$\hat{r} = \frac{\vec{r}}{r}$ $\quad F = \frac{\alpha}{r^3}\langle x, y, z\rangle$

Phys 321 #8

(1) $\dot{T} + \dot{U} = 0$ $\quad T = \frac{1}{2}mv^2 = \frac{1}{2}m(\omega r)^2$

where r = length of pendulum

$\omega = \dot{\theta}$ $\quad U = mgh$ $\quad h = r - r\cos\theta$

$h = r - r + r\frac{\theta^2}{2}$

$U = mgr\frac{\theta^2}{2}$

$$\frac{d}{dt}\left(\frac{1}{2}mr^2\dot{\theta}^2\right) + \frac{d}{dt}\left(mgr\frac{\theta^2}{2}\right) = 0$$

$$\cancel{\frac{1}{2}}\cancel{m}r^{\cancel{2}}\cdot\cancel{2}\dot{\theta}\cdot\ddot{\theta} + \cancel{mgr}\theta\cdot\dot{\theta} = 0$$

$$r\dot{\theta}\ddot{\theta} + g\theta\dot{\theta} = 0$$

$$\dot{\theta}(r\ddot{\theta} + \theta g) = 0 \quad \boxed{\ddot{\theta} = -\frac{g}{l}\theta}$$

$r = l$

$\cancel{\frac{1}{2}m\dot{x}^2 + mgh = \text{constant}}$ $\quad \cancel{h = r\frac{\theta^2}{2}}$

$\cancel{\frac{1}{2}m\dot{x}^2 + mg\frac{\theta^2}{2}l}$ $\quad$ ~~x position~~ $\cancel{= l\cos\theta = l - l\frac{\theta^2}{2}}$

$\cancel{\frac{1}{2}ml^2(-\theta\dot{\theta})^2 + mg\frac{\theta^2}{2}l = C}$ $\quad \cancel{\dot{x} = 0 - l\frac{2\theta\dot{\theta}}{2} = -l\theta\dot{\theta}}$

$\cancel{\frac{d}{dt}\left[\frac{1}{2}ml^2(\theta\dot{\theta})^2 + mg\theta^2 l/2\right] = \frac{d}{dt}C}$

$\cancel{l^2 m\theta\dot{\theta}(\theta\ddot{\theta} + \dot{\theta}^2) + lmg\theta\dot{\theta} = 0}$

$\cancel{l^2 m(\theta^2\dot{\theta}\ddot{\theta} + \theta\dot{\theta}^3) + lmg\theta\dot{\theta} = 0}$

$$\frac{1}{2}m\dot{x}^2 + U(x) = E$$

$$\frac{1}{2}m\dot{x}^2 + mgh = E = \text{constant}$$

$$h = l\frac{\theta^2}{2} \qquad x = l\sin\theta = l\theta$$

$$\dot{x} = l\dot{\theta}$$

$$\frac{1}{2}ml^2\dot{\theta}^2 + mgl\frac{\theta^2}{2} = \text{constant}$$

$$\frac{d}{dt}\left[\tfrac{1}{2}ml^2\dot{\theta}^2 + mgl\frac{\theta^2}{2}\right] = \frac{d}{dt}(\text{constant})$$

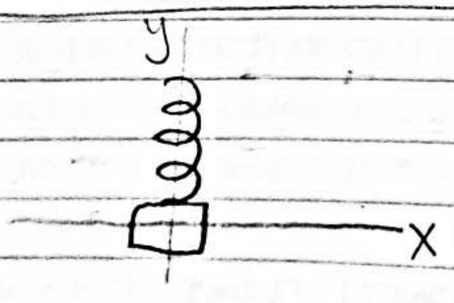
$$\frac{1}{\cancel{2}}ml^2\cancel{2}\dot{\theta}\ddot{\theta} + \frac{mgl\cancel{2}\theta\dot{\theta}}{\cancel{2}} = 0$$

$$l^2\dot{\theta}\ddot{\theta} + gl\theta\dot{\theta} = 0$$

$$\dot{\theta}(l\ddot{\theta} + g\theta) = 0$$

$$\boxed{\ddot{\theta} = \frac{-g}{l}\theta}$$

(2) A)

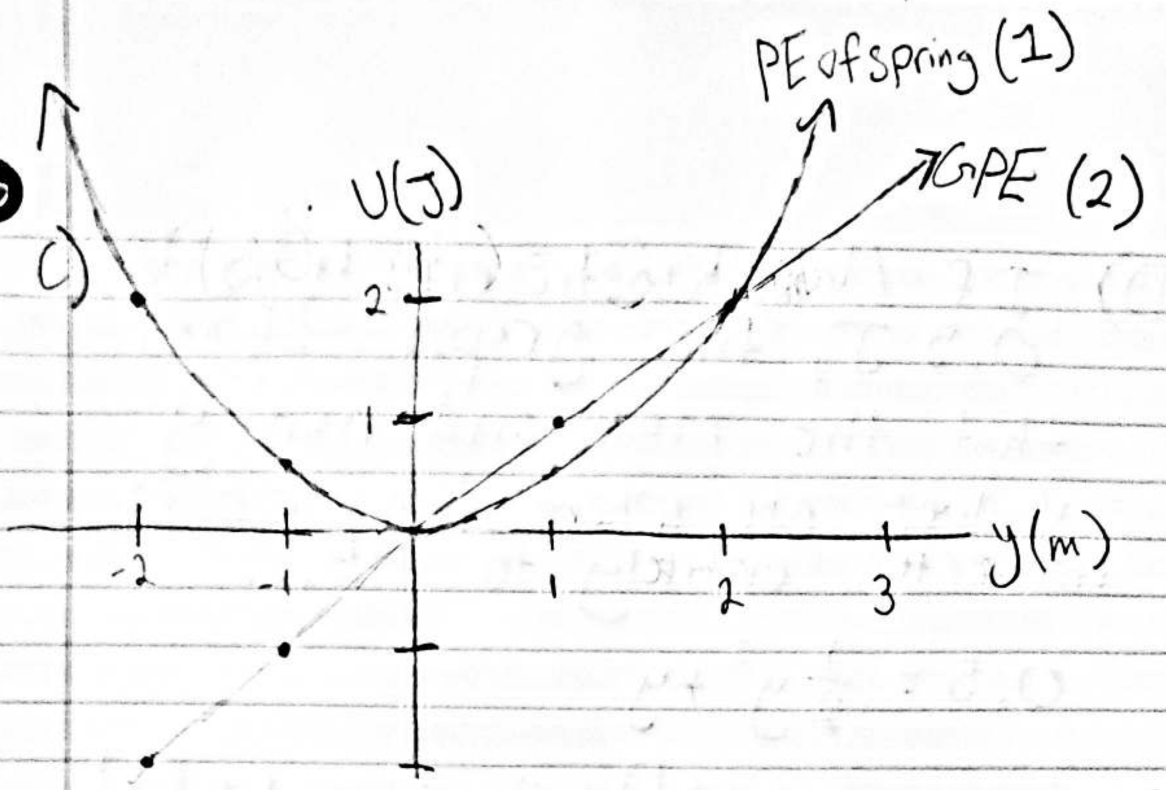


$$mg = Ky_0$$

$$\boxed{y_0 = \frac{mg}{K}}$$

B) $PE = mgy + \frac{1}{2}Ky^2 \qquad K = mg/y_0$

$$\boxed{PE = mgy + \frac{1}{2}mgy^2/y_0}$$

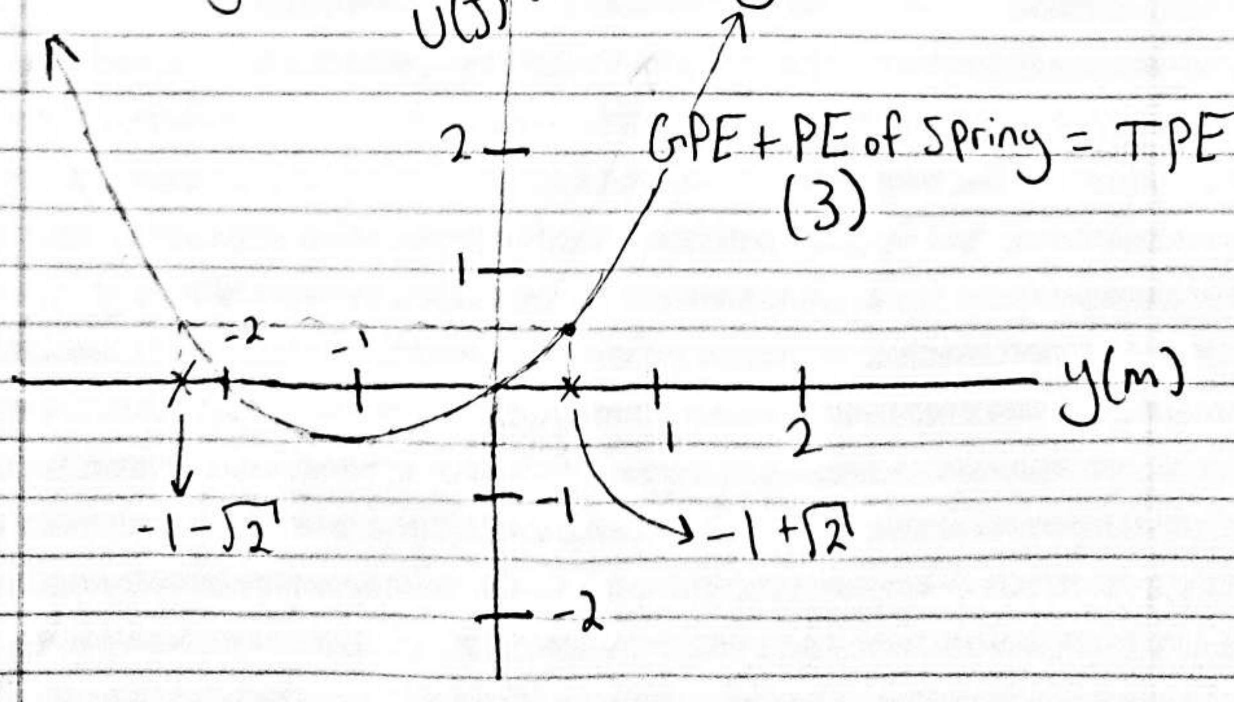


$K=1$ $mg=1$ $y_0 = \frac{1}{1} = 1$

$GPE = y$ PE of spring $= \frac{1}{2y_0} y^2$

$= \frac{1}{2} y^2$

Together... $\frac{1}{2}y^2 + y$



D) If Max Kinetic enery is 0.5 J, the graph shows that the max and min y values are where the KE is converted entirely to PE...

$$0.5 = \frac{1}{2}y^2 + y$$

$$\frac{-1 \pm \sqrt{1+1}}{1} = -1 \pm \sqrt{2}$$

so $\boxed{-1-\sqrt{2} \le y \le -1+\sqrt{2}}$