① A) $KE = \frac{1}{2}m(\dot x^2 + \dot y^2) + \frac{1}{2}I\omega^2$

$= \frac{1}{2}m\dot x^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}m\dot x^2 + \frac{1}{4}MR^2\frac{V^2}{R^2}$

$= \frac{1}{2}m\dot x^2 + \frac{1}{4}m\dot x^2$

$\omega = \frac{V}{r}$ $V = \omega r$

$I = \frac{1}{2}MR^2$ $\omega = \frac{V}{R}$

$PE = m \cdot g \cdot (l - x)\sin\theta$

$\boxed{\mathcal{L} = \frac{1}{2}m\dot x^2 + \frac{1}{4}m\dot x^2 - U = \frac{3}{4}m\dot x^2 - mg(l-x)\sin\theta} \rightarrow -mgl + mgx\sin\theta$

B) $\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot x}\right] \rightarrow \cancel{m}g\cancel{\dot x}\sin\theta = \frac{3}{4}\cancel{m}(2\cancel{\dot x}\ddot x)$

$\boxed{\ddot x = \frac{2}{3}g\sin\theta}$

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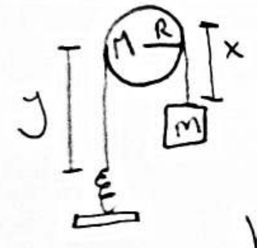
② A)



$y = -x + l - \pi R$

or

$y = -x + \text{const.}$ thus $\dot y = -\dot x$

$KE = \frac{1}{2}m\dot x^2 + \frac{1}{2}I\omega^2$ $\omega = \frac{V}{R}$

$PE = -mgx + \frac{1}{2}k(y_0 - y)^2$

$KE = \frac{1}{2}m\dot x^2 + \frac{1}{4}MR^2\frac{V^2}{R^2}$

$KE = \frac{1}{2}m\dot x^2 + \frac{1}{4}M\dot x^2$

$T = \frac{1}{2}m\dot x^2 + \frac{1}{4}M\dot x^2$

$U = -mgx + \frac{1}{2}k(y_0 + x - l + \pi R)^2$ y_0 = equilibrium length of string on spring side

$\mathcal{L} = \frac{1}{2}m\dot x^2 + \frac{1}{4}M\dot x^2 + mgx - \frac{1}{2}k(y_0 + x - l + \pi R)^2$

$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot x}\right] \rightarrow mg\dot x - kx\dot x + \text{const.} = m\dot x\ddot x + M\dot x\ddot x$

$(mg - kx)\cancel{\dot x} = (m + M)\cancel{\dot x}\ddot x$

$mg - kx = (m + M)\ddot x$

Also it would be good to know equilibrium position

$\boxed{y = c_1 e^{\sqrt{k}\,t/\sqrt{-M-m}} + c_2 e^{\sqrt{k}\,t/\sqrt{-M-m}} + \frac{gM}{K}}$

we need to know initial position and velocity of the mass

③ (R, ϕ, z)

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$x = R\cos\phi$ arclength $= R\phi$

$y = R\sin\phi$ velocity $= R\dot{\phi}$

$U = 0$ since no other forces on particle

$$T = \frac{1}{2}m(R^2\dot{\phi}^2 + \dot{z}^2)$$

or

$$\boxed{\mathcal{L} = \frac{1}{2}m\left[R^2\dot{\phi}^2 + \dot{z}^2\right]}$$

$$T = \frac{1}{2}m(\dot{\phi}^2 R^2 \sin^2\phi + R^2\cos^2\phi\,\dot{\phi}^2 + \dot{z}^2)$$

$$= 1/2\, m(R^2(\sin^2\phi + \cos^2\phi)\dot{\phi}^2 + \dot{z}^2)$$

$$= 1/2\, m(R^2\dot{\phi}^2 + \dot{z}^2)$$