Organizational Capacity and Project Dynamics*

Dana Foarta Michael M. Ting

January 5, 2023

Abstract

This paper provides a dynamic theory of the effects of organizational capacity on public policy. Consistent with prevailing accounts, a bureaucratic organization with higher capacity, i.e., a better ability to get things done, is more likely to deliver projects in a timely, predictable, or efficient fashion. However, capacity also interacts with political institutions to produce far-reaching implications for the size and distribution of public projects. Capacity-induced delays and institutional porousness can allow future political opponents to revise projects in their favor. In response, politicians design projects to avoid revisions, for example by equalizing distributive benefits, or by overscaling projects. We show that higher organizational capacity can increase project size, inequalities in the distribution of project benefits, and delays. The range of capacity levels that produce low social benefits increases with the extent of institutional constraints. This suggests that political systems with high capacity and high institutional constraints are especially vulnerable to inefficient projects.

Keywords: Organizational Capacity, Power Transitions, Project Scale, Project Delays.

JEL codes: D73, D82

*Foarta: Stanford Graduate School of Business and CEPR, email: ofoarta@stanford.edu. Ting: Columbia University, email: mmt2033@columbia.edu. We wish to thank seminar participants at MIT, Stanford University, Bocconi University, University of Padua, SITE Political Economy Theory, and POLECONUK for useful suggestions.

1 Introduction

It is now a truism that organizations are crucial for the outcome of government policies in modern society. Election candidates can make platform promises and legislators can pass laws, but a massive bureaucratic machinery is needed to translate statutes into on-the-ground results.[1] Capturing organizational performance is obviously a formidable task, but practitioners and scholars have increasingly coalesced around the concept of *organizational capacity* as a central determinant. Bodies as varied as the UNDP, USAID, OECD (2011), and the European Centre for Development Policy Management (Keijzer et al., 2011) identify organizational capacity as a key development objective, and scholarly mentions of the term have increased sharply since the 1990s.[2]

The appeal of organizational capacity is clear. Higher capacity — loosely speaking, a better ability to "get things done" — should produce policy outputs that are more timely, more efficient, or of higher quality. Consistent with this perspective, a wide variety of studies have shown that organizations that are under-resourced, under-paid, or prone to political interference produce worse results (e.g., Derthick, 1990; Rauch and Evans, 2000; Gorodnichenko and Peter, 2007; Propper and Van Reenen, 2010). Yet in many political settings, the implications of capacity are less obvious. To take a simple example, suppose that a political system gives broad legal standing to actors who have environmental objections to a construction project. In this setting, a high-capacity bureaucracy might actually encourage litigation and its attendant delays, since victorious litigants can be confident that their proposals will be implemented quickly.

This paper develops a dynamic theory of policymaking that jointly considers organiza-

[1]The Organization for Economic Co-operation and Development estimates that as of 2019, government entities accounted for an average of 18% of member country employment (OECD, 2021).

[2]See "Capacity Development: A UNDP Primer," USAID's "Measuring Organizational Capacity." As of August 2022, Google Scholar returned about 4,880 results for "organizational capacity" between 1990 and 1999, 16,000 between 2000 and 2009, 23,400 between 2010 and 2019, and 15,400 since 2020.

tional capacity and its political and institutional context. Its main objective is to show how these features combine to affect the planning and execution of public policies, in terms of scale, distribution of benefits, and delays. While many elements of our model are standard, the principal hurdle in any such effort is the lack of consensus about how to characterize organizational capacity. A predominant approach in empirical research is to treat capacity as an input into organizational production functions. Such inputs include information (Lee and Zhang, 2017) and perhaps most prominently, human capital (Brown et al., 2009; Dal Bó et al., 2013; Acemoglu et al., 2015; Bolton et al., 2016). Theoretical efforts have thus far adopted widely divergent perspectives on how to incorporate the concept into standard political economy frameworks, ranging from the variance of policy outcomes (Huber and McCarty, 2004), policy valence (Ting, 2011), to agency cost structures (Foarta, 2022).

Our conceptualization of capacity blends many of the insights of existing approaches. Its basis is a discrete Markov process representation of policy projects. Completing a project requires traversing a sequence of bureaucratic stages; for example, research must be completed before construction can begin. Capacity is the probability of progressing from each stage to the next in a given period. If it does not progress, the project remains in the same stage to begin the next period. Benefits are realized upon completion, but each period before completion imposes costs that are increasing in the project's scale. Thus in the absence of renegotiation or political interference, an agency with higher capacity — due to better personnel or technology — reduces costs and variability in delivery times.

The model embeds this process in an institutional environment that gives access to political opponents. At the inception of a project, a representative from one group chooses its scale and an initial distribution of benefits between her group and an opposing group. This distribution may represent a siting choice, or the selection of contractors. After the project begins, groups randomly receive opportunities to attempt to revise the project. Depending on the political system, these opportunities can arise from various sources, for example the election of new politicians or the mobilization of NIMBY groups. Attempting a revision delays project completion by automatically pausing progress. The revision itself succeeds with some probability that corresponds to the openness of the institutional environment to outside intervention. This openness reflects factors such as contracting regulations, the judicial system, or administrative procedures such as the US National Environmental Policy Act (NEPA) review process. A successful revision changes the project's distribution of payoffs to favor the revising party. The original project designer must then take the possibility of strategic revisions into account in choosing the project's scale and payoff split; in particular, one liability of low capacity is the increased opportunity for political intervention during the course of project execution.

A principal attraction of this formulation is its correspondence to the operational realities of implementing many public policies. A good example is the process of constructing large infrastructure projects in the US.[3] The federal government's main mechanism for supporting significant public transportation projects is the Federal Transit Administration (FTA) Capital Investment Grants (CIG) program. CIG administers over $2 billion a year through a competitive grant process, whereby state or local transit agencies propose cost-sharing collaborations with the FTA. Applications must traverse two stages of FTA review before construction can begin. The first, "Project Development," requires a completed NEPA review, approval by local authorities, and secured commitments for at least 30% of non-federal funds. The second, "Engineering," finalizes funding sources and design details, including geotechnical and safety hazard reports. Each phase can be a lengthy undertaking, thus exposing projects to both lawsuits and political turnover.

We find that the interaction between capacity and the institutional environment has significant implications for public projects. Consider starting from a benchmark in which the

[3]The Federal Infrastructure Projects Permitting Dashboard tracks the progress of federally-funded infrastructure projects across major permitting requirements. The Center for an Urban Future provides an overview of the key phases and sources of delay for capital construction projects in New York City.

opposition group never has an opportunity to attempt a revision. In this case, higher capacity has the straightforward effects of reducing completion time and costs, thereby increasing project scale. The initiating politician furthermore awards herself the entire benefit of the project. If the opposition group is given the opportunity to attempt a revision, then the threat of the project being revised has two possible effects. First, it encourages the project initiator to design a larger project. The high running costs of such a project deter revisions due to the prohibitive escalations in total costs. Second, it encourages more equal payoff divisions, as these reduce the gains from revisions. These deterrence effects matter only to politicians who are relatively likely to face future revision attempts: the side that is unlikely to have revision opportunities will typically not attempt revisions, since their revisions are likely to be reversed. Thus, a politically favored initial politician is more likely to achieve her benchmark ideal policy. An unfavored politician is more likely to distort the size and distribution of her projects in order to avoid revisions. When capacity is very low, politicians choose more egalitarian distributions and (to compensate for the reduced project gains) underscaled projects. As capacity increases, they claim an increasing share of project benefits and switch to overscaling. In all cases, high capacity results in winner-take-all allocations.

These results feature no politically-induced delays in equilibrium, but they assume that politicians can freely choose any project scale. Also, they assume that scale increases do not augment running costs so much as to make the project altogether undesirable. In practice, both of these concerns may be present. Scales are often constrained by budgets or physical limitations. Even when physically possible, increasing scale may lead to rapidly raising costs (if the costs are very elastic). Such conditions could make an overscaling strategy unattainable. Modest scales and high capacity imply low running costs, and thereby encourage revisions. The surprising implication is that *higher capacity produces greater obstruction* and delay.

The adjustments that project designers make to avoid revisions have important implications for social welfare. Underscaled and benchmark projects generally provide greater benefits than costs, but overscaled projects can cause the agents to do collectively worse than no project at all. We show that the capacity values that generate such projects both increase and expand with the ease of revisions. A political system that has high organizational capacity and institutional barriers is therefore most prone to overscaling. Consequently, the optimal institutional structure should feature either low capacity and high barriers to completion (i.e., high openness), or high capacity and low barriers.

We finally explore a variant of the model with a more complex project that requires two phases. Here, scales are chosen independently in each phase and the output of the first phase is an "investment" that reduces costs for the project in the second phase. The main result is that the first phase initiator may now invest nothing and effectively cancel a project if she worries about possible overscaling by the opponent. Thus, the prospect of setting project parameters mid-stream can force politicians to internalize welfare consequences to some degree.

Related Literature. A main contribution of this paper is its formalization of organizational capacity as part of a dynamic political process. The execution of policy in our model generates measurable outcomes such as the size, timing, cost, and distributive dimensions of public projects. Several important lines of theoretical work have used related notions of capacity to explore different policy questions. Perhaps most prominently, a recent literature on "state capacity" addresses the ability of the state to achieve macro-objectives such as tax collection and law enforcement (Besley and Persson, 2009; Johnson and Koyama, 2017). One emphasis of this work is the creation of capacity in the shadow of political transitions. By contrast, we address policymaking at the organizational level, taking capacity as given. The granular focus on organizations can be useful because, as many observers have noted, organizational capabilities can vary greatly within a country (Carpenter, 2001).

A series of models by Huber and McCarty (2004, 2006) situates bureaucratic capacity in an explicit institutional setting. They examine the relationship between a legislative principal and a bureaucratic agent, and represent capacity as the variance of possible outcomes following a bureaucratic policy choice. The outcome space in these models is ideological, and the primary outputs include delegation, compliance, and whether legislation is possible. Other institutional theories that model capacity as costs include Foarta (2022) and Turner (2020), who analyze a dynamic electoral setting and policymaking in a separation of powers system, respectively. Aside from a different set of outcomes, another contribution of our present paper is formalizing organizational capacity to generate both variance and costs.

Finally, a now extensive set of theoretical models addresses the dynamics of long-term policies (e.g., Baron, 1996; Battaglini et al., 2012; Callander and Raiha, 2017). Similarly, a growing literature studies the optimal provision of incentives in dynamic environments with multiple stage projects (e.g., Toxvaerd, 2006; Green and Taylor, 2016; Feng et al., 2021). Yet, there is little theoretical work on the political economy of large multistage public investments. Foarta and Sugaya (2021) study the optimal funding for public projects by a lender in a repeated relationship with a local policymaker. Their focus is on how the lender can use the sequential funding of projects to learn and give dynamic incentives to the policymaker. We focus on a setting with multi-stage projects, and on how the expectation of future revisions or cancellations affects initial project characteristics.[4]

Paper Structure. The rest of the paper is organized as follows. The next section discusses how our modeling approach relates to project features observed in practice. Section 3 describes the model, and Section 4 analyzes it and presents the main results. Section 5 extends the model to allow for multiple decisions over project size. Section 6 briefly presents

[4]Focusing on transportation projects specifically, Glaeser and Ponzetto (2018) develop a model of project scale, focusing on voter inattention as the driver for politicians to propose very large projects: increased voter attention to local negative externalities leads to reductions in project scale, and is consistent with evidence of positive correlation between voter education and highway costs.

two examples that illustrate some equilibrium implications. Finally, Section 7 concludes and the Appendix contains all formal derivations and proofs.

2 Motivating Examples

The parameters and mechanisms of our model map into commonly observed features of bureaucracies and public projects. In this section we provide examples of how some of the main components of the model have appeared in the implementation of public policies.

Project Stages and Capacity. The role of the organization is to deliver a completed project by solving problems in a series of stages. In many cases, stages correspond to well-defined organizational practices, such as those involved in US federal contracting:

> [T]he federal contracting process has three separate but related parts: (1) planning (how federal agencies decide what and how much to contract for, when they need given goods or services to be delivered, and what terms and conditions are they subject to); (2) awarding (the background market research, the communications and outreach to prospective contractors, the budgetary criteria, and the precise procedures for awarding competitive bids or making noncompetitive selections); and (3) overseeing (everything from routine reporting requirements to financial audits, field inspections, public comments, and impact studies). (DiIulio, 2014, p. 65)

We model capacity as the probability p that an organization will progress to the next stage in a given period. This parameter perhaps corresponds most closely to prevailing empirical notions of capacity, which often emphasize human capital. Shortfalls in staffing or human capital have frequently been observed to reduce bureaucratic productivity. For example, understaffing at the US Office of Information and Regulatory Affairs has been shown to delay the issuance of federal rules, including the Biden administration's current efforts to update energy efficiency standards for lighting and appliances (Bolton et al., 2016).[5]

Revisions and Delays. Even the most competent public organizations—fully staffed with well-trained, well-paid, and uncorrupt bureaucrats, and equipped with modern technology—face political scrutiny in executing their tasks. As projects become prominent, the opportunities for intervention multiply, and especially so in decentralized institutional systems (Pressman and Wildavsky, 1984).

Our model parameterizes these opportunities in two ways. The first is the likelihood that a party other than the project designer can attempt a revision. Transitions of power due to elections can play this role, but intra-organizational conflict, interest group mobilization, and access to litigation provide openings for contestation as well. The second is the probability that a challenge succeeds. In particular, the extensive reporting requirements of laws such as NEPA and the California Environmental Quality Act provide rationales for reconsidering projects, such as insufficient consideration of alternatives (e.g., Mandelker, 2010). As the model assumes, even unsuccessful challenges can impose costly delays: one report estimated that the 197 NEPA environmental impact statements completed in 2012 took an average of 4.6 years to finalize (US GAO, 2014) (US GAO, 2014). Academic and policy observers have increasingly focused on such regulatory barriers as sources of delay and cost inflation in US infrastructure construction (Smith et al., 1999; Brooks and Liscow, 2022; Mehrotra et al., 2022).[6]

Distribution. Challenges to a project frequently aim to alter its distribution of payoffs. In addition to environmental concerns, revisions may address features such as siting and

[5]See Anna Phillips, "Biden faces delays in undoing Trump's war on efficient dishwashers, dryers and lightbulbs that made him 'look orange'." *Washington Post*, January 9, 2022.

[6]The NYU Transit Costs Project provides a useful overview of the factors that drive transportation costs in modern infrastructure projects.

the set of eligible contractors. The expansion of Atlanta's airport offers a clear example of the latter considerations (Altshuler and Luberoff, 2003). In 1972, the city purchased 10,000 acres to the north of downtown with an eye toward a new facility, but the 1973 election of Maynard Jackson, Atlanta's first black mayor, changed these plans. As a relative outsider, Jackson advocated instead for expanding the existing Hartsfield airport, which was located closer to his political base south of downtown. He additionally took the innovative and controversial step of setting aside 25% of contracts for minority-owned firms (Stone, 1989) (Stone, 1989). With some compromises, this vision largely prevailed and the completed airport was subsequently re-christened with its current name, Hartsfield-Jackson.

3 Model

Consider an environment with infinite, discrete time, $t = 0, 1, 2....$ There are two agents, A and B, representing two distinct political constituencies. Agent A is in control of policy at time 0, and may be thought of as a politician in power at that time. Agent B is the opposition, either another politician or an outside interest group opposed to A. Agent A initiates a long-term project at time 0. Once initiated, the project is run by a non-strategic bureaucracy, and it must go through several stages before reaching completion. In the absence of outside intervention, the bureaucracy moves the project through the required stages, where each stage lasts at least a period. At the end of each period, a transition that switches control to the other agent may occur. Whoever is in control can attempt to change aspects of the ongoing project. The game ends when the project is completed.

The Project. A public project delivers value $v > 0$ per unit produced. It has two main characteristics which are chosen at its initial conception: (1) the scale $s \in [0, s^{\max}]$ with

$s^{\max} > 0$; this captures the number of units that are produced, up to same maximum cap;[7] and (2) the distribution of benefits between the two agents: the fraction $w \geq 0.5$ of the project's payoff that goes to one agent versus the fraction $1 - w$ that goes to the other. The project's inequality is therefore measured by $\Delta = 2w - 1 \in [0, 1]$, where $\Delta = 1$ is maximal inequality and $\Delta = 0$ is the equal division of benefits. The project starts in stage d (the development stage), and it must reach stage e (execution) in order to be completed.

The project delivers its benefits once it reaches stage e. Progression from one stage to the next depends on the organizational capacity of the bureaucracy. Higher capacity allows the bureaucracy to overcome the technical hurdles needed to move the project forward with greater speed. We parameterize capacity by p, the probability with which the project moves from stage d to stage e in any given period. With probability $1 - p$, the project does not progress that period. Every period spent in stage d costs each agent $c(s)$, where we assume the following:

Assumption 1 *The cost function $c(s)$ is continuous, twice differentiable, satisfies $c'(s) > 0$, $c''(s) > 0$, $c(0) = 0$, and has elasticity*

$$\varepsilon(s) \equiv \frac{c'(s) \cdot s}{c(s)} \geq 1.$$

The per-period cost $c(s)$ captures in reduced from the shared costs of a public project: the use of general tax revenue or other public resources for keeping the project running. Hence, the cost is paid by both agents, regardless of the final division of project benefits. The cost is increasing and convex in s. Its elasticity with respect to s, $\varepsilon(s)$, is larger than 1 to ensure that $\frac{c(s)}{s}$ is increasing in s, i.e., larger projects are relatively costlier.

[7]We discuss restrictions on $s^{\max}$ in Section 4.3.

Transitions of Control and Revisions. At the beginning of period 0, agent A chooses the scale of s and payoff division $(w, 1-w)$, where agent A receives fraction w of vs and agent B receives fraction $1-w$. We refer to this distribution of benefits as the project of type Δ^A, as the payoff division favors agent A. Analogously, we denote by Δ^B the project with benefit distribution $(1-w, w)$, where fraction w of the benefit goes to agent B.

At the end of each period t, control over the project may change. With probability r, agent A has control next period. With probability $1-r$, agent B gets control. This captures the probability of the opposition winning political power, or it may simply be the probability with which an opportunity for an appeal arises for the opposition (even under the same political incumbent). The agent in control may then choose to trigger a project revision. Once triggered, a revision freezes the project for the current period, so that it cannot advance to the next stage. With probability q, the revision is successful and changes the project type. There is no additional cost of triggering a review. The parameter q captures institutional or legal barriers, with higher values representing greater openness to amendments of ongoing projects.

Payoffs. A project of type Δ^i completed after $\mathbb{T}$ periods has payoff to agent $i \in \{A, B\}$

$$w \cdot v \cdot s - \mathbb{T} \cdot c(s). \tag{1}$$

Timing. To summarize, the timing is as follows. In period 0, agent A starts a project Δ^A, and chooses its scale s and distribution of benefits $(w, 1-w)$. In each period $t \geq 1$, while the project is in stage d:

1. With probability r, agent A has control over the project; with probability $1-r$, agent B has control.

2. The agent in control chooses whether to trigger a revision.

(a) If a revision is triggered, it succeeds with probability q, and the project type switches from Δ^i to Δ^j, where $i \neq j$, $i, j \in \{A, B\}$; with probability $1 - q$, the revision fails, the project type does not change, and the project remains in stage d for the period.[8]

(b) If a revision is not triggered, then the project moves to stage e with probability p; with probability $1 - p$, it remains in stage d for the period.

3. Each agent pays the project operating cost $c(s)$ for the period.

Once the project reaches stage e, its benefits are realized given the current project type. There is no discounting between periods.

Equilibrium Concept. We derive the Markov Perfect Equilibria of this game with state variables for periods $t \geq 1$ being the current project stage, $S_t \in \{d, e\}$, the agent in control that period, $P_t \in \{A, B\}$, and the project type $\Delta^i \in \{\Delta^A, \Delta^B\}$. In period 0, the state variable is $P_0 = A$. Each period $t \geq 1$, agent P_t chooses a probability of revision $\sigma^{P_t}(\Delta^i) \in [0, 1]$ to maximize her expected utility. In period 0, agent P_0 chooses s and w to maximize her expected utility.

We note that any strategy in which an incumbent i revises a project of type Δ^i is weakly dominated. Thus, $\sigma^i(\Delta^i) = 0$, and we simplify notation by denoting $\sigma^i \equiv \sigma^i(\Delta^j)$, for $i \neq j$.

3.1 Benchmark with No Transitions of Control

Our institutional setting allows opportunities for revisions to an ongoing project. To understand what revisions mean for project characteristics and dynamics, we first analyze the benchmark case where there are no transitions of control ($r = 1$). Agent A starts in control

[8] An alternative effect of a revision could be for the revising agent to directly choose a new benefit distribution $(\hat{w}, 1 - \hat{w})$; we explore this alternative setup in ongoing work and show how current insights translate (details available upon request).

in period 0 and remains in control until the project reaches execution. Agent A has no reason to revise her own project. She chooses $w \in [0,1]$ and $s \in [0, s^{\max}]$ to maximize

$$\max_{s,w} w \cdot v \cdot s - \mathbb{T}(p) \cdot c(s), \tag{2}$$

where $\mathbb{T}(p) = \frac{1}{p}$ is the expected time to project completion. Agent A assigns all the benefits to herself (producing inequality $\Delta^{NT} = 1$) and chooses a scale implicitly given by

$$c'(s^{NT}) = vp. \tag{3}$$

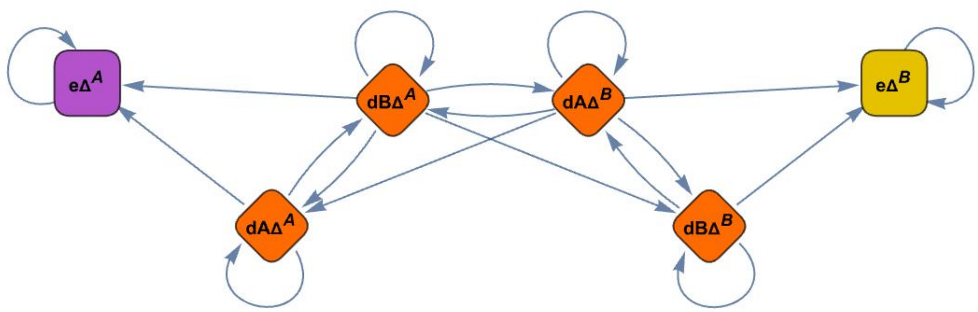
In the full analysis, we can compare how transitions of control and the threat of revisions affect project characteristics relative to Δ^{NT} and s^{NT}.

4 Public Projects under Transitions of Control

We now analyze the full model. Turnovers in control create the possibility of project revisions, which delay progress, lengthen completion times, and raise running costs. The goal of our analysis is to understand if and when revisions occur, and what that implies for the initial characteristics of projects. The answer is not immediately obvious. On the one hand, the expectation of higher running costs due to revisions should decrease the initial scale chosen in period 0 and increase benefit inequality Δ, as each revision can swing the project in one's favor. On the other hand, increasing the initial scale or reducing Δ could be used strategically to discourage revisions by increasing their cost or decreasing their benefits.

To solve for the equilibrium project scale, distribution of benefits, and the path of revisions, we break up the problem into two main steps. First, for a given scale s and payoff inequality Δ, we find the optimal revision strategy for each agent in period $t \geq 1$. Second, we find the s and Δ chosen by agent A at time 0 given the expected continuation play.

Figure 1: Markov Graph of Project Evolution



Note: Illustrates the Markov Process that governs the evolution of the project. Each state registers the project stage (d and e), the agent in control (A or B) and the current project type (Δ^A or Δ^B).

4.1 The Revision Response

In each period $t \geq 1$, the project's evolution into the next period can be represented as a Markov Process with six states, given the possible combination of stage, controlling agent, and project type. The probability of the project moving from its current state to any of the possible states depends on the probability of a transition, r, the bureaucracy's capacity, p, and the revision probabilities σ^A and σ^B. A project at stage e is in an absorbing state, with payoffs given its type, Δ^A or Δ^B. The Markov process is represented in graphical form in Figure 1 and in matrix form in Figure 2. Starting in a state (d, i, Δ^k) with agent i in control and project type Δ^k, the Markov transition probabilities imply an expected probability of reaching stage e with project type Δ^ℓ of $\mathbb{P}(e, \Delta^\ell | d, i, \Delta^k)$ and an expected number of periods needed to reach stage (e, Δ^ℓ) of $\mathbb{T}(e, \Delta^\ell | d, i, \Delta^k)$. We can use these objects to compute the expected utility for each agent, starting from any project stage, for revision strategies σ^A and σ^B.

Figure 2: Project Evolution as a Markov Process

	d, A, Δ^A	d, A, Δ^B	d, B, Δ^A	d, A, Δ^B	e, Δ^A	e, Δ^B
d, A, Δ_A	$(1-p)r$	0	$(1-r)(1-p)$	0	p	0
d, B, Δ^B	$qr\sigma^A$	$(1-p)r(1-\sigma^A)$ $+(1-q)r\sigma^A$	$q(1-r)\sigma^A$	$(1-p)(1-r)(1-\sigma^A)$ $+(1-q)(1-r)\sigma^A$	0	$p(1-\sigma^A)$
d, B, Δ^A	$(1-p)r(1-\sigma^B)$ $+(1-q)r\sigma^B$	$qr\sigma^B$	$(1-p)(1-r)(1-\sigma^B)$ $+(1-q)(1-r)\sigma^B$	$q(1-r)\sigma^B$	$p(1-\sigma^B)$	0
d, B, Δ^B	0	$(1-p)r$	0	$(1-p)(1-r)$	0	p
e, Δ^A	0	0	0	0	1	0
e, Δ^B	0	0	0	0	0	1

Note: Transition matrix for the project. Each state of the Markov Process is given by the project stage (d or e), controlling agent (A or B), and project type (Δ^A or Δ^B).

For agent A, the expected utility given controlling agent $i \in \{A, B\}$ and current project type Δ^k is:

$$U^A(i, \Delta^k|s, w) = \mathbb{P}(e, \Delta^A|d, i, \Delta^k) \cdot w \cdot v \cdot s + \mathbb{P}(e, \Delta^B|d, i\Delta^k) \cdot (1-w) \cdot v \cdot s \\ - [\mathbb{P}(e, \Delta^A|d, i, \Delta^k) \cdot \mathbb{T}(e, \Delta^A|\Delta^k) + \mathbb{P}(e, \Delta^B|d, i\Delta^k) \cdot \mathbb{T}(e, \Delta^B|d, i, \Delta^k)] \cdot c(s). \quad (4)$$

For agent B, the only difference is in the payoffs at each terminal state: fraction $1-w$ of $v \cdot s$ at (e, Δ^A) and fraction w at (e, Δ^B).

Given revision probability σ^i, where $i \in \{A, B\}$, agent $j \neq i$ prefers to revise if her expected utility from doing so is higher than the expected utility from continuing with the current project type. Revisions will be less likely as the scale of the project increases, and running costs increase with it.

Lemma 1 *There exist thresholds $\bar{s}_1 \geq \bar{s}_2 \geq \bar{s}_3$ on project scale s such that*

- *If $\frac{c(s)}{s} \leq \frac{c(\bar{s}_3)}{\bar{s}_3}$, the project is revised every time there is a transition in control: $\sigma^A = \sigma^B = 1$.*
- *If $\frac{c(\bar{s}_3)}{\bar{s}_3} < s \leq \frac{c(\bar{s}_2)}{\bar{s}_2}$, the project is revised only by the agent more likely to be in control: $\sigma^A = 1, \sigma^B = 0$ if $r \geq \frac{1}{2}$, and $\sigma^A = 0, \sigma^B = 1$ if $r < \frac{1}{2}$;*
- *If $\frac{c(\bar{s}_2)}{\bar{s}_2} < s < \frac{c(\bar{s}_1)}{\bar{s}_1}$, then either exactly one agent revises ($\sigma^A = 1, \sigma^B = 0$ or $\sigma^A = 0, \sigma^B = 1$) or there is a mixed strategy equilibrium with $\sigma^A, \sigma^B \in (0, 1)$.*
- *If $s \geq \frac{c(\bar{s}_1)}{\bar{s}_1}$, the project is never revised: $\sigma^A = \sigma^B = 0$;*

The equilibrium regions are represented in Figure 3.[9] Note that Assumption 1 implies $\frac{c(\bar{s}_3)}{\bar{s}_3} \leq \frac{c(\bar{s}_2)}{\bar{s}_2} \leq \frac{c(\bar{s}_1)}{\bar{s}_1}$.

[9]The threshold values are derived in the Appendix as $\frac{c(\bar{s}_1)}{\bar{s}_1} = qv\Delta$, $\frac{c(\bar{s}_2)}{\bar{s}_2} = qv\Delta \cdot \max\left\{\frac{pr}{pr+2q(1-r)}, \frac{p(1-r)}{p(1-r)+2qr}\right\}$, $\frac{c(\bar{s}_3)}{\bar{s}_3} = qv\Delta \cdot \min\left\{\frac{pr}{pr+2q(1-r)}, \frac{p(1-r)}{p(1-r)+2qr}\right\}$.

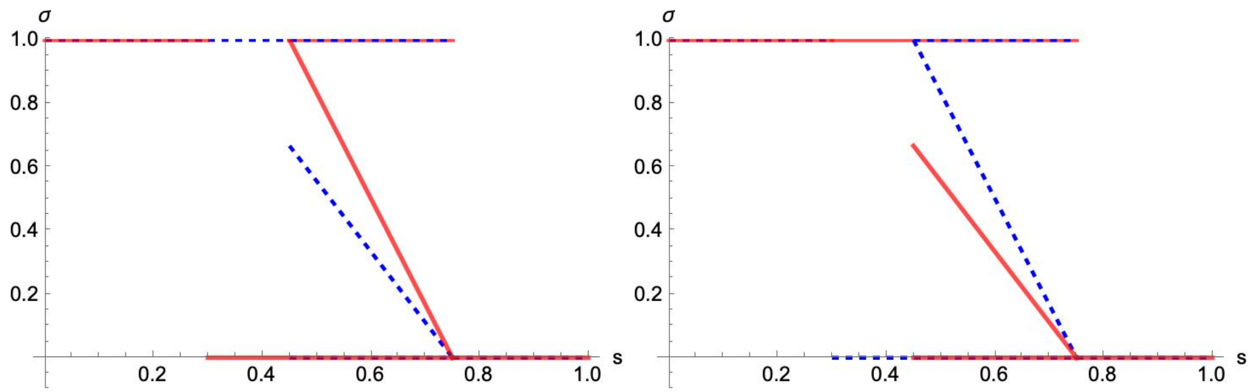
The result boils down to the trade-off implied by a revision: a successful revision leads to a change in payoffs of $vs\Delta$; yet, it comes at the cost of project delays, with total costs proportional to $c(s)$. If $\frac{c(s)}{s}$ is relatively large, then any delay generated by a revision is too costly, regardless of whether the other agent revises. Therefore, no agent revises. If $\frac{c(s)}{s}$ is small, then the implied delay is relatively cheap, regardless of whether the other agent revises. Then, continuing the project is preferable. The value at which $\frac{c(s)}{s}$ is too large to continue is higher for the agent with the higher probability of control in the future. This agent expects a higher chance of reaching execution of her preferred project type; hence, she has a higher expected payoff relative to the running cost.

The MPE in each period $t \geq 1$ is unique outside the $(\overline{s}_2, \overline{s}_1)$ interval. The region of equilibrium multiplicity is the region where profitability of one's revisions depends on the other agent's revision strategy. If only one agent revises, that agent's expected cost under the revision delay is lower than the expected benefit; however, if both agents were to revise, the expected cost would be too high. As we will show below, our qualitative results do not depend on the equilibrium selection in this multiplicity region. In the Appendix, we present the solution for each possible equilibrium selection.

4.2 Initial Project Design

Given the predicted revision response to the project, agent A in period 0 chooses the scale s and the inequality Δ (by choosing w) in order to maximize her expected utility. It is easy to verify that agent A prefers to start with a project of type Δ^A, where she assigns more of the final payoff to herself. The agent's expected value for the project therefore becomes $EU^A(s,w) = r \cdot U^A(A, \Delta^A | s, w) + (1-r) \cdot U^A(B, \Delta^A | s, w)$. Given the strategies described in

Figure 3: Equilibrium Revision Strategies as a Function of Scale s



(a) $r > \frac{1}{2}$ (b) $r < \frac{1}{2}$

Note: Dashed lines depict σ^A and solid lines depict σ^B given $r = 0.6$ (Panel a) and $r = 0.4$ (Panel b), and $v = 3, q = 0.25, p = 0.5, w = 1$.

Lemma 1, the expected utility in period 0 for Agent A can be expressed as

$$EU^A(s, w) = \left[H_1^{AA}(q, p, r) \cdot w + H_2^{AA}(q, p, r) \cdot (1 - w)\right] \cdot s \cdot v - \frac{c(s)}{p} \cdot H_3^{AA}(q, p, r), \qquad (5)$$

where $H_1^{AA}(q, p, r), H_2^{AA}(q, p, r)$ and $H_3^{AA}(q, p, r)$ are functions of q, p, r, with expressions that depend on the equilibrium strategies σ^A, σ^B.[10] They capture, respectively, the probability of agent A obtaining fraction w of the project benefits, the probability of this agent obtaining fraction $1 - w$, and the expected delay in the project's completion. This formulation shows that the expected utility is piecewise linear in w and concave in s whenever $\sigma^A, \sigma^B \in \{0, 1\}$.

Given Agent $A's$ optimal choice of s and w, we can first infer that, under a wide range of cost structures, $\sigma^B = 0$, as long as Agent A is not constrained by the scale cap:

Lemma 2 *Let $s^{\max} \to \infty$. There exists the upper bound $\overline{\varepsilon}(p, q) \geq 4$ on the elasticity of the cost function $c(s)$ such that for $\varepsilon(s) \in [1, \overline{\varepsilon}]$, there are no revisions on the equilibrium path and the finalized project type is Δ^A.*

[10]The superscripts denote the agent in control (A) and the current project type (Δ^A).These expressions are stated explicitly in the Appendix.

Agent A optimally designs the project to avoid revisions down the line. This is possible as long as the running cost does not increase too fast (i.e., the cost elasticity is not too large). Otherwise, the increased scale needed to preempt revisions would also make the entire project too expensive to build.[11]

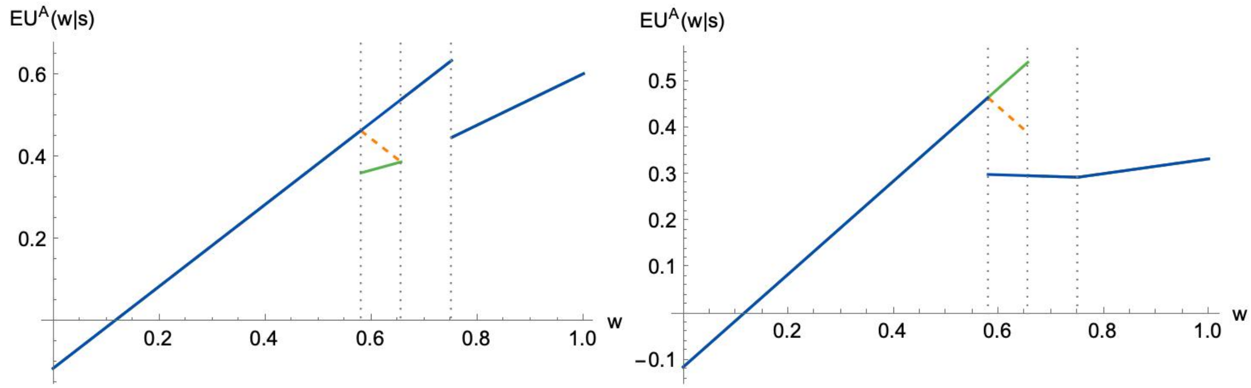
To show the trade-offs involved in using scale or payoff inequality strategically to avoid revisions, it is helpful to consider them sequentially. Take first the choice of payoff division given a fixed scale s. Without the threat of revisions, Agent A would keep all the benefits from the project for herself, by setting $\Delta = 1$. Yet, the more unequal the distribution of benefits, the more likely it is to trigger a revision from the opposition. For any given scale s, agent $A's$ expected payoff from the project increases in Δ, as long as the value Δ is not large enough to prompt the opposition to revise. Once the division is so unequal that it triggers revisions, agent $A's$ expected value drops. Nevertheless, conditional on revisions, agent A favors more inequality. Therefore, the problem for agent A reduces to either choosing Δ just small enough to avoid revisions, or accepting revisions and setting $\Delta = 1$. This trade-off is illustrated in Figure 4.

The inequality choice is made easier by the freedom to set project scale. A larger scale makes running the project costlier. The higher cost of inducing delays in turn discourages revisions. By setting the scale large enough, revisions can be deterred no matter the payoff inequality Δ. The trade-off is that a large scale requires higher running costs until the project reaches execution. Balancing the gain from higher scale with the loss from higher project running costs yields the equilibrium scale, as illustrated in Figure 5.

Putting these two steps together, the following picture emerges: if setting $\Delta = 1$ does not lead to revisions, then agent A can rely on the opposition's high cost of action to implement

[11] The limit on the cost elasticity, $\bar{\varepsilon} \geq 4$ is sufficiently high to accommodate a quadratic cost function, which is our running example for the figures in the paper.

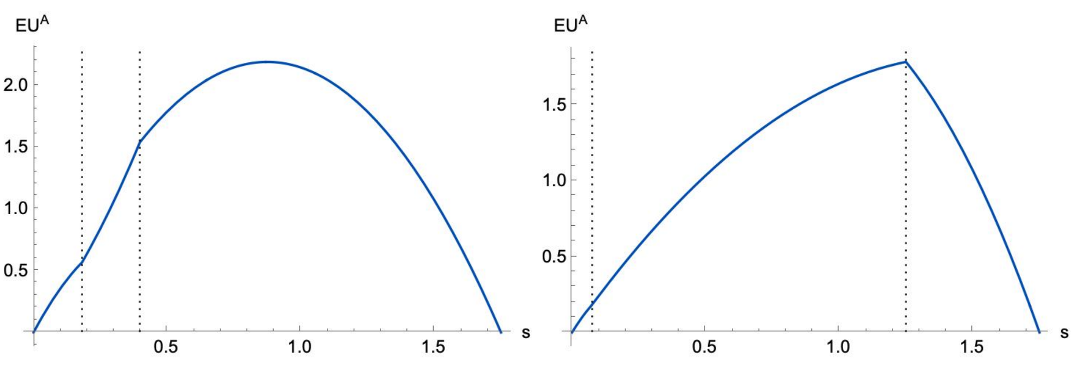
Figure 4: Project Initiator's Expected Utility for a Given s



(a) $r > \frac{1}{2}$ (b) $r < \frac{1}{2}$

Note: Illustrates Agent $A's$ expected utility as a function of w for $s = 0.2$ and $r = 0.6$ (Panel a) or $r = 0.4$ (Panel b). The other parameters are $v = 5, q = 0.25, p = 0.35, s = 0.2$. In the region of multiplicity: In blue, the value function for the equilibrium where only the agent more likely to be in control (advantaged) revises; in green, the equilibrium where only the disadvantaged agent revises; in orange, the mixing equilibrium. The dotted vertical lines represent, in ascending order, the values of w at which $s = \overline{s}_1$, $s = \overline{s}_2$, and $s = \overline{s}_3$.

Figure 5: Project Initiator's Expected Utility



(a) $r = 0.6$ (b) $r > 0.4$

Note: Illustrates Agent $A's$ expected utility as a function of s given $w(s)$ chosen optimally and $r = 0.6$ (Panel a) or $r = 0.4$ (Panel b); $v = 5, q = 0.25, p = 0.35$. The dotted vertical lines represent the thresholds between the following equilibrium regions: $\sigma^A = \sigma^B = 1$ (left); $\sigma^A = 1, \sigma^B = 0$ (middle), $\sigma^A = \sigma^B = 0$ (right).

her unconstrained optimal scale s^{NT}. Otherwise, she can choose a larger scale, in effect overscaling the project, in order to deter revisions. If overscaling is too expensive, she can make the project less unequal, in order to deter revisions by reducing their benefit. Yet, doing so reduces the benefit from the project for Agent A as well. The following result shows what this means for the resulting project scale:

Proposition 1 (Project Scale) *When $\varepsilon(s) \leq \overline{\varepsilon}(p,q)$ and $s^{\max} \to \infty$, there exists threshold $\overline{q}(\varepsilon, p, r) \geq 0$ such that the equilibrium scale s^* satisfies the following:*

- *(Unconstrained project) If $p > \overline{q}$, then s^* equals the benchmark scale under no transitions of control: $s^* = s^{NT}$;*
- *(Overscaled project) If $p \in [\underline{q}, \overline{q}]$, then s^* is strictly higher than the benchmark without transitions: $s^* > s^{NT}$;*
- *(Underscaled project) If $p < \min\{\underline{q}, \overline{q}\}$, then s^* is strictly lower than in the benchmark without transitions $s^* < s^{NT}$.*

We pair this result and its discussion with the corresponding characterization of the equilibrium project inequality:

Proposition 2 (Project Inequality) *When $\varepsilon(s) \leq \overline{\varepsilon}(p,q)$ and $s^{\max} \to \infty$, there exists threshold $\underline{q}(\varepsilon, p, r) \leq \overline{q}(\varepsilon, p, r)$ such that the equilibrium inequality Δ^* satisfies:*

- *(Maximal inequality) If $p \geq \underline{q}$, then inequality is maximal: $\Delta^* = 1$;*
- *(Reduced inequality) If $p < \underline{q}$, both agents receive some share of the project payoff: $\Delta^* < 1$;*

The expressions for thresholds $\overline{q}(\varepsilon, p, r)$ and $\underline{q}(\varepsilon, p, r)$ are given in the Appendix. We note here two main characteristics. First, they both increase in q. Second, they take a simple form when $\varepsilon(s)$ is a constant function. For instance, under the quadratic cost function, where $\varepsilon(s) = 2$, if $r \geq 1/2$, then $\overline{q} = \underline{q} = 0$, whereas if $r < 1/2$, then $\overline{q} = 2q$ and $\underline{q} = q$.

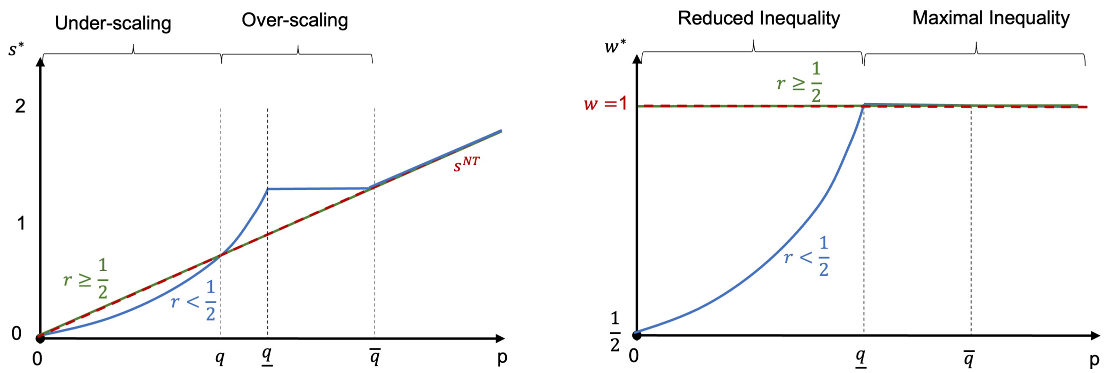
Propositions 1 and 2 show how the strategic use of scale or inequality is intermediated by capacity. When capacity is high, the expected duration of the project is short, and so are the implied running costs. Then, the revision-deterring benefit of a large scale outweighs the increase in running costs. Agent A chooses a large scale, and this alone is enough to deter revisions, without the need to compromise on Δ. In fact, the project scale can be as large as the one chosen by the agent in the benchmark without transitions of control.

As capacity decreases, the expected project runtime and associated costs increase. Agent A would ideally reduce the scale to adjust for these higher costs. Yet, she must keep the scale large enough in order to deter revisions by B. This results in overscaling to fight off potential revisions. Finally, as capacity drops even more and the run time increases further, setting a large scale in order to deter revisions becomes too costly. This is where Proposition 2 shows how reducing inequality acts as a strategic substitute for scaling up. Agent A can save on scale increases by offering the opposition a share of the payoff, thus reducing its benefit from a revision. As A gives away more, her relative cost of running a large project, vis-a-vis her benefit, further increases. This drives her to underscale relative to her unconstrained ideal s^{NT}. We illustrate the equilibrium project characteristics in Figure 6.

An immediate observation coming from Propositions 1 and 2 is that higher capacity increases both project scale and its inequality. As project runtime is expected to be shorter, the project initiator harnesses capacity to her advantage: she uses it to make projects larger and to extract more of the benefits for her group.

Corollary 1 (Effect of Higher Capacity) *Higher bureaucratic capacity p increases equilibrium scale s^* and payoff inequality Δ^*.*

Figure 6: Equilibrium Project Characteristics



(a) Equilibrium Scale (b) Equilibrium Inequality

Note: Equilibrium s (panel a) and w (panel b) for $r = 0.6$ (green), $r = 0.4$ (blue), and $v = 5, q = 0.25$. The red dashed line shows the scale and the payoff division under no transitions.

4.3 Project Revisions in Equilibrium

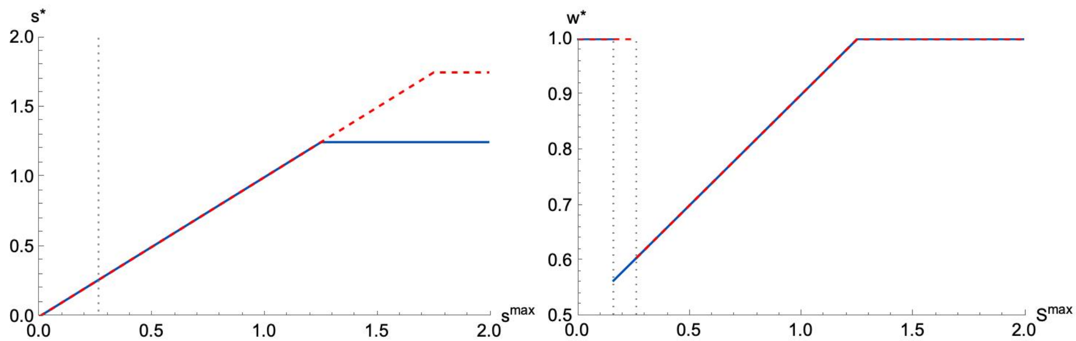
So far, we have shown what happens when agent A has full flexibility to scale up the project, in that $s^{\max}$ is not a binding ceiling. The project's cost structure was also assumed to allow for easy scaling ($\varepsilon(s) \leq \bar{\varepsilon}$). If either of these conditions are not satisfied, revisions may not be avoidable in equilibrium. In what follows, we take up this issue.

Consider first the exogenous ceiling on project scale, $s^{\max}$. This value may be set, for instance, by legal budget caps, hard technological or physical space bounds. If this upper bound binds, the maximum achievable scale for the project may not be large enough to accommodate Agent $A's$ desired overscaling. Then, revisions cannot be avoided in equilibrium.

Proposition 3 (Scale Caps and Equilibrium Revisions) *There exists scale threshold* $\overline{s^{\max}}(v, p, q, r)$ *such that if* $s^{\max} \leq \overline{s^{\max}}$*, then the equilibrium project inequality is maximal* ($\Delta^* = 1$) *and there are revisions on the equilibrium path: each agent revises a project favorable to their opponent* ($\sigma^A = \sigma^B = 1$).

The result is illustrated in Figures 7. Agent A uses scale to strategically deter revisions, up

Figure 7: Limited Scale and Revisions



(a) Equilibrium Scale

(b) Equilibrium Inequality

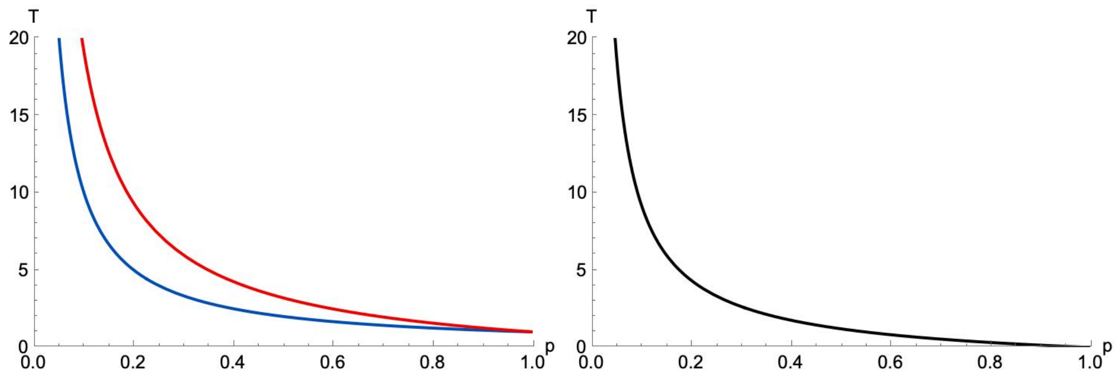
Note: Equilibrium s (Panel a) and w (Panel b) as a function of $s^{\max}$, for $p = 0.5$ (blue solid line) and $p' = 0.7$ (red dashed line) and $q = 0.25, r = 0.4, v = 5$. The dotted vertical lines are the thresholds below which each agent revises in equilibrium for $p = 0.5, p' = 0.7$.

until the needed scale reaches the ceiling $s^{\max}$. At that point, the ability to strategically increase scale is exhausted. If the ceiling $s^{\max}$ is low, the cost of delays is low relative to the potential gain from a revision. Compromising on inequality remains the only tool to deter revisions. Yet, with a low ceiling $s^{\max}$, the needed compromise would have to be exceedingly large. Hence, Agent A gives up trying to deter revisions. Instead, she prefers to make the project highly unequal ($\Delta^* = 1$) and enter a 'winner-take-all' regime where everyone revises the project.

How does this result depend on bureaucratic capacity? For a fixed $s^{\max}$, increasing p increases the probability that revisions occur in equilibrium. As illustrated in Figure 7, the scale needed to deter revisions increases with p. Moreover, as p increases, the expected project duration, and therefore the expected running cost, is smaller. This makes revisions more appealing and their deterrence more difficult.

Corollary 2 (Higher Capacity under Scale Caps) *With $s^{\max} < \infty$, higher bureaucratic capacity p increases the probability of project revisions and delays. Yet, conditional on being in the equilibrium with revisions, higher p reduces expected delays.*

Figure 8: Delay due to Revisions



Note: Left panel plots the expected number of periods to project completion under no delays (blue line) and under revisions by both agents,$\sigma^A = \sigma^B = 1$, (red line). Right panel plots the difference between the two expected times to competition. The plot is for $r = 0.4, q = 0.25$, though the graphs are qualitatively similar for other values.

Figure 8 plots the expected delay induced by revisions in the equilibrium with $\sigma^A = \sigma^B = 1$. The delay is relative to the expected time to completion in the benchmark with no transitions. This delay is inefficient, as it increases running costs.

The second channel for revisions is a highly elastic scaling cost. If $\varepsilon(s)$ is very large, then a small increase in scale is sufficient to inflate the running costs beyond what is desirable for either agent. In that case, Agent A prefers the alternative of starting a small project and trying to capture its entire benefit. Each transition of control then triggers a revision of an unfavorable project, leading to long completion timelines. We can construct such cases by choosing a very elastic cost function.[12]

4.4 Welfare

Our results so far show that the organizational capacity of the bureaucracy has pronounced effects on the strategies of project initiators. Higher values of capacity increase inequality,

[12]For instance, this is the case if $c(s) = s^7$ and $q = 0.35, p = 0.4, 0.47, v = 5$.

while low and medium values result in under- and overscaling. These strategies suggest significant implications for social benefits. In particular, when capacity lies in the interval $[q, \overline{q}]$, projects are both overscaled and unequal, and are therefore especially harmful to the non-initiating agent.

To investigate the aggregate benefits from the project, we consider the problem for a social planner who weighs the two agents equally. This results in a social welfare function

$$W = \frac{1}{2} EU^A(s^*, w^*) + \frac{1}{2} EU^B(s^*, w^*). \tag{6}$$

Given (5) and Propositions 1 and 2, the resulting social welfare function takes a relatively simple form:

$$W = \frac{1}{2} v s^* + \frac{c(s^*)}{p}. \tag{7}$$

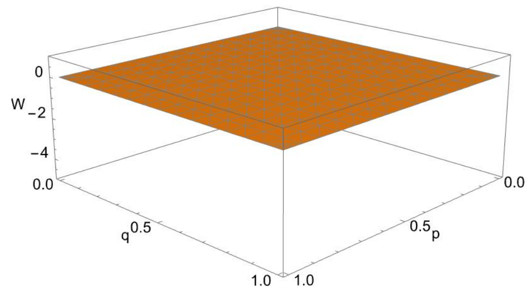
Proposition 4 uses expression (7) to derive the interval $\mathcal{P}$ of capacity values under which equilibrium projects produce lower welfare than no project at all. That is, where $W < 0$.

Proposition 4 (Welfare) *There exists an interval* $\mathcal{P} = (\underline{p}, \overline{p})$ *such that* $W(v, p, q, r) < 0$ *for* $p \in \mathcal{P}$ *and* $W(v, p, q, r) \geq 0$ *for* $p \notin \mathcal{P}$ *if and only if:*

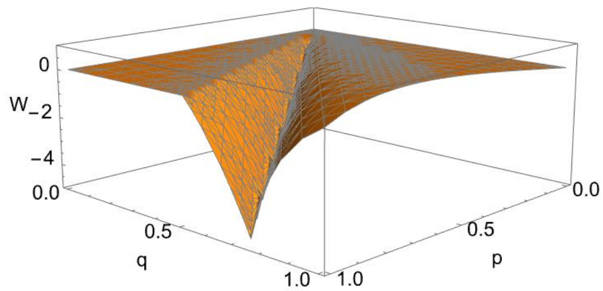
- $\varepsilon(\overline{s}_3) < 2$ *and* $r \geq \frac{1}{2}$, *in which case* $\mathcal{P} = (0, 1)$, *or*
- $r < \frac{1}{2}$, *in which case* $\mathcal{P} \supseteq (\underline{q}, \overline{q})$.

Proposition 4 highlights the two drivers of welfare losses. First, a low cost elasticity ($\varepsilon(s) < 2$) makes large scale projects more desirable. This is socially harmful given that the initiator does not internalize the cost borne by the other agent. The second driver of welfare losses is the strategic overscaling in order to deter revisions. The size of the interval $\mathcal{P}$ is determined by the likelihood of transitions of control and institutional constraints. When the project initiator is more likely to retain control ($r \geq \frac{1}{2}$), the interval $\mathcal{P}$ is empty as long

Figure 9: Welfare



(a) $r > \frac{1}{2}$



(b) $r < \frac{1}{2}$

Note: Welfare as a function of p and q when $c(s) = s^2$, $v = 5$. Left panel has $r = 0.6$; the quadratic cost function, with its elasticity $\varepsilon(s) = 2$ is a special case where equilibrium welfare is constant at $W = 0$. Right panel has $r = 0.4$ and a non-empty interval where $W < 0$.

as the cost elasticity is not too low ($\varepsilon(\overline{s}_3) \geq 2$). For the opposition, the expected gain from revisions is low if they are unlikely to stay in control. The project initiator faces a low threat of revisions, and therefore does not distort the project enough to cause a welfare loss. Even if the equilibrium scale is higher than socially optimal, the additional cost does not outweigh the value created.

The calculus changes if the initiator is less likely to retain control of the project ($r < \frac{1}{2}$). In this case, the expected gains from a revision are larger. To deter revisions, the project initiator responds with larger distortions. The distortion is particularly costly when the project is oversized and highly unequal. Therefore, the interval $\mathcal{P}$ includes the region $[\underline{q}, \overline{q}]$ where projects are oversized and $\Delta = 1$. Next, consider the role played by institutional constraints. We examine the effect of an increase in the likelihood of successful revisions, q:

Corollary 3 *If $\varepsilon(s) = \tilde{\varepsilon} \in \mathbb{R}$, then the bounds of $\mathcal{P}$, $\overline{p}$, $\underline{p}$ are increasing in q. Moreover, whenever $\overline{p} < 1$, the difference $(\overline{p} - \underline{p})$ is increasing in q.*

As legal or institutional challenges become more potent (i.e., q increases), the interval $\mathcal{P}$ both expands and shifts toward higher values of p. This reflects the greater incentive to overscale as the threat of successful revisions increases. As a result, beneficial projects are realized only under very high or very low organizational capacity in high-q polities.[13]

Figure 9 illustrates welfare as a function of p and q when a project initiator is more likely to be in control (Panel a) or less likely to be in control (Panel b). Consistent with Proposition 4, it shows that the values of p and q that induce overscaling are especially bad for welfare. As these values move in tandem, the implication is that systems with high institutional barriers and high capacity are prone to producing poor projects. By contrast, systems with "mismatched" capacity and barriers produce higher welfare, but with some drawbacks. Under low capacity and high barriers, projects are costly and possibly too small. Under high capacity and low barriers, higher social welfare comes at the expense of high inequality.

Welfare under Scale Caps. Strategic overscaling of projects has negative welfare consequences. Scale caps discussed in Section 4.3 impose a ceiling on overscaling. The reduction in scale, however, comes at the cost of revisions on the equilibrium path, which add delays. Given (5), the social welfare function becomes:

$$W = \frac{1}{2}s^* v - \frac{c(s^*)}{p} H_3^{AA}(p, q, r), \tag{8}$$

where $s^* \leq s^{\max}$ and $H_3^{AA}(p, q, r) > 1$ is the expected delay due to revisions. Perhaps surprisingly, the scale caps policy reverses the effects in Proposition 4:

Proposition 5 (Welfare under Scale Caps) *Under a scale cap $s^{\max} < \overline{s^{\max}}$, such that the conditions of Proposition 3 are satisfied, the equilibrium social welfare is always positive: $W \geq 0$.*

[13]The condition of a constant elasticity of the cost function isolates the effect coming from the strategic response to revision threats; otherwise, changes in q could have scaling effects coming through changes in the relative cost of running the project.

Welfare is negative if for $s^* \leq s^{\max}$, $\frac{c(s^*)}{s^*}$ is sufficiently large such that the increased cost due to delays outweighs the benefit of the project. The welfare losses in Proposition 4 are the result of strategic overscaling to deter revisions or of scaling up without internalizing the costs borne by the other agent. Scale caps reverse this effect: the project initiator is forced to underscale the project because of the budget limit. The smaller scale requires a lower project running cost, such that the ratio $\frac{c(s^*)}{s^*}$ is no longer large enough to render $W < 0$. The project still produces a positive social benefit, albeit a small one.

5 Multiple Project Phases

We now adapt the preceding results to a model with two phases. In the basic model, period 0 is distinguished by the ability of the project initiator to choose key program parameters. Inherently complex projects such as those often funded by FTA Capital Improvement Grants typically present multiple opportunities for politicians to revisit basic questions of scale and distribution. For example, in 2011 the Obama administration proposed the $30 billion Gateway Program to upgrade rail infrastructure between New York and New Jersey. Despite favorable FTA reviews, the Trump administration effectively canceled the program, only to have it revived under the Biden administration.[14]

Complex projects often require advance research and planning, and therefore early phases of such projects correspond naturally to investments that reduce subsequent construction or implementation costs. These investments may also provide benefits in their own right, independently of the final project outcome. It is therefore worth asking how the possibility of resetting program parameters mid-stream affects investments, project scale, and revisions. In particular, we examine conditions under which transitions of power may prevent projects from starting at all.

[14] See Matt Hickman, "New York and New Jersey's long-delayed Gateway Program faces a more favorable outlook under Biden presidency." *The Architect's Newspaper*, November 10, 2020.

Each phase of the two-phase model is structurally identical to the basic model. Agent A has control at the start of phase 1, and has control at the start of phase 2 with probability r. Denote the parameters for scale, distribution, and valuation in phase τ by s_τ, Δ_τ, and v_τ, respectively. As in the basic model, s_τ and Δ_τ are chosen in the initial incumbent in each phase, v_τ is exogenous, and project types are determined by players after the initial period of the phase. The phase 1 payoffs thus represent the immediate value of investments such as research contracts or pilot studies. To keep the analysis tractable, when there are multiple equilibria we select the one in which only the favored agent revises.

The phases are dynamically linked through their cost functions. Let the cost of each period in phase τ be $c(s_\tau) = m_\tau s_\tau^2$, where $m_\tau > 0$ and $m_1 = 1$. In phase 2, $m_2 = 1/s_1$, so that early investments in the project reduce future marginal costs. Note that in isolation, phase 1 of the model is identical to the basic game if $s_2 = 0$, and phase 2 of the model is identical to the basic game if $s_1 = 1$.

Within each phase τ, actions following the choice of s_τ only affect payoffs through the division of v_τ. Thus, the agents' incentives following the initial period are similar to those of the one-phase game, and we can exploit the derivations of Section 3 to analyze revisions and the choice of Δ_τ. The second phase primarily affects agent A's incentives in choosing the phase 1 scale, which affects phase 2 costs. Due to the simple structure of m_2 and quadratic costs, s_1 linearly scales A's phase 2 expected payoff. Her phase 1 objective can be expressed as:

$$EU^A(s_1, w_1) + s_1 \tilde{U}^A, \tag{9}$$

where $\tilde{U}^A$ is agent A's phase 2 expected payoff prior to the revelation of the phase 2 initiator.

Using Propositions 1 and 2, this can be expressed as:[15]

$$\tilde{U}^A = \begin{cases} \frac{pv^2\left[2p^2r - pq(5r+3) + 2q^2(3r+1)\right]}{8(p-2q)^2} & \text{if } p < \underline{q} \text{ and } r > 1/2 \\ \frac{pv^2[q(4-5r) - 2p(1-r)]}{8(p-2q)} & \text{if } p < \underline{q} \text{ and } r < 1/2 \\ \frac{v^2\left[p^2r - 4q^2(1-r)\right]}{4p} & \text{if } p \in \left[\underline{q}, \overline{q}\right) \text{ and } r > 1/2 \\ \frac{v^2\left[4qr(p-q) - p^2(1-r)\right]}{4p} & \text{if } p \in \left[\underline{q}, \overline{q}\right) \text{ and } r < 1/2 \\ \frac{pv^2(2r-1)}{4} & \text{if } p \geq \overline{q}. \end{cases} \tag{10}$$

Maximizing (9) with respect to s_1 produces our next result. Roughly speaking, the phase 1 investment is the scale of the one-phase game, s^*, adjusted to reflect $\tilde{U}^A$. Importantly, $\tilde{U}^A$ is negative whenever $r < 1/2$, as well as for some values of p between q and $\overline{q}$ (where $\overline{q} = 2q$ under quadratic costs) when $r > 1/2$. When this happens, the phase 1 scale s_1^* is lower than s^*. Consistent with Lemma 1, s_1^* may even be low enough to induce revisions in equilibrium. Negative values of $\tilde{U}^A$ play a role similar to that of increasing the cost of high project scales in the one-phase model: inhibiting large scales generates projects that are insufficient to deter revisions.

Beyond merely reducing scale, the optimal scale in the initial phase may be zero, which in effect cancels the project. Proposition 6 provides conditions under which this occurs.

Proposition 6 (Project Cancellation) *If $r > 1/2$, then $s_1^* = 0$ only if $p \in [q, \overline{q}]$ and if:*

$$\tilde{U}^A < -\frac{rv\left[p(1-r) + qr\right]}{p(1-r)r + q\left(2r^2 - 2r + 1\right)}. \tag{11}$$

If $r < 1/2$, $s_1^* = 0$ if v_1 is sufficiently low or v_2 is sufficiently high.

For a favored ($r > 1/2$) phase 1 initiator, cancellations occur because of the potential for overscaling. As Figure 6 illustrates, under moderate capacity an unfavored agent B overscales

[15]Note that given $c(s) = m_t s^2$, for $r > \frac{1}{2}$, the thresholds $\underline{q}(v, p, q, r) = \overline{q}(v, p, q, r) = 0$. Hence, we denote by $\underline{q}$ the threshold $\underline{q}(v, p, q, r | r < \frac{1}{2})$ and by $\overline{q}$ the threshold $\overline{q}(v, p, q, r | r < \frac{1}{2})$.

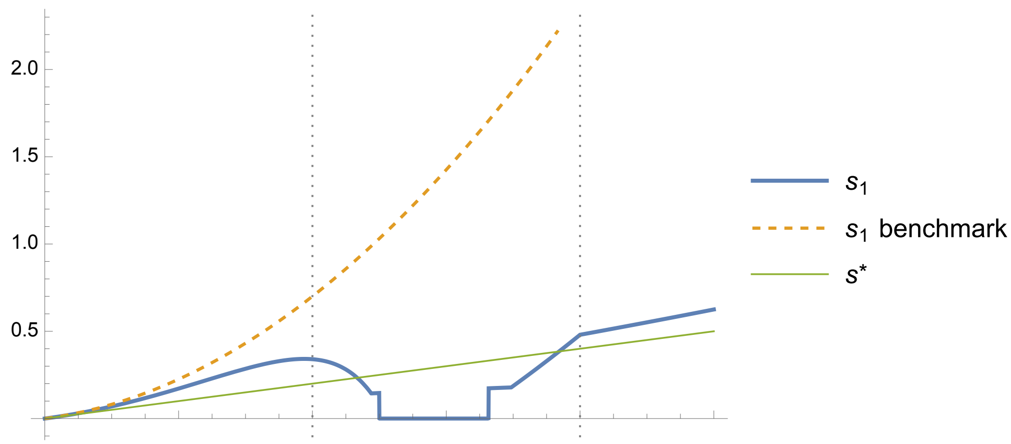
to prevent revisions. This can produce a highly undesirable expected payoff for agent A, especially if she is not overwhelmingly likely to retain power. A highly competitive political environment thereby forces A to internalize in part the social benefits of the project.[16] As Proposition 4 shows, these benefits are minimized at intermediate levels of capacity. By contrast, under low capacity, underscaled projects are relatively efficient and do not invite cancellation. And under high capacity, a favored initiator is likely to benefit from an unequal phase 2 project.

For an unfavored ($r < 1/2$) phase 1 initiator, the main driver of cancellation is the distribution of payoffs over time. Phase 1 produces positive expected payoffs for the initiator, but phase 2 produces negative ex ante expected payoffs at any capacity level. Thus she will simply cancel if v_2 is high relative to v_1.

Figure 10 illustrates the role of cancellations in the $r > 1/2$ case by comparing phase 1 investments against two benchmarks. In the first benchmark, A remains in control with certainty at the beginning of phase 2, but faces the possibility of revision in both phases. As expected, the possibility of losing control over the final project depresses investment. The second benchmark is simply the equilibrium scale s^{NT} in the one-phase game. The initial investment s_1^* may be under- or overscaled relative to this benchmark, depending on agent A's expected phase 2 payoffs. In this example, power transitions are very likely ($r = 0.52$), so the threat of overscaling by B in phase 2 causes underscaling and cancellations when capacity is in the interval $[\underline{q}, \overline{q}]$. This non-monotonicity of project scale with respect to capacity reflects in part the non-monotonicity of social benefits in the one-phase game, as illustrated in Figure 9.

[16]Note, however, that public projects may provide public good benefits to actors besides agents A and B.

Figure 10: Investment with Two Phases



Note: Initial investment (s_1, blue), benchmark investment (dashed) in a setting with where A chooses s_2 and w_2 in phase 2, and investment in the one-phase game ($s^* = s^{NT}$, green), as a function of p. Parameters are $r = 0.52$, $v_1 = 1$, $v_2 = 5$, and $q = 0.4$. Vertical lines are located at the thresholds q and $\overline{q}$, between which overscaling may occur.

6 Applications

Our model produces a range of predictions about the equilibrium implications of changes in organizational capacity (p) and the ability to exploit institutional mechanisms to revise projects (q). This section presents two brief applications to major organizations. The examples are not meant to advance causal claims, but rather show that our results are consistent with seemingly disparate facts about the evolution of key project characteristics.

20th Century US Infrastructure. Propositions 1 and 2 show that as q increases, project designers will prevent revisions by distributing benefits more widely and increasing scales beyond their ideal levels. As Altshuler and Luberoff (2003) relate, changes in the political environment that enabled interest group opposition played a strong role in the trajectory of US infrastructure projects. In the mid-20th century, urban planners exemplified by figures such as Robert Moses operated with relatively few constraints, often promoting automobile-centered ideas for urban renewal with the support of local business interests (Caro, 1974).

> The programs operated, moreover, in relative secrecy, so that those affected often learned of projects just before the bulldozers rolled. In the early years there were no organized interest groups monitoring or learning from these experiences, much less providing potential victims with tactical assistance. Since their cause seemed hopeless, even those most adversely affected generally gave in without a fight. This tendency was accentuated by the fact that the victims were disproportionately poor and black. (Altshuler and Luberoff, 2003, p. 22)

Assisted by the national rise of civil rights and environmental movements, as well as laws such as NEPA and the Clean Air Act, conflicts over infrastructure development rose drastically starting in the late 1960s. The shock of increasingly effective political mobilization affected numerous ongoing projects. For example, a 1982 court order paused construction of the New York Westway in order to protect local fish breeding grounds. The project, which was intended to replace a decaying highway along the west side of Manhattan, was eventually canceled in 1985 after over a decade of development and $200 million in expenditures.[17]

For planners, the response to more effective contestation was not to abandon large projects, but rather to expand their size and distributive reach. Perhaps the most prominent example of this strategy is the Boston Central Artery/Tunnel (CA/T, better known as the "Big Dig"), which replaced an elevated highway in downtown Boston with a technologically advanced tunnel and associated connecting structures. In addition to local interests, stakeholders in CA/T included the Massachusetts and federal governments, which provided its primary funding, as well as neighboring municipalities. The ultimate design reflected their concerns on issues as varied as tunnel size, public transportation, air quality, land takings, parking, and interchange design. The over 1,500 mitigation agreements included wetlands restoration, landfill redevelopment, and the construction of an artificial reef.[18] While the

[17] See Sam Roberts, "The Legacy of Westway: Lessons from its Demise." *New York Times*, October 7, 1985.

[18] See Daniel C. Wood, "Learning From The Big Dig." *Public Roads* 65(1), July/August 2001.

original highway was constructed in five years in the 1950s, the CA/T took over 20 years of planning and construction, at a cost more than double that of early projections.

Inequality in Government Procurement. By Corollary 1, the distributive consequence of increasing p is greater inequality at the project level. Government procurement provides a natural setting for examining this implication. US federal procurement is a highly regulated process that employs hundreds of thousands of personnel. As in the model, revisions play an important role. Losing or excluded bidders can challenge award decisions at either the contracting agency or the Government Accountability Office (GAO), and successful appeals can change awardees, re-open competition, or result in a range of intermediate steps. Recently, about half of the 2,000 or so cases per year heard by the GAO received some form of remediation (US GAO, 2022).

The Competition in Contracting Act mandates a default process of "full and open competition," whereby prospective contractors submit competitive bids that are evaluated according to preset criteria. However, a substantial minority of contracts are awarded on a non-competitive, "sole source" basis. This process is intended for circumstances such as absence of alternate suppliers, emergencies, or one of several public interest criteria. Such contracts require increasing levels of justification and approval as their size grows, but observers have noted that agencies have substantial discretion to adopt them (e.g., Dahlström et al., 2021). Sole-sourcing therefore serves as a plausible proxy for high-Δ projects.

The Department of Defense (DoD) is both the largest user of sole-source contracts and one of the few recent examples of a large-scale increase in organizational capacity in the federal government. In 2009, DoD began a long-term expansion its acquisition workforce, which had declined significantly since the 1990s (Gates et al., 2022). This effort received both extensive resources and exemptions from concurrent DoD hiring freezes, resulting in a workforce growth from about 130,000 to over 180,000 between fiscal years 2009 and 2021.

The added personnel significantly enhanced the ability of program managers to oversee the contracting process (DiIulio, 2014). Importantly, expansion was highly uneven during this period, with no headcount change between fiscal years 2011 and 2014, and a net growth of 15,000 between 2014 and 2017.

The fiscal years 2014 through 2017 coincided with the second term of the Obama presidency, during which Democrats and Republicans split control of government. There was little change in defense spending, but outlays from non-competitive contracts of all sizes grew far faster than those from competitive contracts. For example, among awards worth over $1 million, outlays from competitive contracts (accounting for 53% of the DoD total) decreased by 1.5%, compared to a 34.4% increase from non-competitive contracts.[19] Thus, this era saw dramatic growth in both organizational capacity and less egalitarian projects.[20]

7 Conclusion

Within academic and policy circles, bureaucratic capacity has become a hallmark of good governance. But in contrast to the consensus about its benefits, there is little agreement on its practical definition, and also too little exploration of its implications for key features of public policies. Our theory addresses both of these issues. It models capacity as the transition probability of a simple Markov process, and then situates this process in an institutional environment that features political contestation and institutional rigidities. This basic framework allows us to capture a rich set of outputs, such as the scale, timing, and distributive properties of projects.

The principal equilibrium incentive in the model is the avoidance of revisions, which can delay completion, increase costs, and reduce payoff shares. Depending on capacity

[19] Data from `https://usaspending.gov`. The disparity is somewhat higher for higher-valued contracts.

[20] The acquisition workforce continued to grow during the Trump administration, and the level of sole source contracts remained high, but these developments also coincided with higher defense spending starting in fiscal year 2018.

levels, this generates different political incentives to manipulate project design. In particular, intermediate capacity produces overscaling, while low capacity results in underscaling and more egalitarian payoff divisions. Overall, by reducing opportunities for obstruction, a high capacity bureaucracy encourages larger and less egalitarian projects.

Our framework produces several additional unexpected and potentially testable implications. First, constraints on project scale can increase inequality, revisions, and delays as capacity increases. Second, "matched" levels of capacity and institutional barriers encourage overscaling and produce poor projects from a social welfare perspective. Finally, in complex multi-phase projects, potential transitions of power can result in cancellations when overscaling is a possibility. In short, greater capacity does not unambiguously improve performance, and better projects emerge from profiles of organizational capacity and institutional and technological constraints that discourage the tactical inflation of projects.

Our model treats organizational capacity and the institutional or legal environment as exogenous, but their implications for outcomes raise some basic questions about their origins. We mention several as possibilities for further inquiry. Just as recent work on state capacity has explored the political and economic drivers of investment in taxing powers, it is worth examining the incentives to invest in both the capabilities of agencies that may far outlive them, as well as the institutional context within which social groups determine project outcomes. Next, the openness of an institutional system to revisions could invite more participants, which would be better approximated by having more agents and a richer distributive space. Finally, it may be useful to unpack the capacity parameter p to reflect the realities of modern projects. For example, outside contractors often play major roles in large infrastructure construction, but whether such players enhance capacity, or are symptoms of low capacity, is unclear.[21]

[21] See Ralph Vartabedian, "How California's faltering high-speed rail project was 'captured' by costly consultants." *Los Angeles Times*, April 26, 2019.

References

Acemoglu, D., García-Jimeno, C., Robinson, J.A., 2015. State capacity and economic development: A network approach. American Economic Review 105, 2364–2409.

Altshuler, A., Luberoff, D., 2003. The Changing Politics of Urban Mega Projects. Brookings Institution Press, Washington, DC.

Baron, D.P., 1996. A dynamic theory of collective goods programs. American Political Science Review 90, 316–330.

Battaglini, M., Nunnari, S., Palfrey, T.R., 2012. Legislative bargaining and the dynamics of public investment. American Political Science Review 106, 407–429.

Besley, T., Persson, T., 2009. The origins of state capacity: Property rights, taxation, and politics. American Economic Review 99, 1218–44.

Bolton, A., Potter, R.A., Thrower, S., 2016. Organizational capacity, regulatory review, and the limits of political control. Journal of Law, Economics, & Organization 32, 242–271.

Brooks, L., Liscow, Z.D., 2022. Infrastructure costs. American Economic Journal: Applied Economics forthcoming.

Brown, J.D., Earle, J.S., Gehlbach, S., 2009. Helping hand or grabbing hand? State bureaucracy and privatization effectiveness. American Political Science Review 103, 264–283.

Callander, S., Raiha, D., 2017. Durable policy, political accountability, and active waste. Quarterly Journal of Political Science 12, 59–97.

Caro, R., 1974. The Power Broker: Robert Moses and the Fall of New York. New York: Aflred Knopf.

Carpenter, D., 2001. The Forging of Bureaucratic Autonomy: Networks, Reputations and Policy Innovation in Executive Agencies, 1862-1928. Princeton University Press, Princeton, NJ.

Dahlström, C., Fazekas, M., Lewis, D.E., 2021. Partisan procurement: Contracting with the united states federal government, 2003-2015. American Journal of Political Science 65, 652–669.

Dal Bó, E., Finan, F., Rossi, M.A., 2013. Strengthening state capabilities: The role of financial incentives in the call to public service. The Quarterly Journal of Economics 128, 1169–1218.

Derthick, M., 1990. Agency Under Stress. Brookings Institution, Washington, DC.

DiIulio, J., 2014. Bring back the bureaucrats: Why more federal workers will lead to better (and smaller!) government. Templeton Foundation Press, West Conshohocken, PA.

Feng, F.Z., Taylor, C.R., Westerfield, M.M., Zhang, F., 2021. Setbacks, shutdowns, and overruns. SSRN Working Paper 3775340 .

Foarta, D., 2022. How organizational capacity can improve electoral accountability. American Journal of Political Science URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/ajps.12727.

Foarta, D., Sugaya, T., 2021. Wait-and-see or step in? Dynamics of interventions. American Economic Journal: Microeconomics 13, 399–425.

Gates, S.M., Roth, E., Kempf, J., 2022. Department of Defense Acquisition Workforce Analyses: Update Through Fiscal Year 2021. RAND Corporation, Santa Monica, CA.

Glaeser, E.L., Ponzetto, G.A., 2018. The political economy of transportation investment. Economics of Transportation 13, 4–26.

Gorodnichenko, Y., Peter, K.S., 2007. Public sector pay and corruption: Measuring bribery from micro data. Journal of Public Economics 91, 963–991.

Green, B., Taylor, C.R., 2016. Breakthroughs, deadlines, and self-reported progress: Contracting for multistage projects. American Economic Review 106, 3660–99.

Huber, J.D., McCarty, N., 2004. Bureaucratic capacity, delegation, and political reform. American Political Science Review 98, 481–494.

Huber, J.D., McCarty, N., 2006. Bureaucratic capacity and legislative performance, in: The macropolitics of congress, p. 50.

Johnson, N.D., Koyama, M., 2017. States and economic growth: Capacity and constraints. Explorations in Economic History 64, 1–20.

Keijzer, N., Spierings, E., Phlix, G., Fowler, A., 2011. Bringing the invisible into perspective. Reference paper for using the 5Cs framework to plan, monitor and evaluate capacity and results of capacity development processes. Maastricht: ECDPM. URL: ecdpm.org/5Cs.

Lee, M.M., Zhang, N., 2017. Legibility and the informational foundations of state capacity. Journal of Politics 79, 118–132.

Mandelker, D.R., 2010. The national environmental policy act: A review of its experience and problems. Washington University Journal of Law and Policy 32, 293–312.

Mehrotra, N., Turner, M.A., Uribe, J.P., 2022. Does the US have an infrastructure cost problem? Evidence from the interstate highway system. Unpublished manuscript, Brown University .

OECD, 2011. The Enabling Environment for Capacity Development. URL: https://www.oecd.org/development/accountable-effective-institutions/48315248.pdf.

OECD, 2021. Public Employment and Management 2021. URL: https://www.oecd-ilibrary.org/content/publication/938f0d65-en.

Pressman, J.L., Wildavsky, A., 1984. Implementation: How great expectations in Washington are dashed in Oakland. University of California Press, Los Angeles.

Propper, C., Van Reenen, J., 2010. Can pay regulation kill? Panel data evidence on the effect of labor markets on hospital performance. Journal of Political Economy 118, 222–273.

Rauch, J.E., Evans, P.B., 2000. Bureaucratic structure and bureaucratic performance in less developed countries. Journal of Public Economics 75, 49–71.

Smith, V.K., Von Haefen, R., Zhu, W., 1999. Do environmental regulations increase construction costs for federal-aid highways? A statistical experiment. Journal of Transportation and Statistics 2, 45–60.

Stone, C.N., 1989. Regime Politics: Governing Atlanta, 1946-1988. University Press of Kansas, Lawrence, KS.

Ting, M.M., 2011. Organizational Capacity. Journal of Law, Economics, & Organization 27, 245–271.

Toxvaerd, F., 2006. Time of the essence. Journal of Economic Theory 129, 252–272.

Turner, I.R., 2020. Policy durability, agency capacity, and executive unilateralism. Presidential Studies Quarterly 50, 40–62.

US GAO, 2014. National Environmental Policy Act: Little information exists on NEPA analyses. 14-370.

US GAO, 2022. GAO Bid Protest Annual Report to Congress for Fiscal Year 2022. 23-900462.

Appendix

A Proofs

A.1 Proof for Lemma 1

Given the Markov transition probabilities, the expected utility of Agent A starting in the state where control is held by Agent $i \in \{A, B\}$ and the project type is $\Delta^j, j \in \{A, B\}$, is

$$U^A(i, \Delta^j|\sigma^A, \sigma^B) = sv \cdot [H_1^{ij}(p, q, r) \cdot w + H_2^{ij}(p, q, r) \cdot (1 - w)] - \frac{c(s)}{p} \cdot H_3^{ij}(p, q, r), \quad (12)$$

while the corresponding utility for Agent B is

$$U^B(i, \Delta^j|\sigma^A, \sigma^B) = sv \cdot [H_1^{ij}(p, q, r) \cdot (1 - w) + H_2^{ij}(p, q, r) \cdot w] - \frac{c(s)}{p} \cdot H_3^{ij}(p, q, r), \quad (13)$$

where $H_1^{ij}(p, q, r) = \frac{\Gamma_{ij}}{\Omega}$, $H_2^{ij}(p, q, r) = \frac{\Upsilon_{ij}}{\Omega}$, and $H_3^{ij}(p, q, r) = \frac{\Sigma_{ij}}{\Omega}$, and

$$\begin{aligned}
\Omega &= p(1 - r\sigma^A)(1 - (1 - r)\sigma^B) + q(r\sigma^A + (1 - r)\sigma^B) - 2qr(1 - r)\sigma^A\sigma^B;\\
\Gamma_{AA} &= p(1 - r\sigma^A)(1 - (1 - q)(1 - r)\sigma^B) + qr\sigma^A(1 - (1 - r)\sigma^B);\\
\Gamma_{AB} &= p(1 - (1 - r)\sigma^B)q(1 - r)\sigma^A + qr\sigma^A(1 - (1 - r)\sigma^B)\\
\Gamma_{BA} &= p(1 - r\sigma^A)(1 - qr\sigma^B - (1 - r)\sigma^B) + qr\sigma^A(1 - (1 - r)\sigma^B);\\
\Gamma_{BB} &= p(1 - (1 - r)\sigma^B)qr\sigma^A + qr\sigma^A(1 - (1 - r)\sigma^B);\\
\Upsilon_{AA} &= -pq(1 - r)\sigma^B(1 - r\sigma^A) + q(1 - r)\sigma^B(1 - r\sigma^A)\\
\Upsilon_{AB} &= p(1 - r\sigma^A - q(1 - r)\sigma^A)(1 - (1 - r)\sigma^B) + q(1 - r)\sigma^B(1 - r\sigma^A);\\
\Upsilon_{BA} &= -pqr\sigma^B(1 - r\sigma^A) + q(1 - r)\sigma^B(1 - r\sigma^A);\\
\Upsilon_{BB} &= p(1 - (1 - r)\sigma^B)(1 - (1 - q)r\sigma^A) + q(1 - r)\sigma^B(1 - r\sigma^A);\\
\Sigma_{AA} &= p(1 - p(1 - r)\sigma^B)(1 - r\sigma^A) - 2qp\sigma^A\sigma^B r(1 - r) + q(r\sigma^A + (1 - r)\sigma^B);\\
\Sigma_{AB} &= p(1 + p(1 - r)\sigma^A)(1 - (1 - r)\sigma^B) + 2qp\sigma^A\sigma^B(1 - r)^2\\
&\quad + q(r\sigma^A + (1 - r)\sigma^B);\\
\Sigma_{BA} &= p(1 + pr\sigma^B)(1 - r\sigma^A) + 2qp\sigma^A\sigma^B r^2 + q(r\sigma^A + (1 - r)\sigma^B);\\
\Sigma_{BB} &= p(1 - pr\sigma^A)(1 - (1 - r)\sigma^B) - 2qp\sigma^A\sigma^B r(1 - r) + q(r\sigma^A + (1 - r)\sigma^B).
\end{aligned}$$

Consider first the pure strategy equilibria, $\sigma^A, \sigma^B \in \{0, 1\}$. An equilibrium exists if each agent $i \in \{A, B\}$ prefers to follow his/her prescribed strategy given the other agent j's strategy. For agent i, if $\sigma^i = 1$, then the payoff from revision is

$$EU^{i,R} = rqU^i(i, \Delta^i) + r(1 - q)U^i(i, \Delta^j) + q(1 - r)U^i(j, \Delta^i) + (1 - r)(1 - q)U^i(j, \Delta^j).$$

If $\sigma^i = 0$, then the payoff from project continuation of a project Δ^j is

$$EU^{i,C} = psv(1-w) + (1-p)(1-r)U^i(j,\Delta^j) + (1-p)rU^i(i,\Delta^j)$$

Case 1: $\sigma^A = 1$ and $\sigma^B = 1$. This is an equilibrium if $EU^{i,R} \geq EU^{i,C}$ for $i,j \in \{A,B\}$. These conditions reduce to two upper bounds on $c(s)/s$, such that this equilibrium is sustainable if

$$\frac{c(s)}{s} \leq qv\Delta \cdot \min\left\{\frac{p(1-r)}{p(1-r)+2qr}; \frac{pr}{pr+2q(1-r)}\right\}.$$

Case 2: $\sigma^A = 1$ and $\sigma^B = 0$. This is an equilibrium if $EU^{A,R} \geq EU^{A,C}$ and $EU^{B,R} < EU^{B,C}$. These conditions reduce to two thresholds:

$$\begin{aligned}\frac{c(s)}{s} &\leq qv\Delta,\\ \frac{c(s)}{s} &\geq qv\Delta\frac{p(1-r)}{p(1-r)+2qr}.\end{aligned}$$

Therefore, the equilibrium exists for

$$\frac{c(s)}{s} \in \left[qv\Delta\frac{p(1-r)}{p(1-r)+2qr}, qv\Delta\right].$$

Case 3: $\sigma^A = 0$ and $\sigma^B = 1$. This is an equilibrium if $EU^{A,R} < EU^{A,C}$ and $EU^{B,R} \geq EU^{B,C}$. These conditions reduce to two thresholds:

$$\begin{aligned}\frac{c(s)}{s} &\geq qv\Delta\frac{pr}{pr+2q(1-r)},\\ \frac{c(s)}{s} &\leq qv\Delta.\end{aligned}$$

Therefore, the equilibrium exists for

$$\frac{c(s)}{s} \in \left[qv\Delta\frac{pr}{pr+2q(1-r)}, qv\Delta\right].$$

Case 4: $\sigma^A = 0$ and $\sigma^B = 0$. This is an equilibrium if $EU^{A,R} < EU^{A,C}$ and $EU^{B,R} < EU^{B,C}$. These conditions reduce to the same lower bound $\frac{c(s)}{s} \geq qv\Delta$.

Consider next the case of mixed strategy equilibria.

Case 5: $\sigma^A \in (0,1)$ or $\sigma^B \in (0,1)$. If Agent A mixes with $\sigma^A \in (0,1)$, this requires $U^A(A,\Delta^B|1,\sigma^B) = U^A(A,\Delta^B|0,\sigma^B)$, and thus the equilibrium σ^{B*} is

$$\sigma^{B*} = \frac{p(qsv(2w-1)-c(s))}{(1-r)[p(qsv(2w-1)-c(s))+2qc(s)]}. \tag{14}$$

Similarly, if Agent B mixes, then $U^B(B, \Delta^B|\sigma^A, 1) = U^B(B, \Delta^B|\sigma^A, 0)$. Thus the equilibrium σ^{A*} is

$$\sigma^{A*} = \frac{p(qsv(2w-1) - c(s))}{r[p(qsv(2w-1) - c(s)) + 2qc(s)]}. \tag{15}$$

The mixing probabilities must satisfy $\sigma^B \in [0,1]$ and $\sigma^A \in [0,1]$. Given (14) and (15), this implies

$$\frac{c(s)}{s} \in \left[pqv(2w-1)\max\left\{\frac{1-r}{p(1-r)+2qr}, \frac{r}{pr+2q(1-r)}\right\}, qv(2w-1)\right]. \tag{16}$$

Notice that the above condition allows for an equilibrium with $\sigma^A = 1, \sigma^B \in (0,1)$ if $\frac{c(s)}{s} = \frac{1-r}{p(1-r)+2qr}$ and $\max\left\{\frac{1-r}{p(1-r)+2qr}, \frac{r}{pr+2q(1-r)}\right\} = \frac{1-r}{p(1-r)+2qr}$. Conversely, an equilibrium with $\sigma^A \in (0,1), \sigma^B = 1$ exists if $\frac{c(s)}{s} = \frac{r}{pr+2q(1-r)}$ and $\max\left\{\frac{1-r}{p(1-r)+2qr}, \frac{r}{pr+2q(1-r)}\right\} = \frac{r}{pr+2q(1-r)}$.

Therefore, we have the following bounds in terms of $c(s)/s$ for the equilibrium regions:

$$\begin{aligned}
\frac{c(\bar{s}_1)}{\bar{s}_1} &= qv\Delta, \\
\frac{c(\bar{s}_2)}{\bar{s}_2} &= qv\Delta \cdot \max\left\{\frac{pr}{pr+2q(1-r)}, \frac{p(1-r)}{p(1-r)+2qr}\right\}, \\
\frac{c(\bar{s}_3)}{\bar{s}_3} &= qv\Delta \cdot \min\left\{\frac{pr}{pr+2q(1-r)}, \frac{p(1-r)}{p(1-r)+2qr}\right\}.
\end{aligned}$$

Given $\Delta = 2w - 1$, this implies the following corresponding bounds on w:

$$w_1(s) = \frac{1}{2} + \frac{c(s)}{s}\frac{1}{2qv}, \tag{17}$$

$$w_2(s) = \frac{1}{2} + \frac{c(s)}{s}\frac{1}{2qv} + \frac{c(s)}{s}\frac{1}{pv}\min\left\{\frac{1-r}{r}, \frac{r}{1-r}\right\}, \tag{18}$$

$$w_3(s) = \frac{1}{2} + \frac{c(s)}{s}\frac{1}{2qv} + \frac{c(s)}{s}\frac{1}{pv}\max\left\{\frac{1-r}{r}, \frac{r}{1-r}\right\}. \tag{19}$$

A.2 Proof for Lemma 2

We derive the equilibrium s^* and w^* chosen by Agent A in period 0. The derivation is divided into three parts, and within each part the results are organized into Claims.

Part 1: Properties of the Value Function

Claim 1 *Agent A's expected utility at time 0 is piecewise linear in w for $w \in [0,1]$.*

Proof. Given (12) and (13), with pure strategy equilibria, $\sigma^A, \sigma^B \in \{0, 1\}$,

$$\frac{\partial^2 U^A(i, \Delta^j|\sigma^A, \sigma^B)}{\partial w^2} = \frac{\partial^2 U^B(i, \Delta^j|\sigma^A, \sigma^B)}{\partial w^2} = 0.$$

For mixed strategy equilibria, we have

$$\begin{aligned}\frac{\partial U^A(A, \Delta^A|\sigma^A, \sigma^B)}{\partial w} &= -(1-2p)sv, \\ \frac{\partial U^A(B, \Delta^A|\sigma^A, \sigma^B)}{\partial w} &= -\frac{1-(1-2p)r}{1-r}sv.\end{aligned}$$

Thus, also with mixed strategy equilibria, given (15) and (14),

$$\frac{\partial^2 U^A(i, \Delta^j|\sigma^A, \sigma^B)}{\partial w^2} = \frac{\partial^2 U^B(i, \Delta^j|\sigma^A, \sigma^B)}{\partial w^2} = 0.$$

Let $EU^A(s, w) \equiv r \cdot U^A(A, \Delta^A) + (1-r) \cdot U^A(B, \Delta^A)$. It immediately follows that $EU^A(s, w)$ is linear in w given σ^A, σ^B. ■

Claim 2 *Agent A's expected utility is increasing in w whenever $\sigma^B = 0$.*

Proof. In period 0, we have

$$\begin{aligned}EU^A(s, w|\sigma^A, \sigma^B) = s \cdot \frac{vp}{\Omega} \cdot (1 - r\sigma^A) \cdot (1 - (1-r)\sigma^B) \cdot w \\ + s \cdot \frac{vq}{\Omega} \cdot [r\sigma^A w + (1-r)\sigma^B(1-w) - r(1-r)\sigma^A\sigma^B] \\ - \frac{c(s)}{p\Omega}\left[q(r\sigma^A + (1-r)\sigma^B) + p(1 - r\sigma^A)\right]. \quad (20)\end{aligned}$$

It follows that if $\sigma^A = \sigma^B = 0$, or if $\sigma^A = 1, \sigma^B = 0$ then

$$EU^A(s, w|0, 0) = EU^A(s, w|1, 0) = svw - \frac{c(s)}{p}, \quad (21)$$

and thus

$$\frac{\partial EU^A(s, w)}{\partial w} = sv > 0.$$

■

Claim 3 *Agent A's expected utility is monotone, either increasing or decreasing in w, when-*

ever $\sigma^B = 1$*:*

$$\frac{\partial EU^A(s,w|0,1)}{\partial w}\begin{cases} >0 & \textit{if } p > q\frac{1-r}{r} \\ <0 & \textit{if } p < q\frac{1-r}{r} \end{cases};$$

$$\frac{\partial EU^A(s,w|1,1)}{\partial w}\begin{cases} >0 & \textit{if } p > q\frac{2r-1}{r(1-r)} \\ <0 & \textit{if } p < q\frac{2r-1}{r(1-r)} \end{cases}.$$

Proof. If $\sigma^A = 0, \sigma^B = 1$, then

$$EU^A(s,w|0,1) = sv\frac{q(1-r)(1-w)+prw}{q(1-r)+pr} - \frac{c(s)}{p}\frac{q(1-r)+p}{q(1-r)+pr}, \tag{22}$$

and thus

$$\frac{\partial EU^A(s,w|0,1)}{\partial w} = sv\frac{pr-q(1-r)}{pr+q(1-r)}, \tag{23}$$

which means

$$\frac{\partial EU^A(s,w|0,1)}{\partial w}\begin{cases} >0 & \text{if } p > q\frac{1-r}{r} \\ <0 & \text{if } p < q\frac{1-r}{r} \end{cases}.$$

If $\sigma^A = 1, \sigma^B = 1$, then

$$EU^A(s,w|1,1) = sv\frac{q(1-r)^2+(q(2r-1)+p(1-r)r)w}{q(1-2r(1-r))+pr(1-r)} - \frac{c(s)}{p}\frac{p(1-r)+q}{q(1-2r(1-r))+pr(1-r)}, \tag{24}$$

and thus

$$\frac{\partial EU^A(s,w|1,1)}{\partial w} = sv\frac{q(2r-1)+pr(1-r)}{q(1-2r(1-r))+pr(1-r)}, \tag{25}$$

which means

$$\frac{\partial EU^A(s,w|1,1)}{\partial w}\begin{cases} >0 & \text{if } p > q\frac{2r-1}{r(1-r)} \\ <0 & \text{if } p < q\frac{2r-1}{r(1-r)} \end{cases}.$$

■

Claim 4 *Agent A's expected utility is monotone decreasing in w if the equilibrium is mixing.*

Proof. Given the equilibrium mixing probabilities σ^A, σ^B, we have

$$EU^A(s,w|\sigma^{A*},\sigma^{B*}) = sv(1-w) - c(s)\left(\frac{1}{p}-\frac{1}{q}\right).$$

Hence,

$$\frac{\partial EU^A(s,w|\sigma^{A*},\sigma^{B*})}{\partial w} = -sv < 0.$$

■

Claim 5 *Given any s, agent A's expected utility is continuous in w with the exception of a finite number of discontinuities:*

- *A jump down at $w_3(s)$ if $r > \frac{1}{2}$;*
- *A jump up at $w_2(s)$ if the equilibrium selected when $w \in (w_1, w_2)$ is (i)mixing; (ii) $\sigma^A = 0, \sigma^B = 1$ when $r \geq \frac{1}{2}$; or (iii) $\sigma^A = 1, \sigma^B = 0$ when $r < \frac{1}{2}$.*
- *A jump down at $w_1(s)$ if the equilibrium selected is $\sigma^A = 0, \sigma^B = 1$ when $w \in (w_1, w_2)$;*

Proof.

Case 1: $r \geq \frac{1}{2}$. In this case, the bounds w_2 and w_3 are given by

$$w_2(s) = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{1-r}{r}, \tag{26}$$

and

$$w_3(s) = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{r}{1-r}. \tag{27}$$

The unique equilibrium when $w \in (w_2, w_3)$ is $\sigma^A = 1, \sigma^B = 0$. Therefore, the equilibrium changes at w_3 from $\sigma^B = 0$ to $\sigma^B = 1$. We have

$$\lim_{w \uparrow w_3} EU^A(s, w|0, 0) = \frac{sv}{2} + c(s)\left[\frac{1}{2q} + \frac{2r-1}{p(1-r)}\right],$$

and

$$\lim_{w \downarrow w_3} EU^A(s, w|1, 1) = \frac{sv}{2} + \frac{c(s)}{2}\frac{1}{q}\frac{pr - q(1-r) + q\frac{3r^2}{1-r}}{pr + q(1-r) + q\frac{r^2}{1-r}} + \frac{c(s)}{p}\frac{q\frac{r^2-2r(1-r)}{(1-r)^2} - q - p}{pr + q(1-r) + q\frac{r^2}{1-r}}.$$

Then,

$$\lim_{w \uparrow w_3} EU^A(s, w) - \lim_{w \downarrow w_3} EU^A(s, w) = \frac{c(s)}{p}\frac{2(1-r)(p(1-r) + 2qr)}{q(1 - 2r(1-r)) + pr(1-r)} > 0.$$

If in the region of multiplicity the equilibrium selected is $\sigma^A = 1, \sigma^B = 0$, then the expected utility is continuous and has the same expression as a function of w for all $w \leq w_3$.

If in the region of multiplicity the equilibrium selected is $\sigma^A = 0, \sigma^B = 1$, then the equilibrium σ^B changes at both w_1 and w_2. At w_2 we have

$$\lim_{w \uparrow w_2} EU^A(s, w|0, 1) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{pr},$$

$$\lim_{w \downarrow w_2} EU^A(s,w|1,0) = \frac{sv}{2} + \frac{c(s)}{2q} + \frac{c(s)}{p}\frac{1-2r}{r}.$$

Then,

$$\lim_{w \uparrow w_2} EU^A(s,w) - \lim_{w \downarrow w_2} EU^A(s,w) = -2\frac{c(s)}{p}\frac{1-r}{r} < 0. \tag{28}$$

Hence, there is a jump up at w_2.

At w_1, we have

$$\lim_{w \uparrow w_1} EU^A(s,w|0,0) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{p},$$

$$\lim_{w \downarrow w_1} EU^A(s,w|0,1) = \frac{sv}{2} - (1-r)\frac{c(s)}{2} + c(s)\frac{pr}{2q} - \frac{c(s)}{p}\frac{q(1-r)+p}{q(1-r)+pr}.$$

Then,

$$\lim_{w \uparrow w_1} EU^A(s,w) - \lim_{w \downarrow w_1} EU^A(s,w) = \frac{c(s)(1-pr+q(1-r))}{2q} + \frac{c(s)}{p}\frac{p(1-r)}{q(1-r)+pr} > 0. \tag{29}$$

Hence, there is a jump down at w_1.

If in the region of multiplicity the mixing equilibrium is selected, then at w_1, $\sigma^A = 1, \sigma^B = 0$, and therefore $EU^A(s,w_1|0,0) = EU^A(s,w_1|\sigma^A,\sigma^B)$. Hence, there is no discontinuity at w_1. At w_2, $\sigma^A = 0, \sigma^B = 1$, and therefore $EU^A(s,w_1|0,1) = EU^A(s,w_1|\sigma^A,\sigma^B)$. Hence, there is the same discontinuity at w_2 as in (28).

Case 2: $r < \frac{1}{2}$. In this case,

$$w_2(s) = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{r}{1-r}, \tag{30}$$

and

$$w_3(s) = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{1-r}{r}. \tag{31}$$

As shown above, $EU^A(s,w|0,0) = EU^A(s,w|1,0)$, which implies that there is no discontinuity at w_1 if the equilibrium selected in the multiplicity region is $\sigma^A = 1, \sigma^B = 0$. If the equilibrium selected in the region of multiplicity is $\sigma^A = 0, \sigma^B = 1$, then at w_1 we have the same discontinuity as in (29). If the mixing equilibrium is selected in the region of multiplicity, then at w_1 we have $\sigma^A = 1, \sigma^B = 0$, and therefore $EU^A(s,w_1|0,0) = EU^A(s,w_1|\sigma^A,\sigma^B)$. Hence, there is no discontinuity at w_1.

At w_2, if the equilibrium selected in the multiplicity region is $\sigma^A = 0, \sigma^B = 1$, then there is no discontinuity, as the equilibrium in (w_2, w_3) is also $\sigma^A = 0, \sigma^B = 1$. If the equilibrium selected in the multiplicity region is $\sigma^A = 1, \sigma^B = 0$, then

$$\lim_{w\uparrow w_2} EU^A(s,w|1,0) = \frac{sv}{2} + \frac{c(s)}{2q} + \frac{c(s)}{p}\frac{2r-1}{1-r},$$

$$\lim_{w\downarrow w_2} EU^A(s,w|0,1) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{p(1-r)} - c(s)\frac{2(1-2r)}{pr+q(1-r)}.$$

Then,

$$\lim_{w\uparrow w_2} EU^A(s,w) - \lim_{w\downarrow w_2} EU^A(s,w) = c(s)\frac{2}{p}\frac{p(1-r)+qr}{pr+q(1-r)} > 0.$$

Hence, there is a jump down at w_2.

If the mixing equilibrium is selected in the multiplicity region, then at w_2, $\sigma^A = 1, \sigma^B = \frac{r}{1-r}$.

$$\lim_{w\uparrow w_2} EU^A(s,w|(\sigma^A,\sigma^B)) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{2p(1-r)}$$

Then,

$$\lim_{w\uparrow w_2} EU^A(s,w) - \lim_{w\downarrow w_2} EU^A(s,w) = \frac{c(s)}{2p(1-r)} + c(s)\frac{2(1-2r)}{pr+q(1-r)} > 0.$$

Hence, there is also a jump down at w_2 under mixing.

Finally, at w_3,

$$\lim_{w\uparrow w_3} EU^A(s,w) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{pr} = \lim_{w\downarrow w_3} EU^A(s,w)$$

Thus, there is no jump at w_3 if $r < 0.5$ ■

Claim 6 *It is never the case that* $\frac{\partial EU^A(s,w|0,1)}{\partial w} > 0 > \frac{\partial EU^A(s,w|1,1)}{\partial w}$.

Proof. By Claim 3, given that $\frac{1-r}{r} > \frac{1-2r}{r(1-r)}$, we have

$$\begin{aligned}
&0 < \frac{\partial EU^A(s,w|0,1)}{\partial w} \text{ and } 0 < \frac{\partial EU^A(s,w|1,1)}{\partial w} \text{ if } p > q\frac{1-r}{r},\\
&\frac{\partial EU^A(s,w|0,1)}{\partial w} < 0 < \frac{\partial EU^A(s,w|1,1)}{\partial w} \text{ if } q\frac{1-2r}{r(1-r)} < p < q\frac{1-r}{r},\\
&\frac{\partial EU^A(s,w|0,1)}{\partial w} < 0 \text{ and } \frac{\partial EU^A(s,w|1,1)}{\partial w} < 0 \text{ if } p < q\frac{1-2r}{r(1-r)}.
\end{aligned}$$

■

Part 2: Agent $A's$ choice of $w(s)$ for a given s

Consider the problem for Agent A of choosing w for a given s. Denote this value $w^*(s)$.

Claim 7 *If $r \geq 0.5$, then either $w^*(s) = w_3(s) < 1$ or $w^*(s) = 1$.*

Proof. By Claims 1, 2 and 4, the solution $w^*(s) \in \{w_1(s), w_2(s), w_3(s), 1\}$. If $r \geq 0.5$, then $w^*(s) \notin (w_1, w_2)$ if $w_2(s) \leq 1$, given that the function is monotone in between these bounds, by Lemmas 2-4. By Claim 5, there is either a jump down or continuity at w_1, followed by a jump up or continuity at w_2. Hence, the equilibrium selection in the multiplicity region is irrelevant for the value of $w^*(s)$. Claim 2 shows that $EU^A(s, w)$ is increasing for $w \leq w_1(s)$ and for $w \in [w_2(s), w_3(s)]$. Lemma 6 implies that $\frac{\partial EU^A(s,w|1,1)}{\partial w} > 0$, since $q\frac{1-2r}{r(1-r)} < 0$. Therefore, the expected utility is increasing for all $w \notin (w_1, w_2)$. By Lemma 5, the only discontinuity for $w \notin [w_1, w_2]$ is at $w_3(s)$, where the function jumps down. Therefore, the maximum satisfies $w^*(s) \in \{w_3(s), 1\}$. ■

Claim 8 *If $r < 0.5$,*

- *if in the multiplicity region the equilibrium selected is $\sigma^A = 0, \sigma^B = 1$ or the mixing equilibrium, then $w^*(s) = w_1(s) < 1$ or $w^*(s) = 1$;*
- *otherwise, $w^*(s) = w_2(s) < 1$ or $w^*(s) = 1$.*

Proof. By Claims 1, 2 and 4, the solution $w^*(s) \in \{w_1(s), w_2(s), w_3(s), 1\}$; but Claim 6 implies that the solution cannot be $w_3(s)$. If the selected equilibrium is $\sigma^A = 0, \sigma^B = 1$ in the multiplicity region, then the expected utility function has the same expression for $w \in [w_1(s), w_3(s)]$ and is monotone in this interval, hence the solution cannot be at $w_2(s) \in [w_1(s), w_3(s)]$. By Claim 5, there is jump down at $w_1(s)$ and the expected utility is otherwise continuous. Therefore, $w^*(s) \in \{w_1(s), 1\}$.

If in the multiplicity region the equilibrium selected is the mixing equilibrium, then by Claim 5, the only discontinuity is at $w_2(s)$, where the expected utility jumps down. By Claim 4, the expected utility is decreasing in the mixing region. Hence, $w_2(s)$ cannot be the solution. Therefore, $w^*(s) \in \{w_1(s), 1\}$.

If in the multiplicity region the equilibrium selected is $\sigma^A = 1, \sigma^B = 0$, then by Claim 2, the expected utility is increasing for all $w \leq w_2(s)$. By Claim 5, the only discontinuity is at $w_2(s)$, where the expected utility jumps down. Hence, $w^*(s) \in \{w_2(s), 1\}$. ■

Part 3: Agent $A's$ choice of s

Given $w^*(s)$, we can now move to the selection of s. Denote the optimal scale chosen by A as s^*. Notice that given $w = 1$, the expression for $EU^A(s, 1)$ implied by (12) is strictly concave in s for any combination $\sigma^A, \sigma^B \in \{0, 1\}$. Moreover, it is either concave or increasing convex in s for the σ^A, σ^B corresponding to the mixing equilibrium.

Claim 9 *If $r \geq \frac{1}{2}$, then $w^* = 1$ and*

$$c'(s^*) = \begin{cases} vp, & \text{if } p \geq q \cdot \left(\varepsilon(\bar{s}_3) - \frac{2r}{1-r}\right) \\ c'(\bar{s}_3), & \text{otherwise} \end{cases} . \tag{32}$$

Proof. Consider first the case when the equilibrium selected in the multiplicity region is $(\sigma^A, \sigma^B) = (1, 0)$. By Claim 7, $w^*(s) \in \{w_3(s), 1\}$. We have $w_3 \leq 1$ iff $\frac{c(s)}{s} \leq vq\frac{p(1-r)}{p(1-r)+2qr} = \frac{c(\bar{s}_3(w=1))}{\bar{s}_3(w=1)}$. Thus, if $\frac{c(s)}{s} > \frac{c(\bar{s}_3(w=1))}{\bar{s}_3(w=1)}$, then $w_3 > 1$, and $\sigma^B = 0$ at $w = 1$. The problem for Agent A in this case is

$$\max_{s > \bar{s}_3} sv - \frac{c(s)}{p}. \tag{33}$$

This leads to

$$c'(s^*) = \max\{vp, c'(\bar{s}_3)\}. \tag{34}$$

If $\frac{c(s)}{s} \leq \frac{c(\bar{s}_3)}{\bar{s}_3}$, then $w_3 \leq 1$. At w_3, the expected utility of Agent A is

$$EU^A(s, w_3|1, 0) = \frac{vs}{2} + c(s)\left(\frac{2r-1}{p(1-r)} + \frac{1}{2q}\right), \tag{35}$$

which is strictly convex and increasing in s, reaching the maximum at $\bar{s}_3(w_3)$, where $w_3 = 1$. Thus,

$$EU^A(\bar{s}_3, 1) = \bar{s}_3 \cdot v \cdot \left(1 - \frac{q(1-r)}{p(1-r) + 2qr}\right) \tag{36}$$

At $w = 1$ (with $w_3 < 1$ such that $\sigma^B = 1$), we have $EU^A(s, 1|1, 1) \leq EU^A(s, 1|1, 0)$ since

$$\begin{aligned} EU^A(s, 1|1, 1) &\leq sv\frac{q(1-r)^2 + (q(2r-1) + p(1-r)r)}{q(1-2r(1-r)) + pr(1-r)} \\ &\leq \bar{s}_3 v\frac{p(1-r) + 2qr - q(1-r)}{p(1-r) + 2qr} = EU^A(\bar{s}_3, 1|1, 0). \end{aligned}$$

Therefore, the maximum utility value reached in the region of s values where $w_3 < 1$ is below the utility reached when $s = \bar{s}_3(w = 1)$. Hence, Agent $A's$ optimal choices are $w^* = 1$, and

$$c'(s^*) = \begin{cases} vp, \text{ if } p \geq \bar{q}(\varepsilon, q, r|r \geq 0.5) \\ c'(\bar{s}_3(w = 1)), \text{ otherwise} \end{cases}, \tag{37}$$

where

$$\bar{q}(\varepsilon, q, r|r \geq 0.5) \equiv \max\left\{0, q \cdot \left(\varepsilon(\bar{s}_3(w = 1)) - \frac{2r}{1-r}\right)\right\}. \tag{38}$$

The equilibrium revision strategies are $(0, 0)$ if $w_1 \geq 1$, that is, if $s^* \geq \bar{s}_1$. Otherwise, the revision equilibrium is $(\sigma^A, \sigma^B) = (1, 0)$. For completion, define $\underline{q}(\varepsilon, q, r|r \geq 0.5) = 0$.

Other equilibrium selections under multiplicity. When the equilibrium selected in the multiplicity region is not $(\sigma^A, \sigma^B) = (1, 0)$, the above analysis is unchanged if $w_2(s) \leq 1$. If $w_1(s) \leq 1 < w_2(s)$, i.e., if $\frac{c(\bar{s}_2(w=1))}{\bar{s}_2(w=1)} < \frac{c(s)}{s} \leq \frac{c(\bar{s}_1(w=1))}{\bar{s}_1(w=1)}$, then at $w = 1$, $\sigma^B \neq 0$. In this case, given Claim 5, the solution $w^*(s)$ is $w_1(s)$. Then s is chosen to maximize the following function over s:

$$EU^A(s, w_1(s)) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{p}. \tag{39}$$

If $p \geq 2q$, then $s^* > \overline{s}_1(w=1)$ and therefore the solution cannot be at $w_1 \leq 1$. Instead, $w = 1$ and the solution for s^* is as described above. Otherwise, if $p < 2q$, then the solution to (39) in interior. But in this case, notice that (i) at $\overline{s}_1$, we have $EU^A(\overline{s}_1, 1) = EU^A(\overline{s}_1, w_1)$ and (ii) at $\overline{s}_2$, we have $EU^A(\overline{s}_2, 1) > EU^A(\overline{s}_2, w_1)$, since $\frac{c(\overline{s}_2)}{\overline{s}_2} < vq$. Then note also that (iii) $0 > \frac{\partial^2 EU^A(s,w_1)}{\partial s^2} > \frac{\partial^2 EU^A(s,1)}{\partial s^2}$. Then (i)-(iii) imply that the optimum s satisfies $s \notin (\overline{s}_2(w=1), \overline{s}_1(w=1))$. Therefore, the solution is $w = 1$ and s^* as in the case when the equilibrium selected in the multiplicity region is $(\sigma^A, \sigma^B) = (1, 0)$.

■

Claim 10 *If $r < \frac{1}{2}$, then there exist thresholds $0 \leq \underline{q} \leq \overline{q}$ such that*

$$\begin{cases} c'(s^*) = \frac{vqp}{2q-p}, w^* = w_1(s^*) < 1, \text{ if } p \leq \underline{q} \\ c'(s^*) = c'(\overline{s}^*), w^* = 1, \text{ if } \underline{q} < p < \overline{q} \\ c'(s^*) = vp, w^* = 1, \text{ if } p \geq \overline{q} \end{cases}, \tag{40}$$

where $\overline{s}^* = \overline{s}_1(w=1)$ if the equilibrium selected in the multiplicity region has $\sigma^B \neq 0$, and $\overline{s}^* = \overline{s}_2(w=1)$ if the equilibrium selected in the multiplicity region has $\sigma^B = 0$. The threshold values $\underline{q}, \overline{q}$ also depend on the equilibrium selection in the multiplicity region.

Proof. By Claim 8, $w^*(s) \in \{w_1(s), w_2(s), 1\}$. Consider first the case where the equilibrium selected in the multiplicity region is not $(\sigma^A, \sigma^B) = (1, 0)$, such that $w^*(s) \in \{w_1(s), 1\}$. If $s \geq \overline{s}_1(w=1)$, we have $w_1(s) \geq 1$ and the expected utility at w_1 is

$$EU^A(s, 1|0, 0) = vs - \frac{c(s)}{p}. \tag{41}$$

The solution is

$$c'(s) = \max\left\{vp, c'(\overline{s}_1)\right\}.$$

If $s < \overline{s}_1$, then $w_1(s) < 1$ and the expected utility at w_1 is

$$EU^A(s, w_1|0, 0) = \frac{vs}{2} - c(s)\left(\frac{1}{p} - \frac{1}{2q}\right). \tag{42}$$

The solution for s^* when $w_1(s^*)$ is chosen is

$$c'(s^*) = \begin{cases} \frac{vqp}{2q-p}, \text{ if } p \leq \underline{q}(\varepsilon, q, r|r < 0.5), \\ c'(\overline{s}_1(w=1)), \text{ if } \underline{q}(\varepsilon, q, r|r < 0.5) < p < \overline{q}(\varepsilon, q, r|r < 0.5), \\ vp, \text{ if } p \geq \overline{q}(\varepsilon, q, r|r < 0.5), \end{cases} \tag{43}$$

where

$$\overline{q}(\varepsilon, q, r | r < 0.5) = q \cdot \varepsilon(\overline{s}_1(w=1)), \tag{44}$$

$$\underline{q}(\varepsilon, q, r | r < 0.5) = 2q \cdot \frac{\varepsilon(\overline{s}_1(w=1))}{1 + \varepsilon(\overline{s}_1(w=1))} \tag{45}$$

Notice that the solution can be $w = 1$, with equilibrium revision strategies $(\sigma^A, \sigma^B) = (1, 1)$ only if $q\frac{1-2r}{r(1-r)} < p$ and $s^* \leq \overline{s}_3(w = 1)$. In this case, agent $A's$ expected utility is

$$EU^A(s, 1|1, 1) = vs\frac{r(p(1-r)+qr)}{r(p(1-r)+qr)+q(1-r)^2} - \frac{c(s)}{p}\frac{p(1-r)+q}{r(p(1-r)+qr)+q(1-r)^2}, \tag{46}$$

The solution is at the corner $w = 1$ if $EU^A(s, 1|1, 1) - EU^A(s, w_1|0, 0)) \geq 0$, which is equivalent to

$$\frac{c(s)}{s} \leq \frac{c(\overline{s}_{11})}{\overline{s}_{11}} \equiv vqp \cdot \frac{pr(1-r) - q(1-2r)}{(1-r)r(p-2q)^2 + pq(3-2r)}, \tag{47}$$

where it can be verified that $\overline{s}_{11} < \overline{s}_3(w = 1)$. Else, if $s > \overline{s}_{11}$, then $w_1(s)$ is optimal.

This implies that in the region $s \in [0, \overline{s}_{11}]$, the optimal s is

$$c'(s^*) = \min\left\{vp \cdot \frac{r(p(1-r)+qr)}{p(1-r)+q}, c'(\overline{s}_{11})\right\}, \tag{48}$$

whereas in the region $s > \overline{s}_{11}$, the solution is as in (43).

Having found the solution for $s \leq \overline{s}_{11}$ and for $s > \overline{s}_{11}$, it remains to find the global optimum.

First, if $c'(s^*) = \frac{vqp}{2q-p}$, then note that

$$\frac{p(1-r)+q}{r(p(1-r)+qr)+q(1-r)^2} > \frac{2q-p}{2q}, \tag{49}$$

which means that

$$\frac{\partial^2 EU^A(s|w=1,(1,1))}{\partial s^2} < \frac{\partial^2 EU^A(s|w=w_1,(0,0))}{\partial s^2}, \tag{50}$$

and therefore a sufficient condition for the global maximum to satisfy $s^* \geq \overline{s}_{11}$ is that

$$-\frac{\partial EU^A(\overline{s}_{11}|w=1,(1,1))}{\partial s} < \frac{\partial EU^A(\overline{s}_{11}|w=w_1,(0,0))}{\partial s},$$

which reduces to

$$c'(\overline{s}_{11}) \leq vpq\frac{(3r(1-r)p + q(1-2r+4r^2))}{pq(1-2r)+4q^2-(p-2q)^2 r(1-r)}. \tag{51}$$

Given (47), the above implies an upper bound on the elasticity of $c(s)$ of

$$\varepsilon(s) = \frac{s \cdot c'(s)}{c(s)} \leq \min_{p \in (q\frac{1-2r}{r(1-r)}, 2q), q \in [0,1], r \in [0,0.5]} \left\{ \frac{r(1-r)(p-2q)^2 + pq(3-2r)}{p(1-r)r - q(1-2r)} \cdot \frac{(3r(1-r)p + q(1-2r+4r^2))}{pq(1-2r) + 4q^2 - (p-2q)^2 r(1-r)} \right\}.$$

This bound is decreasing in r, which means that the problem can be reduced to

$$\varepsilon(s) \leq \min_{p \in (0,2q), q \in [0,1]} \left\{ \frac{(p+2q)(3p+4q)}{p(6q-p)} \right\}. \tag{52}$$

Using numerical minimization methods, we can show that this bound for the elasticity is at least 4.19. Therefore, a sufficient condition for the solution to satisfy $s^* \geq \overline{s}_{11}$ is

$$\varepsilon(s) \leq 4 \leq \min_{p \in [0,2q], q \in [0,1]} \left\{ \frac{(p+2q)(3p+4q)}{p(6q-p)} \right\} \equiv \overline{\varepsilon}(p,q). \tag{53}$$

Next, consider the case where $c'(\overline{s}_1) = \frac{vqp}{2q-p}$ and $q < p$. Notice that at $q = p$, we have $s^*_{00} \equiv \arg\max EU^A(s, w_1|0,0)$ that satisfies $c'(s^*_{00}) = vq \leq c'(\overline{s}_1)$. Thus, the maximum value $EU^A(s, w_1|0,0)$ is reached at $s < \overline{s}_1$ for $q = p$. Let s^*_{11} denote the solution for $\arg\max EU^A(s, 1|1,1)$ and let $r = 1/2$, in order to maximize $EU^A(s^*_{11}, 1|1,1)$. The effect of decreasing q at $s = \overline{s}_{11}$ is

$$-\frac{\partial^2 EU^A(s,1|1,1)}{\partial s \partial q} = \frac{vp}{(p+2q)^2}, \tag{54}$$

$$-\frac{\partial^2 EU^A(s,w_1|0,0)}{\partial s \partial q} = \frac{c'(s^*_{00})}{2q^2}, \tag{55}$$

$$-\frac{\partial^3 EU^A(s,1|1,1)}{\partial s \partial q} = 0, \tag{56}$$

$$-\frac{\partial^3 EU^A(s,w_1|0,0)}{\partial s^2 \partial q} = \frac{c''(s^*_{00})}{2q^2}. \tag{57}$$

This implies

$$-\left(\frac{\partial^2 EU^A(s,w_1|0,0)}{\partial s \partial q} + \frac{\partial^2 EU^A(s,1|1,1)}{\partial s \partial q} \right) > 0, \tag{58}$$

while the rate of decrease in the slope of $EU^A(s,w)$ is steeper for $EU^A(s,w_1|0,0)$ compared to $EU^A(s,1|1,1)$. Then, (58) together with (56) and (57) along with condition (53) imply that

$$\max_{s \leq \overline{s}_1} EU^A(s,w_1|0,0) > \max_{s \leq \overline{s}_1} EU^A(s,1|1,1) \text{for } q \in [0,p]. \tag{59}$$

In the third case, if $c'(s) = vp$, then notice that

$$EU^A(s,1|0,0) - EU^A(s,1|1,1) > 0, \tag{60}$$

and

$$\left.\frac{\partial EU^A(s,1|0,0)}{\partial s}\right|_{s=\bar{s}_1} > 0 \implies \left.\frac{\partial EU^A(s,w_1|0,0)}{\partial s}\right|_{s=\bar{s}_1} \geq 0.$$

Then, the $\arg\max_{s \geq \bar{s}_1} EU^A(s,1|0,0)$ is the global maximum.

In sum, under condition (53), the global solution is

$$\begin{cases} c'(s^*) = \frac{vqp}{2q-p}, w^* = w_1 < 1, & \text{if } p \leq \underline{q}(\varepsilon(\bar{s}_1), q, r|r < 0.5), \\ c'(s^*) = c'(\bar{s}_1), w^* = 1, & \text{if } \underline{q}(\varepsilon(\bar{s}_1), q, r|r < 0.5) < p < \bar{q}(\varepsilon(\bar{s}_1), q, r|r < 0.5), \\ c'(s^*) = vp, w^* = 1, & \text{if } p \geq \bar{q}(\varepsilon(\bar{s}_1), q, r|r < 0.5), \end{cases}, \tag{61}$$

where $\bar{q}(\varepsilon, q, r|r < 0.5)$ and $\underline{q}(\varepsilon, q, r|r < 0.5)$ are given in (44)-(45). On the equilibrium path $\sigma^B(s^*, w^*) = 0$.

Other equilibrium selections under multiplicity. Consider next the case where the equilibrium selected in the multiplicity region is $(\sigma^A, \sigma^B) = (1,0)$. In this case, $w^* \in \{w_2(s), 1\}$. The analysis above carries over under the change of threshold from w_1 to w_2. Notice that $EU^A(s, w_2|0,0) \geq EU^A(s, w_1|0,0)$, which means that condition (53) is sufficient to ensure that the global solution is not $w = 1$ and $(\sigma^A, \sigma^B) = (1,1)$. The solution for s^* when w^* becomes

$$\begin{cases} c'(s^*) = \frac{vqp(1-r)}{2q(1-2r)-p(1-r)}, w^* = w_1 < 1 & \text{if } p \leq \underline{q}^{alt}, \\ c'(s^*) = c'(\bar{s}_2(w=1)), w^* = 1 & \text{if } \underline{q}^{alt} < p < \bar{q}^{alt}, \\ c'(s^*) = vp, w^* = 1 & \text{if } p \geq \bar{q}^{alt}, \end{cases}, \tag{62}$$

where

$$\bar{s}_2(w=1) = vq\frac{p(1-r)}{p(1-r)+2qr}, \tag{63}$$

$$\underline{q}^{alt}(\varepsilon, q, r|r < 0.5) = 2q\frac{(1-2r)\varepsilon(\bar{s}_2(w=1)) - r}{(1-r)(1+\varepsilon(\bar{s}_2(w=1)))}, \tag{64}$$

$$\bar{q}^{alt}(\varepsilon, q, r|r < 0.5) = q\left(\varepsilon(\bar{s}_2(w=1)) - \frac{2r}{1-r}\right). \tag{65}$$

and the bound $\frac{c(\bar{s}_{11})}{\bar{s}_{11}}$ is replaced by

$$\frac{c(\bar{s}_{11}^{alt})}{\bar{s}_{11}^{alt}} = vqp \cdot \frac{(1-r)(pr(1-r) - q(1-2r))}{(2qr + (1-r)p)((p+4q)r(1-r) + q(3-2r))}. \tag{66}$$

Finally, if the equilibrium in the multiplicity region is mixing, then the analysis the same as when $\sigma^A = 0, \sigma^B = 1$. ■

A.3 Proof for Proposition 1

The unconstrained scale in the benchmark with no transitions in control: $c'(s^{NT}) = vp$. Hence, given the above derivations, for $r \geq \frac{1}{2}$,

$$s^* \begin{cases} = s^{NT} \text{ if } p \geq \overline{q} \\ > s^{NT} \text{ if } p < \overline{q} \end{cases} .$$

Notice that the case $s^* > s^{NT}$ requires $\epsilon(\overline{s}_3) > 2$.

If $r < \frac{1}{2}$ and the equilibrium selected in the multiplicity region is not $(\sigma^A, \sigma^B) = (1, 0)$, then notice that $vq\frac{q}{2q-p} < vp$ implies $p < q$. Thus,

$$s^* \begin{cases} = s^{NT} \text{ if } p \geq \overline{q} \text{ or } p = q \\ > s^{NT} \text{ if } q < p < \overline{q} \\ < s^{NT} \text{ if } p < q \end{cases} .$$

If $r < \frac{1}{2}$ and the equilibrium selected in the multiplicity region is $(\sigma^A, \sigma^B) = (1, 0)$, then

$$s^* \begin{cases} = s^{NT} \text{ if } p \geq \overline{q}^{alt} \text{ or } p = q \\ > s^{NT} \text{ if } q < p < \overline{q}^{alt} \\ < s^{NT} \text{ if } p < q \end{cases} .$$

A.4 Proof for Proposition 2

The equilibrium payoff inequality is $\Delta^* = 2w^* - 1$. The result follows from Lemma 2, defining $\underline{q}(\varepsilon, q, r | r \geq 0.5) \equiv 0$ and $\underline{q}(\varepsilon, q, r | r < 0.5)$ as in (45).

A.5 Proof for Corollary 1

Follows from Propositions 1 and 2 given the expressions in Claims 9 and 10.

A.6 Proof for Proposition 3

From the proof to Lemma 2, if $r \geq 1/2$ and $\frac{c(s)}{s} \leq \frac{c(\overline{s}_3)}{\overline{s}_3}$, then the optimal choice for agent A is either $w_3(s)$, with corresponding revision strategies $\sigma^A = 1, \sigma^B = 0$, or $w = 1$, with corresponding revision strategies $\sigma^A = \sigma^B = 1$. The latter results in a higher payoff if $EU^A(s,1|1,1) \geq EU^A(s,w_3|1,0)$. That is, if $\frac{c(s)}{s} \leq \frac{c(\overline{s}_{11}^h)}{\overline{s}_{11}^h}$ where

$$\frac{c(\overline{s}_{11}^h)}{\overline{s}_{11}^h} = vpq\frac{(1-r)(q(2r-1)+pr(1-r))}{(p(1-r)+2qr)(pr(1-r)+q(3-6r+4r^2))}. \tag{67}$$

Note that $\frac{c(\overline{s}_{11}^h)}{\overline{s}_{11}^h} \leq \frac{c(\overline{s}_3)}{\overline{s}_3}$. Then, let $s^{\max} < s^*(1,0)$ (the optimal scale for $s \in [\overline{s}_{11}^h, \overline{s}_3(w=1)]$). Then, given that $\partial\frac{EU^A(s,w_3|0,0)}{\partial s} > 0$ for $s \leq s^{\max}$, the optimal s in the interval $[0, s^{\max}]$ is either $s^{\max}$ (such that $\sigma^B = 0$) or $s_{11}^* = \arg\max_{s\leq\overline{s}_{11}^h} EU^A(s,1|1,1)$. Since $EU^A(\overline{s}_{11}^h, w_1|0,0) = EU^A(\overline{s}_{11}^h, 1|1,1)$ and $EU^A(s_{11}^*, 1|1,1) \geq EU^A(\overline{s}_{11}^h, 1|1,1)$, it follows that there exists $\overline{s^{\max}}(v,p,q,r) \in [\overline{s}_{11}^h, s^*)$ such that $EU^A(s_{11}^*, 1|1,1) \geq EU^A(\overline{s^{\max}}, w_3|0,0)$.

If $r < \frac{1}{2}$, the analysis is analogous. Let $s^{\max} < s^*(0,0)$ the optimal scale for $s \in [\overline{s}_{11}, \overline{s}_1]$ (remember that $s^*(1,0) = s^*(0,0)$). Then, given that $\partial\frac{EU^A(s,w|0,0)}{\partial s} > 0$ for $s \leq s^{\max}$, the optimal s in the interval $[0, s^{\max}]$ is either $s^{\max}$ (such that $\sigma^B = 0$) or $s_{11}^* = \arg\max_{s\leq\overline{s}_{11}} EU^A(s,1|1,1)$. Since $EU^A(\overline{s}_{11}, w_1|0,0) = EU^A(\overline{s}_{11}, 1|1,1)$ and $EU^A(s_{11}^*, 1|1,1) \geq EU^A(\overline{s}_{11}, 1|1,1)$, it follows that there exists $\overline{s^{\max}}(v,p,q,r) \in [\overline{s}_{11}, s^*)$ such that $EU^A(s_{11}^*, 1|1,1) \geq EU^A(\overline{s^{\max}}, w_1|0,0)$.

Hence, for $s^{\max} \leq \overline{s^{\max}}$, the equilibrium is $\sigma^A = \sigma^B = 1$, and $w^* = 1$.

A.7 Proof for Corollary 2

Follows from the proof of Proposition 3 that $\overline{s^{\max}}$ is decreasing in p. Conditional on the equilibrium being $\sigma^A = \sigma^B$, notice that the expected time to completion $\frac{H_3^{AA}(p,q,r)}{p}$ is decreasing in p, with $H_3^{AA}(p,q,r)$ defined in the proof to Lemma 1.

A.8 Proof for Proposition 4

Equations (12) and (13) imply that the welfare function given project designer A and project Δ^A is

$$W = \frac{1}{2}sv\cdot[H_1^{AA}(p,q,r) + H_2^{AA}(p,q,r)] - \frac{c(s)}{p}\cdot H_3^{AA}(p,q,r), \tag{68}$$

In equilibrium there are no revisions, so $(\sigma^A, \sigma^B) = (0,0)$ or $(\sigma^A, \sigma^B) = (1,0)$. Then, the welfare function becomes

$$W = \frac{1}{2}s^*v - \frac{c(s^*)}{p}.$$

Thus, $W < 0$ is equivalent to $\frac{c(s^*)}{s^*} > \frac{vp}{2}$.

If $r \geq \frac{1}{2}$. If $s^* = \overline{s}_3$,

$$\frac{c(\overline{s}_3)}{\overline{s}_3} = \frac{vpq(1-r)}{p(1-r)+2qr} < \frac{vp}{2},$$

which means that $W > 0$. If $c'(s^*) = vp$, then[22]

$$W < 0 \Leftrightarrow \varepsilon(s^*) < 2 \text{ and } p > q\left(\varepsilon(\overline{s}_3) - \frac{2r}{1-r}\right). \tag{69}$$

Notice that for $r \geq \frac{1}{2}$, we have $\frac{2r}{1-r} \geq 2$. Thus, if $\varepsilon(s^*) < 2$, then $\overline{q} < 0$. Therefore, the condition $p > q\left(\varepsilon(\overline{s}_3) - \frac{2r}{1-r}\right)$ is always satisfied.

If $r < \frac{1}{2}$. Consider first the case where the equilibrium selection in the multiplicity region has $\sigma^B \neq 0$. If $p \geq \overline{q}(\varepsilon, q, r|r < 0.5)$, then $c'(s^*) = vp$ and the condition for $W < 0$ is as in (69). If $\underline{q}(\varepsilon, q, r|r < 0.5) < p < \overline{q}(\varepsilon, q, r|r < 0.5)$, then $s^* = \overline{s}_1$. Then,

$$W < 0 \text{ if } p > q \max\left\{1, 2\frac{\varepsilon(\overline{s}_1)}{1+\varepsilon(\overline{s}_1)}\right\} = 2\frac{\varepsilon(\overline{s}_1)}{1+\varepsilon(\overline{s}_1)}.$$

Hence, $W < 0$ for $\underline{q}(\varepsilon, q, r|r < 0.5) < p < \overline{q}(\varepsilon, q, r|r < 0.5)$.

If $p \leq \underline{q}(\varepsilon, q, r|r < 0.5)$, then $c'(s^*) = \frac{vqp}{2q-p}$, which leads to $W < 0$ if $p > 2q\frac{\varepsilon(s^*)-1}{\varepsilon(s^*)}$. Thus

$$W < 0 \Leftrightarrow 2q\frac{\varepsilon(s^*)-1}{\varepsilon(s^*)} < p < 2q\frac{\varepsilon(\overline{s}_1)}{1+\varepsilon(\overline{s}_1)}. \tag{70}$$

Putting it all together,

$$W < 0 \text{ if } \begin{cases} \varepsilon(s^*) < 2 \ \& \ r \geq \frac{1}{2}, \\ q\varepsilon(\overline{s}_1) < p < 1 \ \& \ \varepsilon(s^*) < 2 \ \& \ r < \frac{1}{2}, \\ 2q \cdot \min\left\{\frac{\varepsilon(s^*)-1}{\varepsilon(s^*)}, \frac{\varepsilon(\overline{s}_1)}{\varepsilon(\overline{s}_1)+1}\right\} < p < q\varepsilon(\overline{s}_1) \ \& \ r < \frac{1}{2} \end{cases}.$$

Let $E \equiv \min\left\{\frac{\varepsilon(c'^{-1}(\frac{vpq}{2q-p}))-1}{\varepsilon(c'^{-1}(\frac{vpq}{2q-p}))}, \frac{\varepsilon(\overline{s}_1)}{\varepsilon(\overline{s}_1)+1}\right\}$. The interval $\mathcal{P} = (\underline{p}, \overline{p})$ is therefore defined as (assuming the inverse of $c(s)$ exists; else we take $\varepsilon(s^*)$ in the expressions below):

$$\mathcal{P} = \begin{cases} \emptyset \text{ if } r \geq \frac{1}{2} \text{ and } p < \overline{q}(\varepsilon, q, r) \text{ or } \varepsilon(c'^{-1}(vp)) \geq 2, \\ (0,1) \text{ if } r \geq \frac{1}{2} \text{ and } \varepsilon(c'^{-1}(vp)) < 2, \\ (2qE, q\varepsilon(\overline{s}_1)) \text{ if } r < \frac{1}{2} \text{ and } \varepsilon(c'^{-1}(vp)) \geq 2, \\ (2qE, 1) \text{ if } r < \frac{1}{2} \text{ and } \varepsilon(c'^{-1}(vp)) < 2. \end{cases} \tag{71}$$

Other equilibrium selection in multiplicity region. If in the multiplicity region we select the equilibrium with $\sigma^B = 0$, then for $p \geq q^{alt}$, the solution is as in (69). If $\underline{q}^{alt} < p <$

[22] Note that for $c(s) = s^2$, we have $\varepsilon = 2$, hence $W \geq 0$ when $r \geq \frac{1}{2}$.

$\overline{q}^{alt}$, then $s^* = \overline{s}_2$. This means $\frac{c(s^*)}{s^*} = \frac{qvp(1-r)}{p(1-r)+2qr}$, and therefore $\frac{qvp(1-r)}{p(1-r)+2qr} > \frac{vp}{2}$ if $p < \frac{2q(1-2r)}{1-r}$. This leads to

$$W < 0 \Leftrightarrow \underline{q}^{alt} < p < \frac{q}{1-r} \min\{2(1-2r), (\varepsilon(\overline{s}_2)(1-r) - 2r)\}. \tag{72}$$

Finally, if $p \leq \underline{q}^{alt}$, then $W < 0$ if $p > 2q\frac{\varepsilon(s^*)-1}{\varepsilon(s^*)}$.[23]

A.9 Proof for Corollary 3

The values $\underline{p}$ and $\overline{p}$ are given in (71). If $\varepsilon(s) = \tilde{\varepsilon}$, then $\frac{\partial \varepsilon s}{\partial s} = 0$. Then, given (71), it follows that $\underline{p}$ and $\overline{p}$ are (weakly) increasing if q. Moreover,

$$\frac{\partial(\overline{p} - \underline{p})}{\partial q} = \varepsilon \overline{s}_1 - 2E \geq 0.$$

A.10 Proof for Proposition 5

Consider a cap $s^{\max}$ such that we are under the conditions of Proposition 3. Then, in (68), given $s \leq s^{\max}$we have

$$W = \frac{1}{2}sv - \frac{c(s)}{p}\frac{q + p(1-r)(1-p(1-r)-2qr)}{p(1-r)r + q(1-2r+2r^2)}. \tag{73}$$

Thus, $W < 0$ if

$$\frac{c(s)}{s} > \frac{vp}{2}\frac{p(1-r)r + q(1-2r+2r^2)}{q + p(1-r)(1-p(1-r)-2qr)}. \tag{74}$$

If $r < \frac{1}{2}$, a necessary condition for the equilibrium s to satisfy the conditions of Proposition 3 is that $\frac{c(s)}{s} \leq \frac{c(\overline{s}_{11})}{\overline{s}_{11}}$, where $\overline{s}_{11}$ is defined in (47). That is, a necessary condition for $W < 0$ given (74) is

$$\frac{p(1-r)r + q(1-2r+2r^2)}{q + p(1-r)(1-p(1-r)-2qr)} < 2q \cdot \frac{pr(1-r) - q(1-2r)}{(1-r)r(p-2q)^2 + pq(3-2r)} \tag{75}$$

Yet, (75) reduces to the necessary condition that $r \geq \frac{1}{2}$. Thus, for $r < \frac{1}{2}$, with scale caps as in Proposition 3, we have $W \geq 0$.

If $r \geq \frac{1}{2}$, to satisfy the conditions of Proposition 3 we need $\frac{c(s)}{s} \leq \frac{c(\overline{s}^h_{11})}{\overline{s}^h_{11}}$, where $\overline{s}^h_{11}$ is defined in (67). Therefore, a necessary condition for $W < 0$ is that

$$vp\left(\frac{1}{2}\frac{p(1-r)r + q(1-2r+2r^2)}{q + p(1-r)(1-p(1-r)-2qr)} - q\frac{(1-r)(q(2r-1) + pr(1-r))}{(p(1-r)+2qr)(pr(1-r) + q(3-6r+4r^2))}\right) < 0 \tag{76}$$

[23]Note that for $c(s) = s^2$, we have $\varepsilon = 2$ and therefore $W < 0$ for $q < p < \overline{q}^{alt}$.

Yet, for $r \geq \frac{1}{2}$, the expression in (76) is always positive. Hence, for $r \geq \frac{1}{2}$, with scale caps as in Proposition 3, we have $W \geq 0$.

A.11 Proof for Proposition 6

Observe first that because revisions in phase 1 cannot affect payoffs in phase 2, revision strategies are identical to those in the one-phase game. As the result focuses on phase 1 strategies, we omit notation for phases. Moreover, we analyze the case where in the region of multiplicity the equilibrium selected is the one where the agent more likely to be in control revises (agent A if $r \geq \frac{1}{2}$ and Agent B if $r < \frac{1}{2}$).

Part 1: $r > 1/2$. Following the proof of Lemma 2, distribution and revision strategies given s are as follows:

$$\begin{cases} \sigma^A = 1, \sigma^B = 1, \Delta = 1 & s \leq \hat{s}_3 & (a) \\ \sigma^A = 1, \sigma^B = 0, \Delta = w_3 & s \in (\hat{s}_3, \hat{s}_1] & (b) \\ \sigma^A = 0, \sigma^B = 0, \Delta = 1 & s > \hat{s}_1 & (c) \end{cases}$$

where w_3 is given in (19) and:

$$\begin{aligned} \hat{s}_3 &= \frac{pq(1-r)v[p(1-r)r + q(2r-1)]}{(p(1-r)+2qr)\left[p(1-r)r + q\left(4r^2 - 6r + 3\right)\right]}, \\ \hat{s}_1 &= \frac{pq(1-r)v}{p(1-r)+2qr}, \end{aligned}$$

which satisfies $0 < \hat{s}_3 < \hat{s}_1$.

As in the text, denote by $\tilde{U}^A$ A's ex ante expected value of a single phase of play when $m = 1$ (10), the corresponding objective for agent A is:

$$\hat{V}^A(s) = \begin{cases} V_a^A(s) = \left(\frac{rv(p(1-r)+qr)}{p(1-r)r+q(2r^2-2r+1)} + \tilde{U}^A\right)s - \frac{(q+p(1-r))s^2}{p(p(1-r)r+q(2r^2-2r+1))} & s \leq \hat{s}_3, & (a) \\ V_b^A(s) = (\frac{v}{2} + \tilde{U}^A)s + \left(\frac{1}{2q} + \frac{2r-1}{p(1-r)}\right)s^2 & s \in (\hat{s}_3, \hat{s}_1], & (b) \\ V_c^A(s) = (v + \tilde{U}^A)s - \frac{s^2}{p} & s > \hat{s}_1. & (c) \end{cases}$$

We note several properties of $\hat{V}^A(s)$ and its components. It is straightforward to verify that $\hat{V}^A(s)$ is continuous, concave in regions (a) and (c), and convex in region (b). Additionally, $V_a^A(0) = V_b^A(0) = V_c^A(0) = 0$. Finally, $\frac{dV_c^A(s)}{ds} > \frac{dV_a^A(s)}{ds}$. Together, these facts imply that $\hat{V}^A(s)$ can be maximized only at 0, $\hat{s}_1$, $\hat{s}_3$, or s_a or s_c, the interior values of s that maximize $V_a^A(s)$ or $V_c^A(s)$, respectively, if they exist.

Taking first order conditions yields the following candidate interior solutions:

$$s_a = \frac{p}{2}\left(r(v+\tilde{U}^A) + \frac{q(1-r)[(1-2r)\tilde{U}^A - rv]}{p(1-r)+q}\right), \tag{77}$$

$$s_c = \frac{p}{2}(v+\tilde{U}^A). \tag{78}$$

We make three observations that narrow the set of possible solutions. First, the region (c) solution is interior (i.e., $s_c > \hat{s}_1$) if:

$$\tilde{U}^A > \phi_c \equiv -\frac{v(p(1-r)+2q(2r-1))}{p(1-r)+2qr}. \tag{79}$$

Second, $s_a > 0$ if:

$$\tilde{U}^A > \phi_a \equiv -\frac{vr(p(1-r)+qr)}{p(1-r)r+q\left(2r^2-2r+1\right)}. \tag{80}$$

Third, $\hat{V}^A(\hat{s}_3) < \hat{V}^A(\hat{s}_1)$ if:

$$\tilde{U}^A > \phi_b \equiv -\frac{v\left(3p^2(1-r)^2r + pq(1-r)(2r(9r-7)-5) - 2q^2(6(3-2r)r^2-11r)+2)\right)}{2(p(1-r)+2qr)\left(p(1-r)r+q(4r^2-6r+3)\right)}. \tag{81}$$

Under the assumed parameter values, $\phi_b < \phi_a < \phi_c$.

We now derive the optimal s for each possible value of $\tilde{U}^A$. There are four cases.

(i) $\tilde{U}^A > \phi_c$. Because $V_c^A(s) > V_a^A(s)$ for $s > 0$, $s^* = s_c$.

(ii) $\tilde{U}^A \in (\phi_a, \phi_c]$. The possible solutions are s_a and $\hat{s}_1$. Solving $V^A(s_a) = V^A(\hat{s}_1)$ for $\tilde{U}^A$ produces a unique value ϕ_{ac} of $\tilde{U}^A$ such that $\phi_{ac} \in (\phi_a, \phi_c]$ and $s^* = s_a$ if $\tilde{U}^A < \phi_{ac}$ and $s^* = \hat{s}_1$ otherwise. (We omit the expression for ϕ_{ac} due to excessive length.)

(iii) $\tilde{U}^A \in (\phi_b, \phi_a]$. The possible solutions are 0 and $\hat{s}_1$. Observe that $V_c^A(\hat{s}_1) \geq 0$ only for $\tilde{U}^A > \frac{rv(p(1-r)+qr)}{p(1-r)r+q(2r^2-2r+1)}$, but this value of $\tilde{U}^A$ is greater than ϕ_a. Thus the optimal solution must be $s^* = 0$.

(iv) $\tilde{U}^A \leq \phi_b$. The possible solutions are 0 and $\hat{s}_3$. But $\hat{s}_3$ cannot be the solution because $V_a^A(s)$ is decreasing in s for $s > 0$ when $\tilde{U}^A \leq \phi_a$; thus the optimal solution must be $s^* = 0$.

Combining cases produces:

$$s^* = \begin{cases} s_c & \text{if } \tilde{U}^A > \phi_c \\ \hat{s}_1 & \text{if } \tilde{U}^A \in (\phi_{ac}, \phi_c] \\ s_a & \text{if } \tilde{U}^A \in (\phi_a, \phi_{ac}] \\ 0 & \text{if } \tilde{U}^A \leq \phi_a. \end{cases} \tag{82}$$

Comparing $\tilde{U}^A$ with ϕ_a, it is clear that the project will not be cancelled in phase 1 if $p < q$ or $p > 2q = \overline{q}$.

Part 2: $r < 1/2$. Following the proof of Propositions 1 and 2, distribution and revision strategies given s are as follows:

$$\begin{cases} \sigma^A = 1, \sigma^B = 1, \Delta = 1 & s \leq \check{s}_3 & (a) \\ \sigma^A = 0, \sigma^B = 0, \Delta = w_1 & s \in (\check{s}_3, \check{s}_1] & (b) \\ \sigma^A = 0, \sigma^B = 0, \Delta = 1 & s > \check{s}_1 & (c) \end{cases}$$

where $w_1 = \frac{1}{2} + \frac{s}{2qv}$ and:

$$\check{s}_3 = \frac{pqv\left[p(1-r)r - q(1-2r)\right]}{(p^2+4q^2)(1-r)r + pq(4r^2-6r+3)}$$
$$\check{s}_1 = qv,$$

which satisfies $\check{s}_3 < \check{s}_1$.

The corresponding objective for agent A is:

$$\check{V}^A(s) = \begin{cases} V_a^A(s) = \left(\frac{rv(p(1-r)+qr)}{p(1-r)r+q(2r^2-2r+1)} + \tilde{U}^A\right)s - \frac{(p(1-r)+q)s^2}{p[p(1-r)r+q(2r^2-2r+1)]} & s \leq \check{s}_3 & (a) \\ \check{V}_b^A(s) = (\frac{v}{2} + \tilde{U}^A)s + \left(\frac{1}{2q} - \frac{1}{p}\right)s^2 & s \in (\check{s}_3, \check{s}_1] & (b) \\ V_c^A(s) = (v + \tilde{U}^A)s - \frac{s^2}{p} & s > \check{s}_1 & (c) \end{cases}$$

We note several properties of $\check{V}^A(s)$ and its components. In regions (a) and (c), the component functions are identical to those in Part 1 (and thus concave). $\check{V}_b^A(s)$ is convex if $p > 2q$ and concave otherwise. It is straightforward to verify that $\check{V}^A(s)$ is continuous. Additionally, $V_a^A(0) = \check{V}_b^A(0) = V_c^A(0) = 0$. Together, these facts imply that $\check{V}^A(s)$ can be maximized only at 0, $\check{s}_3$, $\check{s}_1$, or s_a, $\check{s}_b$, or s_c, which are the interior values of s that maximize $V_a^A(s)$, $\check{V}_b^A(s)$, or $V_c^A(s)$, respectively, if they exist. Finally, $\check{s}_1$ is positive but $\check{s}_3$ may be negative; solutions in region (a) can exist only if $\check{s}_3 > 0$.

Taking first order conditions yields the following candidate interior solution for region (b), with the interior solutions s_a and s_c for regions (a) and (c) given by (77) and (78), respectively:

$$\check{s}_b = \frac{pq(v + 2\tilde{U}^A)}{4q - 2p}.$$

For $\check{s}_b$ to be interior it must be both positive, which holds if $\tilde{U}^A > -v/2$, and in the interval $(\check{s}_3, \check{s}_1]$, which occurs if $\tilde{U}^A \in \left(\check{\phi}_b^l, \check{\phi}_b^h\right]$, where:

$$\check{\phi}_b^l = \frac{v\left[4q^2(r^2+r-1) - 3p^2(1-r)r - pq(8r^2-6r+1)\right]}{2\left[(4q^2+p^2)(1-r)r + pq\left(4r^2-6r+3\right)\right]}$$
$$\check{\phi}_b^h = v\left(\frac{2q}{p} - \frac{3}{2}\right).$$

We make three observations that narrow the set of possible solutions. First, the region

(c) solution is interior (i.e., $s_c > \check{s}_1$) if:

$$\tilde{U}^A > \check{\phi}_c \equiv v\left(\frac{2q}{p} - 1\right). \tag{83}$$

Second, the region (a) solution is interior (i.e., $s_a \in (0, \check{s}_3)$) if $\tilde{U}^A > \phi_a$ (i.e., condition (80) from Part 1), and $\tilde{U}^A < \check{\phi}_a^h$, where:

$$\check{\phi}_a^h \equiv \frac{\begin{array}{r} v\left[(1-r)r^2\left(p^3(1-r)+4q^3r\right)+p^2qr\left(1-r(5r^2-9r+5)\right)+\right. \\ \left. q^2(1-2r)\left(2p+2q-pr(4r^2-5r+4)\right)\right] \end{array}}{\left((1-r)r(2q-p)-q\right)\left[(1-r)r\left(p^2+4q^2\right)+pq\left(4r^2-6r+3\right)\right]}.$$

The condition $s_a < s_a^h$ is equivalent to $\check{s}_3 > 0$.

Third, if either $\check{s}_b$ or s_c are interior, then A prefers them to $\check{s}_1$, which belongs to both regions (b) and (c). Similarly, if either s_a or $\check{s}_b$ are interior, then A prefers them to $\check{s}_3$.

We now derive the optimal s for each possible value of $\tilde{U}^A$ and q.

(i) $\tilde{U}^A > \check{\phi}_c$. When $\check{\phi}_c$ is interior, the only possible alternative solutions are 0, $\check{s}_3$, and s_a. By the concavity of $V_c^A(s)$, $V_c^A(s_c) > V_c^A(0)$, and $\frac{dV_c^A(s)}{ds} > \frac{dV_a^A(s)}{ds}$ implies that $V_c^A(s_c) > V_a^A(s_a)$ when $s_a > 0$. Finally, straightforward calculation shows that $V_c^A(s_c) > V_a^A(\check{s}_3)$ when $\check{s}_3 > 0$. Thus, $s^* = s_c$.

For subcases (ii)-(v), $p < 2q$, so the objective $\check{V}_b^A(s)$ in region (b) is concave. As $\tilde{U}^A < 0$ for agent A when $r < 1/2$, it it is impossible for condition (83) to be satisfied and thus subcase (i) is irrelevant.

(ii) $\tilde{U}^A \in (\check{\phi}_b^h, \check{\phi}_c]$. In this subcase, $\check{V}_b^A(s)$ is maximized at some $s > \check{s}_1$ and $\check{s}_b$ is not feasible. If $\check{s}_3 \leq 0$, then the optimal s is the region (b) corner: $s^* = \check{s}_1$.

If $\check{s}_3 > 0$, then $\check{V}_b^A(\check{s}_1) > \check{V}_b^A(\check{s}_3)$. By the concavity of $\check{V}_b^A(s)$, $\check{V}_b^A(\check{s}_1) > \check{V}_b^A(0)$, and so the remaining candidate solutions are $\check{s}_1$ and s_a. Performing the necessary substitutions and solving, $\check{V}_a^A(s_a) > \check{V}_b^A(\check{s}_1)$ iff $\tilde{U}^A < \check{\phi}_{a1}$, where:

$$\check{\phi}_{a1} \equiv \frac{v\left[2q^2 - p^2(1-r)r - pq(r^2+2r-2) - 2q\sqrt{2(1-r)(p(1-r)+qr)(p(1-r)+q)}\right]}{p\left[p(1-r)r + q\left(2r^2-2r+1\right)\right]}. \tag{84}$$

It is straightforward to verify that $\check{\phi}_{a1} < \check{\phi}_c$. Thus we have $s^* = \check{s}_1$ if $\tilde{U}^A \in (\check{\phi}_{a1}, \check{\phi}_c]$, and $s^* = s_a$ if $\tilde{U}^A \in (\check{\phi}_b^h, \check{\phi}_{a1}]$, where the latter interval may be empty.

(iii) $\tilde{U}^A \in (\max\{-v/2, \check{\phi}_b^l\}, \check{\phi}_b^h]$. In this subcase, $\check{s}_b$ is a feasible solution, which A obviously prefers to $\check{s}_3$ and $\check{s}_1$. By the concavity of $V_b^A(s)$, $\check{V}_b^A(\check{s}_b) > \check{V}_b^A(0)$, and so the only other possible candidate solution is s_a, if region (a) is non-empty. Thus $\hat{s}_3 \leq 0$ implies that the solution is $\check{s}_b$. Furthermore, $\hat{s}_3 \leq 0$ also implies that $-v/2 > \check{\phi}_b^l$.

If $\hat{s}_3 > 0$, then performing the necessary substitutions and solving, there exists ϕ_{ab} such that $\check{V}_a^A(s_a) > \check{V}_b^A(\check{s}_b)$ if $\tilde{U}^A < \phi_{ab}$, where $\phi_{ab} > \max\{-v/2, \check{\phi}_b^l, \check{\phi}_{a1}\}$ and $\phi_{ab} \in [\phi_a, \phi_a^h]$. (We omit the expression for ϕ_{ab} due to excessive length.) Thus we have $s^* = s_a$ if $\tilde{U}^A \in (\max\{-v/2, \check{\phi}_b^l\}, \phi_{ab}]$, and $s^* = \check{s}_b$ if $\tilde{U}^A \in (\phi_{ab}, \check{\phi}_b^h]$.

(iv) $\tilde{U}^A \leq \max\{-v/2, \check{\phi}_b^l\}$. $\check{V}_b^A(s)$ is strictly decreasing for $s \geq 0$, so if $\check{s}_3 \leq 0$ then the solution is $s^* = 0$. If $\check{s}_3 \geq 0$, the only feasible solutions are the set of region (a) solutions, or $\{0, s_a, \check{s}_3\}$. Thus the solution is $s^* = 0$ for $\tilde{U}^A \leq \phi_a$, $s^* = s_a$ for $\tilde{U}^A \in (\phi_a, \check{\phi}_a^h]$, and $s^* = \check{s}_3$ for $\tilde{U}^A > \check{\phi}_a^h$.

Combining cases (i)-(iv) produces the following optimal scales. For $\check{s}_3 \leq 0$:

$$s^* = \begin{cases} \check{s}_1 & \text{if } \tilde{U}^A \in (\check{\phi}_b^h, \check{\phi}_c] \\ \check{s}_b & \text{if } \tilde{U}^A \in (-v/2, \check{\phi}_b^h] \\ 0 & \text{if } \tilde{U}^A \leq -v/2. \end{cases}$$

And for $\check{s}_3 > 0$:

$$s^* = \begin{cases} \check{s}_1 & \text{if } \tilde{U}^A \in (\check{\phi}_{a1}, \check{\phi}_c] \\ s_a & \text{if } \tilde{U}^A \in (\check{\phi}_b^h, \check{\phi}_{a1}] \\ \check{s}_b & \text{if } \tilde{U}^A \in (\phi_{ab}, \check{\phi}_b^h] \\ s_a & \text{if } \tilde{U}^A \in (\max\{-v/2, \check{\phi}_b^l\}, \phi_{ab}] \\ \check{s}_3 & \text{if } \tilde{U}^A \in (\check{\phi}_a^h, \max\{-v/2, \check{\phi}_b^l\}] \\ s_a & \text{if } \tilde{U}^A \in (\phi_a, \check{\phi}_a^h] \\ 0 & \text{if } \tilde{U}^A \leq \phi_a. \end{cases} \tag{85}$$

It is straightforward to verify that at most one of the regions for which $s^* = s_a$ is non-empty.

For subcases (v)-(viii), $p > 2q$, so the objective $\check{V}_b^A(s)$ in region (b) is convex and $\check{s}_b$ is not a feasible solution. Thus the optimal s is either in region (b) (with possible solutions 0, $\check{s}_3$, $\check{s}_1$), or in region (a), as described in subcase (iv). We first consider the subcase where $\check{s}_3 \leq 0$, so region (a) is empty.

(v) $\tilde{U}^A \leq \check{\phi}_c$ and $\check{s}_3 \leq 0$. The only possible solutions are 0 and $\check{s}_1$. Solving $\check{V}_b^A(\check{s}_1) \geq 0$ for $\tilde{U}^A$ produces:

$$s^* = \begin{cases} s_c & \text{if } \tilde{U}^A > \check{\phi}_c \\ \check{s}_1 & \text{if } \tilde{U}^A \in (v(q-p)/p, \check{\phi}_c] \\ 0 & \text{if } \tilde{U}^A \leq v(q-p)/p. \end{cases} \tag{86}$$

For the remaining subcases, $\check{s}_3 > 0$, so region (a) is non-empty.

(vi) $\tilde{U}^A \in (\check{\phi}_a^h, \check{\phi}_c]$ and $\check{s}_3 > 0$. The only feasible solutions are $\check{s}_3$ and $\check{s}_1$. Performing the necessary substitutions and solving, there exists $\check{\phi}_{13}$ such that $\check{V}_a^A(\check{s}_3) > \check{V}_b^A(\check{s}_1)$ iff $\tilde{U}^A < \check{\phi}_{13}$, where:

$$\check{\phi}_{13} \equiv \frac{v\left[4pq^2(1-2r)^2 - (1-r)r(3p^3 - 8q^3) - p^2q(12r^2 - 14r + 5)\right]}{2p\left[(1-r)r(p^2+4q^2) + pq(4r^2 - 6r + 3)\right]}.$$

It is easily verified that $\check{\phi}_{13} < \check{\phi}_c$. Thus $s^* = \check{s}_3$ if $\tilde{U}^A \in (\check{\phi}_a^h, \check{\phi}_{13}]$, and $s^* = \check{s}_1$ if $\tilde{U}^A \in (\check{\phi}_{13}, \check{\phi}_c]$, where the former interval may be empty.

(vii) $\tilde{U}^A \in (\phi_a, \check{\phi}_a^h]$. In this subcase, the interior solution s_a is feasible. Using expression (84), $\check{V}_a^A(s_a) > \check{V}_b^A(\check{s}_1)$ if $\tilde{U}^A > \check{\phi}_{a1}$. Thus $s^* = \check{s}_1$ if $\tilde{U}^A \in (\check{\phi}_{a1}, \check{\phi}_a^h]$, and $s^* = s_a$ if $\tilde{U}^A \in (\phi_a, \check{\phi}_{a1}]$, where either interval may be empty.

(viii) $\tilde{U}^A \leq \phi_a$. Analogously to subcase (iv), $\check{V}_a^A(s)$ is strictly decreasing for $s \geq 0$, so $s^* = 0$.

Combining cases (i) and (vi)-(viii) produces:

$$s^* = \begin{cases} s_c & \text{if } \tilde{U}^A > \check{\phi}_c \\ \check{s}_1 & \text{if } \tilde{U}^A \in (\check{\phi}_{13}, \check{\phi}_c] \\ \check{s}_3 & \text{if } \tilde{U}^A \in (\check{\phi}_a^h, \check{\phi}_{13}] \\ \check{s}_1 & \text{if } \tilde{U}^A \in (\check{\phi}_{a1}, \check{\phi}_a^h] \\ s_a & \text{if } \tilde{U}^A \in (\phi_a, \check{\phi}_{a1}] \\ 0 & \text{if } \tilde{U}^A \leq \phi_a. \end{cases}$$

It is straightforward to verify that at most one of the regions for which $s^* = s_a$ is non-empty.

Summarizing the conditions for cancellation, when $\check{s}_3 > 0$ A cancels the project in phase 1 if $\tilde{U}^A \leq \phi_a$. When $\check{s}_1 \leq 0$, A cancels when $\tilde{U}^A \leq -v/2$ if $p < 2q$ and $\tilde{U}^A \leq v(q-p)/p$ if $p > 2q$. As $\tilde{U}^A$ is independent of v and negative and decreasing in v_2 when $r < 1/2$, we conclude that A cancels when v_2 is sufficiently large.