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Assignment 1

Part I a) $P(\alpha) = P(\alpha \wedge \beta) + P(\alpha \wedge \neg \beta)$

$= P(\alpha)P(\beta) + P(\alpha)P(\neg\beta)$

$= P(\alpha)[P(\beta) + P(\neg\beta)]$

$= P(\alpha)[P(\beta) + (1 - P(\beta))]$ ← $P(\beta)$'s cancel

$= P(\alpha)(1)$

$= P(\alpha)$

b) Infected $= I$; Result Positive $= R$

We are given...

$P(I)$	0.001	Infection
$P(\neg I)$	0.999	Rate

and

$P(R\mid I)$	0.97	Test
$P(R\mid \neg I)$	0.03	Accuracy

$P(I|R) = ?$

$$P(I|R) = \frac{P(R|I)P(I)}{P(R)} \leftarrow \text{Bayes' Rule}$$

$$= \frac{P(R|I)P(I)}{P(R,I) + P(R,\neg I)} \leftarrow \text{From Part (a)}$$

$$= \frac{P(R|I)P(I)}{P(R|I)P(I) + P(R|\neg I)P(\neg I)} \leftarrow \text{Product Rule} \times 2$$

$$= \frac{0.97(0.001)}{0.97(0.001) + (0.03)(0.999)} \approx 0.0314$$

Part I c) Give Homework = G ; Majority Fail = F

$P(G)$	0.8
$P(\neg G)$	0.2

$P(F\|G)$	0.15
$P(F\|\neg G)$	0.85

$P(\neg G|F) = ?$

$$P(\neg G|F) = \frac{P(F|\neg G)P(\neg G)}{P(F)}$$

$$= \frac{P(F|\neg G)P(\neg G)}{P(F,\neg G)+P(F,G)}$$

$$= \frac{P(F|\neg G)+P(\neg G)}{P(F|\neg G)P(\neg G)+P(F|G)P(G)}$$

$$= \frac{0.85(0.2)}{0.85(0.2)+0.15(0.8)} \approx 0.5862$$

d) i) Using Dice A = A ; Red Face = R

$P(A)$	$1/2$
$P(\neg A)$	$1/2$

$P(R\|A)$	$2/3$
$P(R\|\neg A)$	$1/3$

$$P(R) = P(R|A)P(A) + P(R|\neg A)P(\neg A)$$

$$P(R) = \tfrac{2}{3}(\tfrac{1}{2}) + \tfrac{1}{3}(\tfrac{1}{2})$$

$$P(R) = \tfrac{1}{2}$$

ii) $P(R,R) = P(R|A)^2 P(A) + P(R|\neg A)^2 P(\neg A)$

$$P(R,R) = (\tfrac{2}{3})^2(\tfrac{1}{2}) + (\tfrac{1}{3})^2(\tfrac{1}{2})$$

$$P(R,R) = \tfrac{5}{18}$$

iii) $P(R,R,R) = P(R|A)^3 P(A) + P(R|\neg A)^3 P(\neg A)$

$$P(R,R,R) = (\tfrac{2}{3})^3(\tfrac{1}{2}) + (\tfrac{1}{3})^3(\tfrac{1}{2})$$

$$P(R,R,R) = \tfrac{1}{6}$$

$R_n = R_1, R_2, \dots R_n$

iv) $P(A|R^n) = \dfrac{P(R^n|A)P(A)}{P(R^n)}$

$$= \frac{P(R|A)^n P(A)}{P(R|A)^n P(A) + P(R|\neg A)^n P(\neg A)}$$

$$= \frac{(\tfrac{2}{3})^n(\tfrac{1}{2})}{(\tfrac{2}{3})^n(\tfrac{1}{2}) + (\tfrac{1}{3})^n(\tfrac{1}{2})} = \frac{(\tfrac{2}{3})^n}{(\tfrac{2}{3})^n + (\tfrac{1}{3})^n}$$

$$P(A|R^n) = \frac{(\tfrac{2}{3})^n}{(\tfrac{2}{3})^n + (\tfrac{1}{3})^n}$$

Part I e) i) $P(Cavity \mid Catch) = \frac{P(Cavity \wedge Catch)}{P(Catch)}$

$$= \frac{0.108 + 0.072}{0.108 + 0.016 + 0.072 + 0.144}$$

$P(Cavity \mid Catch) \approx 0.5294$

ii) $P(Cavity \mid Toothache \wedge \neg Catch) = \frac{P(Cavity \wedge Toothache \wedge \neg Catch)}{P(Toothache \wedge \neg Catch)}$

$$= \frac{0.012}{0.012 + 0.064}$$

$P(Cavity \mid Toothache \wedge \neg Catch) \approx 0.1579$

Part II c) i) $P(b, i, \neg m, \neg g, j) = P(b)P(i \mid b, \neg m)P(\neg m)P(\neg g \mid b, i, \neg m)P(j \mid \neg g)$

$$= (0.9)(0.5)(0.9)(0.2)(0)$$

$$= 0$$

$P(b, i, \neg m, \neg g, j) = 0$

ii) $P(j \mid b, i, m) = \frac{P(j, b, i, m)}{P(b, i, m)} = \frac{\sum_g P(j, b, i, m, g)}{P(b, i, m)}$

$f_{J\bar{G}} = f_J(g) f_G(g, b, i, m) + f_J(\neg g) f_G(\neg g, b, i, m)$

$$= \alpha \sum_g P(j \mid g)P(b)P(i \mid b, m)P(m)P(g \mid b, i, m)$$

$$= \alpha P(b)P(m)P(i \mid b, m) \sum_g P(j \mid g) P(g \mid b, i, m)$$

$$= \alpha f_B(b) f_M(m) f_I(b, m) \sum_g f_J(g) f_G(g, b, i, m)$$

$$= \alpha f_B(b) f_M(m) f_I(b, m) f_{J\bar{G}}(b, i, m)$$

$$= \alpha (0.9)(0.1)(0.9)[(0.9)(0.9) + 0(0.1)]$$

$$= \alpha (0.06561)$$

$$= 0.06561 / 0.081$$

$$= 0.81$$

$\alpha = [P(b, i, m)]^{-1}$

$\alpha = [P(b)P(m)P(i \mid b, m)]^{-1}$

$\alpha = [0.9(0.1)(0.9)]^{-1}$

$\alpha = (0.081)^{-1}$