13.10 $\vec{F} = -kx\hat{x} + K\hat{y}$

HW 21

$\vec{F} = -\nabla U = -\left\langle \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y} \right\rangle$

$\boxed{U(x,y) = kx^2 - Ky}$

$\hat{x} = \frac{\vec{x}}{x}$

$\vec{F} = \langle -kx, K \rangle$

$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2$

$\mathcal{L} = T - U = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - kx^2 + Ky$

$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$ $\quad p_y = \frac{\partial \mathcal{L}}{\partial \dot{y}} = m\dot{y}$

$\boxed{T = \frac{p_x^2}{2m} + \frac{p_y^2}{2m}}$

$\boxed{\mathcal{H} = T + U = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + kx^2 - Ky}$

$\frac{\partial \mathcal{H}}{\partial p_x} = \dot{x} = \frac{p_x}{m}$ $\quad \frac{\partial \mathcal{H}}{\partial p_y} = \dot{y} = \frac{p_y}{m}$

$-\frac{\partial \mathcal{H}}{\partial x} = \dot{p}_x = -2kx$ $\quad -\frac{\partial \mathcal{H}}{\partial y} = \dot{p}_y = K$

$p_x = m\dot{x}$ $\quad \dot{p}_x = m\ddot{x}$ $\quad p_y = m\dot{y}$ $\quad \dot{p}_y = m\ddot{y}$

$y = \frac{K}{2m}y^2 + Ay + B$

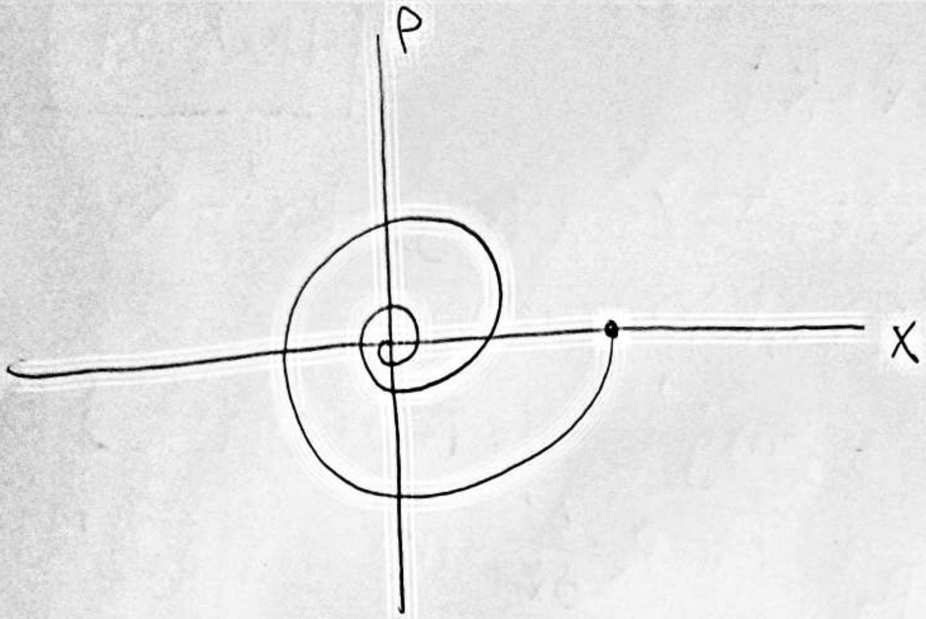
$m\ddot{x} = -2kx$ $\quad m\ddot{y} = K$

In the x direction we have SH motion

In the y direction our force is constant, thus our accel is constant, so we have parabolic motion.

CID 6265

(2)



(3) 13.2b $F_x = -Kx^3$ $F_x = -\nabla U$

$U = \frac{1}{4}Kx^4$

$\mathcal{L} = T - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{4}Kx^4$

$T = \frac{1}{2}m\dot{x}^2$

$P_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$ $\quad \frac{P_x}{m} = \dot{x}$ $\quad T = \frac{P_x^2}{2m}$

$\mathcal{H} = T + U = \frac{P_x^2}{2m} + \frac{1}{4}Kx^4$

$\dot{x} = \frac{\partial \mathcal{H}}{\partial P_x} = \frac{P_x}{m}$ $\quad \dot{x} = \frac{P_x}{m}$ $\quad -\dot{P}_x = \frac{\partial \mathcal{H}}{\partial x} = Kx^3$

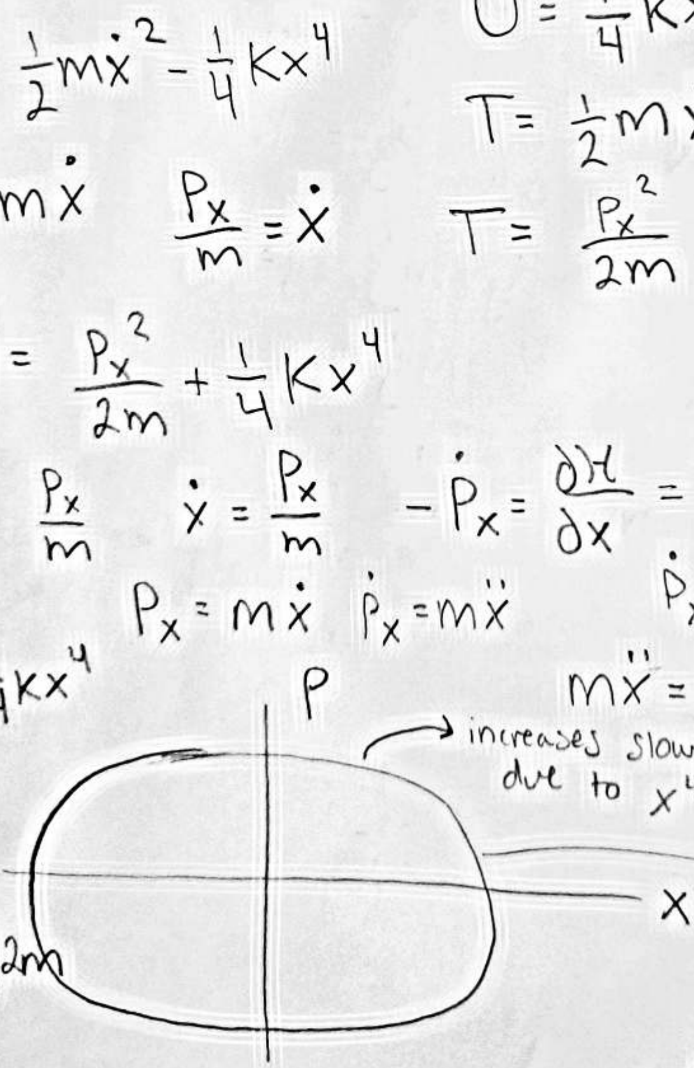
$P_x = m\dot{x}$ $\quad \dot{P}_x = m\ddot{x}$ $\quad \dot{P}_x = Kx^3(-1)$

$E_{tot} = \frac{P^2}{2m} + \frac{1}{4}Kx^4$

$m\ddot{x} = -Kx^3$

distorted ellipses

$P^2 = \left(\frac{1}{4}Kx^4 - E_{tot}\right)2m$



increases slower at low x due to x^4 term

Increases quicker at large x due to x^4 term vs x^2 for normal ellipses.