

Graph Theory

Module 1

Section 4 : Degree of Vertices

July 26, 2023

Degree of a Vertex

Definition (Degree)

The degree of a vertex v of G , $\deg_G(v)$ is the number of edge incident on it except for the loops at v , which are counted twice.

$\delta(G)$ minimum of the degrees of vertices of graph G .

$\Delta(G)$ maximum of the degrees of vertices of graph G .

Regular Graph

Definition (k -regular)

A graph G is k -regular for some non-negative integer k if every vertex of G has degree k .

Definition (regular)

A graph G is regular if it is k -regular for some non-negative integer k .

Definition (cubic)

A 3-regular graph is called a cubic graph.

Definition (1-factor)

A spanning, 1-regular subgraph of graph G is called a 1 factor or a perfect matching of G .

Degree Sequence

isolated vertex is a vertex with degree 0.

pendent vertex is a vertex with degree 1.

pendent edge is an edge incident with a pendent vertex.

Definition (degree sequence)

A finite sequence of degrees of vertices of G which is either in nonincreasing or in nondecreasing order is called a degree sequence.

Definition (graphical sequence)

A finite sequence $d : d_1, d_2, \dots, d_n$ is graphical if there exists a simple graph with degree sequence d .

First Theorem

Theorem (Euler)

The sum of degrees of vertices of a graph is equal to twice the number of its edges.

$$\sum_{i=1}^n d_i = 2m$$

Corollary

The number of odd vertices is even.

Exercises 1

If G, H are isomorphic, then each pair of corresponding vertices of G, H have same degree.

Exercises 2

Let $d : d_1, d_2, \dots, d_n$ be a degree sequence of a graph G and r be a positive integer. Then $\sum_{i=1}^n d_i^r$ is even.

Exercises 3

For every sequence $d : d_1, d_2, \dots, d_n$ with $\sum d_i$ even, there exists a (not necessarily simple) graph with degree sequence d .

Exercises 4

In a group of n persons ($n \geq 2$), there are at least two with the same number of friends.

Exercises 5

Every vertex of a graph G has degree either k or $k + 1$, then the number of vertices of degree k is $(k + 1)n - 2m$.

Thank You