MM5CRT04 Human Rights and Mathematics for Environmental Studies

Module III

Fibonacci Numbers in Nature

August 3, 2023

Contents

The Rabbit Problem

- Fibonacci Sequence
- Lucas Sequence
- Different Types of Fibonacci/Lucas Numbers

Fibonacci Numbers in Nature

- Earth, Illinois, Flowers, Trees, Fruits
- Fibonacci and Bees
- Fibonacci and Subsets
- Fibonacci and Sewage Treatment
- Fibonacci and Atoms
- Fibonacci and Reflection
- Fibonacci, Paraffins and Cycloparaffins
- Fibonacci and Music
- Fibonacci and Compositions with 1‘s and 2‘s

The Rabbit Problem

"Suppose there are two newborn rabbits, one male and the other female. Find the number of rabbits produced in a year if:

1. each pair takes one month to become mature;
2. each pair produces a mixed pair every month, from the second month on; and
3. no rabbits die during the course of the year."

- Liber Abaci, Leonardo of Pisa (popularly known as Fibonacci)

The Rabbit Problem

"Suppose there are two newborn rabbits, one male and the other female. Find the number of rabbits produced in a year if:

1. each pair takes one month to become mature;
2. each pair produces a mixed pair every month, from the second month on; and
3. no rabbits die during the course of the year."

- Liber Abaci, Leonardo of Pisa (popularly known as Fibonacci)

Solution: 144 Pair of Rabbits

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Adults	0	1	1	2	3	5	8	13	21	34	55	89
Babies	1	0	1	1	2	3	5	8	13	21	34	55
Total	1	1	2	3	5	8	13	21	34	55	89	144

Fibonacci Recursive Relation

Fibonacci Sequence : 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...
Refer : `https://oeis.org/A000045`

$$F_n = F_{n-1} + F_{n-2}, \quad F_1 = F_2 = 1 \tag{1}$$

Theorem

No three consecutive Fibonacci numbers can be the lengths of the sides of a nontrivial triangle.

Proof.

By Fibonacci recursive relation and triangular inequality, longest side of a triangle shold be longer than the sum of lengths of the other two sides. □

Latest Result

- LCM(F(m), F(n)) is a Fibonacci number if and only if either F(m) divides F(n) or F(n) divides F(m).
- Every nonunit positive rational number has at most one representation as the quotient of two Fibonacci numbers.

- M. Farrokhi D.G., Assistant Professor Department of Mathematics, Institute for Advanced Studies in Basic Sciences, Zanjan, Iran (Sep 30, 2021)

Lucas Sequence

Lucas Sequence : 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843,. . .
Refer : `https://oeis.org/A000032`

$$L_n = L_{n-1} + L_{n-2}, \quad L(1) = 1, \quad L(2) = 3 \tag{2}$$

Special Fibonacci/Lucas Numbers

- Fibonacci Squares : 1 and 144.
- Lucas Squares : 1 and 4.
- Fibonacci Cubes : 1 and 8.
- Lucas Cube : 1.
- Ubiquitous/Omipresent Fibonacci Number : 89.
- Constant Lucas Companion of 89 : 11.
- $F_7F_8F_9F_{10} = 510510$ is the product of first seven primes.
- Fibonacci $w^2 \pm 1$ numbers : $1, 2, 3, 5, 8$.
- Lucas $w^2 \pm 1$ numbers : $2, 1, 3$.
- Fibonacci $w^3 \pm 1$ numbers : $1, 2$.
- Lucas $w^3 \pm 1$ numbers : $2, 1, 7$.
- Fibonacci triangular numbers : $1, 3, 21, 55$.
- Lucas triangular numbers : $1, 3, 5778$.
- Beastly number 666 in terms of Fibonacci numbers.

Fibonacci in Nature

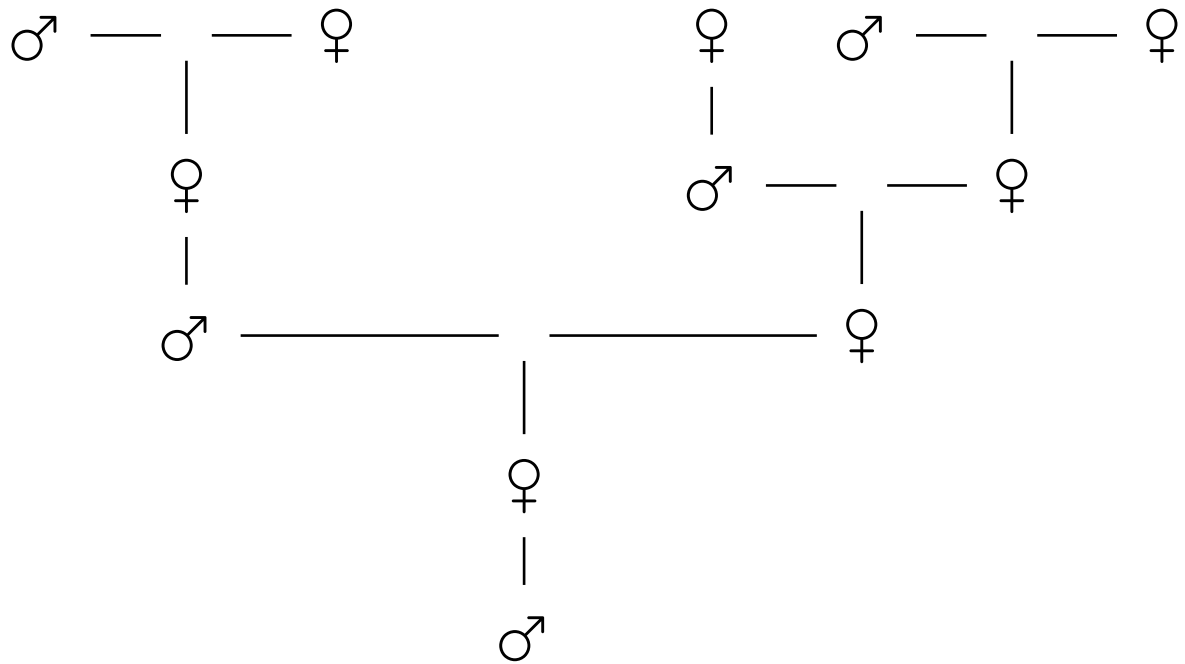
- Diameter of Earth, Jupiter.
- Illinois, USA.
- Number of petals in flowers
- Apple, Star Fish.
- Spiral arrangement of leaves.
- Sunflower, Pineapple, Pinecones, Artichokes.

Fibonacci and Bees

Generation	1	2	3	4	5	6	7	8
Female	0	1	1	2	3	5	8	13
Male	1	0	1	1	2	3	5	8
Total	1	1	2	3	5	8	13	21

Figure: Number of Ancestors of a Drone (Male Bee)

Family Tree of a Drone



Number of Paths in an infinite Beehive

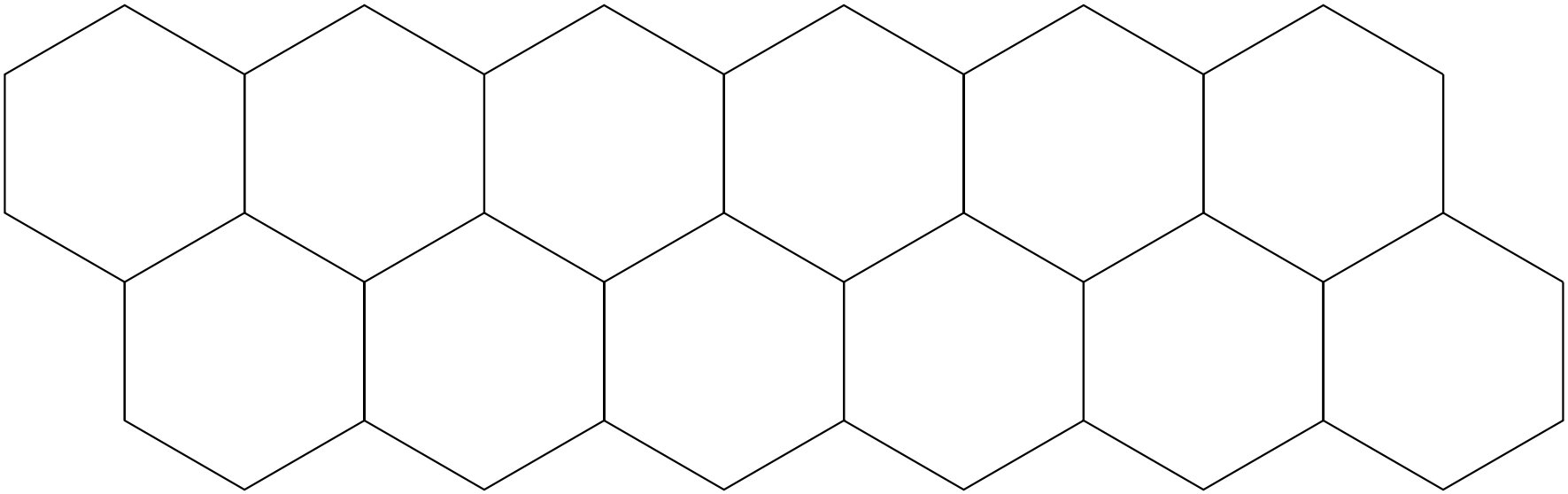


Figure: Infinite Beehive

n	1	2	3	4	5	...	n
b_n	1	2	3	5	8	...	F_n

Figure: Number of Paths

Fibonacci and Subsets

- The number of subsets (including null set) of a set of n points such that consecutive points are not allowed if the points lie on a line.

$$A_1 = 2, \quad A_2 = 3, \quad A_n = A_{n-1} + A_{n-2} = F_{n+2}$$

- The number of subsets (including null set) of a set of n points such that consecutive points are not allowed if the points lie on a circle.

$$B(1) = 2, \quad B(2) = 3, \quad B(3) = 4$$

$$B_n = A_{n-1} + A_{n-3} = F_{n+1} + F_{n-1} = L_n$$

Fibonacci and Sewage Treatment

Fibonacci and Atoms

Fibonacci and Reflection

Fibonacci, Paraffins and Cycloparaffins

Fibonacci and Music

Fibonacci and Compositions with 1‘s and 2‘s

Thank You