



## HW #10

①

A)  $U^{\text{ext}}$  from gravity =  $m_1 g z_1 + m_2 g z_2 + m_3 g z_3$

$$U^{\text{int}} = \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}} + \frac{k q_2 q_3}{r_{23}}$$

$$r_{12} = |\vec{r}_1 - \vec{r}_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$U^{\text{tot}} = U^{\text{ext}} + U^{\text{int}}$$

$$U^{\text{tot}} = m_1 g z_1 + m_2 g z_2 + m_3 g z_3 + \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}} + \frac{k q_2 q_3}{r_{23}}$$

B)  $\vec{r}_1 = \langle a/2, 0, 0 \rangle$   $\vec{r}_2 = \langle 0, \sqrt{3} a/2, 0 \rangle$   
 $\vec{r}_3 = \langle -a/2, 0, 0 \rangle$

$$\vec{F}_2^{\text{tot}} = -\nabla U^{\text{tot}} = \left\langle -\frac{\partial U^{\text{tot}}}{\partial x_2}, -\frac{\partial U^{\text{tot}}}{\partial y_2}, -\frac{\partial U^{\text{tot}}}{\partial z_2} \right\rangle$$

$$\vec{F}_2^{\text{tot}} = \left\langle \frac{-k q_1 q_2 (x_1 - x_2)}{((x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2)^{3/2}} + \frac{k q_2 q_3 (x_2 - x_3)}{((x_2 - x_3)^2 + (y_2 - y_3)^2 + (z_2 - z_3)^2)^{3/2}}, \dots, \dots \right\rangle$$

$$((x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2)^{3/2} = a^3$$

B) continued... place in values for the vector components.

$$\vec{F}_2^{\text{tot}} = \left\langle \frac{-kq_1q_2 \frac{a}{2}}{a^3} + \frac{kq_2q_3 \frac{a}{2}}{a^3}, \right. \\ \left. + \frac{kq_1q_2 \frac{\sqrt{3}}{2} a}{a^3} - \frac{kq_2q_3 \frac{\sqrt{3}}{2} a}{a^3}, \right. \\ \left. -gm_2 \right\rangle$$

c) The triangle is in the x-y plane, thus the force will be directed in this plane.

Of course, this force direction relies on the charges of  $q_1, q_2$  and  $q_3$  to determine the value of the x and y components. But we can confidently say that the force will have no component in the z direction since only gravity exerts in that direction for this problem and we are ignoring it.