1 The Algebra of Complex Numbers

A complex number is of the number of the form a + ib where a and b are real numbers and i is a square root of -1. It doesn't matter which square root is considered as i. Now real numbers are complex numbers with b = 0.

1.1 Arithmetic Operations

Let a+ib and c+id be two complex numbers then their sum defined as x+iy where x = a+c and y = b+d. Similarly, their difference is defined as x+iy where x = a-c and y = b-d.

1.2 Multiplication

Following our usual rule for the product of linear combinations, we get: (a+ib)(c+id) = (ac+iad+ibc-bd) since i is a square root of -1. Therefore, their product is defined as x+iy where x = ac-bd and y = ad+bc.

Division is well-defined

The question is how will you define the division operation for two complex numbers ? The division operation (x+iy)/(a+ib) is well-defined if there exists c+id such that (x+iy) = (a+ib)(c+id). Then from the defintion of product we have, x = ad-bc and y = ac+bd. We know that, if both a and b are non-zero, the system of equations has a unique solution. And, the division operation is well-defined on the grounds of multiplication operation.

1.3 Division

Once the existence is proved, we can compute (x+iy)/(a+ib) in different ways. Thus (x+iy)/(a+ib) = (x+iy)(a-ib)/(a+ib)(a-ib) is much easier to compute.

1.4 Operator extension

The arithmetic operations defined on complex numbers is an extension of those on real numbers. We can see that, (a+i0) + (c+i0) = a+c + i0 = a+c is the same as the addition of real numbers. Similarly, (a+i0)(c+i0) = ac + i0 = ac which is again same as those for real numbers. Clearly, substraction and division of complex numbers are extension of those for real numbers.