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Interest groups, campaign contributions, and probabilistic voting*

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Abstract

This essay develops a simple model to analyze the impact of campaign contributions on electoral-policy decisions of candidates for office. Interest groups here are firms that select contributions under the assumption that candidates' policies and opposing groups' donations remain unaltered. Candidates, however, recognize that their policy choices affect contributions. Campaign contributions are used by candidates to affect policy-oriented voters' perceptions of candidates' positions. In this framework the introduction of campaign contributions may affect candidates' electoral policies, and if they do then they benefit surely *exactly one* of the two interest groups.

1. Introduction

The implications for policy-making of campaign contributions from interest groups are not well understood. In democracies, such groups cannot legally 'buy' policies. But they might nevertheless be able to influence policy through campaign contributions (Adamany, 1977). Contributions may be made either to affect the election outcome or to obtain influence over legislative decision-making by the successful candidate (or both). In the first case, interest groups seek simply to promote the electoral chances of their favored candidate, taking that candidate's policy position as fixed (Jacobson, 1980). In the second case, the groups may not have such a preference but wish to 'buy' access to the successful candidate (Aranson and Hinich, 1979; Drew, 1982). In either case, a group's motivation for making contributions can be because of public issues, such as the legitimacy of capital punishment, or because of private-good incentives, such as the incidence of commodity taxes.

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These possibilities are neither exhaustive nor mutually exclusive. In this essay we focus on the private-good incentive for contributing to a campaign in which the sole issue is the size of the public sector. Also, since there are no new issues to be determined after the election (the model is static), groups contribute only to affect the election outcome.

Suppose that interest groups provide funds only to promote the electoral chances of their favored candidates, taking these candidates' policies as given. Then candidates might have an incentive to adopt positions likely to induce contributions. While money can help candidates become elected, however, policies *per se* matter also – there is a potential trade-off for candidates between choosing policies to attract funds and choosing policies to attract votes. Sections 2 and 3 present a simple (and technically restrictive) model with which to begin analyzing this trade-off. In the model, two candidates compete for office by choosing policy positions in a one-dimensional issue space. Voters have noisy perceptions of candidates' positions, and candidates can alter the extent of the noise through campaign expenditures – via advertising, for example. Voters are risk-averse and so, *ceteris paribus*, it is never in the candidates' interest to be ambiguous if they can avoid it (Shepsle, 1972). The issue is public sector size, reflected through the proportional tax-rate to be imposed to finance some public good. This casting of the problem gives rise naturally to two potential interest groups: producers of private goods and suppliers of the public good (for example, recipients of defense contracts). We presume that the two groups are distinct. Assuming fixed incomes of consumer-voters, then, private-good producers have an interest in relatively low tax-rates and conversely for public-good suppliers. As we indicate earlier, when making campaign contributions, firms do not do so to affect candidates' policies but to affect the relative likelihoods of election. There is thus no explicit exchange relationship between contributors and politicians embedded in the assumptions of the model. But candidates seeking funds recognize that contributions, to some degree, will depend on their policy stance, and so they will take this into account in choosing election strategies.

We examine the impact of contributions on electoral-policy positions in this framework by first identifying, as a benchmark, the equilibrium of the election game under the assumption that no contributions are permitted. Then we compare with this benchmark the resulting equilibrium when contributors may give freely. There turn out to be three basic cases, depending on whether, in the benchmark outcome, potential contributors would have liked to have offered campaign funds had they been so allowed: if no contributions would have been forthcoming, then permitting contributions has *no* impact on policy; if only one candidate would have received any contributions, then *both* policies offered in the election shift to favor the group supporting this candidate; and if both candidates would have received funds, then their respective policies still move to favor only one of the interest groups (in general, the larger).

Section 2 presents the formal model, and Section 3 discusses the results. Section 4 concludes. All proofs are confined to an appendix.

2. Model

2.1 *Economy*

Consider an economy with a finite number of persons, N, sharing identical tastes over two consumable commodities; a pure private good, x, and a pure public good, G. We describe these preferences over (x, G)-bundles by a strictly concave, increasing, and continuously differentiable utility function, U(x, G). Assume $U_{12} \geq 0$ and $\text{Lim.}_{h \to 0} U_h = -\infty$, h = 1, 2. Each person i = 1, 2, ..., N has an endowment, $w_i > 0$, of some nonconsumable resource, say units of labor-power.

There are two firms, A and B, both owned by agents outside the economy of concern (which we may think of as a state in some federal system, for example).[1] Firm A supplies the private good, x, under constant returns to scale, using labor as input. The firm pays α units of the private good for every unit of input: the price α is given exogenously to the economy (A is 'small' relative to the whole system). Firm B uses labor to produce the public good under constant returns. The proportion of aggregate labor-power $t \in [0, 1]$ supplied to firm B is to be determined politically: we interpret the variable t as a proportional tax-rate on endowments. It is harmless to take G as the proportion of total tax-revenue (in units of labor-power) devoted to G-production: this proportion, β, is fixed. The firm appropriates the rest of the tax as recompense. No direct payments for the inputs are necessary since the people cannot be excluded from consuming the public good.

Now, taking $0 < \alpha, \beta < 1$ (by suitable choice of units) and writing w for mean endowment, define the indirect utility function for person i by:

$$u_i(t) = U(\alpha \cdot (1-t) \cdot w_i, \beta twN). \tag{1}$$

The firms' profits are given by:

$$R_A(t) = (1-\alpha) \cdot (1-t) \cdot wN > 0, \quad \text{all } t < 1, \tag{2}$$

and

$$R_B(t) = (1-\beta) \cdot twN > 0, \quad \text{all } t > 0. \tag{3}$$

There are no taxes on firms.

2.2 *Polity*

Two candidates {1,2} compete for office under simple plurality voting and only persons vote.[2] The relevant issue is the proportional tax-rate, so the policy space is [0,1]. In principle, this interval also constitutes the candidates' pure-strategy space. But, it is convenient to restrict the feasible strategy space to $T = [\min._i\{t_i^*\}, \max._i\{t_i^*\}]$, where $t_i^* = \mathrm{argmax}.u_i(t)$. By assumption, $\mathrm{Lim}._{t\to 1}U_1 = \mathrm{Lim}._{t\to 0}U_2 = -\infty$: therefore, $\min._i\{t_i^*\} > 0$ and $\max._i\{t_i^*\} < 1$. This restriction of the available strategy space is for purely technical reasons that will become clear.

There are two sources of uncertainty in the environment: we assume that individual voters and firms are unsure as to exactly what tax-rate either candidate will impose given electoral success, and candidates likewise remain uncertain about voters' decision rules. Consider first voter and firm uncertainty.

Individual voters and firms have common, but noisy, perceptions of candidates' policy positions.[3] Let η_j be a random variable taking values in the real interval $[-z, z]$, where $z \equiv \min.\{\min.\{t_i^*\}, 1-\max.\{t_i^*\}\}$. Assume that η_j has expectation zero and variance $\sigma^2{}_j$, $j = 1, 2$. Thus, candidate j's position in [0, 1] is a random variable $\tau_j = t_j + \eta_j$, such that $t_j \in T$ is j's announced (mean) policy position. By definition of T and z, all realizations of τ_j are feasible; that is, they lie in [0, 1].[4] Let $c_j \geq 0$ be candidate j's level of campaign expenditures (in units of labor-power) during the election. The candidate uses these expenditures to clarify his policy position for voters and firms, using advertising, public meetings and so forth. In particular, assume that $\sigma^2{}_j = s_j(c_j)$, with $s_j'(\cdot) < 0$, $s_j''(\cdot) > 0$, and $s_j(0) > 0$, $j = 1, 2$; thus campaign expenditures reduce the variance of the noise term.

The idea underlying this model of expenditures is that while candidates adopt and state the positions with which they contest the election, they cannot do so without ambiguity. People learn of candidates' positions through the various media, in political debate, and so on. Such intervening variables introduce noise into the signal sent by a candidate, who must then devote resources to articulating a position (recall that with risk-averse voters, it is not in a candidate's interests to be opaque). Notice that under the specification of the functions s_j, any candidate's campaign expenditures affect only that candidate's perceived position. In view of candidates' evident efforts to discredit their opposition, this assumption is not ultimately desirable. Relaxing it, however, simply complicates the argument without adding anything substantive. Assuming direct self-promotion, moreover, the dominant use of campaign funds seems a reasonable approximation to the facts.

Now consider candidate uncertainty. Although candidates know people's policy preferences, these preferences do not fully determine voting behavior. Candidates, therefore, treat voting as probabilistic (see, *inter alia*, Coughlin, 1983, 1984; Hinich, 1978; Hinich, Ledyard and Ordeshook, 1972). Intuitively, the notion is that although economic self-interest affects the way people vote, other variables, such as party identity, candidates' personal characteristics and peer pressure can all sway a decision at the ballot-box. We assume that candidates perceive these variables distributed randomly over the population, and consequently, candidates cannot be sure, *ex ante*, which way a person will vote for any given net utility difference that reflects economic circumstances alone.

Campaign expenditures might affect voting probabilities directly, and not through people's expected utilities as we describe them. While the non-policy variables giving rise to probabilistic voting are randomly distributed over the population and essentially outside candidates' influence (at least, in the short run[5]), campaign expenditures are control variables for the candidates, with concrete implications for how their respective policies are promoted. Additionally, although it is legitimate to assume that increasing campaign expenditures add to a candidate's likelihood of electoral success in *aggregate*, it is not obvious that such increases affect all *citizens'* vote-probabilities in the same direction.[6] Consequently, we model campaign expenditures here as affecting voters' perceptions of candidates' policy positions.

So, in the spirit of Hinich, Ledyard and Ordeshook (1972), let the probability that citizen i votes for candidate j, given $t = (t_1, t_2)$ and $c = (c_1, c_2)$, be,

$$q_{ij}(\cdot \mid t, c) = q_{ij}(Eu_i(\tau_j)) - Eu_i(\tau_k)) \in (0, 1), \; j \neq k.$$

Assume that q_{ij} is a strictly increasing linear function of utility difference for all citizens. The reasons for supposing linearity instead of the more general weak concavity are two. First, both linearity and weak concavity imply global strict concavity of the relevant payoff schedules. Therefore, it is sufficient to examine local properties of the model to infer global results. Since linearity is a legitimate local approximation to any concave function, our assumption is acceptable here. Second, while most of the subsequent analysis follows without qualification if we presume some strict concavity in the q_{ij}, we occasionally require additional assumptions, and these have little or no intuitive basis.[7] In view of these observations, we prefer the linearity assumption.

Notice that the model permits biased voting; that is, $q_{i1}(a) \neq q_{i2}(a)$ is possible. But we assume that everyone votes, so that $q_{i1} + q_{i2} = 1$ for all i. With this assumption of no abstention, it is legitimate to take candidates as seeking to maximize their expected votes, given the opposing candidate's position (cf. Aranson, Hinich & Ordeshook, 1974). Hence, candidate j's problem is to select t_j so as to,

$$\max. \{EV_j(t_j \mid t_k) = \Sigma_N q_{ij}(\cdot)\},$$
$$\text{subject to } t_j \in T;\ j \neq k,\ j, k = 1, 2. \tag{4}$$

2.3 *Campaign contributions*

While we have described the use of campaign contributions (equals expenditures), their supply remains to be discussed. As we argue in the Introduction, we conceive of interest groups as not offering campaign funds on a directly *quid pro quo* basis. Instead, they support candidates or not, taking the respective policy statements as given. Should candidates adopt different stances, then firms have incentive to contribute to at least one contestant in an effort to promote the electoral outcome that they favor.

Let $r = r(t, c) \in [0, 1]$ be the probability that candidate 1 wins the election, given policy announcements t and campaign expenditures c. Since there is no abstention,

$$r(t, c) = \Sigma_{M \in N}\, \Pi_{i \in M} q_{i1}(\cdot \mid t, c) \cdot \Pi_{i \in N \setminus M} q_{i2}(\cdot \mid t, c),$$

where $\mathbf{N} = \{M \mid N \supseteq M, |M| > N/2\}$. This function is not nicely behaved. But, for large N and no abstention, maximizing candidate 1's expected votes is equivalent to maximizing 1's probability of electoral success (Calvert, 1986; Hinich, 1977). Therefore, assuming N large, we can write,

$$r(\cdot, \cdot) = EV_1(\cdot \mid \cdot)/N.$$

Firms allocate campaign contributions to maximize expected profits. Letting firm f's contribution to candidate j be c^f_j, and writing $c^f = (c^f_1, c^f_2)$; firm f's problem is to select c^f so as to,

$$\max. \{E\pi_f(c^f \mid t, c^g) = r \cdot ER_f(\tau_1) + (1 - r) \cdot ER_f(\tau_2) - c^f_1 - c^f_2\},$$
$$\text{subject to } c^f \geq 0;\ f \neq g,\ f, g = A, B. \tag{5}$$

We generate campaign contribution supply functions through solutions to (5). Notice that $c_j = c^A_j + c^B_j$, for $j = 1, 2$; and, unlike voters, firms are risk-neutral.

2.4 *Equilibrium*

The election game with contributions, Γ, consists of two hierarchically linked games. The first of these, $\Gamma_{\{f\}} = (\mathbf{R}^2_+, \mathbf{R}^2_+; \{E\pi_f\})$, is played between firms: given candidate policies and the other firm's campaign contributions, each firm chooses donations to maximize expected profits described by (5). The second game, $\Gamma_{\{j\}} = (T, T; \{EV_j\})$, is played between candidates: given the opposing candidate's policy position, each contestant

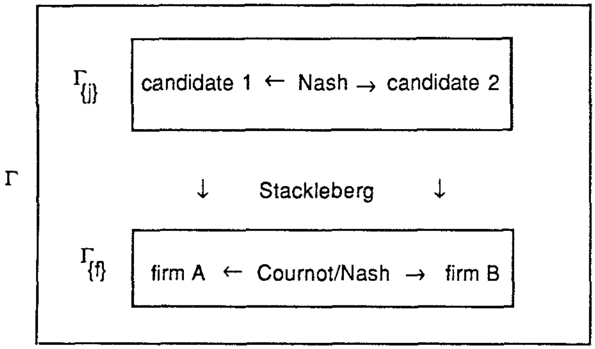

Figure 1. The structure of the election game

selects a position to maximize expected votes described by (4), *taking account* of the firms' implicit contribution-supply functions.

Given an arbitrary pair of policy positions, $t = (t_1, t_2)$, an equilibrium to $\Gamma_{\{f\}}$ is a pair of strategies, $(c^A(t), c^B(t)) \in \mathbf{R}^4_+$, such that:

$$E\pi_f(c^f(\cdot) \mid t, c^g(\cdot)) \geq E\pi_f(c^{f\prime} \mid t, c^g(\cdot)), \forall c^{f\prime} \in \mathbf{R}^2_+; f \neq g, \\ f, g = A, B. \tag{6}$$

An equilibrium to the full game Γ is a list of strategies, $(t^*, c^{A*}, c^{B*}) \in T^2 \times \mathbf{R}^4_+$, such that (6) holds for $t = t^*$ and:

$$EV_j(t_j^* \mid t_k^*) \geq EV_j(t_j \mid t_k^*), \forall t_j \in T; j \neq k, j, k = 1, 2. \tag{7}$$

Thus a firm plays Cournot/Nash with respect to all other players, but a candidate plays Nash against the opposing candidate and Stackleberg with respect to firms; and all agents maximize their payoffs. Figure 1 illustrates the structure of the game.

3. Results

By virtue of the assumptions on the utility function, U, indirect utilities $u_i(\cdot)$ are strictly concave on [0, 1] with unique maxima $\{t_i^*\}$ falling in the subinterval T. Hereafter, we approximate the indirect utilities by quadratic functions $\mu_i(\cdot)$.[8] To provide a benchmark against which to assess the impact of campaign contributions, suppose initially that such contributions are not allowed.

Proposition 1: Suppose that $c^f \equiv 0$, $\forall\, t \in T^2$, f = A, B. Then there exists a unique Nash equilibrium, $t^+ = (t_1^+, t_2^+)$, to the candidate game $\Gamma_{\{j\}}$.

Now admit the possibility of campaign contributions and fix candidate strategies, $t = (t_1, t_2)$. Without loss of generality, assume that $t_1 \leq t_2$.

Lemma 1: (a) $[t_1 = t_2] \Rightarrow [c^A(t) = c^B(t) = 0]$; and (b) $[t_1 < t_2] \Rightarrow [c^A{}_2(t) = c^B{}_1(t) = 0]$.

This fairly obvious Lemma says that firms will not make campaign contributions if candidates select identical policies and that each firm will support at most one candidate (see also Brock and Magee, 1978). In particular, each interest group (firm) supports the candidate spatially closer to its ideal point (but notice that the Lemma involves only *sufficient* conditions for zero contributions). This second property of the model may largely be an artifact of the one-dimensional policy space, coupled with the assumption of only two opposing interest groups. Given the Lemma and that $t_1 \leq t_2$, we can now economize on notation by identifying $c_1 = c^A{}_1 = c^A$ and $c_2 = c^B{}_2 = c^B$ (where the dependency of c^f on t is implicit).

Lemma 2: $\forall\ t \in T^2$ (with $t_1 \leq t_2$, by convention), there exists a unique Cournot/Nash equilibrium, $(c^A(t), c^B(t)) \in \mathbf{R}^4{}_+$, to the contribution game, $\Gamma_{\{f\}}$.

This result claims that the campaign-contribution supply functions are well-defined in the model. In particular, if these functions are differentiable (which is almost always the case: see the appendix), the Lemma allows use of the first-order and second-order conditions generated by solving problems (5) to characterize how (equilibrium) campaign contributions vary with candidates' policy announcements.

Proposition 2: Suppose that $[s_1(c^A(t)) - s_2(c^B(t))]$ is convex in t_1 and concave in t_2. Then there exists a unique equilibrium, $e^* = (t^*, c^{A,*} c^{B*}) \in T^2 \times \mathbf{R}^4{}_+$, to Γ, the election game with contributions.

Whether or not the sufficient condition for this result holds depends on the particular parametric structure of the variance schedules, s_j. For any t, the condition holds at least locally, since by Lemma 2, the difference $[s_1(\cdot) - s_2(\cdot)]$ is then approximately linear in t_j. In any event, we presume hereafter that the condition is satisfied: it is not a necessary condition for existence or uniqueness of equilibrium, and it plays no role in the analysis other than to ensure these properties for Γ.

We now can examine the effects of campaign contributions in the model on electoral policies. Let $e^+ = (t^+, c^{A+}, c^{B+})$; where $t^+ = (t_1{}^+, t_2{}^+)$ is the pair of policy announcements defined in Proposition 1, and $c^{f+} = c^f(t^+)$ is defined in Lemma 2 for policies $(t_1{}^+, t_2{}^+)$. Without loss of generality, take $t_1{}^+ \leq t_2{}^+$. Define the elasticities, $\zeta_j = -(s_j''/s_j')\cdot c_j$, $j = 1, 2$;[9] and let let $\omega_A = c^A/(1-\alpha)$, $\omega_B = c^B/(1-\beta)$.

Proposition 3: (a) $[c^{A+} = 0, c^{B+} = 0] \Rightarrow [e^* = e^+]$;
(b.1) $[c^{A+} > 0, c^{B+} = 0] \Rightarrow [t_j^* < t_j^+, j = 1, 2; \& \ c^{A*} > 0, c^{B*} \geq 0]$;
(b.2) $[c^{A+} = 0, c^{B+} > 0] \Rightarrow [t_j^* > t_j^+, j = 1, 2; \& \ c^{A*} \geq 0, c^{B*} > 0]$;
(c) $[c^{A+} > 0, c^{B+} > 0] \Rightarrow [\{t_j^* \leq (>) \ t_j^+, j = 1, 2\} \Leftrightarrow \{\omega_A \cdot \zeta_1^{-1} \geq (<) \ \omega_B \cdot \zeta_2^{-1} \text{ at } e^+\}; \& \ c^{A*} > 0, c^{B*} > 0]$.

This Proposition says, first, that firms will contribute only if at least one of them would wish to do so at the equilibrium prior to such contributions being permitted (t^+); second, if only one firm would choose to offer funds at t^+, then both candidates move towards that firm's ideal point. And third, if both firms would choose to make contributions at t^+, then candidates locate further leftwards (rightwards) at e* relative to e^+ depending on whether the weighted elasticity of *marginal* variance for candidate 2 at e^+ $(\omega_A \cdot \zeta_2)$, exceeds (falls short of) the weighted elasticity for 1 $(\omega_B \cdot \zeta_1)$. The easiest way to interpret this last condition is to suppose that $\zeta_1 = \zeta_2$ and $\alpha = \beta$. Then the condition says that $t_j^* < (>) \ t_j^+$ for $j = 1, 2$, if and only if $c^{A+} > (<) \ c^{B+}$: in other words, candidates move to favor the largest contributor. This result is entirely reasonable. If $\alpha \neq \beta$ and $\zeta_1 \neq \zeta_2$, the relative sizes of contributions at e^+ have to be appropriately weighted. Notice that it is possible, although unlikely, that permitting contributions has no impact on policy announcements here – *viz.* if equality holds in the condition.

One intuition behind these results is that campaign contributions in the model depend more on the *difference* between candidates' announced policy positions, than they do on the absolute location of these positions. Thus, a candidate who cannot distance himself sufficiently from the opposition, both to attract funds and remain viable in the election, will attempt to associate himself as closely as possible with the other candidate's policy stance: biased voting may 'prevent' complete convergence (the appendix shows that in cases (b.1) & (b.2) of the Proposition, a necessary condition for *both* firms to offer contributions at e* is $|t_2^* - t_1^*| > |t_2^+ - t_1^+|$).

If biased voting derives, say, from party identification, then the result suggests a connection between party identification and the impact of campaign contributions.[10] It is straightforward to check that with unbiased voting, candidate policies would converge in the absence of campaign contributions; that is, $t_1^+ = t_2^+$ (see Hinich, Ledyard and Ordeshook, 1972). By Lemma 1(a), no contributions would be forthcoming and so, by Proposition 3(a), permitting contributions would induce no effects. In this model, then, biased voting drives a wedge between candidates' positions and so generates campaign funds. But the *gap* between policies creates incentives for firms to contribute, and not biased voting *per se*.

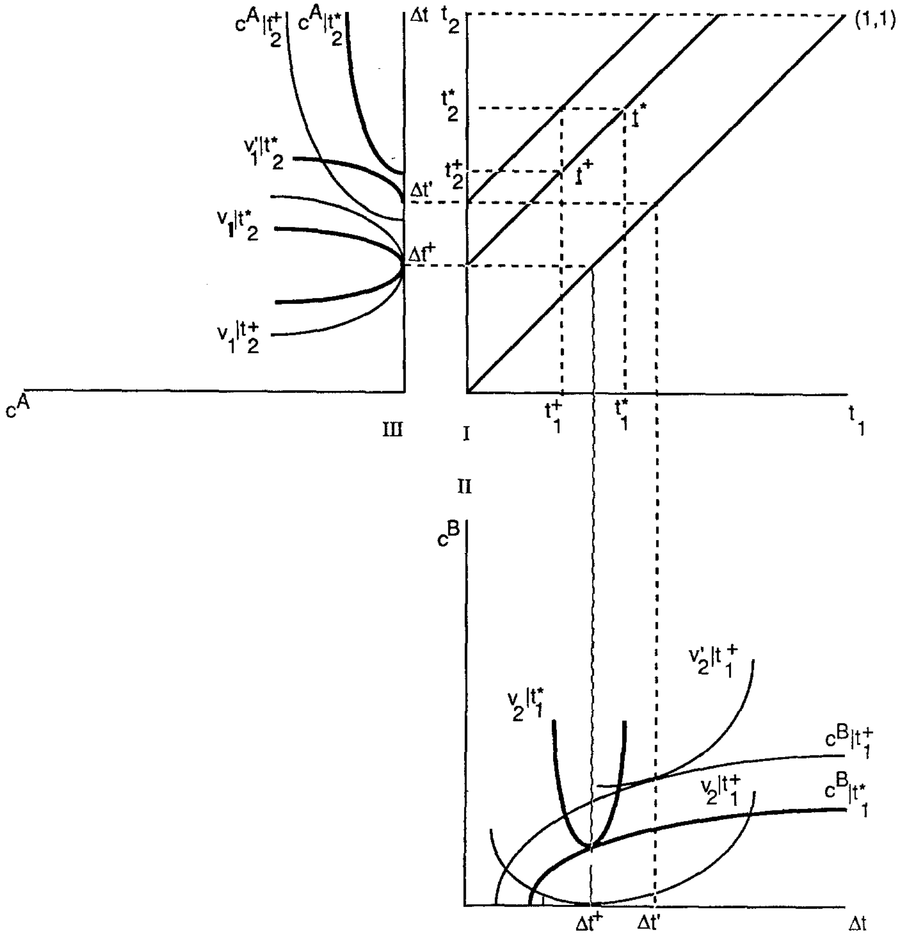

Fig. 2. Illustration for Proposition 3, case (b.2)

The proof of Proposition 3 is straightforward but somewhat lengthy. So, to provide a better grasp of the result's underlying structure, consider Figure 2, which presents a stylized illustration of case (b.2) – that is, if $c^B(t^+) > 0$ and $c^A(t^+) = 0$. Although the model is static, for heuristic purposes we explain the diagram with a 'dynamic' story.

The panels are labelled clockwise, I, II, III, beginning with the top right. Panel I is the space of possible policy pairs, $[0, 1] \times [0, 1]$. By convention, $t_1 \leq t_2$, so that only pairs of policies lying on or above the 45° line through (0, 0) are relevant. Given some point (t_1, t_2) within this region, the intercept of the 45° line through the point gives the difference between the policies, $\Delta t = (t_2 - t_1)$. We then project this difference onto panel II via the 45° line through (0, 0), and onto panel III – which we rotate anti-clockwise through 90° – directly (as drawn, for example, for the point t^+). In panel II (respectively, III), we graph the $\Gamma_{\{f\}}$-*equilibrium* level of firm B's (respectively, A's) contribution to candidate 2 (respectively, 1) as a function of Δt. This graph depends parametrically on candidate policy, t_1 (respectively, t_2). Thus, in panel II (respectively, III), the graph *not* in bold-face describes how the $\Gamma_{\{f\}}$-equilibrium level of c^B (respectively, c^A) varies with Δt, *given* $t_1 = t_1^+$ (respectively, $t_2 = t_2^+$). Also in panel II (respectively, III), we draw candidate 2's (respectively, 1's) iso-vote curve in campaign expenditure/policy difference space for various policy positions of candidate 1 (respectively, 2) – this being a parameter of such curves. So, for example, in panel II we draw the iso-vote curves *not* in bold-face taking $t_1 = t_1^+$, and we draw the curve in bold-face conditional on $t_1 = t_1^*$. Higher curves in both panels correspond to larger expected votes.

Let the pre-contribution equilibrium, $t^+ = (t_1^+, t_2^+)$, be as shown. With no contributions permitted, equilibrium with candidates maximizing votes involves tangency of an iso-vote curve with the horizontal axis at Δt^+ in panel II for candidate 2, and similarly in panel III for candidate 1 (by Lemma 1(a), $\Delta t^+ > 0$ for case (b.2)): for candidate 1 this curve is $v_1 | t_2^+$ in panel III, and for candidate 2 it is $v_2 | t_1^+$ in panel II. Given t^+, the induced contribution-supply functions, as we have just described them, are given by $c^B | t_1^+$ in panel II and $c^A | t_2^+$ in panel III. As the diagram shows, at t^+ firm B would like to offer candidate 2 funds, while firm A does not wish similarly to support candidate 1 (again, it is worth emphasising that these firm decisions are $\Gamma_{\{f\}}$-equilibrium responses).

Allow campaign contributions to be offered. Taking candidate 1's policy stance as given, candidate 2 can improve his expected vote by increasing t_2 up to t_2^*, and thus increase Δt to $\Delta t' = (t_2^* - t_1^+)$. This change in 2's position shifts A's campaign contribution schedule downwards, to $c^A | t_2^*$ and, still taking 1's policy fixed at t_1^+, shifts candidate 1's iso-vote map to the curves drawn in bold-face (panel III). Now hold 2's position fixed at t_2^*. Given t_2^*, candidate 1 can improve expected vote by changing policy from t_1^+. As panel III shows, increasing the difference in the candidates' policies by reducing t_1, can only make 1 worse off. But by increasing t_1 to t_1^*, thus reducing the policy gap back from $\Delta t'$ to Δt^+, candidate 1 moves to a better position. In so doing, 2's iso-vote map shifts to the curves in bold-face and, in this case ((b.2)), shifts firm B's contribution schedule down to $c^B | t_1^*$ (panel II). The policy point, t^*, is the new equilibrium in which B gives funds to 2, A still offers nothing to 1, and both policy announcements have shifted upwards. (Notice, however, that there is no presumption that $v_1 | t_2^+ = v_1 | t_2^*$, and so forth.) Although this final equilibrium to the full game, Γ, has $\Delta t^* = \Delta t^+$, we impose this equivalence only to avoid cluttering the diagram: the model does not allow us to say any more about the impact of permitting campaign contributions than Proposition 3 claims.

4. Conclusion

Insofar as concern over the impact of campaign contributions on policy reflects the possibility that such contributions distort the available electoral choices, the results here suggest that this concern has some justification. In the case in which no funds would be forthcoming were contributions allowed, permitting them has no effect on policy positions. In all other cases the impact on policy relative to what it would have been is unambiguously to push *both* candidates in a direction favorable to one (ordinarily, the larger) donor. In particular, in contrast to the intuition that explicit exchange models suggest, campaign contributions do not induce a more sharply distinguished choice for the electorate by driving candidates further apart and toward the extremes of the issue space (Peltzman, 1976, p. 215; Chamberlin, 1978).

In previous models designed to consider the effects of campaign funding on policy (for example, Ben-Zion and Eytan, 1974; Welch, 1974, 1980), the connection between donations and policy usually is direct: aggregate 'vote-production' and 'contribution-supply' functions being specified at the outset for candidates and contributors separately. There is no policy related voting and to the extent that there are strategic considerations, these are left implicit in the aggregate-vote and donation schedules. The simple model developed here is disaggregated and attempts to derive the vote and contribution functions. In this framework the structure of these schedules turns out to depend critically on the strategic actions of interest groups and candidates in the presence of policy-oriented voting.

Appendix

Proof of Proposition 1: For all i = 1, 2, ..., N, $\mu_i(\cdot)$ is quadratic. Using a Taylor expansion we obtain:

$$E\mu_i(\tau_j) = E[\mu_i(t_j) + \eta_j\mu_i'(t_j) + \eta_j^2\mu_i''(t_j)/2] = \mu_i(t_j) + \mu_i''\sigma_j^2/2$$

where $\mu_i'' < 0$ is a constant. Let $m_i \equiv \mu_i''/2$. Hence, for arbitrary i, j:

$$q_{ij}(E\mu_i(t_j) - E\mu_i(t_k)) \equiv q_{ij}(\mu_i(t_j) - \mu_i(t_k) + m_i\cdot[s_j(c_j) - s_k(c_k)]).$$

By assumption, $c^f_j \equiv 0\ \forall t$, $\forall f$, j; so that $s_j' \equiv 0\ \forall j$. Partially differentiating $EV_j(\cdot\mid\cdot)$ twice – first with respect to t_j and then with respect to t_k – it is easy to confirm that EV_j is strictly concave in t_j and strictly convex in t_k. Since EV_j is certainly continuous and T is compact, the result follows from Theorem 7.4 of Friedman (1977), for example.‖.

Proof of Lemma 1: The Kuhn-Tucker necessary conditions for a solution to the firms' maximization problems are, for j = 1, 2:

$$\partial r/\partial c^A_j\cdot(1-\alpha)\cdot wN\Delta t - 1 + \lambda_{Aj} \leq 0, \text{ with equality surely if } c^A_j > 0, \tag{a1}$$
$$\lambda_{Aj}c^A_j = 0,\ \lambda_{Aj} \geq 0.$$
$$\partial r/\partial c^B_j\cdot(\beta-1)\cdot wN\Delta t - 1 + \lambda_{Bj} \leq 0, \text{ with equality surely if } c^B_j > 0, \tag{a2}$$
$$\lambda_{Bj}c^B_j = 0,\ \lambda_{Bj} \geq 0,\ c^A_j \geq 0.$$

Here, λ_{fj} is the Lagrange multiplier on the constraint, $c^f_j \geq 0$; and $\Delta t = (t_2 - t_1) \geq 0$, by convention. Then part (a) of the Lemma follows immediately from complementary slackness. To check part (b), let $\Delta t > 0$ and consider firm A's choice of c^A. Now,

$$\partial r/\partial c^A_1 = s_1'\cdot\Sigma_N q_{i1}'\cdot m_i/N > 0, \tag{a3}$$
$$\partial r/\partial c^A_2 = -s_2'\cdot\Sigma_N q_{i1}'\cdot m_i/N < 0. \tag{a4}$$

Substituting these into (a1) and using $\lambda_{Aj}c^A_j = 0$, gives $\lambda_{A1} \geq 0$ and $\lambda_{A2} > 0$. The result follows for A. The argument for B is symmetric.‖.

Before proceeding, it is useful to point out that $\partial r/\partial c^A_2 \equiv \partial r/\partial c^B_2$.

Proof of Lemma 2: Fix $t \in T^2$ such that $\Delta t \geq 0$, and consider $E\pi_f$, as defined by (5). By Lemma 1(b), we can set $c^A_2 = c^B_1 = 0$. And to save notation, hereafter write $c^A \equiv c^A_1$ and $c^B \equiv c^B_2$. It is easy to check that $r(t, c)$ is strictly concave and increasing in c^A, and convex and decreasing in c^B. Hence, $E\pi_f$ is concave in c^f and convex in c^g, f ≡ g. Clearly, $E\pi_f \geq 0$ at any solution to f's maximizing problem. Therefore, without loss of generality, we can bound firm f's strategy space above by some finite number, say wN. Since $E\pi_f$ is continuous in c^A and c^B, we again can apply Theorem 7.4 of Friedman (1977) to yield the required existence result.

Proof of Proposition 2: In view of Proposition 1 and Lemma 2, we simply must check that candidates' payoff functions are concave/convex when campaign contributions are permitted to vary. Consider candidate 1, and recall that $\forall i \in N$,

$$q_{i1}(\cdot\mid\cdot,\cdot) = q_{i1}(\mu_i(t_1) - \mu_i(t_2) + m_i\cdot[s_1(c^A(t)) - s_2(c^B(t))]). \tag{a5}$$

By assumption, q_{i1} is concave and increasing in its argument; and $\mu_i(\cdot)$ is strictly concave in

t_j. Also, $m_i < 0$ is constant, and $[s_1(\cdot) - s_2(\cdot)]$ is convex in t_1 and concave in t_2. Hence, $q_{i1}(\cdot)$ is strictly concave in t_1 and strictly convex in t_2. Therefore, $EV_1(\cdot \mid \cdot)$ is concave/convex as required. Since there is no abstention, $q_{12} = 1 - q_{i1}$, $\forall i$. Consequently, $EV_2(\cdot \mid \cdot)$ is apropriately concave/convex also. The result now follows as for Proposition 1.∎.

Since there is no abstention in the model, we can economize on notation by letting q_i be the probability that i votes for candidate 1; so that $q_{i2} = 1 - q_i$, all i.

Proof of Proposition 3: Using Lemma 1(b), and noting that t will be strictly in the interior of T^2 at any equilibrium, (a6) – (a9) must be true at e*:

$$\Sigma q_i' \cdot (\mu_i'(t_1^*) + m_i \cdot [s_1' \cdot \partial c^A/\partial t_1 - s_2' \cdot \partial c^B/\partial t_1]) = 0; \tag{a6}$$

$$\Sigma q_i' \cdot (\mu_i'(t_2^*) + m_i \cdot [s_2' \cdot \partial c^B/\partial t_2 - s_1' \cdot \partial c^A/\partial t_2]) = 0; \tag{a7}$$

$$c^{A*} \cdot [\partial r/\partial c^A \cdot (1-\alpha) \cdot wN\Delta t - 1 + \lambda_A] = 0, \tag{a8}$$

$$\lambda_A c^{A*} = 0; \lambda_A \geq 0, c^{A*} \geq 0;$$

$$c^{B*} \cdot [\partial r/\partial c^B \cdot (\beta - 1) \cdot wN\Delta t^* - 1 + \lambda_B] = 0, \tag{a9}$$

$$\lambda_B c^{B*} = 0; \lambda_B \geq 0, c^{B*} \geq 0.$$

In equations (a6) and (a7), the terms in $\partial c^f/\partial t_j$ describe the marginal change in the $\Gamma_{\{f\}}$-equilibrium level of c^f induced by an incremental change in t_j. As will become clear, these levels are differentiable at e*, and so (a6) and (a7) are well-defined. Now, if $c^f \equiv 0 \; \forall \; t$, then (a6) and (a7) collapse to:

$$\Sigma q_i' \cdot \mu_i'(t_j^+) = 0, j = 1, 2. \tag{a10}$$

To examine the impact of introducing campaign funds on candidates' policy positions, it is enough (because EV_j is strictly concave in t_j) to sign $\partial EV_j/\partial t_j$ at t^+, j = 1, 2. Assume that the contribution supply functions $\{c^f(\cdot)\}$ are differentiable at t^+. Then, in view of (a10), and because $q_i' > 0 \; \forall i$, and $m_i < 0 \; \forall i$, this amounts to signing,

$$Z_1 \equiv [s_1' \cdot \partial c^A/\partial t_1 - s_2' \cdot \partial c^B/\partial t_1] \text{ and } Z_2 \equiv [s_2' \cdot \partial c^B/\partial t_2 - s_1' \cdot \partial c^A/\partial t_2]$$

at t^+ if campaign contributions are permitted. There is a knife-edge case in which $\partial c^f/\partial t_j$ will not be defined at t^+. This case occurs if the profit-maximizing choice of c^f at t^+ is precisely zero in the absence of the nonnegativity constraint. But if this condition occurs, an arbitrarily small (but non-zero) perturbation in the parameters α, β is sufficient to restore differentiability. Consequently, since this case almost never will obtain and is of little intrinsic interest, we assume the differentiability of $\{c^f(\cdot)\}$ at t^+ hereafter.

Case (a): In this instance, $c^{A+} = c^{B+} = 0$. Since (a8) and (a9) must hold, *mutatis mutandis*, at any t such that $\Delta t \geq 0$, we have $\lambda_A > 0$ at t^+ for firm A. Hence,

$$\partial r/\partial c^A \cdot (1-\alpha) \cdot wN\Delta t^+ < 1.$$

The left-hand side of this inequality is the marginal benefit to A of contributing to candidate 1, while the right-hand side is the associated marginal cost. Both are continuous in t, so that the inequality is preserved for incremental changes in policy positions (see panel III of Figure 2 for an illustration). Hence, $\partial c^A/\partial t_j = 0$ at t^+, j = 1, 2. Similarly, $\partial c^B/\partial t_j = 0$ at t^+, j = 1, 2. This establishes case (a).

To prove the remaining cases, it is convenient first to fix t, so that $\Delta t > 0$ and $c^f(\cdot) > 0$, for $f = A, B$. Then, $\lambda_f = 0$, $f = A, B$, and total differentiation of equations (a8) and (a9) for fixed t_k, $k \neq j$, yields this system for $j = 1, 2$:

$$\begin{bmatrix} H_A & 0 \\ 0 & H_B \end{bmatrix} \begin{bmatrix} \partial c^A/\partial t_j \\ \partial c^B/\partial t_j \end{bmatrix} = \begin{bmatrix} -L_{Aj} \\ -L_{Bj} \end{bmatrix}. \tag{a11}$$

In (a11), we define,

$$\begin{aligned} H_A &= (1-\alpha)\cdot w\Delta t\cdot s_1''\cdot\Sigma q_i'\cdot m_i \quad < 0, \\ H_B &= (1-\beta)\cdot w\Delta t\cdot s_2''\cdot\Sigma q_i'\cdot m_i \quad < 0, \end{aligned} \tag{a12}$$

and,

$$\begin{aligned} L_{Aj} &= wN\cdot[\partial r/\partial c^A\cdot\partial\Delta t/\partial t_j]\cdot(1-\alpha), \\ L_{Bj} &= -wN\cdot[\partial r/\partial c^B\cdot\partial\Delta t/\partial t_j]\cdot(1-\beta). \end{aligned} \tag{a13}$$

Then, by Lemma 2 and substituting for $\partial r/\partial c^f$, we can solve (a11) to obtain, for $j = 1, 2$:

$$\partial c^A/\partial t_j = -(s_1'/s_1'').(\partial\Delta t/\partial t_j)/\Delta t, \tag{a14}$$

$$\partial c^B/\partial t_j = -(s_2'/s_2'').(\partial\Delta t/\partial t_j)/\Delta t, \tag{a15}$$

Case (b.1): Here, t^+ is such that $c^{A+} > 0$ and $c^{B+} = 0$. Then, by the argument for case (a), and using (a14),

$$Z_1 = (s_1')^2/s_1''\Delta t = Z_2 > 0. \tag{a16}$$

Therefore,

$$m_i\cdot Z_1 < 0 \;\&\; m_i\cdot Z_2 < 0 \text{ at } t^+. \tag{a17}$$

So, by (a10), $\partial EV_j/\partial t_j < 0$ at t^+, $j = 1, 2$. By concavity of EV_j in t_j, therefore, these inequalities prove that $t_j^* < t_j^+$, $j = 1, 2$, as required.

To check $c^{A*} > 0$ and $c^{B*} \geq 0$, notice first that, since e^* is unique and $t_j^* < t_j^+$, the result of case (a) implies that $c^{f*} < 0$ for at least one firm $f \in \{A, B\}$. Suppose $\Delta t^* \leq \Delta t^+$. Then, because c^B is non-decreasing in t_2, $c^{B+} = 0$ and $t_2^* < t_2^+$ are sufficient to guarantee $c^{B*} = 0$. Hence, $c^{A*} > 0$. Suppose that $\Delta t^* > \Delta t^+$: since $c^{A+} > 0$, $c^{B+} = 0$, and $\partial c^A/\partial t_1 < 0$ at e^+ ((a14)), continuity of the $\Gamma_{\{f\}}$-equilibrium mapping implies that $c^{A*} > 0$. This completes the proof of case (b.1). The argument for (b.2) is symmetric and therefore omitted.

Case (c): In this case, $c^{f+} > 0$, $f = A, B$. Then at t^+,

$$Z_1 = [(s_1')^2/s_1'' - (s_2')^2/s_2'']/\Delta t^+ = Z_2. \tag{a18}$$

From the first-order conditions (a8) and (a9), $c^{f+} > 0$, $f = A, B$, implies:

$$s_1' = (1-\beta)/(1-\alpha)\cdot s_2'. \tag{a19}$$

Let $\rho = (1-\beta)/(1-\alpha)$. By Lemma 1, $\Delta t^+ > 0$ so that:

$$Z_j \geq (<)\, 0 \Leftrightarrow \rho^2 \geq (<)\, s_1''/s_2''. \tag{a20}$$

Therefore, $m_i Z_j \leq (>) 0$, as $\rho^2 \geq (<) s_1''/s_2''$. Using the argument following (a17), we obtain $t_j^* \leq (>) t_j^+$, $j = 1, 2$, as $\rho^2 \geq (<) s_1''/s_2''$. It is now straightforward to check that $c^{f*} > 0$, $f = A, B$. Hence, case (c) follows. This completes the proof of Proposition 3.||

NOTES

1. This assumption implies that the model is not closed. But permitting people to own shares in the firms results in the indirect utilities, that we discuss later, losing the properties required to obtain an equilibrium to the model. Furthermore, once we introduce campaign funding, the objective function of any such firm is not at all well-defined.
2. This assumption is convenient but unnecessary. As it later becomes clear, both firms would vote with probability one for opposing candidates if candidates' policies were distinct; and vote for either with probability 1/2 if policies coincided.
3. Assuming that individuals perceive candidates' positions in the same (noisy) way is unnecessary for the results, but it eases the burden on notation considerably.
4. Without this restriction, the d.f. of the random variable clearly would depend on the location of candidates' policies, which would unnecessarily complicate the issues of interest in this essay. And, evidently, choice of policies in $[0,1]\backslash T$ always will be dominated for office-seeking candidates.
5. See Enelow and Hinich (1984, Ch. 4) on this conjecture.
6. I assume this relationship, incorrectly I now believe, in Austen-Smith (1981). See Jacobson (1980) on the empirical evidence for the impact of campaign expenditures on aggregate vote shares.
7. Specifically, without linearity: (1) for Lemma 2, we would have to assume candidate 1's expected vote to be explicitly convex in 2's campaign expenditures; and (2) for Proposition 3 (the main result), additional restrictions on the signs of the cross-partial derivatives of the candidates' votes with respect to campaign contributions and policy are necessary. Both (1) and (2) involve quantitative restrictions on second-order effects in vote-probabilities.
8. The rationale for adopting a quadratic approximation is to allow us to ignore terms in third or higher order derivatives of $u_i(\cdot)$ – and hence, third and higher moments of the distribution of η_j – in the arguments for our results. Since such terms invariably occur multiplied by numbers smaller than one in absolute value and raised to third or higher orders, this approximation seems justified here.
9. In this definition, we understood c_j as $c_j = c^A{}_j{}^+ + c^B{}_j{}^+$ when considering the benchmark case, e^+.
10. I owe this observation to a referee.

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