Solutions to the Assignment for Week 2

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Exercise 1: Sunk costs, selection, and the competence-loyalty trade-off

(a) Derivation – Joining Condition Under Uncertainty

We analyse the expected utility for an agent deciding whether to join a party by incurring a sunk cost $c > 0$. The outside option w is drawn from a uniform distribution:

$$w \sim U[\underline{w}, \bar{w}]$$

The party offers a fixed reward $R > 0$. Agents are risk-neutral and join if and only if:

$$\mathbb{E}[\max(R, w)] \geq c$$

We distinguish three cases:

Case 1: $R \leq \underline{w}$,

In this case, all realisations of w exceed R, so:

$$\mathbb{E}[\max(R, w)] = \mathbb{E}[w] = \frac{\underline{w} + \bar{w}}{2}$$

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Case 2: $R \geq \bar{w}$

In this case, R exceeds all realisations of w, so:

$$\mathbb{E}[\max(R, w)] = R$$

Case 3: $R \in (w, \bar{w})$

In this intermediate case, the maximum value depends on the realisation of w. We split the expectation accordingly:

$$\begin{aligned}
\mathbb{E}[\max(R, w)] &= \int_{\underline{w}}^{R} \max(R, w) \cdot f(w)\, dw + \int_{R}^{\bar{w}} \max(R, w) \cdot f(w)\, dw \\
&= \int_{\underline{w}}^{R} R \cdot f(w)\, dw + \int_{R}^{\bar{w}} w \cdot f(w)\, dw
\end{aligned}$$

Since $w \sim U[\underline{w}, \bar{w}]$, the density is $f(w) = \frac{1}{\bar{w} - \underline{w}}$. Thus:

$$\begin{aligned}
\mathbb{E}[\max(R, w)] &= \frac{1}{\bar{w} - \underline{w}} \left(\int_{\underline{w}}^{R} R\, dw + \int_{R}^{\bar{w}} w\, dw \right) \\
&= \frac{1}{\bar{w} - \underline{w}} \left[R(R - \underline{w}) + \frac{1}{2}(\bar{w}^2 - R^2) \right]
\end{aligned}$$

This expression gives the closed-form expected value of the maximum between the fixed reward R and a uniformly distributed outside option w when R lies within the support of the distribution.

(b) Interpretation

- **Effect of uncertainty (i.e., width $\bar{w} - \underline{w}$)**: As the width increases, the variance of outside options increases, which makes the expected maximum less sensitive to the sunk cost. This reduces the party's ability to use c as an effective screening device for loyalty.

- **Overlapping outside option distributions**: When the distributions of w for high- and low-competence types overlap, selection based on joining becomes weaker. High-competence agents might sometimes join, while some low-competence types might not, reducing the discriminatory power of the sunk cost.

- **Strength of the selection effect**: Selection is strongest when:
  - The gap between $\underline{w}$ and $\bar{w}$ is small (low uncertainty),
  - The intervals for different types do not overlap,
  - The party reward R is positioned such that only low-outside-option types find joining worthwhile.

  If any of these conditions fails, the ability to screen for loyalty based on sunk costs diminishes.

Part B – Promotion Probabilities and the Loyalty–Competence Trade-off

(a) Derivation – Expected Payoff Under Uncertain Promotion

We now consider a model in which receiving the party reward $R > 0$ is not guaranteed. Instead, each agent of competence type $\alpha \in \{\alpha_L, \alpha_H\}$ receives it with probability $p(\alpha)$. If not promoted, the agent receives only their outside option.

The outside option w is drawn from a type-dependent uniform distribution:

$$w \sim \mathcal{U}[\underline{w}(\alpha), \bar{w}(\alpha)]$$

An agent who joins pays a sunk cost $c > 0$, and their expected utility is:

$$\mathbb{E}_\alpha = p(\alpha) \cdot \mathbb{E}[\max(R, w)] + (1 - p(\alpha)) \cdot \mathbb{E}[w]$$

where:

$$\mathbb{E}[w] = \frac{\underline{w}(\alpha) + \bar{w}(\alpha)}{2}$$

The agent joins if:

$$\mathbb{E}_\alpha \geq c$$

We now compute $\mathbb{E}_\alpha$ in three cases depending on the position of R relative to the support $[\underline{w}(\alpha), \bar{w}(\alpha)]$:

Case 1: $R \leq \underline{w}(\alpha)$

Then $\max(R, w) = w$ always, so:

$$\mathbb{E}[\max(R, w)] = \mathbb{E}[w] = \frac{\underline{w} + \bar{w}}{2}$$

Thus:

$$\mathbb{E}_\alpha = p(\alpha) \cdot \frac{\underline{w} + \bar{w}}{2} + (1 - p(\alpha)) \cdot \frac{\underline{w} + \bar{w}}{2} = \frac{\underline{w} + \bar{w}}{2}$$

$$\Rightarrow \text{Join if } \frac{\underline{w} + \bar{w}}{2} \geq c$$

Case 2: $R \geq \bar{w}(\alpha)$

Then $\max(R, w) = R$ always, so:

$$\mathbb{E}[\max(R, w)] = R$$

$$\Rightarrow \mathbb{E}_\alpha = p(\alpha) \cdot R + (1 - p(\alpha)) \cdot \frac{\underline{w} + \bar{w}}{2}$$

$$\text{Join if } p(\alpha) \cdot R + (1 - p(\alpha)) \cdot \frac{\underline{w} + \bar{w}}{2} \geq c$$

Case 3: $R \in (\underline{w}(\alpha), \bar{w}(\alpha))$

Then we compute the expected maximum as:

$$\mathbb{E}[\max(R, w)] = \frac{1}{\bar{w} - \underline{w}} \left[R(R - \underline{w}) + \frac{1}{2}(\bar{w}^2 - R^2) \right]$$

So the total expected utility becomes:

$$\mathbb{E}_\alpha = p(\alpha) \cdot \left[\frac{1}{\bar{w} - \underline{w}} \left(R(R - \underline{w}) + \frac{1}{2}(\bar{w}^2 - R^2) \right) \right] + (1 - p(\alpha)) \cdot \frac{\underline{w} + \bar{w}}{2}$$

$$\text{Join if this expression } \geq c$$

(b) Interpretation

- **Effect of increasing $p(\alpha_H)$ on the loyalty–competence trade-off:** A higher promotion probability for high-competence agents increases their expected utility from joining, especially when the party reward R is large. This reduces the deterrent effect of their higher outside options. As a result, individuals who would otherwise opt out—due to attractive external opportunities—may now be incentivised to join. This undermines the original screening logic, whereby sunk costs filtered out those with high opportunity costs and low loyalty.

- **When can competence-based promotion undo the selection effect?**
  If the promotion probability $p(\alpha_H)$ is sufficiently high, even agents with superior outside options may find it worthwhile to join. This weakens the ability of sunk costs to select for loyalty. The selection mechanism is most clearly undermined when the expected benefit from competence-based promotion exceeds the sunk cost for high-w individuals.

- **Can strong rewards for competence backfire?**
  Yes. If rewards for competence are too strong, the party risks crowding out loyal but less competent joiners. These individuals, facing low promotion prospects (i.e. low $p(\alpha_L)$), may no longer find joining worthwhile. In turn, the party may attract competent but potentially less loyal candidates—undermining cohesion and long-term organisational loyalty. Thus, strong competence incentives introduce a trade-off between attracting high-performing candidates and maintaining a base of loyal supporters.

Exercise 2: Logrolling

Part A – Utilities and Status Quo Payoffs

1. Utility Under the Default Outcome

Each legislator $i \in \{L, R\}$ has a weighted Euclidean utility function over a policy bundle $(x, y) \in [0, 1]^2$:

$$U_i(x, y) = -\left[\alpha_i(x - x_i)^2 + (1 - \alpha_i)(y - y_i)^2\right]$$

Let the ideal points be:

$$(x_L, y_L) = (0.2, 1.0), \quad (x_R, y_R) = (1.0, 0.2)$$

Assume the default (status quo) policy is the median:

$$(x^*, y^*) = (0.5, 0.5)$$

Let:

$$\alpha_L = \alpha, \quad \alpha_R = 1 - \alpha$$

Then, the utilities under the status quo are:

Legislator L:

$$\begin{aligned} U_L(0.5, 0.5) &= -\left[\alpha(0.5 - 0.2)^2 + (1 - \alpha)(0.5 - 1.0)^2\right] \\ &= -\left[\alpha(0.09) + (1 - \alpha)(0.25)\right] \\ &= -0.25 + 0.16\alpha \end{aligned}$$

Legislator R:

$$\begin{aligned} U_R(0.5, 0.5) &= -\left[(1 - \alpha)(0.5 - 1.0)^2 + \alpha(0.5 - 0.2)^2\right] \\ &= -\left[(1 - \alpha)(0.25) + \alpha(0.09)\right] \\ &= -0.25 + 0.16\alpha \end{aligned}$$

2. Gains from Cooperation

Suppose the legislators strike a deal where each gets their preferred outcome on the issue they care most about. Consider the policy bundle:

$$(x, y) = (0.2, 0.2)$$

Legislator L:

$$\begin{aligned} U_L(0.2, 0.2) &= -\left[\alpha(0.2 - 0.2)^2 + (1 - \alpha)(0.2 - 1.0)^2\right] \\ &= -(1 - \alpha)(0.64) \end{aligned}$$

Legislator R:

$$\begin{aligned} U_R(0.2, 0.2) &= -\left[(1 - \alpha)(0.2 - 1.0)^2 + \alpha(0.2 - 0.2)^2\right] \\ &= -(1 - \alpha)(0.64) \end{aligned}$$

Net Gains from Cooperation

The change in utility from cooperating is:

$$\Delta U_i = U_i(x, y) - U_i(0.5, 0.5)$$

Legislator L:

$$\Delta U_L = -(1-\alpha)(0.64) + [\alpha(0.09) + (1-\alpha)(0.25)]$$

Legislator R:

$$\Delta U_R = -(1-\alpha)(0.64) + [(1-\alpha)(0.25) + \alpha(0.09)]$$

Note that these expressions are symmetric and highlight how both legislators benefit when they each get their preferred outcome on their most salient dimension.

Part B – A Stylised Bargaining Solution (Weighted-Average Approach)

To capture the logic of mutual deference in logrolling, we model the final policy outcome (x^*, y^*) as a weighted average of the legislators' ideal points. The weights reflect how much each legislator cares about each issue.

Let $\alpha \in (0, 1)$ denote the salience of issue x for legislator L. Then legislator R's salience for x is $1 - \alpha$, and vice versa for issue y. The final policy outcome is given by:

$$x^* = \alpha \cdot x_L + (1-\alpha) \cdot x_R, \qquad y^* = (1-\alpha) \cdot y_L + \alpha \cdot y_R$$

1. Deriving $x^*(\alpha)$ and $y^*(\alpha)$

Let the legislators' ideal points be:

$$(x_L, y_L) = (0.2, 1.0), \qquad (x_R, y_R) = (1.0, 0.2)$$

Substituting into the expressions:

$$x^*(\alpha) = \alpha \cdot 0.2 + (1-\alpha) \cdot 1.0 = 1 - 0.8\alpha$$
$$y^*(\alpha) = (1-\alpha) \cdot 1.0 + \alpha \cdot 0.2 = 0.2 + 0.8\alpha$$

2. Interpretation

- As α increases, x^* decreases toward L's ideal point (0.2), while y^* increases toward R's ideal point (0.2).

- When $\alpha = 0.5$, each legislator has equal salience on both issues, and the outcome becomes:

$$x^* = y^* = 0.6$$

- **Who benefits as α increases?** Legislator L gains greater influence over issue x, while legislator R benefits more on issue y. This reflects the trade-off logic at the heart of logrolling.
- **When is the scope for logrolling largest?** When salience is highly asymmetric (i.e., α close to 0 or 1). In such cases, each legislator is willing to make larger concessions on the dimension they care less about, enhancing the potential for mutually beneficial exchange.

Part C – A Dynamic Extension of Logrolling

1. Trade Across Time

We now consider a two-period model in which the legislators bargain over:

- Issue x in period 1
- Issue y in period 2

Let $\delta \in (0, 1)$ denote a common discount factor. The utility functions of the two legislators incorporate salience and discounting:

Legislator L:

$$U_L = -\left[\alpha(x - x_L)^2 + \delta(1 - \alpha)(y - y_L)^2\right]$$

Legislator R:

$$U_R = -\left[(1 - \alpha)(x - x_R)^2 + \delta\alpha(y - y_R)^2\right]$$

The ideal points remain:

$$(x_L, y_L) = (0.2, 1.0), \quad (x_R, y_R) = (1.0, 0.2)$$

Simple Time-Based Exchange

Suppose legislator R gets her ideal outcome on x in the first period, and legislator L gets her ideal outcome on y in the second period. That is:

$$x = x_R = 1.0, \qquad y = y_L = 1.0$$

Then the utilities are:

Legislator L:

$$\begin{aligned} U_L &= -\left[\alpha(1.0-0.2)^2 + \delta(1-\alpha)(1.0-1.0)^2\right] \\ &= -\alpha(0.64) \end{aligned}$$

Legislator R:

$$\begin{aligned} U_R &= -\left[(1-\alpha)(1.0-1.0)^2 + \delta\alpha(1.0-0.2)^2\right] \\ &= -\delta\alpha(0.64) \end{aligned}$$

This outcome illustrates a clean form of intertemporal logrolling: each legislator receives their ideal point on one issue, with temporal separation and discounted future payoffs.

2. Legislator Utilities from Stylised Weighted-Average Outcomes

Suppose the outcomes on each issue are stylised weighted averages of the legislators' ideal points:

$$x^* = \lambda x_L + (1-\lambda)x_R, \qquad y^* = \mu y_L + (1-\mu)y_R$$

We now compute each legislator's total utility as a function of λ, μ, α, and δ.

Let:

$$d_x = x_R - x_L, \qquad d_y = y_L - y_R$$

Legislator L:

$$\begin{aligned} U_L &= -\left[\alpha(x^* - x_L)^2 + \delta(1-\alpha)(y^* - y_L)^2\right] \\ &= -\left[\alpha(1-\lambda)^2(x_R - x_L)^2 + \delta(1-\alpha)(1-\mu)^2(y_R - y_L)^2\right] \\ &= -\left[\alpha(1-\lambda)^2 d_x^2 + \delta(1-\alpha)(1-\mu)^2 d_y^2\right] \end{aligned}$$

Legislator R:

$$\begin{aligned} U_R &= -\left[(1-\alpha)(x^* - x_R)^2 + \delta\alpha(y^* - y_R)^2\right] \\ &= -\left[(1-\alpha)\lambda^2(x_L - x_R)^2 + \delta\alpha\mu^2(y_L - y_R)^2\right] \\ &= -\left[(1-\alpha)\lambda^2 d_x^2 + \delta\alpha\mu^2 d_y^2\right] \end{aligned}$$

These expressions capture each legislator's loss from the stylised compromise relative to their ideal points, weighted by salience and temporal discounting.

3. When Are Both Legislators Strictly Better Off Than Under the Status Quo?

We now compare each legislator's utility under the stylised policy outcomes to their utility under the default (status quo) outcome where $x = y = 0.5$.

Status Quo Utilities Given ideal points:

$$(x_L, y_L) = (0.2, 1.0), \quad (x_R, y_R) = (1.0, 0.2)$$

the utilities under the default are:

$$\begin{aligned} U_L^{\text{SQ}} &= -\left[\alpha(0.3)^2 + \delta(1-\alpha)(0.5)^2\right] = -\left[0.09\alpha + 0.25\delta(1-\alpha)\right] \\ U_R^{\text{SQ}} &= -\left[(1-\alpha)(0.5)^2 + \delta\alpha(0.3)^2\right] = -\left[0.25(1-\alpha) + 0.09\delta\alpha\right] \end{aligned}$$

Stylised Outcome Utilities From earlier:

$$\begin{aligned} U_L &= -\left[\alpha(1-\lambda)^2 d_x^2 + \delta(1-\alpha)(1-\mu)^2 d_y^2\right] \\ U_R &= -\left[(1-\alpha)\lambda^2 d_x^2 + \delta\alpha\mu^2 d_y^2\right] \end{aligned}$$

with $d_x = x_R - x_L = 0.8$, $d_y = y_L - y_R = 0.8$.

Conditions for Improvement Each legislator benefits if their utility under the stylised outcome exceeds the status quo:

$$U_L > U_L^{\text{SQ}}, \quad U_R > U_R^{\text{SQ}}$$

Intuition

- Legislator L benefits when:
  - x^* is close to $x_L = 0.2 \Rightarrow \lambda$ close to 1
  - y^* is close to $y_L = 1.0 \Rightarrow \mu$ close to 1
- Legislator R benefits when:

  - x^* is close to $x_R = 1.0 \Rightarrow \lambda$ close to 0
  - y^* is close to $y_R = 0.2 \Rightarrow \mu$ close to 0

The tension here reflects the core trade-off in logrolling: for both legislators to be better off, the outcome must give each of them a sufficiently favourable position on the issue they care more about. This typically occurs when:

$$\lambda > 0.5 \quad \text{and} \quad \mu < 0.5$$

meaning L wins more on x, and R wins more on y. If discounting δ is not too severe, both legislators can strictly benefit relative to the default.

4. Effect of Uncertainty About the Second Period

Suppose the game ends after period 1 with probability $\pi \in (0, 1)$. This models the risk that period 2 may never occur, due to factors such as a snap election, leadership turnover, or institutional breakdown.

Let $1 - \pi$ denote the probability that period 2 actually occurs. This modifies each legislator's expected payoff from period 2.

Modified Discounting Previously, future payoffs were discounted by $\delta \in (0, 1)$, reflecting time preference. Now, legislators also discount for the possibility that the second period may not happen. The effective discount factor becomes:

$$\delta' = (1 - \pi)\delta$$

Updated Utility Functions Each legislator's utility is now:

$$U_L = -\left[\alpha(x - x_L)^2 + \delta'(1 - \alpha)(y - y_L)^2\right]$$
$$U_R = -\left[(1 - \alpha)(x - x_R)^2 + \delta'\alpha(y - y_R)^2\right]$$

Implications for Cooperation

- A higher probability π of early termination reduces δ', shrinking the expected value of second-period payoffs.

- This especially affects the legislator who is scheduled to gain in the second period. If, for instance, R receives her ideal policy on x in period 1, and L expects to receive her ideal policy on y in period 2, then L becomes less willing to cooperate as π increases.
- The result is a reduced scope for intertemporal logrolls. Legislators may prefer front-loaded agreements or opt out entirely if they cannot credibly expect future reciprocation.

Part D – Institutionalising Logrolls

To sustain intertemporal or cross-issue cooperation, legislators often rely on institutional mechanisms that mitigate the risk of defection. These supports are crucial in dynamic settings where one side may benefit earlier and thus have less incentive to honour future commitments.

1. Repetition and Reputation

In repeated legislative environments, reputation functions as an informal enforcement device (Axelrod, 1984; Greif, 1994).

- Reputational costs discourage short-term opportunism and increase the shadow of the future.
- Legislators who defect from a deal may be excluded from future cooperation.
- Repeated interactions make conditional cooperation more attractive than one-off gains.

2. Side Payments and Cross-Issue Linkage

Logrolls are more enforceable when legislators can compensate each other through side payments or bargaining over multiple dimensions (Weingast and Marshall, 1988).

- Committee influence, spending provisions, or procedural support can be exchanged as compensation.
- This expands the set of mutually acceptable outcomes, especially when issue salience is asymmetric.
- Such linkage aligns with coalition-building models in legislative bargaining.

3. Contracting and Party Enforcement

Political parties can act as enforcement intermediaries, disciplining members who defect from informal or formal deals (Cox and McCubbins, 2005; Carey, 2007).

- Party whips and leadership can monitor and sanction non-cooperation.
- Internal hierarchy enables credible commitments within coalitions.
- Defection can lead to removal from committees or denial of legislative support.

4. Procedural Rules and Vote Bundling

Institutions can facilitate logrolls by structuring how policies are voted on — for example, through bundled votes or omnibus legislation (Krehbiel, 1991; Tsebelis and Money, 1997).

- Closed rules or package deals limit opportunities to defect.
- Institutional sequencing can give each party partial control over different dimensions.
- Formal vote-trading platforms or procedural coordination can enforce exchanges (Jackson and Moselle, 2002).

5. Transparency and Monitoring

Visibility of legislative actions makes it easier to detect and punish defection (Mayhew, 1974; Arnold, 1990).

- Roll-call votes and media scrutiny increase reputational consequences.
- Transparency enables voters and other legislators to hold defectors accountable.
- Monitoring reduces the information asymmetries that otherwise undermine trust.

Exercise 3: Party Competition

Part A – The One-Dimensional Model

Nash equilibrium in the Downsian model (general version) Consider a unidimensional policy space $x \in [0, 1]$. There is a continuum of voters, each with a single-peaked preference over this space. Let the distribution of voter ideal points be described by a strictly increasing and continuous cumulative distribution function $F(x)$. Voters vote for the party closest to their ideal point. There are two parties, A and B, and both are purely vote-seeking: they aim to maximise their vote share by choosing a position $x_P \in [0, 1]$, where $P \in \{A, B\}$.

Median voter theorem (general statement) If the distribution $F(x)$ has a unique median $m \in [0, 1]$, then the policy profile $x_A = x_B = m$ is a pure-strategy Nash equilibrium. That is, convergence to the median voter's ideal point constitutes a set of mutual best responses.

Definition of Nash equilibrium A *pure-strategy Nash equilibrium* is a strategy profile where no party can unilaterally deviate and increase its vote share, given the strategy of the other. That is, each party's choice is a **best response** to the other's, and the strategies are in mutual alignment. This concept aligns with the logic of a fixed point: each player's optimal action maps back to itself when all players optimise simultaneously.

Proof by contradiction Suppose party A locates at the median $x_A = m$, and party B deviates to $x_B \neq m$. Without loss of generality, assume $x_B > m$. Then every voter with an ideal point $x_i < \frac{x_B+m}{2}$ will prefer A. Since m is the median, more than half the electorate lies to the left of $\frac{x_B+m}{2}$, and hence party A secures more than half the votes. Party B would then strictly increase its vote share by moving toward the median, contradicting the assumption that x_B was a best response.

Symmetric reasoning applies if $x_B < m$.

Now consider the case where both parties locate at the median, $x_A = x_B = m$. Then each party wins exactly half the electorate. Any unilateral deviation by either party would result in a strictly lower vote share, as it would be strictly further from the median than its opponent. Hence, no party can profitably deviate.

Conclusion When both parties locate at the unique median of the voter distribution, their strategies are mutual best responses — neither has an incentive to change given the other's choice. This profile satisfies the definition of a pure-strategy Nash equilibrium and constitutes a fixed point in the strategy space under the best-response correspondence. Therefore, convergence to the median is a stable equilibrium outcome under these conditions.

Contextualising the model's assumptions in the United Kingdom

Table 1: UK-specific assessment of contextual, demand-side, and supply-side challenges to the Downsian model, with citations

Challenge	**Modification(s)**	**Assessment in the UK context**
Static (one election, one-shot game)	Consider incentives that arise from effects that materialise in future rounds	The main parties (Labour and Conservatives) engage in long-term reputation building (e.g., rebranding, strategic retreats), especially in anticipation of future electoral cycles. "New Labour", for instance, was a long-term rebranding project (Hay, 1999). Parties also shape expectations through manifesto pledges or leadership changes (Fisher et al., 2014).
One-dimensional policy space	Introduce at least two dimensions of political competition	UK politics reflects both economic and socio-cultural conflict. The Brexit divide exemplifies the second dimension (Evans and Tilley, 2017; Sobolewska and Ford, 2020)
Competition solely over policy platforms, not worldviews.	Sometimes competition might be less about policy platforms, but about views of the world ("ideologies").	Difficult to operationalise and therefore to evaluate
Voters often do not vote completely sincerely, but also strategically.	Allow for strategic voting theoretically; empirically, try to assess divergence between preferences and vote choice	Tactical voting is widespread under FPTP, especially in marginal seats (Eggers and Vivyan, 2020)

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Challenge	**Modification(s)**	**Assessment in the UK context (continued)**
Preference intensity is empirically and theoretically relevant.	Measure preference intensity empirically and allow parties to take it into account when choosing their policy positions	Parties target high-turnout groups like older Leave voters. Issue salience and group mobilisation are key strategic variables.
Parties might have "reputational" constraints / voters might dislike "opportunistic" moves.	Impose constraints on parties' ability to move to new positions on issues where they have strong reputations and/or modify voters' utility function to allow for punishment of pure opportunism	Labour's "march to the middle" under Blair can be seen as an investment into developing a reputation for "competent" economic management, though it likely also diluted Labour's pro-redistributive brand
No uncertainty about parties' positions on a given dimension	Parties might sometimes be able to adopt vague/blurry positions on issues that, for instance, create internal division	Labour's Brexit ambiguity was a strategic response to internal division (Ford et al., 2021). Ambiguity helps maintain party unity but can backfire electorally.
Parties are solely vote-maximising unitary actors.	In addition to vote-/office-seeking motivations, parties may also have policy-seeking ones. They also rely on activists, which can influence their positions.	Internal party dynamics matter (Wager et al., 2022). Momentum shaped Labour's 2017–19 manifestos. Activists constrain leaders' latitude to reposition. As partisan loyalty as weakened, however, the incentives for vote-seeking behaviour have increased, with MPs investing more time in constituency service, questions in the House, and committee activity (Fleming, 2022).

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Challenge	Modification(s)	Assessment in the UK context (continued)
No uncertainty about median voter's position	Sometimes median voter's position is uncertain; this can influence the relative importance of vote- and policy-seeking motivations	Polling volatility and regional divergence (e.g., Scotland vs. England) can create uncertainty.
All parties are assumed to be qualitatively similar.	Allow for some parties to be *issue owners* or niche parties, whereas others are conventional catch-all, mainstream parties	Greens focus on climate; SNP on Scottish independence; Reform UK on national identity. These differ from catch-all strategies.
In multi-party contexts, parties have to consider coalition formation.	Allow beliefs about likely coalitions to influence positional choices	Given that, for now, the UK mainly has single-party, majority governments, such considerations are not particularly important.
Parties can also select the issues they compete on, not just their positions	Political agenda and salience of issues becomes partly endogenous	Conservatives dominated the 2019 campaign by framing it as a "Brexit election". UKIP and Reform UK derive much of their electoral success from their ability to raise the salience of immigration.

Part B - Valence and Party Competition

Location of the Indifferent Voter

Voters are uniformly distributed along the interval $[0, 1]$ and choose the party that maximises their utility:

$$U_i^P = -|x_i - x_P| + v_P$$

where x_i is voter i's ideal point, x_P is party P's position, and v_P is party P's valence (e.g., competence). Assume without loss of generality that $x_A < x_B$.

Let x^* denote the location of the **indifferent voter** between party A and party B. This voter is indifferent between the two parties:

$$U_i^A = U_i^B \Rightarrow -|x^* - x_A| + v_A = -|x^* - x_B| + v_B$$

Since $x^* \in [x_A, x_B]$, the absolute values can be removed directly:

$$-(x^* - x_A) + v_A = -(x_B - x^*) + v_B$$

Simplifying both sides:

$$-x^* + x_A + v_A = -x_B + x^* + v_B$$

$$2x^* = x_A + x_B + (v_A - v_B)$$

$$\Rightarrow \boxed{x^* = \frac{x_A + x_B + (v_A - v_B)}{2}}$$

This expression defines the voter who is exactly indifferent between the two parties. All voters with $x_i < x^*$ will vote for party A, and those with $x_i > x^*$ will vote for party B.

Deriving Each Party's Vote Share

Given the location of the indifferent voter:

$$x^* = \frac{x_A + x_B + (v_A - v_B)}{2}$$

and assuming voters are uniformly distributed on $[0, 1]$, party A receives all votes from voters with ideal points $x_i \leq x^*$, while party B receives the rest.

Since the voter density is uniform, the vote share corresponds directly to the length of the interval each party "wins".

Party A's vote share:

$$V_A = \int_0^{x^*} 1\, dx = x^* = \frac{x_A + x_B + (v_A - v_B)}{2}$$

Party B's vote share:

$$V_B = 1 - V_A = 1 - \frac{x_A + x_B + (v_A - v_B)}{2} = \frac{2 - x_A - x_B - (v_A - v_B)}{2}$$

Summary:

$$\boxed{V_A = \frac{x_A + x_B + (v_A - v_B)}{2}, \quad V_B = \frac{2 - x_A - x_B - (v_A - v_B)}{2}}$$

These vote shares reflect the combined influence of policy positioning (x_A, x_B) and valence advantages (v_A, v_B).

Incentive to Deviate When Party A Locates at the Median

Assume party A chooses the median position:

$$x_A = 0.5$$

Let $x_B \in [0,1]$ be party B's location, and assume $v_A > v_B$, i.e., party A has a valence advantage.

From the previous derivation, the indifferent voter is located at:

$$x^* = \frac{x_A + x_B + (v_A - v_B)}{2} = \frac{0.5 + x_B + (v_A - v_B)}{2}$$

Party B's vote share is:

$$V_B = 1 - x^* = 1 - \frac{0.5 + x_B + (v_A - v_B)}{2} = \frac{1.5 - x_B - (v_A - v_B)}{2}$$

Now consider whether B can improve its vote share by deviating. Suppose B also locates at the median, $x_B = 0.5$. Then:

$$V_B = \frac{1.5 - 0.5 - (v_A - v_B)}{2} = \frac{1 - (v_A - v_B)}{2}$$

This is strictly less than 0.5, since $v_A > v_B$. Thus, when both parties converge to the median, the higher-valence party A wins.

Can B improve its vote share by deviating? Suppose party B moves slightly away from 0.5. Its new vote share becomes:

$$V_B(x_B) = \frac{1.5 - x_B - (v_A - v_B)}{2}$$

Taking the derivative with respect to x_B:

$$\frac{dV_B}{dx_B} = -\frac{1}{2} < 0$$

Thus, increasing x_B (i.e., moving away from the median) **decreases** party B's vote share. Moving leftward (closer to $x_A = 0.5$) also provides no benefit, since A is already there and has a valence advantage.

Conclusion: Party B has no incentive to deviate from the median — but it also cannot win by staying there if $v_A > v_B$. The best it can do is minimise its loss by locating at or near $x = 0.5$, even though it loses the election.

How Does the Importance of Valence Change as x_A and x_B Move Closer Together?

Recall that the indifferent voter's location is:

$$x^* = \frac{x_A + x_B + (v_A - v_B)}{2}$$

and party A's vote share is:

$$V_A = x^* = \frac{x_A + x_B + (v_A - v_B)}{2}$$

Key Insight: The impact of the valence difference $v_A - v_B$ on vote shares becomes more significant as the policy positions x_A and x_B converge.

Explanation:

- When $x_A \approx x_B$, the spatial (policy) component of voter utility becomes nearly equal across parties.
- In that case, voters base their decision almost entirely on the valence term v_P.
- This means even a small difference in perceived competence or integrity (i.e., valence) can swing the election.

Formally, consider the partial derivative of vote share with respect to valence:

$$\frac{\partial V_A}{\partial v_A} = \frac{1}{2}, \quad \frac{\partial V_B}{\partial v_B} = -\frac{1}{2}$$

These effects are constant, but when the spatial component diminishes (i.e., when $x_A \to x_B$), these marginal effects dominate the decision calculus.

Intuition: As the ideological distance between parties narrows, valence becomes the decisive factor in voter choice. This mirrors real-world observations in periods of policy convergence (e.g., the "Third Way" era), where elections were often fought on character, competence, or trustworthiness rather than ideology.

How Does Increasing λ Affect Party A's Optimal Location?

Party A's objective is to maximise the following utility function:

$$U_A = \lambda V_A - (1 - \lambda)(x_A - x_A^{\text{ideal}})^2$$

where:

- V_A is vote share, given by $V_A = \frac{x_A + x_B + (v_A - v_B)}{2}$
- $x_A^{\text{ideal}} \in [0, 1]$ is party A's ideal policy position
- $\lambda \in [0, 1]$ is the weight party A places on votes (vs. policy)

Interpretation:

- The first term captures vote-maximising incentives: party A increases V_A by choosing x_A close to the median or strategically away from x_B, depending on valence.
- The second term penalises deviation from A's ideal point, representing a policy loss.

Effect of λ:

- As $\lambda \to 1$, the party places almost exclusive weight on vote maximisation. The optimal x_A moves closer to the median voter or toward positions that increase V_A given x_B and $v_A - v_B$.
- As $\lambda \to 0$, the party becomes purely policy-seeking. The optimal x_A converges to x_A^{ideal}, regardless of electoral consequences.

Conclusion: An increase in λ leads party A to shift its policy platform away from its ideal point and toward more electorally optimal positions. This includes convergence with the median, counter-positioning against party B, or exploiting a valence advantage.

Which Party Has Greater Flexibility When $\lambda = 0.5$?

Assume both parties weigh votes and policy equally:

$$\lambda = 0.5$$

Each party maximises:

$$U_P = 0.5 \cdot V_P - 0.5 \cdot (x_P - x_P^{\text{ideal}})^2$$

Valence Advantage: Suppose party A enjoys a valence advantage:

$$v_A > v_B$$

From the vote share formula:

$$V_A = \frac{x_A + x_B + (v_A - v_B)}{2}$$

We can see that for any given pair of positions (x_A, x_B), A's vote share is boosted by $v_A - v_B$, independent of spatial proximity.

Implication:

- Party A can afford to deviate further from the median (or from vote-maximising positions) while still maintaining a competitive or even dominant vote share.
- In contrast, party B, with lower valence, must compensate through spatial positioning — moving closer to the median or to voters.

Strategic Flexibility: Valence acts as a form of electoral buffer. It reduces the marginal cost of deviating from the vote-maximising location. Thus, when $\lambda = 0.5$, the party with the valence advantage — here, party A — has: greater freedom to prioritise its policy ideal point, and less electoral risk in doing so.

Can a Party Use Valence to Compensate for Ideological Divergence?

We consider a party P whose utility is given by:

$$U_P = \lambda V_P - (1 - \lambda)(x_P - x_P^{\text{ideal}})^2$$

where V_P is vote share, influenced by both policy proximity and valence.

From earlier, the indifferent voter lies at:

$$x^* = \frac{x_A + x_B + (v_A - v_B)}{2}$$

and the vote share of party A is $V_A = x^*$. Hence, the valence difference $v_A - v_B$ directly shifts the cut-point in favour of the higher-valence party.

Interpretation. Valence increases a party's vote share independently of its spatial position. Therefore, a party with a valence advantage can afford to locate further from the median (closer to its ideal point) while still maintaining electoral competitiveness.

Conditions for Effective Compensation. Valence can meaningfully offset ideological divergence when:

- The valence gap $v_P - v_R$ is sufficiently large;
- The party's ideal point is not too extreme;
- Voter preferences are not sharply bimodal or clustered at the extremes;
- The weight on votes, λ, is not too low.

Conclusion. Yes — valence can act as a substitute for spatial proximity under broad conditions. It allows policy-seeking parties some leeway to deviate from vote-maximising positions without suffering electoral loss.

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