CHAPTER 11

Sovereign Debt*

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Abstract

In this chapter, we use a benchmark limited-commitment model to explore key issues in the economics of sovereign debt. After highlighting conceptual issues that distinguish sovereign debt as well as reviewing a number of empirical facts, we use the model to discuss debt overhang, risk-sharing, and capital flows in an environment of limited enforcement. We also discuss recent progress on default and renegotiation; self-fulfilling debt crises; and incomplete markets and their quantitative implications. We conclude with a brief assessment of the current state of the literature and highlight some directions for future research.

Keywords

Sovereign debt, Debt overhang, Default

JEL classification codes

F34, F32, F21, F41, E62

1. INTRODUCTION: CONCEPTUAL ISSUES

The defining feature of sovereign debt is the limited mechanisms for enforcement. This distinguishes sovereign debt from private debt, whether domestic or international.[1] A private agent or corporation, at least technically, is always subject to a legal authority. Sovereign nations are not. International bonds and bank loans are typically issued or contracted in a major financial center, such as New York or London. As such, they are subject to the legal jurisdiction of the place of issue. If a sovereign debtor fails to make a contracted payment, creditors have limited legal recourse, relying only on overseas legal instruments and reputational considerations. The mechanisms by which countries are induced to follow the terms of contracts, and the implications of limited enforcement for risk-sharing, growth, and other macroeconomic outcomes, is a major theme developed in

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[1] Nevertheless, the lessons derived from the study of sovereign debt are often applicable to other contexts, such as private credit markets in which enforcement is imperfect.

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this chapter. This introductory section lays out some of the conceptual issues that underlie the economics of sovereign debt.

In practice, the standard sovereign debt contract is typically non-contingent.[2] That is, the contract specifies a pre-determined, non-state-contingent sequence of payments in a defined currency due at defined points in time. This notional non-contingency obscures a richer contracting space that comes about through maturity structure, renegotiation, rescheduling, and "haircuts." The question of state contingency is an important theme discussed in this chapter. In addition to limited enforcement, the lack of contingency may reflect asymmetric information. To the extent the government can manipulate the actual or reported behavior of macroeconomic aggregates, contracts with state-contingent payoffs may be prone to moral hazard. Even if the government cannot affect the outcome of the economy, the true state of the economy may not be verifiable to creditors.

The contract (or the legal jurisdiction in which the bond is issued) will detail how the terms can be changed at some future point. For example, collective-action clauses will establish what fraction of bond holders must agree to change the terms of the initial debt contract. There are several conceptual issues involved with renegotiation. One was mentioned in the previous paragraph; namely, renegotiation can allow for ex post state contingency. Another is the normative question of which type of collective-action clauses are best. A third is that in practice, renegotiation is a lengthy and seemingly costly process. This raises the positive question of why this is so. Finally, the fact that debt may be renegotiated or rescheduled makes the definition of default rather subjective. One strict definition of default is failure to make the specified payment at the required date. However, often such payments are renegotiated under the threat of default, with creditors accepting less in place of none.

As a rule, there is no strict seniority in sovereign debt issues (with a few "de facto" exceptions, like credit extended by the IMF). This opens the possibility that existing creditors may see their debt "diluted" by subsequent new bond issuances. This makes long-term sovereign debt vulnerable to capital losses. Moreover, this incentive to dilute has implications for the payoff to voluntary "debt buybacks," (Bulow and Rogoff, 1991), which, as we will discuss, can be considered a reverse dilution of existing bondholders. The lack of strict seniority also raises the question of whether a defaulting government can treat certain creditors preferentially, for example domestic holders of sovereign debt. The extent to which internationally issued bonds are held by domestic residents may influence the net payoff to default (Broner et al., 2010).

In this chapter, we will take up the above themes. There are corollary issues related to sovereign debt we will discuss as well. For example, given a particular contracting space, there may be multiple equilibria, which raises the possibility of self-fulfilling debt

[2] There are a few exceptions, including some of the Brady bond restructurings in the early 1990s and recent bonds issued by Argentina and Greece. Such state-contingent "macro assets" have been advocated by Shiller (1993) and others, although such markets face challenges due to asymmetric information and limited verifiability.

crises. The issue of enforcement quickly leads to the role of reputation, both in regard to debt repayment and spill-overs to other economic activities. One important question is whether default affects private agents' beliefs about property rights more generally and the returns on private investment. Also it is important to consider the preferences of the decision maker, which may be different from a benevolent planner. We will discuss these issues more formally using a basic conceptual framework introduced in Section 3, which is then extended and modified in subsequent sections. Before introducing the framework, we first review several key empirical facts regarding sovereign debt.

2. EMPIRICAL FACTS

In this section we briefly summarize recent empirical research on default and its consequences, the macroeconomic consequences of sovereign debt overhang, and empirical facts regarding bond prices. There has been a recent boom in the collection and analysis of historical data on default. This work has generated novel facts as well as guided the theoretical approach to sovereign debt discussed in the subsequent sections (see Tomz and Wright, 2013, for a recent survey). We list several key findings. The first four concern default and its aftermath. The fifth finding concerns recent evidence on bond spreads. The sixth finding concerns the fact that successful growth episodes are associated with low and declining levels of foreign indebtedness.

1. **Default happens with regularity throughout history:** As an empirical event, default is typically defined as a failure of a government to meet a principal or interest payment on time and/or a rescheduling of debt on terms less favorable to the creditors. Reinhart and Rogoff (2009) emphasize that most countries that are able to raise funds internationally have had one or several default episodes in their history, including major European economies such as England, France, and Germany, and "graduation" to non-default status is extremely rare, even among high-income countries. Moreover, countries rarely default just once; serial default is the norm rather than the exception. Reinhart and Rogoff also emphasize that defaults happen in waves, with many countries being in default simultaneously. Recent episodes of multi-country debt crises include the Great Depression, the Latin American crisis of the 1980s, and the ongoing European debt crisis.
2. **Default often occurs in bad times, but with exceptions:** The fact that default happens most often when output is low provides a natural starting point for thinking about default. Using a newly constructed historical data set, Tomz and Wright (2007) conclude that defaults are more common in bad times than in good, but they also document that there are many exceptions. Specifically, Tomz and Wright document that in their sample of 175 countries, output is on average 1.6 percentage points below trend at the start of a default episode. Nevertheless, more than one-third of their 169

default episodes began when income was at or above trend, and countries frequently fall below trend without defaulting, indicating that a recession is neither necessary nor sufficient for default. Reinhart and Rogoff (2009) document that default crises frequently coincide with major financial crises. The pressure from bank failures and recession on a government's fiscal situation combined with the fact that many financial institutions hold government debt on their balance sheets makes the two types of crises intertwined. From a historical perspective, the fact that the 2008 financial crisis accompanied a sovereign debt crisis in multiple countries is no outlier. In addition to financial crises, default often precedes a large drop in trade (Rose, 2005; Martinez and Sandleris, 2011), and current account reversals/capital flight (Mendoza and Yue, 2012).

3. **Defaults involve a heterogeneous pattern of "haircuts":** Sturzenegger and Zettelmeyer (2008) review debt restructuring episodes from the 1990s and 2000s in six countries and across a number of debt instruments. The defaults in the 1990s and 2000s frequently involved bonds, and therefore differed from the primarily bank-debt crisis of the 1980s. Bond restructurings typically include a public offer of exchange, allowing researchers to compute the implied losses. Sturzenegger and Zettelmeyer (2008) compute the difference in promised payments between the old and new bond offerings in each exchange. A main finding is that these losses varied considerably over the sample. Relative to the face value of outstanding debt, the restructured bonds implied losses ranging from roughly 30% in Uruguay to over 60% for some bond series in Argentina and Russia. The Sturzenegger and Zettelmeyer (2008) sample is relatively small; however, Benjamin and Wright (2008) and Cruces and Trebesch (2011) explore a number of additional restructurings and alternative methodologies and find investor losses of roughly 30–40% on average, and again with considerable heterogeneity across individual default episodes.
4. **Default generates a period of lengthy renegotiation:** Benjamin and Wright (2008) study a large sample of bank-debt and bond renegotiations ranging from 1989 through 2005 involving seventy-three countries and ninety default episodes. In addition to the large and heterogeneous losses discussed above, they document that restructurings are a time-consuming process, taking eight years on average. Moreover, they find that the longer the negotiations, the larger the losses associated with the restructuring. The renegotiation process appears to be sensitive to the behavior of output, with large recessions generating somewhat longer restructurings and final settlement typically occurring when output has returned to trend. Benjamin and Wright (2008) also find that the median country exits restructuring carrying 5% higher debt-to-GDP loads then at the time of default.
5. **Sovereign bond spreads:** Broner et al. (2013) use a sample of emerging market bond yields from 1990 to 2009 to document several facts regarding bond yields and maturities. Specifically, they show that on average spreads over US bonds are higher

for longer maturity bonds, and while all spreads increase during crises, the short-term bond spread increases relatively more so that the yield curve "inverts" during periods of very high average spreads. The authors also document that the maturity of newly issued bonds shorten during crises, as the issuance of debt with more than three-year maturity declines when spreads are high.[3] A standard assumption in the theoretical literature on emerging markets is that foreign investors can hedge idiosyncratic country risk. However, emerging market bond yields exhibit significant co-movement, much more so than the often weak correlation for output. Longstaff et al. (2011) and Borri and Verdelhan (2011) document that global factors, like the return to the U.S. stock market, U.S. corporate bond market, or change in the VIX volatility index explain a large fraction of the common variation in spreads. This evidence suggests that holders of sovereign bonds are being compensated for taking on aggregate risk in addition to idiosyncratic default risk. This is not to say that bond spreads are not correlated with domestic output. Neumeyer and Perri (2005) and Uribe and Yue (2006) document that spreads are strongly countercyclical in emerging markets (see also Edwards, 1984).

6. **Debt overhang and growth:** The standard open-economy growth model predicts that a country with above average growth prospects should attract capital for both investment and consumption smoothing. The empirical pattern, at least for emerging markets since the opening of capital accounts in the 1970s and 1980s, is the opposite. Gourinchas and Jeanne (2007) document what they term the "allocation puzzle"; namely, that countries with above average growth rates are net exporters of capital on average. Aguiar and Amador (2011) show that this pattern is driven by government net foreign assets. In particular, they show that governments of high-growth economies increase net public assets held abroad (foreign reserves minus sovereign debt), while under-performing economies increase their public indebtedness. Moreover, this is not simply high-growth countries paying down a relatively large initial stock of debt nor is it consumption-smoothing at business cycle frequencies. On the other hand, Aguiar and Amador (2011) show that private capital flows accord with the standard intuition; that is, growth is accompanied by an increase in private net foreign liabilities. Alfaro et al. (2011) show that emerging market governments are contracting with other sovereigns, so the allocation puzzle involves governments on both sides of the transactions. Reinhart et al. (2012) document a negative correlation in advanced economies between debt-to-GDP ratios and growth. In sum, the evidence indicates that successful long-term development is not financed through sovereign debt, but rather is associated with a government paying down debt and/or accumulating net foreign assets.

[3] Arellano and Ramanarayanan (2012) confirm these results for a subset of the considered countries using data until 2011.

3. A BENCHMARK FRAMEWORK

In this section we introduce a benchmark limited-commitment environment. The analysis generates a rich set of implications, many of which carry over to the environments considered in subsequent sections. The benchmark framework was initially explored in the closed-economy models of Thomas and Worrall (1988) and Kehoe and Levine (1993). Key conceptual elements can be found in the seminal sovereign debt paper by Eaton and Gersovitz (1982).

Consider a small open economy populated by a representative agent and a government. Time runs discretely and is indexed $t = 0, 1, \ldots$ The economy is subject to exogenous shocks to output, which can be considered endowment or productivity shocks, depending on the context. To set notation regarding shocks and histories, let $s_t \in S$ denote the current state, which follows a finite-state Markov chain starting from some initial state s_0. Let $s^t = (s_0, s_1, \ldots, s_t) \in S^t$ denote a history truncated at time t. Let $\pi(s^t)$ denote the unconditional probability of history s^t, where $\pi(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. The notation $s^{t+j}|s^t$ indicates histories through $t+j \geq t$ that contain s^t, and $s^{t-j} \in s^t$ indicates history s^t truncated at $t-j \leq t$. Similarly, $\pi(s_{t+1}|s^t) = \pi(s^{t+1} = (s^t, s_{t+1})|s^t)$ denotes the probability period $t+1$'s state is s_{t+1} conditional on history s^t. Finally, we let $\sum_{t\geq 0, s^t}$ denote the summation over all $t \geq 0$ and histories $s^t \in S^t$, $\sum_{s^j \in s^t}$ denote the sum over all truncated histories contained in s^t, and $\sum_{\tau \geq t, s^\tau|s^t}$ denote the sum over all infinite histories following s^t. For an allocation series $x =$ consumption, capital, debt, etc., we let $x(s^t)$ denote the allocation at a particular node s^t, and $\mathbf{x} \equiv \{x(s^t)\}_{t\geq 0, s^t} = (x(s^0), x(s^1), \ldots)$ denote the allocation over the infinite history.

There is an international financial market where the final good can be traded intertemporally using a full set of state-contingent assets. Let $Q^*(s^t) = \pi(s^t)/R^t$ denote the international price of a unit of consumption delivered at history s^t in units of period-zero consumption units, where $R = 1+r$ is the gross interest rate in the international financial markets. When the economy is small and its shocks are uncorrelated with the rest of the world's consumption, standard diversification arguments imply risk-neutral pricing. We also assume that international asset markets have full commitment to financial contracts.

Let $c(s^t)$ denote consumption of the representative agent in history s^t. The government's preferences are

$$U(\mathbf{c}) = \sum_{t\geq 0, s^t} \pi(s^t)\beta^t u(c(s^t)), \tag{1}$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is a standard utility function, strictly increasing, concave and satisfying Inada conditions. We assume that the government has sufficient instruments to control the representative agent's decisions, subject to the resource constraints. We postpone discussion of alternative objective functions and how to decentralize the resulting allocation as a competitive equilibrium. The representative agent is endowed with a unit of labor, which it supplies inelastically. To ensure the small open economy's assets remain bounded, we assume $\beta R \leq 1$.

The timing of investment and production is as follows. The economy enters period t with installed capital $k(s^{t-1})$ and a portfolio of state-contingent liabilities $\sum_{s_t} b((s^{t-1}, s_t))$. We use the notation $k(s^{t-1})$ for period-t's capital as it is invested before s_t is realized, with $k(s^{-1})$ standing for the initial period capital stock. Once s_t is realized, the economy hires labor $n(s^t)$ and operates a neoclassical production function $F(s^t, k(s^{t-1}), n(s^t))$. Given that labor supply is inelastic, an endowment economy is nested in this formulation by dropping k as an argument. For simplicity, we drop the labor argument in the production function when convenient. The government then decides the consumption of the representative agent $c(s^t)$, pays liability $b(s^t)$, issues next period's portfolio of state-contingent liabilities, and makes net investment $k(s^t) - (1-\delta)k(s^{t-1})$, where δ is the depreciation rate.

Assuming that the value of production at international prices is finite, the resource constraint of the small open economy can be written as:

$$b_0 \leq \sum_{t \geq 0, s^t} Q^*(s^t)(F(s^t, k(s^{t-1})) - c(s^t) - k(s^t) + (1-\delta)k(s^{t-1})), \tag{RC}$$

where $b_0 \equiv b(s^0)$ is the initial net foreign liability position. The elements of the sum on the right-hand side are net exports at each history, and balance of payments accounting requires that initial net foreign liabilities equals the discounted sum of net exports.

At the end of the period in history s^t, we can define

$$\tilde{b}(s^t) \equiv b(s^t) - Rk(s^{t-1})$$

as the economy's total liability position inclusive of domestically held wealth, where previous period's investment is carried forward at the world interest rate. This is the relevant wealth position at the time decisions regarding consumption, investment, and financial trades are made. There are no inter-temporal adjustment costs to capital, so financial assets and capital can be exchanged at the end of a period before the next shock is realized (but once invested, capital remains in place for a full period). As financial assets span the payouts on physical capital, physical and financial capital are perfect substitutes at the time financial claims are settled and new investment is made, hence we can collapse the two into a single state variable. Therefore, the relevant state variable for the economy is foreign liabilities less installed capital, which is the negative of total wealth held both abroad and at home.

From the above discussion, we can start the economy before initial capital is installed, so that b_0 and $k(s^{-1})$ are chosen simultaneously subject to $\tilde{b}_0 \geq b(s^0) - Rk(s^{-1})$. This exploits the small open-economy assumption to collapse debt and capital into a single state variable. Using $\tilde{b}_0$ as the economy's initial state, we can rewrite (RC) as

$$\begin{aligned}\tilde{b}_0 &\le \sum_{t\ge 0, s^t} Q^*(s^t)\Big(F(s^t, k(s^{t-1})) - c(s^t) - k(s^t) + (1-\delta)k(s^{t-1})\Big) - Rk(s^{-1}) \\ &= \sum_{t\ge 0, s^t} Q^*(s^t)(F(s^t, k(s^{t-1})) - c(s^t) - (r+\delta)k(s^{t-1})), \qquad \text{(RC}'\text{)}\end{aligned}$$

where the second line is a rearranged version of the first.[4]

The critical assumption is that the government has limited commitment: that is, at any point in time, the government can decide to change its policy. In this simple set up, this means that the government can decide not to repay its debt to foreigners and/or expropriate foreign-owned capital invested domestically. We will refer to this as "deviation," rather than "default" to avoid confusion when discussing the empirical implications of the model. In particular, we discuss below interpretations of state-contingent debt as default with partial forgiveness or default with renegotiation. This is distinct from deviation, which as we shall see happens off the equilibrium path.[5]

We let $\underline{V}(s^t, k(s^{t-1}))$ summarize the value of deviation to the government at history s^t with installed capital $k(s^{t-1})$. The fact that capital is a state variable for deviation utility reflects that it cannot be removed within the period. We will sometimes refer to the deviation value using the more evocative terminology of "punishment." The nature of this value will be a key object of interest across different environments. The cases we consider require that the punishment utility is independent of outstanding debt at the time of deviation, and depends on previous equilibrium choices only through the existing capital stock. We further assume that $\underline{V}(s^t, k(s^{t-1}))$ is weakly increasing in $k(s^{t-1})$.

For an allocation to be compatible with the government's ability to deviate, it must deliver present value utility at least as great as $\underline{V}(s^t, k(s^{t-1}))$:

$$\sum_{\tau\ge t, s^\tau | s^t} \pi(s^\tau | s^t)\beta^{\tau - t} u(c(s^\tau)) \ge \underline{V}(s^t, k(s^{t-1})) \quad \text{for all } t, s^t. \qquad \text{(PC)}$$

[4] To obtain this, we can use the fact that for a given t,

$$\begin{aligned}\sum_{s^t\in S^t} Q^*(s^t)k(s^{t-1}) &= R^{-t} \sum_{s^{t-1}\in S^{t-1}} \sum_{s_t\in S} \pi((s^{t-1}, s_t))k(s^{t-1}) \\ &= R^{-t} \sum_{s^{t-1}\in S^{t-1}} \pi(s^{t-1})k(s^{t-1}) \\ &= R^{-1} \sum_{s^{t-1}\in S^{t-1}} Q^*(s^{t-1})k(s^{t-1}).\end{aligned}$$

Using this equivalence in (RC) and rearranging, we have (RC′).

[5] Some authors use the term repudiation or "inexcusable" default rather than deviation, which is entirely consistent, while others use the empirically suggestive term default, which can lead to confusion.

This constraint is commonly referred to as the "participation constraint" or "debt constraint." An allocation that satisfies the sequence of participation constraints ensures that the government would never prefer to deviate. As noted below, this can be construed as a borrowing limit, as the international loan market recognizes that additional debt will not be repaid.

Definition 1. A government-controlled allocation conditional on $\tilde{b}_0$, is defined to be non-negative consumption and capital allocations, $\{\mathbf{c}, \mathbf{k}\}$, that solve the following problem

$$V(\tilde{b}_0) = \max_{\{\mathbf{c},\mathbf{k}\}} \sum_{t\geq 0, s^t} \pi(s^t)\beta^t u(c(s^t)) \quad \text{subject to (RC}'\text{) and (PC).} \tag{\textbf{P}}$$

Note that the solution to Problem (**P**) corresponds to a "self-enforcing" equilibrium in the game between the government, owners of physical capital, and international lenders, or, in the terminology of Chari and Kehoe (1990), a "sustainable plan."

The resource constraint (RC′) implies that V is a strictly decreasing function, assuming we remain in the interior of the constraint set. Viewed recursively, the left-hand side of (PC) can be viewed as the value function conditional on outstanding liabilities at history s^t. The fact that this value is monotonic in $\tilde{b}$, which recall is the sum of net foreign liabilities minus installed capital, implies that the constraint can be viewed as an upper bound on net foreign liabilities (conditional on installed capital) at each history. Conversely, conditional on net foreign liabilities, the constraint implies an upper bound on domestic capital, a point that will feature prominently in our discussion of debt overhang below.

Before studying this problem in detail, we briefly summarize the full-commitment solution. That is, if the government could commit to all financial contracts, the allocation would feature:

(i) Complete risk-sharing, such that $c(s^t) = c_t$ is constant across states at a point in time, and satisfies $\beta R u'(c_t) = u'(c_{t-1})$ inter-temporally; and
(ii) First-best investment, such that $\sum_{s_t\in S} \pi(s_t|s^{t-1})F_k((s_t, s^{t-1}), k(s^{t-1})) = r + \delta$.

The corresponding level of initial consumption can be recovered from the resource constraint (RC′) given the sequence of first-best capital stocks.

3.1. An Endowment Economy

To focus on how limited commitment impedes risk-sharing, let us assume, as a starting point, an endowment economy. That is, there is no capital, and $F(s^t) = y(s^t) = y_t$, where y_t follows a stationary, first-order Markov process that takes discrete values $0 < y^1 < y^2 < \cdots < y^N$. To make the problem more concrete, we specify $\underline{V}(s^t)$. Specifically, we assume that deviation results in total exclusion from international asset markets. If the economy could not trade financial assets at all (autarky), the utility of the representative agent would be:

$$\underline{V}(y) = V^{Aut}(y) \equiv u(y) + \sum_{j=1}^{\infty}\sum_{k=1}^{N} \beta^j \pi(y_{t+j} = y^k | y_t = y) u(y^k). \tag{2}$$

Autarky, as defined above, is the canonical punishment for deviation in the sovereign debt literature (the classic reference is Eaton and Gersovitz (1982)). This punishment is often interpreted as the loss of a country's reputation in international financial markets due to a deviation.

In the environment under consideration, in which the government makes decisions on behalf of a representative agent, the autarky value represents the lowest utility for the government that can be sustained as a self-enforcing equilibrium. Or, as noted above, the autarky value defines a state-contingent upper bound on the economy's liabilities (which, in the endowment economy, equals outstanding sovereign debt). The fact that a self-enforcing equilibrium at the borrowing limit delivers the utility associated with permanent exclusion makes the autarky value (but not the autarkic allocation) a renegotiation proof punishment.[6]

There are some important caveats associated with autarky as a punishment value. If we take the reputational interpretation literally, there remains a question of why the loss of reputation prevents the government from continuing to save in world financial markets. In a seminal paper, Bulow and Rogoff (1989b) build on this insight to construct a celebrated critique of reputational models of debt. In particular, Bulow and Rogoff propose an alternative deviation equilibrium from autarky, one in which countries can never borrow again, but are able to save. That is, they can buy (a sequence of) cash-in-advance insurance contracts in which they pay upfront and are not obligated to pay out in any state the following period. With such contracts available, an economy that has reached its upper bound on liabilities can choose not to repay this debt, but rather use the scheduled payments to buy cash-in-advance contracts. Bulow and Rogoff (1989b) show that this is feasible and generates higher utility, as long as the upper bound on debt is strictly positive. Thus the equilibrium places an upper bound of debt of zero.

This insight has spawned a large literature. We briefly summarize some of the main themes. A straightforward response is to rule out cash-in-advance contracts by appealing to legal enforcement mechanisms in the international financial markets. That is, an unpaid creditor can sue to seize the debtor's overseas assets or exports. In this environment, the legal enforcement implicit in the foreign market's ability to commit to cash-in-advance contracts can also be used to enforce the punishment. In practice, this is reflected in the heavy role of courts, lawyers, and legal contracts involved in adjudicating sovereign debt claims. Direct sanctions under the auspices of a foreign legal authority is the environment favored by Bulow and Rogoff themselves in motivating the re-contracting model

[6] See Wright (2002) and Kletzer and Wright (2000) for the case of one- and two-sided limited commitment, respectively.

discussed below (Bulow and Rogoff, 1989a). However, there is mixed empirical evidence regarding the effect of trade sanctions as an enforcement mechanism (Rose, 2005; Martinez and Sandleris, 2011).[7]

Kletzer and Wright (2000) drop the commitment assumption for foreign creditors and show that debt can be sustained in "anarchy," as in the absence of any legal enforcement there is no mechanism to ensure that foreign commitments implicit in cash-in-advance contracts are honored. Imperfect enforcement of foreign savings vehicles is not necessary, however. Wright (2002) develops an environment in which a limited number of foreign banks can commit (that is, the one-sided limited commitment of our benchmark), but choose not to offer cash-in-advance contracts that disrupt established lending relationships. Others have also appealed to non-legal mechanisms to sustain debt. One prominent theme is that a loss of reputation in debt markets spills over to other economic spheres, depressing trade, output, or investment, without the need for creditor-country courts per se (e.g., Cole and Kehoe, 1998).[8] Relatedly, Amador (2012) argues that, if the government decisions are the result of a political game among distinct agents, then a tragedy of the commons may occur which renders the strategy of using cash-in-advance contracts after a deviation unsustainable, restoring the sustainability of sovereign debt.

While Bulow and Rogoff argued that complete financial autarky may be unrealistic as a punishment without additional legal enforcement to prevent savings, Broner and Ventura (2011) and Broner et al. (2010) argue that autarky may deliver a utility payoff even worse than the one modeled above. Broner and Ventura (2011) note that the failure to enforce international obligations may be associated with a failure to enforce domestic contracts, if the residency of the contracting parties cannot be ascertained. Similarly, domestic residents may hold the government's debt, leading to a potentially damaging redistribution of wealth across domestic agents in the event of a deviation (Broner et al., 2010). This may be particularly severe if domestic banks hold government bonds as assets and face net worth constraints in lending (Gennaioli et al., 2010). Thus, deviation may be associated with a breakdown in domestic risk-sharing that makes the value defined in (2) an upper bound.

That said, the key implications of the benchmark endowment model are robust to alternative enforcement mechanisms. Under standard monotonicity assumptions on the Markov process, $\underline{V}(y)$ in (2) is increasing in y. That is, a high current endowment makes deviation relatively attractive. Moreover, the deviation utility is independent of equilibrium allocations; in particular, it does not depend on the amount of debt outstanding at the time of deviation. As long as $\underline{V}(s^t)$ has these properties, the main implications derived below are robust to alternative punishments. The details of enforcement, and in particular

[7] For an interesting discussion on the role of sanctions versus reputation in sustaining sovereign debt in Spain under Philip II, see Conklin (1998) and Drelichman and Voth (2011).

[8] See Fuentes and Saravia (2010) for evidence with regard to falls in FDI after default.

the severity of the punishment, will determine the level of $\underline{V}$, which in turn determines the equilibrium limits to debt and risk sharing.

We now characterize the constrained-optimal allocation using Lagrangian techniques.[9] Let μ denote the multiplier on the resource constraint (RC), and $\lambda(s^t)\pi(s^t)\beta^t$ denote the sequence of multipliers on the participation constraints. We scale each participation multiplier by a strictly positive number for notational convenience. Note that in the endowment economy case, the problem stated in (**P**) has an objective function that is strictly concave and the constraints are convex. The first-order condition for consumption in state s^t is:

$$\beta^t \left(\pi(s^t) + \sum_{s^j \in s^t} \pi(s^t|s^j)\pi(s^j)\lambda(s^j) \right) u'(c(s^t)) = \mu Q^*(s^t).$$

Note that $\pi(s^t|s^j)\pi(s^j) = \pi(s^t)$ if $s^j \in s^t$, that is, if s^j precedes history s^t. We can use $Q^*(s^t) = \pi(s^t)/R^t$ to simplify this first-order condition:

$$\beta^t R^t \left(1 + \sum_{s^j \in s^t} \lambda(s^j) \right) u'(c(s^t)) = \mu. \tag{3}$$

The right-hand side of (3) is the marginal value of initial assets. If the participation constraints are always slack ($\lambda(s^t) = 0, \forall t$), then $\beta^t R^t u'(c(s^t))$ is a constant. This is the full risk-sharing allocation, in which consumption is not state dependent and varies over time only to the extent that the agent is impatient relative to the world interest rate. However, when $\lambda(s^t) > 0$, the participation constraint is binding[10] and full risk-sharing is not compatible with limited commitment. It can be shown that across states when the constraint binds, consumption is strictly increasing in the current endowment, highlighting the limits to risk sharing imposed by limited commitment. Moreover, the summation on the left-hand side of (3) is non-decreasing, and strictly increasing whenever $\lambda(s^t) > 0$. As the

[9] There are many technical assumptions that lie behind the validity of Lagrangian techniques in infinite-dimensional spaces. Given the infinite sequence of participation constraints, a natural environment is to assume that the set of participation constraints maps allocations into the space of bounded sequences (ℓ_∞). This requires that utility is bounded over the set of feasible allocations (where the difficulty usually lies in ensuring utility is bounded below at zero, as zero consumption is typically feasible). The natural space for multipliers is the space of summable sequences, ℓ_1. However, the dual of ℓ_∞ is larger than ℓ_1. Fortunately, for many environments of economic interest, it can be shown that the Lagrange multipliers are indeed elements of ℓ_1. See Dechert (1982) and Rustichini (1998) for details. A final requirement is that the participation constraint set includes an interior feasible allocation (if the constraint set is convex and we are characterizing a global optimum), or satisfies a local regularity condition (similar to the full-rank Jacobian condition in finite dimensions) if necessary conditions for an interior optimum are the object of interest. The standard reference is Luenberger (1969). Throughout the chapter, we assume the existence of an interior optimum and the validity of Lagrangian techniques without further comment.

[10] We use the term "binding" to indicate the constraint's multiplier is strictly positive, which, from complementary slackness, requires the constraint is satisfied with equality. With this usage, the constraint is not considered "binding" if the constraint is satisfied with equality but the multiplier is zero.

right-hand side is constant, this implies that the marginal utility of consumption is falling over time faster than βR. In particular, when $\beta R = 1$, consumption is non-decreasing, and strictly increasing whenever the participation constraint binds.

Another view of this implication comes from the Euler equation. Consider $s^{t+1} = (s^t, s_{t+1})$ and evaluate (3) at s^t and s^{t+1} to obtain:

$$\beta R \left(1 + \frac{\lambda(s^t, s_{t+1})}{\Lambda(s^t)}\right) u'(c(s^t, s_{t+1})) = u'(c(s^t)), \tag{4}$$

where $\Lambda(s^t) \equiv 1 + \sum_{s^j \in s^t} \lambda(s^j)$. If the participation constraint is slack at s_{t+1}, then $\beta R u'(c(s^t, s_{t+1})) = u'(c(s^t))$, which is the full-commitment Euler equation from state s^t to state s^{t+1}. When the participation constraint binds, $\lambda(s^{t+1}) > 0$, and marginal utility in s^{t+1} is distorted down relative to s^t (and consumption is distorted up). This captures that limited participation provides an incentive for consumption to grow over time. This feature of the optimal allocation is often referred to as "back loading," as consumption is higher in the later periods.

The intuition for back loading is as follows. The country always has the option of deviating and enjoying $\underline{V}(y_t)$. To ensure continued participation, the allocation at each history must deliver at least this utility, which requires a certain stream of consumption. Additional consumption in a particular period helps satisfy this requirement. Moreover, it also helps satisfy the requirement in all previous periods as well. This is because the left-hand side of the constraint (PC) is forward looking; it is the discounted sum of all future utility. At the margin, therefore, consumption in the future is preferable as it relaxes all preceding participation constraints. The math of the first-order condition (3) reflects this feature by including the cumulative sums of Lagrange multipliers from all previous periods.

This is perhaps a sophisticated way of saying that limited commitment provides an incentive to save. However, it also says more. When $\beta R = 1$, we see that consumption never declines, regardless of the state of nature. That is, current consumption always provides a floor for future consumption. Moreover, the fact that μ is finite implies the expression on the left-hand side of (3) converges. For $\beta R = 1$, this implies that $\lim_{t\to\infty} \lambda(s^t) = 0$, and $c(s^t)$ converges to a constant. In the limit, therefore, the economy achieves perfect risk sharing (see Ray, 2002), for a general statement of this result). Along the transition, consumption increases at each node at which the participation constraint binds. As $\underline{V}(y)$ is increasing in y, it reaches a maximum at $y = y^N$. Once the highest endowment is realized, consumption is constant thereafter and full risk-sharing is attained (Worrall, 1990).

This last result requires $\beta R = 1$. When $\beta R < 1$, we have that $\lim_{t\to\infty} \beta^t R^t = 0$, and from equation (3), we see that the cumulative sum therefore must diverge, that is, $\lambda(s^t)$ is strictly positive infinitely often. In particular, whenever the participation constraint is slack, consumption is below the previous period's due to impatience as shown by (4). However, consumption cannot fall indefinitely, as this will eventually violate the participation constraint. When the constraint binds, consumption's fall is mitigated, or even reversed if the current endowment realization is high enough. Moreover, as (PC) holds with equality when $\lambda(s^t) > 0$, we see that continuation utility is state dependent (as $\underline{V}(y_t)$ is state dependent). The combination of front loading due to impatience and back loading due to limited commitment implies consumption will converge to an ergodic distribution, which in general will be non-degenerate.

The model predicts that large debt positions impede risk-sharing. The resource constraint at any history requires that the present discounted value of net exports equals outstanding debt. Heuristically, a large stock of outstanding debt lowers the present value of consumption, making the participation constraint relevant in more states. This generates the empirical prediction that, all else equal, a large stock of outstanding debt is associated with more volatile consumption. If a country is patient, it will respond to this by saving. This is a general implication of many limited-commitment environments—the presence of a borrowing constraint provides an incentive to save. Placed in an international general equilibrium, this effect can be used to micro-found $\beta R < 1$. Perhaps more important for the discussion of risk-sharing, we see that borrowing today not only requires repayment in the future (as in the full-commitment environment), it may also generate more volatile consumption going forward due to impaired risk-sharing, a form of "debt overhang" onto consumption volatility.

The participation constraint is also informative about net payments (the trade balance). In particular, suppose the constraint is binding at a particular point in time, t. Rearranging (PC), this implies:

$$u(y_t) - u(c(s^t)) = \sum_{\tau \geq t+1, s^\tau | s^t} \pi(s^\tau | s^t) \beta^{\tau - t} \left[u(c(s^\tau)) - u(y_\tau) \right].$$

Note that the right-hand side must be greater than or equal to zero to satisfy the participation constraint in every state at $t + 1$. This implies that $y_t \geq c(s^t)$. That is, the country does not receive net inflows when its constraint is binding. Or, perhaps more intuitively, the constraint only binds if the country is expected to make a non-negative payment—the country is never tempted to renege when it is due payments from the rest of the world.

This plus risk-sharing considerations imply that the participation constraint tends to bind when the endowment is relatively high. Or, more precisely, fix a history through $t - 1$ and consider two endowment realizations $y^j < y^k$ at time t. If the participation constraint binds for y^j, then it also binds for y^k. This reflects the fact that a high endowment makes deviation attractive, plus the risk-sharing requirement that a high-endowment state is the time to repay creditors (given state-contingent asset markets). While intuitive, this implication has often led to the confusing and incorrect statement that the model predicts that "default happens in high-endowment states." The incentive to deviate in a high-endowment state is met, in equilibrium, with a reduction in the amount of net exports required in those states, ensuring continued participation. From the budget constraint, this must be balanced with lower net imports in other states. In particular, a binding participation constraint in high-endowment states reduces the amount of transfers the country receives in low-endowment states. That is, it acts as a constraint on borrowing in bad times, a natural and general implication of limited commitment.

To summarize the results so far, we see that limited commitment impedes risk-sharing, and does so particularly severely when the country is heavily indebted. The natural response to the lack of commitment is to save, which, if the country has sufficient patience, will ultimately lead to first-best risk-sharing; otherwise, consumption fluctuates with output indefinitely. When the country's participation constraint binds, net exports are non-negative, implying that borrowing is limited in other states of the world. The limited risk-sharing, volatility of consumption, and the negative consequences of indebtedness are all implications with clear counterparts in the data.

A natural question is whether these allocations can be implemented with realistic financial contracts and how to interpret empirical "default" episodes. The environment admits alternative interpretations of state-contingent contracts and default episodes. Grossman and Van Huyck (1988) argue that the state-contingent assets can be interpreted as partial defaults that are excused. That is, the lenders can observe the state and forgive a certain portion of the outstanding debt conditional on the state. A similar point is made by Bulow and Rogoff (1989a), who argue that sovereign debt can be renegotiated ex post under the threat of legal sanctions. This makes debt payments de facto state contingent, although the participation constraint is based on an explicit bargaining protocol rather than the reputational autarky value introduced above. They point out that the face value of debt can be set to match the highest possible payment, and lower payments are decentralized as default and renegotiation.[11]

While these interpretations are consistent with the fact that observed defaults are not punished severely (e.g., sovereign foreign assets are protected and economies regain access to financial markets), they suggest that some element of default is a high-frequency occurrence. At the other extreme, a narrow notion of default focuses on the model's prediction for a binding participation constraint in the lowest endowment state. If the constraint binds in the lowest state, it also binds for all continuation values. From the participation constraint, we then have that net exports are zero. This implies that the country is in autarky for that one period. Moreover, as pointed out above, the country will not exit this autarkic state until it makes a payment. While it refuses to make the payment, it continues to be denied net inflows and remains at the autarky value. This pattern predicts a punishment phase followed by partial repayment. However, the punishment phase lasts only until an endowment above the lowest state occurs, making it relatively short-lived, depending on the persistence of the endowment process. Another difficulty with this interpretation

[11] In both Grossman and Van Huyck (1988) and Bulow and Rogoff (1989a) renegotiation is either not time-consuming, or, via backwards induction, prolonged renegotiation takes place off the equilibrium path. We discuss other bargaining outcomes in Section 4.

of default is that if the country is relatively patient (i.e., $\beta R = 1$), a binding constraint with $y_t = y^1$ can only occur in the initial period(s), after which, savings (and state-contingent assets) will prevent the constraint from ever binding again in the low-endowment state. If the economy is impatient (βR is less than one), the economy may revisit this worst-outcome value due to borrowing and repeated low-endowment realizations.

3.2. Debt Overhang in a Production Economy

The preceding analysis concerned an endowment economy, and it contained one example of "debt overhang"; namely, risk-sharing is impeded by a large stock of outstanding debt. In a production economy, limited commitment predicts that output and growth is also adversely affected by debt. We should note at the start that in this subsection we consider allocations that are on the constrained Pareto frontier between investors/lenders and the domestic government. There is an early literature on sovereign debt overhang, such as Sachs (1989) and Krugman (1988), that explores Pareto inefficiencies that arise due to debt overhang. In these models, debt is assumed to be exogenous, and debt relief is shown to enhance investment and in some cases generate a Pareto improvement. In the analysis that follows, debt overhang arises in a model of endogenous debt dynamics due to the limited ability of the government to commit. The "ex post" constrained efficiency of the equilibrium allocation implies that a Pareto improvement cannot be engineered through debt forgiveness.

In this subsection, we assume that the economy operates the neoclassical production function $F(s^t, k(s^{t-1}), n(s^t))$, using the notation introduced at the beginning of this section. Recall as well that capital is a state variable in the deviation utility $\underline{V}(s^t, k(s^{t-1}))$. This allows the government to expropriate and redistribute capital following a deviation. That is, limited commitment extends to the protection of property located within the country. A natural benchmark for $\underline{V}(s^t, k(s^{t-1}))$ is the closed-economy neoclassical growth model. Specifically, following a deviation on promises to creditors or investors, the economy reverts to autarky but continues with the existing capital stock and technology. This is the deviation considered in Marcet and Marimon (1992). A harsher alternative is that some of the expropriated capital is rendered unusable without foreign involvement and/or the production function is operated with less efficiency. A perhaps even tougher environment can be constructed assuming that private (domestic) entrepreneurs are required to operate capital and can invest abroad following a deviation. In this case, the combination of capital flight and a government without commitment on capital taxation prevents the economy from accumulating domestic capital. Several of these alternatives are explored numerically in Aguiar and Amador (2011). While the precise modeling of the deviation value matters quantitatively, the conceptual points developed below rest on the assumption that $\underline{V}$ is strictly increasing in the domestic capital stock and is independent of equilibrium debt.

We consider the government-controlled allocation. The government's problem is given by (**P**) subject to (RC′) and (PC). We are confronted with a non-convex participation constraint due to the presence of a choice variable $k(s^t)$ on the right-hand side of (PC). In this case, there is not a general method for addressing this issue. Thomas and Worrall (1994) and Aguiar et al. (2009) provide some restrictions on the relative concavity of u and F that ensure the constraint is globally convex. Nevertheless, we can gain important insights by characterizing necessary conditions for an interior optimum without verifying their sufficiency.

As in the endowment economy, we use Lagrangian techniques. We continue to let μ denote the multiplier on the resource constraint (RC′) and $\lambda(s^t)\pi(s^t)\beta^t$ denote the multipliers on the participation constraints (PC). The first-order condition for consumption is the same as in the endowment economy (equation 3). The first-order condition for $k(s^t)$ is

$$\mu \sum_{s^{t+1}|s^t} Q^*(s^{t+1}) \left(F_k(s^{t+1}, k(s^t)) - r - \delta\right) = \sum_{s^{t+1}|s^t} \beta^{t+1}\pi(s^{t+1})\lambda(s^{t+1})\underline{V}_k(s^{t+1}, k(s^t)).$$

Dividing through by $\pi(s^t)$ and using the definition of $Q^*(s^{t+1})$, we have

$$\mu \sum_{s_{t+1}} \pi(s_{t+1}|s^t) \left(F_k(s^{t+1}, k(s^t)) - r - \delta\right) = \sum_{s_{t+1}} \beta^{t+1}R^{t+1}\pi(s_{t+1}|s^t)\lambda(s^{t+1})\underline{V}_k(s^{t+1}, k(s^t)).$$

The left-hand side is the expected marginal product of capital minus its opportunity cost, scaled by the marginal value of period-0 resources. In a world of full commitment, this will equal zero. The right-hand side reflects the distortions of limited commitment. In particular, if $\lambda(s^{t+1}) > 0$ in period $t+1$, then investment in period t is distorted. As $\underline{V}_k > 0$, we have that $\lambda(s^{t+1}) > 0$ implies that the expected marginal product of capital across states in period $t+1$ is strictly greater than the opportunity cost of capital; that is, investment is distorted down today if the participation constraint binds in any state tomorrow. The intuition is that a large capital stock raises the value of deviation ($\underline{V}_k > 0$), providing an increased incentive for the government to renege on debt promises. To relax the participation constraint, the government underinvests. This can be decentralized by a higher tax on capital income (Aguiar et al., 2009; Aguiar and Amador, 2011), where "taxes" may take the form of statutory taxes as well as bribes, permits and red tape, or *anticipated* ex post expropriations of capital income by politicians. Another direct implication of the non-negativity of the right-hand side is that capital is never greater than the first-best level; that is, over investment in this environment never enhances commitment.

The fact that the constraint in $t+1$ matters for period t investment provides a link between borrowing and investment volatility. Aguiar et al. (2009) explore this link in detail, and we briefly summarize their results. Consider two alternative histories in period t, s_1^t and s_2^t, with output higher in s_1^t, and identical initial wealth positions so resources are greater in s_1^t. To make the point in its simplest form, assume *iid* shocks so that the first-best investment level is independent of histories. In the low-output state s_2^t, the government would like to smooth the consumption by borrowing against high states in the future. Conversely, the government in state s_1^t would like to borrow less, or even use the extra output to pay down debt, raising consumption and risk-sharing opportunities going forward. In order to relax the borrowing constraint in the former case, the government invests less in s_2^t. That is, low output leads to low investment, indicating that limited commitment can prolong the impact of a transitory shock.

Moreover, this effect is most severe when the economy is heavily indebted. In particular, an economy can avoid distorted investment by maintaining a sufficiently low level of outstanding debt. For example, take the case of $\beta R = 1$. As in the endowment case, the first-order condition for consumption implies that the economy eventually saves to a point where the participation constraint no longer binds. In the production economy, this implies that investment ultimately achieves the first-best level, but only when debt is low enough. If $\beta R < 1$, then the economy converges to an ergodic distribution in which investment cycles indefinitely at a level always strictly below the first best. The implication that high levels of sovereign debt enhances the cyclicality and reduces the level of investment is consistent with the empirical results on the poor macroeconomic performance of heavily indebted economies. A particularly striking case study is Argentina's debt crisis of 2001, which coincided with the start of a sequence of government expropriations of private capital income.[12]

The analysis thus far highlights the deleterious implications of debt overhang in a world of limited commitment. The efficient allocation, when the country's government is sufficiently patient, addresses this by paying down debt. This begs the question of why so many countries stagnate in a heavily indebted state. One explanation is that some countries, due perhaps to demographics or mortality, discount at a relatively high rate. As we saw, if $\beta R < 1$, the efficient allocation does not achieve a debt level low enough to support first-best investment or risk-sharing. However, assuming $\beta R < 1$ is not completely satisfactory as an explanation. For example, many countries eventually do pay down their debt, but do so over a very long period of time. In the terminology of Reinhart and Rogoff, countries do "graduate" from debt overhang, but the process is a prolonged one. Note that a low value of β does not necessarily imply slow convergence to the steady-state or ergodic distribution; that is, even though a low value of β implies that saving is depressed, this is balanced by a reduction in the level of steady-state capital.

Aguiar and Amador (2011) provide an alternative explanation for the fact that successful countries reduce net foreign liabilities, but the speed at which this occurs may vary across economies. Specifically, they propose a model of political turnover in which political actors rotate in and out of power according to a Markov process. Motivated by the work of Persson and Svensson (1989) and Alesina and Tabellini (1990), participants in the political process are assumed to prefer consumption during their incumbency. Specifically, an incumbent places a premium $\tilde{\theta} > 1$ on consumption while in power. Aguiar and Amador consider an environment in which output is deterministic, but political turnover

[12] Restrepo-Echavarria (2013) explores the ability of the model to explain Latin America's "lost decade" following the 1980s debt crisis.

is stochastic, isolating the frictions due to political uncertainty. Let $p_{t,t+j}$ denote the probability that the incumbent in period t is also in power in $t+j$. The preferences of the incumbent at period t are:

$$\tilde{W}_t = \tilde{\theta} u(c_t) + \sum_{\tau > t} \beta^{\tau - t} \left(p_{t,\tau} \tilde{\theta} + 1 - p_{t,\tau} \right) u(c_\tau).$$

Considering the simplest case in which the political turnover process is *iid* across political participants (i.e., $p_{t,t+j} = p, \forall j > 0$), incumbent utility becomes:

$$\begin{aligned} W_t \equiv \frac{\tilde{W}_t}{p\tilde{\theta} + 1 - p} &= \theta u(c_t) + \sum_{\tau > t} \beta^{\tau - t} u(c_\tau) \\ &= \theta u(c_t) + \beta V_{t+1}, \end{aligned}$$

where $\theta = \frac{\tilde{\theta}}{p\tilde{\theta}+1-p}$, and we have renormalized the incumbent utility. Note that V_{t+1} is the utility of private agents in the economy.

In this environment, the current incumbent discounts between today and tomorrow at the rate $\beta/\theta < \beta$, but discounts across future periods at the geometric rate β. In this sense, the incumbent has hyperbolic or quasi-geometric preferences in the spirit of Laibson (1994). The fact that the current incumbent discounts in a non-geometric fashion is a natural consequence of stochastic political turnover; for the incumbent, the current period is special because it is in power with probability one. Any future period must be down-weighted by the probability that the current incumbent will lose office in the interim, but also reflect the fact that it may return to power as well. The *iid* environment implies that the conditional probability of incumbency is the same across any two future periods, and therefore those are weighted equally using the undistorted discount factor β. Aguiar and Amador (2011) show how to generalize this to persistent political processes. Political economy distortions are captured by θ, with $\theta = 1$ reflecting a benevolent government and $\theta > 1$ reflecting an incumbent who strictly favors front-loading consumption.

If the incumbent makes decisions regarding debt repayment or capital taxation, the relevant participation constraint is

$$W_t \geq \underline{W}(k_{t-1}), \tag{5}$$

where $\underline{W}(k_{t-1})$ is the deviation utility of the current incumbent given invested capital k_{t-1}. This utility incorporates any punishment the political process may impose on deviation, as well as the consequences of financial autarky. We continue to assume that $\underline{W}'(k_t) > 0$, so that additional capital makes deviation less costly.[13] An efficient allocation

[13] We also implicitly use the concavity assumption that $(F'(k) - r - d)/\underline{W}'(k)$ is declining in k. This is satisfied, for example, by a broad class of deviation value functions in the neighborhood of the first-best capital.

in this environment is to maximize (**P**) subject to (RC′) and the sequence of incumbent participation constraints (5).

The first-order condition for consumption is:

$$\beta^t R^t \left(1 + \sum_{j=0}^{t} \lambda_j + (\theta - 1)\lambda_t\right) u'(c_t) = \mu, \tag{6}$$

where λ_t denotes the scaled multiplier on incumbent t's participation constraint and μ denotes the multiplier on the resource constraint. This condition is similar to (3), the first-order condition absent political economy frictions, save for the presence of $(\theta - 1)\lambda_t$ on the left-hand side of (6). This additional term reflects that for the incumbent political party, current consumption is particularly valued. This additional term does not overturn the implication that if $\beta R = 1$, then $\lim_{t\to\infty} \lambda_t = 0$, as the cumulative sum continues to imply back loading of consumption. However, the presence of the additional term does influence dynamics. To see this transparently, consider the case of $\beta R = 1$ and assume risk neutral preferences: $u'(c) = 1$. (Aguiar and Amador, 2011, show how the results extend to the general environment with $\beta R \leq 1$ and concave utility). In this case, the cumulating sum on the left-hand side of (6) is constant over time, implying the following first-order dynamics:

$$\lambda_{t+1} = \left(1 - \frac{1}{\theta}\right)\lambda_t, \tag{7}$$

with $\lambda_0 = \frac{\mu - 1}{\theta} \geq 0$. This implies that λ_t converges to zero at a rate $1/\theta$, so convergence is governed by the extent of political distortions reflected in θ.

As in the benchmark case, the participation-constraint multiplier λ determines the distortions in investment, as the first-order condition for capital takes the same form (absent productivity shocks):

$$F'(k_t) - (r + \delta) = \lambda_{t+1} \underline{W}'(k_t). \tag{8}$$

Therefore the magnitude of political economy distortions also governs the convergence of capital to the first-best level. That is, a more politically distorted economy grows at a slower rate. The environment exhibits the standard implication of limited commitment that the participation constraint is relaxed as the sovereign accumulates wealth. Aguiar and Amador (2011) use this insight to generate an environment where growth occurs by reducing the government's net foreign liabilities, consistent with the "allocation puzzle" empirical facts discussed in Section 2. However, countries experience differential growth rates depending on the extent of political economy distortions. This is consistent with the fact that some countries experience long periods of stagnation in which debt is high and growth is low, while other countries exhibit extremely high levels of growth all the while serving as net exporters of capital. The model suggests that political economy distortions do not preclude an economy from eventually achieving high-income status, but does suggest that the process will be a long one. Moreover, Aguiar and Amador (2011) show that the more distorted economy must achieve a lower level of steady-state debt to support the same level of capital as a less distorted economy, consistent with the "debt intolerance" pattern documented by Reinhart et al. (2003) in which less-developed economies encounter macroeconomic difficulties at lower levels of external debt than high-income economies.

We conclude our discussion of the benchmark economy with a few comments on how to decentralize the constrained-efficient allocation as a competitive equilibrium. In regard to investment, the previous discussion highlights that the stock of physical capital may influence the government's incentive to renege on debt and tax promises. This can be decentralized with a tax on the returns to investment, as discussed in Kehoe and Perri (2004) in a two-country general equilibrium model and Aguiar et al. (2009) in a small-open-economy environment. Equation (8) directly implies that the returns to capital must be distorted relative to the opportunity cost if the participation constraint is strictly binding (whether the government is benevolent or not), and a tax on capital income is a natural decentralization of this wedge.

The decentralization of the endowment economy of Section 3.1 has been studied by Jeske (2006) and Wright (2006) in an environment in which heterogeneous private agents insure endowment risk with each other and with foreign financial markets. These authors assume complete enforcement of financial contracts signed by two domestic agents, but limited enforcement of private international contracts. If a private agent defaults on a foreign debt commitment, the agent retains access to domestic financial markets. This is a weak punishment, as the agent can use other domestic agents (and their continued access to foreign financial markets) to insure its endowment shocks, and in this sense the incentives to deviate are greater for an individual agent than they are for a benevolent government that is punished by aggregate financial autarky. Jeske (2006) shows that the undistorted competitive equilibrium is not efficient. Wright (2006) shows that a subsidy to foreign borrowing can be used to decentralize the constrained efficient allocation. The need for a subsidy reflects the fact that the competitive equilibrium's inefficiency stems from debt constraints that are too tight. By making repayment less burdensome, a subsidy to foreign debt allows the individual to increase its borrowing from abroad. This, plus complete and fully enforced domestic asset markets, allows the individuals to achieve the constrained efficient allocation in a competitive equilibrium.

4. RICHER NOTIONS OF "DEFAULT"

A drawback of the benchmark model is the modeling of "default." In fact, there are two concepts in the model that could be interpreted as default. In the terminology of Grossman and Van Huyck (1988), there is excusable default as well as repudiation (or inexcusable default). Repudiation is what occurs off the equilibrium path and delivers utility $\underline{V}(s^t)$. Excusable default is when a state-contingent payment is made, which Grossman and Van Huyck (1988) interpret as a partial default and state-contingent haircut. In this section, we enrich both concepts. In regard to inexcusable default, we can observe a severing of the creditor–debtor relationship in equilibrium by introducing unobservable shocks to the outside option $\underline{V}$. Regarding the notion of excusable default and haircuts, a literature is developing on generating haircuts endogenously after a non-trivial bargaining problem. We review some of the key concepts in Section 4.2.

4.1. Equilibrium Default

In the benchmark model, insurance contracts are rich enough that there is never a severing of a risk-sharing relationship in equilibrium. In practice, we do see periods of limited access to international financial markets after a failure to make a scheduled payment. As noted above, one interpretation of the fully contingent set of contracts is a reduced form of incomplete contracts combined with state-contingent renegotiation. In the next subsection, we take up renegotiation more formally. In this subsection, we consider an environment where markets are incomplete because some shocks are not observable, which can generate an endogenous separation between creditors and debtors.

Specifically, assume that there are unobservable (to lenders) shocks that enter as arguments to the outside option $\underline{V}$. Such an environment has been explored in the corporate context by Cooley et al. (2004) and in a general setting in Hopenhayn and Werning (2008). Let s_t denote the shock to productivity as before, and introduce z_t as a shock to the deviation utility that is not observable by foreign creditors. The unobserved shock could represent the economy's vulnerability to direct sanctions or financial autarky (e.g., the vulnerability of the domestic banking system to sovereign default), which is known to the government but not to lenders, or the political consequences the incumbent faces in deciding whether to repay or deviate (e.g., the political consequences of the wealth transfers stemming from default). In this subsection, we use the term default to be synonymous with opting for the deviation utility $\underline{V}$ to highlight the precise notion of default we have in mind.

The environment follows that of the benchmark endowment economy. Let s_t index the endowment, which for simplicity we assume to be *iid* and drawn from a discrete set S. Let z denote an *iid* continuous random variable that is independent of s and has support Z. Let $\underline{V} : Z \times S \to \mathbb{R}$ denote the value of deviation as a function of the outside option shock and the endowment shock, which we assume is strictly increasing in both arguments. Conditional on s, the variable z indexes the government's outside option for default. Inverting this mapping, let $F_s(v) = \Pr\{\underline{V}(z, s) \leq v|s\}$ denote the probability that the realized z is such that $\underline{V}(z, s) \leq v$ conditional on s, and let $f_s(v) = F_s'(v)$. This problem is relatively tractable given that z affects the outside option only and therefore is not something that can be credibly revealed absent separation. The problem of unobserved shocks that directly influence payoffs within the creditor–debtor relationship is treated, for example, by Atkeson (1991), Tsyrennikov (2012), and Dovis (2012) in the sovereign debt context, and by Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007), and Quadrini (2004) in the corporate bond context.

We write the problem recursively, letting the utility of the representative foreign creditor be the objective function and the utility of the debtor be the state variable. In particular, let $B(v)$ denote the expected net present value of payments to the creditor(s) conditional on the debtor enjoying utility greater than or equal to v, prior to the realization of today's shocks. This "inverts" the government's strictly decreasing value function that maps promised debt payments into discounted utility. We assume that $\beta R = 1$. The timing of default is that the decision is made after observing s and z, but before receiving transfers from financial markets:

$$B(v) = \max_{\{v(s),c(s),w(s)\}} \sum_s \pi(s) F_s(v(s)) \Big(y(s) - c(s) + \beta B(w(s)) \Big)$$

subject to:

$$\sum_s \pi(s) \left(F_s(v(s))v(s) + \int_{v(s)}^{\infty} \tilde{v} dF_s(\tilde{v}) \right) = v, \tag{9}$$

$$u(c(s)) + \beta w(s) = v(s). \tag{10}$$

Equation (9) is a "promise keeping" constraint that ensures that the debtor receives v in expectation. Equation (10) reflects that we have broken up the problem into choosing state-contingent utility $v(s)$, and then allocating that utility over consumption today $c(s)$ and future promises $w(s)$ for notational convenience. Note the creditor receives net payments $y(s) - c(s)$ only if the government decides not to default in that state, and receives zero otherwise. Therefore payments are discounted by $F_s(v(s))$ in each state. Note also that the concavity of the problem is not guaranteed without suitable assumptions on $F_s(v)$.

Let us assume the support Z is such that $\underline{V}(z, s) \leq u(y(s)) + \beta \sum_{s'} \pi(s')u(y(s'))/(1-\beta)$ for all z, s.[14] Under this assumption, in an efficient allocation, if $y(s) - c(s) + \beta B(w(s)) \leq 0$, then $F_s(v(s)) = 1$: that is, if the country in state s expects to receive a positive net present value from abroad, then it will not default, independent of the realization of the shock z.

We proceed now under the premise that $B(v)$ is differentiable and the optimum is characterized by first-order conditions. Let μ denote the multiplier on the promise-keeping constraint and $\pi(s)\gamma(s)$ on the $v(s)$ constraints.[15] The first-order conditions for

[14] This assumption can be interpreted as stating that the variation in the outside option is due to a random non-negative cost of default in addition to financial autarky.

[15] In the present context, promise keeping can be written as an inequality constraint, as the creditor can always choose to deliver more without violating the other aspects of the problem. This ensures the multipliers are non-negative.

$v(s)$, $c(s)$, and $w(s)$, considering the cases where $F_s(v(s)) \in (0, 1)$, are:

$$f_s(v(s))\Big(y(s) - c(s) + \beta B(w(s))\Big) = -F_s(v(s))\mu + \gamma(s)$$
$$F_s(v(s)) = u'(c(s))\gamma(s)$$
$$-F_s(v(s))B'(w(s)) = \gamma(s).$$

The envelope condition is $B'(v) = -\mu$. Combining conditions and rearranging gives:

$$\frac{1}{u'(c(s^{t-1}, s))} - \frac{1}{u'(c(s^{t-1}))} = \frac{f_s(v(s^{t-1}, s))}{F_s(v(s^{t-1}, s))}\Big(y(s^{t-1}, s) - c(s^{t-1}, s) + \beta B(w(s^{t-1}, s))\Big), \tag{11}$$

for all states s and all histories s^{t-1}. In the equation above, we have used the condition that $-B'(w(s)) = 1/u'(c(s))$ in period t and $t-1$. The left-hand side is the distortion to consumption smoothing; absent distortions from limited commitment, this should be zero as $\beta R = 1$. On the right-hand side is the benefit from reducing the probability of default at the margin. The first term is the elasticity of the probability of default with respect to promised utility, and this is multiplied by discounted net payments promised to creditors. In states in which this term is positive (that is, states s in which the country is a debtor and defaults with positive probability), there is an incentive to have $c(s^{t-1}, s) > c(s^{t-1})$. That is, there is an incentive to distort consumption to reduce default at the margin, and in particular to shift consumption toward states when the probability of default is particularly elastic and net repayments are particularly large. Note that in states where the country is a saver, then, as discussed above, the country does not default for any z, and thus consumption is constant: $c(s^{t-1}, s) = c(s^{t-1})$. The resulting pattern of increasing consumption is similar to the benchmark model: the economy has an incentive to pay down its debt and increase consumption over time. Moreover, the option to default distorts risk-sharing across endowment states. In contrast to the benchmark model, there will be default with positive probability and the probability of default is greater when debt is high. While under suitable assumptions it is the case that default is more likely when $y(s)$ is particularly low,[16] the independent shock z weakens the correlation between output and default, consistent with the empirical facts that default is more likely in recessions, but occurs with some probability in booms as well.

4.2. Renegotiation

One of the drawbacks of the benchmark model is the treatment of the default *process*. As noted above, several papers have motivated a complete-markets asset structure with

[16] For example, assume $\underline{V}(z, s)$ depends only on z and $f(v)/F(v)$ is weakly decreasing in v. Assuming $B(v)$ is concave, differentiation of (11) implies that $v(s)$ is increasing in $y(s)$, or that $F(v(s))$ is decreasing in $y(s)$. The assumption that the outside option is independent of $y(s)$ is extreme, but the intuition that imperfect risk-sharing combined with incomplete markets generates default in bad states will reappear in the quantitative models discussed in Section 6.

default and partial repayment, as in the papers of Grossman and Van Huyck (1988) and Bulow and Rogoff (1989a). While useful as foundations for rich risk-sharing possibilities implemented with non-contingent contracts, they do not speak to the delays observed in actual default episodes. The bargaining models of Fernandez and Rosenthal (1990) and Yue (2010), while generating many useful insights in regard to risk sharing and debt dynamics, have limited implications for endogenous delay in equilibrium. As noted in the empirical review (Section 2), the average default episode lasts eight years, calling for a richer model of debt renegotiation.

We briefly review some of the recent contributions in regard to sovereign debt renegotiation. The models of Bi (2008a) and Benjamin and Wright (2008) share the emphasis on limited commitment that we emphasized in the benchmark model, but also emphasize the role of incomplete markets. In their framework, the creditor and debtor bargain over the surplus of the relationship. As in our benchmark environment, limited commitment prevents the debtor from fully pledging future income. However, in the benchmark, the debtor could make state-contingent promises that were self enforcing, allowing it to credibly pledge more in high states. In a state-contingent world, there is no incentive to delay negotiation until after output shocks are realized, as all payments can be made contingent on the histories of shocks. In the absence of state-contingent assets, the debtor can only pledge a non-contingent amount. In an incomplete-markets environment, there is therefore an incentive to delay. In particular, the parties would like to delay until expectations of future output are high in order to credibly pledge these endowment streams.[17,18]

Sovereign debt typically involves multiple creditors, and particularly so when debt takes the form of bonds rather than bank loans, which is the recent trend. Renegotiation of debt therefore requires some level of coordination among creditors. The difficulty involved with coordination may raise the costs of renegotiation and prolong debt restructuring. Pitchford and Wright (2012) show that there is an incentive to hold out in debt negotiations, as the last to agree to a settlement has disproportionately large bargaining power due to its ability to veto the entire settlement. Limited commitment plays a role here as well, as the borrower cannot commit to treat hold outs more harshly than those that settle early. Pitchford and Wright (2012) argue that this incentive to hold out can create delays in debt restructuring. One proposed solution to such hold outs is collective-action

[17] Both Bi (2008a) and Benjamin and Wright (2008) have a rich model of the bargaining process, including stochastic variation in bargaining power, and both papers include a quantitative evaluation of the respective model's ability to match key empirical facts. The Benjamin and Wright (2008) model also predicts the evolution of debt during the restructuring process.

[18] An additional explanation for why debt renegotiations or default durations are prolonged is that the parties have asymmetric information. Two recent papers in this regard are D'Erasmo (2011) and Bai and Zhang (2012). Cole et al. (1995) present a model in which the government's type is not observable and varies over time; to signal a switch to a relatively patient government, a payment is made and the country exits default. A similar spirit but a different application than renegotiation underlies Sandleris (2008), in which the decision not to default is a signal regarding the government's private information on the health of the economy.

clauses (CAC's), in which restructuring can be implemented by a sub-set of bondholders (usually a super-majority). Pitchford and Wright (2012) show somewhat paradoxically that this may serve to increase delay, as negotiation is costly and with only a subset of bondholders required for settlement, there is an incentive to free ride on negotiation costs.

Bolton and Jeanne (2007) and Bolton and Jeanne (2009) analyze CAC's with an eye toward ex ante efficiency. Making settlement more costly raises ex post restructuring costs, but may ease ex ante incentive problems. In particular, difficult restructuring may induce a borrower to repay when it otherwise would default. Thus an ex post inefficient restructuring process may ease the fundamental inefficiency due to limited commitment. However, Bolton and Jeanne show that this mechanism must be used with care as it may be taken one step too far. In particular, there is an individual incentive for one or a group of creditors to make their bonds relatively hard to restructure; this is particularly relevant in a dynamic setting as the government is tempted to dilute existing bondholders by issuing harder-to-restructure new bonds. This could make some bonds de facto senior to other issues, as bonds that are more difficult to restructure have a greater chance for repayment. In equilibrium, this externality generates bond issues that are excessively difficult to renegotiate, potentially lowering ex ante welfare.

5. SELF-FULFILLING DEBT CRISES

In the benchmark model of Section 3 we characterized efficient equilibria; that is, we solved a planning problem subject to a break-even constraint for creditors and capital owners. However, that model often admits other equilibria which are not on the constrained Pareto frontier. The multiplicity of equilibria raises the possibility of self-fulfilling debt crises in which agents "switch" to a Pareto-inferior equilibrium.

We illustrate the possibility of self-fulfilling crises in a simple, two-period model and then discuss extensions to a dynamic setting. Consider a small open economy with constant tradable output endowment y. There are two periods $t = 1, 2$, with preferences given by $u(c_1)+\beta u(c_2)$. Let $\beta = R^{-1}$, so that the government discounts at the world interest rate.

The country begins period one with outstanding liabilities b. The timing is as follows[19]: the government begins the first period by issuing new bonds b' to competitive bondholders with discount factor $\beta = R^{-1}$, given an equilibrium price schedule $q : \mathbb{R} \to \mathbb{R}_+$. While the government takes the schedule q as given, it chooses new debt issuances b' internalizing the shape of the price schedule. After selling the newly issued bonds, the

[19] We have followed the timing introduced by Cole and Kehoe (2000), which is different from the timing usually assumed in the quantitative literature discussed in Section 6. In Cole and Kehoe, the price schedule is offered before the government makes its default decision, while the typical assumption in the quantitative literature is that the price schedule is offered after the government makes its within-period default decision. The key distinction is whether the price schedule (that is, the price as a function of newly issued debt, b') is conditional or unconditional on non-default within the period.

country decides to repay or not the legacy debt b. Failure to pay the legacy debt sends the country into financial autarky, where it faces a reduction in endowment of amount τ every period. In particular, default in period one yields value:

$$\underline{V}(q(b')b') = u(y - \tau + q(b')b') + \beta u(y - \tau).$$

Note that the timing assumption implies that the government retains the money raised by new bond issues $q(b')b'$ whether or not it defaults in the first period. In the second period, if the government has previously defaulted it simply consumes $y - \tau$. If the country did not default in the first period, it decides whether to repay b'; that is, it chooses the maximum of repayment consumption $y - b'$ and default consumption $y - \tau$.

To summarize the optimal default decision let $D_2 : \mathbb{R} \to \{0, 1\}$ be the policy function for default in period 2. Specifically, $D_2(b') = 1$ if $b' > \tau$, and equals 0 otherwise. Let $D_1 : \mathbb{R} \times \mathbb{R} \times Q \to \{0, 1\}$ be the policy function for default in period 1 conditional on new debt b', legacy debt b, and the equilibrium price schedule $q \in Q$, where Q is the set of functions mapping $\mathbb{R}$ into $\mathbb{R}_+$[20]:

$$D_1(b'; b, q) = \begin{cases} 1 \text{ if } u(y + q(b')b' - b) + \beta u\left(y - (1 - D_2(b'))b' - D_2(b')\tau\right) < \underline{V}(q(b')b') \\ 0 \text{ otherwise.} \end{cases}$$

We define equilibrium as follows:

Definition 2. An equilibrium given initial debt b is a price schedule, q, default policy functions D_1 and D_2, and a debt choice b^* such that:

(i) Given b and q, D_1 and D_2 are defined by the government's optimal default decision for each b';

(ii) Given b and q and the optimal default policies, the government chooses new debt optimally:

$$b^* = \underset{b'}{\operatorname{argmax}} \left\{(1 - D_1(b'; b, q))u(y + q(b')b' - b) + D_1(b'; b, q)u(y + q(b')b' - \tau)\right.$$
$$\left. + \beta(1 - D_1(b'; b, q)D_2(b'))u(y - b') + \beta D_1(b'; q, b)D_2(b')u(y - \tau)\right\};$$

(iii) Given D_1 and D_2, q satisfies $q(b') = \beta(1 - D(b'; b, q))$ for all b', where $D \equiv D_1 \times D_2$.

The first condition of equilibrium states that the government makes its default decision in order to maximize utility, whether default occurs in the first period or second, and this decision is made after new debt issuances b'. The second condition states that the government chooses new debt optimally. Note that at the time it issues new debt in the

[20] To streamline the exposition, we do not formally limit the debt domain to ensure consumption is non-negative (that is, $b' \leq y$ and $b - q(b')b' \leq y$). It should be understood that default will be chosen over negative consumption. This decision may be relevant in equilibrium if $b > y$ and the government cannot issue new debt at a positive price, making default its only feasible option.

first period, the government cannot commit to a default decision for that period, and so recognizes that default will be chosen optimally ex post. The third condition states that investors break even for any bond issuance b'. The fact that condition three holds for all b' implies equilibrium satisfies a perfection requirement; that is, even if the government chose a sub-optimal level of debt, investors break even.

The following proposition states that the model can have multiple equilibria:

Proposition 1 (Self-Fulfilling Debt Crises). *Let $\underline{b}$ be the unique value such that $u(y - \underline{b}) + \beta u(y) = (1 + \beta)u(y - \tau)$. Let $\overline{b}$ be the unique value such that*

$$(1+\beta)u\left(y - \frac{\overline{b}}{1+\beta}\right) = u\left(y - \tau + \frac{\beta\overline{b}}{1+\beta}\right) + \beta u(y - \tau).$$

If $b \in (\underline{b}, \overline{b}]$, then there exists at least two equilibria, one of them with $q(b') = 0$ for all b', and another one with an equilibrium schedule such that $q(b/(1 + \beta)) = \beta$.

This proposition captures the concept of a self-fulfilling debt crisis. A price schedule $q(b') = 0$ for all b' is one in which lenders are unwilling to purchase bonds of any amount. That is, the government is unable to issue new bonds and existing bond holders demand immediate repayment. For $b > \underline{b}$, we have that $u(y - b) + \beta u(y) < (1 + \beta)u(y - \tau)$, and so the government's best response to such a roll-over crisis is to default (for any b'), and this confirms the zero price. On the other hand, there is an equilibrium which supports positive lending up to $\overline{b}$. In particular, if lenders were willing to buy new bonds and $b \leq \overline{b} < \tau(1 + \beta)$, then the government would prefer to issue new bonds and then repay outstanding debt as long as it can issue $b' = b/(1 + \beta)$ at the price $q(b') = \beta$ and outstanding debt is below $\overline{b}$. Issuing $b' = b/(1 + \beta)$ at a price of β implements the full-commitment solution and delivers a utility that favors repayment over default. Hence, the solution in part (iii) would be $b^\star = b/(1 + \beta)$. The price schedule q can be extended to off-equilibrium debt issuances such that equilibrium conditions (i) and (ii) hold.

In this manner, we can construct two equilibria with distinct price schedules and that generate distinct equilibrium allocations, as long as initial debt $b \in (\underline{b}, \overline{b}]$. In particular, one equilibrium features an inability to roll-over debt and an immediate default, while the other features the ability to issue new bonds and avoid default. In this range of initial debt, the government is vulnerable to self-fulfilling expectations about its willingness to repay. Proposition 1 does not guarantee that $\underline{b} < \overline{b}$, so the relevant region for multiple equilibria may not always exist. However, there exists a $\tau^* < y$ such that if $\tau \in (\tau^*, y)$, then $\underline{b} < \overline{b}$.[21]

An important point is that while the government is vulnerable for $b > \underline{b}$, a price schedule of zero is not an equilibrium if $b \leq \underline{b}$. That is, if initial debt is low enough, the government is not subject to self-fulfilling crises. While b is a primitive in the two-period

[21] To see this, note that $\overline{b} < \tau(1 + \beta)$ and that $\overline{b}$ is increasing in $\tau \in [0, y)$, which can be seen from the definition of $\overline{b}$. Note also that $\underline{b} \leq y$. Define τ^* so that $\underline{b} = y$, which from the definition of $\underline{b}$ must be strictly less than y. Therefore, for $\tau \in (\tau^*, y)$ we have $\overline{b} > y > \underline{b}$.

model, in a fully dynamic model debt levels are endogenous state variables. Cole and Kehoe (2000) build a dynamic equilibrium model that embeds the above analysis. One important modeling device is that as long as the government's liability position lies in the range where a self-fulling crisis is possible, then there exists a constant and strictly positive probability that such a crisis occurs. A main result of their analysis is that the government has an incentive to save its way out of the crisis zone $(\underline{b}, \bar{b}]$. That is, the government responds to the vulnerability to self-fulfilling debt crises by reducing its debt.[22]

A number of extensions to the Cole and Kehoe model have been made in the recent literature. Conesa and Kehoe (2011) allow shocks to income. In particular, they suppose the economy is in recession and faces a constant hazard of recovery. In this case, saving exacerbates the consumption impact of the recession and the government may opt to remain in the crisis zone hoping that a recovery occurs before a debt crisis. They refer to this as "gambling for redemption."

A second extension concerns nominal bonds. The option to inflate away the real value of nominal debt provides an alternative to outright default in the event of a debt crisis. Aguiar et al. (2012) show that issuing nominal bonds has an ambiguous effect on vulnerability to a self-fulfilling debt crisis. Specifically, if the government's commitment to low inflation is high absent a crisis, then nominal bonds have a desirable state-contingent feature; in good times, the real return is high, while in the event of a crisis, the government inflates away part of the real value of the bonds. As creditors prefer partial repayment to outright default, the ability to respond with inflation generates a superior outcome to real bonds. However, if the commitment to low inflation is weak even in good times, the government loses the state-contingency potentially allowed by nominal bonds. In particular, the government has a temptation to inflate ex post even in normal times, and this will be reflected in lower bond prices (or higher interest rates) ex ante, making repayment that much more burdensome. This effect may be large enough to dominate, generating a larger crisis zone for nominal bonds. Aguiar et al. (2012) use this fact to rationalize why many emerging markets with weak inflationary regimes issue bonds in foreign currency, while economies like the U.S., the U.K., and Japan issue large amounts of domestic currency bonds at low nominal interest rates and seemingly without risk of self-fulfilling crises.

Chatterjee and Eyigungor (2012b) quantitatively explore the benefit of long-maturity bonds in an environment prone to self-fulfilling crises. The paper contrasts the temptation to dilute existing bond holders in an incomplete-markets setting (a point discussed in detail in the next section) with the protection long-maturity bonds provide from rollover crises. The calibrated model of Chatterjee and Eyigungor (2012b) indicates that even a small likelihood of a rollover crisis implies that a country would seek to limit its short-term debt to the point where the probability of a rollover crisis is endogenously reduced

[22] The one caveat to this result is if initial debt is so large that the transition to $\underline{b}$ may be long enough that the government is better off remaining in the crisis zone indefinitely.

to a small value. Moreover, the use of long-maturity bonds, despite the costs rising from debt dilution, reduces the reliance of debt reduction as the optimal response to potential rollover crises.

6. INCOMPLETE-MARKET MODELS AND THEIR QUANTITATIVE IMPLEMENTATION

In this section, we discuss how well models of sovereign debt perform quantitatively as well as some additional conceptual issues that arise in models of incomplete markets. The primary paradigm for quantitative analysis is the model of Eaton and Gersovitz (1982). In particular, a small open-economy trades a non-contingent bond in order to insure itself against endowment or productivity shocks. The only state contingency spanned by the asset markets is through the option to default.

We first introduce a simple version of the model and then discuss some of the extensions in the literature. The model follows early quantitative versions of the Eaton-Gersovitz model explored by Aguiar and Gopinath (2006), Arellano (2008), and Hamann (2002).

Consider a small open economy that has a stochastic endowment stream y_t, which follows a Markov process. The government has preferences given by (1) and trades a one-period bond with risk neutral investors. Time is discrete and the timing within a period is as follows. At the start of period t, the government has outstanding liabilities b_t. It observes the endowment shock y_t and then decides whether to repay b_t or default. If it defaults, it enjoys deviation utility $\underline{V}(y_t)$ to be defined below. If it repays, it issues new bonds b_{t+1} at price $q(b_{t+1}, y_t)$. The price of a bond is a stationary function of outstanding debt and the current endowment shock. As discussed in Section 5, the literature has different timing conventions regarding whether new bonds are auctioned before or after the current period default decision is made. The standard assumption in the quantitative debt literature is that the bond price schedule is contingent on no default in the current period.

Let $V^{ND}(b, y)$ denote the value of choosing to repay its debts when it starts the period with an amount of debt b and an income y:

$$V^{ND}(b, y) = \max_{b'} \left\{ u(y + q(b')b' - b) + \beta \mathbb{E}[V(b', y')|y] \right\} \tag{12}$$

and $b'(b, y)$ denotes an associated optimal debt-issuance policy.

Let $\underline{V}(y)$ be the value of default, conditional of the current income value y:

$$\underline{V}(y) = u((1 - \tau)y) + (1 - \lambda)\beta \mathbb{E}[\underline{V}(y')|y] + \lambda \beta \mathbb{E}[V(0, y')|y]. \tag{13}$$

Then, $V(b, y)$, the government's value function at the start of the period conditional on outstanding debt due b and current endowment shock y, is the maximum of $V^{ND}(b, y)$

and the default value $\underline{V}(y)$:

$$V(b, y) = \max_{D \in [0,1]} \{(1 - D) V^{ND}(b, y) + D \times \underline{V}(y)\} \tag{14}$$

and $D(b, y)$ denotes an associated optimal default policy. Note that neither $b'(b, y)$ nor $D(b, y)$ may be uniquely defined.

A few things to note in the definition of $\underline{V}$. First, the economy is excluded from asset markets in the period of default and suffers a loss in output τy. This is designed to capture direct sanctions and other output consequences from default. In equation (13), the loss is proportional to output, which is the formulation used by Aguiar and Gopinath (2006). Arellano (2008) allows τ to vary with the level of the endowment according to a function $\tau(y)$. A second feature of equation (13) is that with probability $\lambda \in [0, 1)$, the country can regain access to financial markets, starting anew with a clean credit rating and zero debt. This is a reduced form for the default process discussed in Section 4. In the simplest quantitative models, the haircut is set at one hundred percent so the country emerges with zero debt.

A few further remarks on the direct costs τ are in order. At a conceptual level, debt can be sustained in equilibrium through financial exclusion alone, as shown by Eaton and Gersovitz (1982). As a quantitative matter, the amount of debt that can be supported by the threat of financial autarky is relatively small in this benchmark economy. This is related to the fact that in a closed-economy representative-agent model, aggregate consumption fluctuations at business cycle frequencies have relatively small welfare implications, a point made using a simple numerical example by Lucas (1987). Aguiar and Gopinath (2006) extend the Lucas example to show that financial autarky is not a harsh punishment in a small open endowment economy, and thus very little debt is sustainable in equilibrium. A second point is that the output costs of default may depend on the level of output. This is the formulation proposed by Arellano (2008). While default provides a crude form of state contingency, as described in previous sections there is a fair amount of contingency built into the renegotiation process. A simple way to incorporate this into an otherwise incomplete-markets environment is to assume that default is punished (disproportionately) less severely if it occurs in low-output states. Finally, a recent paper by Mendoza and Yue (2012) endogenizes the link between the state-contingent output costs of default and the reputational costs of financial autarky using a model of trade credit.

The break-even constraint for risk neutral lenders is

$$q(b', y) = \frac{\mathbb{E}[1 - D(b', y') | y]}{R}. \tag{15}$$

Let us define an equilibrium:

Definition 3. A recursive equilibrium is a price schedule $q(b', y)$, value functions $V(b, y)$, $V^{ND}(b, y)$, and $\underline{V}(y)$, and policy functions $b'(b, y)$ and $D(b, y)$ such that: (i) the government optimizes given the price schedule, that is $\underline{V}(y)$, $V(b, y)$, $V^{ND}(b, y)$ solve equations (12), (13), and (14) and b' and D are the resulting policy functions; and (ii) creditors earn R in expectation given the government's equilibrium policy functions, that is, equation (15) holds.

The quantitative literature typically computes an equilibrium as follows. For clarity, assume that $\lambda = 0$, so that default leads to permanent financial autarky. Note that the value function $V^{ND}(b, y)$ depends on the bond-price schedule q via the budget constraint. In fact, V^{ND} is weakly increasing in q for all b and y. When $\lambda = 0$, $\underline{V}(y)$ is independent of q. Therefore, $V^{ND}(b, y) - \underline{V}$ is weakly increasing in q. We can construct an equilibrium by iterating on the following operator. Let $B = [\underline{b}, \overline{b}]$ denote the state space of debt and Y the state space of y. Define the operator T on the space of functions that map $B \times Y \to [0, R^{-1}]$ by:

$$Tq = \frac{\mathbb{E}_y \left(1 - \chi_{\{V^{ND}(b,y;q) < \underline{V}(y)\}}\right)}{R}, \tag{16}$$

where we make the dependence of V^{ND} on q explicit. As the indicator function $\chi_{\{V^{ND}(b,y;q)<\underline{V}(y)\}}$ is weakly decreasing in q, if $q \geq q'$ then $Tq \geq Tq'$. Thus the operator is monotone. As $q \leq \frac{1}{1+r^\star}$, the typical algorithm starts with this upper bound and iterates on the operator until convergence to a fixed point.[23] If $\lambda > 0$, then the analysis is complicated by the fact that $\underline{V}(b)$ depends on q through the re-entry value function V. As a computational matter, the algorithm can still be used to search for a solution, although the monotonicity of the operator is not guaranteed.

While straightforward to compute, the model yields few analytical insights without further restrictions. The value function V is the maximum of two other value functions and in general is not concave or differentiable. However, some intuition for the quantitative results can be obtained by stripping the model down to a two-period decision problem. In particular, assume the country owes b and has endowment y in the current period, and chooses b' to be repaid in the second period. Let $F(y')$ denote the cdf of next period's endowment conditional on y, which takes values in $[\underline{y}, \overline{y}]$. In the second period, the government can repay the debt and enjoy utility $u(y' - b')$, or default and enjoy utility $u((1-\tau)y'))$. The country will default if $b' > \tau y$.

The government's problem in the first period is:

$$\max_{b'} u(y + q(b')b' - b) + \beta \int_{\underline{y}}^{\frac{b'}{\tau}} u((1-\tau)y')dF(y') + \beta \int_{\frac{b'}{\tau}}^{\overline{y}} u(y' - b')dF(y'). \tag{17}$$

The first-order condition is:

$$u'(c)\left(q(b') + q'(b')b'\right) = \beta \int_{\frac{b'}{\tau}}^{\overline{y}} u'(y' - b')dF(y'), \tag{18}$$

[23] For more on computational algorithms, see Hatchondo et al. (2010).

where $c = y + q(b')b' - b$. The default decision implies an equilibrium price schedule:

$$q(b') = \frac{1 - F\left(\frac{b'}{\tau}\right)}{R}. \tag{19}$$

Using this and rearranging the government's first-order condition gives:

$$u'(c)\left(1 + \frac{q'(b')b'}{q(b')}\right) = \beta R \mathbb{E}\left\{u'(y' - b')dF(y')|b' \leq \tau y\right\}, \tag{20}$$

where the expectation is conditional on not defaulting. The right-hand side is the expected marginal cost of repaying the debt conditional on repayment, discounted by relative impatience. The left-hand side is the marginal value of an additional unit of debt inclusive of the price effect; that is, the marginal utility of consumption times one plus the elasticity of the bond-price schedule with respect to new debt. The elasticity of the bond price reflects that the government internalizes the effect of new debt issues on the price it faces. This elasticity is non-positive, and the greater in magnitude the less the government is willing to borrow at the margin.

If the bond-price schedule is very elastic, the government has a strong desire to save (or not borrow). This is the same effect discussed in Section 5 to describe why the government saves its way out of the crisis zone in a model of self-fulfilling debt crises. While in the current context we consider a single equilibrium price schedule, the fact that this schedule may be elastic provides the same incentive to save.

To replicate debt levels high enough to induce frequent default in this basic framework, calibration typically involves βR significantly below one. This counters the incentive to save in response to limited commitment and generates realistic debt levels. A motivation for this assumption is that the governmental decision maker is relatively impatient due to political turnover.

Moreover, the elasticity is sensitive to the "marginal" probability of default as well as the "average." In particular, the elasticity of the price schedule can be written

$$\frac{q'(b')b'}{q(b')} = \frac{-f\left(\frac{b'}{\tau}\right)\frac{b'}{\tau}}{1 - F\left(\frac{b'}{\tau}\right)}, \tag{21}$$

where $f(y') = F'(y')$. A high probability of default implies a small denominator, reflecting a large average probability of default $F(b'/\tau)$. The numerator reflects the pdf at the debt level under consideration. If the variance of y' is relatively high (or the pdf has a fat lower tail), there is substantial mass below the key threshold, lowering the ratio of marginal to average. Therefore, volatility generates more frequent default. Aguiar and Gopinath (2006) build on the empirical work of Aguiar and Gopinath (2007) to argue that emerging markets have large shocks to the trend growth rates, which raises the probability of default in equilibrium. This rationalizes why volatility in trend growth and frequency of default occur together in an economy.

As noted above, Arellano (2008) generates plausible default probabilities by introducing state-contingent punishments; that is, having τ depend on the realization of y'. This makes default more responsive to output shocks and therefore relatively less responsive to outstanding debt, lowering the elasticity of the bond-price schedule as well as making non-contingent debt more attractive as an insurance option. Such a nonlinear output cost has been derived endogenously by Mendoza and Yue (2012).

An important property of the bond-price elasticity in the benchmark dynamic setting with persistent shocks is that it may vary with the output shock. In particular, the quantitative models of Aguiar and Gopinath (2006) and Arellano (2008) generate a counter-cyclical net export process. That is, when endowment is low and absent default, the economy is a net saver on average. This undermines risk-sharing, which in an endowment economy calls for net inflows in low-income states. The reason this occurs is that the costs of borrowing on average (q) and at the margin ($|q'|$) are counter-cyclical, providing a relatively strong incentive to reduce debt in low-endowment states.

Note that the above example implied default in the final period if $b' > \tau y$; that is, the government defaults in low endowment states conditional on debt. It may be tempting to conclude that the fully dynamic quantitative models generate default in bad states solely due to the fact that the output cost is increasing in endowment. Although this is true in the final-period default decision in our example (and is an important contributing mechanism in the quantitative models), the loss of access to credit markets is also relevant in dynamic quantitative models. In particular, concavity of u implies that the marginal burden of net repayment of debt is higher the smaller the current endowment, all else equal. Arellano (2008) uses this insight in an infinite horizon environment with *iid* shocks to show that if default is optimal for an endowment y, it will also be optimal for $y' < y$, holding debt constant. Note that default occurs only if the country cannot generate a net inflow by borrowing (otherwise, it could consume more by borrowing, and then default the next period), and it is the burden of net repayment that is more costly in low endowment states. The fact that a country may be required to make a net payment even in the lowest endowment states reflects the incompleteness of the asset markets. Conversely, for a high-endowment shock, the desire to smooth consumption by paying down debt (or increasing assets) makes default less attractive, as default prevents carrying the high endowment into the future via asset markets. The implications under persistent shocks are muddied by the fact that the endowment realization also influences the bond prices going forward; nevertheless, in the calibrated models popular in the literature, the consistent prediction is that conditional on b there is a threshold endowment above which the country repays and below which the country defaults.

While the calibrated models of Aguiar and Gopinath (2006) and Arellano (2008) provide quantitative insights regarding which economies may be prone to default and why, the simplicity of the models cannot address many of the facts discussed in Section 2. Filling these gaps is an active area of current research. We briefly summarize several innovations. Yue (2010) quantifies a model of one-shot renegotiation, capturing the fact that default begins a bargaining process that leads to partial repayment. Benjamin and Wright (2008) calibrate the dynamic bargaining model discussed in Section 4, matching the key empirical relationships between length of default and subsequent repayment.

The single shock model cannot explain why default may occur in relatively good times (*conditional* on debt levels), as it sometimes does in practice. Cuadra and Sapriza (2008) and Hatchondo et al. (2009) introduce political uncertainty with heterogeneous potential incumbents as an additional source of volatility, similar in spirit to the model of Cole et al. (1995). A hybrid of the Cole et al. model and the Grossman and Van Huyck (1988) complete-markets model is quantified in Alfaro and Kanczuk (2005). The additional shocks, particular to the discount factor, generate a more empirically plausible level of debt and default, highlighting the tension between political economy distortions and the incentive to save in response to limited commitment.

A series of recent papers have relaxed the one-period bond assumption by considering instead bonds with longer maturities, and have shown that the introduction of longer maturities significantly improves the quantitative properties of the models. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012b) replace single-period bonds with bonds of longer duration in a tractable manner.[24] In particular, Hatchondo and Martinez (2009) assume bonds pay a geometrically declining sequence of coupons $(1, \delta, \delta^2, \ldots)$ indefinitely, while Chatterjee and Eyigungor (2012b) adapt a "perpetual youth" framework to bonds, assuming that a bond matures next period with constant hazard $\lambda = 1 - \delta$. A law of large numbers assumption generates a predictable stream of aggregate payments given the stock of outstanding bonds. The parameters δ and λ are primitives of the environment and do not vary over time. Either modeling approach renders bond maturity a stationary variable, allowing for longer durations without having bonds issued in different periods carrying heterogeneous maturities.

The equilibrium bond-price schedule in this framework is no longer characterized by the simple operator (16). In particular, the return to a bond depends on whether the government defaults, as in (16), but in addition depends on the price of bonds next period absent default. This reflects the capital gain or loss a bond holder experiences over the life span of the bond. Chatterjee and Eyigungor (2012b) discuss issues related to existence and computability of an equilibrium in this environment.[25]

A key element of the long-maturity framework is that non-maturing bonds that were issued in previous periods are subject to dilution. That is, existing bond holders will take a capital loss on their bonds if the government's subsequent decisions raise the probability

[24] For an earlier quantitative model of maturities see Bi (2008b).

[25] For an alternative approach at dealing with the computational issues that arise in sovereign debt models, see Pouzo and Presno (2012).

of future default. The government lacks commitment regarding future bond issuance and bond holders are assumed to lack a mechanism with which to punish dilutions; in particular, the models do not consider trigger strategies or direct sanctions regarding bond dilutions, distinguishing them from outright default. Moreover, mirroring actual practice, bonds issued at an earlier period are not senior to subsequently issued bonds. That is, otherwise identical bonds issued at different dates have the same legal standing in the event of default, reflecting the *pari passu* clause that is standard in sovereign bond contracts.[26]

As a capital loss for existing bond holders is an implicit transfer to the government (abstracting from any deadweight loss of default costs), there is an incentive for the government to issue new bonds to dilute the existing bonds. This point was highlighted by Bulow and Rogoff (1991) in reverse; namely, Bulow and Rogoff argued it is sub-optimal for a sovereign to repurchase its own debt on secondary markets. A repurchase generates a capital gain for existing bond holders, which is an implicit transfer from the government to bond holders. This insight has gained renewed attention as the Greek government bought back outstanding debt at a premium (relative to initial market prices) in 2012.

Long-term bonds and the associated movements in price over the life of the bond implies the bonds have different hedging properties than short-term debt. Arellano and Ramanarayanan (2012) propose a quantitative model with both short-term and long-term debt. Having both bonds available allows richer insurance possibilities than in a one-bond model. Moreover, a portfolio of bonds of different maturities allows a richer enforcement mechanism. Recall that the benchmark model of Section 3 allowed punishment of any deviation from the equilibrium allocation. Deviations (depending on the decentralization) could involve failure to pay debt, as in the current environment, but it could also take the form of not "saving" in anticipation of future liabilities that may come due. In the current incomplete-markets environment, the government is only punished if it fails to make a debt payment. A combination of short-term and long-term debt can replicate a pattern of near-term and long-term payments to financial markets, better mimicking the complete-markets allocation.[27]

Arellano and Ramanarayanan (2012) exploit the spanning and incentive-provision possibilities of multiple-maturity bonds to rationalize the fact that countries shorten maturities when a debt crisis is likely. The advantages of short-term debt are greatest when default is likely in the short-run, generating an increase reliance on short-term debt for new issues, consistent with the empirical evidence of Broner et al. (2013) and Arellano and Ramanarayanan (2012). Broner et al. (2013) take a different view,

[26] Chatterjee and Eyigungor (2012a) explore the benefits of allowing seniority to be enforced in the environment of Chatterjee and Eyigungor (2012b), documenting that enforceable seniority would substantially reduce default and increase equilibrium borrowing and welfare.

[27] Alfaro and Kanczuk (2009) discuss a similar point while introducing reserves into the model and studying the resulting portfolio problem. Interestingly, Alfaro and Kanczuk (2009) found that the standard quantitative model with reserves could not account for the significant amount of reserves that countries hold in the data. More recently, Bianchi et al. (2012) have shown that introducing long-maturity bonds also helps improve the ability of the model in this dimension.

arguing that this pattern reflects changing risk premia on the part of the lenders, rather than hedging motives on the part of the borrower. Both of these papers highlight the result that *temporary* increases in the probability of default during a time of crisis can account for the inversion of the yield curve observed in the data.

Quantitative models of sovereign debt is an active area of research and has already generated a number of important insights. We conclude with a few caveats on this literature, which can also serve as an indicator of where future research is warranted. The quantitative models often lack microfoundations for key assumptions. In particular, the limitations on financial contracts are typically taken as primitives. There is a parallel literature on optimal contracting subject to frictions such as limited enforcement and asymmetric information, as discussed in previous sections. These models are often qualitative, and have proven difficult to map into the data and quantify, although some attempts have been made (for example, Tsyrennikov, 2012). Bridging this gap is an important open question in the sovereign debt literature. Secondly, while the quantitative models are designed with empirical targets in mind, it is often not clear how to interpret the data through the lens of the model. For example, Dias et al. (2011) highlight that the data on debt stocks is a mixture of face values and market values, with different maturities, durations, and coupon payments, that are difficult to aggregate into a parsimonious set of state variables that appear in a quantitative model. More generally, when comparing the quantitative predictions of sovereign debt models to the data, it is important that we are comparing conceptually similar objects.

7. CONCLUDING REMARKS

This chapter has provided an overview of the economics of sovereign debt, with a particular emphasis on the implications of limited enforcement that distinguishes this market. The set of models reviewed offer important insights into a variety of phenomena, including: the role of reputations versus legal enforcement mechanisms; the implication that limited commitment generates debt overhang onto macroeconomic outcomes such as investment, growth, and volatility; the often slow process of graduation to non-frequent-defaulter status and the associated role played by debt overhang and political economy frictions; the possibility of unobserved or unverifiable shocks in limiting risk-sharing; the vulnerability to self-fulfilling debt crises; the difficulties in renegotiating debt in a timely and efficient manner; and the ability of theoretical models to quantitatively match key empirical patterns. The recent literature has generated a substantial number of important insights along these dimensions. That said, more progress is needed on mapping the theoretical models to the data. This includes exploring decentralizations that rely on realistic assets, legal mechanisms, and reputational concerns, combined with a coherent theory of equilibrium selection as many models in the literature support multiple equilibria. Any such decentralization has implications for prices that can be compared to the growing empirical literature on bond spreads. This process will provide the microfoundations for the growing quantitative literature that has begun to match empirical prices and quantities, but often relying on ad hoc assumptions that restrict equilibrium objects such as financial contracts and the output costs of default. As noted at the end of the previous section, the counterpart to this agenda of bringing theory to data is ensuring the measured quantities from the data are the conceptually appropriate counterparts to the model's equilibrium objects.

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