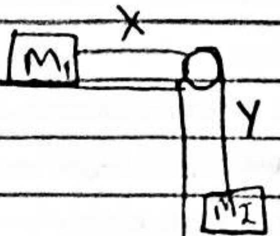
HW 18

① 7.50



$f(x,y) = x+y = const.$

$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{y}^2$

$PE = -mgy$

$L = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{y}^2 + m_2 g y$

$\frac{\partial L}{\partial x} + \lambda\frac{\partial f}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} \rightarrow 0 + \lambda = \frac{d}{dt}[m_1\dot{x}] \rightarrow \lambda = m_1\ddot{x}$

$\frac{\partial L}{\partial y} + \lambda\frac{\partial f}{\partial y} = \frac{d}{dt}\frac{\partial L}{\partial \dot{y}} \rightarrow m_2 g + \lambda = \frac{d}{dt}[m_2\dot{y}] \rightarrow m_2 g + \lambda = m_2\ddot{y}$

$x+y = const.$ $\quad \frac{d}{dt}[x+y] = \frac{d}{dt}const. \rightarrow \dot{x}+\dot{y} = 0$

$\lambda = -m_1\ddot{y}$ $\quad \rightarrow \ddot{x}+\ddot{y} = 0$

$mg + \lambda = m_2\ddot{y} \rightarrow m_2 g = \ddot{y}(m_1+m_2) \rightarrow \boxed{\ddot{y} = \frac{m_2 g}{m_1+m_2}}$ $\quad \ddot{x} = -\ddot{y}$

$m_2 g + \lambda = -m_2\ddot{x}$

$\lambda = m_1\ddot{x} \longrightarrow m_2 g = -\ddot{x}(m_1+m_2) \rightarrow \boxed{\ddot{x} = \frac{-m_2 g}{m_1+m_2}}$

$\lambda = \frac{-m_1 m_2 g}{m_1+m_2}$ $\quad -\lambda\frac{\partial F}{\partial x} = \boxed{\frac{+m_1 m_2 g}{m_1+m_2} = F_x^{cnstr}}$ $\quad \boxed{\frac{-m_1 m_2 g}{m_1+m_2} = F_y^{cnstr}}$

Newton approach

$m\ddot{x} = F_T \quad m\ddot{y} = +mg - F_T$

note that since they are connected, $\ddot{x} = \ddot{y}$

$m_1 a = F_T \quad M_2 a = mg - F_T$

$m_2 a = m_2 g - m_1 a$

$a(m_2+m_1) = m_2 g \quad a = \frac{m_2 g}{m_1+m_2} = \ddot{x} = \ddot{y}$ ✓

so $m_1 a = F_T \rightarrow \frac{m_1 m_2 g}{m_1+m_2} = F_T$ ✓

where F_T is positive for $x \rightarrow m_1\ddot{x} = +F_T$

negative for $y \rightarrow m_2\ddot{y} = m_2 g - F_T$

Just like we found with the Lagrangian

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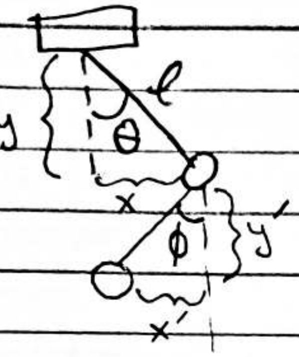

$x = l\sin\theta \qquad x' = l\sin\theta + l\sin\phi$

$y = l\cos\theta \qquad y' = l\cos\theta + l\cos\phi$

clockwise angle measurements are negative

$\dot{x} = l\cos\theta\,\dot{\theta} \qquad \dot{x}' = l\cos\theta\,\dot{\theta} + l\cos\phi\,\dot{\phi}$

$\dot{y} = -l\sin\theta\,\dot{\theta} \qquad \dot{y}' = -l\sin\theta\,\dot{\theta} - l\sin\phi\,\dot{\phi}$

$$T = \frac{1}{2}m(l\cos\theta\,\dot{\theta})^2 + \frac{1}{2}m(l\sin\theta\,\dot{\theta})^2 + \frac{1}{2}m(l\cos\theta\,\dot{\theta} + l\cos\phi\,\dot{\phi})^2 + \frac{1}{2}m(-l\sin\theta\,\dot{\theta} - l\sin\phi\,\dot{\phi})^2$$

$PE = -mgl\cos\theta$

$PE' = -mgl\cos\theta - mgl\cos\phi$

$U = -mgl\cos\theta - mgl\cos\theta - mgl\cos\phi$

$\mathcal{L} = T - U$

→ Plugged into Mathematica

$$\boxed{\mathcal{L} = \frac{1}{2}lm\left(l\dot{\phi}^2 + 2l\dot{\theta}^2 + 2g\cos\phi + 2l\dot{\phi}\dot{\theta}\cos(\phi-\theta) + 4g\cos(\theta)\right)}$$