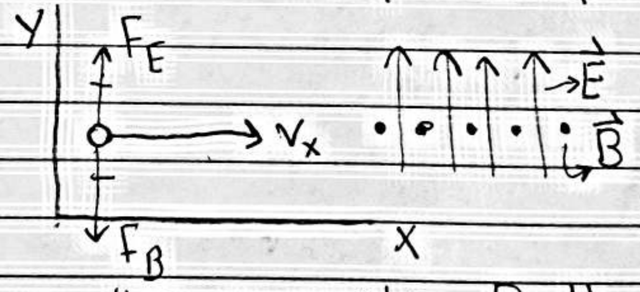
"I Promise that I have abided the rules of the exam with complete integrity."
signed: Jacob A. Clintz

(1) A) My friend is almost correct. First of all, to use the center of mass to find Kinetic energy your system must be rigid if it is multiparticle. Then, this system may also have rotational motion which would have to be added to the total KE. Thus, we have
$$KE = \frac{1}{2}MV_{CM}^2 + \frac{1}{2}I\omega^2.$$

B) We would need an equal force opposing the electric force to keep the particle undeflected.



In my diagram the magnetic field would need to be in the direction of out of the page towards us with magnitude...

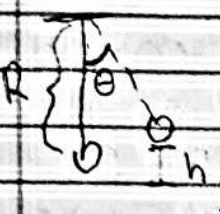
$$F_E = qE \qquad F_B = qVB \qquad \boxed{B = \frac{E}{V}}$$

$$F_E = F_B \qquad qE = qVB$$

First by energy....

(c) $\dot{T} + \dot{U} = 0$ for conservation

$$\frac{d}{dt}\left[\tfrac{1}{2}mv^2\right] + \frac{d}{dt}\left[mgh\right] = 0$$



$$h = R - R\cos\theta \qquad \cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$h = R - R + R\frac{\theta^2}{2} = R\frac{\theta^2}{2}$$

$$V = \omega r$$

$$\frac{d}{dt}\left[\tfrac{1}{2}mv^2\right] \rightarrow \frac{1}{2}m\left[2v\frac{dv}{dt}\right]$$

$$= \frac{1}{2}m\left[2\dot{\theta}\ddot{\theta}R^2\right] = m\dot{\theta}\ddot{\theta}R^2$$

$$m\dot{\theta}\ddot{\theta}R^2 + \underbrace{\frac{d}{dt}\left[mg\left(R\frac{\theta^2}{2}\right)\right]}_{\frac{mgR}{\cancel{2}}\left[\cancel{2}\theta\dot{\theta}\right]} = 0$$

$$\cancel{m}\dot{\theta}\ddot{\theta}R^{\cancel{2}} + \cancel{m}g\cancel{R}\theta\dot{\theta} = 0 \rightarrow \dot{\theta}(\ddot{\theta}R + g\theta) = 0$$

$$\boxed{\ddot{\theta} = -\frac{g}{R}\theta}$$

Now by 2nd Law...

$$\tau = I\alpha \rightarrow \tau = mR^2\ddot{\theta}$$

$\tau = -mR\theta g$ since $\tau = RF\sin\theta$

$F = mg \qquad \sin\theta \approx \theta$

$\tau = Rm\theta g$

thus $-\cancel{R}\cancel{m}\theta g = \cancel{m}R^{\cancel{2}}\ddot{\theta}$

$$\boxed{\ddot{\theta} = -\frac{g}{R}\theta}$$

They are the Same

D) Newton's Laws do not apply when speeds approach relativity.
For instance, the force placed on an object will be measured for a different time from a rest frame in comparison to the measurement made in a frame going by at $v = 0.95c$.
The same force being measured for different times violates newton's laws calling for equal and opposite reactive forces.

$90-\alpha$

$90-\alpha+90+\alpha$

(2) A) In all situations with no outside forces, $P_i = P_f$ for linear momentum.

$P_i = mV_i \quad P_f = (m+3M)V_f$ → for the center of mass

$$(m+3M)V_f = mV_i \qquad \boxed{V_f = \frac{mV_i}{(m+3M)}}$$

This V_f is in the same direction as V_i, or the direction the clay was moving in before the collision. (This conserves momentum components).

B) Here, we will compare angular momentums. With no outside forces, $L_i = L_f$.

magnitude of L_i

$$L_i = \vec{r} \times \vec{P_i} = r_\perp mV_i = LmV_i\cos(\alpha)$$

$$L_f = I\omega \qquad I = mL^2 + \frac{1}{3}ML^2 + \frac{1}{3}ML^2 + \frac{1}{3}ML^2$$

$$I = mL^2 + ML^2$$

since  $dm = \frac{M}{L}dx$ where $\frac{M}{L} = \alpha$

$$I_{rod} = \int_0^L x^2 \frac{M}{L} dx = \frac{M}{L}\frac{1}{3}x^3\Big|_0^L = \frac{1}{3}ML^2$$

thus $LmV_i\cos(\alpha) = (ML^2 + mL^2)\omega$

$$\boxed{\omega = \frac{mV_i\cos(\alpha)}{ML + mL}}$$

c) $T_i = \frac{1}{2} m v_i^2$ $\quad T_f = \frac{1}{2} I \omega^2$ with no linear KE due to being fixed in place.

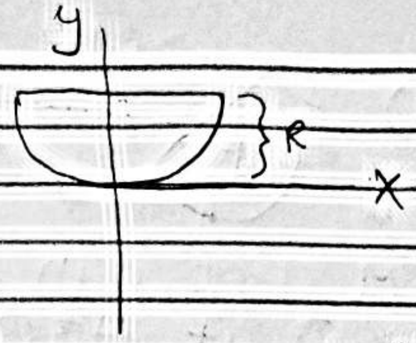
$$\frac{T_i}{T_f} = \frac{\cancel{\frac{1}{2}} m v_i^2}{\cancel{\frac{1}{2}} I \omega^2} = \frac{m v_i^2}{I \omega^2}$$

$$= \frac{m v_i^2}{(mL^2 + ML^2) \cdot (m v_i \cos(\alpha) / (L(M+m)))}$$

$$\rightarrow \frac{\cancel{(m+M)} L^{\cancel{2}} \cdot (m v_i \cos(\alpha))}{\cancel{L} \cancel{(m+M)}} = L(m v_i \cos(\alpha))$$

thus $\longrightarrow$ $$\frac{\cancel{m} v_i^{\cancel{2}}}{L \cancel{m} \cancel{v_i} \cos(\alpha)} = \boxed{\frac{v_i}{L \cos(\alpha)}} = \frac{T_i}{T_f}$$

(3) A) center of mass



$X_{CM} = 0$

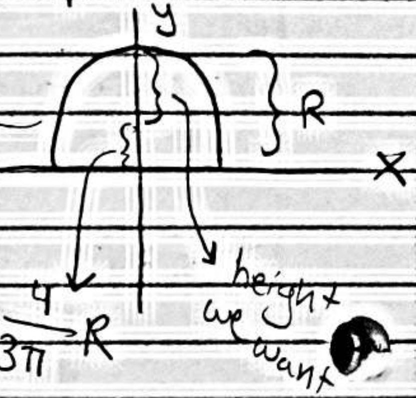
$y_{CM} = \frac{1}{M}\int y\,dm$

$dm = \sigma\,dA \qquad \sigma = \frac{M}{A} \qquad dA = r\,dr\,d\phi$

$y = r\sin\phi$

$y_{CM} = \frac{1}{A}\int r^3 \sin\phi\,dr\,d\phi$

let's flip it this way!



$A = \frac{1}{2}\pi R^2$

$y_{CM} = \frac{1}{A}\int_0^{\pi}\int_0^{R} r^2 \sin\phi\,dr\,d\phi$

$\int_0^R r^3\,dr = \frac{1}{3}r^3\Big|_0^R = \frac{1}{3}R^3$

$\int_0^{\pi} \sin\phi\,d\phi = -\cos\phi\Big|_0^{\pi} = 1+1 = 2$

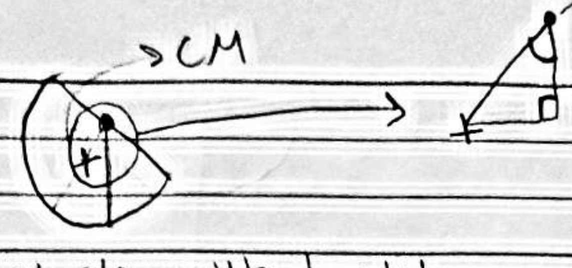
Thus $y_{CM} = \frac{1}{\frac{1}{2}\pi R^2}(2)\left(\frac{1}{3}R^3\right) = \frac{4}{3\pi}R$

However, this is with cylinder upside down. Thus, the actual height above the ground will be $R - \frac{4}{3\pi}R$

height of CM $= R - \frac{4}{3\pi}R$

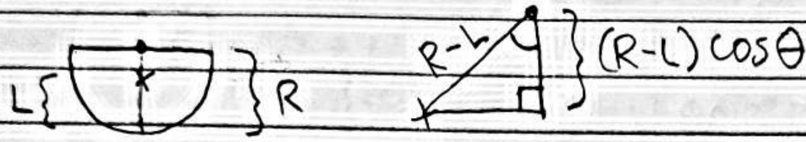
volume of cylinder $= \pi R^2 L \qquad M = \pi R^2 L \rho$

B)



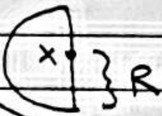
The top of the cylinder at its center does not change height

CM starts with height as 0 at $\theta = 0$



CM starts with height L, or $R-(R-L)$

ends with R



$PE = Mgh_{cm}$

$$\boxed{PE = Mg\left(R-(R-L)\cos\theta\right)}$$

C) Taylor series expand about $\theta = 0 = a$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(\theta)^n$$

$$U = Mg(R-(R-L)\cos\theta)$$

$$U = MgR - Mg(R-L)\cos\theta$$

$$U' = Mg(R-L)\sin\theta$$

$$U'' = +Mg(R-L)\cos\theta$$

$$U(\theta) \approx Mg(R-(R-L)\cos 0) + \frac{Mg(R-L)\sin(0)}{1}(\theta) + \frac{Mg(R-L)\cos(0)}{2}(\theta^2)$$

thus $U(\theta) \approx MgL + 0 + \frac{Mg(R-L)}{2}\theta^2$

c) continued...

$$U(\theta) \approx \underbrace{MgL}_{} + \frac{1}{2}(Mg(R-L))\theta^2$$

since this is a constant we can ignore it.

which is Simple harmonic potential energy

thus $U(\theta) \approx \frac{1}{2}(Mg(R-L))\theta^2$ for small angles

Which is of the form

$U = \frac{1}{2}kx^2$ where $x = \theta$, $k = Mg(R-L)$

Thus it exhibits Simple harmonic motion

D)

$$\text{frequency} = \sqrt{\frac{\text{"Spring const."}}{\text{"mass"}}} \rightarrow \text{moment of inertia}$$

$$= \sqrt{\frac{\cancel{M}g(R-L)}{\cancel{M}R^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)}}$$

$$\boxed{\text{frequency} = \sqrt{\frac{(R-L)g}{R^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)}}}$$