5

Strategic voting in multimember districts

Compared to the attention they have lavished on strategic voting in single-member simple plurality elections, scholars have neglected strategic voting in multimember districts. We do have the Leys-Sartori conjecture – the thesis that strategic voting will be politically significant, acting to reduce the number of competitors, under PR systems with low district magnitudes, high thresholds, or other features that militate against the success of small parties. But there is not much systematic empirical evidence to back this claim up. Indeed, neither Leys nor Sartori cite or adduce any evidence at all; their conjecture is based on their own insight and an informal appeal to logic.

In this chapter, I consider three different multimember electoral systems: the single nontransferable vote (SNTV) system; the largest-remainders proportional representation (LRPR) system; and the divisor-based proportional representation (DBPR) system. For each system, I show how one can adapt the model of strategic voting used in the previous chapter to cover multimember elections. Key assumptions about voters remain the same. What changes are the rules of the election in which voters participate.

The main goals of this chapter are two. First, I will show that, for each of the multimember systems considered, a direct generalization of Duverger's Law, which I call the "$M + 1$ rule," exists. The $M + 1$ rule states that no more than $M + 1$ candidates (in the case of SNTV) or lists (in the case of LRPR and DBPR) can be viable – i.e., proof against strategic voting – in an M-seat district. When $M = 1$, this yields the local version of Duverger's Law developed in the previous chapter (because all three systems reduce to plurality rule in single-member districts). For $M > 1$, the PR results provide a specific quantitative version of the Leys-Sartori conjecture, while the SNTV results formalize a thesis first advanced by Reed (1991).

Second, I shall provide empirical evidence that strategic voting does in practice play the role assigned to it in theory. In particular, relevant data are marshaled from multimember districts in Japan (SNTV), Colombia (LRPR), and Spain (DBPR).

Three subsidiary themes of this chapter are as follows. First, I shall show that strategic voting need not necessarily have a reductive impact. That is, there exist modalities of strategic voting that do not have the classical vote-concentrating effect upon which Duverger and the subsequent literature have focused. An extended consideration of one kind of "nonstandard" strategic voting is undertaken for the case of Chile.

Second, I shall emphasize that, even when strategic voting does exert a reductive influence, it only imposes an upper bound on the number of viable candidates or lists.[1] The local version of Duverger's Law states that ordinary plurality elections usually lead to *two* viable candidates. But the only explicit argument in favor of this prediction is that third-party candidates will suffer substantially from strategic voting (and hence may not enter to begin with). This argument only says that there cannot be three or more: It puts an upper bound on the number of viable candidates, not a lower bound.

Third, I shall argue that strategic voting ought to fade out in multimember districts when the district magnitude gets much above five. The logic behind this argument is simply that it gets harder and harder to satisfy the informational assumptions of the model as district magnitude increases. This does not provide a very precise idea about when strategic voting ought to fade out, but empirically (in Japan, Colombia, and Spain at least) it seems to be above magnitude 5.

The layout of the chapter is structural rather than thematic. That is, I consider each of the three main multimember electoral systems – SNTV, LRPR, and DBPR – in turn, running through the five themes just highlighted for each system (rather than marching down the themes, with comments on each system under the thematic headings).

5.1 SNTV

Perhaps the simplest way to describe SNTV is to say that it is identical in all respects to the Anglo-American system of single-member districts operating under plurality rule, except that the district magnitude (the

[1] I believe that the first person to make a point along these lines was Wildavsky (1959:307, n. 11): "At best, however, Duverger's law argues for the discouragement of local multipartism rather than necessarily for the maintenance of local bipartism. Communities ... may well find sufficient political expression through a single party."

number of members elected from each district) is not fixed at one but instead is larger. In both systems, the nation is divided into a number of geographically defined constituencies from each of which a prespecified number of representatives are returned. In both systems, each voter has one (nontransferable) vote to cast. In both systems, the winning candidates in each district are the top M vote-getters, where M is the district magnitude.[2] From this perspective, the SMSP system of voting can be thought of as a special case of SNTV, corresponding to $M = 1$.

This formal similarity between SMSP and SNTV extends to the nature of strategic voting incentives under the two systems. Steven Reed (1991) was the first to note this in his article extending Duverger's Law to the Japanese case. In Reed's formulation, both strategic voting in the mass electorate and prudent withdrawals by candidates (those who fear bearing the brunt of strategic voting) serve to push the number of viable candidates in M-seat Japanese districts toward $M + 1$. In previous work, I have formalized the strategic voting part of Reed's argument, showing that in the equilibria of a pure model – with short-term instrumentally rational voters possessed of rational expectations, as in Chapter 4 – typically at most $M + 1$ candidates can expect to get positive vote shares (Cox 1994).

The reason for saying that "*typically* at most $M + 1$ candidates can expect to get positive vote shares" is that strategic voting equilibria under SNTV come in both Duvergerian and non-Duvergerian varieties, just as do equilibria under SMSP. Duvergerian equilibria correspond to situations in which there is a clear gap separating the first from the second loser, so that the latter is perceived as having virtually no chance of competing for the last-allocated seat, and hence suffers strategic desertion. Non-Duvergerian equilibria correspond to situations in which it is not clear *ex ante* who will be the first loser and who the second, with the result that neither suffers from strategic desertion, and the number of viable candidates exceeds $M + 1$.

The reason for saying "*at most* $M + 1$ candidates can expect to get positive vote shares" is to emphasize that all a consideration of strategic voting gets one is an upper bound on the number of competitors. Reed, like Duverger, offers his $M + 1$ rule as a point estimate of the number of competitors, not an upper bound. But if all one appeals to is the wasted vote argument, then having just M candidates is certainly an allowable

[2] The Japanese did impose an additional requirement: In order to win a seat, a candidate had to garner more than a legally defined "minimum vote." The minimum was set at such a low value, however, that no candidate who finished in the top M places in a district failed to attain it. Taiwan has a similar requirement.

outcome (when there are just M candidates, all of them will win and voters have no incentives to vote strategically). It is true that there is a plausible auxiliary argument, having nothing to do with strategic voting, that suggests that there will be more than M candidates. After all, if there are only M candidates then a potential entrant might very well see his or her chances of securing a seat as pretty good, and therefore enter.

I would just note three things about this auxiliary argument. First, neither Duverger nor Reed make this argument explicitly, nor does anyone else in the electoral studies literature, as far as I know. Second, in the $M = 1$ case, there have been pockets of one-party rule in the U.S. in which uncontested races are not particularly rare. Third, Greenberg and Shepsle (1987) show that M-candidate equilibria can in principle arise under SNTV. The logic of their model is that M candidates, the current entrants, may have adopted positions such that no new entrant can win a seat. In this case, no new competitor (at least of the seat-seeking kind) will wish to enter. The Greenberg-Shepsle model thus constitutes a theoretical criticism of the implicit argument upon which Reed and Duverger rely, in that they do find conditions under which just M candidates enter.[3]

Having noted how the $M + 1$ rule generalizes from the SMSP case to SNTV, I can now note one way in which strategic voting under SNTV differs from that under SMSP: It need not always benefit stronger candidates at the expense of their weaker opponents. Votes cast for candidates with more votes than they need to guarantee a seat are also wasted, from a short-term instrumentally rational voter's point of view. She could divert her vote from such a strong candidate, without causing that candidate to lose, and instead cast it for the best of the *marginal* candidates (i.e., those competing for the marginal or last-allocated seat).[4] Such a decision is risky, in that if too many supporters of the strong candidate think in this way, then the "strong" candidate may lose! Thus, there is some reason to expect that voters will be a bit timid in deserting supermarginal candidates. Nonetheless, there is also some temptation to do so, and so another type of strategic voting is at least theoretically possible under SNTV: the strategic desertion of strong or supermarginal candidates.

[3]In practice, this argument does not seem to be important, at least in the Japanese case, but it is not clear to me that it can be dismissed in general. I should also note that for PR systems the argument that there should be at least $M + 1$ entrants is substantially less compelling (because the entrants in the PR case would be lists, not candidates).

[4]In the case of single-member districts, of course, the "marginal" candidates are simply the two front-runners. More generally, the marginal candidates are those on the boundary between winning and losing: the last winner and the first loser.

Strategic voting in multimember districts

Is there in fact evidence that voters in Japan vote strategically, deserting either weak or strong candidates? The answer is affirmative. I shall consider in turn the evidence marshaled by Reed (1991), Cox (1994), and Cox and Shugart (1995).

Runners-up and second runners-up

Reed uses a test, similar to that employed by Galbraith and Rae (1989) in the British case, that focuses on the percentage of candidates whose vote totals decline, by their order of finish in the previous poll. As Reed (p. 351) notes, "a model of sophisticated voting would predict that second runners-up and lower finishers should lose votes," if they run again, because "they have little chance of winning," hence "some of their voters [will] abandon them." As it turns out, aggregate electoral statistics from postwar elections (down to 1986) confirm this expectation: "Whereas no more than half of the runners-up lose votes, over 70 per cent of the second runners-up lose ground. Runners-up tend to gain votes but second runners-up almost always decline."

The bimodality hypothesis

As noted above, a pure model of strategic voting admits of both Duvergerian and non-Duvergerian equilibria, corresponding to situations in which voters do and do not fully coordinate their strategies, and in which the second loser does and does not fall substantially behind the first loser. In light of this, a theoretically interesting statistic is the ratio of the second to the first loser's vote total, the SF ratio introduced in Chapter 4. Under Duvergerian equilibria, the SF ratio will be near zero. Under non-Duvergerian equilibria, the SF ratio will be near unity. Thus, if one were to compute the ratio for a number of districts, the resulting distribution of SF values should be bimodal.

I have tested this bimodality hypothesis empirically in the case of Japan, using district-level electoral returns over the period 1958–1990.[5] The procedure, in the case of 3-seat districts, was as follows. First, I computed the ratio of the vote total of the second loser (fifth-place candidate) to the vote total of the first loser (fourth-place candidate), for all districts with at least five candidates. Then, I produced a histogram to summarize the distribution of the resulting SF ratios (Figure 5.1). Results for 4- and 5-seat districts (the other frequently occurring types of district in Japan) are given in Figures 5.2 and 5.3.

[5] The data used in this analysis are from Steven Reed's compendium *Japan Election Data: The House of Representatives 1947-1990* (Ann Arbor: Center for Japanese Studies, 1992). A machine-readable version of the dataset can be found on the web site of the Lijphart Elections Archive at http://dodgson.ucsd.edu/lij.

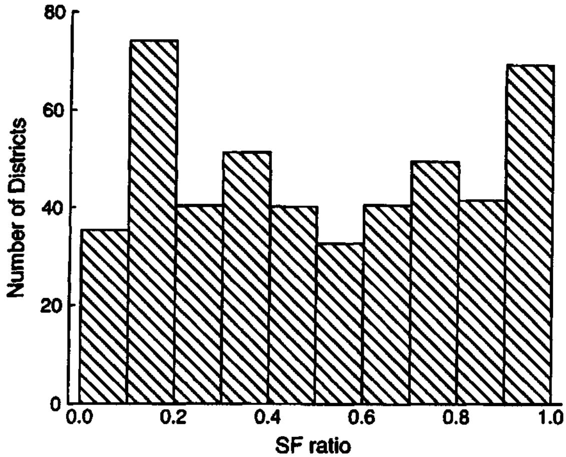

Figure 5.1. Testing the bimodality hypothesis in 3-seat districts: Japanese lower house elections, 1958–1990

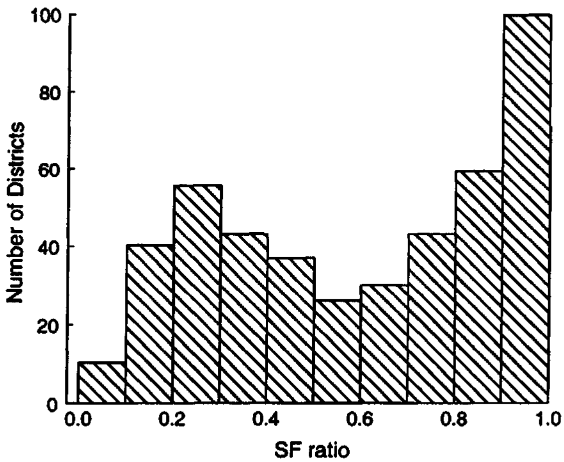

Figure 5.2. Testing the bimodality hypothesis in 4-seat districts: Japanese lower house elections, 1958–1990

As can be seen, the SF distribution appears to be bimodal in each case. SF values near .5 are rare, relative to those near 1 or 0. That is, it is much more common to have either a close or a distant second loser than an "in-between" second loser. Moreover, the closer is the first loser to the last winner, hence the more likely it is that a few more votes might change the outcome, the further from .5 is the SF ratio (i.e., the stronger is the tendency for the ratio to be either near 1 or near 0).

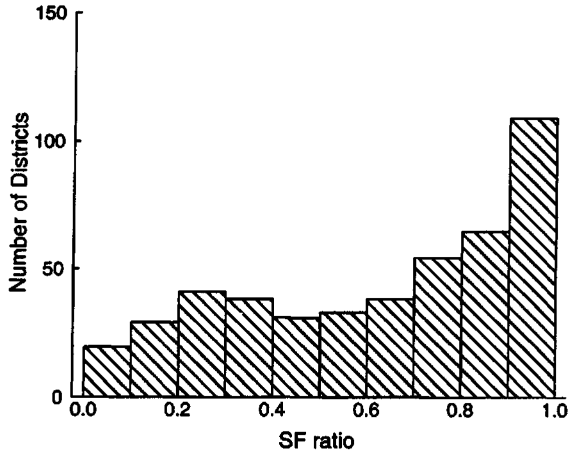

Figure 5.3. Testing the bimodality hypothesis in 5-seat districts: Japanese lower house elections, 1958–1990

Are the distributions displayed in Figures 5.1, 5.2, and 5.3 significantly bimodal? One can reject the null hypothesis that the distribution is *uni*modal in the first two cases, but not at conventional levels of significance in the third. One cannot reject the null hypothesis that the distribution is *bi*modal, in favor of the alternative that it is multimodal, in any of the cases.[6] All told the evidence is as it appears to be to the naked eye: The first two figures really are bimodal; the third is harder to call but has some tendency toward bimodality.

A second-order pattern apparent in the data is that the height of the mode near zero – i.e., the mode that corresponds to Duvergerian equilibria in which strategic voting has a substantial impact – declines as district magnitude increases. My interpretation of this is that the quality of voter information regarding candidate chances declines with district magnitude. In particular, it is harder to be sure who is trailing in a more crowded field in which small vote percentages can win a seat. Think, in the extreme, of a high-magnitude system like Israel's, in which a party needs

[6] I used dip or depth tests (see Hartigan and Hartigan 1985) to test unimodality. In the third case, the probability of observing the degree of bimodality visible in Figure 5.3 is a bit below .2, under the null hypothesis that the distribution is really unimodal. I used a kernel density-based test proposed by Silverman (1981) to pit the null hypothesis of bimodality against the compound alternative of more than two modes, finding *p*-values of .22, .26, and .98 for the first, second, and third figures respectively.

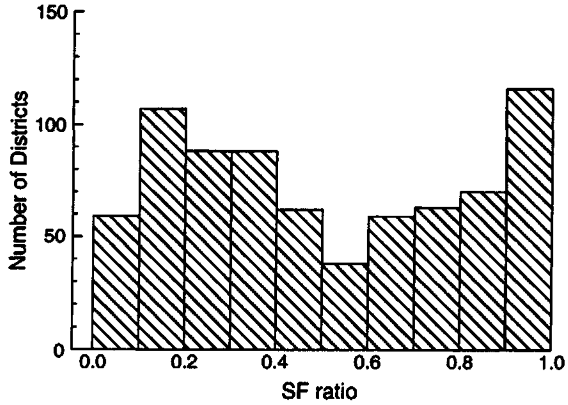

Figure 5.4. Testing the bimodality hypothesis in low-turnover districts: Japanese lower house elections, 1960–1993

only 1.5% of the vote to win a seat. 1.5% is less than the sampling error of almost all polls and so it is hard to see how Israeli voters could be extremely confident that a party with, say, 1% support in the polls was in fact hopeless. Absent consistent beliefs about a party's hopelessness, however, incentives to strategically desert this party dissipate. Thus, large-magnitude systems should in general depress the level of strategic voting, by destroying the primary informational prerequisite of such voting.[7]

Strategic desertion of the strong

Evidence that Japanese voters strategically desert leading candidates – those with more support than they need – is much less compelling than that they desert weak candidates. Nonetheless, there is some evidence consistent with the model. First, there is an over-time trend within the dominant Japanese party, the Liberal Democrats, toward fewer seats being lost on account of votes "wasted" on strong candidates (Cox and Niou 1994). Second, there is a statistically significant tendency for fewer votes to be "wasted" on leading candidates when the margin of victory of the last winning candidate is narrower, indicating that votes switched from leading to marginal candidates are more likely to affect the outcome (see Cox 1994).

[7]In Chapter 10 I will consider some evidence that Israeli voters do vote strategically, but for different reasons than those modeled here.

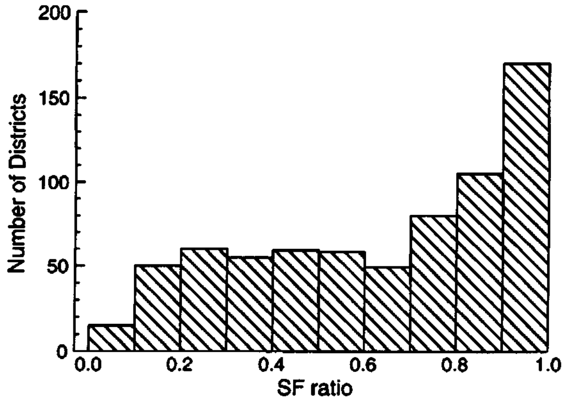

Figure 5.5. Testing the bimodality hypothesis in high-turnover districts: Japanese lower house elections, 1960–1993

The informational prerequisites of strategic voting

If voters have little information about the electoral prospects of the various candidates, then one should expect little strategic voting because it will never be clear who is trailing. In this section, I test this idea by examining the role of electoral history in stabilizing and coordinating electoral expectations. If voters are faced with little change in the field of candidates running from election to election, so that everyone can simply extrapolate from "last time" to "this time," then the informational requirements of the model are more likely to be met than if the field of candidates changes substantially.

Do the data conform to the expectation just articulated? Cox and Shugart (1995) have explored this question both graphically and via probit analysis, using Japanese lower house electoral data from 1960 to 1993.

Consider the graphical approach first. Let $NEWPER_{jt}$ be the number of *new* candidates in district *j* at election *t* (i.e., candidates who did not run in the same district at $t - 1$), divided by the district magnitude. The mean value of NEWPER in the 1,487 district elections held 1960–1993 is .5, or half a new candidate per seat in the typical contest. If one divides the Japanese observations into two categories, those with NEWPER > .5 (indicating an above-average level of turnover in candidates) and those with NEWPER ≤ .5 (indicating a below-average level of candidate turnover), one finds a much more marked bimodality in the SF distributions for the low turnover races than for the high turnover races (see Figures 5.4 and 5.5).[8]

To back up the visual impression conveyed by Figure 5.4, Cox and Shugart ran a probit analysis of the probability that a particular contest would produce a low SF value, controlling for the district magnitude and the total number of candidates running. They find that more candidates of any kind, and especially more new candidates, significantly reduce the probability that the second loser will be well out of the running.

The theory of strategic voting developed here implies that races that are easier to handicap should also be those in which the electorate's expectations are more coordinated concerning who does and does not have a chance at victory. Coordinated expectations, in turn, should lead to greater strategic desertion of weak lists/candidates, leading to low SF ratios. The analysis of Japanese elections lends credence to this chain of reasoning. Races that are plausibly more difficult to handicap – with more candidates per seat, and especially more new candidates per seat – are also those in which the SF ratio tends to be high. The findings regarding new candidates corroborate experimental evidence provided by Forsythe et al. (1993) on the importance of election histories in facilitating voter coordination.

5.2 LRPR

Under closed-list LRPR, votes are converted into seats as follows. The electoral quota, Q, is multiplied by the total number of valid votes cast in the district, V, to produce a quota or "price" denominated in votes.[9] Seats are then allocated to lists in two stages. In the first stage, each list "buys" as many seats as its vote total permits, given the "price" per seat, QV. This leaves each list with a certain number of seats and a certain

[8] This result is consistent with a rational entry model in which candidates *anticipate* strategic voting, and thereby obviate its necessity in practice. Under this view, some districts are pretty cut-and-dried at the time at which entry decisions must be made, with little chance of a challenger unseating an incumbent. In such districts, few challengers with high opportunity costs, which is to say few strong challengers, enter the race. The second loser is thus typically a weak candidate, the strong ones having decided not to risk strategic desertion, and this produces a low SF ratio. In contrast, other districts may have several open seats or otherwise look to be good opportunities. Here, there will typically be several strong challengers among whom it is difficult to distinguish, and the SF ratio will accordingly end up near unity. I do not have the information that would be needed to separate this story from the strategic voting story told in the text.

[9] Thus, as used here, Q is a proportional quota (where it is assumed that $1/(M + 1) < Q \leq 1$), with QV denoting the "raw votes quota." In practice, many systems start with a specification of the raw votes quota.

balance of unexpended votes, called the remainder. In the second stage, seats not allocated in the first stage are allocated, one to a customer, to the lists with the largest remainders. After seats have been allocated to the lists, they are reallocated to the candidates on the lists in accord with the order of appearance of the names on each list. Thus, if list j gets 5 seats, the first 5 candidates on that list receive seats.

Applying LRPR in single-member districts ($M = 1$) is equivalent to SMSP, if $Q > 1/(M + 1)$. For, given that $Q > 1/(M + 1) = 1/2$, the only seat at stake in the district will go to the candidate with the most votes, either in the first stage (if that candidate commands a sufficient majority of votes) or in the second stage (if he does not). Thus, SMSP elections can be thought of as a special case of LRPR.

Applying LRPR with a large quota ($Q = 1$) is equivalent to SNTV. With a quota equal to unity, no seats are allocated in the first stage (unless one candidate gets all the votes, a possibility I shall ignore) and each list's remainder is simply its vote total. Thus, the second-stage allocation of seats to the lists with the largest remainders is equivalent to allocation by plurality rule: The top M vote-getting lists will win the seats at stake. Those seats then go to the top name on each winning list. As this is essentially the allocation rule used under SNTV, one can consider SNTV to be LRPR with a quota $Q = 1$.[10]

The formal similarity of SNTV and LRPR elections helps one to extend several of the results obtained in the previous section for SNTV to the case of LRPR (cf. Cox and Shugart 1995).[11] In particular, under LRPR there are still two different kinds of strategic vote: one in which voters abandon hopeless or submarginal lists (those that are expected neither to win a quota seat nor to be competitive for the last-allocated remainder seat), and one in which they abandon strong or supermarginal lists (those that have remainders above the expected least winning remainder).[12] There are also still two kinds of equilibria: Duvergerian

[10]The only difference is that, under LRPR, the top-of-list candidates do have running mates below them on the list, while in SNTV each candidate runs his own campaign. As none of the lists win more than one seat, however, it is essentially an SNTV contest between the tops-of-list.

[11]It is not too misleading to think of the matter in the following way (although this does not turn out to work cleanly for the proofs): In an LRPR contest in which expected vote shares are known with considerable precision, everyone can calculate the number of quota seats that each list is expected to win, and the number of remainder seats, say μ, that will be left to be allocated in the second stage. The allocation of these seats will then look essentially like the allocation of (seats in an SNTV election, with each party's expected remainder taking the place of its expected vote in the SNTV model.

[12]Tsebelis (1986) is the first paper of which I am aware that stresses the latter kind of strategic voting.

equilibria, in which the M+2nd list (ranked in order of vote totals) is clearly behind the $M + 1$st list, and thus suffers the loss of all instrumental support; and non-Duvergerian equilibria, in which the M+2nd list is close enough to the $M + 1$st as to be indistinguishable, and thus holds on to its instrumental supporters. Finally, there is still an upper bound implied on the number of viable lists, on the assumption that Duvergerian equilibria are more common: There should generally be no more than $M + 1$ viable lists.[13]

What differs between the SNTV and LRPR cases is the practical meaning of the results generated. For SNTV, the results apply to candidates and the $M + 1$ upper bound has some bite to it: The number of candidates is always at least M, so a prediction that the effective number should be at most $M + 1$ is both non-obvious and constraining. For LRPR, the results apply to lists and because lists can win more than one seat the upper bound has much less bite to it, especially as the district magnitude increases. Taagepera and Shugart (1989:144) have found that most real-world observations fall within +/–1 of the equation $N_{eff} = 2.5 + 1.25 log_{10} M$ (where N_{eff} is the effective number of parties competing). If we ignore several facts – that their data include mostly non-LRPR systems, that they compute the effective number of parties at the national rather than district level, that they use an *effective* magnitude for each system (a kind of adjusted average of the district magnitudes) rather than the actual magnitude defined at the district level – this suggests that the $M + 1$ upper bound will rarely be binding in districts of magnitude greater than three (since for $M > 3$, $M + 1 < (2.5 + 1.25 log_{10} M) + 1$). Thus, something else other than strategic voting must explain the reduction of the effective number of parties below $M + 1$ observed in most large-magnitude PR systems.[14]

[13]For, suppose that $K > M + 1$. Then (absent non-Duvergerian equilibria) there will necessarily exist at least $K - M \geq 2$ lists that are expected to win neither a quota nor a remainder seat, and Theorem 2 in Cox and Shugart (1995) applies to show that all but one of these lists must have zero expected vote shares.

[14]The pure model refers to the "number of lists/candidates that receive positive vote shares," and it has been suggested to me that this is what should be counted in empirical tests. But if one entertains the notion that the model's assumptions might not be met perfectly, as I think one must, then one wants to count the "number of parties that get a positive share of the vote from the short-term instrumentally rational part of the electorate." I do not pretend to know how to do this, but looking at the effective number of parties seems a reasonable first cut. Thus, I have compared the theoretical upper bounds in the text to Taagepera and Shugart's estimates of the effective number of parties.

What might this "something else" be? On the one hand, it might involve the direct response of other agents – contributors, potential entrants, activists – to the particular electoral structure in which they find themselves. If so, then one could continue to think of the features of the electoral system alone as producing the relationship between M and N_{eff}. On the other hand, the factors that reduce N_{eff} well below $M + 1$ might not be directly tied to particular electoral structures at all. Perhaps, for example, there are economies of scale in advertising and/or creating habitual attachments in the electorate, and these kick in independently of electoral structure to limit the effective number of lists. By this accounting, strategic voting would be a more limiting factor than would be the problem of overcoming voter ignorance at low district magnitudes, while just the reverse would be true at high district magnitudes. Complementing this economies-of-scale view would be consideration of the cleavage structure of society, with societies divided into larger numbers of pre-existing, cohesive, and hence easily mobilized, groups producing more parties than those with fewer such cleavages, especially at high district magnitudes (see Chapter 11).

Strategic voting in Colombia

In this section, I consider the empirical usefulness of the results just sketched.[15] There are two main patterns that the model predicts strategic voting will produce: First, trailing lists will be deserted by all supporters; second, leading lists will be deserted by excess supporters. In examining how these two predictions fare, I shall look at Colombian electoral data from the period 1974–1990.

Why Colombia? As explained in Cox and Shugart (1995), it is not easy to find an appropriate LRPR case for analysis. Most PR systems use a divisor formula (e.g., d'Hondt's). Many of those that do use LRPR couple it with upper tiers or other complicating features. Colombia is a good case because all seats are allocated at the district level, each party regularly presents multiple lists in each district (at both House and Senate elections), and most lists do not win quota seats. Thus, Colombian LRPR contests are similar to SNTV contests between the heads of each list, so that the correspondence to the formal conditions required in the theorems (Cox and Shugart 1995) is fairly close.

[15]Before proceeding I should note a caveat analogous to that registered in Chapter 4: Even if the model's predictions are borne out, this will not prove that strategic voting is the primary causal agent. The problem is that *any* class of agents who care about the outcome of the election will tend to allocate whatever resources they control to marginal lists.

The bimodality hypothesis

I shall again investigate the SF ratio. Under Duvergerian equilibria, the SF ratio will be zero, since the second losing remainder will be pushed down to zero by strategic desertion,[16] while under non-Duvergerian equilibria the SF ratio will be unity, since the first and second lists with losing remainders will have virtually identical remainders. Thus, the prediction is that the SF distribution will be bimodal, with one mode near unity and another near zero. Lists that are not ahead of the list expected to have the greatest losing remainder either will be "close enough" so that they are still seen as having a chance at the last remainder seat, in which case they will hold on to their support and produce a large value of the SF ratio, or will be "too far" behind, in which case all their *instrumental* supporters will desert them, leaving only a rump of noninstrumental support.

In analyzing the Colombian data, we present detailed results only for districts of magnitude one to five, pooling 55 districts from the House with 88 districts from the Senate. At higher district magnitudes in Colombia, there is very little evidence of strategic voting. This is consistent with the trends observable in the Japanese data and again suggests that strategic voting phenomena fade out rapidly as the district magnitude increases past five (cf. Sartori 1968:279).

A histogram of SF values from 143 elections held in small-magnitude Colombian House and Senate districts over the period 1974-1990 is presented in Figure 5.6. As can be seen, it appears to be bimodal, although the mode near zero is considerably smaller than that near unity.

Does the degree of bimodality observed in Figure 5.6 pass some threshold of statistical significance? If one thought that, with everyone voting sincerely, the SF ratio ought to be uniform, or unimodal with a peak near .5, then these nulls could be rejected at conventional levels of significance. One can also reject other null hypotheses that entail at least as many observations with SF ratios in the middle tertile as there are in the upper or lower tertiles.

As another benchmark, consider the ratio r_{j-1}/r_j: the $(j-1)th$ list's remainder divided by the jth list's remainder. When $j = M + 1$, this is the SF ratio. But for $j \neq M + 1$, we have other ratios. As it turns out, for $j < M + 1$ all of the ratios r_{j-1}/r_j are distributed unimodally with considerably thinner tails than posited in the hardest-to-reject scenario. I propose testing the null hypothesis that the distribution of the SF ratio is no different than the distribution of r_{j-1}/r_j, for some $j < M + 1$ (which $j < M + 1$

[16] This is true only if the list expected to have the second largest losing remainder has a lower expected vote total than the list expected to have the largest losing remainder. Most of the Colombian data, as it turns out, satisfy this condition.

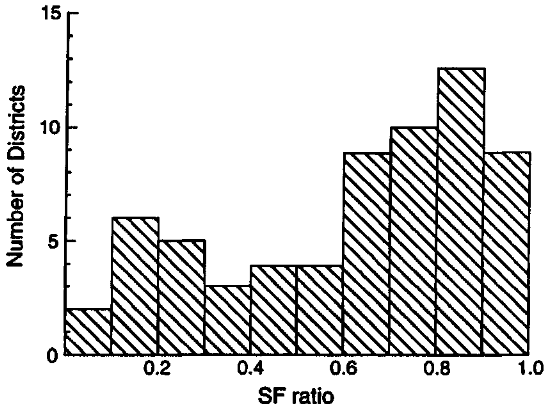

Figure 5.6. Testing the bimodality hypothesis in low-magnitude districts: Colombian lower house elections, 1974–1990

is selected turns out not to matter). The null hypothesis, in other words, says that there is nothing unusual or special about the SF ratio as compared to other remainder ratios involving adjacent candidates.

One can reject this null hypothesis at conventional levels of statistical significance. The distribution in Figure 5.6, in other words, is not like the other remainder ratio distributions. Where they have thin and declining tails toward zero, the SF distribution has at least a very thick tail and probably what it appears in the figure to have: a second mode near zero. We interpret this difference as evidence of strategic voting. (I should note that an analogous test for the Japanese case comes to a similar conclusion. For example, the ratio of the second-place candidate's vote total to the first-place candidate's vote total is distributed in a thoroughly unimodal fashion.)

Is there any other evidence consistent with the idea that Colombians vote strategically in their small-magnitude districts? It turns out that the closer is the least winning remainder to the greatest losing remainder, hence the more likely that a few more votes might change the outcome, the stronger is the tendency for the SF ratio to be either near 1 or near 0. One way to document this is to regress the absolute value of the difference between the SF ratio and .5 on the percentage margin separating the last winning and first losing remainders. Theorem 1 (of Cox and Shugart) suggests that SF values near .5 should be particularly unlikely if the race for the last seat is a close one. The results from such a regression (controlling for district magnitude) show that SF values from closer races do tend to be further from .5 on average than those from less close races.

An effect of the magnitude found would be expected under the null hypothesis of no effect about six times in a hundred.

Strategic desertion of strong lists

Evidence that Colombian voters strategically desert leading lists is much weaker than the evidence for strategic desertion of trailing lists.[17] There is a mild decline in the percentage of votes wasted on leading lists from 1974 to 1986 (with 1990, when the dominant Liberal party was divided over a proposed constitutional revision, reverting to earlier levels). And, if one regresses the percentage of votes wasted on leading lists on the "margin of victory" (least winning remainder minus greatest losing remainder, as a percentage of total vote), one finds a positive effect. Unlike in Japan, however, this effect is not statistically discernible from zero.

5.3 DBPR

I conjecture that PR systems operating under a divisor-based seat allocation formula, such as the d'Hondt method, are similar in their strategic voting equilibria to those operating under a largest-remainders formula. In particular, DBPR elections should obey an $M + 1$ rule, whereby there are rarely more than $M + 1$ viable lists, just as LRPR elections do. This point is potentially important because a fair number of the world's PR systems (especially in Latin America) are low-magnitude divisor systems, in which the $M + 1$ upper bound might be constraining.[18]

In this section, I propose to do three things. First, I shall review evidence from the Spanish case that voters under low-magnitude DBPR do vote strategically to the detriment of smaller parties, consistent with the $M + 1$ rule. Second, I consider the nature of strategic voting under the open-list DBPR system of Chile, where typically the number of lists falls below $M + 1$ and yet, as will be seen, opportunities to vote strategically still arise. The point of this investigation is to highlight modes of strategic voting that do not have the concentrative effect upon which students of electoral systems since Duverger have typically focused. Third, I also briefly note some opportunities for strategic voting that arose under the open-list system of election once used in French labor elections (again with fewer lists than the upper bound of $M + 1$, again without a clear concentrative effect).

[17]In part this may be because the relevant conditions of the formal theorems in Cox and Shugart (1995) are met in the sample only about 75% of the time.

[18]Tables 3.1–3.3 show that Argentina, Cape Verde, Chile, the Dominican Republic, Ecuador, Spain, and Turkey all combine PR-d'Hondt with median magnitudes below 6.

Strategic voting in Spain

The Spanish electoral system adopted in 1977 is a good example of a PR system in which larger parties regularly get larger seat shares than vote shares (cf. Gunther and Montero 1994). Over the period 1979–1989, Spain's has consistently been among the least proportional of all European electoral systems. The primary features that fuel this disproportionality are the use of the d'Hondt method of seat allocation (which is more favorable to large parties than most other PR allocation formulas), the presence of many districts of small magnitude (31 districts with magnitude 5 or smaller), the absence of any upper tiers, and the use of a 3% threshold at the district level. In the smaller Spanish districts, one might expect to see strategic voting similar to that seen in the smaller Japanese districts (operating under SNTV) or the smaller Colombian districts (operating under LRPR).

And in fact strategic voting does appear to be substantial in the smaller Spanish districts. The best evidence of this is provided by Gunther (1989), based on surveys conducted after the general elections in 1979 and 1982. Examining the reported votes of respondents with strong partisan preferences, Gunther finds that relatively large percentages of those preferring the large parties reported voting for them, while relatively small percentages of those preferring the small parties reported voting for them. Moreover,

> This *prima facie* case for the presence of sophisticated voting is strongly corroborated when the voting behavior of these respondents is broken down by province in accord with the number of deputies sent to the *Cortes* from each district. Respondents with highly-favorable attitudes toward the third- and fourth-place parties in large provinces were about twice as likely to vote for them as sympathizers of those same parties in small provinces (Gunther 1989:842).

Gunther's definition of what counts as a "large" and "small" province depends in a natural way on the size of the party being investigated. The Communist Party of Spain (PCE), for example, had a national vote total which, if replicated in each district, would have given it a seat in districts of magnitude five or above, but no seats in the smaller districts. The smaller Popular Alliance (AP) could only expect to get a seat in districts electing six or more deputies (again, on the assumption that it would get its national level of support in each district). Thus, the dividing line between "small" districts (in which a party's supporters should have feared wasting their votes) and "large" districts (in which a party's supporters should not have feared wasting their votes) varied systematically across parties, depending on their national vote shares.

What is striking about Gunther's results, as he notes, is the Spanish voter's apparent ability to figure out these rational cutpoints.

It is possible, of course, that Spanish voters in some of the "small" districts knew that their party had stronger support there than it did in the nation as a whole, and accordingly did not desert. Alternatively, some voters in "large" districts may have known that their party was weaker than it was in the nation as a whole, and accordingly voted strategically. So Gunther's analysis may understate the true ability of Spanish voters to calculate the right time to cast a strategic vote. Can a district-level analysis of SF ratios, similar to that performed for Japan and Colombia above, provide any corroborating evidence?

In a d'Hondt system, the SF ratio refers to the ratio of the second losing quotient to the first losing quotient. But in the Spanish data, the list possessing the second losing quotient won seats in all but six provinces in 1986. This is in sharp contrast to the Colombian data, where most of the second losing remainders were owned by lists that had *not* won a seat (and in even sharper contrast to the Japanese data, where all of the candidates owning the second losing vote totals of course did not win seats). As argued briefly in the Colombian case, there is less reason to expect bimodality in the SF distribution if the second losing remainder or quotient is owned by a list that has won seats. If the list with the second largest remainder or quotient does not win seats, then one can imagine that polling results before the election might have indicated this. Voters then have an easy calculation: "My list is so small that my votes are wasted on it." (This is the calculation that Gunther imputes to his survey respondents.) But if one's favorite list is a large list, clearly expected to win one or more seats, then one has to calculate the expected distribution of seats to *each* party, calculate the first and second losing quotients in the district, and discover that one's party owns the second losing quotient.[19] Even then, one might worry that if too many copartisans decide to desert the favored party, in order to use their wasted votes elsewhere, then the party will lose one of the seats that it was expected to win. So it is not obvious that we should expect any bimodality in the SF ratios in Spain; the prediction is similar to the weak prediction in the Japanese and Colombian cases that lists with "excess" votes will be relieved of them.

As it turns out, the SF distributions for Spain from the 1982, 1986, and 1989 elections are all distinctly unimodal. And there are really too few cases in which the second losing quotient was owned by a losing

[19]As noted before in the text, elites might get into the act and publicize the logic of the wasted vote. But they certainly have a harder sell in the case under consideration than if the list is small.

party (21) to yield much of a test.[20] There is thus no aggregate evidence in the PR-d'Hondt case comparable to that from the SNTV and LRPR cases. For this case, then, one has to rely on Gunther's surveys, which provide more direct evidence of strategic voting in any event.

Strategic voting in Chile

The Chilean electoral system (for the lower house) uses two-seat districts. Each Chilean voter has one vote to cast, which she must cast for an individual candidate. Candidates are grouped for purposes of seat allocation into lists, each list having two candidates. Votes are counted, and seats allocated, in two stages. First, the votes for all candidates on a given list are summed, to give a list vote total. These vote totals are then translated into an allocation of seats among the lists by the d'Hondt divisor method of PR (on which, more presently). Second, plurality rule is used to allocate each list's seats among that list's candidates. Typically, a Chilean list will win only one seat, in which case that seat goes to the candidate on that list receiving the greatest number of votes.

An example may help to clarify matters. Suppose that there are two candidates on the A list (A1 and A2) and two candidates on the B list (B1 and B2). The vote totals for these candidates are 15,000, 16,000, 7,000, and 20,000, respectively. In this case, list A's vote total is 31,000, while list B's is 27,000. The d'Hondt method of seat allocation proceeds as follows. First, one divides each list's vote total by the numbers from 1 to M, to produce a number of quotients equal to the number of lists times the district magnitude. In this case, there are two lists chasing after $M = 2$ seats, and so list A's quotients are 31,000 and 15,500; while list B's quotients are 27,000 and 13,500. Second, one takes the M largest quotients produced in the first stage, and allocates one seat per quotient to the party owning each such quotient. In the present case, that means that each list is given a single seat: List A gets a seat by virtue of its first quotient being the largest, list B by virtue of its first quotient being the second largest. Candidate A2 gets the seat awarded to list A, because he has more votes than his running mate. Similarly, candidate B2 gets the seat awarded to list B.

The Chilean system gives a strong incentive to coalition and in practice has given rise to two large alliances, the *Unión por El Progreso,* con-

[20] In these 21 cases it turns out that the SF ratio is always greater than .7. This is not strongly inconsistent with the bimodality hypothesis, in that there are no observations in the midrange (.4 to .6). But on the other hand there is no *aggregate* evidence of strategic voting, either.

sisting of center-right and right-wing parties; and the *Concertación*, consisting of the center-left and left-wing parties. The nature of the incentive to coalesce can be seen by considering a case in which two conservative parties, that together garner 60% of the vote, offer a joint list. Suppose that there are two liberal parties, that garner 25% and 15% of the vote, respectively. If the liberal parties offer a joint list, then that list will win a seat (because 25+15 = 40, which is more than half of 60). But if each liberal party runs its own list, then the conservative alliance will win both seats in the district (because the two largest quotients will be 60 and 60/2 = 30, both owned by the conservative list).

What is the nature of strategic voting incentives under the Chilean rules? I shall focus upon the typical situation, in which the main contenders are a joint list from the *Unión por El Progreso*, on the right, and a joint list from the *Concertación*, on the left.

It will help to introduce a little notation to structure the discussion. Let L_1 and L_2 denote the number of voters who most prefer the first and second leftist candidates, with $L_1 \geq L_2$; and C_1 and C_2 denote the number of voters who most prefer the first and second conservative candidates, with $C_1 \geq C_2$. Assume that all voters have a unique most-preferred candidate and let $L = L_1+L_2$, $C = C_1+C_2$. So the total number of voters is $T = L + C$. Each voter j must choose a strategy v_j from the set {Vote for the first leftist, Vote for the second leftist, Vote for the first conservative, Vote for the second conservative}.

Some insight into strategic voting under the Chilean system can be had without developing a full incomplete-information model of the kind used hitherto. Indeed, I shall focus on a simpler (complete-information) model and a simpler question: When will the situation in which all voters vote sincerely (which I shall denote by $v^* = (v_1^*, \ldots, v_T^*)$) be a strong equilibrium (or core point)?

A vector of voting strategies $v = (v_1, \ldots, v_T)$ is a strong equilibrium if no coalition of voters can alter their voting strategies in such a way as to make all of them better off, on the assumption that all other voters continue to act as specified in v. I shall further simplify the analysis by assuming that all voters have separable preferences: They either rank the two leftist candidates one-two (which leftist is ranked first being unconstrained), or they rank them three-four. Even with this restriction, there still turn out to be strategic voting incentives.[21]

One situation in which an incentive to vote strategically can arise occurs when $.5C < L < 2C$ and $2/3(L - .5C) > C_1 - C_2$. The first condition guarantees that both the conservative and the leftist list will win one seat, if everyone votes sincerely. The second condition says that the leftists

[21] Without this restriction other possibilities open up as well.

have $2/3(L - .5C)$ excess votes which they could give to the second conservative.[22] How would this gambit affect the outcome? The leftist list would still win a seat, since $L{-}2/3(L - .5C) > C{+}2/3(L - .5C)$. The $2/3(L - .5C)$ leftist voters could be chosen, moreover, so that the allocation of the leftist seat was unaffected. (For example, taking at least as many votes from the second as from the first leftist to finance the "raid" on the conservatives would leave the first leftist winning the leftist seat.) Finally, the $2/3(L - .5C)$ extra votes for the second conservative would be sufficient to at least put him into a tie with his running mate, and possibly to elect him over that running mate, because $2/3(L - .5C) > C_1 - C_2$.[23] Thus, *if* leftist voters generally prefer the second conservative to the first, they have an incentive to "raid" the conservative list.[24]

Of course, this strategy is quite risky if the parties are not sure of their vote totals. It would be particularly risky if one list attempted to transfer nearly all of their excess votes to the other, in an attempt to affect which candidate there won, because this would leave the raiding list with just over half the votes of the raided list. A little miscalculation in the total votes received by each list would then give both seats to the raided list.

With the restriction to two lists and separable preferences, the case just considered is essentially the only one in which incentives to vote strategically arise, as the following proposition goes some way toward establishing:

> *Proposition:* The sincere strategy vector v^* is a strong equilibrium if either:
> (1) $L < .5C$; or
> (2) $L > 2C$; or
> (3) $L = .5C$; or
> (4) $L = 2C$; or
> (5) $.5C < L < 2C$ and $2/3(L - .5C) < C_1 - C_2$ and $2/3(C - .5L) < L_1 - L_2$.

A sketch proof of this assertion for the main cases, (1), (2), and (5), goes as follows. In case (1), the two conservative candidates will win under v^*. There is nothing that the leftist voters can do to alter this and the

[22] More precisely, they could take the integer portion of $2/3(L{-}.5C)$, if this is nonintegral, or the next lowest integer, if this is integral. These votes are "excess" in the sense that transferring them from one list to the other leaves both lists winning just one seat.

[23] If $2/3(L - .5C)$ is an integer, one larger than $C_1 - C_2$, then the leftists can only spare $2/3(L - .5C)$-1 votes and thus can only put the weaker Conservative into a tie.

[24] A symmetric case arises regarding conservative incentives to "raid" the leftist list. Again, the key consideration is whether the conservatives have enough *surplus* votes (more than needed to secure their seat under the d'Hondt allocation rule) to affect the allocation of the leftist seat among the leftist candidates.

conservative voters have no incentive to do so, as the outcome corresponds to their most-preferred outcome. So everyone may as well vote sincerely. Case (2) is symmetric, with the leftists and conservatives switching roles. In case (5), the leftist list and the conservative list will each win one seat. Leftist voters do not have enough surplus votes to "raid" the conservative list (because $L - .5C < C_1 - C_2$) and similarly for the conservatives.[25]

Is there any evidence of strategic raiding of the kind suggested above in Chilean elections? In the 1989 elections, the larger list was in a position to raid the smaller (having more excess votes than separated the candidates on the second-place list) in only about half the districts. Moreover, there were only ten cases where the larger list would have needed to use only a small percentage (less than 10%) of its excess votes in order to affect who won the smaller list's seat, and in several of these districts the larger list either won both seats or was near to doing so, which would obviously remove any incentive to raid. All told, then, there were only about five districts in which the conditions for strategic raiding were favorable. And in these five cases supporters of the larger lists may not have cared much who won the other list's seat; or may not have had enough information about the lists' relative standings to act on the strategic opportunity they faced. The only case that I know of where strategic calculations are alleged to have come into play – the 1993 election in the *Las Condes* constituency – involved national considerations.[26]

Strategic voting in French labor elections

Rosenthal (1974) describes a species of strategic voting under open-list DBPR in which French labor unions attempt to raid one another's lists in a fashion somewhat similar to the Chilean case. The elections concern who will serve as factory representatives for the workforce. Rival unions put up lists, typically with as many names on the list as there are representatives to be elected from a given factory. Workers select a single list, but are then allowed to strike out as many names on the list as they see

[25]The proposition states a sufficient condition for the sincere strategy vector v^* to be a strong equilibrium. It is almost a necessary condition as well, except for some messy minor cases that I will not go into.

[26]In this constituency, the leader of the more moderate right-wing party faced stiff opposition from a strong candidate of the more extreme right-wing party, as well as an incumbent candidate of one of the *Concertación* parties. Faced with the prospect of a parliament in which the more moderate opposition leader was not present, it is alleged that leftist voters who were not from the party of the left-wing incumbent voted in sufficient numbers to give both seats to the right. See Lagos (1996).

fit.[27] Seats are allocated first to the union lists, by PR-d'Hondt;[28] and then to the candidates on each list, by plurality rule (with ties broken by list order). Thus, for example, if a list wins 3 seats, the 3 candidates on the list receiving the most votes win the seats, where a candidate's vote total equals the number of voters selecting the candidate's list, less the number that have struck out his or her name.

How do unions manipulate this system? As described in Rosenthal (1974), a union may instruct a small band of its militants to support another union's list but to strike out the top names. The result, if the other union's supporters typically strike no one off, is that the top names (i.e., the union leaders) are defeated. The total number of seats that the other union wins is not changed but the occupants of those seats are altered. In order to protect against this strategy, the to-be-raided union may instruct its members to strike off the last names on the list. The bulk of Rosenthal's analysis concerns the equilibrium to this game of strategy, which apparently the French unions have been playing for some time.

5.4 CONCLUSION

This chapter has investigated strategic voting equilibria in three main electoral systems: SNTV, LRPR, and DBPR. In multimember districts operating under SNTV or PR, strategic voting can refer to the strategic desertion of both candidates/lists that are "too weak" and candidates/lists that are "too strong." Outcome-oriented voters in multimember districts desert weak candidates/lists for the same reason that they do in single-member districts – a fear of wasting their votes (cf. Leys 1959; Sartori 1968). They desert strong candidates/lists when those candidates/lists have one or more of the M seats sewn up but there are other seats still up for grabs; for then the voter's vote has a much greater chance of affecting the outcome if cast for one of the "marginal" candidates/lists: those on the edge between winning and losing the last-allocated seat.

The equilibrium levels of strategic voting entailed in a pure model imply that all three systems – SNTV, LRPR, and DBPR – impose the same upper bound on the number of viable competitors, K: namely, $K \leq M + 1$.[29] For SNTV, this is in accordance with Reed's "$M + 1$ rule" (Reed

[27]Thus, in effect, each worker selects a list and then casts approval votes for the candidates on the list. Cf. Brams and Fishburn (1983).

[28]Each list's vote total is roughly equal to the number of workers selecting the list; for details see Rosenthal (1974).

[29]How to measure the "number of viable competitors" operationally is another matter. See footnote 14.

1991); for PR, the upper bound provides a quantification of the Leys-Sartori conjecture.

There is evidence that elections held in small-magnitude Japanese (SNTV), Colombian (LRPR), and Spanish (DBPR) districts have followed the $M + 1$ rule (or tended toward agreement with it). In the case of large-magnitude PR, however, the $M + 1$ upper bound appears not to be binding, revealing that empirically observed effective numbers of lists are depressed below this upper bound by forces other than strategic voting.

I have also stressed three subsidiary themes in this chapter: First, that the logic of strategic voting leads only to an upper bound on the number of candidates or lists, as taken for granted above; second, that there are nonstandard modalities of strategic voting that do not have the classical vote-concentrating effects assumed by Duverger and the subsequent literature in electoral studies; third, that strategic voting fades out in multimember districts when the district magnitude gets above five. The last of these points relates to the informational requirements of the model, and in particular the rational expectations assumption.

Violations of the rational expectations assumption are largely a matter of the level of information about electoral prospects in the election at hand. Strategic voting should decline as voters' expectations about who will win and who will lose are less clear and less coordinated. Voters' expectations should be less clear and coordinated: (1) the greater is electoral volatility (so that expectations about "this time" cannot be grounded in simple extrapolations from "last time"); (2) the fewer are the relevant polls published in the mass media (so that expectations cannot be grounded on simple extrapolations from polls);[30] and (3) the larger is the district magnitude (since a given vote percentage means more, in terms of a chance at a seat, as district magnitude increases, a voter requires more information to become confident that a given list is really out of the running as M increases). The first two points, concerning the usefulness of electoral history and contemporaneous polls in coordinating voters' expectations, have been validated by a series of interesting experiments conducted by Forsythe et al. (1993). The last point, concerning the impact of district magnitude, points to a conclusion similar to that drawn by Sartori (1968:279): "The general rule is that the progression from maximal manipulative impact [via strategic voting] to sheer ineffectiveness follows, more than anything else, the size of the constituency."

[30]Many countries outlaw the publication of polling results during some portion of the campaign, e.g., France (last 7 days), Italy, Spain, Peru, Portugal, Belgium, Japan, Lithuania (throughout the campaign), Bulgaria (last 14 days), the Czech Republic (last 7 days), and Poland (last 7 days). The more such laws are actually observed, the more difficult it may be for voters to vote strategically, at least if who is ahead and who behind has not become clear before the ban on polls begins.