Real Analysis

Module 3

Section 7.17
Uniform Convergence & Differentiation

January 19, 2022

Uniform Convergence & Differentiation

Theorem (7.17)

Suppose $\{f_n\}$ is a sequence of functions differentiable on $[a, b]$ and such that the sequence $\{f_n(x_0)\}$ converges for some $x_0 \in [a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then $\{f_n\}$ converges uniformly on $[a, b]$ to a function f and

$$f'(x) = \lim_{n\to\infty} f_n'(x), \quad \forall x \in [a, b]$$

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Proof.

Let $\varepsilon > 0$.

We have, sequence $\{f_n(x_0)\}$ converges for some $x_0 \in [a, b]$.

Choose natural number N such that $\forall n, m \geq N$,

$$|f_n(x_0) - f_m(x_0)| < \frac{\varepsilon}{2}, \quad \text{since every convergent sequence is Cauchy}$$

$$\left|f_n'(t) - f_m'(t)\right| < \frac{\varepsilon}{2(b-a)} \quad \text{since } f_n' \text{ converges to } f \text{ uniformly on } [a, b]$$

$\square$

Thank You