① $m\ddot{x} = -b\dot{x} - cV\dot{x}$

$= -b\dot{x} - c\sqrt{\dot{x}^2+\dot{y}^2}\,\dot{x}$

$m\ddot{y} = mg - b\dot{y} - c\sqrt{\dot{x}^2+\dot{y}^2}\,\dot{y}$

② a) $\sum_{i=0}^{n} \frac{f^{(n)}(c)}{n!}(x-c)^n$

$f(x) = \ln(1+x)$

$f(x) = \ln(1+c) + \frac{1}{1+c}(x-c) - \frac{1}{2(1+c)^2}(x-c)^2$

b) $f(x) = \cos(x)$

$f(x) = \cos(c) - \sin(c)(x-c) - \frac{1}{2}\cos(c)(x-c)^2$

③ a) $m\ddot{y} = -mg + b\dot{y}$

or

$m\dot{V}_y = -mg - bV_y$

$\dot{V}_y = -g - \frac{b}{m}V_y$ $\qquad r = \frac{-b}{m}$

$V_y(t) = Ae^{-b/m\,t} + B$ $\qquad 0 = -g - \frac{b}{m}B$

$B = -\frac{mg}{b}$

$V_y(t) = Ae^{-b/m\,t} - \frac{mg}{b}$

$V_y(0) = V_{y0} = A - \frac{mg}{b}$ $\qquad A = V_{y0} + V_{ter}$

$$\boxed{\begin{aligned} V_y(t) &= (V_{y0}+V_{ter})e^{-b/m\,t} - V_{ter} \\ y(t) &= -\frac{m}{b}(V_{y0}+V_{ter})(1-e^{\frac{-b}{m}t}) - V_{ter}t \end{aligned}}$$

b) $\boxed{y_{max} = \frac{m}{b}\left(V_0 - V_{ter}\ln\left|\frac{V_{ter}+V_0}{V_{ter}}\right|\right)}$

$V_y(t) = 0 = (V_{y0} + V_{ter})e^{-b/m\,t} - V_{ter}$

$\frac{V_{ter}}{V_{y0}+V_{ter}} = e^{-b/m\,t}$

$\boxed{-\frac{m}{b}\cdot\ln\left|\frac{V_{ter}}{V_{y0}+V_{ter}}\right| = t}$

c) $y_{max} = \frac{m}{b}\left(V_0 - V_{ter}\ln\left|1+\frac{V_0}{V_{ter}}\right|\right)$

as $b \to 0$

$V_0 - \frac{mg}{b}\ln\left|1+\frac{bV_0}{mg}\right|$

$y_{max} = \frac{1/2\,V_0^2}{g}$

$\ln\left|1+\frac{V_0}{V_{ter}}\right| \approx \frac{V_0}{V_{ter}} - \frac{1}{2}\frac{V_0^2}{V_{ter}^2}$

$V_{ter}(\;\text{"}\qquad\text{"}\;) = V_0 - \frac{1}{2}\frac{V_0^2}{V_{ter}}$

$y_{max} = \frac{m}{b}\left(\cancel{V_0} - \cancel{V_0} + \frac{1}{2}\frac{V_0^2}{V_{ter}}\right)$

$= \frac{\cancel{m}}{\cancel{b}}\left(\frac{1}{2}\frac{\cancel{b}}{\cancel{m}g}\cdot V_0^2\right)$

$\boxed{y_{max} = \frac{1}{2}\frac{V_0^2}{g}}$