IEMS 351 Homework 2

Fall 2024

Due: The homework will be due by midnight (11:59PM) on Oct. 11.

Problem 1

Given the scatter plot below, using a simple linear regression model may not be a proper choice. Please do the following:

1. **Provide an alternative one-dimensional regression model**, $m(z; x) =?$. (Here x is a model parameter and z is a feature.)
2. Given the regression model that you provide and data points $(z_1, y_1), (z_2, y_2), \ldots, (z_N, y_N)$, **write down the optimization problem that minimizes the least-square error.** The letter m should not appear in the optimization problem.

Hint: What function decreases first and then increases? There is no unique answer to the regression model.

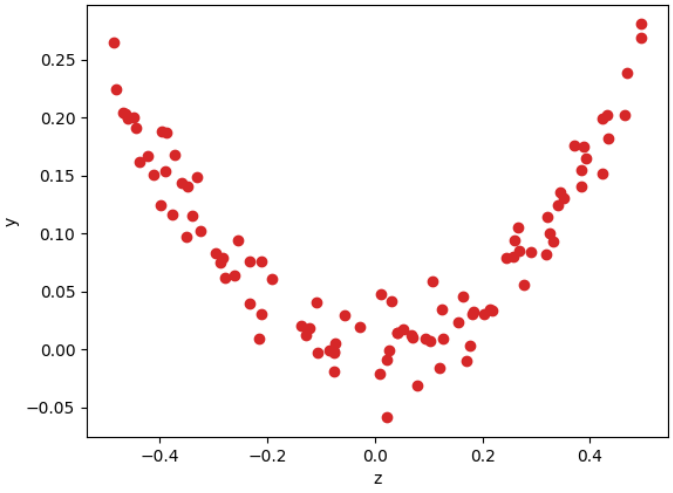

Figure 1: Problem 1: Scatter plots

Problem 2

1. Write down the definition of global minimizer of $f : \mathbb{R} \mapsto \mathbb{R}$.
2. Write down the definition of local minimizer of $f : \mathbb{R} \mapsto \mathbb{R}$.

Problem 3

1. Use Python to plot the graph of $f(x) = x^2$.
2. Use $(\lambda x + (1-\lambda)y)^2 - \lambda x^2 - (1-\lambda)y^2 = \lambda(\lambda - 1)(x-y)^2$ and the definition of convex function to explain why $f(x) = x^2$ is a convex function.

Problem 4

1. In the class, we know that $f(x) = x^2$ is a convex function. Please provide a different univariate convex function.
2. Use Python to plot the graph of the convex function that you provide.
3. Let g denote the convex function that you provide. Is $h(x) = f(x) + g(x)$ a convex function? Please explain why.