# Annotations : Elementary topoics in Differential Geometry, John A. Thorpe

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# Chapter 1

# Graphs and Level Sets

§1 pp. 1 **Given a function $f : U \to \mathbb{R}$ where $U \subset \mathbb{R}^{n+1}$, its level sets are the sets $f^{-1}(c)$ . . .**

The set of points in the domain of $f$ at which $f$ attains the value $c$ is the level set at height $c$.

§1 pp. 1 **The graph of a function $f : U \to \mathbb{R}$ is the subset of $\mathbb{R}^{n+2}$ defined by . . .**

The graph of function $f$ is the set of points $\{(x_1, x_2, \cdots, x_{n+1}, f(x_1, x_2, \cdots, x_{n+1}))\}$.

# Chapter 2

# Vector Fields

**A vector at point $p \in \mathbb{R}^{n+1}$ is a pair $\bar{v} = (p, v)$ where $v \in \mathbb{R}^{n+1}$.** §2 pp. 6
For example : $(1, 2, 2, 4)$ is a vector at $(1, 2)$ defined by $v = 2p$. However, $(1, 2, 2, 4)$ is not a 4-dimensional point in this context. But, the vector $(2, 4)$ with it's tail shifted from $(0, 0)$ to $(1, 2)$.

**The vectors at $p$ form a vector space $\mathbb{R}^{n+1}_p$ of dimension $n + 1$ . . .** §2 pp. 6
The vector space $\mathbb{R}^{n+1}_p$ is identical with $\mathbb{R}^{n+1}$.

**. . . our rule of addition does not permit the addition of vectors at different points of $\mathbb{R}^{n+1}$.** §2 pp. 6
The addition of two vectors at different points is not defined.

**A vector field $\bar{X}$ on $U \subset \mathbb{R}^{n+1}$ is a function which assigns to each point of $U$ a vector at that point.** §2 pp. 6
Consider a function $X : U \to \mathbb{R}^{n+1}$. Suppose $X(p) = v$ and $X(q) = w$. Then $\bar{X}(p) = (p, v)$ and $\bar{X}(q) = (q, w)$.

**A vector field $\bar{X}$ on $U$ is smooth if the associated function $X : U \to \mathbb{R}^{n+1}$ is smooth.** §2 pp. 7
A vector space is smooth if all partial derivatives of the component functions $X_i : U \to \mathbb{R}$ exists and are continuous.

**Associated with each smooth function $f : U \to \mathbb{R}^{n+1}$ is a smooth vector field on $U$ called the gradient $\nabla f$ of $f$ . . .** §2 pp. 8
Suppose all partial derivatives exists and are continuous for $f : U \to \mathbb{R}$. Let $p \in U$, then $\nabla f(p) = (p, \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_{n+1}})$
For example : Let $f(x, y) = xy^2$. Then $\nabla f(p) = (p, v)$ where $v = (y^2, 2xy)$.
ie, $\nabla f$ assigns to the vector $(1, 3)$ a vector $(9, 6)$ at $(1, 3)$.