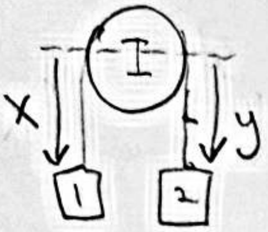
HW 20

1



$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} I \omega^2 \qquad \omega = \frac{v}{r}$$

$$\omega = \frac{\dot{x}}{r}$$

$$x = -y + \text{const.} \qquad I = \frac{1}{2} M R^2$$

$$\dot{x} = -\dot{y} \qquad \dot{y} = -\dot{x}$$

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{4} M \dot{x}^2$$

$$U = -m_1 g x - m_2 g y = -m_1 g x + m_2 g x$$

$$\mathcal{L} = T - U = \frac{1}{4} M \dot{x}^2 + \frac{1}{2}(m_1 + m_2)\dot{x}^2 - (m_2 - m_1) g x$$

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = \frac{1}{2} M \dot{x} + m_1 \dot{x} + m_2 \dot{x} = \dot{x}\left(\frac{1}{2} M + m_1 + m_2\right)$$

$$\mathcal{H} = p_x \dot{x} - \mathcal{L} \qquad \dot{x} = \frac{p}{\frac{1}{2} M + m_1 + m_2}$$

$$\mathcal{H} = \frac{p^2}{\frac{1}{2} M + m_1 + m_2} - \left[\frac{1}{2} \cancel{\left(m_1 + m_2 + \frac{1}{2} M\right)} \frac{p^2}{\cancel{\left(\frac{1}{2} M + m_1 + m_2\right)^2}} - (m_2 - m_1) g x \right]$$

$$\mathcal{H} = \frac{p^2}{\frac{1}{2} M + m_1 + m_2} - \left[\frac{1}{2} \frac{p^2}{\left(\frac{1}{2} M + m_1 + m_2\right)} - (m_2 - m_1) g x \right]$$

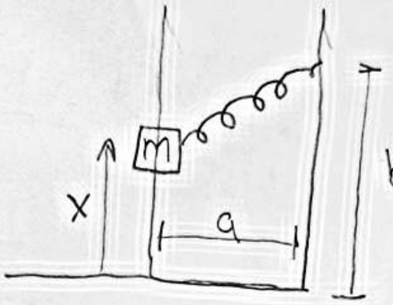
$$\boxed{\mathcal{H} = \frac{1}{2} \frac{p^2}{\left(\frac{1}{2} M + m_1 + m_2\right)} + (m_2 - m_1) g x}$$

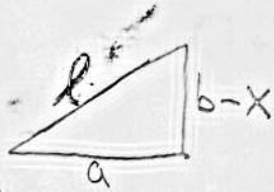
$$\frac{\partial \mathcal{H}}{\partial x} = (m_2 - m_1) g = -\dot{p} \qquad \frac{\partial \mathcal{H}}{\partial p} = \frac{p}{\frac{1}{2} M + m_1 + m_2} = \dot{x}$$

$$-\dot{p} = \left(\frac{1}{2} M + m_1 + m_2\right) \ddot{x} (-1)$$

$$-\ddot{x} = \frac{(m_2 - m_1) g}{\frac{1}{2} M + m_1 + m_2} \qquad \boxed{\ddot{x} = \frac{-(m_2 - m_1) g}{\frac{1}{2} M + m_1 + m_2}}$$

(2) $T = \frac{1}{2} m \dot{x}^2$





$U_{mass} = mgx$

$U_{spring} = \frac{1}{2} K (L - \ell)^2$

ℓ = current length of spring

L = equilibrium length

$\ell^2 = a^2 + (b-x)^2$

$\ell^2 = a^2 + b^2 - 2bx + x^2$

$\ell = \sqrt{a^2+b^2-2bx+x^2}$

$U_{tot} = mgx + \frac{1}{2} K (L - \ell)^2$

$\mathcal{L} = T - U = \frac{1}{2} m \dot{x}^2 - mgx - \frac{1}{2} K (L - \ell)^2$

$P = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$ $\quad T = \frac{1}{2} m \left(\frac{P}{m}\right)^2 = \frac{P^2}{2m}$

$\dot{x} = \frac{P}{m}$

$\mathcal{H} = T + U = \frac{P^2}{2m} + mgx + \frac{1}{2} K (L - \ell)^2$

$\boxed{\dot{x} = \frac{\partial \mathcal{H}}{\partial P} = \frac{P}{m}}$ already found!

$$\boxed{\dot{P} = -\frac{\partial \mathcal{H}}{\partial x} = -mg + \frac{K(-2b+2x)\left(L - \sqrt{a^2+b^2-2bx+x^2}\right)}{2\sqrt{a^2+b^2-2bx+x^2}}}$$

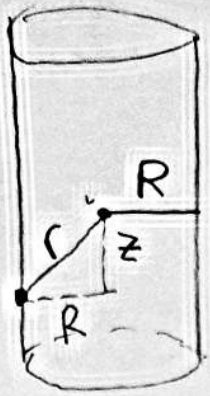
(Derivative $\frac{\partial \mathcal{H}}{\partial x}$ taken in mathematica

$\dot{x} = \frac{P}{m}$ $\quad \boxed{x = \frac{Pt}{m} + C}$ $\quad$ also $m\ddot{x} = -\frac{dU}{dx}$

$P = m\dot{x}$

$$\dot{P} = m\ddot{x} = -mg + \frac{K(-2b+2x)\left(L - \sqrt{a^2+b^2-2bx+x^2}\right)}{2\sqrt{a^2+b^2-2bx+x^2}}$$

13.13



2 degrees of freedom

$$U(\vec{r}) = -\int_0^r -kr\hat{r}\cdot d\vec{r}$$

$$d\vec{r} = \langle dx, dy, dz\rangle$$

$$\hat{r}\cdot r = \vec{r}$$

$$\vec{r} = \langle x, y, z\rangle$$

$$U(\vec{r}) = \int_0^r kx\,dx + \int_0^r ky\,dy + \int_0^r kz\,dz$$

$$U(\vec{r}) = \frac{1}{2}kr^2 + \frac{1}{2}kr^2 + \frac{1}{2}kr^2 = \frac{3}{2}kr^2$$

$$r^2 = R^2 + z^2 \quad r = \pm\sqrt{R^2+z^2}$$

$$U = \frac{3}{2}k\left(R^2+z^2\right) \quad T = \frac{1}{2}m\left(V_z + V_\phi\right) \quad V_z = \dot{z}$$

$$T = \frac{1}{2}m\left[R^2\dot{\phi}^2 + \dot{z}^2\right] \quad V_\phi = R\dot{\phi}$$

$$\mathcal{L} = T - U = \frac{1}{2}m\left[R^2\dot{\phi}^2 + \dot{z}^2\right] - \frac{3}{2}k\left(R^2+z^2\right)$$

$$P_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = mR^2\dot{\phi} \quad \dot{\phi} = \frac{P_\phi}{mR^2} \quad P_z = \frac{\partial\mathcal{L}}{\partial\dot{z}} = m\dot{z} \quad \dot{z} = \frac{P_z}{m}$$

$$T = \frac{1}{2}\cancel{m}\left[\cancel{R^2}\frac{P_\phi^2}{m^{\cancel{2}}R^{\cancel{4}}} + \frac{P_z^2}{m^{\cancel{2}}}\right] = \frac{P_\phi^2}{2R^2m} + \frac{P_z^2}{2m}$$

$$\boxed{\mathcal{H} = \frac{P_\phi^2}{2mR^2} + \frac{P_z^2}{2m} + \frac{3}{2}k\left(R^2+z^2\right)}$$

$$\dot{P}_z = -\frac{\partial\mathcal{H}}{\partial z} = 3kz \quad \dot{P}_\phi = -\frac{\partial\mathcal{H}}{\partial\phi} = 0 \quad \ddot{\phi}mR^2 = 0$$

$$\boxed{\dot{P}_z = 3kz} \quad \dot{P}_\phi = 0 \quad \boxed{\phi(t) = C_1 + tC_2}$$

$$\dot{z} = \frac{\partial\mathcal{H}}{\partial P_z} = \frac{P_z}{m} \quad \dot{\phi} = \frac{\partial\mathcal{H}}{\partial P_\phi} = \frac{P_\phi}{mR^2}$$

$$\boxed{\dot{z} = P_z/m} \quad \dot{\phi} = \frac{P_\phi}{mR^2} \quad P_\phi = \dot{\phi}mR^2$$

$$\dot{P}_\phi = \ddot{\phi}mR^2$$

motion around the cylinder will vary linearly. momentum is conserved angularly

$$\boxed{P_z = m\dot{z}} \quad \dot{P}_z = m\ddot{z}$$

$$\boxed{3kz = m\ddot{z}}$$

The motion in z dir. will be simple harmonic