Differential Geometry

Module I

Chapter 1 : Level Sets and Graphs

June 14, 2021

Level Set

Definition

Let function $f : U \to \mathbb{R}$ where $U \subset \mathbb{R}^{n+1}$. Let c be a real number. Then the **Level set** of f at height c is the set of all points in U with image c.

$$f^{-1}(c) = \{(x_1, x_2, \cdots, x_{n+1}) \in U : f(x_1, x_2, \cdots, x_{n+1}) = c\} \quad (1)$$

Graph

Definition

Let function $f : U \to \mathbb{R}$ where $U \subset \mathbb{R}^{n+1}$. Then,

$$graph(f) = \{(x_1, x_2, \cdots, x_{n+2}) \in \mathbb{R}^{n+2} : f(x_1, x_2, \cdots, x_{n+1}) = x_{n+2}\} \tag{2}$$

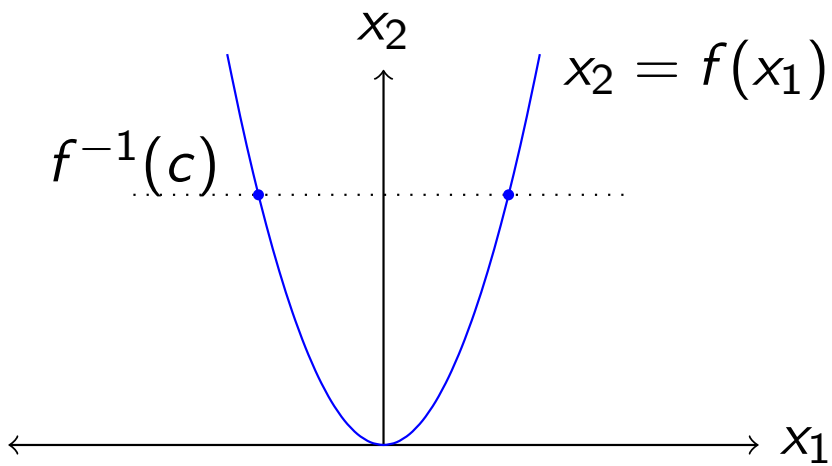

Figure: Graph of $f(x_1) = x_1^2$ and Level set $f^{-1}(c)$