated district (where k=0.5), but that in embedded districts the size of the largest party is indeed reduced according to the impact of extra-district politics.

Figure 10.3 is a scatterplot of the seat share of the largest party (y-axis) and the actual number of seat-winning parties in each district. The solid light gray line is the simple model, unadjusted for national politics, i.e., $s'_1 = N'_{S0}{}^{-.5}$ (Equation 10.2). It is obvious that the data trend is below this line. What explains this pattern? Here is where the nationally adjusted Equation 10.3 comes in. This equation is plotted with the thick dotted curve. Like its counterpart in Figure 10.1, this curve does not appear straight because it includes the embeddedness parameter, k. Therefore, it sags lower when the district contains a smaller share of the assembly's total seats. The curve – and

[9] If we run a regression to test Equation 10.2, we get: $\log s'_1 = -0.00061$ **–0.594** $\log N'_{S0}$. The 95 percent confidence interval is –0.622 to –0.566. Thus the logical value of –0.5 for an unadjusted model is not within the confidence interval. This establishes that we need the embeddedness adjustment.

[10] Summaries of regression results are found in the appendix to this chapter. The very high R^2 in this and some other results in this chapter are due to the large concentration of data points from M=1 districts, where N'_{S0} and s'_1 are constrained always to be one.

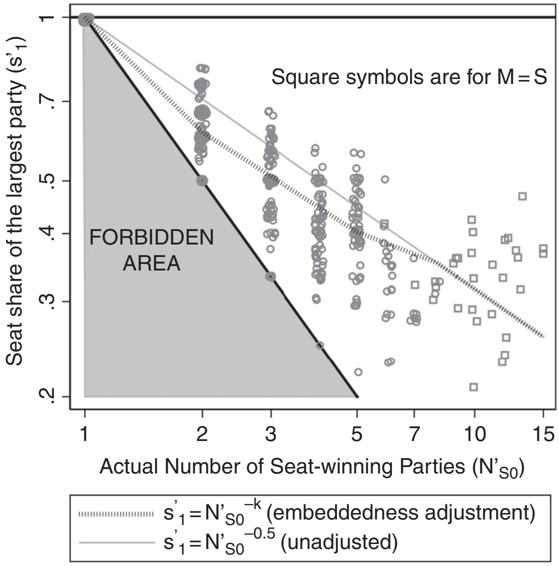

FIGURE 10.3 The number of seat-winning parties and the seat share of the largest party, district level

others like it in this chapter – is not a best fit; rather it represents the logical model.[11]

The observations that cluster well below the straight line are mostly from the lower-magnitude districts in which politics from outside the district has the most potential to impact the distribution of seats within a district. The extra-district political effects exert downward pressure on the size of the largest party. However, as we approach higher values of N'_{S0}, we are encountering more cases of districts that are large and thus represent a larger share of the assembly. By the time we reach $N'_{S0} \geq 7$, we have only cases of M=S, as depicted with the squares. At this point, the curve for Figure 10.3 has become identical to Equation 10.2 because there is no extra-district factor to account for when all seats are allocated in a single nationwide district.

Equation 10.3, and the graphed result in Figure 10.3, is consistent with parties' allocation of resources to districts where they are weak, as we hypothesized. However, the impact of such resource shifting is somewhat different from what we predicted in Taagepera and Shugart (1993). There, we suggested it would mean parties tending to win seats that they would not win if

[11] More specifically, it is a smoothed curve plotted through the result of Equation 10.3 for the mean value of k at any given value of N'_{S0}. The value of k differs for data points at a given value of the x-variable whenever they involve a different S.

the district were isolated, hence M^k parties instead of $M^{0.5}$. What we have instead is $M^{0.5}$ correctly identifying the trend in the number of seat-winning parties (N'_{S0}), but the seat share of the largest tending to be $s'_1 = {N'_{S0}}^{-k}$, instead of $s'_1 = {N'_{S0}}^{-.5}$. In other words, the effect of extra-district politics does not so much increase the chances of some party going from zero seats to one as it does the chances that one or more of the $M^{0.5}$ seat-winning parties, other than the largest, picks up an additional seat (or seats).[12]

The implication of this finding of the extra-district effect on the largest party is that the knowledge that some party other than the locally strongest is a significant player in national politics may increase that party's support. In fact, it is apparently the second party in the district that tends to benefit from the extra-district effect.[13] The upshot is that the gap between the two largest parties tends to be reduced significantly from what it would be if there were no extra-district effect. In other words, *national politics makes district politics more competitive than it might otherwise be* – another way in which all politics is national. For some details, see the chapter appendix.

Linking the Number of Parties, and Party Sizes, to the Effective Number

The next step is to derive the model that was shown in Figure 10.1 at the start of this chapter, linking the district magnitude and the effective number of seat-winning parties, N'_S. To complete these steps, we first need to connect the seat share of the largest party (s'_1) to district magnitude (M), and then we will connect it to N'_S. The first of these links should be to replace N'_{S0} in Equation 10.3 with $N'_{S0} = M^{.5}$, which would give us:

$$s'_1 = (M^{.5})^{-k} = M^{-.5k}. \tag{10.4}$$

When we run regression on the logs to test Equation 10.4, we find:

$$\log s'_1 = -0.00035 \ \mathbf{-0.508}(k\log M) \quad [R^2 = .912].$$

The relationship is shown in Figure 10.4, using the same symbol formatting as previous graphs: the solid light gray line represents the expectation without taking account of embeddedness ($s'_1 = M^{-0.25}$), whereas the thicker dotted curve represents Equation 10.4. Obviously we see a fair degree of scatter, but the pattern is confirmed; in particular, if we failed to consider a district's embeddedness, we would overstate the size of the largest party in most districts.

12 If we graph the embeddedness parameter in the relationship of N'_{S0} and M, we find that it predicts too high. Often the value of N'_{S0} is greater than $M^{0.5}$ (see Figure 1.1), yet the average trend is closer to $N'_{S0}= M^{0.5}$ than it is to $N'_{S0}=M^k$. See chapter appendix for details.

13 We do not attempt to model the size of the second party systematically, as doing so becomes cumbersome.

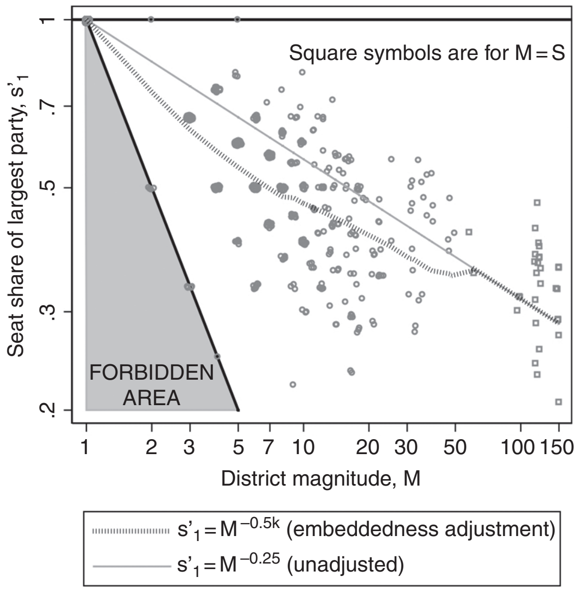

FIGURE 10.4 The seat share of the largest party and district magnitude, district level

Now we can proceed to the final step, from s_1 to N_S, and then to M. Previous analysis[14] has led to the equation, $N_S = s_1'^{-4/3}$, for nationwide values. Then we can proceed to the interlocking models, by trying:

$$N'_S = (M^{-.5k})^{-4/3} = M^{2k/3} \tag{10.5}$$

The regression equation that we obtain is:

$$\log N'_S = 0.00027 + \mathbf{.668}(k \log M) \quad [R^2 = .937].$$

This is the curve depicted with the dotted curve in Figure 10.1 at the start of this chapter. It is clearly a better fit than the unadjusted model that does not take account of embeddedness (the solid line in Figure 10.1), which is $N'_s = M^{1/3}$.

Without the logic developed in this chapter, we would never have been able to explain the nonlinear pattern in the district-level results. With our concept of embeddedness, we are able to understand that the district-level effect depends not only on district magnitude, but also on the share of the assembly seats any given district represents. We have thus completed a chain of logical relationships for seat-winning parties at the district level. We are ready to turn to the votes.

[14] Discussed in Chapter 7; there the focus was nationwide, but there is no difference in the connection between N_S and s_1 at the two levels. Both quantities are already affected separately by embeddedness, hence the connection between the two remains the same.

HOW WE DERIVE VOTES FROM SEATS AT THE DISTRICT LEVEL

Up to now, this chapter has focused on the district-level distribution of seats. We have demonstrated how parties' seats are shaped by the nationwide system in which a district is embedded. In this section we turn our attention to the votes at the district level; as we did at the nationwide level in Chapter 8, we now deduce votes from seats. An important conclusion of this section is that the votes are affected by the district's embeddedness even more than the seats.

In particular, the system that is generally thought to be most predictable and "simplest" actually is much more affected by extra-district factors, and hence harder to predict. We are referring to the M=1, plurality, or FPTP system. Widely seen – from the so-called Duverger's law – as supporting the concentration of votes on two major parties, FPTP systems in fact are often more fragmented than moderate PR systems, *even at the district level*. Why is this? We offer an argument for this strange and – for most scholars of electoral and party systems – inexplicable result. What we find is that it is especially true under FPTP that politics in the district level is heavily shaped by the nationwide system. All politics is apparently national, *especially* when M=1 in a large assembly.

As we did in Chapter 8, we start thinking about the votes by reference back to the seats distributions. The basic reason is that it is seats that are physically constrained by institutions – principally district magnitude and assembly size. We build our models of district-level votes distribution using a concept that was already introduced in Chapter 8: the *pertinent vote-earning parties*. The concept is fundamentally the same in district as in national politics.

Pertinent Vote-Earning Parties at the District Level

At the national level (Chapter 8), we established that Equation 8.2,

$$N_{V0} = N_{S0} + 1,$$

was a reasonable estimate for the idea of "number of pertinent vote-earning parties," which we are unable to measure directly. In Chapter 8, we built our tests around the following:

$$N_V = (N_{S0} + 1)^{2/3}.$$

When we turn to the district level, of course, we need to take embeddedness into account. Before we can do so, we must recall (from earlier in this chapter) that we did not need to account for embeddedness to derive the district-level number of seat-winning parties, N'_{S0}. However, we did need it to connect the effective number of seat-winning parties, N'_S, to district magnitude, M (Equation 10.5). Therefore, the implication is that our logical model for the effective number of seat-winning parties, $N_S = {N_{S0}}^{2/3}$

(Table 9.2), could not work at the district level (i.e., as $N'_S = N'_{S0}{}^{2/3}$). However, the resolution that suggests itself is the following, with inclusion of the embeddedness function:

$$N'_S = N'_{S0}{}^{4k/3}.$$

This is confirmed by OLS regression.[15] The same relationship logically must hold for votes as for seats, meaning we should have $N'_V = N'_{V0}{}^{4k/3}$. In that case, what we expect is:

$$N'_V = (N'_{S0} + 1)^{4k/3} \tag{10.6}$$

We will be testing Equation 10.6 shortly. But wait! There is a complication, and it has to do with the M=1 districts – those districts that are supposed to be the most straightforward, according to the so-called Duverger's law. While the relationship between N'_S and N'_{S0} can vary with k, the national embeddedness function, when M>1, such variation is impossible when M=1. After all, in such a district it must always be the case that $N'_{S0} = N'_S = 1$, independent of how many other districts, of whatever magnitude, the assembly electoral system may contain. We showed in Figure 10.2, at the outset of this chapter, that a district's N'_V has a systematic relationship to the *size of the assembly* in which the district is embedded. A key remaining task of this chapter is to explain this relationship. First, however, let us test Equation 10.6 across the full range of simple electoral systems.

In Figure 10.5, we see the graph of Equation 10.6, in the thick dotted curve. The thick dotted curve plots, for the median value of k at any N'_{S0}, the regression result, which is an uncannily strong confirmation of the model:

$$logN'_V = -.00311 + 1.3355[klog(N'_{S0} + 1)] \quad [R^2 = .233].$$

With the expectation (Equation 10.6) being $4k/3$, we could hardly ask for a better approximation to the logical model. It is also clear that the unadjusted model (solid light gray curve) seriously underestimates what N'_V tends to be, for any given number of seat-winning parties. We must take embeddedness into account.

Despite the good overall fit of Equation 10.6, we should acknowledge that the estimate is visually high for N'_{S0}=2. Any attempt to adjust the model to make the curve dip down for cases where there are two seat-winning parties (and perhaps also for three-party districts) would make the model unnecessarily complex. The model fits N'_{S0}=1 well; it predicts N'_V = 2.543 where the actual mean N'_V = 2.5954. There are many more cases of N'_{S0}=1 than of N'_{S0}=2 (or N'_{S0}=3, for that matter). The implication is that either our sample of two-party districts is unusual in some respect, or that there is a systematic "balancing" factor when there are just two parties. Such

[15] We get log N'_S = 0.0000 + **1.304**(k log N'_{S0}) [R^2=0.986].

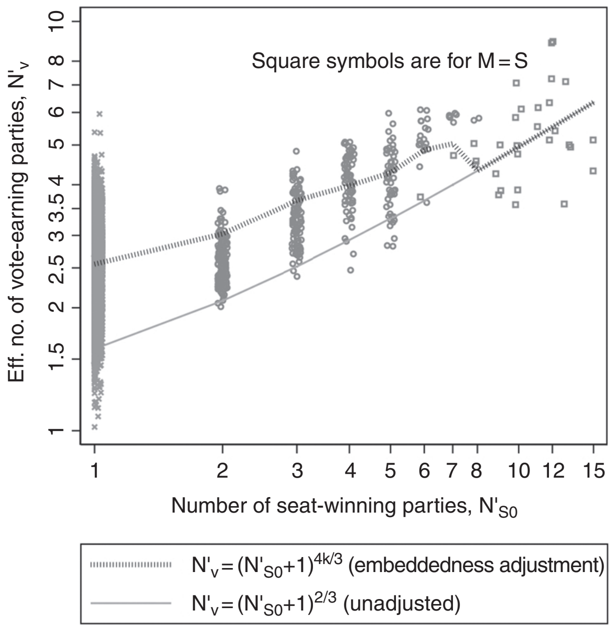

FIGURE 10.5 The actual number of seat-winning parties (N'_{S0}) and the effective number of vote-earning parties (N'_V), with incorporation of district-embeddedness function (k) The thin gray curve is the model without embeddedness: $N'_V = (N'_{S0}+1)^{2/3}$. The thick dotted curve represents Equation 10.7, incorporating embeddedness: $N'_V = (N'_{S0}+1)^{4k/3}$.

a factor would be a further generalization of embeddedness, in that it would imply that voters wanting to vote against the strongest party in their district may tend to boost the second party's votes, pushing N'_V below 2.5 for cases where exactly two parties win seats.

The striking thing is that no such second-party favoring is evident under M=1 (such districts represent the overwhelming share of N'_{S0}=1 cases); otherwise we would not see the large cluster of observations at N'_{S0}=1 for which N'_V >3. The implication is that voters face less pressure to boost the second party when only the largest can win than they do when both of them can.[16] While testing this voter response would take us well beyond the scope of this chapter, we offer it as a corrective to the usual assumptions about strategic voting–desertion of trailing parties or candidates to support the preferred one among the top two. Such desertion apparently happens less when M=1 than expected, and perhaps less than when two (but not more) parties win locally.

[16] There are no districts in our dataset that were contested by only a single party.

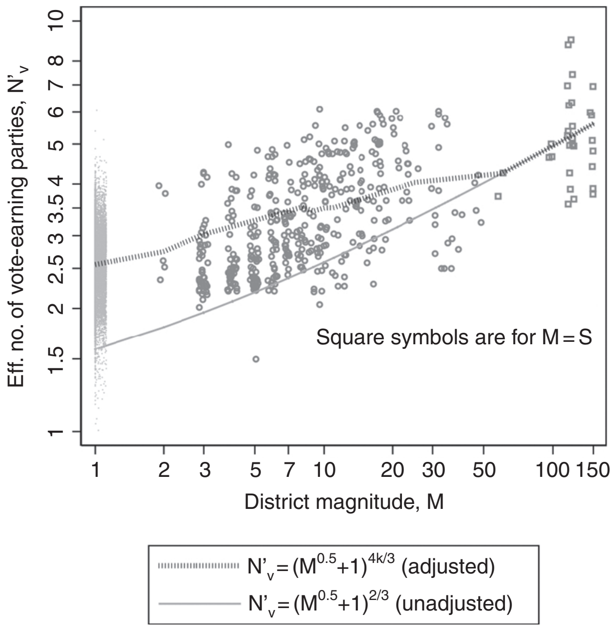

FIGURE 10.6 District magnitude and the effective number of vote-earning parties (N'_V), with incorporation of district-embeddedness function (k)

Now we are ready to connect N'_V to M. Given that we have N'_{S0}=$M^{0.5}$, it should be as simple as inserting $M^{0.5}$ in place of N'_{S0} in Equation 10.6; in other words:

$$N'_V = (M^{0.5} + 1)^{4k/3} \tag{10.7}$$

In Figure 10.6 we graph district-level effective number of vote-earning parties (N'_V) against district magnitude (M). We see that we clearly must take into account embeddedness. If we failed to account for the assembly size, S, we would have to conclude that our effort at model-building had failed. The solid light gray line in Figure 10.6 shows such a model: N'_V=$(M^{0.5}+1)^{2/3}$, without embeddedness. This is visibly fairly accurate for M=S (squares), but when M<S, the vast majority of district-level observations are above this line. The reason is embeddedness. Moreover, the curve for Equation 10.7 is not as steep as the unadjusted model, precisely because extra-district politics pushes up N'_V in lower-magnitude districts, which represent a smaller share of the system's total S seats, than it can when the district is a larger share of the assembly in which it is embedded.[17]

[17] In districts with three to five seats we again see our model with embeddedness predicting somewhat higher than the mean, probably due to the "two-party" effect that we discussed in reference to the trend in Figure 10.5. Most cases of exactly two parties occur within this magnitude range.

Nowhere, however, is the embeddedness factor more in evidence than when M=1. Once again, we see a high density of single-seat districts at values of N'_V >3; evidently there are many districts that are quite "non-Duvergerian" in their result, yet there are also many such districts with very low values of N'_V. In other words, *the effective number of vote-earning parties is much less predictable precisely in those cases where the district magnitude is most constraining*.

We already saw at the start of this chapter, in Figure 10.2, that we can approximate the trend N'_V for FPTP systems when we graph it against assembly size. The line that we saw in Figure 10.2 represents the equation,

$$N'_V = 2^{(2+.8304\ \ logS)/3} = 1.587S^{0.08333} = 2^{2/3}S^{1/12} \quad [M = 1]. \qquad (10.8)$$

Equation 10.8 follows from Equation 10.6, $N'_V = (N'_{S0} + 1)^{4k/3}$, when M=1 and hence N'_{S0} =1. (Recall that k=0.5+0.2076 log(S/M)/$M^{0.25}$ simplifies to k=0.5+0.2076 log$S$ when M=1. Hence 4k=2+.8304 log$S$. The rest is algebraic reformulation.) Thus, consistent with the theoretical claim that stands behind our embeddedness function, k, we see that *the size of the national assembly systematically affects mean district-level* $\mathrm{N'_V}$ *in FPTP systems*. The effective number of parties in an individual one-seat district is, on average, two, raised to an exponent in which the only variable input is the assembly size.

Of course, individual districts in any given election can vary wildly around the mean; every district has its own local issues and different interest groups within the constituency, as well as a unique set of candidates with their own personal vote. No broad model could possibly claim to predict what a specific district's effective number of vote-earning parties will be.[18]

What this analysis has shown is that when attempting to account for the district-level N'_V, one actually can start with a nationwide factor, assembly size. Districts then differ from a baseline predictable from knowing how many total districts there are in the election. This is a remarkable finding – national politics shapes the district level systematically, even under simple FPTP where the district's electoral rule is supposedly the most constraining.

APPLICATION TO PRESIDENTIAL DEMOCRACIES

The models and graphs shown in this chapter so far have been based solely on parliamentary democracies with simple electoral systems, aside from the FPTP graph (Figure 10.2), which included some presidential cases. In the case of

[18] One thing we can conclude is that the variance around the predicted values from our logical model is *not* accounted for by social diversity, as defined by Milazzo et al. (in press). The authors generously provided us with their data, and we find it explains precisely none of the deviation of a given district's N'_V from the value derived from Equation 10.8. (Details in our online appendix. www.cambridge.org/votes_from_seats)

presidential democracies, the assembly politics is not the only political game in the system, and it might be that the assembly electoral-system variables are swamped by the all-important competition for the presidency. Accordingly, we ran the regressions testing our logical models on a wider dataset including presidential democracies. The models continue to work, albeit with more scatter. *In no case is there a statistically significant difference between the parliamentary and presidential subsets.* The regressions for parliamentary and presidential systems are shown side-by-side in the appendix to this chapter.

That there is no systematic impact of presidentialism on the relationship between district magnitude and the effective number of parties is an important finding. It goes against much conventional wisdom about how presidential competition – such as the notion of presidential candidates' "coattails" – shapes assembly competition. We might have expected no ability of variables grounded purely in the assembly electoral system to predict patterns in party systems of presidential democracies. Already we expressed the idea that the Seat Product could predict the nationwide effective number of parties about as well in presidential systems as in parliamentary (Chapter 7). The impact of the legislative assembly electoral system extends to the district-level patterns as well. In Chapter 11, we will take this finding even farther – showing that even the effective number of presidential candidates can be predicted with astonishing accuracy (albeit imperfectly) by knowing the district magnitude and assembly size.

CONCLUSION

The findings of this chapter may come as a surprise to anyone familiar with almost any of the literature on electoral systems over the preceding five decades or more. It is well "known" that one-seat districts with plurality rule tend to have a "two-party" system, and that this effect is especially notable at the district level. Such a claim is pretty much the essence of the so-called Duverger's law. Meanwhile, according to the same common wisdom, PR electoral systems tend to have multiparty systems.

We show that this is a highly incomplete characterization. Duverger's propositions are overdue for an update, which we could provide only after constructing models based on nationwide assembly size (S) and mean district magnitude (M) – and, as in this chapter, extending and testing them at the district level. Specifically, we find that the districts in large countries with FPTP electoral systems actually tend to have a higher effective number of vote-earning parties (N'_V) than do the districts of moderate magnitude in many PR systems. More importantly, *this is as we expect*, based on our logical models.

In this chapter, we started with a very basic model of the number of parties, of any size, that can be expected to win in a district, given its magnitude.

TABLE 10.1 *District level equations for the Seat Product Model.*

District seats		
$N'_{S0} = M^{1/2}$ [no k!]		
$\mathbf{s'_1 = M^{-k/2}}$	$s'_1 = N'_{S0}{}^{-k}$	
$N'_S = M^{2k/3}$	$N'_S = N'_{S0}{}^{4k/3}$.	$N'_S = s'_1{}^{-4/3}$
District vote-seat interaction		
$N'_{V0} = (N'_{S0}+1)$		
$v'_1 = (N'_{S0}+1)^{-k}$	$v'_1 = (s'_1{}^{-1/k} + 1)^{-1/k}$	
$N'_V = (N'_{S0}+1)^{4k/3}$	$N'_V = (s'_1{}^{-1/k} + 1)^{4k/3}$	$N'_V = [N'_S{}^{3/(4k)} + 1]^{4k/3}$
District votes		
$\mathbf{N'_{V0} = (M^{1/2}+1)}$		
$v'_1 = (M^{1/2}+1)^{-k}$	$v'_1 = N'_{V0}{}^{-1/k}$	
$N'_V = (M^{1/2}+1)^{4k/3}$	$N'_V = N'_{V0}{}^{4k/3}$	$N'_V = v'_1{}^{-4/3}$

The basic models are shown in bold; the rest follows from basic laws (see Table 9.2) and algebra. District embeddedness function: k=0.5+0.2076log(S/M)/$M^{.25}$.

This number tends to be the square root of the district magnitude (as shown already in Chapters 1 and 7), independent of the larger assembly electoral system in which the district is embedded. However, other key indicators of the district-level party system can be understood only by incorporating this "embeddedness" factor. This complex factor, dubbed "k" is based on the share of the total assembly represented by the seats elected in a given district. It thus is equal to the square root of the magnitude if there is only one district for the entire country's assembly (i.e., no embeddedness), and becomes a larger fraction as magnitude is smaller *and* the assembly is larger.

Table 10.1 offers a summary of the key equations developed in this chapter. They closely parallel those shown in Table 9.2 for nationwide outcomes, except for the fact that the impact of assembly size enters only through the exponent, where k is included (except for N'_{S0}!). Then in Table 10.2, we offer a demonstration of what the predicted values are from the key equations. All of the examples in Table 10.2 are based on an assembly of 270 seats (a value near the median for our sample). Further exploration of how the embeddedness function works out in practice may be found in the chapter appendix.

When we take embeddedness into account, we see that the largest party tends to be smaller, and the effective number of seat-winning parties higher, than would be the case without this factor. We suggest that this effect results from the impact of *nationwide politics* on the district, whereby parties other than the locally strongest bring in resources to try to blunt the main local party's advantage. Of course, to the extent that parties engage in this sort of behavior

TABLE 10.2 *Average expectations at various levels of* M, *when* S=270

M	1	2	5	10	20	45	90	135	270
$k=0.5+0.2076\log(270/M)/M^{0.25}$	1.00475	.87189	.74051	.66710	.61096	.56237	.53216	.5183	.5
$s'_1 = M^{-k/2}$	1	.7392	.5511	.4639	.4005	.3429	.3020	.2805	.2467
$v'_1 = (M^{1/2}+1)^{-k}$	.4984	.4637	.4191	.3862	.3540	.3171	.2863	.2687	.2395
$N'_S = M^{2k/3}$	1	1.496	2.213	2.784	3.388	4.167	4.935	5.447	6.463
$N'_V = (M^{1/2}+1)^{4k/3}$	2.531	2.786	3.188	3.555	3.993	4.624	5.299	5.767	6.723

successfully, they tend to make local politics look somewhat more like national politics. This is what we mean by the phrase "all politics is national."

Our logical model of how embeddedness works allows for a more systematic understanding of N'_V. We can predict N'_V quite reliably by combining the concept of "strivers are winners, plus one," which we developed for the nationwide party system (Chapter 8), with the district "embeddedness" factor introduced here. When we take these steps, we see that the number of "pertinent" vote-earning parties in a district can be estimated as the number of parties that won at least one seat in the district, plus one ($N'_{S0}+1$). Then we can raise this quantity to the power, $4k/3$, where k is the embeddedness factor. Moreover, this works quite well for predicting the average even under FPTP, allowing us to conclude that the effective number of vote-earning parties when M=1 is strongly conditioned by the assembly size.

In other words, our *district embeddedness is an especially strong factor in FPTP systems*. In practice, this finding means that politics is quite "national" despite the presence of the district magnitude that is supposedly most conducive to local politics and to a Duvergerian "law-like" tendency towards two-party politics. The long-overdue update to these tendencies that we introduce is to reveal that a large assembly is a key factor in pushing N'_V well upward beyond two. We see this play out in Canada, India, and the UK. All politics is national – even in the one-seat districts of simple plurality electoral systems.

We have now completed Part III of the book, our main set of chapters devoted to the nationwide and district effects of electoral systems on the interparty dimension – i.e., outputs such as the degree of fragmentation of the votes and seats, the size of the largest party, and deviation from proportionality.[19] Our focus so far has been primarily on parliamentary systems, in which the balance of power in the assembly determines parties'

[19] We did not show a model for district-level deviation from proportionality, because details remain to be worked out, even as the basic steps followed in Chapter 9 (national level) can be replicated at district level. See our online appendix. www.cambridge.org/votes_from_seats

bargaining power for executive formation. We have noted that our models do indeed work for presidential systems, too, but scatter tends to be higher. Perhaps this is not surprising, given that executive power in such systems does not depend on the balance of partisan power in the assembly, but on a separate nationwide electoral contest. In Chapters 11 and 12, we explore the impact of variables unique to presidential systems. These are then followed by two chapters that go deeper into district-level politics, by analyzing the intraparty dimension.

Appendix to Chapter 10

CONSTRUCTING DISTRICT EMBEDDEDNESS EXPONENT k

We start this appendix with an unusual caveat: this is the one section in the book that the authors themselves hesitate reading over again. It feels like taking indispensable but bitter medication. Yes, the model is sound, and we need it, if we want to advance into the territory of nationally embedded districts, which was the basic theme of Chapter 10. The calculations have been checked and rechecked. But the reasoning is strenuous. Do not feel bad, if you give up and just decide to judge the tree by its fruits – the degree of agreement with actual district level data.

If districts were equivalent to mini-states with a single statewide electoral district (M=S), matters would be simple: just replace MS in the nationwide equations with M^2. But they are not. They are embedded in a wider nationwide context that intervenes so as to raise the effective number of parties[20] and depress the largest party's seat and vote shares. Let us make it more specific for the effective numbers of seat-winning and vote-earning parties.

The effective numbers of parties in a district unaffected from the outside would follow the nationwide models N_S=$(MS)^{1/6}$ and N_V=$[(MS)^{1/4}+1]^{2/3}$, but with S=M. Hence we would have N_S=$M^{1/3}$ and $N'_V = (M^{1/2}+1)^{2/3}$. Yet we actually observe higher values, as if the basic building block $M^{0.5}$ had been replaced by M^k, where k is larger than 0.5. How much larger, this should depend on how much M is smaller than S. The nationwide "interference" on district-level politics logically should be strongest when district magnitude is small compared to assembly size, yet larger than 1. At M=1, the number of seat-winning parties is bound to be one, with or without nationwide interference, although of course the effective number of vote-earning parties would still be

[20] We showed in Chapter 7 that the simple, unadjusted model for the number of seat-winning parties, N'_{S0}=$M^{0.5}$, fits well enough, and thus we keep to this simpler formulation. However, as discussed in the main text of Chapter 10, the adjustment derived in this appendix is needed to explain a host of other outcomes at the district level.

larger than one. At the opposite extreme, with a single nationwide electoral district (M=S) there is no interference from the outside.

This means that some function of M and S should be added to 0.5, the exponent in the basic building block. The embeddedness exponent then is:

$$k = 0.5 + f(M, S).$$

Replacing 0.5 with k (and hence 1 with $2k$) in the equations for district-level effective numbers of parties, they become

$$N'_S = M^{2k/3} \text{ and } N'_V = (M^{1/2} + 1)^{4k/3}.$$

Now it is a matter of determining the form of function $f(M,S)$ and hence k. The outcome, shown as Equation 10.1, is baffling – even to us:

$$k = 0.5 + 0.2076\log(S/M)/M^{.25}.$$

The path to reach this outcome is convoluted. We'll try to make it as simple as possible, but this is still one of the most complex parts of the book.

Try the simplest first: assume that $f(M,S)$ is a constant: k=0.5+a. But this cannot be all there is, because the supplement to 0.5 must vanish when M takes its highest possible value, M=S. Inserting log(S/M) is the simplest way to satisfy this constraint, given that log1=0. Then

$$k = 0.5 + a\log(S/M).$$

Next, we ask what happens when M takes its lowest possible value, M=1? Then the broad format is simplified to

$$k = 0.5 + a\log S \quad [\text{for } M = 1].$$

We could determine a empirically – except that it cannot be done with seats, because the logical N'_S=1 is satisfied with any value of k. This is why we must appeal to district votes data to determine the value of constant a. We will get to this.

For the moment, however, return to the broad format k=0.5+$a$log(S/M). For district level effective number of seat-winning parties, this means that N'_S=$M^{2k/3}$ becomes

$$N'_S = M^{1/3+(2/3)a\log(S/M)}.$$

In a logarithmic graph N'_S versus M, such as Figure 10.1, this pattern is an arc (the thick dotted curve) above the simple line N'_S= $M^{1/3}$ (shown in the graph as the thin straight line). The curve joins this line at M=1 and M=S, but the one incorporating nationwide embeddedness is higher throughout the range, 1<M<S. How high above the line should this arc be? Here we have no logical answer. Empirically, this height can be simply regulated by dividing $a$log(S/M) by M^b and adjusting the empirical constant b. The broad format,

$$k = 0.5 + a\log(S/M)/M^b,$$

thus emerges from logical considerations and preference for maximal simplicity. To determine the value of constant b, we could use the M>1 data for either N'_S or N'_V – and they had better yield the same result! But first we must pin down the value of constant a.

Recall that at M=1we have simply k=0.5+$a$log$S$. Then N'_S=1, with or without nationwide impact, so it cannot give us any information about k. But for votes we have

$$N'_V = (M^{1/2} + 1)^{4k/3} = 2^{4k/3} = 2^{2/3+(4/3)a\log S}.$$

It is simpler to deal with the decimal logarithm of N'_V:

$$logN'_V = \log 2[2/3 + (4/3)a\log S] = 0.201 + 0.401a\log S.$$

These coefficients, 0.20069=(2/3)log2 and 0.40137=(4/3)log2, emerge from the 0.5 in k=0.5+$a$log$S$. This means they are theoretically set and not subject to empirical manipulation.[21] Is logN'_V really linearly related to log$S$? Figure 10.2 shows that this is so indeed, even while scatter is huge – as one would expect for individual districts. The best fit is:

$$\log N'_V = 0.127 + 0.113\log S. \quad [R^2 = .17].$$

But the empirical intercept 0.127 differs from the required 0.201. So the equation must be recalculated with intercept stipulated as 0.201. This alters the best-fit line very little. The result is

$$\log N'_V = 0.201 + 0.0828\log S,$$

and hence (after taking the antilog of 0.201)

$$N'_V = 1.59S^{0.083}.$$

Combining 0.083log$S$ and previous 0.401$a$log$S$ leads to a=0.2065.

This is an empirical estimate. Does it have a logical foundation? This may well be so. The total number of candidates running in FPTP districts worldwide also increases with increasing assembly size.[22] This leads to hints that the exponent in the equation above might have to be 1/12=0.08333 on logical grounds. Then constant a in k=0.5+$a$log(S/M)/M^b should be:

[21] The model is set up in terms of decimal logarithms. All numerical values, including that of constant a, would be off by a factor ln$x$/log$x$=2.3026 when natural logarithms are used.

[22] Why this is so might become more tangible in the light of the cube root law of assembly sizes (see Chapter 2). To the extent this law holds, the population of a district in system of FPTP would be close to P'=S^2. A larger district population might well engender more people to run as independents or minor party candidates.

$$a = 1/(16\log 2) = 0.2076.$$

We'll use this number rather than the empirically obtained a=0.2065. However, some kinks in the model that leads to this conclusion remain to be ironed out; so it's too early to present it in this book.

Now that constant a in $k=0.5+a\log(S/M)/M^b$ has been settled, it is time to address b. By trial and error we find that the average patterns in Fig. 10.1 (N'_S versus M) and 10.6 (N'_V versus M) are best satisfied when we set b at 0.25, approximately. Finally, the general formula for k, at any S and M, is

$$k = 0.5 + 0.2076\log(S/M)/M^{.25}.$$

To repeat: here 0.5 is the value in the absence of nationwide impact; log(S/M) assures that k remains 0.5 when M=S; the coefficient 1/(16log2)=0.2076 emerges from logical concerns about the total number of candidates in M=1 elections; and $1/M^{0.25}$ results from empirical fit of district level effective numbers at given M. The broad format is determined by logical reasoning, while coefficient b=0.25 was found empirically and a=0.2076 was reinforced by incomplete logical considerations. The remaining challenge in logical model building is to reinforce the logic for a and explain why b=0.25.

EVIDENCE THAT EMBEDDEDNESS DOES NOT AFFECT THE NUMBER OF SEAT-WINNING PARTIES

The goal here is two-fold. The first goal is to give us a feel of how k decreases as M increases, for a medium-size assembly. The second and more important goal is to present evidence that, in contrast to the effective numbers of parties and the largest seat and vote shares, the actual number of seat-winning parties (N'_{S0}) does not receive a boost from the district being embedded in a larger nationwide assembly electoral system. This contrast is rather surprising.

In **Table 10.A1** we show several values of district magnitude and what k would be for a hypothetical constant S of 270. We use 270 because it is approximately the mean for our sample of multidistrict simple PR systems. The first column indicates the magnitudes, and the second one the number of actual districts we have in the data sample with that M. (The actual districts, of course, may come embedded in a wide range of assembly sizes.) The third column then shows what N'_{S0} is predicted to be, using $N'_{S0} = M^{0.5}$. From the third and fourth columns we see the actual mean N'_{S0} for the districts in our sample (again, regardless of S), followed by the ratio of the actual value to that predicted from $N'_{S0} = M^{0.5}$. That these ratios tend to be close to 1.00 confirms the good fit of the *unadjusted* model for N'_{S0}; that is, not taking into account the impact of national politics on the district.

TABLE 10.A1 *How district magnitude shapes the number of parties, with and without embeddedness*

M	No. of cases	$N'_{S0}=M^{0.5}$	Actual mean N'_{S0}	Ratio, if $M^{0.5}$	k if S=270	$N'_{S0}=M^k$	Ratio if M^k
3	45	1.73	2.09	1.206	0.81	2.43	0.860
4	45	2.00	2.20	1.100	0.77	2.90	0.758
5	49	2.24	2.14	0.958	0.74	3.29	0.651
6	40	2.45	2.75	1.123	0.72	3.63	0.758
7	29	2.65	2.93	1.108	0.70	3.92	0.747
8	27	2.83	2.89	1.021	0.69	4.19	0.690
9	21	3.00	3.33	1.111	0.68	4.43	0.753
10	30	3.16	3.63	1.149	0.67	4.65	0.782
11	10	3.32	3.30	0.995	0.66	4.85	0.680
12	17	3.46	3.47	1.002	0.65	5.04	0.689
14	17	3.74	4.00	1.069	0.64	5.38	0.743
16	10	4.00	4.00	1.000	0.63	5.69	0.702
Mean				1.070			0.734

The last three columns show what the impact of nationwide politics would be, first by demonstrating how k decreases, for a constant S=270, from nearly 0.81 at low M (three-seat district) to 0.63 when M=16. If we were to continue with even larger magnitudes, while holding S=270, we would find k =0.58 at M =35, k =0.56 at M =50, and of course, k=0.5 if there were just one district with M=S=270. In the opposite direction we reach k=1.005 at M=1. Yes, k can surpass 1, at large S and low M.

The final column of Table 10.A1 shows very clearly that we do not need the adjustment for embeddedness (k) to predict N'_{S0}. All ratios of N'_{S0} predicted from M^k are clearly worse than the predictions from $M^{0.5}$. However, in this chapter, we saw that it is necessary to adjust for the embeddedness of the district when we are attempting to predict the effective numbers of parties and the largest seat and vote shares in a district. Hence the need for the calculations that produce this exponent, as outlined in the previous section of this appendix.

DISTRICT-LEVEL REGRESSION RESULTS AND HOW PRESIDENTIAL SYSTEMS ARE DIFFERENT (OR ARE NOT)

In this section we report the regression results of the district-level effects tested in this chapter. In all cases, the unit of analysis is the "list" within a district, even if this differs from party (i.e., where there are alliances). When M=1, the distinction does not matter (see Chapter 2), but for M>1 it sometimes does.

TABLE 10.A2 *Comparing regression results for parliamentary and presidential systems*

		Parliamentary			Presidential			
Output	Input	Intercept	B coeff.	R^2	Intercept	B coeff.	R^2	Significant difference?
N'_{S0}	$\log M$	.000399	.509	.954	(.0103)	(.554)	.8872	No
N'_{S0}	$k\log M$	–.000440	(.818)	.942	.00489	(.862)	.903	No
s'_1	$k\log N'_{S0}$	4.15*e*–06	–.958	.955	–.000723	–1.039	.941	No
s'_1	$k\log M$	–.000148	–.494	.912	(–.0098)	–.5734	.767	No
N'_S	$k\log M$	.0000396	.648	.937	(.00800)	.722	.848	No
N'_V	$k\log(N'_{S0}+1)$	–.00311	1.336	.233	–.169	1.613	.292	No
N'_V	$k\log(M^{0.5}+1)$	–.0123	1.369	.203	–.0564	1.308	.093	No

Parentheses indicate a result contrary to model; i.e., an intercept significantly different from zero, or a coefficient significantly different from our logical model prediction (for which see main text). Significance reported at $p \leq .05$.

We have three countries for which we have district-level data on the number and size of lists, but in which those lists may contain candidates of multiple parties: Brazil, Chile, and Finland (on the latter case, see the examples of alliances in Chapter 6). This matter is taken up in detail in Chapter 14, where we unpack the lists down to their component parties. For the purposes of this chapter, we are concerned with how the electoral system works on those entities to which the proportional allocation formula is applied: in simple PR systems, that is the list.

In Table 10.A2, we show the results for the parliamentary and presidential subsamples side by side. As noted in Chapter 10, for all of the district-level relationships tested, there is no evidence of a systematically different relationship under presidentialism. Table 10.A2 allows for a comparison the coefficients and statistical fit for parliamentary and presidential data samples. In the last column it indicates whether the difference across executive formats is itself statistically significant.[23] In all cases, the answer is *no*.

For the first outcome indicated, N'_{S0}, we show tests of the models both with and without embeddedness. We see that for the regression of N'_{S0} and $k\log M$, neither executive type produces the expected coefficient of –1.00. This confirms that we do not need to consider embeddedness for the actual

[23] Significant difference was determined by examining marginal effects across the range of the independent variable in a regression in which both samples were pooled, and differentiated with a dummy variable interacted with the independent variable (as recommended by Brambor, Clark, and Golder 2006).

number of seat-winning parties. Using the unadjusted log$M$ produces a better result, notwithstanding that the predicted parameters for presidentialism are just outside of the confidence intervals of the regression estimates.

The table also shows tests of several other output variables, sometimes with more than one input, in line with the way we introduced the logical sequences in this chapter. As explained in the chapter text, for all these other outputs, we do need to take into account embeddedness. In all cases, the results for presidential systems are statistically indistinguishable from those for parliamentary.