HW 33 CID 6265

(1) $M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$ $\quad -K = \begin{bmatrix} -k & k \\ k & -2k \end{bmatrix}$

$\det[K-\omega^2 M] = 0 = \det\begin{bmatrix} +k-\omega^2 m & -k \\ -k & +2k-\omega^2 m \end{bmatrix}$

$(+k-\omega^2 m)(+2k-\omega^2 m) - k^2 = 0$

$+2k^2 - k\omega^2 m - \omega^2 m 2k + \omega^4 m^2 - k^2 = 0$

$= k^2 - 3km\omega^2 + m^2\omega^4 = 0$

$\omega_1 = \dfrac{\sqrt{\frac{+3k}{m} - \frac{\sqrt{5}k}{m}}}{\sqrt{2}}$ $\quad \omega_2 = \dfrac{\sqrt{\frac{+3k}{m} + \frac{\sqrt{5}k}{m}}}{\sqrt{2}}$

First ω_1

$K - \omega_1^2 M = \begin{bmatrix} k - \frac{3k-\sqrt{5}k}{2} & -k \\ -k & +2k - \frac{3k-\sqrt{5}k}{2} \end{bmatrix}$

so $k\begin{bmatrix} 1 - \frac{3-\sqrt{5}}{2} & -1 \\ -1 & +2 - \frac{3-\sqrt{5}}{2} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$

$a_1\left(1 - \frac{3-\sqrt{5}}{2}\right) - a_2 = 0$

$-a_1 + a_2\left(2 - \frac{3-\sqrt{5}}{2}\right) = 0$

$a_1 = \frac{1}{2}(1-\sqrt{5})$

$a_2 = \frac{1}{2}(-3+\sqrt{5})$

or $a_1 = \frac{1}{2}(1+\sqrt{5})$

$a_2 = 1$

$\vec{a} = \begin{bmatrix} \frac{1}{2}(1+\sqrt{5})A \\ A \end{bmatrix}$

Thus $q_1 = \frac{1}{2}(1+\sqrt{5})A\cos(\omega_1 t - \delta)$

$q_2 = A\cos(\omega_1 t - \delta)$

where $\omega_1 = \dfrac{\sqrt{3k/m - \sqrt{5}k/m}}{\sqrt{2}}$

now $\omega_2 \ldots$

$$K - \omega_2^2 M = \begin{bmatrix} k - \frac{3k+\sqrt{5}k}{2} & -k \\ -k & 2k - \frac{3k+\sqrt{5}k}{2} \end{bmatrix}$$

so $$k\begin{bmatrix} 1 - \frac{3+\sqrt{5}}{2} & -1 \\ -1 & 2 - \frac{3+\sqrt{5}}{2} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

$$a_1\left(1 - \frac{3+\sqrt{5}}{2}\right) - a_2 = 0$$

$$-a_1 + a_2\left(2 - \frac{3+\sqrt{5}}{2}\right) = 0$$

$a_1 = \frac{1}{2}(1+\sqrt{5})$ $\quad \vec{a} = \begin{bmatrix} \frac{1}{2}(1-\sqrt{5}) \\ 1 \end{bmatrix}$

$a_2 = \frac{1}{2}(-3-\sqrt{5})$

or $a_1 = \frac{1}{2}(1-\sqrt{5})$

$a_2 = 1$

$$q_1 = \frac{1}{2}(1-\sqrt{5})A\cos(\omega_2 t - \delta)$$

$$q_2 = A\cos(\omega_2 t - \delta)$$

where $\omega_2 = \dfrac{\sqrt{3k/m + \sqrt{5}k/m}}{\sqrt{2}}$

② A) $T_m = \frac{1}{2}m((\dot{x}_2+\dot{x}_1)^2)$ $U_m = \frac{1}{2}k(x_2-x_1)^2$

$T_{2m} = \frac{1}{2}2m\dot{x}_1^2$ $U_{2m} = \frac{1}{2}kx_1^2$

$L = \frac{1}{2}m(\dot{x}_2+\dot{x}_1)^2 + m\dot{x}_1^2 - \frac{1}{2}k(x_2)^2 - \frac{1}{2}kx_1^2$

$\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = m(\dot{x}_2+\dot{x}_1)+2m\dot{x}_1$ $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_1}\right) = m\ddot{x}_2 + 3m\ddot{x}_1$

$\frac{\partial \mathcal{L}}{\partial x_1} = -kx_1$ thus, $-kx_1 = 3m\ddot{x}_1 + m\ddot{x}_2$ ✓

$\frac{\partial \mathcal{L}}{\partial \dot{x}_2} = m(\dot{x}_2+\dot{x}_1)$ $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right) = m\ddot{x}_2 + m\ddot{x}_1$

$\frac{\partial \mathcal{L}}{\partial x_2} = -kx_2$ thus $-kx_2 = m\ddot{x}_2 + m\ddot{x}_1$ ✓

B) $M = \begin{bmatrix} 3m & m \\ m & m \end{bmatrix}$ $-k = \begin{bmatrix} -k & 0 \\ 0 & -k \end{bmatrix}$

$$\det(k-\omega^2 M) = \det\begin{bmatrix} k-\omega^2 3m & -\omega^2 m \\ -\omega^2 m & k-\omega^2 m \end{bmatrix} = 0$$

$$(k-\omega^2 3m)(k-\omega^2 m) - \omega^4 m^2 = 0$$

$$= k^2 - 4km\omega^2 + 2m^2\omega^4 = 0$$

$\omega_1 = \sqrt{\frac{k}{m} + \frac{k}{\sqrt{2}m}}$ $\omega_2 = \frac{\sqrt{\frac{2k}{m} - \frac{\sqrt{2}k}{m}}}{\sqrt{2}}$ First ω_1

so

$$k\begin{bmatrix} 1-\frac{3m}{m}-\frac{3m}{\sqrt{2}m} & -\frac{m}{m}-\frac{m}{\sqrt{2}m} \\ -\frac{m}{m}-\frac{m}{\sqrt{2}m} & 1-\frac{m}{m}-\frac{m}{\sqrt{2}m} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

$$a_1\left(-2-\frac{3}{\sqrt{2}}\right) - a_2\left(1+\frac{1}{\sqrt{2}}\right) = 0$$

$$-a_1\left(1+\frac{1}{\sqrt{2}}\right) - a_2\frac{1}{\sqrt{2}} = 0$$

$a_1 = 1-\sqrt{2}$ $\quad a_2 = 1$ $\quad$ thus $\vec{a} = \begin{bmatrix}(1-\sqrt{2})A \\ A\end{bmatrix}$

and

$$x_1 = (1-\sqrt{2})A\cos(\omega_1 t - \delta)$$
$$x_2 = A\cos(\omega_1 t - \delta)$$

where $\omega_1 = \sqrt{\frac{k}{m} + \frac{k}{\sqrt{2}\,m}}$

<u>now ω_2</u> ...

$$k\begin{bmatrix} 1-\frac{(2-\sqrt{2})3m}{2m} & -\frac{(2-\sqrt{2})m}{2m} \\ -\frac{(2-\sqrt{2})m}{2m} & 1-\frac{(2-\sqrt{2})m}{2m}\end{bmatrix}\begin{bmatrix}a_1 \\ a_2\end{bmatrix} = 0$$

$$a_1\left(-2+\frac{3}{\sqrt{2}}\right) - a_2\frac{(2-\sqrt{2})}{2} = 0$$

$$-a_1\frac{(2-\sqrt{2})}{2} + a_2\frac{1}{\sqrt{2}} = 0$$

$a_1 = 1+\sqrt{2}$ $\quad a_2 = 1$ $\quad$ thus $\vec{a} = \begin{bmatrix}(1+\sqrt{2})A \\ A\end{bmatrix}$

and

$$x_1 = (1+\sqrt{2})A\cos(\omega_2 t - \delta)$$
$$x_2 = A\cos(\omega_2 t - \delta)$$

where

$$\omega_2 = \frac{\sqrt{\frac{2k}{m} - \frac{\sqrt{2}k}{m}}}{\sqrt{2}}$$