Measurement of Tax Progressivity

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Everyone knows that some taxes are progressive and others are regressive, and that there are degrees of each. Yet there is no generally accepted index of how progressive or regressive any given tax is. The purpose of this article is to present a widely useful index of tax progressivity, explore its properties, and to apply it to the analysis of the progressivity of a number of *U.S.* taxes. The index, inspired by and related to the Gini ratio, varies from +1 at the extreme of progressivity where the entire tax burden is borne by members of the highest income bracket, through 0 for a proportional tax, to −1 at the extreme of regressivity at which the entire tax burden is borne by members of the lowest income bracket. When the index is applied to 1970 data, the most highly progressive *U.S.* tax proves to be the federal corporate income tax with an index of +.32. The most regressive are sales and excise taxes with an index of −.15.

A useful property of the index is that the index of progressivity of a tax system consisting of two or more taxes is a weighted average of their individual indexes. On this basis, the entire 1970 *U.S.* tax system was very slightly progressive with an index of +.070.

Comparison of the 1970 values of the index with those of 1966 reveal that, although there was virtually no change in the progressivity of the *U.S.* tax structure as a whole, there were interesting changes in progressivity of individual taxes. In particular, the federal tax structure became somewhat more progressive, but the change was almost exactly offset by declining progressivity of state and local taxes.

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The nature of the index is presented in Section I, illustrated by its application to 1966 tax data compiled by Joseph Pechman and Benjamin Okner. In addition, progressivity of 1966 taxes is compared with that of 1970 by comparing 1966 values of the index with those calculated from 1970 tax data compiled by Okner. Section II concludes with discussion of a number of properties of the index.

I. The Index of Tax Progressivity (S)

The nature of the index of tax progressivity is best illustrated by example. For this purpose, the data of Table 1 will be employed. Column 1 contains the percentage of families accumulated in order of income, marked off in deciles. Column 2 contains the corresponding accumulated percent of total income. The remaining columns contain the accumulated percent of total tax revenue contributed by these same families. For example, the third line of the table, corresponding to the 30 percent of families with the lowest incomes, indicates that these families received only 8.13 percent of total family income, contributed 2.90 percent of total revenue raised by the individual income tax, bore 4.38 percent of the corporate income tax, and so on.

The index of progressivity developed from these data is related to the Lorenz curve and the Gini concentration ratio. Although these measures are generally familiar, it will facilitate the presentation to begin with a short review of their nature. Data for the Lorenz curve of income distribution are contained in the first two columns of Table 1. When accumulated percent of total family income is plotted vertically against accumulated percent of families on the horizontal axis, we obtain the familiar Lorenz curve of Figure 1. It will be recalled that if income were exactly

TABLE 1—ACCUMULATED *U.S.* INCOME AND TAX BURDEN BY POPULATION DECILES, 1966

Population Decile	Cumulated Percentage: Adjusted Family Income	Individual Income Tax	Corporate Income Tax	Property Taxes	Sales and Excise Taxes	Payroll Taxes	Personal Property & Motor Vehicle Taxes	Total Federal Taxes	State and Local Taxes	Total Taxes
1	1.21	0.16	0.53	0.85	2.13	0.70	1.72	0.54	1.45	0.81
2	3.88	0.89	1.97	3.18	6.25	3.02	5.42	2.10	4.48	2.83
3	8.13	2.90	4.38	6.89	12.22	8.32	11.50	5.39	9.08	6.51
4	13.92	6.60	7.21	10.96	19.90	16.24	19.70	10.41	14.81	11.74
5	21.16	12.00	10.38	15.33	29.07	26.60	27.47	17.01	21.40	18.34
6	30.22	19.51	13.87	20.19	40.02	39.34	37.14	25.37	29.28	26.56
7	40.02	28.22	17.91	25.78	51.07	52.25	47.61	34.47	37.68	35.45
8	52.29	40.28	23.59	33.19	64.43	67.28	60.81	46.15	48.20	46.78
9	67.45	56.03	32.15	44.33	79.38	83.75	76.83	60.62	61.35	60.85
10	100	100	100	100	100	100	100	100	100	100
Addendum:										
Average tax rate		8.5	3.9	3.0	5.1	4.4	0.3	17.6	7.6	25.2

Source: Calculated from data in Pechman and Okner, and Okner. Adjusted family income is variant 1c. Tax burden was calculated for each population decile by applying decile tax rates to adjusted family income for each decile. Results were then converted to percentages and accumulated.

equally distributed, the Lorenz curve would follow the diagonal line *OB*, but because the poorest 10 percent of families receive less than 10 percent of total income, the curve sags below the diagonal, following *OCB*. The greater the inequality of income, the farther the Lorenz curve bows away from the diagonal.

The Gini ratio measures income concentration by the proportion of the area of triangle *OAB* that is contained in the sector

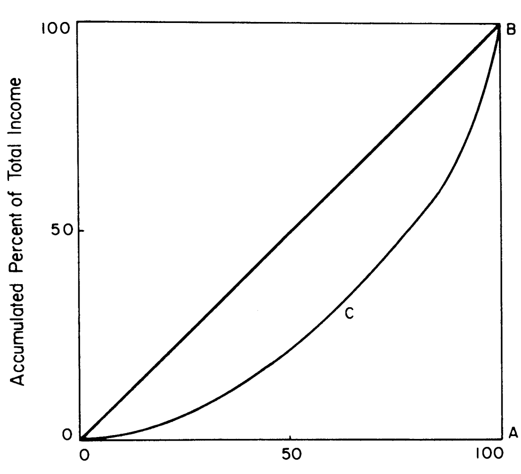

FIGURE 1. LORENZ CURVE OF *U.S.* FAMILY INCOME

bounded by the diagonal line *OB* and the curve *OCB*. Thus the Gini ratio can vary between 0 for income equality to 1 for the extreme inequality in which all income is concentrated in a single family.

To measure the progressivity of a tax, we employ a figure similar to a Lorenz curve, but one in which the accumulated percent of tax burden is plotted vertically against the accumulated percent of income on the horizontal axis. Such Lorenz curves are plotted in Figure 2 to represent the individual income tax and all sales and excise taxes combined. If the income tax were strictly proportional to income, the poorest 10 percent of all families, who earn 1.21 percent of all family income, would bear 1.21 percent of the income tax burden. The poorest 20 percent with 3.88 percent of family income would pay 3.88 percent of the tax, and so on. Thus the curve plotted for such a proportional tax would follow the diagonal line *OB*. Since, however, the income tax is progressive, the percentage of tax burden borne by the lowest income groups is smaller than their share of total income and the curve sags below the diagonal. Thus the curve *OCB* corresponds to the income tax.

In contrast, the percent of total tax burden imposed on low-income families by a

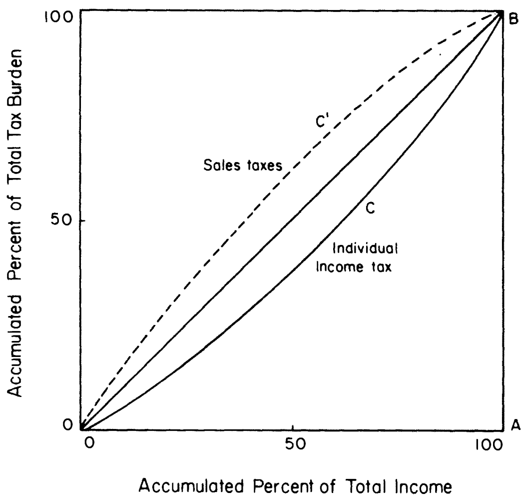

FIGURE 2. LORENZ CURVES FOR INDIVIDUAL INCOME TAX AND FOR ALL SALES AND EXCISE TAXES

regressive tax exceeds their percentage share of total income, so the curve $OC'B$, corresponding to the sales tax, arches above the diagonal.

Analogously to the Gini ratio we define the index of progressivity S in terms of K, the area of the triangle OAB, and L, the area $OABC$, contained between the Lorenz curve and the horizontal axis OA, so that

$$(1) \quad S = (K - L)/K = 1 - (L/K)$$

For a proportional tax, $L = K$, so $S = 0$. Since the curve corresponding to a progressive tax sags below the diagonal, the area L is smaller than K. As a result, the index S is positive for progressive taxes. In the limiting case where the highest income bracket bears the entire tax burden, the Lorenz curve lies along the sides OA and AB, so $L = 0$, and $S = +1$.

With a regressive tax, the Lorenz curve arches above the diagonal. This makes the area L larger than K, so S is negative. In the extreme case of regressivity, $L = 2K$ and $S = -1$. In other words, the index of progressivity S varies between $+1$ in the limiting case of progressive tax, through 0 for proportional taxes, to -1 in the limiting case of regressivity. Inspection of Figure 2 indicates that the values of S for income and for sales taxes are roughly equal in absolute magnitude, but with a positive index corresponding to the progressive income tax and a negative value corresponding to the regressive sales tax. This is borne out by measurement (see below) which yields for the income tax, $S = +.17$, but for the sales tax, $S = -.16$.

Figure 3 presents Lorenz curves for the six taxes studied by Pechman and Okner as of 1966. As would be expected from the known concentration of wealth as compared to income, taxes on corporate income and on property were much the most progressive *U.S.* taxes. Payroll taxes—mostly for social security and unemployment insurance—were the most regressive, although by only a small margin over sales taxes and taxes on personal property and motor vehicles. These findings are borne out by calculated indexes of progressivity as presented in Table 2. Progressivity of *U.S.* 1966 taxes varied from $S = +.36$ for the corporate income tax to $S = -.17$ for payroll taxes.

The second column of Table 2 shows corresponding values of S when calculated from *U.S.* tax data as of 1970 as compiled by Okner. Comparison of the two columns reveals interesting changes in the progressivity of individual taxes between the two dates. Whereas the individual income tax

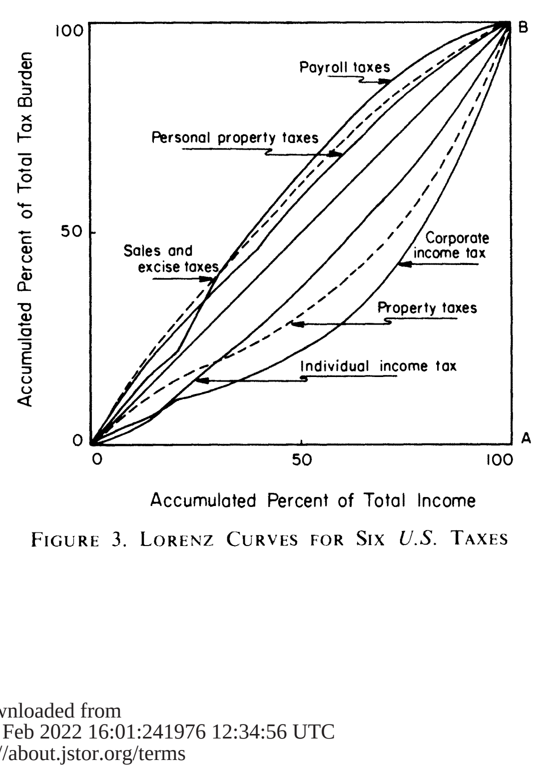

FIGURE 3. LORENZ CURVES FOR SIX *U.S.* TAXES

TABLE 2—PROGRESSIVITY OF *U.S.* TAXES, 1966 AND 1970

	Index (S) 1966	1970
Individual income tax	.17	.19
Corporate income tax	.36	.32
Property tax	.23	.18
Sales and excise taxes	−.16	−.15
Payroll taxes	−.17	−.13
Taxes on personal property and motor vehicles	−.12	−.09
All federal taxes	.087	.091
All state and local taxes	.045	.027
All taxes	.074	.070

became slightly more progressive, the corporate income tax became somewhat less progressive. Property taxes became considerably less progressive, while payroll taxes became considerably less regressive. Sales and excise taxes and taxes on personal property became less regressive.

Taken as a whole, the system of federal taxes became slightly more progressive over the period, but the state and local tax system became slightly less progressive. The two changes were nearly exactly offsetting, however, and the *U.S.* tax system as a whole retained in 1970 the same slight progressivity it had had in 1966.

II. Properties of the Index

A. *Mathematical Representation*

It facilitates exposition to represent the accumulated percent income, measured on the horizontal axis, as a variable y that ranges from 0 to 100. The ordinate of the Lorenz curve representing the corresponding accumulated percent of total tax burden for a given tax x, then becomes $T_x(y)$. In these terms, the area under the curve corresponding to tax x is given by

$$L_x = \int_0^{100} T_x(y)\, dy \tag{2}$$

Recalling that the area of triangle OAB has been designated K, we see that the index of progressivity of tax x is given by

$$\begin{aligned} S_x &= 1 - (L_x/K) \\ &= 1 - (1/K)\int_0^{100} T_x(y)\, dy \end{aligned} \tag{3}$$

B. *Calculation of the Index*

In practice, of course, the values of $T_x(y)$ are known for only a few discrete values of y. In Table 1, indeed, values are given for only 11 values of y: for $y_1, y_2, \ldots, y_{10}$, corresponding to the population deciles, and for $y_0 = 0$. But this information is adequate to provide a close approximation to the value of the integral as:

$$\begin{aligned} L_x &= \int_0^{100} T_x(y)\, dy \\ &\approx \sum_{i=1}^{10} (1/2)[T_x(y_i) + T_x(y_{i-1})](y_i - y_{i-1}) \end{aligned} \tag{4}$$

This approximation is easily calculated from data like that of Table 1, and has been employed in the compilation of Table 2. The area of the triangle, K, is of course the same for all taxes. Since the triangle has base and altitude of 100, $K = 5{,}000$ throughout.

C. *Response to Transfer of Tax Burden Among Families*

The index has the following important property: any change in tax law that transfers part of the tax borne by any family to another family with higher income increases S. Likewise any transfer of tax burden to a lower income family reduces S. Since this property of the index is quite obvious, it is sufficient to demonstrate it informally. Suppose a change in law transfers p percent of total tax burden from families in the third population decile of the income distribution to families in the seventh decile. Such a transfer leaves unaffected the values of $T_x(y_i)$ associated with deciles 1 and 2 and with deciles 7 through 10, but subtracts p from values of $T_x(y_i)$ associated with deciles 3 through 6. As can be seen from the approximation (4), the result is a reduction of L and an increase in S. Similarly, transfer of tax burden from high income to low income families increases L and reduces S.

D. *Systems of Taxes*

Although the degree of progressivity of individual taxes is often of interest, it is more important from a policy point of view to treat the tax system as a whole. Another useful property of the index of progressivity is that the index for a system of two or more taxes is the weighted average of the indexes for the individual taxes, with respective average tax rates as weights. Average tax rate is defined for this purpose as the ratio of total dollar revenue yield of the tax to total dollar family income.

In other words, where S_x and S_z are indexes for taxes x and z, and r_x and r_z are the respective average tax rates, the index of progressivity of the two taxes taken together as a tax system, S_{xz}, is given as

$$(5) \qquad S_{xz} = (r_x S_x + r_z S_z)/(r_x + r_z)$$

To see why this is so, recall that the ordinate of the Lorenz curve for tax x, that is, $T_x(y)$, is the accumulated percentage of total revenue from tax x borne by those families whose incomes accumulate to y percent of total income. If total dollar income for all families combined is Y, then total revenue from the tax in question is $r_x Y = R_x$. In these terms, the total burden of tax x borne by those families whose incomes accumulate to y percent of total becomes

$$(6) \qquad \$_x = R_x T_x(y)/100 = r_x Y T_x(y)/100$$

Similarly, the total dollar burden of tax z on these same families is

$$(7) \qquad \$_z = r_z Y T_z(y)/100$$

It immediately follows that the percent of total burden of the two taxes combined that is borne by these families is

$$(8) \quad T_{xz}(y) = (\$_x + \$_z)/(R_x + R_z) = [r_x T_x(y) + r_z T_z(y)]/(r_x + r_z)$$

It follows that

$$(9) \quad S_{xz} = 1 - (1/K) \int_0^{100} T_{xz}(y)\,dy = 1 - (1/K)\left[r_x \int_0^{100} T_x(y)\,dy + r_z \int_0^{100} T_z(y)\,dy\right] /(r_x + r_z)$$

It requires only slight algebraic manipulation to complete the proof.

When the individual tax indexes in Table 2 are averaged, using the average tax rates provided in the addendum to Table 1, the results for the entire tax system were $S = .077$ in 1966 and $S = .069$ in 1970, as close to the value calculated directly for the total tax system as rounding permits. When the entire *U.S.* tax system is treated as a whole, highly progressive taxes average with regressive taxes to produce a system that is very nearly proportional.

Since the subsystem of federal taxes is more heavily weighted with highly progressive taxes than is the state and local system, it proves to be the more progressive of the two, but the difference is surprisingly small, as can be seen when the two are plotted together in Figure 4. The values of S are

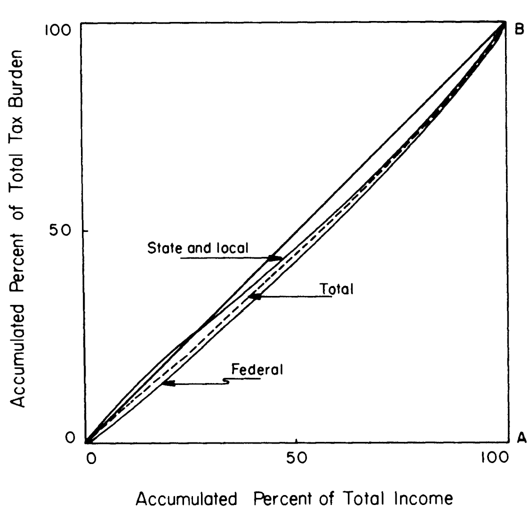

FIGURE 4. CURVES FOR FEDERAL, STATE AND LOCAL, AND TOTAL TAXES

recorded in Table 2. Note again that the value of S for the total tax system is the weighted average of the values for the federal and state and local subsystems.

E. *Distribution of Income and the Value of S*

In interpreting the index of progressivity, it must be borne in mind that the value obtained for any given tax is affected by the initial distribution of income among families. One simple way to see that this must be so is to consider a poll tax. The percent of poll tax burden borne by families with accumulated percent of income y is approximately proportional to the percent of families in that income bracket. Thus the Lorenz curve represenging the poll tax would be identical to the Lorenz curve of Figure 1, but with the axes interchanged. This makes the index S for the poll tax simply the negative of the Gini ratio for the income distribution. The poll tax represents an extreme instance, but similar considerations apply to any other tax.

This aspect of the index serves to emphasize that income distribution is central to the very concept of progressivity. There is nothing inherently regressive about a sales tax or even a poll tax. They are regressive because income is unequally distributed, and the more unequally income is distributed, the more regressive they become. Comparison of indexes for different taxes properly reflects the nature of these taxes in terms of the income distribution of the society within which they are applied. By the same token, however, the dependence of the index on income distribution presents an important qualification to its use in comparison of tax progressivity among different societies with different income distributions.

F. *The Index as an Average*

The index S measures the average progressivity of a tax or tax system across the entire income range, yet some taxes are progressive over one range of incomes and

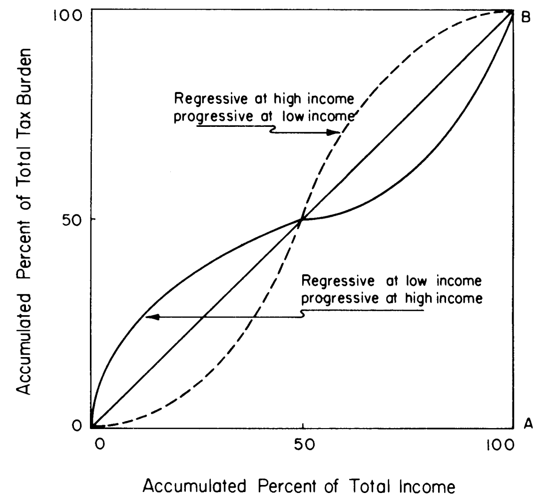

FIGURE 5. LORENZ CURVES OF TWO TAXES WITH $S = 0$

regressive over another. Figure 5 compares the curves of two taxes, one that is regressive at its lower but progressive at its upper range, the other is the opposite. The S ratio for such taxes depends on the algebraic sum of the areas between the curve and the diagonal line, with areas below the diagonal taken with positive, and those above the diagonal with negative sign. Thus both the taxes in Figure 5 would be described by $S = 0$, and would be classified as proportional taxes.

This is a familiar problem with any sort of average. Distributions with equal means can have widely different variances; those with equal variances can have widely different skewness. The S index is no exception to the general rule that it is impossible to capture completely a complex phenomenon by a single measurement, but it still represents a widely useful measure of tax progressivity when carefully applied.

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