Buying Supermajorities

TIM GROSECLOSE *Ohio State University*
JAMES M. SNYDER, JR. *Massachusetts Institute of Technology*

Minimal winning coalitions have appeared as a key prediction or as an essential assumption of virtually all formal models of coalition formation, vote buying, and logrolling. Notwithstanding this research, we provide a model showing that supermajority coalitions may be cheaper than minimal winning coalitions. Specifically, if vote buyers move sequentially, and if the losing vote buyer is always granted a last chance to attack the winner's coalition, then minimal winning coalitions will generally not be cheapest, and equilibrium coalitions will generally not be minimal winning. We provide results relating equilibrium coalition size with preferences of the legislators and vote buyers, and we show that minimal winning coalitions should occur in only rare cases. We discuss these results in light of empirical work on coalition size and suggest other possible avenues for testing our model.

It is obvious to most students of positive political theory that minimal winning coalitions are cheaper to build than supermajority coalitions. If a coalition builder must pay for each member added to the coalition, then s/he should never pay more than the smallest number required to win. Minimal winning coalitions have appeared either as a key prediction or as an essential assumption of almost all formal models of coalition formation, vote buying, and logrolling (Baron and Ferejohn 1989, Denzau and Munger 1986, Gamson 1961, Koehler 1972, Koford 1982, Riker 1962, Shepsle 1974, Snyder 1991).

As an empirical matter, however, oversized coalitions seem to be at least as prevalent as minimal winning coalitions. Divisions on legislative roll calls are seldom near 50–50 (Damgaard 1973, Hinkley 1972, Lutz and Murray 1975, Lutz and Williams 1976, Uslaner 1975), and more refined searches for "minimal winning tendencies" typically fail as well. For example, the "effectiveness" of a majority party appears to be strictly increasing with the party's size, even after it is much larger than minimal winning (Francis 1970, Hinkley 1972, LeBlanc 1969, Moore 1969), and "defections" do not increase as the majority party increases in size (Uslaner 1975). Moreover, in the context of parliamentary coalition government formation, nonminimal winning coalitions frequently form (Browne 1973, Laver and Schofield 1990).

Theorists have responded to this evidence by providing reasons for the formation of nonminimal winning coalitions. One reason is uncertainty (Koehler 1972, 1975, Riker 1962). If there is some chance that legislators who have promised their votes will fail to honor their promise, whether due to reneging or to abstaining for unavoidable reasons, then vote buyers will typically pad their majorities and aim for an expected victory margin of more than one vote. Another possible reason, at least for distributive policy arenas, is the existence of a legislative norm of "universalism." Legislators might prefer to support all distributive projects proposed each session rather than live in an environment in which new minimal winning coalitions form each session and pass only their own projects (Klingaman 1969, Weingast 1979). A third reason is the cost of "ideological diversity," leading to minimal *connected* winning coalitions that may be larger than minimal winning coalitions (Axelrod 1970).[1] In this paper we provide another reason: Nonminimal winning coalitions may actually be *cheaper* than minimal winning coalitions. Specifically, we show that if two vote buyers compete for a majority in a legislature, and if they make offers to purchase votes sequentially, then minimal winning coalitions generally will not be cheapest, and equilibrium coalitions will generally not be minimal winning. In fact, equilibrium supermajorities will be the rule rather than the exception.[2]

The intuition behind this result is as follows. If the first vote buyer has bribed a minimal winning coalition, then the second vote buyer needs to bribe only one member of the first vote buyer's coalition in order to invade successfully. If the first vote buyer has bribed more than a minimal winning coalition, however, then the second vote buyer must bribe at least two members of the first vote buyer's coalition. Thus, by bribing more than a minimal winning coalition, the first vote buyer can decrease the amount of the bribe paid to each member of his or her original coalition, while keeping constant the amount the second vote buyer must pay to invade successfully. If the savings due to decreasing the bribes are greater than the costs of bribing another legislator, then the first vote buyer is better off bribing a supermajority. We show that the savings are usually greater than the costs.

The recent House vote on the North American Free Trade Agreement (NAFTA) provides an illustration.

Tim Groseclose is Assistant Professor of Political Science, Ohio State University, and Visiting Professor, Massachusetts Institute of Technology, and James M. Snyder, Jr., is Associate Professor of Political Science, Massachusetts Institute of Technology, Cambridge, MA 02139.

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[1] Baron and Ferejohn (1989) note that nonminimal winning coalitions are possible in their random-recognition, divide-the-dollar, majority-rule bargaining game, when motions must be seconded before a vote is taken. Their calculations show, however, that supermajority coalitions can occur only when the legislature is very small and the discount rate is quite high. Accordingly, they conclude that their model basically supports Riker's (1962) size principle.

[2] Our results build on the work of Groseclose (1996), who presents an example showing that supermajorities can be cheaper than minimal winning coalitions, and Snyder (1990), who presents a model in which the vote buyer defending the status quo would generally build a supermajority coalition.

According to many observers, President Clinton and Republican House leaders traded favors for votes. The final vote of 234 to 200, however, indicates that these leaders constructed a coalition that was 16 votes larger than minimal winning. While this behavior is an anomaly for many previous models, it is consistent with ours. The president and the Republican leaders faced significant opposition from certain Democrats, such as Majority Leader Richard Gephardt and Chief Whip David Bonior. Since Gephardt and Bonior could have attempted to invade the pro-NAFTA coalition by buying votes themselves, it is conceivable that the pro-NAFTA leaders consciously built a supermajority coalition to deter such an effort. The costs are much higher for invading a properly constructed supermajority coalition than the optimal minimal winning coalition. By making the costs of invasion high enough, the pro-NAFTA leaders may have convinced Gephardt and Bonior to concede the issue rather than spend their resources.

Two assumptions are crucial for our theoretical result. First, we assume there are two competing vote buyers instead of one. In the limiting case when the reservation price of the second vote buyer is zero, our model becomes the single-vote-buyer model. In this case our results are the same as in previous papers (e.g., Denzau and Munger 1986): The first vote buyer buys only a minimal winning coalition.

Second, we assume that vote buyers move sequentially. Although we make this assumption primarily for analytic convenience (if vote buyers were to move simultaneously, then pure strategy equilibria typically would not exist), under several interpretations it accords well with actual coalition building. For example, the status quo is a favored alternative, since all proposed bills must eventually defeat the status quo in a vote. If payments for votes on a bill are typically written into the bill itself (as special conditions, allowances, exemptions, transition rules, and so on) and thus become public when the bill is reported, and if it is costly to change bills, then defenders of the status quo are effectively able to move last.

The assumption of sequential offers also makes sense when coalitions are viewed in a dynamic context, and the problem of *maintaining* a winning coalition is taken into account as well as the problem of forming one. For example, suppose an organized interest group builds a legislative coalition to pass a bill in one session of the legislature and must maintain this coalition in subsequent sessions, when groups that oppose the bill may have opportunities to counterattack and try to rescind it. Then the group supporting the policy effectively moves first, and in building its coalition it should defend against a wide range of possible attacks. One interpretation of our results, therefore, is that supermajorities are generally cheaper than minimal winning coalitions when the costs of maintaining winning coalitions are important. Supermajorities are cheaper in the long run.

THE MODEL AND TWO EXAMPLES

A legislature is to decide by majority rule between two alternatives, a status quo (s) and a new policy (x), both elements of $\mathbb{R}$. Each legislator has preferences over how s/he votes and also over a "political resource," which we call money.[3] For each legislator i, let $v(i) = u_i(x) - u_i(s)$ denote the intensity of i's preference for voting for x over s, measured in money. We call $v(i)$ the reservation price of i. Without loss of generality, label legislators so that $v(i)$ is a nonincreasing function.[4]

Two voters buyers, A and B, have preferences over x and s. The vote buyers could be interest groups, a president or governor, or a party or committee leaders. Without loss of generality, assume that A prefers x over s and that B has the opposite preference. Let $W_A = U_A(x) - U_A(s)$ be A's willingness to pay for x measured in money, and let $W_B = U_B(s) - U_B(x)$ be B's willingness to pay for s. Each vote buyer wants to minimize the total bribes paid, while passing his or her preferred policy in the legislature, but each would prefer to concede the issue than pay more than his or her willingness to pay.

For each legislator i, let $a(i)$ denote A's offer to i, and let $b(i)$ denote B's offer to i. We call $a(\cdot)$ and $b(\cdot)$ the bribe offer functions. Each legislator takes the bribe offers as given and then votes for the alternative that gives him or her the greater payoff. Since legislators only have preferences over how they vote, not over which alternative wins, once the bribe offers are known, each legislator has a dominant voting strategy.

We assume that the vote buyers move sequentially, with A moving first and B, defender of the status quo, moving last. All actors become informed of bribes once they are offered; in particular, B is perfectly informed about the bribes made by A when it is her turn to offer bribes.

We wish to characterize the subgame-perfect, pure-strategy Nash equilibria of the game. To avoid open-set nonexistence problems and the need to use some type of ε-equilibrium solution concept, we assume that all unbribed legislators who are indifferent between the alternative x and the status quo s vote for s and that all bribed legislators who are indifferent between x and s (given the offered bribes) vote for the outcome preferred by the vote buyer who bribes them last. These are purely

[3] Thus, legislators are primarily concerned with the position-taking aspect of their voting decisions, not with policy outcomes. This is consistent with the view of Mayhew (1974), Fenno (1978), and others.

[4] One may think of legislators' preferences for policies and bribes as indirect preferences induced from other goals. For example, suppose each legislator cares only about his or her reelection probability, and this depend positively on how often the legislator's votes are consistent with constituents' preferences and on total campaign contributions from interest groups, or favors from legislative leaders (pork barrel projects, campaign appearances, contributions from leadership PACs, and so on). The assumptions we make about preferences are consistent with maximizing reelection probabilities; we simply assume the function mapping these two intermediate goals into probabilities is linear and separable. See Denzau and Munger (1986) for further discussion of the relationship between direct and indirect utility functions of legislators.

We allow v_i to be infinite. That is, legislators can have such an intense preference for x or s that no bribe of any size will make them change their vote. Of course, if a majority of the legislators have such intense preferences, then no vote buying takes place, and the model becomes trivial.

technical assumptions, which simplify the analysis and exposition.

The following simple example illustrates why it may be optimal for a vote buyer to build a nonminimal winning coalition.

EXAMPLE 1. *The legislature contains seven members, all indifferent between the two proposals, that is,* $v(i) = 0$ *for all* i. *Since* B *will move second and attack the weakest part of* A*'s coalition,* A *will offer the same bribe,* a, *to all legislators he bribes. Let* $m + 4 \geq 0$ *be the size of the coalition* A *bribes. If* $m = 0$, *then* B *can defeat* x *by paying* $a + \varepsilon$ *to one member of* A*'s coalition. Thus,* A *must set* $a \geq W_B$ *to win, and* A*'s total bribes are then* $4W_B$. *If* $m = 1$, *then* B *must pay* $a + \varepsilon$ *to two members of* A*'s coalition to defeat* x. *Thus,* A *must set* $a \geq W_B/2$ *to win, and his total bribes are then* $5W_B/2$. *This is* less *than* $4W_B$, *so* A *is better off buying a supermajority of size 5 than a minimal winning coalition. If* $m = 2$, *then* B *must pay* $a + \varepsilon$ *to three members of* A*'s coalition, so* A *must set* $a \geq W_B/3$ *to win, and his total bribes are then* $2W_B$. *In fact,* A *minimizes his total payments by bribing all seven legislators. Here,* $m = 3$, *and* A*'s total bribes are* $7W_B/4$. *Clearly, there is an equilibrium where* x *wins if and only if* $W_A \geq 7W_B/4$. *If* $W_A < 7W_B/4$, *then there are only equilibria in which no bribes are paid and* s *wins.*

It should be clear that this example is easily generalized to a legislature of any size n, so we state the following without proof.

COMMENT 1. *Suppose the number of legislators* n *is odd, all legislators are initially indifferent between* x *and* s, *and* $W_A \geq 2nW_B/(n + 1)$. *Then, in equilibrium,* A *bribes* all *legislators, with* $a(i) = 2W_B/(n + 1)$ *for all* i.

At least two features of this equilibrium are worth noting. First, if A faced no opposition, then clearly he would only bribe a minimal winning set of legislators.[5] Thus, the existence of an opposing vote buyer is necessary to generate the nonminimal winning coalition strategy.[6] Second, for large n, A's total payments are approximately $2W_B$. Thus, in order for an equilibrium to exist in which x wins, W_A must be (approximately) at least twice as large as W_B. Finally, there is an implicit assumption that A and B cannot collude. Otherwise, A could offer B an amount $W_B + \varepsilon$ in exchange for B's promise not to compete in buying votes. This would be cheaper for A, and Pareto optimal from the vote buyers' point of view, although not from the legislators' point of view. We assume that legal prohibitions or intense antagonism between A and B make this cooperative solution impossible.

It is also easy to produce examples in which A bribes a coalition that is larger than minimal winning but smaller than universal. Consider the following case, in which all legislators are initially opposed to x.

EXAMPLE 2. *The legislature contains seven members, each with* $v(i) = -1$. *Assume* $W_B = 3$, *and assume* W_A *is large enough that an equilibrium exists in which* x *wins. In such an equilibrium,* A *will offer the same bribe,* a, *to all legislators he bribes. Let* $m + 4$ *be the size of the coalition* A *bribes. If* $m = 0$, *then* A *must set* $a = 1 + W_B = 4$ *to win, and his total bribes are 16. If* $m = 1$, *then* A *must set* $a = 1 + W_B/2 = 5/2$ *to win, and his total bribes are 25/2. If* $m = 2$, *then* A *must set* $a = 1 + W_B/3 = 2$ *to win, and his total bribes are 12. Finally, if* $m = 3$, *then* A *must set* $a = 1 + W_B/4 = 7/3$ *to win, and his total bribes are 49/4. Thus, the optimal strategy is for* A *to bribe six of the seven legislators, neither a minimal winning nor a unanimous coalition.*

Examples 1 and 2 provide an interesting alternative to the standard explanations of the tendency for omnibus public works bills to have universalistic or near universalistic support (Collie 1988, Weingast 1979). These bills usually do not arouse strong opposition, even among legislators who do not receive projects. They typically involve only a small part of the budget, and commentators sometimes refer to them as legislative grease. That is, they are viewed not as part of the main machinery of public policy but only as an accessory that helps it to run smoothly. As a result, legislative preferences are much like those of example 1 or example 2—all legislators are indifferent or nearly indifferent to the bill's passage. The main opposition to such bills usually comes from ideological groups opposed to government spending, such as the National Taxpayers' Union, or ideological conservatives such as James Buckley (see Krehbiel 1991 and citations therein for a discussion of a famous case in which Buckley opposed the 1973 Senate omnibus rivers and harbors bill). As examples 1 and 2 show, the optimal way to prevent such opponents from invading the coalition is to pay small bribes (projects) to a very large proportion of the legislature.

In fact, the two examples may provide an explanation to a puzzle Collie has raised about the issue: If a norm of universalism is the principle actually at work in omnibus bills, then why do not *all* congressional districts receive projects in such bills? For example, Ferejohn (1974) reports that omnibus public works bills typically contain projects affecting 350–400 districts (about 82–90%). Example 2 provides an answer. If legislators who do not receive projects are not indifferent to the bill's passage but instead are slightly opposed, say, because they care (only slightly) about the fact that the bill adds to the tax burden of their constituents, then the optimal coalition size will typically not be universalistic. In example 2, for instance, the optimal coalition size is just over 85%.

Finally, we should point out that many of the assumptions of our model can be relaxed without qualitatively affecting the results presented below. First, although our model assumes only two periods of vote buying, it is easily generalized to any finite number of periods, as long as the two vote buyers still alternate in making bribes. In such a multiperiod model the optimal strategy

[5] Technically, when there is no opposing vote buyer, A's optimal strategy must be defined as the limit of an open set, since we assume that indifferent legislators vote for s. Alternatively, we could assume there is some minimum positive bribe, say, a penny.

[6] Of course, uncertainty about how legislators may vote (or abstain), even when bribed, also may lead A to bribe a supermajority. It is unlikely, however, that such uncertainty would lead to a very large supermajority, at least in a large legislature. The law of large numbers would reduce the overall uncertainty considerably.

of the vote buyer who moves last is to wait until the last round before making any bribes. The competing vote buyer therefore only makes bribes in the second to last round. The outcome is exactly as in our two-round game, except there are prior periods during which the voter buyers wait each other out.

Second, although our model somewhat arbitrarily allows the defender of the status quo a last-mover advantage, this can be supported as an equilibrium result in a more general model in which neither vote buyer is given such an advantage. Suppose a first round is added in which vote buyers first simultaneously decide whether to initiate the vote buying game and then play the game as we specify. If only one vote buyer chooses to initiate in the first round, then s/he is required to move first; if both choose to initiate, then they flip a coin to see who moves first; and if neither chooses to initiate, then the game is not played, no roll call votes are taken, and the status quo remains in place. It is easy to show that in this game the vote buyer who prefers the status quo will never initiate (this buyer can always afford to sit back and wait). Thus, if the game takes place, the vote buyer preferring the status quo necessarily moves last.[7]

Third, we assume that legislators do not have preferences over which policy wins but only over how they vote. That is, our legislators are strictly Mayhewian position takers. The model would change very little, however, if the legislators were also given preferences over outcomes. The reason is that such preferences only matter if a legislator is pivotal; otherwise, a legislator's voting strategy depends only on position-taking considerations. Thus, whenever a vote buyer buys at least one vote more than s/he needs to win, all legislators become nonpivotal, and the outcome-based part of legislators' preferences disappear. All that would remain is the position-taking part of their preferences, as our model assumes.

Fourth, in the formal part of our analysis we treat bribes as if they are simple cash transfers from vote buyers to legislators. It may be more natural, however, to treat them as particularized benefits written into the bill under consideration. The main effect of such a change is that legislators would no longer be indifferent about the bribes received by other legislators; bribes would produce externalities. For instance, suppose that bribes are particularistic projects written into the bill, and these projects are funded by the general tax system, so that every dollar in bribes means one more dollar in taxes spread evenly among the legislators. Then, legislators would strictly prefer that their colleagues receive fewer bribes. Also, given a fixed set of bribes, each legislator's preference for the bill shifts downward by the amount s/he must pay in increased taxes.

To illustrate, return to example 1, but treat bribes by A as district-specific projects written into the bill rather than cash transfers. Also, assume that taxes are apportioned equally, so each legislative district pays one-seventh of the total costs of these projects. Suppose A wants to minimize the total value of the projects, perhaps due to a direct ideological distaste for such projects (they are "pork"), or perhaps because he faces an overall project budget constraint that must be divided across a number of bills. Then it is straightforward to show that the optimal strategy for A is to pay five legislators $7W_B/4$ dollars in bribes, and in equilibrium each legislator pays $5W_B/4$ dollars in taxes.[8] Treating A's bribes as projects added to bill x produces the same results as treating these bribes as cash payments and shifting each v_i down by $5W_B/4$ dollars, the amount of taxes each legislator must pay to finance the particular projects.

With just one exception, all the results presented below would be qualitatively unchanged if we replace the assumption that bribes are money payments with the assumption that bribes are written into bills. The exception is that universalistic coalitions are never optimal under the bribes-as-legislation assumption. When bribes are written into legislation, they must be financed by taxes. Thus, assuming legislators perceive the full tax cost of the bribes, when coalitions are universalistic the bribes act only as transfers between legislators. As a consequence, the bribes do not raise the overall support of legislators for a bill and therefore do nothing to ward off an opposing vote buyer from invading. Thus, when bribes are written into legislation, universalistic coalitions are never optimal strategies for vote buyers.

MORE GENERAL RESULTS

In this section, we characterize equilibrium vote buying under more general configurations of legislators' preferences. Mathematically, it is simpler to assume there is a continuum of legislators rather than a finite number. Therefore, suppose the set of legislators can be indexed by a uniform distribution on the interval $[-1/2, 1/2]$. Clearly, the median voter is at zero. Let $v(z)$ be the reservation-price function (see Figure 1). Assume that $v(\cdot)$ is nonincreasing and differentiable. The strategies of A and B are functions $a(\cdot)$ and $b(\cdot)$ on $[-1/2, 1/2]$. We focus on cases in which W_A is large enough that x wins in equilibrium. (The equilibria in which s wins are less interesting. Since B does not face a potential invader, the results are the same as in a one-buyer model.) Accordingly, let $m + 1/2$ be the fraction of legislators, both bribed and unbribed, who vote for x. Thus, m represents the "excess" size of A's coalition, relative to a minimal winning coalition. In order to defeat x, B must pay bribes to at least m members of A's coalition.

[7] Alternatively, we might assume that if neither player initiates, then no bribes are offered, but a vote is taken in the legislature to see which policy wins. Then, the buyer who favors the alternative initially preferred by a majority in the legislature (rather than the buyer who favors the status quo) has the advantage.

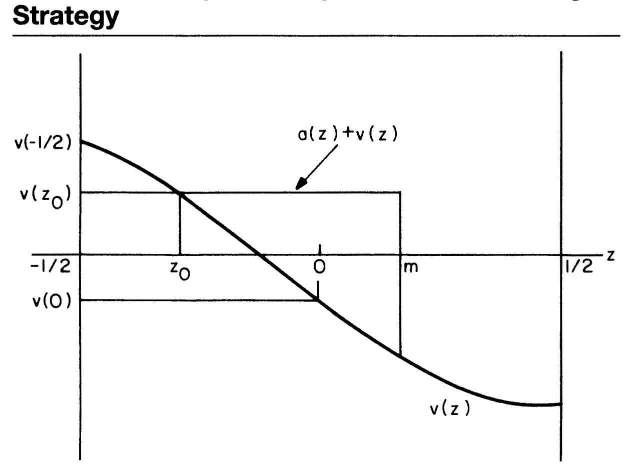
[8] We assume that since B is defending the status quo, rather than proposing new legislation, if she attacks then she must pay bribes in cash. Although this introduces an asymmetry between the two vote buyers, it seems plausible in many cases. For example, suppose A is an appropriations subcommittee chair who may write projects into a bill as bribes. The opposition to A's bill, if any, is likely to come from the president or minority party leaders in the House. These opponents do not have the same resources as the subcommittee chair. It is more likely that their bribes would take the form of campaign appearances in upcoming elections, promises of help on future legislation, White House visits, and so on, rather than amendments to A's bill.

We call A's bribe offer function $a(\cdot)$ a *leveling* strategy if there is a legislator z_0, such that $v(z) + a(z) = v(z_0)$ for all bribed legislators, that is, all z such that $a(z) > 0$ (see Figure 1). When there is a continuum of legislators, we say the strategy is leveling if $v(z) + a(z) = v(z_0)$ for *almost all* bribed legislators, that is, for all bribed legislators except a set of measure zero. Thus, A leaves B with a level field of legislators from which to choose when deciding whom to bribe. Whenever there are equilibria in which x wins, there is always one in which A plays a leveling strategy. Under some specifications for $v(\cdot)$ and W_B, however, there are also equilibria in which A plays a nonleveling strategy.

The intuition behind a leveling strategy is similar to the "no-soft-spots" theory of international relations (Dresher 1966). Suppose a nation, considering how best to defend itself from enemies, has only two places through which it can be attacked, a plain and a mountain pass. Optimal placement requires that more troops be put along the plain, since this point is easier to invade. In fact, since an invader would choose the weakest point of entry, the defender's optimal strategy is to allocate troops such that each point is equally difficult to invade, that is, no spot is any softer than another. A similar idea is true for coalition building. Since an opposing leader would attack legislators whose support for the bill is weakest, a vote buyer's best strategy is to make each bribed coalition member equally expensive for the opponent to invade.

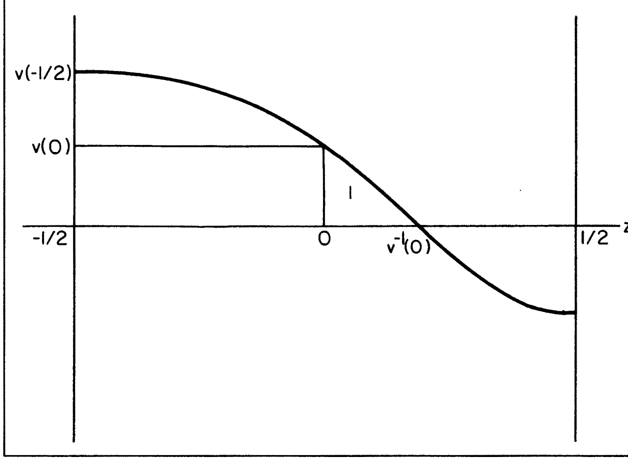
There are three types of equilibrium in which x wins. In the first case, W_B is very small relative to the initial level of legislative support for x, and A can win without bribing any legislators. In the second case, W_B is in an intermediate range relative to the initial support levels of the legislature. A buys votes but does not necessarily adopt a leveling strategy. Furthermore, A only bribes legislators who already support x, although some must be bribed in order to raise their level of support high enough to prevent B from trying to buy their votes. In the last case, W_B is large relative to the initial level of support for x, and A always adopts a leveling strategy. The three cases are summarized in the following proposition.[9]

FIGURE 1. Payments by *A* under a Leveling Strategy



Note: Each legislator bribed by A costs the same for B to buy, $v(z) + a(z) = v(z_0)$.

FIGURE 2. Proposition 1, Case 1: Area I > W_B by Assumption, So Total Payments by A = Zero



PROPOSITION 1. *Suppose* $a^*(\cdot)$ *and* $b^*(\cdot)$ *constitute an equilibrium in which x wins. Then, exactly one of the following cases holds:*

1. *if* $v(0) > 0$ *and* $W_B \le \int_0^{v^{-1}(0)} v(z)dz$, *then* $a^*(z) = 0$ *for all* z;
2. *if* $v(0) > 0$ *and* $\int_0^{v^{-1}(0)} v(z)dz < W_B < v(0)v^{-1}(0)$, *then* $a^*(z)$ *satisfies* $a^*(z) = 0$ *for* $z \notin [0, v^{-1}(0)]$, $a^*(z) \le v(0) - v(z)$ *for all* $z \in [0, v^{-1}(0)]$, *and* $\int_0^{v^{-1}(0)} [v(z) + a^*(z)]\, dz = W_B$;
3. *if* $v(0) \le 0$ *or* $W_B \ge v(0)v^{-1}(0)$, *then* $a^*(\cdot)$ *is a leveling strategy, with* $a^*(z) = W_B/m - v(z)$ *for all* z *such that* $a^*(z) > 0$, *where* m *satisfies* $m > max\{0, v^{-1}(0)\}$ *and* $W_B/m > v(0)$.

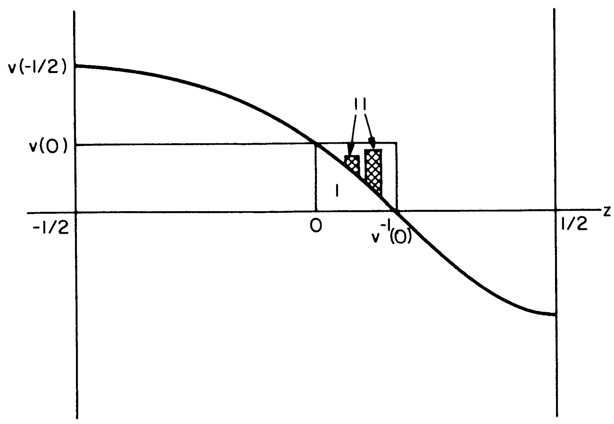
PROOF. All proofs are in Appendix A.

The three cases of the proposition are illustrated in figures 2 and 3. In Figure 2 no bribes are made. Here, the cheapest way for B to invade is to buy all legislators in the interval $[0, v^{-1}(0)]$, which costs an amount equal to area I in the figure. Since this area is greater than W_B by assumption, B prefers to concede the vote.

[9] We do not characterize B's equilibrium strategies, because they are less interesting. Also, they are more complicated, because B must condition her choices on A's strategies. Here is a rough description of one strategy for B that supports the equilibrium in proposition 1: If A offers bribes such that B can counter with bribes which ensure that s defeats x in the legislature and which also cost B an amount less than W_B, then B chooses any bribe offer function $b(\cdot)$ that minimizes total bribes and ensures that s defeats x. If A offers bribes such that B cannot counter with bribes ensuring that s wins except by offering total bribes greater than or equal to W_B, then B chooses $b(z) = 0$ for all z.

Other strategies for B are possible, however. For example, we could amend the second part of the strategy above to read: If A offers bribes such that B cannot counter with bribes ensuring that s wins except by offering total bribes greater than or equal to W_B, then B chooses $b(z) = a(z) + v(z) - \varepsilon$ for all z, where $\varepsilon > 0$. Here, B makes positive offers, but none of the offers will be accepted (given A's offers), and so B's total payments will be zero. Of course, all of B's optimal strategies will yield the same equilibrium policy and the same payments to legislators.

FIGURE 3. Proposition 1, Case 2: Area II Represents Payments by *A*. At the Optimum, I + II = W_B



In Figure 3 area II represents the bribes paid by A. First note that no point in this area is above the line through $v(0)$. Payments above $v(0)$ would make some bribed members of A's coalition more expensive for B to buy than legislator 0. Accordingly, B would ignore such legislators in an invasion, so A could decrease his total bribes while keeping B's costs of invading constant. Thus, such payments are suboptimal. Next, note that if B were to invade, she must buy $v^{-1}(0)$ legislators, and the cheapest way to do this is to buy all legislators in the $[0, v^{-1}(0)]$ interval. The costs to B are thus represented by the sum of areas I and II. Thus, A must set area II (the total bribes he pays) plus area I (the initial support of these legislators) so that their sum is at least W_B. Any strategy that satisfies this, and which also lies below $v(0)$, constitutes an optimal strategy. Intuitively, there is not a unique optimum because if B invades she must bribe *every* member of A's bribed coalition. Thus, A can always rearrange bribe levels among these legislators without changing his own total payments or B's cost to invade. One optimal choice is a leveling strategy, but other strategies are possible as well. For example, the bribes in Figure 3 do not constitute a leveling strategy, but they are optimal.

It is more interesting to note that in Figure 3 (and all situations in which case 2 of the proposition holds) A makes payments only to legislators who initially support x, so each legislator votes exactly as s/he would have voted if no bribes at all were paid. A's payments are made only to prevent B from invading his existing majority. Empirically, this means that in some cases it may be extremely difficult to detect bribes, since roll call votes may be unchanged by them.

Case 3 of the proposition was illustrated in Figure 1 for the subcase in which $v(0)$ is less than 0 (so only a minority of the legislature initially supports x). The subcase in which $v(0) > 0$ is similar. A always adopts a leveling strategy because A's bribed coalition must contain more than $v(0)$ members. Thus, if B invades, then (in contrast to case 2 of the proposition) she only buys *part* of A's bribed coalition. If one member of A's bribed coalition were more expensive than others, then B would ignore this member when invading. As a consequence, A makes each bribed member equally expensive to B, that is, he adopts a leveling strategy.

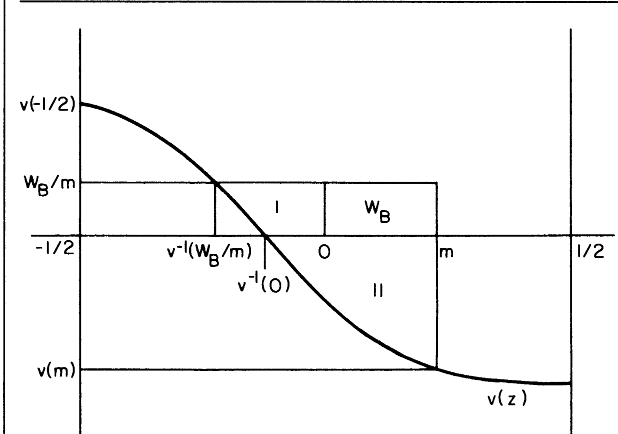
Several potentially testable hypotheses about coalition building follow directly from proposition 1. For example, there exist equilibria in which a majority of the legislature initially supports a vote buyer's position, yet the vote buyer still makes bribes. Also, there exist equilibria in which a vote buyer bribes legislators who initially support his position. In fact, in case 2 equilibria of this type, the vote buyer *only* bribes legislators who initially support her issue position. These results contrast sharply with the predictions of the one-buyer model.

Since one typically imagines that bribes take place when a majority of the legislature is initially opposed to the vote buyer, we consider this case in more detail. In the notation of our model, this means $v(0) \leq 0$. This is a subcase of case 3.

Suppose $v(0) \leq 0$, and consider the following two conditions: $v(-1/2) \geq W_B/m$, and $v(-1/2) < W_B/m$ (see figures 4 and 5). The term W_B/m is the minimum amount B must pay to buy the vote of a legislator in A's coalition. In the first case, the leftmost legislator ($z = -1/2$) costs B more than W_B/m, even if s/he does not receive a bribe from A. Thus, A does not need to bribe this legislator, or other legislators with z close enough to 1/2, in order to prevent B from trying to buy their votes in an invasion. We call this a nonflooded coalition. In the second case, the leftmost legislator costs B less than W_B/m if A does pay a bribe. In this case, A bribes every member of his coalition. We call this a flooded coalition. In either case, A's optimal leveling strategies are completely characterized by a single parameter, the excess coalition size, m.

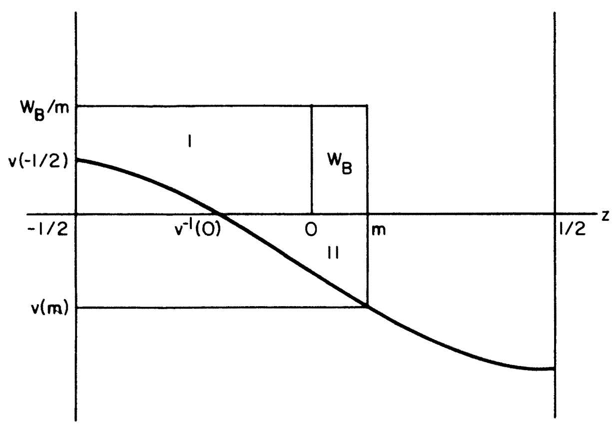
We now characterize the conditions under which flooded and nonflooded coalitions are optimal. With a nonflooded coalition, $a^*(z) = W_B/m - v(z)$ for all $z \in [v^{-1}(W_B/m), m]$, and $a^*(z) = 0$ otherwise. With a

FIGURE 4. Payments by *A* When *A* Constructs a Nonflooded Coalition



Note: Total payments = I + II + W_B.

FIGURE 5. Payments by *A* When *A* Constructs a Flooded Coalition



Note: Total payments = I + II + W_B.

flooded coalition, $a^*(z) = W_B/m - v(z)$ for all $z \in [-1/2, m]$, and $a^*(z) = 0$ otherwise. To buy back a member of A's coalition, B must pay $a^*(z) + v(z)$. This is the initial level support for x, $v(z)$, plus the payment offered by A, $a^*(z)$. To prevent B from invading, A must make offers such that each bribed legislator costs W_B/m to B. Thus, $a^*(z) = W_B/m - v(z)$ for all bribed legislators.

With a nonflooded coalition, A's total payments (T_A) are, therefore:

$$T_A = \int_{v^{-1}(W_B/m)}^{m} a^*(z)dz$$

$$= \int_{v^{-1}(W_B/m)}^{m} [W_B/m - v(z)]dz$$

$$= W_B - (W_B/m)v^{-1}(W_B/m) - \int_{v^{-1}(W_B/m)}^{m} v(z)dz$$

$$= W_B + [-(W_B/m)v^{-1}(W_B/m) - \int_{v^{-1}(W_B/m)}^{v^{-1}(0)} v(z)dz] + \int_{v^{-1}(0)}^{m} - v(z)dz. \quad \mathbf{(1)}$$

The three terms in equation (1) correspond to areas W_B, I, and II in Figure 4. In the case of a flooded coalition, A's total bribe payments are

$$T_A = \int_{-1/2}^{m} [W_B/m - v(z)]dz$$

$$= W_B + W_B/2m - \int_{-1/2}^{m} v(z)dz$$

$$= W_B + [W_B/2m - \int_{-1/2}^{v^{-1}(0)} v(z)dz] + \int_{v^{-1}(0)}^{m} - v(z)dz. \quad \mathbf{(2)}$$

The three terms in equation (2) correspond to areas W_B, I, and II in Figure 5.

A will choose m^* to minimize T_A. This yields the following proposition (see figures 6 and 7).

Proposition 2. *Suppose* $v(\cdot)$ *is nonincreasing and differentiable, and* $v(0) \leq 0$. *Then* m^* *is unique, and exactly one of the following holds:*

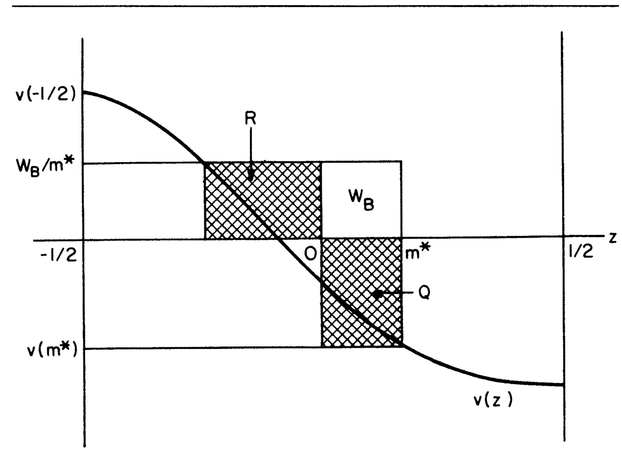
1. A *constructs a nonflooded, nonuniversalistic coalition, in which case* $m^* \geq W_B/v(-1/2)$, $m^* < 1/2$, *and* m^* *satisfies* $-(W_B/m^*)v^{-1}(W_B/m^*) = -m^*v(m^*)$;
2. A *constructs a flooded, nonuniversalistic coalition, in which case* $m^* < W_B/v(-1/2)$, $m^* < 1/2$, *and* m^* *satisfies* $(W_B/m^*)(1/2) = -m^*v(m^*)$;
3. A *constructs a universalistic coalition, in which case* $m^* = 1/2$.

Proposition 2 establishes the uniqueness of the optimal coalition size m^*. Cases 1 and 2 are illustrated in figures 6 and 7. The conditions describing these two cases also have a geometric interpretation. Specifically, they imply that at an interior m^*, two particular rectangular areas must be equal, as shown in figures 6 and 7.

An especially interesting and somewhat counterintuitive result is that in cases 1 and 2 $\partial m^*/\partial W_B > 0$. This is easily seen by differentiating the conditions defining m^* with respect to W_B (see Appendix B). Thus, A's optimal coalition size *increases* as B's willingness to pay increases. The intuition underlying this result becomes clearer by considering the limiting case. When W_B is zero, then A faces no vote buying opposition and therefore only bribes a minimal winning coalition (as in Denzau and Munger 1986 and Snyder 1991).

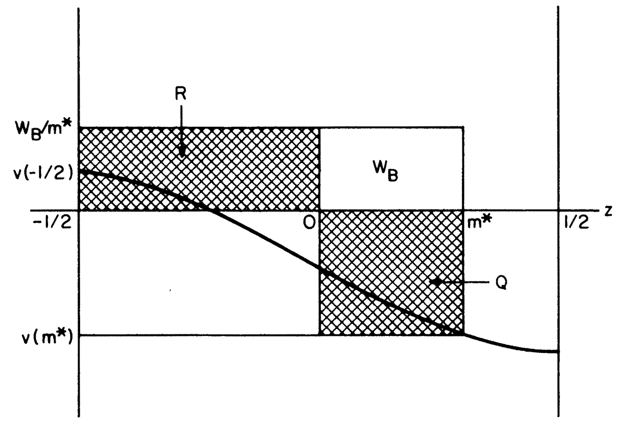
We can characterize m^* more fully in special cases. In the next proposition, we consider the case in which $v(\cdot)$ is linear, that is, $v(z) = \alpha - \beta z$, with $\beta \geq 0$.

FIGURE 6. Payments by *A* When a Nonflooded Coalition Is Optimal



Note: At the optimum, $R = Q$.

FIGURE 7. Payments by *A* When Flooded Coalition Is Optimal



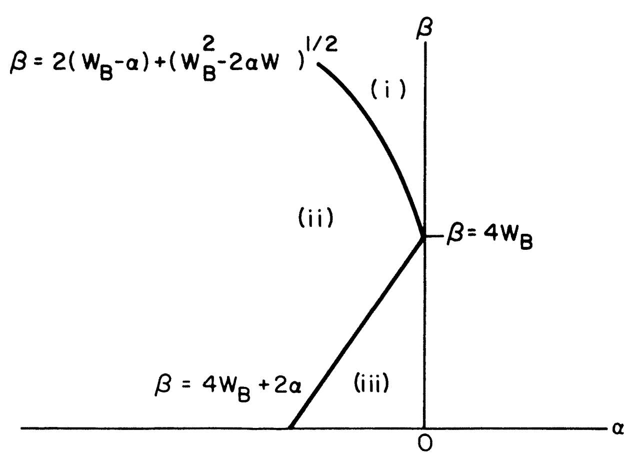
Note: At the optimum, $R = Q$.

PROPOSITION 3. *Suppose* $v(z) = \alpha - \beta z$, *with* $\beta \geq 0$ *and* $\alpha \leq 0$. *Then, the types of coalitions formed and* m^* *are characterized as follows* (see Figure 8).

1. A *constructs a nonflooded, nonuniversalistic coalition iff* $\beta \geq 2(W_B - \alpha) + (W_B^2 - 2\alpha W_B)^{1/2}$. *In this case,* $m^* = (W_B/\beta)^{1/2}$.
2. A *constructs a flooded, nonuniversalistic coalition iff* $4W_B + 2\alpha < \beta < 2(W_B - \alpha) + (W_B^2 - 2\alpha W_B)^{1/2}$. *In this case,* m^* *solves* $\beta(m^*)^2 = \alpha m^* + W_B/2m^*$.
3. A *constructs a flooded, universalistic coalition iff* $\beta \leq 4W_B + 2\alpha$. *In this case,* $m^* = 1/2$.
4. A *never constructs a nonflooded, universalistic coalition.*

Several features of proposition 3 are worth noting. First, m^* is a continuous function of α, β, and W_B. This is easily verified by checking the boundaries between the cases, $\beta = 2(W_B - \alpha) + (W_B^2 - 2\alpha W_B)^{1/2}$ and $\beta = 4W_B + 2\alpha$. Also, m^* is differentiable except at the boundaries. In subcase 1, $\partial m^*/\partial\alpha = 0$, and $\partial m^*/\partial\beta < 0$; in subcase 2, $\partial m^*/\partial\alpha > 0$, and $\partial m^*/\partial\beta < 0$. For fixed α, as β rises, the initial level of support for x in the legislature declines (β is the negative of the slope of v). Also, for fixed β, as α falls, the initial support for x declines. Thus, in either subcase 1 or 2, as the initial legislative support for x declines, A's optimal coalition size shrinks.

FIGURE 8. Values of α and β Corresponding to Cases 1, 2, and 3 of Proposition 3



It is also illuminating to consider the special case in which $\beta = 0$ (all legislators have the same preferences). If $\beta = 0$ and $\alpha \geq -2W_B$, then A constructs a universal coalition. This is similar to example 1 given in a previous section of the paper (where $\alpha = 0$), in which A bribes the entire legislature. If $\beta = 0$ and $\alpha < -2W_B$, then $m^* = (W_B/-2\alpha)^{1/2} < 1/2$. This is similar to example 2 in the previous section.

Another interesting result is the following. The legislators in the interval $[v^{-1}(W_B/m^*), v^{-1}(0)]$ represent the bribed friends of A, that is, legislators who receive payments from A even though they initially support x. It is straightforward to show that in subcase 1 of proposition 3, $v^{-1}(0) - v^{-1}(W_B/m^*) = m^*$. That is, given a linear v function, the size of the bribed friends of A is always equal to the excess majority of A's coalition, m^*. As an illustration of this result, consider the House vote on NAFTA, discussed earlier. The bill passed by a vote of 234-200, suggesting that President Clinton and the Republican House leaders "bought" 16 votes more than the required number of 218. To the extent that the relative willingness to pay among legislators was approximately linear,[10] the equation above implies that approximately 16 members received a favor even though they would have voted for the bill without the favor.

Finally, we should point out that the characterization in proposition 3 holds for some cases with $\alpha > 0$ as well as for $\alpha \leq 0$. (In the interest of continuity, we proved proposition 3 using results from part of proposition 2. Since proposition 2 requires $v(0) \leq 0$, we include the restriction $\alpha \leq 0$ in the statement of proposition 3.) If $\alpha > 0$ and $W_B > v^{-1}(0)v(0) = \alpha^2/\beta$, then proposition 3 holds. If $\alpha > 0$ and $W_B \leq \alpha^2/\beta$, then $m^* = v^{-1}(0) = \alpha/\beta$.

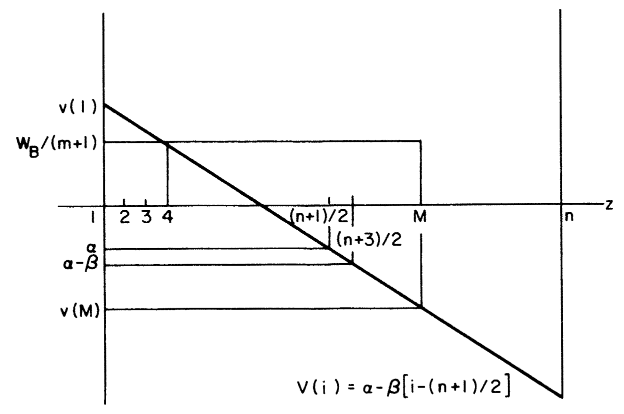
BACK TO FINITE LEGISLATURES

In this section, we derive some necessary conditions for minimal winning coalitions to form in equilibrium in a finite legislature. These conditions are quite strong, suggesting that in most plausible cases, minimal winning coalitions in legislatures are unlikely.

Let n be the number of legislators, n odd. Suppose legislators are uniformly distributed over $\{1, 2, \ldots, n\}$, and suppose $v(i) = \alpha - \beta[i - (n + 1)/2]$ for all i, with $\beta \geq 0$ and $\alpha \leq 0$. Thus, $v(\cdot)$ is again a linear function, with $v((n + 1)/2) = \alpha$. As above, let m be the "excess size" of A's coalition, relative to a minimal winning coalition. Also, let M be the total size of A's coalition (see Figure 9). That is, given bribe offer functions $a(\cdot)$ and $b(\cdot)$ such that x wins, let $m + (n + 1)/2 = M$ be the

[10] Actually, all that is required is that the function is approximately linear over the region in which legislators are bribed.

FIGURE 9. Payments by *A* with a Finite Legislature and Linear *v* Function



number of legislators who vote for x. We then have the following proposition.

PROPOSITION 4. *Suppose* $v(i) = \alpha - \beta\,[i - (n + 1)/2]$, *with* $\beta \geq 0$ *and* $\alpha \leq 0$. *If* $a^*(\cdot)$ *and* $b^*(\cdot)$ *constitute an equilibrium in which* x *wins, then* $m^* = 0$ *only if* $W_B \leq [1/3 + (28/9)^{1/2}]\beta < (2.1)\beta$.

Proposition 4 shows that when $v(\cdot)$ is linear, minimal winning coalitions will be rare. Recall that β gives the difference in the intensity of preference for x over s (measured in money) between two adjacent legislators. The proposition states that as long as vote buyer B is willing to pay more than twice this amount, then A will buy a supermajority. For example, if the median legislator requires a payment of \$10 to vote for x, and the legislator just to his or her right requires \$12, then A will buy a supermajority as long as B's willingness to pay for s is at least \$4.2 (a surprisingly small sum). Put differently, if B's willingness to pay for s is equal to or greater than that of the rightmost legislator (which is true if B *is* the rightmost legislator), then A always buys a supermajority if the legislature contains seven or more members.[11]

It is also worth noting that the cut-off value for W_B does not depend on α. That is, A's decision to buy or not buy a supermajority depends only on the *relative* preference intensities among legislators, not the absolute intensities.

Table 1 illustrates more generally how coalition size varies as a function of the slope and intercept parameters of the $v(\cdot)$ function. Notice that when W_B is about twice as great as $v(n)$, vote buyer B is willing to pay approximately twice as much to maintain s as the most pro-s legislator, and A's optimal coalition size is about 60%. When B's willingness to pay for s over x is about 20 times as great as that of the most pro-s legislator, A's optimal coalition size is more than 80%.[12]

[11] In a seven-member legislature, the rightmost member is 3 units to the right of the median, which is greater than 2.1.

[12] Because the legislature is finite, the numbers in Table 1 do not exactly satisfy the equal-area condition noted after proposition 2 and shown in figures 6 and 7.

TABLE 1. An Example Showing Equilibrium Majority Sizes and Total Payments
(number of legislators = 99)

W_B	α	β	$v(99)$	m^*	$T_A(m^*)$
10	−0.5	0.01	−0.99	25	57.35
10	−0.5	0.05	−2.95	13	35.69
10	−0.5	0.10	−5.40	9	30.00
10	−0.5	0.50	−25.00	3	22.50
10	−0.5	1.00	−49.50	2	20.00
10	−0.5	5.00	−245.50	0	16.50
10	0.0	0.01	−0.49	31	19.69
10	0.0	0.05	−2.45	13	19.30
10	0.0	0.10	−4.90	9	19.00
10	0.0	0.50	−24.50	3	18.00
10	0.0	1.00	−49.00	2	17.00
10	0.0	5.00	−245.00	0	15.00
50	−0.5	0.01	−0.99	49	148.50
50	−0.5	0.05	−2.95	31	132.31
50	−0.5	0.10	−5.40	21	121.16
50	−0.5	0.50	−25.00	9	105.00
50	−0.5	1.00	−49.50	6	100.00
50	−0.5	5.00	−245.50	2	88.00
50	0.0	0.01	−0.49	49	99.00
50	0.0	0.05	−2.45	31	98.44
50	0.0	0.10	−4.90	21	97.80
50	0.0	0.50	−24.50	9	95.00
50	0.0	1.00	−49.00	6	93.00
50	0.0	5.00	−245.00	2	85.00
100	−0.5	0.01	−0.99	49	247.50
100	−0.5	0.05	−2.95	42	243.85
100	−0.5	0.10	−5.40	31	229.50
100	−0.5	0.50	−25.00	13	207.14
100	−0.5	1.00	−49.50	9	200.00
100	−0.5	5.00	−245.50	3	184.50
100	0.0	0.01	−0.49	49	198.00
100	0.0	0.05	−2.45	44	197.78
100	0.0	0.10	−4.90	31	196.88
100	0.0	0.50	−24.50	13	193.00
100	0.0	1.00	−49.00	9	190.00
100	0.0	5.00	−245.00	3	180.00

Notes: W_B is vote buyer B's willingness to pay for policy s over policy x. α and β are the intercept and slope, respectively, of the legislator reservation price function, v.
m^* is the "excess size" of A's coalition in equilibrium; $m^* = 0$ means that A buys a minimum winning coalition, $m > 0$ means that A buys a supermajority, and $m^* = 49$ means that A buys a universalistic coalition. $T_A(m^*)$ is total bribes paid by A given at m^*.

CONCLUSION

The model studied here demonstrates that, in situations with competing vote buyers, equilibrium vote-buying behavior will typically not result in minimal winning coalitions. Indeed, coalitions will often be quite large and sometimes will even be universalistic. The reason is that, given the threat of counterattacks by other vote buyers, supermajority coalitions are often cheaper than minimal winning coalitions.

There are, of course, other reasons vote buyers might build supermajority coalitions. The most obvious is uncertainty: If potential voter buyers are uncertain about legislators' preferences, then they will generally pad their majorities somewhat to increase the probability of winning a simple majority.

Analyses of several factors might prove useful in distinguishing among various motives for forming supermajorities. The first is coalition size across policy areas. Recall that the winning vote buyer buys a larger coalition in the legislature, the *greater* the other vote buyer's willingness to pay. In the limit, if a vote buyer faces no opposition, then s/he buys a minimal winning coalition. Agricultural policy may provide an interesting case. For most of its history, agricultural policy in the United States is probably best characterized as an issue area with one-sided vote buying by the farm bloc. More recently, however, food processors and consumer groups have organized more forcefully to oppose farmers, although farmer interests are still powerful. What has happened to the size and structure of the coalitions that support agricultural bills? In other areas, such as labor policy and environmental policy, there have typically been organized interest groups on both sides. What has happened to coalition sizes as the relative resources of various groups have changed over time?

The second factor is coalition size as a function of legislative size and turnover. If uncertainty is the main force driving the formation of supermajorities, then coalition size (as a percentage of the legislature) should be inversely related to the total number of legislators and positively related to turnover. Assuming enough "independence" in decision making across legislators, the law of large numbers will tend to make voting outcomes more certain in larger legislatures, so vote buyers can achieve a high probability of victory with coalitions that are only slightly larger than half the legislature. (One factor working in the opposite direction is that vote buyers may have less information about each member in larger legislatures.) Also, assuming less is known about the preferences of new legislators than about veteran members, a large amount of turnover should lead vote buyers to pad their majorities more heavily.

The third factor is majorities in legislative elections. Incumbents running for reelection to Congress and most state houses typically win with large majorities. Of course, incumbents probably attempt to pad their majorities because of electoral uncertainty—turnout, the strength of partisan tides, and the emergence of scandals are all variable. Yet, these majorities also may be the product of incumbents' strategic efforts to maintain invulnerable coalitions of support, against even the best challengers, at the lowest possible cost. The size of an incumbent's majority depends at least in part on his or her activities on behalf of constituents, that is, the allocation of effort across constituents with respect to ombudsman-like services, securing federal contracts and grants, and so on. We may be able to distinguish between the two motives by studying the size of electoral majorities and measures of constituency service, as functions of such factors as the size of the constituency, variability in turnout, voter turnover (possibly due to redistricting), and the distribution of partisan support.

Finally, at least two theoretical extensions of the paper are worth further study. One is the consideration of alternative forms of agreements between vote buyers and legislators. For example, instead of the simple form of vote buying that we consider, suppose payments and votes are conditional on an outside event, such as whether a majority has already voted for the bill. That is, a vote buyer may pay some legislators to act as "hip pocket" votes—they vote with him or her only if their vote is needed to pass the bill. For instance, Marjorie Margolies-Mezvinsky seems to have been such a vote on Clinton's 1993 tax bill. She agreed to wait by the vote counter during the roll call and vote for the bill if Clinton did not have a majority near the end of the time limit. This is exactly what she did, and Clinton subsequently rewarded her by speaking at a special entitlements conference that Margolies-Mezvinsky sponsored in her district (King, Zeckhauser, and Feldman 1995, Woodward 1994). How large would optimal coalitions be under this form of vote buying? Would vote buyers be able to build coalitions more cheaply? Are there theoretical reasons suggesting that this type of vote buying should be commonplace in legislatures?

A second extension is to study the outcomes when bribes must be written into the bills being considered by the legislature rather than paid out as "cash." As noted above, this could cause legislators to care not only about their own bribes but also about the bribes received by other legislators. For example, if bribes consist mainly of special projects included in a bill, and these projects are funded by general taxes, then legislators who prefer lower taxes would like to limit the bribes received by other legislators. We argued earlier that this modification of the basic model probably does not qualitatively affect the results. In the case we considered, however, the bribes paid by the vote buyer favoring the bill were written into the bill, while the side favoring the status quo was assumed to pay bribes in cash. It would be interesting to extend the analysis to the case in which all vote buyers must pay bribes by writing them into legislation.

A third extension is to endogenize bill selection. A natural way to model this is to add a prior stage to our game in which a player strategically chooses a bill. The willingness to pay of each vote buyer and the reservation prices of the legislators will then depend on the bill chosen. Of course, once a bill is selected, willingness to pay and reservation prices are fixed, and the analysis above applies. Yet, an extended analysis could answer many questions that our model does not address. What are the equilibrium bills that various proposers would offer? Does there exist a status quo that cannot be defeated by any bill? That is, when vote buying is added to the unidimensional spatial model, does an undefeated policy exist, as in the median voter theorem? If such a stable policy exists, is it unique? How is it characterized? Finally, if the proposer is a legislator but the vote buyers are outside interest groups, would the proposer tend to propose his or her ideal point, or would s/he propose something else in order to extract more bribes from the interest groups? That is, would proposers concentrate solely on maximizing their policy preferences, or would they also tailor their proposals to extort money from vote buyers?

APPENDIX A

Proof of Proposition 1. To prove proposition 1, we first prove the following lemma.

Lemma: Suppose $a^*(\ )$ and $b^*(\ )$ constitute an equilibrium in which A wins. Also, suppose $v(0) < 0$. Then A buys a supermajority.

Proof: Since A wins, he must bribe at least $v^{-1}(0)$ legislators, otherwise his coalition is less than a majority. Next, it is easily shown that optimal strategies for A will involve bribing all legislators in a set $[z_0, \varepsilon]$ for some $z_0 \in [-1/2, v^{-1}(0)]$ and some $\varepsilon \geq 0$. Thus, A's coalition is $[-1/2, \varepsilon]$, and it is a supermajority iff $\varepsilon > 0$. To invade, B must buy ε legislators. Since B is willing to pay a total of W_B to the ε cheapest legislators of A's coalition, A's optimal strategy is to make each member of his coalition cost at least W_B/ε to B. Consequently, A's bribe to any legislator in $[v^{-1}(0), \varepsilon]$ will be at least $W_B/\varepsilon - v(z)$. Since A bribes each legislator in $[v^{-1}(0), \varepsilon]$, his total costs, T_A, are at least

$$\int_{v^{-1}(0)}^{\varepsilon} [W_B/\varepsilon - v(z)]\, dz$$

Since $\forall\, z \in [v^{-1}(0), \varepsilon]\ v(z) \leq 0$,

$$T_A \geq \int_{v^{-1}(0)}^{\varepsilon} W_B/\varepsilon\, dz$$

$$= [\varepsilon - v^{-1}(0)]W_B/\varepsilon$$

$$= W_B - v^{-1}(0)W_B/\varepsilon.$$

Since $v^{-1}(0) < 0$, T_A approaches infinity as ε approaches zero. Hence $\varepsilon = 0$ cannot be an optimal strategy. Therefore A must buy a supermajority. *Q.E.D.*

Proof of Proposition 1. Case (1). If $W_B \leq \int_0^{v^{-1}(0)} v(z)dz$, then A can prevent B from invading without bribing any legislators. Thus $a^*(z) = 0\ \forall\, z$ is A's optimal strategy.

Case (2). Since $\int_0^{v^{-1}(0)} v(z)\, dz < W_B$, A must bribe some legislators to prevent an invasion by B. First, we show that A can prevent B from invading by bribing only legislators in $[0, v^{-1}(0)]$ and paying them $a^*(z) = v(0) - v(z)$. With these payments, the cost to B to invade is $v(0)v^{-1}(0)$. Since this is greater than W_B, B does not invade. From this it follows that there exists a strategy in which A (i) only bribes legislators in $[0, v^{-1}(0)]$, (ii) pays bribes not greater than $v(0) - v(z)$ to each z, (iii) pays a total of W_B in bribes, and (iv) makes the cost for B to invade W_B.

Next, we show that A's optimal strategy is to bribe only legislators in $[0, v^{-1}(0)]$. To see this, first suppose A bribes a set of legislators, $\mathbf{C}$, outside this interval, where $\mathbf{C}$ has positive measure and $\mathbf{C} \subset (v^{-1}(0), 1/2]$. Under this strategy A must first pay each member of $\mathbf{C}$ a bribe of $-v(z)$ to make them willing to vote for x. But A must also pay these and other legislators a total of W_B to prevent an invasion by B. A's total payment is thus $W_B + \int_{\mathbf{C}} - v(z)\, dz > W_B$, hence it is not optimal for A to bribe any legislators in $(v^{-1}(0), 1/2]$. Next, suppose A bribes a legislator $z_0 \in [-1/2, 0)$. Define $\mathbf{D}$ as the set of $v^{-1}(0)$ cheapest legislators for B to buy. If $z_0 \notin \mathbf{D}$, then B would not bribe z_0 if she invades. Thus, A could strictly decrease the payment to z_0, while keeping the costs for B to invade constant. If $z_0 \in \mathbf{D}$, then $\exists\, z_1 \in [0, v^{-1}(0)]$ such that $z_1 \notin \mathbf{D}$. This implies that z_1 receives a bribe from A, but if B invaded she would not bribe z_1. Again, A could strictly decrease the payment to z_1, while keeping the costs for B to invade constant. Thus, it is not optimal for A to bribe any legislators in $[-1/2, 0)$.

Finally, suppose A pays more than $v(0) - v(z)$ to some legislator $z \in [0, v^{-1}(0)]$. If this occurs, then B can buy a legislator just to the left of legislator 0 for a cheaper price. As a consequence A could have paid less in bribes, while keeping the costs to B to invade constant.

Case (3-a). If $v(0) < 0$, then by the lemma A must buy a supermajority. Let $1/2 + m$ be the size of A's coalition, which is easily shown to be $[-1/2, m]$. To invade, B's optimal strategy is to buy the m cheapest members of this coalition. Define b as the cost to B of buying the m^{th} the cheapest member of A's coalition. First, notice that no member of A's paid coalition will cost more than b. That is, $\forall\, z \in [-1/2, m]$, either $a^*(z) = 0$ or $v(z) + a^*(z) \leq b$. This follows from the fact that if any bribed member of A's coalition costs more than b, then A could decrease the bribe slightly and keep B's cost of invading the same, while strictly decreasing his own costs.

Next, we show that all members of A's bribed coalition (actually, all members with the possible exception of a set of measure zero) cost at least b dollars for B to buy. That is, $\forall\, z$, $v(z) + a^*(z) \geq b$. To see this, first suppose to the contrary that there exists a positive-measure set of legislators in A's coalition whose cost to B is strictly less than b. This implies that $\exists\, \varepsilon \in (0, W_B/m)$ and a positive measure set $\mathbf{F}$ such that $\forall\, z \in \mathbf{F}$, $v(z) + a^*(z) < b - \varepsilon$. Let $\mathbf{E} = (v^{-1}(0), m]$, and denote the measures of $\mathbf{E}$, $\mathbf{F}$, and $\mathbf{E}\backslash\mathbf{F}$, respectively, by $|\mathbf{E}|$, $|\mathbf{F}|$, and $|\mathbf{E}\backslash\mathbf{F}|$. Also, let $\delta = \min\{W_B/m - \varepsilon, \varepsilon|\mathbf{F}|/(|\mathbf{F}| + |\mathbf{E}\backslash\mathbf{F}|)\}$.

Consider the following strategy for A, which we show is superior to the current one. First, decrease payments to $\mathbf{E}\backslash\mathbf{F}$ by δ. (This can be done since $\delta \leq W_B/m$ and it can be shown that all members of $\mathbf{E}\backslash\mathbf{F}$ must receive a bribe of at least W_B/m in order for A to win.) Note that the m^{th} cheapest member of A's coalition now costs at least $b - \delta$ dollars for B to buy. B will buy at most $m - |\mathbf{F}|$ members of $\mathbf{E}\backslash\mathbf{F}$. Hence, this revision in A's strategy decreases B's costs by at most $(m - |\mathbf{F}|)\delta$ dollars. Next, let A increase the bribe to each member of $\mathbf{F}$ by $\delta|\mathbf{E}\backslash\mathbf{F}|/|\mathbf{F}|$ dollars. Since $\delta \leq \varepsilon|\mathbf{F}|/(|\mathbf{F}| + |\mathbf{E}\backslash\mathbf{F}|)$, each member of $\mathbf{F}$ will still be among the m cheapest for B to buy. Hence, this raises the costs to B by $\delta|\mathbf{E}\backslash\mathbf{F}|$ dollars. The net increase in costs to B is at least $\delta(|\mathbf{E}\backslash\mathbf{F}| - m + |\mathbf{F}|)$. Since $|\mathbf{E}\backslash\mathbf{F}| \geq |\mathbf{E}| - |\mathbf{F}| \geq m - v^{-1}(0) - |\mathbf{F}|$, and $v^{-1}(0) < 0$, the net increase in costs to B is strictly positive. However, the net change in costs to A is $-\delta|\mathbf{E}\backslash\mathbf{F}| + (\delta|\mathbf{E}\backslash\mathbf{F}|/|\mathbf{F}|)|\mathbf{F}| = 0$. Thus, the revised strategy for A is superior, and it follows that for each member of A's bribed coalition $v(z) + a^*(z) = b$. That is, A adopts a leveling strategy.

Case (3-b). Now suppose $v(0) > 0$ and $W_B \geq v(0)v^{-1}(0)$. This implies that A cannot prevent an invasion by B without bribing legislators outside the $[0, v^{-1}(0)]$ range. Hence, A must bribe (i) a positive-measure set of legislators $\subset (v^{-1}(0), 1/2]$ or (ii) a positive measure set of legislators $\subset [-1/2, 0)$ and no legislators (or at most a set of zero measure) $\in (v^{-1}(0), 1/2]$.

Suppose case (i) applies. Note that A's coalition size above a majority, m, is strictly greater than $v^{-1}(0) > 0$. Let b equal the cost to B of bribing the m^{th} cheapest legislator in A's coalition. It is easily shown that A will not make any member of his bribed coalition cost more than b. To show that none will cost less than b, suppose, to the contrary, that a positive measure set of legislators cost less than b. Next, since (i) applies, there exist bribed legislators whose initial support for x is negative; i.e., for them $v(z) < 0$. A's bribe to these legislators includes two parts: a "base" amount, $-v(z)$, to make them want to vote for x, and a "rent," $a^*(z) + v(z)$, to prevent B from attacking them. A can delete some of these members from his coalition, pocket the "base" amount of the bribe, and transfer the "rent" part of the bribe to legislators whose cost to B is less than b. By doing this A keeps the cost to B the same, while decreasing his own costs.

(Formally, case (i) implies there exist two positive-measure sets, $\mathbf{G}$ and $\mathbf{H}$, that satisfy: (i) $\mathbf{G} \cap \mathbf{H} = \varnothing$; (ii) $\forall\, z \in \mathbf{H}$, $v(z) < 0$; and (iii) $\forall\, z \in \mathbf{G}$, $a^*(z) + v(z) < b - \varepsilon$, for some $\varepsilon \in (0, b)$. Let A take a subset of $\mathbf{H}$, $\bar{\mathbf{H}}$, of measure $(\varepsilon/b)\min\{|\mathbf{G}|, |\mathbf{H}|\}$. Let A delete these members from his coalition. This decreases his total bribe by $\int_{\bar{\mathbf{H}}} a^*(z)\, dz$. Next let A increase the bribe to each member of $\mathbf{G}$ by $\eta = (1/|\mathbf{G}|) \int_{\bar{\mathbf{H}}} a^*(z) + v(z)\, dz$. Since each member of $\bar{\mathbf{H}}$ has $a^*(z) + v(z) \leq b$,

$$\eta \leq (1/|\mathbf{G}|)b|\bar{\mathbf{H}}| = (1/|\mathbf{G}|)b(\varepsilon/b)|\mathbf{G}| = \varepsilon.$$

Hence, if B attacks, she will still buy each member of $\mathbf{G}$. Thus, the transfer from $\bar{\mathbf{H}}$ to $\mathbf{G}$ causes the costs to B to remain constant. However, the net change in A's payment is

$$|\mathbf{G}|\eta - \int_{\bar{\mathbf{H}}} a^*(z)\, dz = \int_{\bar{\mathbf{H}}} v(z)\, dz,$$

which is negative, since $v(z) < 0$, $\forall\, z \in \bar{\mathbf{H}}$.)

Now suppose case (ii) applies. Here, the size of A's coalition is $1/2 + m$, where $m = v^{-1}(0) > 0$. As before, let b equal the m^{th} cheapest member of A's coalition for B to buy. No bribed member of A's coalition can cost more than b. Otherwise, A could decrease the bribe to such a member without decreasing B's cost of invading.

Next, it must be the case that $b \geq v(0)$. To see this, note that the cost to B to invade is not greater than $mb = v^{-1}(0)b$. If $b < v(0)$, then the costs to invade are less than $v^{-1}(0)v(0) \leq W_B$. Since A makes the costs at least W_B to invade, this is a contradiction. It now follows that $\forall\ z \in [0, v^{-1}(0)]$, $a^*(z) + v(z) \leq b$. Now, since a positive-measure set of legislators $\subset [-1/2, 0)$ are bribed, it follows that more than m legislators have costs of b or less for B to bribe. Because of this, one can show, as in (3-a), that there exists a way for A to increase the bribe to a positive-measure set of legislators who cost less than b, while decreasing the bribe to all other bribed legislators. Furthermore, A can do this in such a way to keep his total payments the same while increasing the cost for B to invade. This implies that in this case as well, A's optimal strategy is a leveling one.

For cases (i) and (ii) b is defined as the m^{th} cheapest legislator for B to buy. Because A adopts a leveling strategy, each bribed legislator costs b for B to buy. Thus the costs to B are mb. Since A sets these equal to W_B, $b = W_B/m$, and $a^*(z) + v(z) = W_B/m$.

Next note that if legislator 0 is bribed, $a^*(0) + v(0) = W_B/m$. Hence, $v(0) < W_B/m$. If legislator 0 is not bribed, he is the m^{th} cheapest legislator for B to bribe. By definition of b, $v(0) = b = W_B/m$. Either way, $v(0) \leq W_B/m$.

All that remains to show is that $m \geq v^{-1}(0)$. Note that A will pay $W_B/m - v(z)$ to each member of his bribed coalition which is $(v^{-1}(W_B/m), m]$. Thus A pays total bribes of

$$\int_{v^{-1}(W_B/m)}^{m} W_B/m - v(z)\, dz.$$

Differentiating this with respect to m and evaluating at $m = v^{-1}(0)$ gives a nonpositive value. Thus A will not increase his total payment by setting $m \geq v^{-1}(0)$. *Q.E.D.*

Proof of Proposition 2. For m close to 0, A's coalition must be flooded. Inspecting equation (2), it is obvious that $T_A \to \infty$ as $m \to 0$. Thus, $m^* > 0$.

Consider first the case of a nonflooded coalition, so $m \geq W_B/v(-1/2)$. Differentiate (1) with respect to m and simplify to obtain $\partial T_A/\partial m = (W_B/m^2)v^{-1}(W_B/m) - v(m)$. Setting this equal to zero produces the condition given in case 1 of the proposition. Differentiating again yields $\partial^2 T_A/\partial m^2 = -(2W_B/m^3)v^{-1}(W_B/m) - (W_B^2/m^4)\ dv^{-1}/dv\,(W_B/m) - dv/dz(m)$. This is positive, since $v^{-1}(W_B/m) < 0$ for all $m > 0$, and $dv^{-1}/dv(W_B/m) = 1/[dv/dz(v^{-1}(W_B/m))]$, and $dv/dz \leq 0$ (v is nonincreasing by assumption). Thus, there is at most one optimum m^* satisfying $m^* \geq W_B/v(-1/2)$, and $m^* < 1/2$ iff $\partial T_A/\partial m\,(m^*) = 0$.

Next, consider the case in which A's coalition is flooded, so $m < W_B/v(-1/2)$. Differentiating (2) with respect to m and simplifying yields $\partial T_A/\partial m = -W_B/2m^2 - v(m)$. Setting this equal to zero produces the condition given in case 2 of the proposition. Differentiating again yields $\partial^2 T_A/\partial m^2 = W_B/m^3 - dv/dz(m)$, which is positive since $dv/dz(m) \leq 0$. Thus, there is at most one optimum satisfying $m^* < W_B/v(-1/2)$, and $m^* < 1/2$ iff $\partial T_A/\partial m(m^*) = 0$.

Finally, to see that there is only one optimum, suppose instead that there are two: one in which A's coalition is nonflooded, m_a, and one in which his coalition is flooded, m_b. Then, $m_a > W_B/v(-1/2) > m_b$, which implies that $v(m_a) < v(m_b)$ and $-v^{-1}(W_B/m_a) < 1/2$. If m_b satisfies the first-order condition for an interior flooded coalition, then $-W_B/2m_b^2 - v(m_b) = 0$. But then the conditions $v(m_a) < v(m_b)$ and $-v^{-1}(W_B/m_a) < 1/2$ imply that $(W_B/m_a^2)v^{-1}(W_B/m_a) - v(m_a) > -W_B/2m_b^2 - v(m_b) = 0$, so m_a cannot satisfy the first-order condition for an interior nonflooded coalition, a contradiction. Also, $m = 1/2$ is not optimal, since this requires $(4W_B)v^{-1}(2W_B) - v(1/2) \leq 0$, but $(4W_B)v^{-1}(2W_B) - v(1/2) > -W_B/2m_b^2 - v(m_b) = 0$. *Q.E.D.*

Proof of Proposition 3. Since v is decreasing, proposition 2 implies that T_A is convex and m^* is unique. Suppose A constructs a nonflooded coalition. Substituting $v(m) = \alpha - \beta m$ and $v^{-1}(W_B/m) = \alpha/\beta - W_B/\beta m$ into the first-order condition in case 1 of proposition 2 gives $-(W_B/m)[\alpha/\beta - W_B/\beta m] = -m(\alpha - \beta m)$. Rearranging yields $-m\alpha + \beta m^2 - W_B^2/\beta m^2 + W_B\alpha/\beta m = 0$, or $(m/\beta)[-\alpha\beta + \beta^2 m - W_B^2/m^3 + W_B\alpha/m^2] = 0$. Adding $W_B\beta/m$ and $-W_B\beta/m$ to the term in brackets yields $(m/\beta)[W_B\beta/m - \alpha\beta + \beta^2 m - W_B^2/m^3 + W_B\alpha/m^2 - W_B\beta/m] = 0$, which can be factored as $(m/\beta)[\beta(W_B/m - \alpha + \beta m) - (W_B/m^2)(W_B/m - \alpha + \beta m)] = 0$, or $(m/\beta)(W_B/m - \alpha + \beta m)(\beta - W_B/m^2) = 0$. The first two terms in this product are strictly positive (the second term is positive because $m > v^{-1}(W_B/m)$), so the equation holds iff $m^* = (W_B/\beta)^{1/2}$. Case 1 of proposition 2 shows that this is an interior solution for a nonflooded, nonuniversalistic coalition iff

$$m^* = (W_B/\beta)^{1/2} \geq W_B/(\alpha + \beta/2) \qquad \textbf{(A-1)}$$

and

$$m^* = (W_B/\beta)^{1/2} < 1/2. \qquad \textbf{(A-2)}$$

Since $a + \beta/2 > 0$ (otherwise, $v(-1/2) < 0$ and only flooded coalitions are possible), (A-1) implies

$$\alpha + \beta/2 \geq (W_B\beta)^{1/2}. \qquad \textbf{(A-3)}$$

Squaring both sides, multiplying by 4, and adding $4(a - W_B)^2 - 4\alpha^2 - 4W_B\beta$ yields

$$[\beta + 2\alpha - 2W_B]^2 \geq 4W_B^2 - 8\alpha W_B. \qquad \textbf{(A-4)}$$

Next, note that (A2) and (A3) imply that the term in brackets in (A4) is positive. Thus, taking square-roots and rearranging yields

$$\beta \geq 2(W_B - \alpha) + (4W_B^2 - 8\alpha W_B)^{1/2}. \qquad \textbf{(A-5)}$$

Finally, since $\alpha \leq 0$, it is easily shown that (A-5) implies (A-1) and (A-2). To show that (A-5) implies (A-1), reverse the steps above; to show that (A-5) implies (A-2) set $\alpha = 0$ in (A-5). Thus, a nonuniversalistic, nonflooded coalition is optimal iff (A-5) holds, proving case 1 of the proposition.

To show case 4, suppose A constructs a nonflooded, universal coalition. If A's coalition is nonflooded, then $v(-1/2) > W_B/m^*$ by definition. Substituting for $v(-1/2)$ and $m^* = 1/2$ (since the coalition is universal) gives $\alpha + \beta/2 > 2W_B$. Using the definition of T_A for a nonflooded coalition and differentiating, $\partial T_A/\partial m(1/2) = (4W_B/\beta)(\alpha - 2_B) - \alpha + \beta/2$. Since the coalition is universal, this is nonpositive. But this implies $\beta \leq 4W_B$, which contradicts $\alpha + \beta/2 > 2W_B$.

Next we prove case 3 of the proposition. First, to show sufficiency, suppose the coalition is universal and flooded. Then $\partial T_A/\partial m\ (1/2) = \beta/2 - \alpha - 2W_B$. Since the coalition is universal, this must be nonpositive, which implies $\beta \leq 4W_B + 2\alpha$. To show necessity, assume

$$\beta \leq 4W_B + 2\alpha. \qquad \textbf{(A-6)}$$

Next, the coalition is either flooded or nonflooded. If it is flooded, then $\partial T_A/\partial m(1/2) = \beta/2 - \alpha - 2W_B \leq 0$, which, by (A-6), implies the coalition is universal. If it is nonflooded, then $\partial T_A/\partial m(1/2) = (4W_B/\beta)(\alpha - 2W_B) - \alpha + \beta/2$. This is nonpositive, since (A-6) and $\alpha \leq 0$ imply that $\beta \leq 4W_B$, which implies the coalition is universal. Thus, whether the coalition is flooded or nonflooded, (A-6) implies it is universal. By case 4 it must be flooded. This establishes case 3 of the proposition.

Finally, the only remaining possibility is that A's coalition is flooded and nonuniversal. Cases 1, 3, and 4 imply that this occurs iff $4W_B + 2\alpha < \beta < 2(W_B - \alpha) + (W_B^2 - 2\alpha W_B)^{1/2}$. Substituting $v(m) = \alpha - \beta m$ into the definition of T_A for flooded coalitions and checking the first-order condition shows that m^* must satisfy $\beta(m^*)^2 = \alpha m^* + W_B/2m^*$. This concludes the proof. *Q.E.D.*

Proof of Proposition 4. Given A's bribe offer function $a(\cdot)$, B must pay $b(i) > a(i) - v(i)$ to all legislators $i \in \{(n + 1)/2, \ldots, m + (n + 1)/2\}$ in order to defeat x. It is straightforward to prove that A's equilibrium bribe offer function must be a leveling strategy (the proof follows the same line of argument as the proof of case 3 of proposition 1). Let $a(\cdot)$ be a leveling strategy such that x wins. Let $M = \max\{i \mid a(i) > 0\}$, and let $m = M - (n + 1)/2$. Note that B must pay $m + 1$ legislators in order to defeat x. Let i_0 solve $\min\{i \mid v(i) \leq W_B/(m + 1)\}$ or, equivalently, $\min\{i \mid i \geq [\alpha - W_B/(m + 1)]/\beta + (n + 1)/2\}$, and let $\delta(m) = i_0 - [\alpha - W_B/(m + 1)]/\beta - (n + 1)/2$. Then, A's total payments are $T_A = \Sigma_{i=i_0}^{m+(n+1)/2} [W_B/(m + 1) - v(i)] = \Sigma_{i=i_0}^{m+(n+1)/2} [W_B/(m + 1) - \alpha + \beta\,(i - (n + 1)/2)] = [m + (n + 1)/2 - i_0 + 1][W_B/(m + 1) - \alpha + \beta i_0 - \beta(n + 1)/2] + \Sigma_{i=0}^{m+(n+1)/2-i_0} \beta_i = (\beta/2)\,[m + (n + 1)/2 - i_0 + 1][2W_B/\beta(m + 1) - 2\alpha/\beta + i_0 - (n + 1)/2 + m] = (\beta/2)[m + W_B/\beta(m + 1) - \alpha/\beta + 1 - \delta(m)][m + W_B/\beta(m + 1) - \alpha/\beta + \delta(m)]$. It is straightforward to show that T_A is convex in m, so $m^* = 0$ if and only if $T_A(0) \leq T_A(1)$. Substituting and rearranging, $T_A(0) \leq T_A(1)$ iff $3W_B^2 - 2\beta W_B - 8\beta^2 \leq -8\alpha\beta + 4\alpha W_B + 4\beta^2[\delta(1) - \delta(1)^2 - \delta(0) + \delta(0)^2]$. Since $\delta(m) \in [0, 1]$ for all m, the term in square brackets is less than or equal to 1/4, so the right-hand side of this inequality is less than or equal to $4\alpha(W_B - 2\beta) + \beta^2$. Thus, $m^* = 0$ implies $3W_B^2 - 2\beta W_B - 9\beta^2 \leq 4\alpha(W_B - 2\beta)$. If $W_B > [1/3 + (28/9)^{1/2}]\beta$, then the left-hand side of this inequality is positive, and the right-hand side is negative (recall that $\alpha < 0$), a contradiction. Thus, $m^* = 0$ only if $W_B \leq [1/3 + (28/9)^{1/2}]\beta$. *Q.E.D.*

APPENDIX B

Proof of comparative static stated following proposition 2.

Consider proposition 2, subcase (2), in which m^* is defined by $-(W_B/m^*)v^{-1}(W_B/m^*) = -m^*v(m^*)$. Differentiating this equation with respect to W_B yields $-[v^{-1}(W_B/m^*) + (W_B/m^*)\ dv^{-1}/dv\ (W_B/m^*)][1/m^* - \partial m^*/\partial W_B\ [W_B/(m^*)^2]] = -\partial m^*/\partial W_B\ [v(m^*) + m^* dv/dz(m^*)]$.

Rearranging gives $\partial m^*/\partial W_B = [v^{-1}(W_B/m^*)/m^* + (W_B/(m^*)^2)\ dv^{-1}/dv\ (W_B/m^*)]/[(W_B/(m^*)^2)v^{-1}(W_B/m^*) + (W_B^2/(m^*)^3)dv^{-1}/dv\ (W_B/m^*) + v(m^*) + m^*\ dv/dz(m^*)]$. Both terms in the numerator are negative, as are all four terms in the denominator, so $\partial m^*/\partial W_B > 0$. An even simpler set of calculations shows that $\partial m^*/\partial W_B > 0$ also holds for subcase 3.

In interpreting $\partial m^*/\partial W_B$, changes in W_B should be viewed as the result of changes in vote buyer B's preferences over x and s, not due, for instance, to a change in the actual location of x or s.

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