HW #6

3.29 $I = \frac{2}{5} M R^2$

$I_0 = \frac{2}{5} M_0 R_0^2 \qquad I_f = \frac{2}{5} M_f R_f^2$

$L_0 = I_0 \omega_0 \qquad L_f = I_f \omega_f$

$M = DV$
$M = D\left(\frac{4}{3}\right)\pi R^3$

$L_0 = L_f$

$\omega_f = \omega_0 I_0 / I_f$

$\omega_f = \frac{\omega_0 M_0}{M_f}\left(\frac{R_0}{R_f}\right)^2 = \frac{\omega_0 \cancel{M_0}}{\cancel{M_0} \cdot 8}\left(\frac{1}{2}\right)^2 = \boxed{\frac{\omega_0}{32}}$

$M_f = 8M_0$

3.34 $y(t) = V_0 t - \frac{1}{2} g t^2$

for $y(t) = 0 = V_0 t - \frac{1}{2} g t^2 \qquad t = \frac{2V_0}{g}$

Angular momentum $L = I\omega_0$

no torque, gravity acts on CM
Thus L is conserved

full spin $= 2\pi$
n number spins $= 2\pi n = \omega_0 \cdot t$

$\cancel{2}\pi n = \frac{\omega_0 \cancel{2} V_0}{g}$

$V_0 = \frac{g \pi n}{\omega_0}$

$r = x$

$\rho = \text{constant}$

(3) a) $I = \int r_\perp^2 \, dm$

$dm = \rho \, dV$

$dm = \rho \, dx\, dy\, dz$

$dm = \rho \, dx$ $\boxed{1D}$

$\frac{m}{L} = \rho$

$$I = \int_{-L/2}^{L/2} \rho x^2 dx$$

$$= \frac{x^3}{3}\Big|_{-L/2}^{L/2} = \frac{L^3}{24} + \frac{L^3}{24} = \frac{L^3}{12}$$

$$I = \rho \frac{L^3}{12} = \boxed{\frac{mL^2}{12}}$$

b) $I = \int r_\perp^2 \, dm$ $\quad r_\perp^2 = r^2$

$dm = \rho \, dA$

$dA = dr\, d\theta\, r$

$$\int_0^{2\pi} d\theta = 2\pi$$

$$I = \rho_0 \int r^3 (1 - r/R)^2 \, dr\, d\theta$$

$$\int_0^r \frac{r^5}{R^2} - \frac{2r^4}{R} + r^3 \, dr$$

$$I = \frac{2\pi \rho_0 R^4}{60}$$

$$\frac{r^6}{6R^2} - \frac{2r^5}{5R} + \frac{r^4}{4}\Big|_0^R$$

$$\frac{R^4}{6} - \frac{2}{5}R^4 + \frac{1}{4}R^4$$

$$= R^4/60$$

$$M = \rho_0 \int r(1 - r/R)^2 \, d\theta\, dr$$

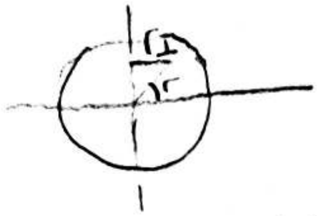
$$\int_0^R \frac{r^3}{R^2} - \frac{2r^2}{R} + r^1 \, dr$$

$$\frac{r^4}{4R^2} - \frac{2r^3}{3R} + r^2/2 \Big|_0^R = \frac{1}{4}R^2 - \frac{2}{3}R^2 + \frac{1}{2}R^2$$

$$= R^2/12$$

$$\int_0^{2\pi} d\theta = 2\pi \qquad M = \rho_0 \frac{2\pi R^2}{12} = \rho_0 R^2 \pi/6$$

$$I = \frac{\pi \rho_0 R^2}{6} \cdot \frac{R^2}{5} = \boxed{MR^2/5}$$



$$r_\perp^2 = r^2\cos^2\theta$$

$$\rho_0 = \frac{M}{A} = \frac{M}{\pi r^2}$$

$$I = \int r_\perp^2\, dm \qquad dm = \rho_0\, dA$$

$$dA = r\,d\theta\,dr$$

$$I = \rho_0 \int r^3 \cos^2\theta\, d\theta\, dr$$

$$\rho_0 \int_0^{2\pi}\int_0^{R} r^3 \cos^2\theta\, d\theta\, dr$$

$$= \frac{1}{4}\pi\rho_0 R^4 \qquad \rho_0 = \frac{M}{\pi R^2}$$

$$= \frac{1}{4}\frac{\pi M}{\pi R^2}R^4 = \boxed{\frac{1}{4}MR^2}$$