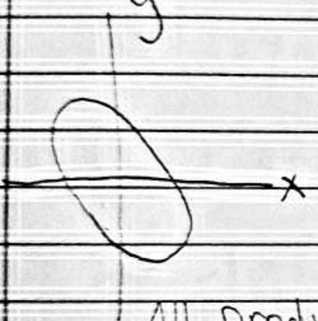
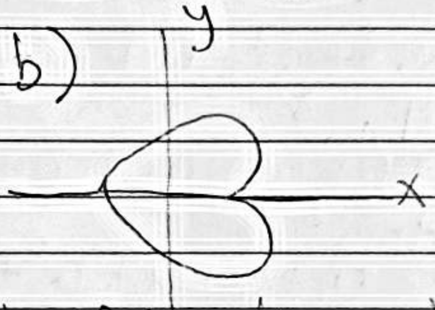
(1)

HW 28
CID 6265

a)



b)

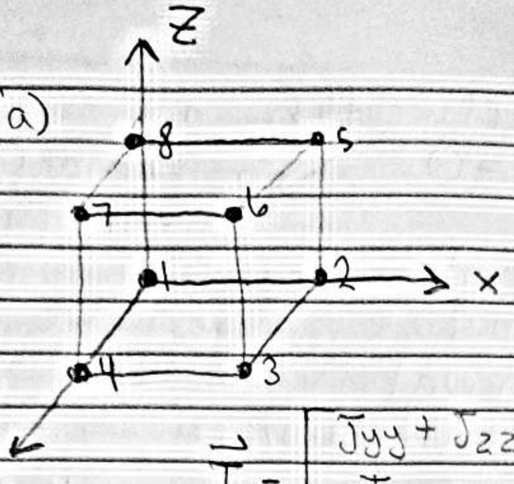


All products of inertia and moments of inertia including the z coordinate will be zero for both Laminas.

a) $J_{zy} = J_{zx} = 0 \qquad J_{xy} \neq 0 \qquad J_{zz} = 0$

b) $J_{zy} = J_{zx} = 0 \qquad J_{xy} = 0 \qquad J_{zz} = 0$

(2) (a)



$$\vec{I} = \begin{bmatrix} J_{yy}+J_{zz} & -J_{xy} & -J_{xz} \\ -J_{xy} & J_{zz}+J_{xx} & -J_{yz} \\ -J_{xz} & -J_{yz} & J_{xx}+J_{yy} \end{bmatrix}$$

$$J_{xy} = \int xy\rho\, dV = \sum m_i x_i y_i$$

$$J_{xy} = \sum_{i=1}^{8} m_i x_i y_i$$

$$= m(0 + \cancel{a(0)} + a^2 + \cancel{a(0)} + \cancel{a(0)} + a^2 + \cancel{a(0)} + \cancel{0(a)})$$

$$= 2ma^2$$

$$J_{xx} = m(0 + a^2 + a^2 + 0 + a^2 + a^2 + 0 + 0)$$

$$= 4ma^2$$

$$J_{yy} = m(0 + 0 + a^2 + a^2 + 0 + 0 + a^2 + a^2)$$

$$= 4ma^2$$

$$J_{zz} = m(0 + 0 + 0 + 0 + a^2 + a^2 + a^2 + a^2)$$

$$= 4ma^2$$

$$J_{xz} = m(0 + 0 + 0 + 0 + 0 + a^2 + a^2 + 0)$$

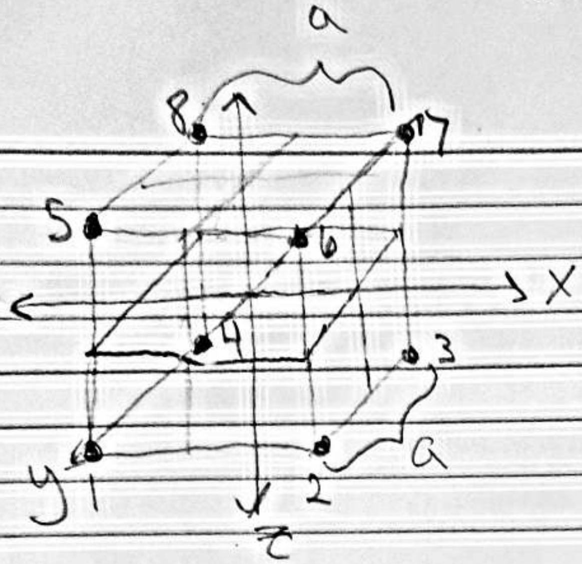
$$= 2ma^2$$

$$J_{yz} = m(0 + 0 + 0 + 0 + 0 + 0 + a^2 + a^2)$$

$$= 2ma^2$$

$$\vec{I} = \begin{bmatrix} 8ma^2 & -2ma^2 & -2ma^2 \\ -2ma^2 & 8ma^2 & -2ma^2 \\ -2ma^2 & -2ma^2 & 8ma^2 \end{bmatrix}$$

$$J_{xy} = m\left(\cancel{-\frac{a^2}{4} + \frac{a^2}{4} - \frac{a^2}{4} + \frac{a^2}{4} - \frac{a^2}{4} + \frac{a^2}{4} - \frac{a^2}{4} + \frac{a^2}{4}}\right)$$



The cube has symmetry about $y=0$, $x=0$, and $z=0$. Thus all products of inertia are zero.

$$J_{yy} = m\left(\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + 4\left(\frac{a}{2}\right)^2\right)$$

$$J_{yy} = m\left(a^2 + a^2\right) = 2ma^2$$

$$J_{xx} = m\left(\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + 4\left(\frac{a}{2}\right)^2\right)$$

$$= 2ma^2$$

$$J_{zz} = m\left(\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + 4\left(\frac{a}{2}\right)^2\right)$$

$$= 2ma^2$$

$$\vec{I} = \begin{bmatrix} 4ma^2 & 0 & 0 \\ 0 & 4ma^2 & 0 \\ 0 & 0 & 4ma^2 \end{bmatrix}$$