

## Complexities in Electoral Systems: Do Simple Models Work Anyway?

A theme of this book has been that we can develop logical quantitative expectations about the impact of electoral systems, *provided the system is relatively simple*. Many of the logical models we have devised and regressions we have undertaken have been based on systems we have classified as “simple,” while others have included one class of complex system, two-tier proportional. Our reason for not attempting to model more complex systems is that, the more additional complications are added to rules, the more difficult it is to make meaningful predictions. Moreover, if we are unable to offer predictions as political scientists, committed as we are to logical quantitative models, we are even less able to offer advice to practitioners other than, “Keep it simple.” Nonetheless, electoral system designers often do not keep things simple, either because they have sincere preferences for a system that is complex, or, more often, they must make compromises with other interests that have diverging preferences.

In Chapter 3, we identified several features of electoral systems that render them complex. We have already found that one of these, a compensatory upper tier, can be included in an extended Seat Product Model, making such systems broadly predictable in their impact on party systems (Chapter 15). Another complex system, the single nontransferable vote (SNTV), was a theme of Chapters 13 and 14, but we focused on its effects on intraparty outcomes, not its effect on the interparty dimension. Other complex rules have been left out till now. In this chapter we bring them back in, asking to what extent their effects on interparty outcomes could be systematically predicted. First we consider ranked-choice (transferable) voting, nontransferable voting, and two-round systems. We ask whether these more complex formulas nonetheless might be predictable almost as if they were simple systems. The answer for this set of complex systems is a cautious yes, except in the case of nontransferable voting systems.

Then we turn our attention to a common complicating feature found in many contemporary electoral systems, legal thresholds. We develop a logical model of the impact of legal thresholds. We find that it is successful for some types of systems, but not for others. In particular, as long as the system is otherwise

simple in its structure or is a two-tier compensatory system, the Seat Product Model is generally more accurate than the threshold model. Yet for other more complex systems, the threshold model shows some promise. Understanding the effect that thresholds have (or do not have) is important, given that in recent decades many new systems have incorporated various threshold provisions.

#### SINGLE-TIER SYSTEMS WITH COMPLEX FEATURES: HOW SIMPLE?

In Chapter 3 we discussed several rules that introduce complexity aside from legal thresholds or composite, two-tier, systems. These latter features are discussed later in this chapter, but first we turn to rules that might render a single-tier system – one in which all seats are allocated in districts of some magnitude – too complex for the Seat Product Model to be useful.

More complex rules include two-round majority (or majority-plurality), and systems that use ranked-choice ballots: the Alternative Vote (AV, or instant runoff) and the single transferable vote (STV) variant of PR. Additionally, while we stated in Chapter 3 that SNTV is arguably the simplest electoral formula that could be used when  $M > 1$ , it is not at all simple in the incentives it gives to parties attempting to maximize their own seat total, as we discussed in Chapters 13 and 14. In this section, we look at the main examples of each of these systems and ask whether the Seat Product Model performs well for them or not. We would not expect it to do well, because it was designed for simple systems (and, in its extended form, two-tier compensatory PR). However, it is an empirical question. Actually that question could be reframed as, do these more complex systems function *as if they were simple*?

In Table 16.1 we see seven single-tier complex electoral systems in six countries.<sup>1</sup> For each system, we compare the actual SPM predictions of their effective number of seat-winning parties ( $N_S$ ) and seat share of the largest party ( $s_1$ ) to the actual values. The intention here is to determine whether, were we to take any one of these systems' Seat Product,  $MS$ , and treat as if it were a simple system, would our predictions be off the mark? In order to facilitate making this determination, the cases are sorted in Table 16.1 according to increasing  $MS$ .

#### Single-Seat Districts with Complex Rules

The first entries in the table are for the three  $M=1$  systems in Australia and France. The formulas used in these countries are quite different from one another, and from plurality. As introduced in Chapter 3, Australia uses the alternative vote (AV), with its ordinal ballots, whereas France uses the majority-plurality formula in two rounds. AV might be thought of as the closest system

<sup>1</sup> The French use of a two-round system was interrupted by the one-time use of list PR in 1986. For this reason, and the large expansion of assembly size, we show two electoral-system periods for France under a two-round system.

TABLE 16.1 *Single-tier systems with complex formulas: actual versus predicted effective number of seat winning parties ( $N_s$ ) and seat share of largest party ( $s_1$ )*

Country	Years	No. of elections	Formula	MS	Actual $N_s$	$N_s$ from SPM	Ratio for $N_s$	actual $s_1$	$s_1$ from SPM	Ratio for $s_1$
Australia	1949–2011	25	AV	134	2.48	2.26	1.10	0.505	0.542	0.93
France	1958–1981	7	Two rounds	497	3.50	2.81	1.25	0.453	0.46	0.98
France	1988–2007	5	Two rounds	577	2.85	2.89	0.99	0.497	0.451	1.10
Colombia	1991–1998	2	SNTV	756	3.09	3.04	1.02	0.5	0.434	1.15
Japan	1952–1993	16	SNTV	1936	2.76	3.53	0.78	0.531	0.388	1.37
Vanuatu	1983–2004	6	SNTV	146	3.79	2.29	1.66	0.456	0.536	0.85
Ireland	1954–2007	14	STV	601	2.79	2.91	0.96	0.478	0.449	1.06
Malta	1966–2008	10	STV	321	1.995	2.62	0.76	0.526	0.486	1.08

Are single-tier complex systems similar to simple systems? Values are mean for entire time period indicated

Note: Mean values for  $N_s$  in Colombia exclude 1994 and 2002, due to a large “other” category preventing accurate calculation.

one could have to FPTP (plurality) without having FPTP (e.g., Lijphart 1991), whereas most scholars see the French two-round system as meaningfully distinct from FPTP. In fact, when Duverger (1986) updated his “law” he included the two-round system along with PR as a system that promotes multipartism.

Despite the important distinctions between AV and two-round majority-plurality, the statistics in Table 16.1 suggest that if we pretended these were simple FPTP systems, our predictions would not be far off. The partial exception is the earlier period in France, where actual  $N_S$  exceeds our SPM-derived prediction by a factor of 1.25. Yet the seat share of the largest party,  $s_1$ , is spot-on, as are, essentially, both values for the more recent period in France and in Australia. What this implies is that the reason France tends to have a higher effective number of seat-winning parties, and lower seat share of the largest party, relative to Australia is not the electoral formula, but the very different assembly sizes.

Even with FPTP, we would expect France to have  $N_S$  closer to 3.0 than to 2.0, and a largest party just short of majority status, because its assembly is so large. Meanwhile, we would expect Australia to have  $N_S$  under 2.5, and generally a majority party, because its assembly is not so large. Thus we cannot say that AV and two rounds are fundamentally different from FPTP, in terms of their impact on the assembly party system, *once assembly size is considered*. Because there are so few actual examples of these systems, we would caution that one should not assume that their performance would be similar if adopted elsewhere.

The two systems diverge substantially from one another, however, if we look at the electoral party system. Using the MS-based formula derived in Chapter 8, we would expect Australia to have  $N_V$  around 2.68 for the elections shown in Table 16.1. The actual average is 2.90, only marginally higher. For France, on the other hand, the average over the 1988–2007 period would be expected, from the formula, to be  $N_V = 3.27$ . In reality it has been 5.49, dramatically higher.<sup>2</sup> The two-round system used in France encourages many parties to run in the first round, and then line up behind alliance partners before the second round – a game not possible in the AV “instant runoff.” The result is to encourage fragmentation of the vote, making it all the more striking that the assembly party system results about where we would expect if it were a FPTP system.

With this information about higher electoral fragmentation in France, we can understand better now what we saw in Chapter 11, in which France had several outlier elections in its values of  $N_V$  (and also  $N_P$ , for presidential candidates, which we derive by way of  $N_V$ ). We included France in Chapters 11 and 12 on the basis of the information revealed here, in Table 16.1, that shows its party system looks like that of a simple electoral system for the *assembly* parties ( $N_S$ ). Thus the

<sup>2</sup> We base the calculation of actual  $N_V$  on first-round votes in France and first-preference votes in Australia.

French two-round system is de facto almost simple for seats, but not for votes, due to many parties entering first-round elections (for president as well as assembly). Many of these vote-earning parties likely would not exist, or would be more minor, if the system were simple FPTP, yet they are winning seats at about the rate we would expect from FPTP.

### Multiseat Districts with SNTV or STV

As for the  $M > 1$  formulas in Table 16.1, two of three examples of SNTV show a largest party that is larger than expected (if we took the system to be simple PR). Correspondingly, Japan's  $N_S$  was much lower than expected, although Colombia's was near expectation and Vanuatu's tends to be smaller than expected. The advantage that large parties often obtain under SNTV is an established attribute of the system (Cox 1997: 243–249; Shugart, Moreno, and Fajardo 2007), because a governing party has superior access to resources that enable it to divide the vote efficiently – the vote-management strategies that we have discussed. (Apparently this is not possible in Vanuatu, where fragmentation is particularly high.)

It would seem that SNTV is easier to predict on the intraparty dimension (as we showed in Chapter 13) than on the interparty dimension. Perhaps this is unsurprising, given that SNTV is a purely candidate-based system, meaning that collective party vote totals are not relevant for determining the outcome across parties. Yet the system provides parties with strong incentives to organize themselves so as to avoid having excess candidates or too-unequal vote distribution among candidates. This is an easier task for large parties (due to resources permitting coordination) and also for small parties (coordinate on one). The middle-sized parties are likely to get squeezed, allowing SNTV to be simultaneously advantageous to large parties<sup>3</sup> and superproportional (Shugart, Moreno, and Fajardo 2007).

The statistics in Table 16.1 suggest that we should not treat SNTV as if it were just a simple PR system; for STV, on the other hand, there is somewhat more reason to treat it as such. For the country with the longest experience of STV, Ireland, both indicators are almost exactly as the SPM predicts, under the assumption of a simple system. The same can be said for the largest party size in Malta, even though from the SPM's perspective, the nearly pure two-party system in Malta is a surprise.<sup>4</sup>

The exercise in this section suggests that ranked-ballot and two-round systems, but not SNTV, perform in a way not fundamentally different from simple systems with the same Seat Product. We need to add the caution,

<sup>3</sup> Although, again, not in practice in Vanuatu.

<sup>4</sup> What about  $N_V$ ? In Ireland, the votes also turn out almost precisely as the  $MS$ -based formula would lead us to expect (actual 3.18 versus expected 3.28). In Malta, considerably lower (2.07 versus 3.01).

however, that with these formulas being relatively rare, we are unable to say that another country, or subnational jurisdiction, adopting either of these formulas would have a similar experience.

### Other Complex Formulas for Multiseat Districts

The systems listed in Table 16.1 do not completely exhaust the complex formulas used in single-tier systems with multiseat districts. We discussed several other examples in Chapter 3. Most of them are rare, but we will discuss briefly two other types: two rounds in multiseat districts and multiple nontransferable vote (MNTV), also known as bloc vote or unlimited vote.

In the set of countries for which we have data, there is one case of  $M>1$ , two-round majority-plurality. That is Mali, which had a period of democratic rule before a military coup in 2012. The first democratic election took place in 1992, with a second in 1997. For these elections, the mean assembly size was 138, and mean magnitude around 2.4. If we calculated a Seat Product from these values, it would be 323. By the Seat Product Model, then, we would predict  $N_S=2.63$  and  $s_1=0.484$ . That is, those would be the predicted values *if Mali had used a simple system*.

Mali's system was not, however, simple – not only due to the two-round formula, but also due a further complicating characteristic: this formula was used in multiseat districts in which the voter was given a single party-list vote (Reilly et al. 2005: 170). As Taagepera (2007: 92) argues, for multiseat plurality, the mean magnitude should be first raised to the power, negative one, before calculating the Seat Product. This would imply, averaging the two Malian elections, a Seat Product of

$$M^FS = (2.4^{-1})138 = 58.1,$$

where the exponent,  $F$ , is an adjustment for allocation formula.<sup>5</sup> If we use this adjusted Seat Product, we would expect  $N_S = 1.97$  and  $s_1=0.60$ . Note the dramatic difference when we change the assumption of a simple system and account for the use of multiseat plurality (or, in this case, majority-plurality). We go from a largest party just short of half the seats to one that is expected to have a three-fifths super-majority.

In fact, the actual values, averaged over these elections, were  $N_S = 1.78$  and  $s_1=0.73$ . Looking at the individual elections reveals an even worse result, from the standpoint of a country's politics adapting to electoral-system design. In 1992 the country had  $N_S = 2.24$  and  $s_1=0.496$ ; in the second election, the leading party, the Alliance for Democracy in Mali, capitalized on both incumbency and a large-party-favoring electoral system to give  $N_S = 1.31$  and

<sup>5</sup> For simple systems, including FPTP and the standard PR formulas,  $F=1$ , as explained by Taagepera (2007: 92).

$s_1=0.87$ .<sup>6</sup> It is almost as if the political elites and voters responded to the system in 1992 as it were low-magnitude PR, but then in the second use of the system, the potential space for any significant opposition simply collapsed under an electoral system that left no room for parties other than the ruling one. Colomer (2014) commented before the 2012 coup that Mali had political institutions that weakened the viability of its democracy.<sup>7</sup> We can agree with this assessment, albeit retroactively.

Another rare form of multiseat district formula is multiple nontransferable vote (MNTV). In this system, the voter has votes for as many candidates as there are seats in the district. Is this equivalent to multiseat plurality, greatly advantaging the largest party? Or is it more like SNTV, encouraging strong candidate-centered politics? Our weaselly answer is, it depends. We offer two short case studies to demonstrate.

In Thailand, MNTV formerly was used for an assembly of 360 seats (increased to 390 in 1996) using a mean district magnitude of around 2.5. The system was in place in the 1980s and 1990s. In only one election of this period was  $s_1$  larger than 0.30; in four of them it was under 0.25. Similarly,  $N_S$  was over 6.00 in several elections before falling to 4.32 in 1996;  $N_V$  was, of course, even higher. Although the effective numbers were lower at the district level – that is, much of the high fragmentation at the national level was a result of there being many regional parties (Hicken 2009: 110–112) – the system clearly did not contribute to nationwide consolidation of parties, as we might expect from a system that resembles a multiseat plurality system.

Studies of Thai party politics in this era emphasize how little party cohesion there was, and how correspondingly personalistic the politics was (Hicken 2009: 94–97). So, the Thai experience looks like a case both of highly candidate-centered politics (more akin to SNTV), and without any dominant party (unlike some actual SNTV cases, notably Japan). Clearly, many voters either did not use all the votes allotted them, or used them for candidates of different parties. The result was utter fragmentation, which was a contributing factor in the major constitutional reform of 1997 that sought to encourage consolidation of the party system (MacIntyre 2003: 118–119). As we will see later in this chapter, the MMM system adopted under the new constitution “worked” by that standard, although not by others.

Our second example of MNTV comes from the Legislative Council election in the Palestinian Territories of 2006. In this case, the system was even more complex; there was a nationwide party-list component in addition to the basic districts. It is the latter component that concerns us here. It consisted of sixty-six seats in districts that ranged in magnitude from one to nine, averaging 4.1. The contest pitted the established ruling party, Fatah, against Change and

<sup>6</sup> The party had 63 percent of the votes in 1997, up from 48 percent in 1992.

<sup>7</sup> Although Colomer’s paper was published in 2014, a working paper version making this claim about Malian institutions was circulating months before the May 2012 coup.

Reform, a party put together for the election by Hamas, the armed militant Islamic Resistance Movement. In the separate party-list vote, Change and Reform bested Fatah only 44.5 percent to 41.4 percent. Yet Change and Reform won forty-five (68.2 percent) of the seats in the MNTV tier (and seventy-four overall, when its share of the sixty-six noncompensatory PR seats was added). Thus one party was strongly advantaged, earning over two-thirds of the MNTV-elected seats, almost as if it was a party-centered electoral system.

What explains the difference, relative to Thailand? In the Palestinian case, voters tended towards voting for the full slate of candidates of one party. This was even truer for Change and Reform than for Fatah (Abdel-Ghaffar, et al., n.d.). The result was that most districts were clean sweeps, and were especially likely to be so if Change and Reform was the party with the local plurality.<sup>8</sup> Thus we can conclude that when MNTV is used in a context in which the leading party has a strongly party-motivated constituency, it is almost as if it was multiseat block plurality. The result, in the Palestinian case, was a surprise sweeping victory by a party committed more to armed struggle than to parliamentary politics, and the process soon broke down into a spiral of violence.

By contrast, the Thailand case shows that the MNTV formula need not be so party-centric: if voters cast their votes based on individual candidates, seats in any given district may be won by multiple parties. These two cases of MNTV thus represent extremes of the possible outcomes. That such similar electoral formulas can produce such divergent results, depending on context, is probably a good reason to avoid using MNTV, and to prefer simplification.

### **Magnitude Variance in PR**

Another potentially complicating feature in single-tier PR systems is magnitude variance. Taagepera (2007) suggested that highly unequal magnitude of districts could place a system outside the category of “simple.” Most districted PR systems rely on existing administrative divisions to serve as electoral districts, and because these divisions vary in population, they may vary widely in magnitude, as well. How much does such variation complicate predicting their party fragmentation? We are unable to find any evidence that unequal *M* results in any worse performance of the SPM.<sup>9</sup> We now turn to the impact of thresholds, in either single-tier or composite systems.

<sup>8</sup> Abdel-Ghaffar, et al., (n.d.) show that, contrary to some claims in media sources, Fatah did not lose because it split and presented competing slates. There were some districts where independents who were Fatah defectors cost the mainstream party seats, but the overall loss was caused by Change and Reform being more popular, having voters more likely to cast most or all their available votes on its candidates, and the resulting ability to sweep most or all seats in several multiseat districts.

<sup>9</sup> The feature can be conceptualized as a deviation from a norm – here, equal *M*. If we apply the measure of deviation from proportionality ( $D_2$ , see Chapters 4 and 9) to each district’s difference



## A LOGICAL MODEL FOR IMPACT OF THRESHOLDS

The remaining forms of complex electoral system that we will investigate include those that have a legal threshold. A threshold may result in a party being ineligible to win seats even if it had sufficient votes to win were it not for the threshold. In other words, legal thresholds can counteract the effects of a large magnitude, a high share of compensation seats, a permissive formula, or other aspects of an electoral system that would favor the representation of small parties. In order to assess whether the Seat Product Model offers guidance to the party system that results from a given electoral system, even when a threshold intervenes between votes and seats, we need to attempt to develop a logical model of the impact of legal thresholds. We do so in this section. Then we pit the predictions of the threshold model against those of the SPM on several sets of complex electoral systems.

## First Approximation to the Model of Thresholds

Suppose seats in the national assembly are allocated by nationwide proportionality, restricted by a nationwide threshold in terms of a minimal percentage of votes. How does this legal threshold affect the number and size of parties in the assembly?

A higher legal threshold can be expected to reduce the number of parties and thus increase the seat shares of the surviving parties. By so doing, it can also be expected to reduce the effective number of parties and indirectly prolong the duration of governmental cabinets. But going beyond directionality, to which quantitative extent would legal thresholds affect these quantities?

Our objective in this section is to construct a quantitatively predictive logical model, connecting the aforementioned outputs to the nationwide legal threshold. We will then test it on several classes of systems that employ legal thresholds, and compare the threshold model to the SPM.

The starting point for the SPM was the observation that, in a single district of  $M$  seats, at least one party must win seats, and at most  $M$  parties can win seats (see Figure 1.2 and Chapters 7 and 9). In an assembly of  $S$  seats, allocated by PR with no legal threshold, this corresponds to a range from 1 to  $S$ . When a sufficiently high legal threshold ( $T$ ) is introduced, the plausible number of seat-winning parties is reduced. The number of parties reaching the threshold can in this way be at most the integer part of  $1/T$ . For instance, with a threshold of  $T=4$  percent, at most  $1/0.04=25$  parties could gain representation; whereas if the threshold were to be increased by 0.01 percent, to  $T=4.01$  percent, we get  $1/T=23.94$ , and only 23 parties

from the system's mean  $M$ , and then regress this against the SPM prediction, we get  $R^2$  of zero. Literally, 0.000. Magnitude variance has no measurable impact on either  $N_S$  or  $s_1$ . This does not preclude the possibility that it has other effects, for instance favoring certain parties over others (Monroe and Rose 2002, Kedar et al. 2016).

could surmount the threshold. On the average, this integer part is  $(1/T)-1/2$ , but as long as  $T$  is less than 20 percent, approximation by  $1/T$  introduces an error of estimate of less than 10 percent. The number of seat-winning parties is thus restricted to the range,

$$1 < N_{S0} < 1/T.$$

As in the basic party size model, without any further information, our best guess is that the number of seat winning parties,  $N_{S0}$ , would be around

$$N_{S0} \approx (1/T)^{1/2} = T^{-1/2}.$$

[16.1; nationwide PR with electoral threshold larger than  $1/S$ ]

For this model to apply, the electoral threshold must equal or surpass  $1/S$ . If not, then  $N_{S0} = S^{1/2}$  prevails, as it then imposes a more stringent limitation. The actual legal thresholds on top of nationwide PR always exceed  $1/S$ , except in the Netherlands, where they have equaled  $1/S$  since 1935.

We can test Equation 16.1 via regression on the logs. When we do, it is at first not promising, as we obtain  $\log N_{S0} = 0.346 - 0.328 \log T$ . The result is absurd, as it must be the case that if  $T=1$  (i.e., 100 percent of the votes), only one party can win – the party that obtains all of the votes, which logically also gets 100 percent of the seats. Yet the regression result predicts  $N_{S0} = 2.2$  when  $T=1$ . We can re-estimate the regression by suppressing the constant term. When we do so, we obtain:

$$\log N_{S0} = -0.515 \log T.$$

This is obviously approximately Equation 16.1. Thus we have some confirmation of the logic. However, it must be considered weakly supported, relative to the SPM, because of the need to suppress the regression constant term. Such a step was not required for testing the effect of the Seat Product,  $MS$ . The difference in success of model testing may be because  $MS$  has a wide range and we had hundreds of elections to test it on. In the case of legal thresholds, we have a narrow range of  $T$ : excluding cases of no legal threshold,<sup>10</sup> the range is only 0.67 percent to 10 percent. We have only fifty-eight elections (in ten countries) on which to test it. Given the data limitations, then, we can be satisfied that the logic is valid and attempt to extend it to the more important quantities,  $N_S$  and  $s_1$ .

From Chapter 7 (and Table 9.2), we have  $s_1 = N_{S0}^{-0.5}$ . Combining this relationship with Equation 16.1, we should have:

$$s_1 \approx T^{1/4}. \quad (16.2)$$

<sup>10</sup> We are unable to include these in our regression because Equation 16.1 applies only when  $T \geq 1/S$ .

The final step, to the effective number of seat-winning parties, is possible because of the formula  $N_S \approx s_1^{-4/3}$  (see Table 9.2). Thus we expect:

$$N_S \approx T^{-1/3}. \quad (16.3)$$

We can test these via regression on the logs. Once again, we must use no-constant regression. For both of these equations we have more elections with the complete data, giving us 159 observations. The results are:

$$\log s_1 = 0.265 \log T \text{ and } \log N_S = -0.371 \log T.$$

These numbers are close enough to the predicted 0.250 and  $-0.333$  to give the logic some credence, with the above caveats about the range of the data and the number of cases. Figure 16.1 shows graphs testing the Threshold models. The left panel shows the impact of thresholds on the seat share of the largest party ( $s_1$ ), while the right panel shows the effective number of seat-winning parties ( $N_S$ ). The straight lines represent Equations 16.2 and 16.3, respectively. (The dashed curves will be explained in the next section.) We see that both models capture the basic trend but need fine tuning. At large  $T$ , median  $s_1$  is clearly lower than predicted, and correspondingly, median  $N_S$  is much higher. Where do these models go off balance?

## Second Approximation to the Model of Thresholds

We saw that  $N_{S0} = T^{-1/2}$  is largely confirmed, with regression producing an exponent  $-0.515$ . The next logical step, to  $s_1$ , introduces discrepancy, which does not seem to increase later, as we proceed to  $N_S$ . So let us take a closer look at this step.

In Chapter 7 we observed that the largest share cannot be less than the mean share,  $1/N_{S0} = T^{1/2}$ , nor larger than 1, thus  $T^{1/2} \leq s_1 < 1$ . This “1” was an overestimate: at least one seat would have to be left to each of the other seat-winning parties. Yet this was such a minor adjustment that we could neglect it. When thresholds are imposed, this is no longer the case: with  $T=0.10$ , each seat-winning party must have not just one seat, but 10 percent of all seats! This is not peanuts! So the stipulation,  $T^{1/2} \leq s_1 < 1$ , must be corrected. Subtracting the minimal share  $T$  for each party but the largest, we get

$$T^{1/2} \leq s_1 < 1 - (N_{S0} - 1)T = 1 - (T^{-1/2} - 1)T = 1 - T^{1/2} + T.$$

The geometric mean of the extremes now predicts

$$s_1 = T^{1/4}(1 - T^{1/2} + T)^{1/2}, \quad (16.4)$$

which reduces the largest share, compared to the previous  $s_1 = T^{0.25}$ . If the previous relationship between  $s_1$  and  $N_S$  still holds, then

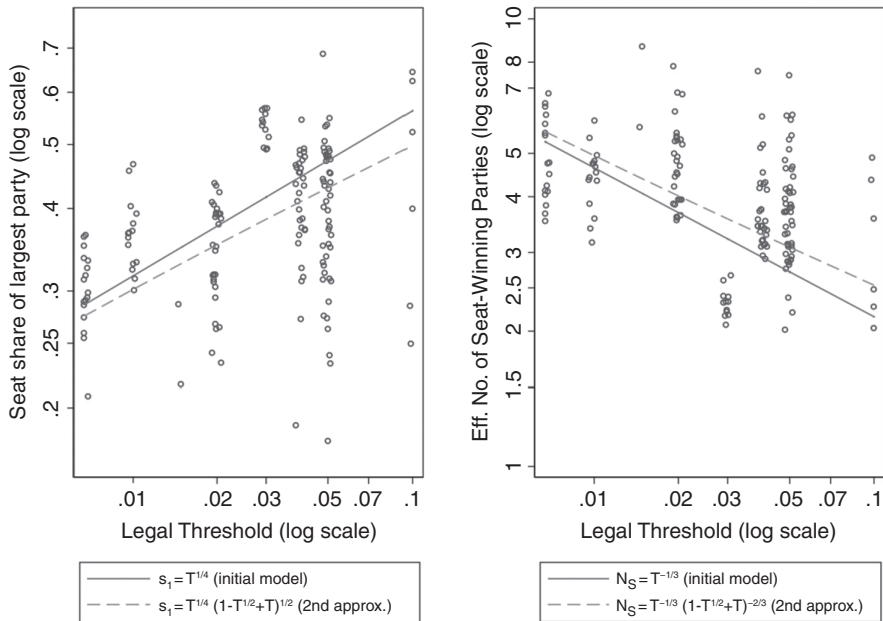


FIGURE 16.1 The effect of legal threshold on the seat share of the largest party ( $s_1$ , left panel) and the effective number of seat-winning parties ( $N_S$ , right panel)

$$N_S = T^{-1/3} (1 - T^{1/2} + T)^{-2/3}. \quad (16.5)$$

These relationships are shown as dashed curves in Figure 16.1. They are visibly closer to the average data pattern, compared to the straight lines, but still do not seem to go far enough. We may still miss some logical detail.

Despite their shortcomings, we will subject Equations 16.4 and 16.5 to empirical test alongside the Seat Product Model in the next section. In doing so, we break the various complex systems into distinct categories, whereas the graphs in Figure 16.1 (and the preceding regressions) pool all of the cases. We will see that the threshold model has value for some classes of complex systems, whereas for others the SPM is preferred.

### IS THE THRESHOLD MODEL BETTER THAN THE SEAT PRODUCT MODEL?

In this section, we put the threshold model up against the Seat Product Model. We want to know if we are better able to explain the actual effective number of seat-winning parties and largest party sizes by knowing the system's threshold, or by using the Seat Product Model. We find that it depends on just how complex the system is. The closer it is to a simple system, the better off we are

using the SPM, in general. However, as a system gets more complex, we find some more promise from the threshold model, but a highly complex system remains almost completely unpredictable.

### Single-Tier PR Systems

We will start this contest of the models with the simplest electoral systems that nonetheless have thresholds. These are systems that allocate their seats in a single tier, yet have a nationwide threshold.

For each system shown in Table 16.2 we see the actual values for both  $N_S$  and  $s_1$ , as well as for  $MS$  and the threshold. We indicate a ratio for each model, calculated as actual value divided by predicted value. This allows us to see at a glance whether one model or the other is better at predicting the party-system measures of these electoral systems, which would be simple were it not for their thresholds.

If one model's ratio is notably better than the other, it is in bold for a given electoral system. If both are close to 1.00, both are in bold. The line at the bottom of the table shows us that, for all these systems as a group, the SPM performs better for  $N_S$ , while the threshold model performs better for  $s_1$ . In all these cases the threshold ought to be sufficient to override the effect of the Seat Product. Yet, surprisingly, the predictions of the threshold model are no better than those of the SPM. Elimination of small parties through imposition of thresholds does not seem to affect the largest share or the effective number of parties to any marked degree. Thus adding a separate threshold model to our prediction arsenal seems superfluous – unless further improvement can be made to the threshold model.

In general, the SPM performs as if the systems shown in Table 16.2 were simple, without a threshold. There are some notable exceptions, however, where the threshold model performs better. For instance, in Ukraine the very high Seat Product would imply an extremely fragmented party system (SPM-derived predictions of  $N_S=7.66$  and  $s_1=0.217$ ). The actual party system in these elections was quite close the prediction of the threshold model. In Turkey the threshold model is correct that the largest party should have about a majority of seats, whereas the SPM would predict it to be below 40 percent.

A system like Turkey's since 1995 contains an internal contradiction: on the one hand, a mildly permissive Seat Product, while on the other hand a highly restrictive threshold. We would caution that when a system is designed with such countervailing features, it would be hard to predict which way it will work out. We will return to a discussion of the unusual features of this Turkish system later in this chapter. The general picture of these single-tier systems is that we would place more confidence in the SPM than in the threshold model. At least when  $MS$  is not exceedingly high (as in Ukraine) or the threshold is not greater than 5 percent (as in Turkey), our best guess at the impact of the system on  $N_S$  or  $s_1$  is to just ignore the threshold and treat it as a simple system.

TABLE 16.2 *Comparing model predictions to actual values for single-tier systems that have legal thresholds*

Country	Years	No. of elections	MS	Legal threshold (percent)	Actual $N_s$	Ratio for $N_s$ from SPM	Ratio for $N_s$ from T	actual $s_1$	Ratio for $s_1$ from SPM	Ratio for $s_1$ from T
Netherlands	1956–2010	17	22500	0.67	4.92	<b>0.93</b>	0.88	0.302	<b>1.06</b>	1.10
Israel	1951–1992	12	14400	1	4.41	0.89	0.89	0.381	1.26	1.26
Netherlands	1946–1952	3	10000	1	4.6	<b>0.99</b>	0.93	0.31	<b>0.98</b>	<b>1.03</b>
Israel	1996–1999	2	14400	1.5	7.16	1.45	1.64	0.25	0.83	0.76
Israel	2003–2009	3	14400	2	6.93	1.41	1.73	0.262	0.87	0.74
Ukraine	2006–2007	2	202500	3	3.36	0.44	0.94	0.401	1.85	<b>1.04</b>
Sweden	1948–1968	7	1888	4	3.11	<b>0.88</b>	0.95	0.488	<b>1.25</b>	<b>1.19</b>
Croatia	2000–2011	4	2114	5	2.99	0.84	0.97	0.385	<b>1.00</b>	0.90
Czechia	2002–2010	3	2858	5	3.76	<b>1.00</b>	1.22	0.345	<b>0.93</b>	0.80
Latvia	1995–2011	6	2000	5	5.43	1.53	1.76	0.258	0.67	0.60
Poland	2001–2011	4	5879	5	3.42	<b>0.80</b>	1.11	0.41	1.21	<b>0.95</b>
Slovakia	1998–2010	4	22500	5	4.93	<b>0.93</b>	1.60	0.318	<b>1.11</b>	0.74
Turkey	1983–1991	3	1921	10	2.71	0.77	1.07	0.526	1.35	<b>1.06</b>
Turkey	1995–2015	7	3625	10	3.04	0.78	1.20	0.48	1.24	<b>0.96</b>
		<b>77</b>				<b>0.97</b>	1.21		1.12	<b>0.94</b>

Now we will consider whether the threshold or Seat Product Model performs better for two-tier compensatory PR. Most of these systems have thresholds in addition to their composite electoral formula. Thus we need to consider the relative accuracy of both models.

## Two-Tier Compensatory Systems

In Table 16.3, we carry out our contest between the SPM and Threshold models on the class of two-tier compensatory systems. This group includes the mixed-member proportional (MMP) systems of Germany and New Zealand, as well as systems with PR in both tiers (see Chapter 3 for definitions).

Leaving aside their threshold provisions, two-tier compensatory systems are the simplest systems among those that are composites of different rules. In Chapter 15 we developed an extended form of the SPM (Equation 15.2) that takes into account the Seat Product of the basic tier and the “tier ratio” (the ratio of seats in the compensatory tier to the entire assembly). If the table indicates zero for tier ratio, it is a remainder-pooling system (see Chapter 3 for definitions of the subtypes). Which performs better on these two-tier systems, the extended SPM, or the threshold model?

We see that for these systems it is a close call as to which model is better. The SPM is, on average, slightly better for  $N_S$ . However, the threshold model performs better for  $s_I$ . For individual countries there is a rather strong tendency of the threshold model to get  $s_I$  closer to its actual values, but countries vary in which model does better for  $N_S$ . Thus, if asked how to estimate the impact of a proposed two-tier compensatory system, we would be inclined to turn to the SPM, in its extended form for  $N_S$ , but the threshold model for  $s_I$ . If we were to use both models, we might arrive at a reasonable range of what to expect from this class of complex system.

## Composite Systems That Are Noncompensatory

Now we consider mixed-member majoritarian (MMM) systems, those in which the list-PR tier is noncompensatory (explained in Chapter 3). We do not expect the SPM to apply to these systems, but why not try? In Table 16.4, we perform the same contest as in the preceding section, but on the MMM systems.

We have fewer of these systems, and fewer elections within them, on which to base our test. Moreover, they do not comprise a coherent category. Only four of the systems shown in Table 16.4 are strictly MMM systems, in which there is no adjustment to list allocations based on how a party has performed in the districts where plurality or majority is used. These are Japan (since 1996), South Korea, Lithuania, and Thailand (in 2001 and 2005).

The other systems incorporate a partial compensation mechanism: Hungary, Italy, and Mexico. Each of these has been classified in some sources as MMP, but such a classification is misleading because the systems are neither designed

TABLE 16.3 *Comparing model predictions to actual values for two-tier compensatory systems that have legal thresholds*

Country	Years	No. of elections	$MS_B$	tier ratio (t)	Legal threshold (percent)	Actual $N_S$	Ratio for $N_S$ from SPM	Ratio for $N_S$ from T	actual $s_1$	Ratio for $s_1$ from SPM	Ratio for $s_1$ from T
Cyprus	1985–2011	6	522	0	2	3.56	1.25	<b>0.89</b>	0.353	0.77	<b>1.00</b>
Denmark	1953–1968	6	816	0.229	2	3.79	<b>1.01</b>	0.94	0.407	<b>1.10</b>	<b>1.15</b>
Denmark	1971–2005	14	1104	0.229	2	5.03	1.27	1.25	0.34	<b>0.96</b>	<b>0.96</b>
Denmark	2007–2011	2	1877	0.229	2	5.47	1.26	1.36	0.266	0.80	0.75
Bulgaria	1991–2005	5	1858	0	4	2.65	0.75	<b>0.81</b>	0.497	1.27	<b>1.21</b>
Sweden	1970–2010	13	3395	0.112	4	3.74	<b>0.87</b>	<b>1.14</b>	0.424	1.27	<b>1.03</b>
Germany	1953–1987	10	247	0.5	5	3.07	0.78	<b>1.00</b>	0.485	1.36	<b>1.13</b>
Germany	1990–2009	6	314	0.5	5	3.70	<b>0.90</b>	1.20	0.41	1.18	<b>0.95</b>
Iceland	2003–2009	3	567	0.143	5	3.84	<b>1.17</b>	1.25	0.381	<b>0.93</b>	<b>0.89</b>
New Zealand	1996–2011	6	69	0.433	5	3.29	<b>1.09</b>	<b>1.07</b>	0.436	<b>1.00</b>	<b>1.01</b>
		<b>71</b>					<b>1.04</b>	1.09		1.06	<b>1.01</b>



to offer full compensation, nor capable of it in practice. (The mechanism is different in each, further complicating conclusion drawing.)

For the pure MMM systems, the SPM (extend version, as if the system were MMP) does not perform too badly, but the mean of actual  $N_S$  is lower than expected and the mean of actual  $s_1$  correspondingly higher. The variance from expectation is small enough, on average, that we might say just use the SPM were we asked to assess the impact of a proposed MMM system. On the other hand, that MMM would tend to result in lower  $N_S$  and higher  $s_1$  is consistent with the design of the system, which is intended to boost larger parties.

Such consolidation of the party system was certainly the explicit goal of reformers in Thailand (MacIntyre 2003: 118–119), following their experience with extreme fragmentation under MNTV (reviewed earlier in this chapter). The Thai electoral reform to MMM was therefore a case of “mission accomplished”; in fact, more so than the averages in Table 16.4 imply.

In the first postreform election in 2001,  $N_S=3.06$  and  $s_1=0.524$ . In the second election, the leading party, Thai Rak Thai, won over three fourths of the seats on just 56.4 percent of the vote, and  $N_S=1.65$ . Thai democracy broke down a few years later. Similar to the example we saw earlier with Mali, the adoption of a majoritarian system (in this case MMM) resulted in one party quickly attaining dominance. The electoral system turned dominance in votes into a near wipeout of the opposition in seats. These examples should give considerable pause to electoral-system designers in young or unstable democracies. Combining complexity and majoritarian features in such cases poses risks not worth taking.

Of our four examples of pure MMM, only one has  $N_S$  considerably higher than the SPM would predict, were it MMP. That case is Lithuania, which is also the only one with two-round majority in its single-seat districts. This factor may keep more parties in the game, despite the lack of compensatory allocation of the list-PR component. As for the threshold model, two of the MMM systems do not have a legal threshold. For the two that do, the threshold model is not especially useful in practice.

For the MMM systems that have partial compensation, the actual values (reported in the bottom part of Table 16.4) look unpredictable. This is perhaps not surprising, because each system has a *sui generis* design. This is precisely the situation in which we are unable to make predictions of effects. Actually, one might as well say the same for all the systems in Table 16.4. With MMM systems, whether or not they have partial compensation, small variations in the system might produce widely different effects. Unlike two-tier compensatory systems, including MMP, there is no coherent design to MMM systems that would make them conducive to generalization. The more that designers of systems choose unusual systems like MMM, the harder it is to know what to expect. We would suggest that MMM is best avoided for this reason. While it is plausible that MMP offers a “best of both worlds” (Shugart and Wattenberg, 2001), such a claim would be more difficult to sustain for MMM.

TABLE 16.4 *Comparing model predictions to actual values for two-tier noncompensatory systems that have legal thresholds*

Country	Years	No. of elections	MS <sub>B</sub>	tier ratio (t)	Legal threshold (percent)	Actual N <sub>S</sub>	Ratio for N <sub>S</sub> from SPM	Ratio for N <sub>S</sub> from T	actual s <sub>1</sub>	Ratio for s <sub>1</sub> from SPM	Ratio for s <sub>1</sub> from T
<i>Pure MMM</i>											
Japan	1996–2009	5	300	0.38	0	2.62	0.716		0.545	1.442	
Korea	1988–2004	3	231	0.202	0	2.77	0.930		0.444	1.007	
Lithuania	1992–2008	5	71	0.496	5	4.18	1.302	1.356	0.398	0.954	0.926
Thailand	2001–2005	2	400	0.2	5	2.36	0.724	0.766	0.65	1.578	1.512
		15					0.918			1.245	
<i>MMM with partial compensation</i>											
Hungary	1994–2010	6	176	0.544	5	2.72	0.697	0.882	0.513	1.421	1.193
Italy	1994–2001	3	475	0.246	4	6.32	1.806	1.924	0.258	0.683	0.629
Mexico	2000–2009	3	300	0.4	2	2.78	0.745	0.692	0.446	1.199	1.265

## WORKING AROUND COMPLEXITY: A CASE STUDY OF TURKEY

When a system has complex features, not only may outcomes be harder to predict, but also we may observe political actors undertaking unusual work-around solutions. We offer a brief example of this phenomenon from Turkey. As we saw in the section on thresholds, Turkey has the highest legal threshold, at 10 percent.

Were it not for the threshold, the Turkish system in place since 1995 would be mildly permissive, given  $MS$  over 3500. Moreover, it would seem to permit parties with regional concentration to win seats, given that the large assembly is divided into many multiseat districts. Yet regional parties are severely undercut by the fact that the threshold is not only high, but is applied nationwide. Despite the allocation of all seats in districts, as if it were a simple districted PR system, a party is barred from winning seats unless it clears 10 percent of the nationwide vote (Bahcik 2008) – even in cases where it might be the leading vote-winning party in a district.

The contradictory features of the Turkish system mean that in some years it has accommodated many parties, such as 1999 when  $N_S=4.41$  and the largest party had just under 25 percent of the seats; five parties cleared the threshold. Yet in other years it has been enormously majoritarian in effect, such as 2002 when the leading party, the Justice and Development Party (AKP), won 66 percent of seats on only 34.3 percent of votes – a result that rates as the most exaggerated inflation of a leading party's support in our database for any country that uses a proportional allocation formula. The large overrepresentation of the AKP was made possible because only two parties cleared the threshold, and more than 40 percent of the votes were cast for parties that fell below the threshold, including one that narrowly missed, with 9.5 percent. By any objective democratic standard, Turkey has a threshold that is too high. It might even be said to be undemocratic – it is so restrictive.<sup>11</sup>

Two district-level examples demonstrate the perverseness of such a system. In 2002 in the district of Agri, the AKP won three of five seats on a mere 17.6 percent of the vote. The leading party in the district, the DHP, had 35 percent of the vote, yet won no seats. Two seats were won by the CHP, despite its having only 9.6 percent of the vote. The DHP was similarly shortchanged in Van, where it won 40.9 percent of the votes but no seats, while the AKP won six of the seven seats on only 25.8 percent and the fourth-place CHP the other seat on just 5.2 percent. These cases, where a party won a plurality of the vote but no seats, are extreme examples, but demonstrate how a legal threshold may leave an electoral system outside our definition of “simple” (see Chapter 2), by violating the rank-size principle by which the relative sizes of parties in voting are reflected in the allocation of seats. If the first party gets no seats, this basic

<sup>11</sup> It was challenged in the European Court of Human Rights (2008), which declined to rule against the provision. See [http://hudoc.echr.coe.int/eng-press#{"display":\["1"\],"dmdocnumber":\["837654"\]}](http://hudoc.echr.coe.int/eng-press#{) (accessed July 30, 2016).

concept of simplicity is obviously violated! These examples are a more extreme case of what we noted in Chapter 1 in Poland's 2015 election, where a list sometimes has district-level votes sufficient to elect one or more candidates yet wins none because of a nationwide threshold. Any single-tier districted system that has a nationwide threshold introduces the risk of such unusual outcomes.

Some Turkish opposition groups eventually found an ingenious way around the threshold, however, by taking advantage of a provision permitting independent candidacy (Bahcik 2008, Eccarius-Kelly 2008). The main party of the country's Kurdish minority, the Democratic Society Party, ran such candidacies in specific districts in which they have regional concentration. Running as parties, they would have been shut out by the high nationwide threshold, but running as individual candidates, they were able to compete and sometimes win.<sup>12</sup>

Table 16.5 offers an example of how this worked in the eight-seat district of Van in the 2011 election. The AKP earned 40.26 percent of the vote, which allowed it to elect four candidates from its (closed) party list. No other party that cleared the nationwide threshold had sufficient votes in the district for a seat. The remaining four seats were won by independent candidates, all of them Kurdish. These candidates combined for nearly half the district's votes (48.6 percent). They presented four candidates, whose votes were sufficiently equally divided to allow all four to be elected.<sup>13</sup> These Kurdish politicians thus used *SNTV-like strategies*,<sup>14</sup> such as we discussed in Chapters 13 and 14. It was as if the electoral system was two entirely different systems side-by-side: closed-list PR for parties that earn 10 percent nationwide, and SNTV at the local level for those who do not expect to clear the nationwide threshold. Using this strategy, Kurdish candidates were able to win twenty-six seats in the Turkish Grand National Assembly in 2007 and thirty-six in 2011 (5.1 percent and

<sup>12</sup> For purposes of determining a district's allocation, an independent candidate's votes are compared to the D'Hondt divisors of those party lists that have passed the nationwide threshold. Thus the independents are treated exactly as are the qualifying lists, except that any one independent's votes can never qualify for a second seat.

<sup>13</sup> They needed to avoid their fourth candidate falling behind the fifth D'Hondt divisor for the AKP. In the event, Gür was just under 7,000 votes ahead of this figure. How did the party achieve such even distribution? It was due to its local organization, which made it possible to visit Kurdish voters' homes and instruct them on which specific candidate to vote for in their area. Information from Giriş Tarihi, "Örnek pusula, şablon ve yorgan ipli seçim taktiği" [Tactics with Sample Ballot, Template and String], June 14, 2011. [www.sabah.com.tr/yasam/2011/06/14/ornek-pusula-sablon-ve-yorgan-ipli-secim-taktigi](http://www.sabah.com.tr/yasam/2011/06/14/ornek-pusula-sablon-ve-yorgan-ipli-secim-taktigi) [last accessed November 4, 2016]. The title of the article refers to campaign workers providing a string of a certain length to drape over the sample ballot and show the voter (particularly if illiterate) the location on the ballot of the name of the candidate for whom they should vote. Research and translation assistance provided by İpek Bahçeci.

<sup>14</sup> We acknowledge a comment from Steven Verbanck for first calling our attention to this practice. [www.fruitsandvotes.wordpress.com/2007/07/22/turkish-ruling-party-gains-in-votes-declines-in-seats/#comment-2734](http://www.fruitsandvotes.wordpress.com/2007/07/22/turkish-ruling-party-gains-in-votes-declines-in-seats/#comment-2734)

TABLE 16.5 *Example of how independents were elected in one Turkish district: Van, 2011*

Party or candidate	Votes	Votes %	Seats
Justice and Development	171,665	40.26	4
Kemal Aktaş	65,447	15.35	1
Özdal Üçer	51,357	12.04	1
Aysel Tuğluk	49,339	11.57	1
Nazmi Gür	41,212	9.67	1
CHP	15,945	3.74	0
MHP	12,734	2.99	0

Other parties totaling 18,703 votes and other independents totaling 3,974 not shown  
Total votes cast: 426,402

6.5 percent, respectively).<sup>15</sup> These seats were won across sixteen and nineteen districts, respectively.

The example of the Kurdish opposition not only shows the steps political actors sometimes take to circumvent overly complex systems, but also reminds us that SNTV is the “simplest” system for multiseat districts (see Chapter 2). Running as if the system were SNTV, these political forces were able to be represented in an otherwise restrictive system. While the adaptation was ingenious, and looks like something out of a textbook on intraparty vote management (as discussed in Chapters 13 and 14), such strategy would have been unnecessary had Turkey adopted a simpler electoral system to start with.

## CONCLUSIONS

In this chapter, we delved further into several electoral systems that we initially ruled out of our wider effort at developing systematic quantitative logical models. These systems were left out because they violate basic criteria for “simple” systems outlined in Chapter 2. The definition of a simple system states that all *S* seats in the assembly must be allocated within districts, and that the rule used in those districts must respect the relative vote shares of parties in the allocation of seats. The definition leaves aside those systems that have two tiers of allocation, two rounds of voting, ranked-choice ballots, or high thresholds, among other complex features. Only the simple systems readily lend

<sup>15</sup> In June, 2015, the Kurdish nationalists teamed up with a national left-wing party and jointly cleared the threshold. They did so again in November of 2015, when a snap elections was called because no government was formed in the assembly elected earlier in the year. A further post-script is that Turkish democracy came under severe stress in July, 2016, with a failed coup and widespread purge by the AKP afterwards.

themselves to quantitative logical modeling, such as represented by the Seat Product Model (SPM).

In Chapter 15 we had already seen how one type of complex system could be included in an extended version of the SPM: if it is a two-tier compensatory system its allocation principles still lend themselves to a general model. However, many other complex systems do not, leaving it an empirical question whether they might function *as if* they were simple. In the case of single-tier systems with nonsimple formulas, we found some evidence that they function like simple systems. For instance, both the French two-round system and Australia's Alternative Vote have tended to produce assembly outcomes that look like FPTP, once we account for their assembly sizes, as the SPM does. We were able to draw the same conclusion for the Single Transferable Vote, at least in the case of Ireland. However, because each of these system types is quite rare, we must be cautious about generalizing from the cases reviewed in this chapter. That the most prominent examples of AV and STV might work in practice much like FPTP and list PR, respectively, is encouraging, given the high regard many reformers and scholars have for these systems (Bowler and Farrell 2006). AV and STV may have some advantages over their simpler cousins while still having systematic effects on party systems, although, again, their rarity makes it hard to be sure.

Other single-tier complex systems, however, do not function as if they were just simple districted systems. The Single Nontransferable Vote, for one, seems to have outcomes as unpredictable on the interparty dimension from the SPM as they proved (in Chapters 13 and 14) predictable for their intraparty effects. Other, still more unusual and complex single-tier systems are even more difficult to understand systematically.

Then in this chapter we developed a model of how systems with legal thresholds shape assembly party systems. We pitted this model against the SPM for a several system types. We found that the SPM performs better than the threshold model in predicting the effective number of seat-winning parties in both districted PR systems (with thresholds of 5 percent or lower) and for two-tier compensatory systems. However, the threshold model performs slightly better for the largest party's seat share.

This chapter lets us draw some important conclusions for institutional engineering. Some degrees of complexity do not prevent our being able to make systematic predictions. This is useful information if we are called upon to advise electoral-system designers. While we maintain an overall preference for simplicity, some of the most common provisions incorporated into electoral laws – compensatory upper tiers and modest legal thresholds – apparently do not greatly upset the fundamental impact of variables like district magnitude and assembly size. Nor, perhaps, do ranked-choice ballots, a tool much admired by electoral-reform advocates and some political science experts on electoral systems. Other designs such as mixed-member majoritarian (MMM) systems, by contrast, do not lend themselves to systematic prediction of their effects.

The analysis in this chapter could be helpful, then, for practitioners of politics: *keep it simple, but if you must complicate, do so minimally*. Moreover, the experiences of some newer democracies reviewed in this chapter highlight the special risks that stem from introducing complexity aimed at making a system more majoritarian. We discuss these implications for practitioners further in our next, concluding chapter.