① A) $\ddot{x} + 2\beta\dot{x} + \frac{k}{m}x = 0$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \qquad r^2 + 2\beta r + \omega_0^2 = 0$$

$$\frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2}$$

$$x(t) = C_1 e^{(-\beta + \sqrt{\beta^2 - \omega_0^2})t} + C_2 e^{(-\beta - \sqrt{\beta^2 - \omega_0^2})t} \qquad r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

$$\boxed{x(t) = e^{-\beta t}\left(C_1 e^{\sqrt{\beta^2 - \omega_0^2}\,t} + C_2 e^{-\sqrt{\beta^2 - \omega_0^2}\,t}\right)}$$

B) $\sqrt{\beta^2 - \omega_0^2}$ must be imaginary

or $\beta < \omega_0$

Thus $\sqrt{-1\underbrace{(\omega_0^2 - \beta^2)}_{\text{positive}}} \rightarrow i\sqrt{\omega_0^2 - \beta^2} = i\omega_1$

$$\omega_1 = \sqrt{\omega_0^2 - \beta^2}$$

thus $x(t) = e^{-\beta t}\left(C_1 e^{i\omega_1 t} + C_2 e^{-i\omega_1 t}\right)$

C) $x(t) = e^{-\beta t}\left[C_1 \cos(\omega_1 t) + C_2 \sin(\omega_1 t)\right]$

or

$$x(t) = A e^{-\beta t} \cos(\omega_1 t - \delta)$$

$$e^{-\beta t} = 0.1 \rightarrow x(t) = \underbrace{A(0.1)}_{10\% \text{ of } A} \cos(\omega_1 t - \delta)$$

$$\ln(0.1) = -\beta t$$

$$t = \frac{-\ln(0.1)}{\beta} = \frac{\ln\left(\frac{1}{0.1}\right)}{\beta} = \boxed{\frac{\ln(10)}{\beta} = t}$$

5.23 $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2$

$$\frac{dE}{dt} = \frac{1}{2}(2\dot{x}\ddot{x})m + \frac{1}{2}(2x\dot{x})K$$

$$\frac{dE}{dt} = \dot{x}\ddot{x}m + x\dot{x}K$$

$m\ddot{x} + b\dot{x} + Kx = 0 \qquad m\ddot{x} + kx = -b\dot{x}$

$$\dot{x}\ddot{x}m - \frac{dE}{dt} + x\dot{x}K = 0$$

$$\frac{dE}{dt} = \dot{x}(\underbrace{m\ddot{x} + Kx}_{m\ddot{x} + Kx = -b\dot{x}})$$

$$\frac{dE}{dt} = -b\dot{x}^2$$

notice that $b\dot{x}$ is force $= F$
Thus we have $F \cdot \dot{x}$, where $\dot{x} = V$
Instantaneous rate at which force does work (power) $= FV$

$$\frac{dE}{dt} = -b\dot{x}\dot{x} = -F_{dmp}\dot{x} = -F_{dmp}V$$

$-F_{dmp}V \rightarrow \frac{-W_{dmp}}{t}$

$=$ minus rate at which energy is dissipated by F_{dmp}.