Assignment – Week 1

Jacob Edenhofer*

30/04/2025

Preliminary remarks

You are expected to complete **two tasks for this week**, ideally of **different types** (e.g., one essay, one simulation, one formal derivation) — though this is not a strict requirement. The deadline is **midnight on the 2nd of May**. You are strongly encouraged to explore additional tasks — this is an opportunity to deepen your understanding by tackling what most interests or challenges you.

The rationale behind offering many different tasks is twofold: to give you **maximum choice**, and to help you develop a sense of what it means to **examine models carefully**. Whether through formal derivation, coding, or conceptual writing, the aim of these assignments is to stretch yourselves intellectually. Try things, fail at them, and then try again. **Hitting walls is not only normal — it is essential** when working with formal models and doing research more generally. Much of the value lies in grappling with structure, assumptions, and logic — not just in arriving at the "correct" answer.
Please feel free to choose the level of difficulty that suits you best. Some tasks are more involved than others, and it is entirely fine to focus on those you find most manageable or intriguing. **There is absolutely no reason to feel bad about choosing easier options** — this material is tricky, and any serious, good-faith effort is genuinely appreciated.

This is a **formative course** — your marks do not count, so this is also a chance to take risks and venture out of your comfort zone. If time is tight and you cannot get to some of the more challenging exercises, **that is completely fine**. Do what you can, and feel free to keep the sheet for future reference.

The use of **generative AI is actively encouraged**. It can help illuminate reasoning gaps, accelerate coding, or suggest alternative formulations. That said, a word of caution:

*jacob.edenhofer@nuffield.ox.ac.uk

outsourcing your thinking too early or too often can undermine your learning. The long-run payoff comes from engaging deeply first, and only then consulting AI to refine or test your ideas. I will not police the use of AI — the responsibility for learning is ultimately yours.

Submission instructions: Please email your completed tasks to me by the deadline. Ideally, submit your work as a single **integrated document** — such as an `R Markdown` or `Jupyter Notebook` file - combining code, output, and explanatory text. This format is especially useful for simulations and makes your reasoning easier to follow. You may also submit essays in Word, LaTeX, or plain Markdown. Although zipped folders are acceptable (for example, if you submit multiple files or raw code), they are **not encouraged**.

A stylised model of self-enforcing democracy (Przeworski, 1991; Przeworski et al., 2015; Fearon, 2011)

1. **Recurring costs c:**
   Derive the compliance condition when costs recur, i.e. when they are $\sum_{t=0}^{\infty} \delta^t c = \frac{c}{1-\delta}$. Provide an intuitive justification for assuming recurring costs. In one or two sentences, reflect on how your results compare to the case of a one-time cost. What implications does this have for political stability?

2. **Comparative statics:**
   Derive the signs of $\frac{dS^*}{d\delta}, \frac{dS^*}{dc}, \frac{dS^*}{dk}, \frac{dS^*}{dp}$ using the implicit function theorem. Clearly state how the sign depends on the slope of $\pi(S)$ and the difference between the probability of winning the next election and prevailing in a violent fight.

3. **Essay-based empirical illustration (max. 500 words):**
   Choose one real-world example to illustrate and interpret the comparative statics from the self-enforcing democracy model introduced in class (see table in slide deck). These can be:

   - A cross-country comparison (e.g., comparing democratic stability in two different countries), or
   - A within-country, over-time comparison (e.g., the same country during two distinct historical periods).

   Your goal is to connect the model's parameters to concrete features of your cases. That is, "mapping parameters" means interpreting each of the model's elements in light of political, institutional, or historical realities. For example:

- How would you interpret the probability of winning a violent conflict (p) in your cases — what factors shape it?
- What might the costs of fighting (c) look like in context — economically, socially, or institutionally?
- How costly is regular political participation (k), and what drives this?
- Do political actors appear patient or short-sighted — how would you interpret the discount factor (δ)?
- What determines the perceived probability of winning an election (π), and how does it vary?

Then, explain how the values of these parameters differed between your two examples. What changed, and why? Did the likelihood of peaceful power transfer increase or decrease as a result?

Note: You are not expected to model this formally or provide precise estimates — instead, focus on building a clear, well-reasoned narrative that uses the model to shed light on real-world political dynamics.

4. **Essay-based discussion of the microfoundation of $\pi(S)$ (max. 1000 words):** In the self-enforcing democracy model, $\pi(S)$ — the probability of electoral victory — is often treated as an exogenous object. In this essay, you are asked to consider what it would mean to *microfound* this object: that is, to derive $\pi(S)$ as the outcome of strategic interactions between political actors, rather than assuming its shape or behaviour in an ad hoc way.

   Your task is to provide an intuitive explanation for why $\pi(S)$ might be hump-shaped — that is, increasing in electoral stakes S at first, but decreasing beyond some point. What assumptions about preferences, beliefs, risk attitudes, or strategic behaviour of incumbents and opposition actors could plausibly lead to this non-monotonic relationship?

   Hint: For possible mechanisms or modelling strategies, feel free to consult the ideas outlined in the bonus question below — especially the discussion of contest models.
   Bonus question: Can you outline a simple formalisation of your argument? You don't need to solve the model — just sketch the players, their utility functions, key constraints, and the logic of interaction. A promising route is to model elections as a contest:

- Both incumbent and opposition expend effort to win; effort is costly and affects the probability of victory.
- As S (the electoral stakes) increases, both sides are more willing to fight — up to a point.
- Beyond that point, the opposition may scale back effort (e.g., due to rising marginal costs, fear of repression, or coordination failure), leading to a decline in $\pi(S)$.

This kind of logic can produce a hump-shaped $\pi(S)$ even in relatively simple models.

If you want to pursue the contest route, a good starting point is:

- Hirshleifer (1989)
- Konrad (2009)

You're welcome to pursue alternative microfoundations, but make sure your story clearly links actors' strategic incentives to the shape of $\pi(S)$.

5. **Simulation task:**
Using the example discussed in lecture, plot the compliance condition for each of the following parameters: the discount factor δ, the probability of success in conflict p, and the cost of political engagement k. For each parameter, compare two values (e.g., high vs. low), while holding all other parameters constant.

Ideally, produce one plot per parameter, showing both cases on the same graph (with clearly labelled lines). If that proves difficult, two side-by-side plots are fine. Be sure to:

- Label axes clearly,
- Indicate which line corresponds to which parameter value (e.g. in a legend),
- Specify the parameter values used.

In one or two sentences (in a separate document or embedded), briefly explain what the plots show and why the changes in each parameter affect the compliance condition in the way they do.

Analysing our simplified version of the Ferejohn (1986) model

1. **Asymmetric shock extension (uniform version):**
Suppose that in our simplified version of the Ferejohn (1986) model, performance outcomes are given by $o = e + \varepsilon$, where the noise term ε follows a uniform distribution over an asymmetric interval:

$$\varepsilon \sim \text{Uniform}(-a, b), \quad \text{with } a, b > 0, \ a \neq b$$

That is, ε has constant density on the interval $[-a, b]$, with:

$$f(\varepsilon) = \frac{1}{a+b}, \quad \text{for } \varepsilon \in [-a, b]$$

(a) Derive the incumbent's first-order condition for optimal effort, assuming voters use a cutoff rule $o \geq o^*$.

(b) What happens to the optimal level of effort when the distribution becomes more negatively skewed (i.e., when a increases relative to b)? What if the distribution becomes more positively skewed?

(c) Explain your result intuitively: Why might politicians work harder in environments with a higher chance of bad shocks?

Hint: Use the fact that the incumbent maximises:

$$\max_{e} \ R \cdot \Pr(o \geq o^*) - c(e) \text{ where } \Pr(o \geq o^*) = \Pr(\varepsilon \geq o^* - e)$$

2. **Voter loss asymmetry:**
In the Ferejohn model, voters decide whether to reelect an incumbent based on the observed outcome $o = e + \varepsilon$. Suppose voters cannot observe the incumbent's true effort e, only the noisy outcome o. They apply a cutoff rule: *reelect iff (if and only if) $o \geq o^*$, otherwise replace.*

Now, assume voters incur asymmetric losses depending on the type of mistake:

- Loss $L_1 > 0$ if they wrongly **reelect** a low-effort incumbent,
- Loss $L_2 > 0$ if they wrongly **replace** a high-effort incumbent,

- No loss if the re-election decision is "correct".

Assume:

- The incumbent exerts effort e (fixed for this exercise),
- The noise term follows a normal distribution: $\varepsilon \sim \mathcal{N}(0, \sigma^2)$,
- High effort means $e = e_H$, and low effort means $e = e_L$, with $e_H > e_L$.

(a) Write the expected voter loss as a function of the cutoff o^*, assuming voters believe there is a 50% chance the incumbent exerted high effort and 50% low effort.

(b) Using the standard normal cumulative distribution function $\Phi(\cdot)$, express the two types of expected losses:

- The probability of mistakenly **reelecting** a low-effort incumbent,
- The probability of mistakenly **replacing** a high-effort incumbent.

(c) Suppose $L_1 > L_2$. Should voters set a higher or lower cutoff o^*? Why?

(d) Provide a political interpretation of this asymmetry: In what types of real-world settings might voters fear one kind of mistake more than the other?

Hint for (b): For a given effort level e, the outcome $o = e + \varepsilon$ is normally distributed with mean e and variance σ^2. So, the probability that $o < o^*$ is:

$$\Pr(o < o^* \mid e) = \Phi\left(\frac{o^* - e}{\sigma}\right)$$

3. **Essay: Are stationary cutoff rules suboptimal?**
Read the intuitive parts of Acharya et al. (2025) and Gieczewski and Li (2024). These papers argue that voters who apply a fixed performance threshold (stationary cutoff) may fail to incentivize effort effectively, especially when political or economic environments evolve.

Write a short essay (max. 1000 words) engaging with at least two of the following points — ideally the first and at least one other:

(a) In your own words, summarise the main reasons why stationary cutoff rules can be suboptimal. Focus on the strategic behaviour of incumbents and any intertemporal dynamics mentioned in the readings.

(b) Reflect on how time-varying or adaptive cutoffs might improve accountability. What kind of cutoff trajectory (e.g., declining, increasing, responsive) might work better — and under what conditions?

(c) Incumbents are empirically more likely to be reelected–a phenomenon known as the incumbency advantage. Do these models provide a rational explanation for this pattern? Why or why not? How does this theoretical logic compare to common explanations based on voter bias, partisanship, or information asymmetries?

(d) Finally, consider any downsides or challenges: What could go wrong if voters try to adjust the cutoff too much or too fast? Might dynamic cutoffs also reduce accountability in some situations, and why?

Note: You are not required to do formal modelling. Focus on clarity of argument, use examples where helpful, and feel free to critique the assumptions of the underlying models.

4. **Simulation of dynamic cutoffs: Effort and reelection over time**

In the Ferejohn model, incumbents choose effort e_t in each period to maximise the probability of reelection, given a voter-set performance threshold k_t. Suppose now that voters become more lenient over time, and the cutoff declines linearly:

$$k_t = k_0 - \alpha t, \text{with } \alpha > 0$$

Your task is to write a simulation of this model over $T = 20$ periods. Assume the following:

- The performance outcome in each period is: $o_t = e_t + \varepsilon_t$
- Noise: $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$
- Cost of effort: $c(e) = \frac{1}{2}e^2$
- Benefit of reelection: $R = 1$
- The incumbent chooses e_t in each period to solve:

$$\max_{e_t} \quad R \cdot \Pr(o_t \geq k_t) - c(e_t)$$

- Use the fact that: $\Pr(o_t \geq k_t) = 1 - \Phi\left(\frac{k_t - e_t}{\sigma}\right)$

Tasks:

(a) Write a function that, for given k_t, returns the optimal e_t^*. You may use grid search or numerical optimisation.

(b) Simulate the model over 20 periods using:

- $k_0 = 1.0$, $\alpha = 0.02$, $\sigma = 0.5$
- For each period, compute k_t, e_t^*, and Pr(reelection)

(c) Plot the time series of:

- The cutoff k_t
- The optimal effort e_t^*
- The reelection probability $\Pr(o_t \geq k_t)$

(d) Discuss your results: How does declining voter stringency affect effort and accountability? Do you observe 'shirking' over time? Why or why not?

Technical hint: If you use Python (e.g., NumPy, SciPy, Matplotlib) or R, you can use 'scipy.optimize.minimize_scalar' (Python) or 'optimize' (R) to find optimal effort in each period.

Analysing our simplified version of the Fearon (1999) model

1. **Simulating selection in elections: Fearon's model made simple**
In this exercise, you will simulate a simplified version of Fearon's electoral model. The goal is to help you explore the logic of political selection: how voters try to choose good types, how bad types may mimic good ones to stay in power, and how this affects voter welfare.

Setup:

- Each politician is either a **good type** or a **bad type**.
- Good types always choose $x = 0$
- Bad types choose $x > 0$: the higher the x, the worse the outcome for voters, but the better the politician feels (e.g. more rent-seeking, less effort).
- The performance signal observed by voters is:

$$z = -x^2 + \varepsilon, \quad \text{where } \varepsilon \sim \mathcal{N}(0, 1)$$

- The signal z is noisy but worse when x is far from 0.
- Voters reelect the incumbent if $z > k$, where k is a fixed cutoff.
- Bad types want to be reelected, so they may "mimic" good types by choosing a lower x, but that comes at a personal cost.

Your task:

(a) **Fix parameters:**
- Cutoff $k = -0.5$
- Bad types choose a fixed effort level $x_B = 0.5$
- Simulate a group of 1000 politicians
  - Case 1: 30% are bad types
  - Case 2: 70% are bad types
- Good types choose $x = 0$
- For each politician, simulate $z = -x^2 + \varepsilon$, with $\varepsilon \sim \mathcal{N}(0, 1)$

(b) **Apply the reelection rule:** Reelect if $z > k$. For each case (30% and 70% bad types):
- What fraction of all politicians are reelected?
- Among those reelected, what fraction are actually bad types?

(c) **Voter welfare:** Assume voter utility depends on the policy choice:

$$U = -\mathbb{E}[x^2 \mid \text{reelected}]$$

- That is, voters prefer to reelect politicians with low x^2
- Compute the average x^2 among those reelected
- What does this tell you about how the share of bad types affects welfare?

Hint: You can use a simple loop or vector to assign politician types, compute x, draw ε, and calculate z. Use conditional logic to apply the cutoff rule. Use averages to compute reelection shares and voter utility.

Interpretation:

- What happens when voters believe most politicians are bad?
- Does stricter voting help voters avoid bad types, or just cause them to reelect no one?

- When do bad types successfully "pass" as good types?

2. **Simulation: How signal noise affects voter decisions** This exercise introduces you to a simplified version of Fearon's idea that voters learn about politician 'quality' from noisy performance signals. You will simulate how well a simple voting rule works when signals are more or less noisy.

   Basic setup:

   - There are two types of politicians:
     - **Good types** produce a high signal: on average $z = 1.0$
     - **Bad types** produce a low signal: on average $z = -1.0$
   - Voters observe noisy signals:

   $$z = \mu + \varepsilon, \quad \text{where } \varepsilon \sim \mathcal{N}(0, \sigma^2)$$

   - The noise level σ determines how reliable the signal is. A higher σ means more uncertainty.
   - Voters apply a simple decision rule:
     - **Reelect the politician if** $z > 0$

   Your task:

   (a) Simulate a group of 1000 politicians. Assume:
     - 50% are good types, 50% are bad types.

   (b) For each noise level—$\sigma = 0.5$, $\sigma = 1.0$, and $\sigma = 2.0$—do the following:
     - For each politician:
       * If they are a good type, draw $z \sim \mathcal{N}(1.0, \sigma^2)$
       * If they are a bad type, draw $z \sim \mathcal{N}(-1.0, \sigma^2)$
     - Apply the voter rule: reelect the politician if $z > 0$

   (c) For each noise level σ, compute the following:
     - The overall **reelection rate**
     - The share of **good types who are reelected** (true positives)
     - The share of **bad types who are reelected** (false positives)

   (d) Reflect briefly:

- What happens to voter accuracy when signals become noisier?
- Do voters make more mistakes when σ is large? Why?
- What might this imply for real-world voting decisions when information is limited?

Hint: You can use a for-loop or vector to simulate 1000 politicians. Assign each politician a type (good or bad), draw their signal z, and then apply the rule $z > 0$. Use simple counts or averages to compute the required statistics.

3. **Essay: Selection versus sanctioning**
Based on the chapter by Fearon (1999), write a short essay (max. 800 words) addressing the following:

(a) What is the difference between viewing elections as mechanisms of *selection* versus *sanctioning*? How do the models differ in terms of what voters and politicians are assumed to know and control?

(b) Fearon argues that real-world voter behaviour often fits the selection model better than the sanctioning model. Summarise the key empirical observations he gives to support this claim.

(c) According to the mixed model, why can trying to select good types undermine the incentives to behave well in office?

(d) Do you think democratic elections function better as selection or sanctioning devices in practice?

Note: Your focus should be on clear explanation and thoughtful reflection, not technical modelling.

4. **Essay: Are term limits good for accountability?** In electoral models like those of Ferejohn and Fearon, reelection incentives help motivate good performance. But many democracies impose term limits, which prevent incumbents from seeking reelection after a fixed number of terms. This creates both benefits and costs.

In this essay, you are asked to explore the trade-offs associated with term limits. Your task is to think carefully about how term limits affect political incentives in repeated elections when signals are noisy and types are unobservable.

Write a short essay (max. 1000 words) engaging:

(a) Briefly summarise why the prospect of reelection creates incentives for effort in two-period models like Ferejohn's. What happens in the final period of a politician's career under term limits?

(b) Now suppose that voter signals are very noisy (e.g., due to shocks, limited information, or media bias). Why might voters fail to remove bad types even after one or two terms? How might term limits help in this case?

(c) Can term limits improve accountability by "flushing out" low-quality incumbents? What kinds of political environments would make this logic more persuasive?

(d) On the other hand, what are the costs of term limits? How do they affect incentives to work hard or invest in long-term projects? What happens if a good type is forced out?

(e) Overall, under what conditions do you think term limits are more helpful or harmful for political accountability?

(f) Optional: Suppose we had better ways of reducing signal noise (e.g., audits, independent media, policy metrics). Would the case for term limits be stronger or weaker?

Note: You are not expected to model this formally. Focus on reasoning carefully through incentives and trade-offs, and use theory and real-world logic to support your answer.

References

Acharya, Avidit, Elliot Lipnowski, and João Ramos, "Political Accountability under Moral Hazard," *American Journal of Political Science*, 2025, *69* (2), 641–652.

Fearon, James D., "Electoral Accountability and the Control of Politicians: Selecting Good Types versus Sanctioning Poor Performance," in Adam Przeworski, Bernard Manin, and Susan C. Stokes, eds., *Democracy, Accountability, and Representation*, Cambridge University Press, 1999, pp. 55–97.

_ , "Self-enforcing Democracy," *The Quarterly Journal of Economics*, 2011, *126* (4), 1661–1708.

Ferejohn, John, "Incumbent Performance and Electoral Control," *Public Choice*, 1986, *50* (1-3), 5–25. Accessed April 30, 2025.

Gieczewski, Germán and Christopher Li, "Incumbent Performance and Electoral Control: A Comment," *Quarterly Journal of Political Science*, 2024, *19* (3), 331–353.

Hirshleifer, Jack, "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success," *Public Choice*, 1989, *63* (2), 101–112.

Konrad, Kai A., *Strategy and Dynamics in Contests*, Oxford University Press, 2009.

Przeworski, Adam, *Democracy and the Market: Political and Economic Reforms in Eastern Europe and Latin America*, Cambridge University Press, 1991.

_ **, Gonzalo Rivero, and Tianyang Xi**, "Elections as a Conflict Processing Mechanism," *European Journal of Political Economy*, 2015, *39*, 235–248.