Computational Astrophysics Project
Dynamical Hardening of Black Hole Binaries
in Active Galactic Nuclei

Jacob Bieker Zorry Belcheva Giannis Politopoulos Dylan Natoewal
Benjamin Gilliam Lindsey Oberhelman

April 2019

1 Introduction

For our final project we aim to test a theoretical model for merging black hole binaries in order to help explain the recent observations of the LIGO gravitational wave observatory. This project was motivated by the increasing number of gravitational wave detections from stellar-mass binary black hole mergers [1]. We investigate a possible source of gravitational waves: black hole binaries harden, i.e. spiral-in faster, when passing through the accretion disk of an Active Galactic Nucleus (AGN). The hardening rate is directly affected by the initial orbital parameters of the binaries and the hydrodynamical effects caused by traversing the disk. We intend to determine the timescale over which these binary systems merge under the combined influence of gravitational dynamical encounters and hydrodynamic interactions with the accretion disk of a supermassive black hole. We simulate a dense accretion disk and a collection of black hole binary pairs in orbit around a central supermassive black hole (SMBH). General relativistic effects are not included in the simulation. Our project is inspired by open questions raised in the work done by Yang et al. (2019) and others.

2 Initial Parameters

2.1 Merging criterion and limits on binary orbital separation

Due to computational limitations, we have assumed that are no stars are orbiting the supermassive black hole, and limit the simulation to binaries of black holes only orbiting a SMBH with a massive, optically thick, geometrically thin accretion disk. To avoid relativistic effects, we define a binary black hole system to be "merged" when the distance between them is less than or equal to 100 Schwarzschild radii. The Schwarzschild radius of an object with mass M is given by

$$R_S = \frac{2GM}{c^2}, \tag{1}$$

where G is the gravitational constant and c is the speed of light. This gives $100R_S \approx 9000$km for a $30M_\odot$ black hole. This is nearly 30 times larger than the minimal separation of ~ 300 km measured for the merging black holes of LIGO's first gravitational wave detection [1]. On the other hand, while creating the black hole binaries, we need to set an upper limit on their separation in order to prevent them from being dissociated by the gravitational potential of the SMBH. In other words, the binary black holes need to be within each other's gravitational spheres of influence. For an estimate of this, we use the Hill Radius, given by

$$r_{\rm H} \approx a(1-e)\sqrt[3]{\frac{m}{3M}}. \tag{2}$$

Here a and e are the semi-major axis and eccentricity of the obit of the binary *around* the SMBH, henceforth the outer binary, m is the total mass of the binary black holes, henceforth the inner binary, and M is the mass of the SMBH. We use $0.5 \times r_{\rm H}$ as the initial separation for the binary black holes. As a rough estimate, for two $10M_\odot$ black holes orbiting a SMBH with $M = 10^6 M_\odot$ at $a = 1000AU$, the Hill

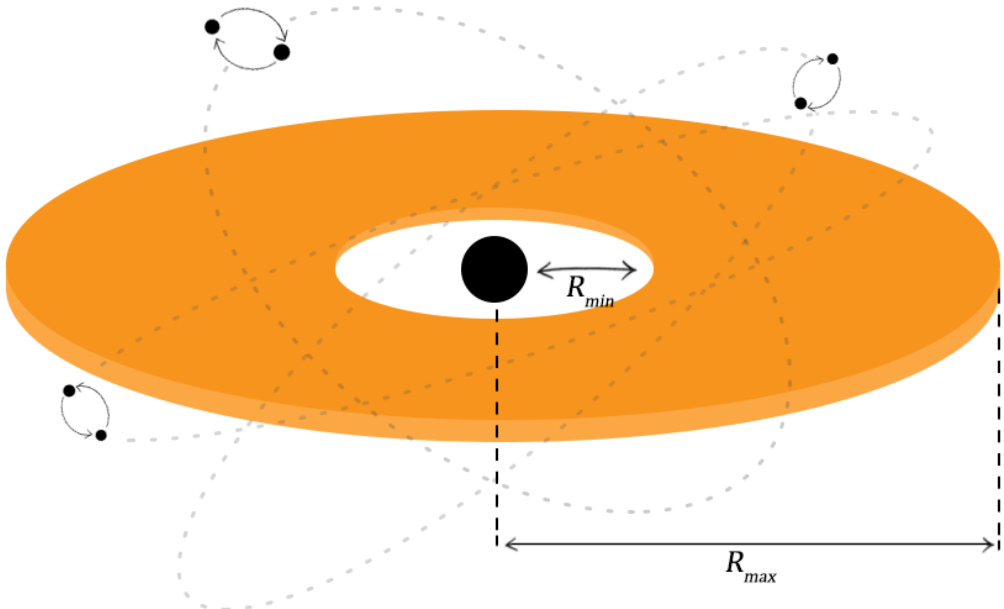

Figure 1: Schematic of the system: accretion disk around a supermassive black hole, with binary pairs of stellar mass black holes orbiting in random Keplerian orbits around them.

radius is $\sim$ 18 AU. To avoid creating binaries that are close to the merging criterion, the lower limit on their initial separation is set to be $10^5 \times R_{S,\text{binary}}$, where $R_{S,\text{binary}}$ is the Schwarzschild radius that a black hole with the combined mass of the binary would have. To summarize, when creating a binary black hole pair, the inner binary semi-major axis would be randomly chosen from a uniform distribution in the interval $(10^5 \times R_{S,\text{binary}}, 0.5r_H)$, which, for example, for a pair of two $10M_\odot$ black holes makes $(0.04, \sim 9)$ AU. The eccentricity of the orbits is fixed at a constant value of 0.6.

2.2 Accretion disk

Due to its topological similarity to a protoplanetary disk, we model the accretion disk around the black hole using the class `ProtoPlanetaryDisk()`, already implemented in AMUSE. It is handled by the SPH hydro code `Gadget2`. The dimensions of the disk are set to avoid the relativistic region close to the SMBH, and also match with measurements from recent papers. More details on the inner and outer bounds are given in the next section. It is of mass 10% of the SMBH mass, and is modelled with 10^5 hydrodynamical particles.

2.3 Positions of the binaries and the accretion disk around the SMBH

Figure 1 shows representation of the system being modelled. The region of close proximity to the SMBH is, of course, largely influenced by general relativistic effects. We are not exploring this region in our simulation, and hence we want to avoid placing binaries there in the first place because this would produce inaccurate results. As illustrated in the figure, for both the accretion disk and the binary black hole pairs, no particles are initially placed inside a spherical region with radius $R_{min} = 100R_{S,\text{SMBH}}$ i.e. 100 times the Schwarzschild radius of the SMBH. For a SMBH with mass $10^6 \times M_\odot$, this is ~ 0.02 AU. The accretion disk size, as well as the upper boundary of the outer binary's semi-major axis is set to $R_{max} \sim 1$ parsec.

For reference, figures 2 and 3 show 3D plots of the initial positions of our binary black holes with and without the disk, respectively.

2.4 Full list of initial conditions

Full summary of the conditions used for the initial state of the system are given in table 1.

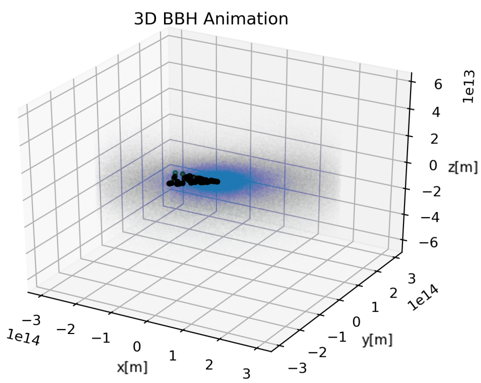

Figure 2: Initial positions of the binaries (in black) with the hydrodynamical disk.

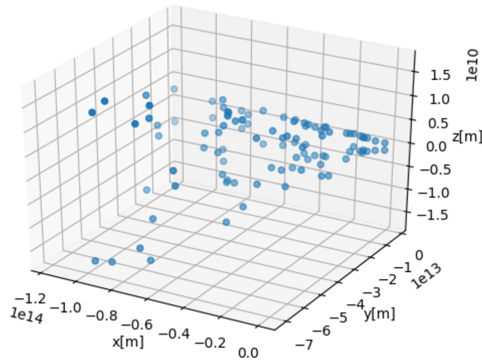

Figure 3: Initial positions of binaries without disk.

Initial Parameters	
SMBH Mass[3]	$10^6 M_\odot$
Binary Black Hole Mass[2, 4]	30 $M_\odot$
Number of Binaries	50
Initial Separation	0.5 $\times r_H$
Merge Condition	100 $\times R_{S,\mathrm{BH}}$
Disk Density	Power Law
Disk Size	$10^{-5} \sim 1$ pc
Disk Mass[5]	10% $\mathrm{M_{SMBH}} = 10^5 M_\odot$
Total Time	100 Myr
Timestep	0.1 Myr

Table 1: Initial conditions of the system.

3 Code implementation

3.1 Setup

The elements of the system (SMBH, AGN disk and black hole binaries) and the included physics as discussed in section 2 are implemented in three steps. In the first step all elements are created. The SMBH is used to create a (background) potential for the whole system. A summary of initial conditions can be seen in Table 1. The second step initializes the gravity code for the SMBH and the binary black holes as well as the hydrodynamics code for the accretion disk. And in the third step the elements are connected (gravitationally only) using Bridge within the AMUSE framework. The potential of the SMBH affects both the accretion disk and the black hole binaries. The disk only affects the binaries and the binaries do not affect any other elements. Interactions between pairs of binaries are included, but in total they are insignificant due to the pairs being too far from each other.

3.2 Usage

The code of our simulation is written in a way that allows easy changes in the initial conditions and all (explorable) parameters. The `main.py` puts everything together in a couple of steps. When running the file, the user can change the initial conditions via the option parser. The SMBH is initialised using its own class which has an accretion disk parameter that can be set to `True` or `False`. The binaries are initialised through the `BinaryBlackHole` class. There is also a class for the accretion disk. `BinaryBlackHolesWithAGN.py` calls these objects to create the full system.

4 Running the simulation

The simulation we aimed to run requires extensive computational resources. In order to run it, we wrote a formal proposal and applied for computing time on Cartesius, the Dutch national supercomputer. After clearing the requisite bureaucratic hurdles, we were granted 20,000 CPU hours for our task. However, we were unable to use the AMUSE software suite installed on Cartesius. When running our simulation, we received a permission error. `Gadget2` was trying to write data to the read-only directory where AMUSE is installed. Because of the limited time remaining this required a change in strategy. We opted to reduce the resolution of the simulation by increasing the timestep, thereby enabling the simulation to be run on the Sterrewacht machines. However, increasing the timestep greatly reduces the accuracy of the simulation, conflicting with energy conservation and other effects coming from the discontinuities in time.

5 Results

We predicted that the black hole binaries would tighten due to interaction with an AGN accretion disk, but our results seem to contradict this. Figures 2 and 3 show the evolution of the Euclidean distances between each binary pair. Each line is a different pair of black holes. These plots were produced with 50 binaries. Figure 4 is with the hydrodynamic disk included, and Figure 5 is without the disk and acts as a control sample to measure our results. Figures 4 and 5 show the separation (physical separation between the black holes of a binary) for each binary black hole pair (with and without the disk, respectively). It is shown that the separation of the binaries in a system with an accretion disk steadily increases over time, whereas without a disk the binaries remain in (somewhat) stable orbits with slight perturbations. There is a lot of noise in the figure, suggesting that with a proper treatment of the accretion disk, an adequate measurement of the merging rate increase would first have to take out this noise. Decreasing the timestep would most likely be beneficial for this.

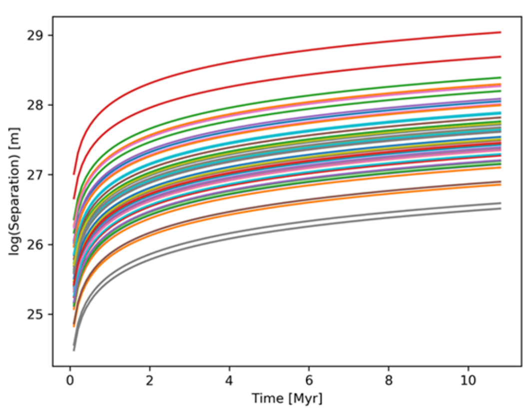

Figure 4: Each binary pair separation orbiting the AGN with disk.

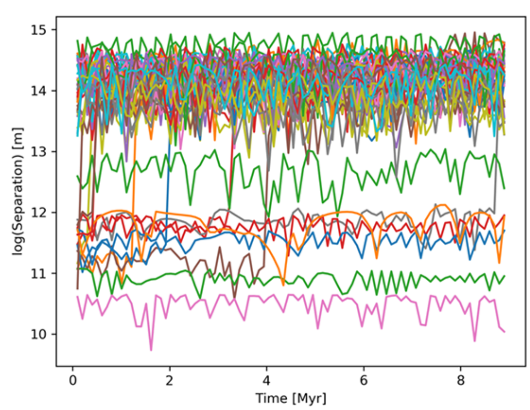

Figure 5: Each binary pair separation orbiting the AGN without disk.

6 Discussion

From our results it is clear that our binaries do not merge but appear to diverge from each other, thus we cannot confirm that interactions between the accretion disk of an AGN and a binary black holes cause them to harden. Although the binaries are affected by the disk (compare figures 4 and 5), it was not the expected result. We strongly suspect that our results are not valid. The work done by Bartos, et al. (2017) indicates that interactions between the AGN disk and the binaries indeed cause the binaries to harden [6].

Without a disk the simulation produces expected results; stable binary black hole orbits (Figure 5). The oscillations (noise) can be seen as a minor perturbations to otherwise stable orbits. This suggests that the simulation works well for systems without an accretion disk.

The mistake in our code when including an accretion disk appears to be systematic, occurring across all binaries. Furthermore, we were unable to implement the hydrodynamic drag which would cause the inner binary to lose energy, reducing the orbital period of the binary and resulting in a merge.

Because we were constrained by time, we were not able to simulate the General Relativity and gravitational waves as a way to make the BHs lose energy, hence we cannot fully replicate the true physical conditions of the system, which makes out simulation not exactly realistic. However, it should not matter too much since even including GR and GW, without the disk interactions, the timescales for inspiral are too long. Also, relativistic effects only become of importance when the black holes are really close to each other, in the last time steps before merging.

7 Conclusion

We conclude that our simulation is incomplete and unable to give accurate predictions of the behaviour of black hole binaries orbiting a SMBH with an accretion disk. However, our results also show that the simulation works well without an accretion disk.

A higher resolution simulation with a timestep shorter than the orbital period of the black hole binary would likely produce better, more insightful results.

In the future, it is worth using the time allocated for our simulation on a supercomputer to do a run with shorter time steps. This would allow us to investigate better the effect of the accretion disk on the merging of the black holes, and separate the noise from the measurements more effectively.

Since we recorded the full particle history of the simulation, we will perform further analysis on the data. We will explore energy conservation, and better statistical strategies to extract a potential signal and find the systematic error in the code. The full particle history can be found in the directory:
`/disks/strw14/bieker/Comp_Astro_Final_Project`

8 Possible improvements

Here we list a couple of potential improvements that would help make the system more physically correct. Some are more straightforward to implement than others, and some may not have a large effect on the overall result of our simulation, however it is important to at least keep in mind that the following are not a part of the current simulation:

- General relativity. AMUSE has codes that include relativistic effects, however we have not used those, which is why we avoid a generous region around all black holes.
- Refine the merging criterion to further explore the merger time-interval. Binary black holes are said to begin merging after the innnermost stable circular orbit, which occurs at a distance of around 3 Schwarzschild radii, an order of magnitude closer together than our current criterion.
- Include single stellar black holes to see how they evolve, and if they evolve into binaries. Also, include stars and treat binaries of a black hole and a star in a similar fashion. This would include implementing stellar evolution, and more advanced channeling and bridging between the codes, and hence make it very computationally expensive.
- Due to software limitations, we were not able to simulate gravitational waves. It has been shown that higher mass binaries merge faster due to gravitational waves (Peter et. al 1964). Future work could involve implementing gravitational waves into the simulation and further testing of higher mass binaries. BBHs near SMBHs are also subject to secular Kozai–Lidov processes, which can alter the inclination and eccentricity of their orbit, potentially decreasing the merger time.
- Explore the regions in space that the merging black holes occupy prior merging. Leigh et al. (2017) claim that migration traps in the disk around a SMBH provides a good environment for dynamical hardening of the binaries. Such migration traps occur when the orbit of a binary is in direct contact with the accretion disk. Due to the black holes losing energy every time they cross the disk because of friction, if their orbit intersects the disk, they are likely to harden much faster than a binary outside of the disk. In order to resolve this, the hydro code needs to resolve the particles in better detail.

- Generate the binary black holes from a wider mass distribution. At the moment, all of the black holes in the binaries have masses of 30 $M_{\odot}$. However, most of the LIGO observations infer black holes of mass 20-35 $M_{\odot}$. It is currently unclear how these formed in the first place, but exploring this hardening scenario might provide some insight on their formation.
- Fix the apocenter (the furthest point of an object in an elliptical orbit to its center of attraction) of the black hole binaries to 10 percent of the Hill radius. This will reduce the amount of randomness introduced to the experiment.
- For each passing through the disk, the binary black holes accrete some material and gain mass. We currently do not know whether this mass change produces a significant impact on the inner binary orbits, or not.
- Future work to refine this analysis will additionally take into account that some BHs are already in binaries before moving into the AGN disk, changing the mass and mass ratio distributions. Spin and the speed of migration within the disk can also affect the final distributions, as can repeated mergers within migration traps.

References

[1] The LIGO Scientific Collaboration, Abbott, B. P., Abbott, R., Abbott, T. D., et al. (2016) "The Rate of Binary Black Hole Mergers Inferred from Advanced LIGO Observations Surrounding GW150914" `https://arxiv.org/abs/1602.03842`

[2] Yang, Y., Bartos, I., Haiman, Z., et al. (2019) "AGN Disks Harden the Mass Distribution of Stellar-mass Binary Black Hole Mergers" `https://arxiv.org/abs/1903.01405`

[3] Greenhill, J. L. (2000) "Accretion and Outflow in the Circinus AGN" `https://arxiv.org/abs/astro-ph/0010277`

[4] Leigh, N., et al. (2017) "On the rate of black hole binary mergers in galactic nuclei due to dynamical hardening" `https://arxiv.org/abs/1711.10494`

[5] Lodato et al. (2002) "Non-Keplerian rotation in the nucleus of NGC 1068: evidence for a massive accretion disk?" `https://arxiv.org/abs/astro-ph/0211113`

[6] Bartos, I., Kocsis, B., Haiman, Z., Márka, S. (2017) "Rapid and Bright Stellar-mass Binary Black Hole Mergers in Active Galactic Nuclei" `https://arxiv.org/abs/1602.03831`