Annotations : Complex Analysis 2nd Edition, Lars Valerian Ahlfors

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Chapter 1

The Algebra of Complex Numbers

§1.1 pp. 1 **Addition and multiplication do not lead out from the system of complex numbers.**

If we make a larger number system containing real numbers, then we want to add/multiply real numbers in the new system the same way as before.

§1.1 pp. 2 **Once the existence of quotient has been proved, its value can be found in a simpler way.**

Suppose $x, y \in \mathbb{C}$. x/y exists only if there exists a unique number $z \in \mathbb{C}$ such that $x = yz$. For example, $3/5$ exists only if there exists x such that $5x = 3$. And $3/5$ is undefined in $\mathbb{Z}$ since there is no integer x such that $5x = 3$. At the same time, $3/5$ is undefined since 0.6 is not an integer is a very bad logic.

§1.2 pp. 3 **. . . But these values cannot be combined arbitrarily, for the second equation (4) is not a consequence of (5).**

Suppose $\beta \neq 0$, then we have four possible soultions to (5). But, all of them does not satisfy $2xy = \beta$. It turns out that x and y has same sign if $\beta > 0$. And, x and y has different signs if $\beta < 0$.

§1.2 pp. 4 **. . . it is not possible to distinguish between the positive and negative square root of a complex number.**

Every non-zero complex number have two complex numbers as square roots. But, complex numbers can not have an order relation. Therefore, we cannot name those roots positive or negative.

§1.3 pp. 4 **So far our approach to complex numbers has been completely uncritical.**

Before defining the operations on $\mathbb{C}$, we should have proved the existence of such a number system (which is algebraically unique). Author delayed that proof, probably because he doesn't want it to be a distraction.