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COMMITMENT VS. FLEXIBILITY

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We study the optimal trade-off between commitment and flexibility in a consumption–savings model. Individuals expect to receive relevant information regarding tastes and thus they value the flexibility provided by larger choice sets. On the other hand, they also expect to suffer from temptation, with or without self-control, and thus they value the commitment afforded by smaller choice sets. The optimal commitment problem we study is to find the best subset of the individual's budget set. This problem leads to a principal–agent formulation. We find that imposing a minimum level of savings is always a feature of the solution. Necessary and sufficient conditions are derived for minimum-savings policies to completely characterize the solution. We also discuss other applications, such as the design of fiscal constitutions, the problem faced by a paternalist, and externalities.

KEYWORDS: Intertemporal preferences, commitment, flexibility, hyperbolic discounting, social security, temptation, self-control.

1. INTRODUCTION

A COMMONLY ARTICULATED JUSTIFICATION for government involvement in retirement income is the belief that an important fraction of the population saves inadequately when left to their own devices (Diamond (1977)). From a worker's perspective most pension systems, pay-as-you-go and capitalized systems alike, effectively impose a minimum-savings requirement. One purpose of this paper is to see if such minimum-savings policies are optimal in a model where agents suffer from the temptation to overconsume.

More generally, if people suffer from temptation and self-control problems, what should be done to help them? Current models that emphasize such problems lead to a simple but extreme answer: It is optimal to completely remove all future choices. In particular, in the intertemporal choice framework it is best to commit individuals to a particular consumption path, removing all future savings choices—full commitment is optimal. In these models, the desire to commit is simply overwhelming.

Eliminating all ex post choices is unlikely to be a good idea when new information regarding preferences or other variables is expected to arrive in the

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future. In these circumstances, individuals value the flexibility to act on their information. Indeed, in the absence of temptation or self-control problems maintaining all future savings choices implements the optimal allocation—full flexibility is optimal and strictly preferable to full commitment.

This paper studies the design of optimal commitment devices in situations where eliminating all choices is not necessarily optimal. We introduce a value for flexibility and study the resulting trade-off with commitment, defined as the removal of some future choices. Our model combines a preference for flexibility and a preference for commitment by introducing taste shocks into both a time-inconsistent quasi-hyperbolic discounting framework (Phelps and Pollack (1968), Laibson (1997)) and the temptation and self-control model of Gul and Pesendorfer (2001). The resulting preferences belong to a class introduced by Dekel, Lipman, and Rustichini (2001).

The individual we model suffers from temptation for higher present consumption. Each period a taste shock is realized that affects the individual's desire for current versus future consumption.[2] Importantly, taste shocks are privately observed by the individual. If, instead, taste shocks were observable and verifiable by an outside party, one could simply contract upon them in a way that avoids all temptation and achieves the unconstrained ex ante optimum. However, when the shocks are private information, only the agent can act upon them, introducing a trade-off between commitment and flexibility. Commitment is valued because it reduces temptation; flexibility is valued because it allows the use of valuable private information.

The optimal commitment problem we study selects a subset of the individual's budget set to maximize ex ante utility, taking into account the ex post temptation problem individuals experience facing that set. The commitment problem does not allow insurance or transfers across taste shocks. Although this restriction is not without loss of generality, it is a natural starting point for at least three reasons. First, it is useful to isolate the problem of commitment—defined as a reduction of choices from the individual's budget constraint—from the problem of insurance or redistribution, which is beyond the scope of this paper. Second, individuals may have access to commitment technologies, such as an illiquid asset, but not insurance contracts. Thus, it is important to understand what the ideal commitment device, not featuring insurance, looks like. Finally, the possibility of transferring resources across different types is simply absent in some reinterpretations of our model discussed in Section 5.

Commitment devices are valuable in this framework for two distinct reasons. First, by affecting the allocation toward higher savings, they counteract the overconsumption from temptation. In the time-inconsistent quasi-hyperbolic

[2]Our analysis focuses on taste shocks, but the crucial feature is the arrival of any new information relevant to the savings decision. Flexibility would also be valuable if one modeled health, employment, and income shocks. As we later show in detail, with constant absolute risk aversion preferences, a model with income shocks is isomorphic to a model with taste shocks.

model this is the only gain. In addition, in the model with costly self-control, commitment devices may reduce the self-control costs of resisting the temptation.

We set up the optimal commitment problem as a principal-agent problem, where the "principal" has the individual's ex ante preferences and the "agent" has the ex post preferences.

A very simple commitment device in this setting is a minimum-savings rule, which restricts individuals to save above some level, allowing complete flexibility otherwise. Facing such a rule, individuals with low enough taste shocks will be unconstrained, saving above the imposed minimum; those with high enough taste shocks will be constrained by the minimum-savings level, and as a result they all choose the same consumption and savings bundle.

Our main result concerns the optimality of such simple commitment devices. We provide a necessary and sufficient condition on the distribution of taste shocks for a minimum-savings rule to completely characterize the solution to the commitment problem. To establish this result it is necessary to ensure two things. First, that it is not desirable to have individuals who consume strictly below the budget line—that is, "money burning" is not optimal. Second, that the best subset of the budget line is simply all those points above some minimum-savings level.

More generally, we show that the optimal commitment device always shares a key feature with minimum-savings rules: types above some threshold have the same consumption and savings bundle. This bunching result has a strong economic intuition. If instead agents at the very top are separated, then they would surely be consuming more than what the principal would for any taste shock. Thus, at the top there is no trade-off between commitment and flexibility, and locally no flexibility is provided.

Our analysis is also useful for other applications, quite different from the consumption–savings model we focus on, and we discuss some examples. The first concerns fiscal constitutional design, where citizens value government spending, but ruling administrations value it even more. Our results translate to conditions for simple spending caps to be optimal. Second, we discuss paternalism, whereby a principal cares about an agent but has some disagreement with the agent's preferences. Our results may then be relevant for thinking about minimum-schooling laws. Finally, we discuss an environment where individuals impose consumption externalities on each other: a utilitarian planner maximizes average welfare and internalizes these externalities, but individuals acting privately do not. These examples illustrate how our results may be applicable to other situations featuring a trade-off between commitment and flexibility.

This paper relates to several choice-theoretical papers in the literature. Models with time-inconsistent preferences solved as a competitive game, as in Strotz (1956), were the first to formalize a value for commitment. In particular, the hyperbolic discounting model has proven useful for modeling the possibility of undersaving and the desirability of commitment devices

(Phelps and Pollack (1968), Laibson (1997)). In a series of recent papers, Gul and Pesendorfer (2001, 2004a, 2004b) and Dekel, Lipman, and Rustichini (2001, 2004) have provided axiomatic foundations for preferences that value commitment and have derived useful representation theorems. Kreps (1979) provided an early axiomatic foundation for a preference for flexibility and showed that these preferences can always be represented by including taste shocks in an expected-utility framework.

Our paper contributes to work on optimal social security design, especially that which incorporates a concern for possible undersaving by individuals. To the best of our knowledge, modeling the trade-off between commitment and flexibility is novel in this context. For example, Laibson (1998) studies corrective Pigouvian taxation in a deterministic representative–agent model with quasi-hyperbolic discounting. Interestingly, Krusell, Kuruşçu, and Smith (2005) show that linear taxation is not an effective instrument for resolving the temptation problems faced by agents who have some self-control. In contrast, the pure commitment mechanisms we consider here would attain the first-best allocation in these environments; this underscores the importance of modeling a desire for flexibility. On the other hand, with nondegenerate taste shocks, our focus on pure commitment mechanisms, which do not allow for transfers across types, is restrictive. A natural next step is to incorporate a value for flexibility (e.g., with taste shocks) while also allowing for transfers.

Other work features similar trade-offs between some form of commitment and flexibility. For example, following Holmström (1977, 1984) many papers have addressed the problem of managerial delegation to a biased but informed agent. Early work proceeded under various simplifying assumptions: quadratic payoff functions, a one-dimensional action to be delegated, and the delegation set restricted to be an interval; recent work has relaxed the latter assumption (Melumad and Shibano (1991), Martimort and Semenov (2004)). Also related is Athey, Atkeson, and Kehoe (2005), who emphasize a trade-off between rules and discretion in the context of a time-inconsistent benevolent government. We believe that our results and methods, which apply powerful Lagrangian optimization techniques, may prove useful for these and other applications.

The rest of the paper is organized as follows. Section 2 lays out the basic model with quasi-hyperbolic preferences. Section 3 studies optimal commitment and derives the main results. Section 4 extends the results to preferences that display temptation and self-control. We discuss other interpretations of our model and applications of our results in Section 5. The final section concludes.

2. BASIC CONSUMPTION–SAVINGS PROBLEM

In this section, we introduce the basic consumption–saving setup with time-inconsistent preferences. There are two periods and a single consumption good each period. We denote first- and second-period consumption by c and k, respectively. Given total resources y, the consumer is constrained by the budget set $B \equiv \{(c, k) \in \mathbb{R}_+^2 \mid c + k \leq y\}$, where we have normalized the net interest rate to zero.

In the first period individuals receive a taste shock θ from a bounded set Θ with distribution function $F(\theta)$, normalized so that $\mathbb{E}[\theta] = 1$. The taste shock affects the marginal utility of current consumption: higher θ makes current consumption more valuable. Taste shocks are assumed to be the individual's private information.

We follow Strotz (1956), Phelps and Pollack (1968), Laibson (1997), and others by modeling the agent in each period as different *selves* with different preferences. For the ensuing games played between selves we consider subgame perfect equilibria as our solution concept.

The utility for *self*-1 from periods $t = 1, 2$ with taste shock θ is then

$$\theta U(c) + \beta W(k),$$

where $U : \mathbb{R}_+ \to \mathbb{R}$ and $W : \mathbb{R}_+ \to \mathbb{R}$ are increasing, concave, and continuously differentiable, and $0 < \beta \leq 1$. Utility for *self*-0 from periods $t = 1, 2$ is given by

$$\mathbb{E}[\theta U(c) + W(k)].$$

This setup represents a two-period version of quasi-geometric discounting. We associate $1 - \beta$ with the strength of temptation toward present consumption.

There is *disagreement* among the different *selves* on discounting, but *agreement* regarding taste shocks. The tension is between tailoring consumption to the taste shock and *self*-1's constant desire for higher current consumption. This tension generates the trade-off between commitment and flexibility from the point of view of *self*-0. Indeed, this is the central feature of the model, which can be reinterpreted and applied to other situations with similar trade-offs (see Section 5).

The taste shock distribution can be interpreted in two ways. Under an objective interpretation, it represents the actual probability distribution over ex post ordinal preferences. Under a subjective interpretation, in contrast, the distribution encompasses both subjective probability assessments on ordinal preferences and the cardinality from state-dependent utility.

Taste shocks are a tractable way to introduce a value for flexibility, and may also capture the significant variation in consumption and savings behavior observed in the data, after conditioning on all available variables. Other shocks, such as unobservable income or health, can also generate a value for flexibility. Indeed, a model with privately observed income shocks is equivalent to a model with privately observed taste shocks when the utility function is exponential. We discuss such equivalence in Section 5.4.

A useful benchmark allocation is the ex ante *first-best allocation*, $(c^{fb}(\theta), k^{fb}(\theta))$, defined by the solution to $\max_{(c,k)\in B}\{\theta U(c) + W(k)\}$. This allocation would be feasible if taste shocks were not private information and were contractible. Another benchmark allocation is that obtained with full flexibility or no commitment: *self*-1 is constrained only by the resource constraint and solves $\max_{(c,k)\in B}[\theta U(c)+\beta W(k)]$. We denote the unique solution to this problem by $(c^f(\theta), k^f(\theta))$.

3. OPTIMAL COMMITMENT WITHOUT SELF-CONTROL

Commitment entails reducing the set of available choices. The optimal commitment problem is to choose a subset $C\subset B$ of the budget set that maximizes the expected utility of *self*-0 given that choices are in the hands of *self*-1, that is, that the allocation is the outcome of a subgame perfect equilibrium. Formally, we choose $C\in B$ so as to maximize $\int[\theta U(c(\theta))+W(k(\theta))]\,dF(\theta)$ subject to $c(\theta), k(\theta)\in \arg\max_{(c,k)\in C}(\theta U(c)+\beta W(k))$.

Finding the best subset C is equivalent to the following principal–agent problem directly over allocations $c(\theta)$ and $k(\theta)$:

$$\max_{c,k}\int\big[\theta U(c(\theta))+W(k(\theta))\big]\,dF(\theta)$$

subject to

$$\theta U(c(\theta))+\beta W(k(\theta))\geq \theta U(c(\theta'))+\beta W(k(\theta'))\quad \text{for all } \theta,\theta'\in\Theta, \tag{1}$$

$$c(\theta)+k(\theta)\leq y\quad \text{for all } \theta\in\Theta. \tag{2}$$

Given total resources y, the problem is to maximize expected utility from the point of view of *self*-0 (henceforth, the principal) subject to the constraint that θ is private information of *self*-1 (henceforth, the agent). The incentive compatibility constraint (1) ensures that the agent reports the shock truthfully.[3]

3.1. *Two and Three Types*

We begin by studying the optimal commitment problem with only two taste shocks and then turn to the case with a continuum. When taste shocks take only two possible values, the optimum can be fully characterized as follows.

PROPOSITION 1: *Suppose $\Theta=\{\theta_l,\theta_h\}$ with $\theta_l<\theta_h$. There exists a $\beta^*\in(\theta_l/\theta_h,1)$ such that for $\beta\in[\beta^*,1]$ the first-best allocation is implementable. Otherwise*:

[3]Several recent papers study principal–agent problems where the agents have nonstandard preferences. For example, Della-Vigna and Malmendier (2004), Eliaz and Spiegler (2004), Esteban and Miyagawa (2005), and Sarafidis (2005) study optimal nonlinear pricing contracting problems with agents who suffer from time-inconsistency or self-control problems; none of these papers examines the design of optimal commitment devices as in this paper. Some authors have studied the problem of commitment through the manipulation of information or memory instead of explicit contracts (e.g., Carrillo and Mariotti (2000), Benabou and Tirole (2002)).

(i) *If* $\beta \geq \theta_l/\theta_h$, *separation is optimal*, *i.e.*, $c^*(\theta_h) > c^*(\theta_l)$ *and* $k^*(\theta_h) < k^*(\theta_l)$.

(ii) *If* $\beta \leq \theta_l/\theta_h$, *bunching is optimal*, *i.e.*, $c^*(\theta_l) = c^*(\theta_h)$ *and* $k^*(\theta_l) = k^*(\theta_h)$.

In both cases, *the optimum can be attained without burning money*: $c^*(\theta) + k^*(\theta) = y$ *for* $\theta = \theta_h, \theta_l$.

The proof is given in the Appendix.

The result that the first-best allocation is incentive compatible for low enough levels of temptation relies on the discrete difference in taste shocks and does not hold with a continuum of shocks. For higher temptation the first-best allocation is no longer incentive compatible and the proposition shows that the solution takes one of two forms. For intermediate levels of temptation it is optimal to separate the agents. To achieve separation the principal must offer bundles that yield to the agent's ex post desire for higher consumption, giving them higher consumption in the first period than the first best. For high enough temptation, however, separating the agents requires too much first-period consumption and bunching both types becomes preferable. Bunching resolves the commitment problem at the expense of flexibility. The optimal amount of flexibility depends negatively on the degree of disagreement relative to the dispersion of taste shocks. The proposition also shows that the optimum can be attained on the frontier of the budget set, so that money burning is not required.

Unfortunately, with more than two types, extending these conclusions is not straightforward. For example, consider three taste shocks, $\theta_l < \theta_m < \theta_h$, with respective probabilities p_l, p_m, and p_h. In this case bunching may occur between any consecutive pair of shocks. Money burning for the middle type may be optimal if p_m is small enough and $\beta \in (\beta^*, \theta_l/\theta_m)$, where β^* is as defined by the proposition above with two types, θ_l and θ_h. This captures the intuition that if the middle shock occurs with very low probability, money burning is not very costly and might be preferable for incentive purposes. If $\beta \notin (\beta^*, \theta_l/\theta_m)$ money burning is never optimal for small enough p_m. However, we have found numerically that when $\beta < \beta^*$ money burning may be optimal for an intermediate range of p_m.[4] These results help illustrate that money burning is a possible feature of the solution and that conditions on the distribution are required to rule it out.

3.2. *Continuous Distribution of Types*

For the rest of the paper we assume that the distribution of types is represented by a continuous density $f(\theta)$ over the bounded interval $\Theta \equiv [\underline{\theta}, \bar{\theta}]$. It is convenient to change variables from $(c(\theta), k(\theta))$ to $(u(\theta), w(\theta))$, where $u(\theta) \equiv U(c(\theta))$ and $w(\theta) \equiv W(k(\theta))$, and we term either pair of functions an

[4]More precise statements and proofs of these results are available in an online supplementary document (http://www.econometricsociety.org/ecta/supmat/5090results.pdf).

allocation. Let $C \equiv U^{-1}$ and $K \equiv W^{-1}$, which are then increasing and convex functions.

We now characterize the incentive compatibility constraints (1). Facing a direct mechanism given by $(u(\theta), w(\theta))$, an agent with taste shock θ maximizes over the report and obtains utility $V(\theta) \equiv \max_{\theta' \in \Theta}\{(\theta/\beta)u(\theta') + w(\theta')\}$. If truth-telling is optimal, then $V(\theta) = (\theta/\beta)u(\theta) + w(\theta)$ and by integrating the envelope condition $V'(\theta) = u(\theta)/\beta$ one obtains the standard integral condition

$$\text{(3)} \qquad \frac{\theta}{\beta}u(\theta) + w(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1}{\beta}u(\tilde{\theta})\,d\tilde{\theta} + \frac{\theta}{\beta}u(\underline{\theta}) + w(\underline{\theta}).$$

Incentive compatibility of (u, w) also requires u to be a nondecreasing function of θ: agents who are more eager for current consumption cannot consume less. Thus, condition (3) and the monotonicity of u are necessary for incentive compatibility. As is standard, these two conditions are also sufficient.

The principal's problem is thus to maximize $\int_{\underline{\theta}}^{\bar{\theta}}(\theta u(\theta) + w(\theta))f(\theta)\,d\theta$ subject to the budget constraint $C(u(\theta)) + K(w(\theta)) \leq y$, the incentive compatibility constraint (3), and monotonicity $u(\theta') \geq u(\theta)$ for $\theta' \geq \theta$. Note that this problem is convex because the objective function is linear and the constraint set is convex.

Substituting the incentive compatibility constraint (3) into the objective function and the resource constraint, and integrating by parts allows us to simplify the problem by dropping the function $w(\theta)$, except for its value at $\underline{\theta}$. Consequently, the principal's problem reduces to finding a function $u : \Theta \to \mathbb{R}$ and a scalar $\underline{w}$ that solves[5]

$$\text{(4)} \qquad \max_{\underline{w}, u \in \Phi} \left\{ \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w} + \frac{1}{\beta}\int_{\underline{\theta}}^{\bar{\theta}}(1 - G(\theta))u(\theta)\,d\theta \right\},$$

subject to

$$\text{(5)} \qquad W\big(y - C(u(\theta))\big) + \frac{\theta}{\beta}u(\theta) - \frac{\theta}{\beta}u(\underline{\theta}) - \underline{w} - \frac{1}{\beta}\int_{\underline{\theta}}^{\theta} u(\tilde{\theta})\,d\tilde{\theta} \geq 0$$

for all $\theta \in \Theta$,

where

$$\Phi = \big\{\underline{w}, u \mid \underline{w} \in W(\mathbb{R}_+), u : \Theta \to U(\mathbb{R}_+), u \text{ nondecreasing}\big\}$$

and

$$G(\theta) \equiv F(\theta) + \theta(1 - \beta)f(\theta).$$

[5]The objective function and the left-hand side of the constraint are well defined for all $(\underline{w}, u) \in \Phi$, because monotonic functions are integrable and the product of two integrable functions, $1 - G(\theta)$ and $u(\theta)$, is integrable (Rudin (1976, Theorems 6.9 and 6.13)).

Any allocation $(\underline{w}, u) \in \Phi$ uniquely determines an incentive compatible direct mechanism using (3). An allocation $(\underline{w}, u)$ is *feasible* if $(\underline{w}, u) \in \Phi$ and the budget constraint (5) holds.

3.3. *Minimum Savings*

This section shows that minimum-savings rules are necessarily part of the optimum.

Bunching at the top can be achieved by removing bundles previously offered for types above some threshold $\hat{\theta}$, who then move to the bundle of $\hat{\theta}$, which is the one nearest still available. That is, for any feasible allocation $(\underline{w}, u)$ and $\hat{\theta} \in \Theta$, take the allocation $(\underline{w}, \hat{u})$ given by $\hat{u}(\theta) = u(\theta)$ for $\theta < \hat{\theta}$ and $\hat{u}(\theta) = u(\hat{\theta})$ for $\theta \geq \hat{\theta}$. Thus, bunching the upper tail is always feasible; we now show that it is also always optimal.

PROPOSITION 2: *An optimal allocation $(\underline{w}, u^*)$ satisfies $u^*(\theta) = u^*(\theta_p)$ for $\theta \geq \theta_p$, where θ_p is the lowest value in Θ such that*

$$\int_{\hat{\theta}}^{\bar{\theta}} (1 - G(\tilde{\theta}))\, d\tilde{\theta} \leq 0$$

for $\hat{\theta} \geq \theta_p$. It is optimal for the budget constraint (5) *to hold with equality at θ_p.*

PROOF: The contribution to the objective function from types with $\theta \geq \theta_p$ is $(1/\beta)\int_{\theta_p}^{\bar{\theta}}(1-G(\theta))u(\theta)\, d\theta$. Substituting $u(\theta) = \int_{\theta_p}^{\theta} du + u(\theta_p)$ and integrating by parts, we obtain

$$u(\theta_p)\frac{1}{\beta}\int_{\theta_p}^{\bar{\theta}}(1 - G(\theta))\, d\theta + \frac{1}{\beta}\int_{\theta_p}^{\bar{\theta}}\int_{\theta}^{\bar{\theta}}(1 - G(\tilde{\theta}))\, d\tilde{\theta}\, du. \tag{6}$$

Note that, for the second term, $\int_{\theta}^{\bar{\theta}}(1 - G(\tilde{\theta}))\, d\tilde{\theta} \leq 0$ for all $\theta \geq \theta_p$. It follows that it is optimal to set $du = 0$ or, equivalently, $u(\theta) = u(\theta_p)$ for $\theta \geq \theta_p$.

When $\theta_p = \underline{\theta}$, all types are pooled at the same bundle and it is clearly not optimal to be in the interior of the budget set. If θ_p is interior, then the first term in (6) is zero, so $u(\theta_p)$ can always be increased up to the point where the budget constraint binds without affecting the objective function. Thus, it is optimal not to have money burning at θ_p. *Q.E.D.*

This result states that, for any bounded distribution of taste shocks, a positive mass of upper agents gets the same bundle of consumption and savings, which lies on the budget line. A minimum-savings rule that binds for some types has the property that top types are bunched. Thus, this section of the allocation can be implemented by a minimum-savings rule that is binding for precisely these agents. It follows that minimum-savings are necessarily part of the optimum.

To gain some intuition for this result, note that *self*-1 with taste shock $\theta \leq \beta\bar{\theta}$ shares the preferences of *self*-0 with a higher taste shock equal to θ/β. That is, the indifference curves of $\theta u + \beta w$ and $(\theta/\beta)u + w$ are equivalent. Informally, these types can make a case for their preferences. In contrast, *self*-1 types with $\theta > \beta\bar{\theta}$ display a blatant desire for current consumption from *self*-0's point of view. That is, there is no possible taste shock for *self*-0 that justifies *self*-1's preferences. Separating such types requires consumption to increase with θ, but this cannot be optimal since they are overconsuming from *self*-0's point of view. Thus, these agents should be bunched. In other words, at the very top of the distribution, for $\theta \geq \beta\bar{\theta}$, there is no trade-off between commitment and flexibility. The result shows that bunching goes further, that is, in the neighborhood of $\beta\bar{\theta}$, the value of commitment continues to dominate that of flexibility: $\theta_p < \beta\bar{\theta}$.[6]

3.4. *Simple Minimum-Savings Policies*

We showed above that minimum-savings are necessarily part of the optimum. We now investigate whether minimum-savings policies may fully characterize the optimum. The results with discrete types suggest the need for some condition on the distribution of taste shocks. The following condition turns out to be exactly what is needed.

ASSUMPTION A: *For all $\theta \leq \theta_p$, $G(\theta) \equiv (1-\beta)\theta f(\theta) + F(\theta)$ is nondecreasing.*

When the density f is differentiable, Assumption A, is equivalent to a lower bound on its elasticity:

$$\theta \frac{f'(\theta)}{f(\theta)} \geq -\frac{2-\beta}{1-\beta}.$$

The lower bound is negative and continuously decreasing in β. The condition is satisfied for any density f with $\theta f'/f$ bounded below when β is close enough to 1. Moreover, many densities satisfy this condition for all $\beta \in [0,1]$. For example, it is trivially satisfied for all density functions that are nondecreasing, and holds for the exponential distribution, the log-normal, and the Pareto and Gamma distributions for a subset of their parameters.

[6]The assumption that taste shocks are bounded above, equivalent to assuming that consumption in the second period is bounded away from zero under full flexibility, ensures that θ_p is well defined. For distributions with unbounded support, θ_p may not be well defined and full flexibility may be optimal; for example, with a Pareto distribution $F(\theta) = 1 - (b/\theta)^\alpha$ for $x \geq b$, implying $G(\theta) = 1 + ((1-\beta)a - 1)(b/\theta)^\alpha$. For $\alpha \geq (1-\beta)^{-1}$ one obtains that $\theta_p = b$, so it is optimal to pool all agents. However, for $\alpha < (1-\beta)^{-1}$ there is no solution to θ_p and, it turns out, it is optimal to provide full flexibility.

It is important to recall the two possible interpretations for taste shocks when considering Assumption A. Given the state-dependent utility function, $\theta U(c)+\beta W(k)$, an objective interpretation of the distribution of shocks, $F(\theta)$, implies that it can be identified from ex post behavior. For example, if individuals have full flexibility and choose freely along the budget line, the observed distribution of consumption and savings choices, $c^f(\theta)$ and $k^f(\theta)$, identifies the distribution of taste shocks, given the utility functions and the temptation parameter. In contrast, under a subjective interpretation, information regarding taste shocks must be elicited directly ex ante from the individual, which is likely to be empirically more challenging.

Our next result shows that, under Assumption A, agents with $\theta \leq \theta_p$ are offered their ex post unconstrained optimum from the budget line and agents with $\theta \geq \theta_p$ are bunched at the unconstrained optimum for θ_p. That is, the optimal mechanism offers the whole budget line to the left of the point $(c^f(\theta_p),\ k^f(\theta_p))$ and corresponds to a simple minimum-savings rule that imposes $k \geq k^f(\theta_p)$. Denote the proposed allocation in terms of utility assignments by $(\underline{w}^*, u^*)$, with $\underline{w}^* = W(k^f(\underline{\theta}))$, $u^*(\theta) = U(c^f(\theta))$ for $\theta \leq \theta_p$ and $u^*(\theta) = U(c^f(\theta_p))$ for $\theta > \theta_p$.

We next show that this simple allocation is optimal if and only if Assumption A holds. Our strategy involves applying Lagrangian theorems, which require verifying that our problem is sufficiently convex and differentiable. Once this is established, the argument is simple: we impose the necessary and sufficient first-order conditions at the conjectured allocation and back out the implied Lagrangian multipliers; the required nonnegativity of these multipliers turns out to be equivalent to Assumption A.[7,8]

Define the Lagrangian function as

$$
\begin{aligned}
L(\underline{w}, u|\Lambda) \equiv{}& \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w} + \frac{1}{\beta}\int_{\underline{\theta}}^{\bar{\theta}} (1-G(\theta))u(\theta)\,d\theta \\
&+ \int_{\underline{\theta}}^{\bar{\theta}} \biggl(W\bigl(y - C(u(\theta))\bigr) + \frac{\theta}{\beta}u(\theta) \\
&\qquad - \left(\frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w}\right) - \int_{\underline{\theta}}^{\theta} \frac{1}{\beta}u(\tilde{\theta})\,d\tilde{\theta} \biggr)\,d\Lambda(\theta),
\end{aligned}
$$

where the function Λ is the Lagrange multiplier associated with the incentive compatibility constraint.[9] Without loss of generality we set $\Lambda(\bar{\theta}) = 1$. Note

[7]One virtue of this approach is that we do not need to restrict the maximization with ad hoc "technical conditions" such as piecewise differentiability or continuity.

[8]Our approach allows us to incorporate the monotonicity condition implied by incentive compatibility directly. It differs from the common approach of neglecting monotonicity and trying to guarantee that the solution to the relaxed problem turns out to be monotone.

[9]Intuitively, the Lagrange multiplier Λ can be thought of as a cumulative distribution function that determines the importance of the resource constraints. If Λ is representable by a density λ,

that we do not need to incorporate the monotonicity constraint into the Lagrangian. Instead, we work directly with Φ, which includes the monotonicity condition. Integrating the Lagrangian by parts yields

$$L(\underline{w}, u|\Lambda) = \left(\frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w}\right)\Lambda(\underline{\theta}) + \frac{1}{\beta}\int_{\underline{\theta}}^{\bar{\theta}} (\Lambda(\theta) - G(\theta))u(\theta)\, d\theta$$
$$+ \int_{\underline{\theta}}^{\bar{\theta}} \left(W\big(y - C(u(\theta))\big) + \frac{\theta}{\beta}u(\theta)\right) d\Lambda(\theta).$$

The next lemma exploits the convexity of the problem to show that appropriate first-order conditions are necessary and sufficient for optimality.

LEMMA OF OPTIMALITY: (i) *If an allocation* $(\underline{w}_0, u_0) \in \Phi$ *is optimal with* u_0 *continuous, then there exists a nondecreasing* Λ_0 *such that the following first-order conditions in terms of Gateaux differentials*,[10]

$$\partial L(\underline{w}_0, u_0; \underline{w}_0, u_0|\Lambda_0) = 0, \tag{7}$$

$$\partial L(\underline{w}_0, u_0; h_{\underline{w}}, h_u|\Lambda_0) \leq 0, \tag{8}$$

hold for all $(h_{\underline{w}}, h_u) \in \Phi$ *and* h_u *continuous*.

(ii) *Conversely, if there exists a nondecreasing* Λ_0 *such that the first-order conditions* (7) *and* (8) *hold for all* $(h_{\underline{w}}, h_u) \in \Phi$, *then* $(\underline{w}_0, u_0)$ *is optimal*.

See the Appendix for the proof.

Using the second expression for the Lagrangian, the Gateaux differential at the proposed allocation $(\underline{w}^*, u^*)$ is given by

$$\begin{aligned} &\partial L(\underline{w}^*, u^*; h_{\underline{w}}, h_u|\Lambda) \\ &\quad = \left(\frac{\underline{\theta}}{\beta}h_u(\underline{\theta}) + h_{\underline{w}}\right)\Lambda(\underline{\theta}) + \frac{1}{\beta}\int_{\underline{\theta}}^{\bar{\theta}} (\Lambda(\theta) - G(\theta))h_u(\theta)\, d\theta \\ &\qquad + \frac{\theta_p}{\beta}\int_{\theta_p}^{\bar{\theta}} \left(\frac{\theta}{\theta_p} - 1\right) h_u\, d\Lambda(\theta) \end{aligned} \tag{9}$$

then the constraints can be incorporated as the familiar integral of the product with the density function $\lambda(\theta)$. Although this is a common approach, in general, Λ may have points of discontinuity. Indeed, the multiplier we construct has two points of discontinuity.

[10]Given a function $T : \Omega \to Y$, where $\Omega \subset X$, and X and Y are normed spaces, if for $x \in \Omega$ and $h \in X$ the limit

$$\lim_{\alpha \downarrow 0} \frac{1}{\alpha}[T(x + \alpha h) - T(x)]$$

exists, then it is called the Gateaux differential at x with direction h and is denoted by $\partial T(x; h)$.

for all $(h_{\underline{w}}, h_u) \in \Phi$. The next proposition uses this lemma to prove that a minimum-savings rule is the optimum under Assumption A.

PROPOSITION 3: *The minimum-savings allocation* $(\underline{w}^*, u^*)$ *is optimal if Assumption* A *holds*.

PROOF: We show that there exists a nondecreasing multiplier Λ^* such that the proposed $(\underline{w}^*, u^*)$ satisfies the first-order conditions (7) and (8) for all $(h_{\underline{w}}, h_u) \in \Phi$. Let $\Lambda^*(\underline{\theta}) = 0$, $\Lambda^*(\theta) = G(\theta)$ for $(\underline{\theta}, \theta_p]$, and $\Lambda^*(\theta) = 1$ for $\theta \in (\theta_p, \bar{\theta}]$. Note that Λ^* is not continuous; it has an upward jump at $\underline{\theta}$ and a jump at θ_p. We need to show that the jump at θ_p is upward. Indeed,

$$\lim_{\theta \downarrow \theta_p} \Lambda^*(\theta) - \Lambda^*(\theta_p) = 1 - G(\theta_p) \geq 0,$$

which follows from the definition of θ_p. To see this, note that if $\theta_p = \underline{\theta}$, the result is immediate, because then Λ^* would jump from 0 to 1 at $\underline{\theta}$. Otherwise, by definition θ_p is the lowest $\hat{\theta}$ such that $\int_{\hat{\theta}}^{\bar{\theta}} (1 - G(\tilde{\theta}))\, d\tilde{\theta} \leq 0$ for all $\theta \geq \hat{\theta}$, which implies that $1 - G(\theta_p) \geq 0$.

Substituting the proposed multiplier Λ^* into the Gateaux differential (9) yields

$$\begin{aligned} \partial L(\underline{w}^*, u^*; h_{\underline{w}}, h_u | \Lambda^*) &= \frac{1}{\beta} \int_{\theta_p}^{\bar{\theta}} (1 - G(\theta)) h_u(\theta)\, d\theta \\ &= \frac{1}{\beta} \int_{\theta_p}^{\bar{\theta}} \left[\int_{\theta}^{\bar{\theta}} (1 - G(\tilde{\theta}))\, d\tilde{\theta} \right] dh_u(\theta), \end{aligned}$$

where the last equality follows by integrating by parts, which can be done given the monotonicity of h_u and by the definition of θ_p. This Gateaux differential is zero at the proposed allocation and, by the definition of θ_p, it is nonpositive for all h_u nondecreasing. It follows that the first-order conditions (7) and (8) are satisfied for all $(h_{\underline{w}}, h_u) \in \Phi$. *Q.E.D.*

Proposition 3 shows that the optimal allocation can be very simple and implemented by imposing a minimum level of savings. The next proposition shows that more complicated schemes are optimal if Assumption A does not hold.

PROPOSITION 4: *If Assumption* A *does not hold*, *then no minimum-savings rule is optimal*. *That is*, *the allocation* $(\underline{\hat{w}}, \hat{u})|_{x_p}$, *defined by* $\hat{u}(\theta) = U(c^f(\theta))$ *for* $\theta < x_p$, $\hat{u}(\theta) = U(c^f(x_p))$ *for* $\theta \geq x_p$ *and* $\underline{\hat{w}} = W(k^f(\underline{\theta}))$, *is not optimal for any* $x_p \in \Theta$.

PROOF: Let $a, b \in \Theta$ be such that $a < b < \theta_p$ and $G(a) > G(b)$ (so Assumption A does not hold).

The proof proceeds by contradiction. Suppose that $(\underline{\hat{w}}, \hat{u})|_{x_p}$ is optimal for some x_p. Then by part (i) of the Lemma of Optimality, there has to exist a nondecreasing Lagrange multiplier $\hat{\Lambda}$ such that the conditions for optimality (7) and (8) are satisfied at the proposed allocation for all $(h_{\underline{w}}, h_u) \in \Phi$ and h_u continuous. Condition (8) with $h_u = 0$ requires that $\hat{\Lambda}(\underline{\theta}) = 0$ because $h_{\underline{w}}$ is unrestricted. Using $\hat{\Lambda}(\underline{\theta}) = 0$ and integrating (9), with x_p in place of θ_p, by parts (Theorem 6.20 in Rudin (1976) guarantees this step given that h_u continuous) leads to

$$\partial L(\underline{\hat{w}}, \hat{u}; h_{\underline{w}}, h_u|\hat{\Lambda}) = \gamma(\underline{\theta})h_u(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \gamma(\theta)\, dh_u(\theta) \tag{10}$$

with

$$\gamma(\theta) \equiv \frac{1}{\beta}\int_{\theta}^{\bar{\theta}} (\hat{\Lambda}(\tilde{\theta}) - G(\tilde{\theta}))\, d\tilde{\theta} + \frac{x_p}{\beta}\int_{\max\{\theta, x_p\}}^{\bar{\theta}} \left(\frac{\tilde{\theta}}{x_p} - 1\right) d\hat{\Lambda}(\tilde{\theta}).$$

By condition (8), it follows that $\gamma(\theta) \leq 0$ for all $\theta \in \Theta$ is necessary for optimality.

Then (7) implies that $\gamma(\theta) = 0$ for $\theta \in [\underline{\theta}, x_p]$, i.e., wherever $\hat{u}$ is strictly increasing. It follows then that $\hat{\Lambda}(\theta) = G(\theta)$ for all $\theta \in (\underline{\theta}, x_p]$. The proposed allocation $(\underline{\hat{w}}, \hat{u})$ thus determines a unique candidate multiplier $\hat{\Lambda}$ in the separating region $(\underline{\theta}, x_p]$. This implies that $x_p \leq b$ otherwise the associated multiplier $\hat{\Lambda}(\theta)$ would be decreasing: $\Lambda(a) > \Lambda(b)$. Integrating by parts the second term of the previous equation, we obtain

$$\gamma(\theta) = \frac{1}{\beta}\int_{\theta}^{\bar{\theta}} (1 - G(\tilde{\theta}))\, d\tilde{\theta} + \frac{1}{\beta}(\theta - x_p)(1 - \Lambda(\theta)) \quad \text{for all } \theta \geq x_p.$$

By definition of θ_p there must exist a $\theta \in [x_p, \theta_p)$ such that the first term is strictly positive; since $\Lambda(\theta) \leq 1$, the second term is nonnegative; hence $\gamma(\theta) > 0$, contradicting a necessary condition for optimality. Hence no minimum-savings rule is optimal. *Q.E.D.*

A minimum-savings allocation does not entail money burning. Recall that with three types, money burning may be optimal. A situation with three types can be approximated by continuous types taking a sequence of continuous densities that become increasingly peaked around θ_l, θ_m, and θ_h. However, the distributions in the sequence would eventually violate Assumption A, which requires a density with bounded slope. Thus, with continuous distributions that violate Assumption A, money burning may be optimal.

Even if one restricts attention to allocations that do not involve money burning, an improvement over the minimum-savings policy can be constructed by removing intervals in the separating regions wherever the monotonicity condition in Assumption A fails.[11] This construction also yields intuition into Assumption A. Suppose its condition is not satisfied for $\theta_a < \theta < \theta_b \leq \theta_p$. When one removes the open interval between $c^f(\theta_a)$ and $c^f(\theta_b)$, all types with $\theta \in (\theta_a, \theta_b)$ move from their unconstrained optimum to one of the two extremes $c^f(\theta_a)$ or $c^f(\theta_b)$. The change in welfare depends critically on how many of such types moved to the left versus the right, because welfare rises from those moving left and falls from those moving right, for a small enough interval. The slope of the density function affects precisely this, explaining its role in Assumption A.

Taken together, the previous two propositions imply that minimum-savings policies completely characterize the optimum if and only if the distribution of taste shocks satisfies Assumption A, which is the case for a wide class of distributions. Recall that for low enough levels of temptation, for β close to 1, Assumption A is satisfied for essentially all distributions. In this sense, simple minimum-savings policies are especially likely to be optimal for modest levels of temptation.[12]

Our result is also relevant for thinking about the market's provision of commitment devices. Indeed, simple market arrangements may be able to mimic the optimal one. Under Assumption A, the optimal allocation can be implemented with a particular form of illiquid asset. Suppose the consumer initially, at $t = 0$, divides his wealth between two assets: liquid and illiquid. Both assets have the same rate of return, but funds invested in the illiquid asset cannot be used for consumption at $t = 1$; they can only be consumed at $t = 2$. Thus, investing in the illiquid asset represents a self-imposed minimum level of savings and in this way the individual can implement the optimal allocation.

In an earlier version of this paper (Amador, Werning, and Angeletos (2003)) we showed that all our results extend to finitely many periods with independent and identically distributed taste shocks. By using a dynamic programming argument, each stage is similar to the two-period problem in (4) and (5). Minimum-savings are always part of the solution, and a simple minimum-savings policy completely characterizes the solution if and only if Assumption A holds. This result also establishes that the commitment afforded by the illiquid asset structure studied by Laibson (1997) may, in fact, be fully optimal.

We turn next to comparative statics with respect to the strength of temptation. As β decreases, θ_p decreases so that more types are bunched and the minimum-savings level increases.

[11]A formal statement and proof of this "drilling" result is contained in the supplementary material (Amador, Werning, and Angeletos (2006)).

[12]Of course, for small levels of temptation the minimum-savings level will also be small, because θ_p converges to $\bar{\theta}$ as $\beta \to 1$.

PROPOSITION 5: *The bunching point θ_p increases with β. The minimum-savings level $k_{\min} = y - C(u(\theta_p))$ decreases with β.*

PROOF: That θ_p is weakly increasing follows directly from its definition. To see that $k_{\min}$ is decreasing, note that it solves $(\theta_p/\beta)U'(y-k_{\min})/W'(k_{\min}) = 1$ and that an interior θ_p solves $\theta_p/\beta = \mathbb{E}[\theta \mid \theta \geq \theta_p]$. Combining these, we obtain

$$\mathbb{E}[\theta \mid \theta \geq \theta_p]\frac{U'(y-k_{\min})}{W'(k_{\min})} = 1.$$

Given that $\mathbb{E}[\theta \mid \theta \geq \theta_p]$ is increasing in θ_p, the result follows from the concavity of U and W. *Q.E.D.*

4. OPTIMAL COMMITMENT WITH SELF-CONTROL

In this section we study an individual facing temptation, but with some power of self-control. This model captures the idea that individuals may partially resist temptations, but that it is costly to do so. In this framework commitment devices are valuable not only because they affect equilibrium behavior, but also because they may reduce the costs of exerting self-control.

Dekel, Lipman, and Rustichini (2001, 2004) and Gul and Pesendorfer (2001) consider ex ante preferences defined over choice sets made available to the agent ex post. Our specification for the ex ante utility of set C is

$$(11) \qquad P(C) = \mathbb{E}\Big[\max_{(c,k)\in C}\big(\theta U(c) + W(k) + \varphi(\theta U(c) + \beta W(k))\big) - \varphi \max_{(c,k)\in C}(\theta U(c) + \beta W(k))\Big],$$

which is adopted from Krusell, Kuruşçu, and Smith (2005). The parameter $\varphi > 0$ captures the cost of self-control, whereas $(1-\beta)$ captures the strength of the temptation to consume in the current period. This specification has the convenient property that as $\varphi \to \infty$, preferences converge to the quasi-hyperbolic model. Indeed, we shall show that slightly modified versions of our previous results and analysis from the quasi-hyperbolic case apply here.

Up to this point we have allowed only the taste shock θ to be uncertain. We now pursue a generalization that allows the levels of temptation and self-control costs to be uncertain as well. There are two motivations for such a generalization. First, in a recent paper Dekel, Lipman, and Rustichini (2004) provided natural examples that illustrate the need for uncertain temptation and provide axiomatic foundations for it. Second, the generalization also allows us to capture the commonly held view that differences in savings may be partly due to differences in temptation or self-control costs (Diamond (1977)). We assume θ, β, and φ are drawn from a continuous joint distribution over some bounded rectangular support $[\underline{\theta}, \bar{\theta}] \times [\underline{\beta}, \bar{\beta}] \times [\underline{\varphi}, \bar{\varphi}]$, where $\underline{\beta} > 0$.

The optimal commitment problem can be stated as maximizing $P(C)$ by choosing a subset $C \subset B$, where $B = \{(c, k) \mid c + k \leq y\}$ is the budget constraint. As before, we seek to rewrite this as a principal–agent problem. The objective function in (11) can be written as

$$\mathbb{E}\Big\{(1+\beta\varphi) \max_{(c,k)\in C}[(\theta/\hat{\beta})U(c) + W(k)] \\ - \varphi\beta \max_{(c,k)\in C}[(\theta/\beta)U(c) + W(k)]\Big\},$$

where we let $\hat{\beta}$ be $(1+\beta\varphi)/(1+\varphi)$. Define the random variables $\hat{z}$ and z by $\hat{z} \equiv \theta/\hat{\beta}$ and $z \equiv \theta/\beta$, and let the extended support $\hat{\Theta}$ be the union of the supports for z and $\hat{z}$, so that $\hat{\Theta} \equiv [\underline{x}, \bar{x}] \equiv [\underline{\theta}(1+\underline{\phi})/(1+\bar{\beta}\underline{\phi}), \bar{\theta}/\underline{\beta}]$. Let an allocation over the extended support $\hat{\Theta}$ be given by a pair of functions $u : \hat{\Theta} \to U(\mathbb{R}_+)$ and $w : \hat{\Theta} \to W(\mathbb{R}_+)$.

The principal–agent formulation of the commitment problem is to find an allocation that maximizes

$$\mathbb{E}\big[(1+\beta\varphi)(\hat{z}u(\hat{z}) + w(\hat{z})) - \beta\varphi(zu(z) + w(z))\big] \tag{12}$$

subject to $C(u(x)) + K(w(x)) \leq y$ and

$$xu(x) + w(x) \geq xu(x') + w(x') \quad \text{for all } x, x' \in \hat{\Theta}. \tag{13}$$

Let $\alpha(x) \equiv \mathbb{E}[(1+\beta\varphi)|\hat{z} = x]$ and $\kappa(x) \equiv \mathbb{E}[\beta\varphi|z = x]$. Denote by $h_1(\hat{z})$ and $h_2(z)$ the densities of $\hat{z}$ and z, respectively. By the law of iterated expectations, we have that the new objective function (12) can be written as

$$\int_{\hat{\Theta}} (xu(x) + w(x))\hat{g}(x)\, dx, \tag{14}$$

where the density $\hat{g}(x) \equiv \alpha(x)h_1(x) - \kappa(x)h_2(x)$ can be negative or positive. This alternative expression for the utility function (11) corresponds to the signed measure representation theorem of Dekel, Lipman, and Rustichini (2001).

The incentive compatibility constraints (13) are equivalent, as before, to

$$xu(x) + w(x) = \underline{x}u(\underline{x}) + \underline{w} + \int_{\underline{x}}^{x} u(x')\, dx' \tag{15}$$

with the monotonicity constraint that u be nondecreasing. We can now substitute (15) into the objective function (14) and the resource constraints. Let $\hat{G}(x) = \int_{\underline{x}}^{x} \hat{g}(z)\, dz$ (where $\hat{G}(\bar{x}) = 1$). Integrating the objective function by parts then yields the program

$$\max_{(\underline{w}, u(\cdot))\in\hat{\Phi}} \left\{ \underline{x}u(\underline{x}) + \underline{w} + \int_{\hat{\Theta}} [1 - \hat{G}(x)]u(x)\, dx \right\} \tag{16}$$

subject to

$$W\big(y - C(u(x))\big) + xu(x) - \underline{x}u(\underline{x}) - \underline{w} - \int_{\underline{x}}^{x} u(x')\,dx' \geq 0, \tag{17}$$

where

$$\hat{\Phi} \equiv \big\{\underline{w}, u \mid \underline{w} \in W(\mathbb{R}_+), u : \hat{\Theta} \to U(\mathbb{R}_+), u \text{ nondecreasing}\big\}.$$

With the problem mapped into a version that is formally equivalent to the problem at the end of Section 3.2, the following propositions are direct translations of our previous results.

PROPOSITION 6: *An optimal allocation* $(\underline{w}^*, u^*)$ *satisfies* $u^*(x) = u^*(x_p)$ *for* $x \geq x_p$, *where* x_p *is the lowest value in* $\hat{\Theta}$ *such that*

$$\int_{\hat{x}} (1 - \hat{G}(x))\,dx \leq 0$$

for all $\hat{x} \geq x_p$. *It is optimal for the budget constraint to hold with equality at* x_p.

Define the full flexibility allocation as $(u^f(x), w^f(x)) \in \arg\max_{u,w}\{xu + w\}$ subject to $C(u) + K(w) \leq y$. Let the proposed allocation be given by $\underline{w} = w^f(\underline{x})$, and $u^*(x) = u^f(x)$ if $x < x_p$ and $u^*(x) = u^f(x_p)$ if $x \geq x_p$. We introduce the following assumption analogous to that of Assumption A.

ASSUMPTION B: *For all* $x \leq x_p$, $\hat{G}(x)$ *is nondecreasing*.

The next proposition states that minimum-savings rules are optimal under Assumption B.

PROPOSITION 7: *The allocation* $(\underline{w}^*, u^*)$ *is optimal if Assumption* B *holds*. *If Assumption* B *does not hold*, *no minimum-savings rule fully characterizes the optimum*.

PROOF: The proof of the first statement is identical to that in Proposition 3, except that the multiplier Λ does not jump at the bottom $\underline{x}$ (because here $\hat{G}(x)$ is zero at $\underline{x}$). The second statement follows the proof of Proposition 4. *Q.E.D.*

We now discuss two results that obtain when the strength of temptation and the self-control costs are not random. The first result connects the condition behind Assumption A (from Section 3) with Assumption B: it shows that relative to the time-inconsistent quasi-hyperbolic model, the possibility of self-control strengthens the case for minimum-savings policies to characterize the full optimum. The second result shows that, as is natural, higher temptation and lower self-control costs raise the minimum-savings level, increasing commitment at the expense of flexibility. Proofs for both results are contained in the Appendix.

PROPOSITION 8: *When β and φ are certain, Assumption* B *holds if $G(\theta)$ is nondecreasing on $[\underline{\theta}, \hat{\beta}x_p]$.*

PROPOSITION 9: *The minimum savings point $K(w(x_p))$ decreases with β and increases with φ.*

5. OTHER APPLICATIONS

In this section we discuss applications of our results to situations distinct from the intertemporal consumption model that also feature a trade-off between commitment and flexibility. The last subsection extends the bunching at the top result to utility functions that are not additively separable.

5.1. *Optimal Fiscal Constitutions*

Consider an economy where a ruling government decides the allocation of resources between private and public consumption. Ex post, the government obtains valuable information regarding the social value of public services, but is biased toward higher public spending. Ex ante, society faces the constitutional problem of restricting the fiscal choices available to its government.

The welfare of the citizens is given by $\theta U(g) + W(c)$, where c denotes private consumption and g denotes public services. The government, on the other hand, wishes to maximize $\beta^{-1}\theta U(g) + W(c)$, where $\beta^{-1} > 1$ parameterizes the government's bias towards public spending. The realization of the value of public services θ is private information of the government. The resource constraint is $B = \{(c, g) \in \mathbb{R}^2_+ \mid c + g \leq y\}$. A fiscal constitution is a subset $C \subseteq B$ that constrains the government to choose $(c, g) \in C$.

Note that our restriction to pure commitment mechanisms, with no transfers across types, seems especially natural in this application. The optimal constitution is a subset C that maximizes society's welfare given the (mis)behavior of the government. Proposition 2 shows that it is always optimal to limit government spending. Proposition 3 implies that, under Assumption A, only an upper cap on government spending is needed.

5.2. *Optimal Paternalism*

Few would argue that parents are not, at times, literally paternalistic toward their children. Much government regulation—such as minimum-schooling laws, and drinking and drug restrictions or prohibitions—is also largely justified on paternalistic grounds. Paternalism involves disagreement regarding preferences *between* individuals, instead of *within* an individual as is the case with temptation. However, the crucial feature in our model is a form of disagreement, not any particular source for this disagreement. Consequently, our results can be applied to some paternalistic situations as well.

As an example, consider the case of a child who must divide time between schooling s and leisure l, constrained by a time endowment $s + l \leq 1$. The child has utility function $\theta U(l) + \beta W(s)$ with $\beta < 1$. The parameter θ affects the marginal valuation of leisure and is the child's private information. The paternalist—parent or government—cares about the child, but has a different preference over his allocation of time and maximizes $\theta U(l) + W(s)$; she values schooling relatively more than the child does.

This setup focuses on the time allocation dimension for which pure commitment mechanisms that rule out transfers across types are natural. The problem faced by the paternalist maps directly into our setup. Our result then provides conditions under which imposing a minimum-schooling level is optimal.[13]

5.3. *Externalities*

There are two consumption goods, c and k. The population is composed of a continuum of agents indexed by θ, distributed according to $F(\theta)$. The utility of agent θ is given by

$$V(\theta) \equiv \theta U(c(\theta)) + \beta W(k(\theta)) + (1 - \beta) \int W(k(\tilde{\theta}))\, dF(\tilde{\theta}),$$

where $\beta < 1$ and $(c(\theta), k(\theta))$ represents the allocation in the population. The last term captures a positive externality generated by the consumption of good k. For example, it may represent the possible externality imposed from the appearance of neighbors' houses.

Agents do not internalize the externality and maximize $\theta U(c) + \beta W(k)$. A utilitarian planner, however, maximizes

$$\int V(\theta)\, dF(\theta) = \int \big[\theta U(c(\theta)) + W(k(\theta))\big]\, dF(\theta).$$

This welfare function is equivalent to one without externalities, but where a utilitarian planner assigns utility $\theta U(c) + W(k)$ to agent θ. If the only instrument available to the government is the removal of consumption opportunities, then this maps directly into our framework. Our main result then provides

[13]An interesting extension, which we have not explored here, would add a consumption good and allow for transfers across types in this good. In such a model, a natural conjecture is that the optimal mechanism would feature some monetary incentives to schooling.

conditions for the optimality of a rule that imposes a minimum level of consumption for the good, generating positive externalities.[14]

5.4. *An Income Shock Interpretation for Taste Shocks*

Returning to the intertemporal consumption application, suppose that instead of taste shocks, the individual experiences an income shock q in the first period, with distribution $F(q)$ over $Q \equiv [\underline{q}, \bar{q}]$. We focus on pure commitment mechanisms that offer no insurance, so the budget constraint of each consumer imposes

$$c(q) + k(q) \leq y + q \quad \text{for all } q \in Q.$$

We assume the realization of q is private information to the agent. The planner does observe the savings decision $k(q) = y + q - c(q)$. The agent's incentive constraints are

$$\begin{aligned} &u(c(q)) + \beta w(k(q)) \\ &\quad \geq u(c(q') + q - q') + \beta w(k(q')) \quad \text{for all } q, q' \in Q. \end{aligned}$$

To obtain a perfect mapping to our taste shock framework, we adopt the exponential utility function $u(c) = -e^{-c}$. The incentive constraints are then equivalent to

$$\begin{aligned} &u(c(q) + q - q) + \beta w(k(q)) \geq u(c(q') + q - q') + \beta w(k(q')), \\ &e^q u(c(q) - q) + \beta w(k(q)) \geq e^q u(c(q') - q') + \beta w(k(q')). \end{aligned}$$

Define $\theta \equiv e^{-q}$, and let $\hat{c}(\theta) \equiv c(-\ln(\theta)) + \ln(\theta)$ and $\hat{k}(\theta) \equiv k(-\ln(\theta))$. Then the problem can be written as

$$\max_{\hat{c}, \hat{k}} \mathbb{E}\big[\theta u(\hat{c}(\theta)) + w(\hat{k}(\theta))\big]$$

subject to

$$\begin{aligned} &\theta u(\hat{c}(\theta)) + w(\hat{k}(\theta)) \geq \theta u(\hat{c}(\theta')) + w(\hat{k}(\theta')), \\ &\hat{c}(\theta) + \hat{k}(\theta) \leq y. \end{aligned}$$

[14] Perhaps this relates to housing codes, which restrict the use of homeowners' property. However, more generally, the restriction to no transfers may be less natural for some cases given the more standard Pigouvian tax approach to externalities. For example, in the case of pollution, monetary incentives have been employed in addition to maximum quantity restrictions.

The problem is then identical to our main setup. This example is important because it provides an objective reinterpretation of the taste shocks. Assumption A imposes a restriction on the distribution of income shocks, which might be identified directly, or indirectly from observable savings behavior.

5.5. *Bunching for More General Utility Functions*

We now extend the bunching result to preferences that are not additively separable between c and k. In particular, let $U(c,k,\theta)$ and $V(c,k,\theta)$ denote the utility functions for the agent and principal, respectively. As before, $\theta \in \Theta \equiv [\underline{\theta}, \bar{\theta}]$.

We assume that the taste shocks provide an ordering in that higher θ tilts preferences, for both agent and principal, toward higher current consumption. We also assume that the agent, relative to the principal, is biased toward current consumption, at least at the top. These assumptions can be formalized as single-crossing conditions.

ASSUMPTION C: *The utility functions $U(c,k,\theta)$ and $V(c,k,\theta)$ satisfy the following conditions*:

(i) *If $U(c_a,k_a,\theta) \geq U(c_b,k_b,\theta)$ for $c_a > c_b$, then $U(c_a,k_a,\theta') > U(c_b,k_b,\theta')$ for all $\theta' \in \Theta$ such that $\theta' > \theta$.*

(ii) *If $V(c_a,k_a,\theta) \geq V(c_b,k_b,\theta)$ for $c_a > c_b$, then $V(c_a,k_a,\theta') > V(c_b,k_b,\theta')$ for all $\theta' \in \Theta$ such that $\theta' > \theta$.*

(iii) *There exist a $\theta_b < \bar{\theta}$ such that for any $c_a > c_b$, if $V(c_a,k_a,\bar{\theta}) \geq V(c_b,k_b,\bar{\theta})$, then $U(c_a,k_a,\theta_b) > U(c_b,k_b,\theta_b)$.*

The first two conditions state that higher types single-cross lower types for both utility functions. The third condition ensures a form of bias at the top: It states that there exists an interior taste shock, such that the preferences of the agent with this shock single crosses that of the planner with the highest taste shock. Note that these conditions are all satisfied in the additively separable case considered previously.

For any allocation (c,k), define $(\hat{c},\hat{k})$ as

$$(\hat{c}(\theta),\hat{k}(\theta)) = \begin{cases} (c(\theta),k(\theta)), & \text{if } \theta \leq \theta_b, \\ (c(\theta_b),k(\theta_b)), & \text{if } \theta > \theta_b. \end{cases}$$

The following result states that some bunching is always optimal.

PROPOSITION 10: *Suppose Assumption C holds. Then, for any feasible allocation (c,k), the allocation $(\hat{c},\hat{k})$ is a feasible improvement.*

The proof is given in the Appendix.

The proof of this result relies on the fact that an allocation that separates all types must be offering bundles for the highest types that ensure that these are overconsuming, from the principal's point of view (this is the role of condition (iii)). Removing an upper portion leads these types to bunch at the remaining bundle with the highest available current consumption (the role of assumption (i)). This reallocation is also preferred by the principal (the role of assumption (ii)).

6. CONCLUSIONS

Our consumer values commitment to avoid the temptation of current consumption, and flexibility to respond to taste shocks. The resulting trade-off makes the design of an optimal commitment device nontrivial.

We find that a minimum-savings rule is always part of the optimal commitment policy. Moreover, a minimum-savings rule completely characterizes the optimum when a condition on the distribution of taste shocks is satisfied. The minimum-savings level then increases with the strength of temptation. These results are robust to the way temptation is modeled, and can be extended to situations with uncertain levels of temptation and self-control, as well as to longer time horizons.

Our model and results can be applied to other situations that feature similar trade-offs between commitment and flexibility, such as paternalism, the design of fiscal constitutions to control government spending, and externalities. Another potential application is to problems of time inconsistency of government policy to examine the trade-off of rules vs. discretion.

To isolate the problem of commitment as one that reduces available choices from the budget set, this paper ignored the possibility of transfers across types. An interesting direction for future research is to consider insurance and taxes that allow these transfers so as to provide a more complete characterization of the optimal tax and social security policies for the class of environments we have considered in this paper.[15]

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[15]Preliminary work along these lines can be found in Amador, Angeletos, and Werning (2004).

APPENDIX

PROOF OF PROPOSITION 1: With $\beta = 1$, the incentive constraints are slack at the first-best allocation. Define $\beta^* < 1$ to be the value of β for which the incentive constraint of agent θ_l holds with equality at the first-best allocation. Then for $\beta > \beta^*$ both incentive constraints are slack at the first-best allocation and $\beta^* > \theta_l/\theta_h$ follows because

$$\begin{aligned}
\beta^* &\equiv \theta_l \frac{U(c^{fb}(\theta_h)) - U(c^{fb}(\theta_l))}{W(y - c^{fb}(\theta_l)) - W(y - c^{fb}(\theta_h))} \\
&> \theta_l \frac{U'(c^{fb}(\theta_h))(c^{fb}(\theta_h) - c^{fb}(\theta_l))}{W'(y - c^{fb}(\theta_h))(c^{fb}(\theta_h) - c^{fb}(\theta_l))} \\
&= \theta_l \frac{U'(c^{fb}(\theta_h))}{W'(y - c^{fb}(\theta_h))} = \frac{\theta_l}{\theta_h}.
\end{aligned}$$

Now, consider the case where $\beta > \theta_l/\theta_h$ and suppose that $c(\theta_h) + k(\theta_h) < y$. Then an increase in $c(\theta_h)$ and a decrease in $k(\theta_h)$ that holds $(\theta_l/\beta)U(c(\theta_h)) + U(k(\theta_h))$ unchanged and increases $c(\theta_h) + k(\theta_h)$ up to y, increases the objective function. Such a change is incentive compatible because it strictly relaxes the incentive constraint of the high type pretending to be a low type, leaving the incentive constraint of the low type unchanged. It follows that we must have $c(\theta_h) + k(\theta_h) = y$ at an optimum.

Similarly, $c(\theta_l) + k(\theta_l) < y$ cannot be optimal, because lowering $c(\theta_l)$ and raising $k(\theta_l)$ while holding $\theta_l U(c(\theta_l)) + \beta W(k(\theta_l))$ constant would then be feasible. Such a variation does not affect the incentive constraint of the low type and relaxes the incentive constraint of the high type, yet it increases the objective function because $\theta_l U(c(\theta_l)) + W(k(\theta_l))$ increases. This also shows that separating is optimal in this case, proving part (i). Analogous arguments establish part (ii). *Q.E.D.*

Lemma of Optimality and First-Order Conditions

We first show that the maximization of the Lagrangian is a necessary and sufficient condition for optimality of an allocation. This is stated in the following two results:

RESULT (i) —Necessity: *If an allocation* $(\underline{w_0}, u_0) \in \Phi$ *with* u_0 *continuous is optimal, then there exists a nondecreasing* Λ_0 *such that the Lagrangian is maximized*:

$$\begin{aligned}
(18) \qquad & L(\underline{w_0}, u_0; \underline{h_w}, h_u | \Lambda_0) \\
& \leq L(\underline{w_0}, u_0; \underline{w_0}, u_0 | \Lambda_0) \quad \textit{for all } (\underline{h_w}, h_u) \in \Phi, \quad h_u \textit{ continuous}.
\end{aligned}$$

RESULT (ii)—Sufficiency: *An allocation* $(\underline{w_0}, u_0) \in \Phi$ *is optimal if there exists a nondecreasing* Λ_0 *such that*

$$L(\underline{w_0}, u_0; \underline{h_w}, h_u|\Lambda_0) \leq L(\underline{w_0}, u_0; \underline{w_0}, u_0|\Lambda_0) \quad \textit{and} \quad \textit{all } (\underline{h_w}, h_u) \in \Phi. \tag{19}$$

PROOF: Our optimization problem maps into the general problem studied in Sections 8.3 and 8.4 of Luenberger (1969): $\max_{x\in X} Q(x)$ subject to $x \in \Omega$ and $G(x) \in P$, where Ω is a subset of the vector space X, $Q:\Omega \to \mathbb{R}$ and $G:\Omega \to Z$; where Z is a normed vector space, and P is a positive nonempty convex cone in Z.

For Result (ii), set

$$\begin{aligned}
&X = \{\underline{w}, u \mid \underline{w} \in W(\mathbb{R}_+) \text{ and } u:\Theta \to \mathbb{R}\},\\
&\Omega = \big\{\underline{w}, u \mid \underline{w} \in W(\mathbb{R}_+),\\
&\qquad\qquad u:\Theta \to U(\mathbb{R}_+),\ \text{and } u \text{ is nondecreasing}\big\} \equiv \Phi,\\
&Z = \Big\{z \,\Big|\, z:\Theta \to \mathbb{R} \text{ with } \sup_{\theta\in\Theta} |z(\theta)| < \infty\Big\}\\
&\quad \text{with the norm } \|z\| = \sup_{\theta\in\Theta} |z(\theta)|,\\
&P = \{z \mid z \in Z \text{ and } z(\theta) \geq 0 \text{ for all } \theta \in \Theta\}.
\end{aligned}$$

We let the objective function in (4) be Q and let the left-hand side of the resource constraint in (5) be defined as G. Result (ii) then follows immediately because the hypotheses of Theorem 1 in Luenberger (1969, p. 220) are met.

For Result (i), modify Ω and Z to require continuity of u:

$$\begin{aligned}
&\Omega = \big\{\underline{w}, u \mid \underline{w} \in W(\mathbb{R}_+), u:\Theta \to U(\mathbb{R}_+),\ \text{and}\\
&\qquad\qquad u \text{ is continuous and nondecreasing}\big\},\\
&Z = \{z \mid z:\Theta \to \mathbb{R} \text{ and } z \text{ is continuous}\},\\
&\quad \text{with the norm } \|z\| = \sup_{\theta\in\Theta} |z(\theta)|,
\end{aligned}$$

with X, P, Q, and G as before. Note that Q and G are concave, that Ω is convex, that P contains an interior point (e.g., $z(\theta) = 1$ for all $\theta \in \Theta$), and that the positive dual of Z is the set of nondecreasing functions on Θ by the Riesz representation theorem (see Luenberger (1969, Chap. 5, p. 113)). Finally, if $\underline{w_0}, u_0$ is optimal within Φ and $\underline{w_0}, u_0 \in \Phi \cap \{u \text{ is continuous}\}$, then $\underline{w_0}, u_0$ is optimal within the subset $\Phi \cap \{u \text{ is continuous}\} \equiv \Omega$. Result (i) then follows because the hypotheses of Theorem 1 in Luenberger (1969, p. 217) are met. *Q.E.D.*

Once we have obtained Results (i) and (ii), to prove the Lemma of Optimality, we need to show that the maximization conditions in (18) and (19) are equivalent to the appropriate first-order conditions. We first show that these first-order conditions can indeed be computed. The following lemma helps do this.

LEMMA A.1—Differentiability of Integral Functionals with Convex Integrands: *Given a measure space* (Θ, Θ, μ) *and a function* $\psi : X \times \Theta \to \mathbb{R}$, *where* $X \subset \mathbb{R}^n$, *suppose the functional* $T : \Omega \to \mathbb{R}$, *where* Ω *is some subset of the space of all functions mapping* Θ *into* X, *is given by* $T(x) = \int_\Theta \psi(x(\theta), \theta)\mu(d\theta)$. *Suppose that* (i) *for each* $\theta \in \Theta$, $\psi(\cdot, \theta) : X \to \mathbb{R}$ *is concave*; (ii) *that the derivative* ψ_x *exists and is a continuous function of* (x, θ); *and that* (iii) $x + \alpha h \in \Omega$ *for* $\alpha \in [0, \varepsilon]$ *for some* $\varepsilon > 0$. *Then the h-directional Gateaux differential* $\partial T(x; h)$ *exists and is given by*

$$\partial T(x; h) = \int_\Theta \psi_x(x(\theta), \theta)h(\theta)\mu(d\theta)$$

if the right-hand side expression is well defined.

PROOF: Adding and subtracting $\int_\Theta \psi_x(x(\theta), \theta)h(\theta)\mu(d\theta)$ from the definition of the Gateaux differential,

$$\begin{aligned}
&\partial T(x; h) \\
&\quad = \int_\Theta \psi_x(x(\theta), \theta)h(\theta)\mu(d\theta) \\
&\quad\quad + \lim_{\alpha \downarrow 0} \int_\Theta \biggl[\frac{1}{\alpha}\bigl[\psi(x(\theta) + \alpha h(\theta), \theta) - \psi(x(\theta), \theta)\bigr] \\
&\quad\quad\quad - \psi_x(x(\theta), \theta)h(\theta)\biggr]\mu(d\theta).
\end{aligned}$$

We seek to show that the last term is well defined and vanishes.

For $\alpha < \varepsilon$ one can show that

$$\begin{aligned}
&\left|\frac{1}{\alpha}\bigl[\psi(x(\theta) + \alpha h(\theta), \theta) - \psi(x(\theta), \theta)\bigr] - \psi_x(x(\theta), \theta)h(\theta)\right| \\
&\quad \le \left|\frac{1}{\varepsilon}\bigl[\psi(x(\theta) + \varepsilon h(\theta), \theta) - \psi(x(\theta), \theta)\bigr] - \psi_x(x(\theta), \theta)h(\theta)\right|
\end{aligned} \tag{20}$$

by concavity of $\psi(\cdot, \theta)$. Whereas $\psi(x(\theta) + \varepsilon h(\theta), \theta)$, $\psi(x(\theta), \theta)$, and $\psi_x(x(\theta), \theta)h(\theta)$ are all integrable by hypothesis, it follows that $\frac{1}{\varepsilon}[\psi(x(\theta) + \varepsilon h(\theta), \theta) - \psi(x(\theta), \theta)] - \psi_x(x(\theta), \theta)h(\theta)$ is also integrable. Because a function is integrable if and only if its absolute value is integrable, then (20) provides the required integrable bound to apply Lebesgue's dominated convergence theorem, implying

$$\lim_{\alpha\downarrow 0}\int_{\Theta}\left[\frac{1}{\alpha}\left[\psi(x(\theta)+\alpha h(\theta),\theta)-\psi(x(\theta),\theta)\right]\right.$$
$$\left.-\psi_x(x(\theta),\theta)h(\theta)\right]\mu(d\theta)$$
$$=\int_{\Theta}\left[\lim_{\alpha\downarrow 0}\frac{1}{\alpha}\left[\psi(x(\theta)+\alpha h(\theta),\theta)-\psi(x(\theta),\theta)\right]\right.$$
$$\left.-\psi_x(x(\theta),\theta)h(\theta)\right]\mu(d\theta)=0$$

by definition of ψ_x. It follows that $\partial T(x;h)=\int_{\Theta}\psi_x(x(\theta),\theta)h(\theta)\mu(d\theta)$.

Q.E.D.

We can apply Lemma A.1 because the Lagrangian functional is the sum of three terms that can be expressed as integrals with concave differentiable integrands. Whereas the Lagrangian functional is defined over a convex cone Φ, hypothesis (iii) of the lemma is met with any $\varepsilon\leq 1$ for any $x\in\Phi$ and $h=y-x$ for $y\in\Phi$.

Furthermore, in our case $\int\psi_u(u(\theta),\theta)h_u(\theta)\,d\Lambda(\theta)$ is well defined for any u and h_u such that $(\underline{w},u)\in\Phi$ and $(h_{\underline{w}},h_u)\in\Phi$ for some $\underline{w},h_{\underline{w}}\in\mathbb{R}$. This follows because u and h_u are nondecreasing on Θ, they are measurable and bounded, and by standard arguments, $\psi_u(u(\theta),\theta)h_u(\theta)$ is also measurable and bounded, and thus integrable.

These arguments establish that we can write the Gateaux differential of the Lagrangian for $(\underline{w},u),(h_{\underline{w}},h_u)\in\Phi$ as

$$\partial L(\underline{w},u;h_{\underline{w}},h_u|\Lambda)$$
$$=\left(\frac{\underline{\theta}}{\beta}h_u(\underline{\theta})+h_{\underline{w}}\right)\Lambda(\underline{\theta})+\frac{1}{\beta}\int_{\underline{\theta}}^{\bar{\theta}}(\Lambda(\theta)-G(\theta))h_u(\theta)\,d\theta$$
$$+\int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{\theta}{\beta}-W'\big(y-C(u(\theta))\big)C'(u(\theta))\right]h_u\,d\Lambda(\theta),$$

which collapses to (9) at the proposed allocation.

Finally, the following lemma, which is a simple extension of a result in Lemma 1 in Luenberger (1969, p. 227), allows us to characterize the maximization conditions of the Lagrangian obtained in Results (i) and (ii) by the appropriate first-order conditions.

LEMMA A.2—Optimality and First-Order Conditions: *Let f be a concave functional on P, a convex cone in X. Take $x_0\in P$ and define $H(x_0)\equiv$*

$\{h : h = x - x_0$ and $x \in P\}$. Then $\delta f(x_0, h)$ exists for $h \in H(x_0)$. Assume that $\delta f(x_0, \alpha_1 h_1 + \alpha_2 h_2)$ exists for $h_1, h_2 \in H(x_0)$ and $\delta f(x_0, \alpha_1 h_1 + \alpha_2 h_2) = \alpha_1 \delta f(x_0, h_1) + \alpha_2 \delta f(x_0, h_2)$ for all $\alpha_1, \alpha_2 \in R$.

A necessary and sufficient condition that $x_0 \in P$ maximizes f is that

$$\delta f(x_0, x) \le 0 \quad \text{for all } x \in P,$$
$$\delta f(x_0, x_0) = 0.$$

In our case, all the hypotheses of Lemma A.2 are met for the Lagrangian, because it is a convex functional over a convex cone, and because Lemma A.1 verifies the differentiability requirement, as discussed above. Thus, we obtain that a necessary and sufficient condition for the Lagrangian to be maximized at $(\underline{w}_0, u_0)$ over Φ is

$$\partial L(\underline{w_0}, u_0; \underline{w_0}, u_0 | \Lambda_0) = 0,$$
$$\partial L(\underline{w_0}, u_0; h_{\underline{w}}, h_u | \Lambda_0) \le 0$$

for all $(h_{\underline{w}}, h_u) \in \Phi$.

Given Results (i) and (ii), the proof of the Lemma of Optimality follows.

PROOF OF PROPOSITION 8: Let $F(\cdot)$ be the cumulative distribution function of the taste shocks. We want to show that if $G(x) \equiv F(x) + x(1-\beta) f(x)$ is nondecreasing, then $\hat{G}(x) = (1 + \beta\varphi) F(\hat{\beta} x) - \beta\varphi F(\beta x)$ is nondecreasing. After letting $\lambda = 1/\varphi$ and differentiating, we obtain

$$\Delta(x, \lambda) \equiv \left(\frac{\lambda + \beta}{\beta}\right) \hat{\beta}(\lambda) f(\hat{\beta}(\lambda) x) - \beta f(\beta x) \ge 0$$

and note that $\Delta(x, 0) = 0$. Substituting the definition of $G(\cdot)$ yields the alternative expression

$$\Delta(x, \lambda) = \frac{\lambda + \beta}{\beta(1-\beta)x} \big[G(\hat{\beta}(\lambda)x) - F(\hat{\beta}(\lambda)x)\big] - \beta f(\beta x).$$

Define

$$\tilde{\Delta}(x, \lambda, z) \equiv \frac{\lambda + \beta}{\beta(1-\beta)x} \big[G(z) - F(\hat{\beta}(\lambda)x)\big] - \beta f(\beta x). \tag{21}$$

Note that $\tilde{\Delta}(x, \lambda, z)$ increases in z and that $\tilde{\Delta}(x, \lambda, \hat{\beta}(\lambda)x) = \Delta(x, \lambda)$.

To prove $\Delta(x, \lambda) \ge 0$, we write

$$\Delta(x, \lambda) = \tilde{\Delta}(x, \lambda, \hat{\beta}(\lambda)x) = \tilde{\Delta}(x, 0, \hat{\beta}(\lambda)x) + \int_0^\lambda \tilde{\Delta}_\lambda(x, \tilde{\lambda}, \hat{\beta}(\lambda)x)\, d\tilde{\lambda} \tag{22}$$

and proceed to show that both terms on the right-hand side are nonnegative.

To see the sign of the first term in (22), note that because $\tilde{\Delta}$ is increasing in z,

$$\tilde{\Delta}(x, 0, \hat{\beta}(\lambda)x) \geq \tilde{\Delta}(x, 0, x) = \tilde{\Delta}(x, 0, \hat{\beta}(0)x) = \Delta(x, 0) = 0.$$

For the integral term in (22) we compute the integrand by differentiating (21) and rearranging using the definition of $G(\cdot)$:

$$\begin{aligned}
&\tilde{\Delta}_\lambda(x, \lambda, z)\\
&\quad = \frac{1}{\beta(1-\beta)x}\\
&\qquad \times \left[G(z) - G(\hat{\beta}(\lambda)x) + \frac{\lambda}{1+\lambda}\hat{\beta}(\lambda)x(1-\beta)f(\hat{\beta}(\lambda)x)\right].
\end{aligned}$$

Thus, for $z \geq \hat{\beta}(\tilde{\lambda})x$, we have $\tilde{\Delta}_\lambda(x, \tilde{\lambda}, z) \geq 0$. It follows that, for $\tilde{\lambda} \in [0, \lambda]$, we have $\hat{\beta}(\lambda)x \geq \hat{\beta}(\tilde{\lambda})x$ and, therefore, $\tilde{\Delta}_\lambda(x, \tilde{\lambda}, \hat{\beta}(\lambda)x) \geq 0$. Thus, the integral term in (22) is nonnegative. Given that $\hat{\beta}(\lambda)x_p(\lambda)$ is nondecreasing in λ, we need $G(x)$ to be nondecreasing up to $\hat{\beta}x_p$. *Q.E.D.*

PROOF OF PROPOSITION 9: Writing $1 - \hat{G}(x) = (1 + \beta\varphi)(1 - F(\hat{\beta}x)) - \beta\varphi(1 - F(\beta x))$, integrating, and rearranging yields

$$\begin{aligned}
&\int_{x_0}^{\bar{x}} (1 - \hat{G}(z))\,dz\\
&\quad = (1+\varphi)\hat{\beta}\int_{x_0}^{\bar{x}}(1 - F(\hat{\beta}x))\,dx - \beta\varphi\int_{x_0}^{\bar{x}}(1 - F(\beta x))\,dx\\
&\quad = (1+\varphi)\int_{\hat{\beta}x_0}^{\bar{\theta}}(1 - F(\theta))\,d\theta - \varphi\int_{\beta x_0}^{\bar{\theta}}(1 - F(\theta))\,d\theta\\
&\quad = \int_{\beta x_0}^{\bar{\theta}}(1 - F(\theta))\,d\theta - (1+\varphi)\int_{\beta x_0}^{\hat{\beta}x_0}(1 - F(\theta))\,d\theta\\
&\quad = \int_{\beta x_0}^{\bar{\theta}}(1 - F(\theta))\,d\theta - \int_0^{x_0(1-\beta)}\left(1 - F\left(\frac{y}{1+\varphi} + \beta x_0\right)\right)dy.
\end{aligned}$$

The second equality uses the change in variables $\theta = \hat{\beta}x$ for the first integral, $\theta = \beta x$ for the second, and the fact that $\hat{\beta}\bar{x} > \beta\bar{x} = \bar{\theta}$. The third equality simply rearranges the integrals. The fourth equality performs the change of variables $y = (1+\varphi)(\theta - \theta_0)$ using the fact that $1 + \varphi = (1-\beta)/(\hat{\beta} - \beta)$.

The comparative static with respect to φ is now straightforward: An increase in φ raises the integrand $1 - F(y/(1+\varphi) + \beta x_0)$ so that x_p must fall with φ.

To obtain the comparative static with respect to β, we differentiate the last expression:

$$\begin{aligned}
&\frac{\partial}{\partial\beta}\int_{x_0}^{\bar{x}}(1-\hat{G}(z))\,dz \\
&\quad=\left[F(\hat{\beta}x_0)-F(\beta x_0)+\int_0^{x_0(1-\beta)} f\left(\frac{1}{1+\varphi}y+\beta x_0\right)dy\right]x_0 \\
&\quad>0,
\end{aligned}$$

implying that x_p rises with β.

Finally, note that the minimum-savings $k_{\min}$ is defined as the solution to

$$x_p\frac{U'(y-k_{\min})}{W'(k_{\min})}=1,$$

so that comparative statics for x_p translate directly into $k_{\min}$. In particular, $k_{\min}$ is increasing in φ and decreasing in β. *Q.E.D.*

PROOF OF PROPOSITION 10: First note that part (i) of the single-crossing assumption implies that for an allocation (c,k) to be incentive compatible, $c(\theta)$ has to be nondecreasing.

To show that $(\hat{c},\hat{k})$ is feasible, first note that if the resource constraints were satisfied at the original allocation (c,k), they are also satisfied at $(\hat{c},\hat{k})$. For incentive compatibility, note that $(\hat{c},\hat{k})$ remains incentive compatible for all types $\theta\le\theta_b$ given that (c,k) is incentive compatible. For type θ_b we have that

$$U\big(c(\theta_b),k(\theta_b),\theta_b\big)\ge U\big(c(\theta),k(\theta),\theta_b\big)\quad\text{for all }\theta\le\theta_b.$$

Given that $c(\theta)$ is nondecreasing, it follows from part (i) of the single-crossing assumption that

$$U\big(c(\theta_b),k(\theta_b),\theta'\big)\ge U\big(c(\theta),k(\theta),\theta'\big)\quad\text{for all }\theta\le\theta_b\le\theta'.$$

The new allocation $(\hat{c},\hat{k})$ is thus incentive compatible. We now show that it is an improvement over the original allocation.

Note that

$$U\big(c(\theta_b),k(\theta_b),\theta_b\big)\ge U\big(c(\theta),k(\theta),\theta_b\big)\quad\text{for all }\theta>\theta_b.$$

From monotonicity and from part (iii) of the single-crossing assumption, it follows that

$$V\big(c(\theta_b),k(\theta_b),\bar{\theta}\big)\ge V\big(c(\theta),k(\theta),\bar{\theta}\big)\quad\text{for all }\theta>\theta_b.$$

Using part (ii) of the single-crossing assumption,

$$V\big(c(\theta_b), k(\theta_b), \theta\big) \geq V\big(c(\theta), k(\theta), \theta\big) \quad \text{for all } \theta > \theta_b.$$

So, in the new feasible allocation $(\hat{c}, \hat{k})$, the value to the planner has weakly improved for all types. The new allocation $(\hat{c}, \hat{k})$ that bunches types above θ_b is then a weak improvement over (c, k). *Q.E.D.*

REFERENCES

AMADOR, M., G.-M. ANGELETOS, AND I. WERNING (2004): "Corrective vs. Redistributive Taxation," Mimeo, M.I.T.

AMADOR, M., I. WERNING, AND G.-M. ANGELETOS (2003): "Commitment vs. Flexibility," Working Paper 10151, National Bureau of Economic Research.

——— (2006): "Supplement to 'Commitment vs. Flexibility'," *Econometrica Supplementary Material*, 74, http://www.econometricsociety.org/ecta/supmat/5090results.pdf.

ATHEY, S., A. ATKESON, AND P. KEHOE (2005): "The Optimal Degree of Monetary Policy Discretion," *Econometrica*, 73, 1431–1476.

BENABOU, R., AND J. TIROLE (2002): "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics*, 117, 871–915.

CARRILLO, J., AND T. MARIOTTI (2000): "Strategic Ignorance as a Self-Disciplining Device," *Review of Economic Studies*, 67, 529–544.

DEKEL, E., B. L. LIPMAN, AND A. RUSTICHINI (2001): "Representing Preferences with a Unique Subjective State Space," *Econometrica*, 69, 891–934.

——— (2004): "Temptation Driven Preferences," Mimeo, Boston University.

DELLA-VIGNA, S., AND U. MALMENDIER (2004): "Contract Design and Self-Control: Theory and Evidence," *Quarterly Journal of Economics*, 119, 253–402.

DIAMOND, P. (1977): "A Framework for Social Security Analysis," *Journal of Public Economics*, 8, 275–298.

ELIAZ, K., AND R. SPIEGLER (2004): "Contracting with Diversely Naïve Agents," CEPR Discussion Paper 4573.

ESTEBAN, S., AND E. MIYAGAWA (2005): "Optimal Menu of Menus with Self-Control Preferences," Mimeo, Columbia University.

GUL, F., AND W. PESENDORFER (2001): "Temptation and Self-Control," *Econometrica*, 69, 1403–1435.

——— (2004a): "Self-Control and the Theory of Consumption," *Econometrica*, 72, 119–158.

——— (2004b): "Self-Control, Revealed Preference and Consumption Choice," Mimeo, Princeton University.

HOLMSTRÖM, B. (1977): "On Incentives and Control in Organizations," Ph.D. Thesis, Stanford Graduate School of Business.

——— (1984): "On the Theory of Delegation," in *Bayesian Models in Economic Theory*, ed. by M. Boyer and R. E. Kihlstrom. Amsterdam: North-Holland, 115–141.

KREPS, D. (1979): "A Representation Theorem for Preference for Flexibility," *Econometrica*, 47, 565–577.

KRUSELL, P., B. KURUŞÇU, AND A. SMITH (2005): "Temptation and Taxation," Mimeo, Princeton University.

LAIBSON, D. (1997): "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 112, 443–477.

——— (1998): "Life-Cycle Consumption and Hyperbolic Discount Functions," *European Economic Review*, 42, 861–871.

LUENBERGER, D. G. (1969): *Optimization by Vector Space Methods*. New York: Wiley.
MARTIMORT, D., AND A. SEMENOV (2004): "Communication by Interest Groups and the Organization of Lobbying," Mimeo, University of Toulouse.
MELUMAD, N. D., AND T. SHIBANO (1991): "Communication in Settings with No Transfers," *The RAND Journal of Economics*, 22, 173–198.
PHELPS, E. S., AND R. POLLACK (1968): "On Second Best National Savings and Game-Equilibrium Growth," *Review of Economic Studies*, 35, 185–199.
RUDIN, W. (1976): *Principles of Mathematical Analysis* (Third Ed.). New York: McGraw-Hill.
SARAFIDIS, Y. (2005): "Inter-Temporal Price Discrimination with Time Inconsistent Consumers," Mimeo, INSEAD.
STROTZ, R. H. (1956): "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, 23, 165–180.