# CSCI 8920 Decision Making Under Uncertainty
# Assignment 1: Probability and Bayesian Networks

## General Information

**Deadline:** as shown on eLC
**Worth:** 100 pts

## The Assignment

The purpose of this assignment is to understand and become familiar dealing with probabilities, and get acquainted with Bayesian networks. Your grade will be based on the correctness of the solutions. Please be as specific as possible while writing the answers.
Note: This assignment is not a group project and everybody should work on it individually. Your answers should be your own work. *In order to complete this assignment successfully, you must first carefully read Chapters 13 and 14 of the Russell and Norvig textbook.*

### Problems

Part I Fun with Probabilities

(a) (8 points) Prove that the following holds for all propositions $\alpha$ and $\beta$:

$$Pr(\alpha) = Pr(\alpha \wedge \beta) + Pr(\alpha \wedge \neg\beta).$$

(b) (10 points) Suppose that a tuberculosis (TB) skin test is 97% accurate. Suppose that 1 in 1,000 of the subjects who get tested is infected. Now suppose that a patient gets a positive test result. What is the probability that he is infected? Show your steps.

(c) (10 points) A forgetful teacher is supposed to give a homework assignment to his students each week. The probability that he will forget to give the homework on a given week is 0.2. If the teacher gives the homework, the probability that a majority of the students will still fail the weekly quiz is 0.15. If the students don't receive the homework, the probability that a majority of the students will fail the weekly quiz is 0.85. A majority of the students failed this week's quiz. What is the probability that the teacher forgot to give the homework to the students? Show your steps.

(d) (12 points) A dice $A$ has four red faces and two white faces. A dice $B$ has two red faces and four white faces. You flip a coin once, if heads the game continues with dice $A$, otherwise it continues with dice $B$.

i. On rolling the dice, what is the probability that a red face appears on the dice?

ii. At the second rolling of the same dice, what is the probability that a red face appears?

iii. If the first two rolls show a red face, what is the probability that the third roll is also red?

iv. If the first $n$ rolls show a red face, what is the probability that you are using dice $A$?

(e) (10 points) Given the full joint distribution shown in Fig. 13.3 of the textbook, calculate

i. $Pr(Cavity|catch)$

ii. $Pr(Cavity|toothache \wedge \neg catch)$

Show your steps.

Part II Reasoning Using Bayesian Networks

(a) (5 points) Your first task will be to download, install and become familiar with a popular tool called **Netica** for building Bayesian networks (BN) and reasoning using BNs. Please download and install the Windows version or Mac version of Netica from *http://www.norsys.com/download.html*. **The free version of Netica is limited to BNs with no more than 15 nodes.** Once Netica is installed, please familiarize yourself with its operation and other capabilities using the example BNs that come bundled with it.

(b) (35 points) You will model a problem domain involving autonomous mobile robots participating in perimeter patroling using a BN in Netica.

We will consider the setting where a team of two mobile robots, $i$ and $j$, are tasked with tracking a fugitive hiding among civilians.

Let the operation theater be a grid of $4 \times 4$ sectors. Robots $i$ and $j$ recognize the fugitive using a face recognition software, which tends to be 80% accurate (ie. the probability of the fugitive being correctly recognized is 0.80, as is the probability of a non-fugitive being correctly not recognized.) Having received specific intelligence, the fugitive is also identified using the clothes that he is wearing. However, this way of identifying the fugitive is only 75% accurate at best. Robot $i$ must also keep away from $j$ in order to cover the maximum ground. A robot may receive a proximity signal when another robot is within a Manhattan distance of 2 sectors. The likelihood of this signal is 95% when the distance is less than or 1 sector, but gets weaker with the likelihood falling to 80% when the distance is 2 sectors. Finally, the fugitive's strategy is to keep the robot $j$ in its sight, without itself being seen. Hence, he will always be within a horizontal or vertical distance of 1 sector from $j$'s location with each location being equi-probable. Please use your best judgment in entering probabilities not specified above, and state these in the README file.

Using your BN built in Netica, please answer the following questions:

i. If a possible fugitive in sector (2,1) is positively recognized using his face but his clothes are not identified, what is the probability that the fugitive is in (2,1)? How does this probability change when the clothes are also identified? What is the most probable location(s) of the robot $j$?

ii. If robot $i$'s proximity indicator goes off when it is in location (2,2), what are the probabilities that the fugitive is located in each of the 16 sectors? What then becomes the most likely location of a fugitive if he is recognized by face only in (2,3) and by clothes only in (1,2).

iii. If the proximity indicator goes off when robot $i$ is in sector (2,3) and the fugitive is spotted both by face and clothes in (2,3) as well, what is the most likely location(s) of the robot $j$.

(c) (10 points) Consider the Bayesian network shown in Fig. 14.23 of the textbook. Do *not* use Netica for this question.

i. Calculate the value of $P(b, i, \neg m, \neg g, j)$ using the Chain rule.

ii. Use **Variable Elimination** to calculate the probability that someone goes to jail given that they broke the law, have been indicted,

and face a politically motivated prosecutor.

## What and how to hand it in

You'll hand in the *typed or handwritten* answers to Part I and Part II.c in class to the instructor by the deadline. Please be sure to indicate the question numbers alongside the answers. The document should include your name, student id, and all the answers.

You'll submit Part II.b of your assignment in a *single zipped file* by the deadline via eLC. In the zip file, please include a valid Netica's dne file that represents your BN, and in a separate README file, the answers to the questions in Part II.b.

Assignments that are **late** but within a day of the deadline will be penalized 33% of the total number of points. Assignments submitted later than one day will not be accepted.