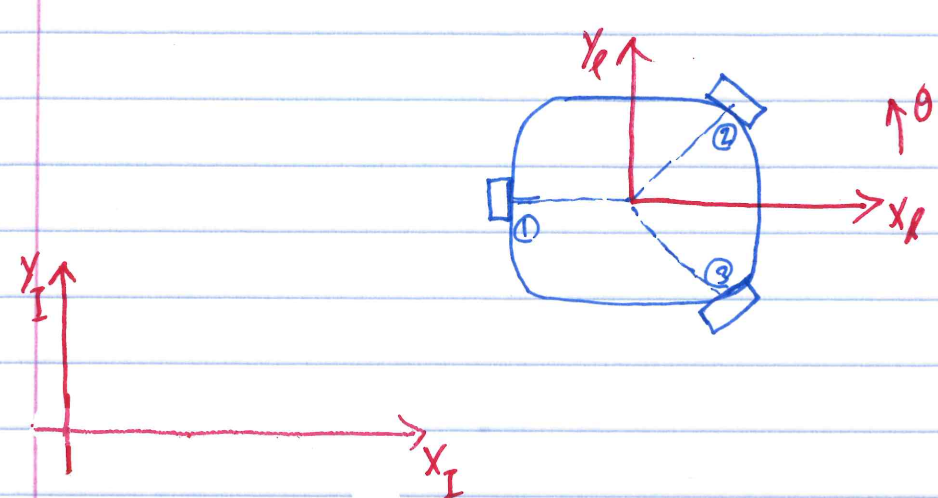
Odometry:

Consider the robot schematics shown below



The kinematic relationship describing the robot motion in local coordinate frame can be written as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3l} & -\frac{1}{3l} & -\frac{1}{3l} \end{bmatrix} \begin{bmatrix} R.\dot{\varphi}_2 \\ R.\dot{\varphi}_1 \\ R.\dot{\varphi}_3 \end{bmatrix}$$

$R = 0.03\text{ m}$ & $l = 0.19\text{m}$

$$= \frac{2\pi R}{60} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3l} & -\frac{1}{3l} & -\frac{1}{3l} \end{bmatrix} \begin{bmatrix} rpm_2 \\ rpm_1 \\ rpm_3 \end{bmatrix} \qquad (1)$$

To find wheel velocities corresponding to a desired $\dot{x}, \dot{y}, \dot{\theta}$

$$\begin{bmatrix} rpm_2 \\ rpm_1 \\ rpm_3 \end{bmatrix} = \frac{60}{2\pi R} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3l} & -\frac{1}{3l} & -\frac{1}{3l} \end{bmatrix}^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \qquad (2)$$

To express the robot motion in the global coordinate frame, we need to use

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3l} & -\frac{1}{3l} & -\frac{1}{3l} \end{bmatrix} \begin{bmatrix} R\dot{\varphi}_2 \\ R\dot{\varphi}_1 \\ R\dot{\varphi}_3 \end{bmatrix} \quad (3)$$

Odometry is about determining the pose of the robo i.e. $[x, y, \theta]^T$ based on an initial pose for the robot an the individul wheel velocities, which can, in turn, be relate to encoder measurements.

The relationship between the change in encoder count and the distance traveled by the wheels is

$$D = 2\pi R \frac{\Delta tick}{N} \quad (4)$$

$\Delta tick \longrightarrow$ change in the tick number

$N \longrightarrow$ ticks per revolution

The distance traveled by the wheel can be related to its rotational velocity as below

$$D = R\dot{\varphi}\, dt \quad (5)$$

The derivatives in Eq. (3) will be approximated using the following relationship

$$\begin{aligned} x[k+1] &= x[k] + \dot{x}\, dt \\ y[k+1] &= y[k] + \dot{y}\, dt \\ \theta[k+1] &= \theta[k] + \dot{\theta}\, dt \end{aligned} \Rightarrow \begin{bmatrix} x[k+1] \\ y[k+1] \\ \theta[k+1] \end{bmatrix} = \begin{bmatrix} x[k] \\ y[k] \\ \theta[k] \end{bmatrix} + \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} dt \quad (6)$$

$$\Rightarrow \begin{bmatrix} x(k+1) \\ y(k+1) \\ \theta(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \theta(k) \end{bmatrix} + \begin{bmatrix} \cos(\theta(k)) & -\sin(\theta(k)) & 0 \\ \sin(\theta(k)) & \cos(\theta(k)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3l} & -\frac{1}{3l} & -\frac{1}{3l} \end{bmatrix} \begin{bmatrix} R\dot{\varphi} \\ R\dot{\varphi} \\ R\dot{\varphi} \end{bmatrix} \quad (7)$$

$\times d$

Using Eq. (5), the eq. (7) can be rewritten as

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \theta(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \theta(k) \end{bmatrix} + \begin{bmatrix} \cos(\theta(k)) & -\sin(\theta(k)) & 0 \\ \sin(\theta(k)) & \cos(\theta(k)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3l} & -\frac{1}{3l} & -\frac{1}{3l} \end{bmatrix} \begin{bmatrix} D_2 \\ D_1 \\ D_3 \end{bmatrix} \quad (8)$$

where eech of the D values canbe calculated using encoder measurements in Eq. (4).

Equation 8 above can be used to give an estimate of the robot position in the global frame, based on an initial value of the position and the distance traveled by each wheel.