Annotations : Mathematical Analysis 2nd Edition, Tom M. Apostol

Jacob Antony
jacobantony987@gmail.com

December 26, 2020

Chapter 1

The Real & Complex Number Systems

§1.2-11 is the axiomatic characterisation of the set of all real numbers.

§1.1 pp. 1 **We assume there exists a nonempty set $\mathbb{R}$ of objects, called real numbers ...**

We take a collection of objects and write a few rules. Now we imagine a new world filled with those objects which strictly follows our new rules.

§1.2 pp. 1 **... we assume the existence of two operations, called addition and multiplication ...**

We can not define operations on a collection of undefined objects. Therefore, we add a few more rules and indirectly define the operations required for our purpose.

§1.3 pp. 2 **We also assume the existence of a relation $<$ ...**

These objects have an order which is again defined indirectly. And we also have rules on the effect of operations on this order.

§1.6 pp. 4 **A set of real numbers is an inductive set if ... A real number is a positive integer if it belongs to every inductive set ...**

Here, integers is defined from real numbers using principle of induction.

§1.17 pp. 12 **The fact that a real number might have two different decimal representations ...**

A real number has two different decimal representations if and only if it is rational and $\gcd(q, 10) \neq 1$ where q is its quotient.

Chapter 2

Some basic notations of set theory

$(a, b) = \{\{a\},\ \{a, b\}\}$ Def 2.1 pp. 33

The ordered pairs are defined using sets.

Define an n-tuple using sets

If two function F and G satisfy the inclusion relation $G \subseteq F$, we say that G is a restriction of F or that F is an extension of G. §2.6 pp.35

Relations are set of ordered pairs (x, y). And functions are relations with the property that there is no two ordered pairs with the same first member. And if $(x, y) \in G$, then we write $y = G(x)$.

Suppose $G \subseteq F$. Then every ordered pair in G is also there in F. $(x, y) \in G \implies y = G(x) = F(x)$. That is, G agrees with F everywhere it is defined.

If the function F is one-to-one on its domain, then $\check{F}$ is also a function. Thm 2.10 pp.37

Given a function F, F is a relation. And the converse relation of F given by $\check{F} = \{(y, x) : (x, y) \in F\}$. If F is one-to-one, then there is no two ordered pairs in F with the same second member. Thus $\check{F}$ will have no two ordered pairs with the same first member. Therefore, $\check{F}$ is a function.

$H \circ (G \circ F) = (H \circ G) \circ F$ **always holds whenever each side of the equation has a meaning.** §2.8 pp.37

LHS has meaning only if $\mathscr{R}(F) \subset \mathscr{D}(G)$ and $\mathscr{R}(G \circ F) \subset \mathscr{D}(H)$. RHS has meaning only if $\mathscr{R}(G) \subset \mathscr{D}(H)$ and $\mathscr{R}(F) \subset \mathscr{D}(H \circ G)$.

Existence of $(H \circ G) \circ F$ is a stronger notion than that of $H \circ (G \circ F)$

. . . Such a composite function is said to be a subsequence of s. §2.9 pp. 38

Given a sequence s, a subsequence s_k is obtained by dropping some terms of s. But, we are not allowed to re-order the remaining terms of the sequence.

§2.10 pp. 38 **Also, if $A \sim B$ and if $B \sim C$, then $A \sim C$.**
Given $A \sim B$, then there exists a bijection $f : A \to B$. Also $B \sim C$, thus there exists another bijection $g : B \to C$. Clearly, $f \circ g : A \to C$ is a bijection and therefore $A \sim C$.

§2.11 pp. 38 **if $\{1, 2, \cdots, n\} \sim \{1, 2, \cdots, m\}$, then $m = n$.**
Let $N = \{1, 2, \cdots, n\}$ and $M = \{1, 2, \cdots, m\}$. Given, $N \sim M$, thus there exists a bijection $f : N \to M$. $|M| = |f(N)|$ since f is surjective. And $|f(N)| = |N|$ since f is injective. Clearly, $|M| = |N|$.

§2.11 pp. 38 **. . . an infinite set must be similar to some proper subset of itself . . . whereas a finite set cannot be . . .**
A set which is not finite is defined as infinite set. However, the existence of equi-numerous proper subset is a important characterisation for infinite sets.

Thm 2.16 pp. 39 **Let S . . . countable set . . . $A \subseteq S$. If A is finite, there is nothing to prove, . . . assume that A is infinite (which means S is also infinite.)**
If A is finite, then A is countable. If A is infinite, this proof requires a sequence with range S. There exists an infinite sequence with distinct terms of S only if S is infinite. But, S is countable doesn't necessarily mean that S is infinite.

Thm 2.17 pp. 40 **A situation like $s_n = 0.1999\cdots$ and $y = 0.2000\cdots$ cannot occur here because . . .** In §1.17, it is proved that there exists only two different infinite decimal represenation for any real number.