Accurate computation of the Hotelling Observer for the evaluation of image reconstruction algorithms in helical, cone-beam CT

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Abstract— **A task-based method for evaluating cone-beam CT image reconstruction algorithms is proposed. The task is signal-known-exactly/background-known-exactly detection. The aim is to compute the efficiency of image reconstruction with respect to this task. The efficiency is the ratio of the square of the signal detectability by the ideal observer in the reconstructed image and in the projection data. As reconstruction, here, is a non-invertible linear operator, the efficiency is less than or equal to one. For the model used to describe the projection data the ideal observer is equivalent to the Hotelling observer. To obtain Hotelling observer performance, the Hotelling template is computed in both the projection data and reconstructed image domains. Under the assumption of uncorrelated noise, the data domain Hotelling template computation is straight-forward; however, its computation in the reconstructed image space is complicated by having a very large non-diagonal covariance. In this work, an efficient, accurate method is developed for evaluating the Hotelling template in the reconstructed image space in cone-beam CT. Obtaining this template, enables the accurate computation of the efficiency of the cone-beam CT image reconstruction algorithm.**

I. Introduction

THE development of useful image quality metrics is essential for the optimization of any imaging system. Such metrics are particularly important now for guiding the design of image reconstruction algorithms for cone-beam computed tomography (CT) systems. Recently there has been a big push for the development of such algorithms. Presently, there are numerous algorithms that can, under ideal conditions, yield exact volume images for a variety of cone-beam CT scanning configurations. Under realistic conditions, however, considering data discreteness and noise, the performance of these reconstruction algorithms can differ. Complete characterization of the reconstruction algorithm in terms of basic metrics such as resolution or response to noise can be difficult because CT systems are not shift invariant, and as a result such imaging system properties vary within the imaging volume. The approach taken here is to employ task-based metrics that are currently well-developed and are becoming standard, see chapters 13 and 14 in "Foundations of Image Science" by Barrett and Myers [1]. The challenge in applying the task-based approach to cone-beam CT image reconstruction stems from the sheer size of the projection data sets and the anisotropy of the reconstructed volume image properties. We address both of these issues in this proceedings article.

Specifically, we aim at computing the ideal observer efficiency [2], [3] for signal detection in helical cone-beam CT reconstructed images. The detection task considered is a signal known exactly/back ground known exactly (SKE/BKE) task. The efficiency is the squared signal to noise ratio (SNR) for detection in the reconstructed volume divided by the squared SNR for detection in the raw projection data. The data model considered in this work involves Gaussian statistics, where the covariances are the same for signal present and signal absent cases. Under this approximation, the ideal observer is equivalent to the Hotelling observer. Although the reconstruction operation is linear, the SNR for signal detection in the reconstructed volume will in general be less than that in the projection data domain. The drop in SNR results from inconsistencies in the data and

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the fact that the reconstruction operator is not invertible. As different algorithms make use of the inconsistent data differently, one can expect that the drop in SNR will also vary with reconstruction algorithm.

II. Method

Based on a standard model for the data function in CT, we describe the method for computing the ideal observer efficiency for signal detection in the reconstructed volume.

The data model for the CT system relates the data vector g to the object function $f(\vec{r})$:

$$\mathbf{g_i} = \int_{-\infty}^{\infty} \mathbf{dlf}\left(\tilde{\mathbf{s}}_\mathbf{i} + \mathbf{l}\hat{\theta}_\mathbf{i}\right) \ + \ \mathbf{n_i} \text{ where } \mathbf{i} \in [\mathbf{1}, \mathbf{N}], \tag{1}$$

where $\mathbf{g_i}$ and $\mathbf{n_i}$ are random variables. The data measurement $\mathbf{g_i}$ is the line integral over $f(\vec{r})$ along the ith ray defined by the x-ray source location s_i and ray direction $\hat{\theta}_i$ with noise $\mathbf{n_i}$ added. A reasonable approximation of the CT noise model is that the measurements are independent, and they follow a Gaussian distribution with variance:

$$(K_g)_{i,i} = \alpha < \mathbf{g_i} > +\beta. \tag{2}$$

The covariance of the sinogram, K_g, is assumed diagonal, where the diagonal elements have a constant background noise level β plus a component proportional to the data mean $< \mathbf{g_i} >$. The set of projection data collected is the minimum for accurate reconstruction by the back-projection filtration (BPF) image reconstruction algorithm [4].

The implementation of the reconstruction algorithm is not discussed here, we simply denote it as an $M \times N$ matrix A, where M is the number of voxels in the image and N is the number of measured rays in the projection data. The image reconstruction algorithm is a linear discrete-to-discrete transform, $y = Ag$, yielding an image y by applying the matrix A to a data vector g.

In order to compute the efficiency ratio for detection, the Hotelling template is found for the observers acting on both the projection data and the reconstructed image:

$$e = SNR_y^2/SNR_g^2, \text{ where} \tag{3}$$
$$SNR_y^2 = s_y^T w_y^{(hot)} \quad SNR_g^2 = s_g^T w_g^{(hot)}.$$

SNR_y and SNR_g are respectively the SNRs in the reconstructed volume and the projection data; s_y and s_g are respectively the reconstructed signal and the projected signal; and $w_y^{(hot)}$ and $w_g^{(hot)}$ are respectively the Hotelling templates in reconstructed image space and in the the projection data. The Hotelling template $w_g^{(hot)} = s_g/K_g$ is simple to calculate, because the data covariance is diagonal. Computing the Hotelling template $w_y^{(hot)} = s_y/K_y = (As_g)/(AK_gA^T)$ is more problematic, because A can be as large as $10^9 \times 10^9$, making direct computation of K_y impractical. Furthermore, inversion of K_y itself is not feasible. Instead, as suggested in Refs. [1], [5] we employ a channelization approach.

The aim here is to use channelization to find the Hotelling template $w_y^{(hot)}$ in such a way that no symmetry restrictions are imposed. The SNR for a single channel w in the reconstructed image space is given by

$$SNR_w^2 = (w^T s_y)^T (w^T s_y)/(w^T AK_gA^T w). \tag{4}$$

If $w = w_y^{(hot)}$, the SNR is maximized and $SNR_w = SNR_y$. If not, we can use the gradient of this expression to generate efficient channels with which to represent $w_y^{(hot)}$

$$u = \nabla_w SNR_w^2 = 2\frac{w^T s_y}{w^T AK_gA^T w} s_y - 2\left(\frac{w^T s_y}{w^T AK_gA^T w}\right)^2 AK_gA^T w. \tag{5}$$

If $|u|$ is small then w is close to the Hotelling template, otherwise u itself becomes a good channel to add to the expansion set for $w_y^{(hot)}$. The procedure for finding $w_y^{(hot)}$ is iterative. We expand w in channels $w = \sum_{i=1}^{N_c} c_i u_i$. At first N_c is 1, and $u_1 = Aw_g^{(hot)}$. A second channel u_2 is generated by applying Eq. (5). The expansion set for w is increased to include the second channel. The coefficients for both channels are found by evaluating the two channel Hotelling observer. w is updated, and substituted into Eq. (5) to generate a third channel. The iteration is stopped when $|u|$ drops below a specified threshold. In the example below the iteration number is taken to 20 to get a reasonable estimate for $w_y^{(hot)}$, but in practice far fewer iterations are needed because we only need SNR_y to compute efficiency.

III. Results

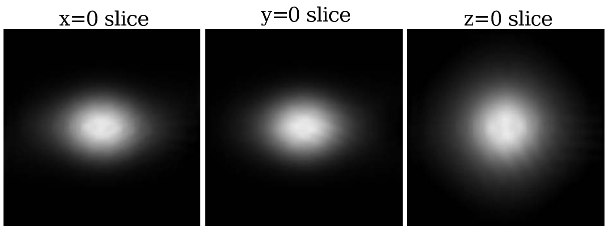

Fig. 1. Slices of the Hotelling template in the reconstructed volume space.

In the simulation presented here, the helical scanning configuration is specified by the parameters: radius 50 cm, source-to-detector distance 80 cm, helical pitch length 10 cm, the x-ray source is scanned over a single helical turn, the detector size is 10x10 cm^2, projection data dimension is 512 views by 50x50 detector bins. The detector dimensions are small relative to normal CT scanning, because it is only necessary that the field-of-view (FOV) cover the signal to evaluate the SKE/BKE detection task. For actual image reconstruction or the evaluation of more elaborate model observers, the FOV may need to be increased.

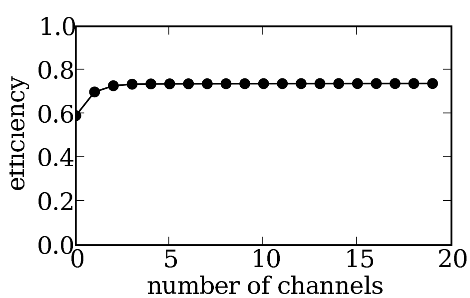

Fig. 2. Computed efficiency for SKE/BKE signal detection by the BPF reconstruction algorithm as a function of the number of channels.

The signal considered here is a small Gaussian lump of width 1 cm and amplitude 0.05, and the background is a larger Gaussian lump of width 5 cm and unit amplitude. The noise model is given in Eq. (2) with $\alpha = 0.02$ and $\beta = 0.02$. Both the signal and background are centered on the axis of the helical scan. Even though the signal and background are spherically symmetric, the Hotelling template will not have spherical symmetry because of the geometry of the helical scan and discreteness of the data.

For this scanning configuration and detection task, slices of the 3D Hotelling template are shown in Fig. 1. The resulting template is non-spherical. The template is compressed along the z-axis, the axis of the helical scan. To demonstrate the convergence of the channelization procedure, the efficiency is plotted as a function of the number of channels in Fig. 2. The efficiency of the BPF algorithm for this particular configuration and task is approximately 75%.

The SKE/BKE ideal observer efficiency is a practical metric. As a rule of thumb, the computation time for the efficiency is comparable to the reconstruction time for a complete volume image. The fact that multiple reconstructions are needed for the iteration is offset somewhat by the reduction of FOV to the known signal. Of course the efficiency is a function of both the scanning configuration and specifics of the detection task, so multiple evaluations will in general be required.

IV. Conclusion

We have developed a task-based metric for the evaluation of 3D cone-beam CT image reconstruction algorithms. The metric measures, in some sense, the loss of information pertaining to signal detection caused by the reconstruction algorithm. Though the example here used spherically symmetric Gaussian distributions, there is no requirement on the symmetry of the signal. For the example shown with one set of scanning and algorithm parameters, the efficiency of the algorithm is 75%. This efficiency may be improved by further algorithm development in cone-beam CT image reconstruction. Extension of the efficiency metric shown here will be extended to more realistic detection tasks, such as having the presence of statistical backgrounds.

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