(1) $F = q\vec{v} \times \vec{B}$

$$m\ddot{x} = qB\dot{y}$$
$$m\ddot{y} = -qB\dot{x}$$

$$m\frac{\partial}{\partial t}\dot{v}_x = qB\frac{\partial}{\partial t}v_y$$

$$m\ddot{v}_x = qB\dot{v}_y \qquad \dot{v}_y = \frac{-qBv_x}{m}$$

$$m\ddot{v}_x = qB\left(\frac{-qBv_x}{m}\right)$$

$$\ddot{v}_x = -\frac{q^2B^2}{m^2}v_x$$

wave equation

$$\ddot{v}_x = -\alpha v_x \qquad \alpha = \frac{q^2B^2}{m^2}$$
$$r^2 = -\alpha$$
$$r = \pm i\sqrt{\alpha}$$

$$\boxed{v_x = A\cos(\sqrt{\alpha}x) + D\sin(\sqrt{\alpha}x)}$$

$$v_y = \frac{m\dot{v}_x}{qB} \qquad \dot{v}_x = -\sqrt{\alpha}A\sin(\sqrt{\alpha}x) + \sqrt{\alpha}D\cos(\sqrt{\alpha}x)$$

$$v_y = \frac{\cancel{m}}{\cancel{qB}}\left(-\frac{\cancel{qB}}{\cancel{m}}A\sin(\sqrt{\alpha}x) + \frac{\cancel{qB}}{\cancel{m}}D\cdot\cos(\sqrt{\alpha}x)\right)$$

$$\boxed{v_y = -A\sin(\sqrt{\alpha}x) + D\cos(\sqrt{\alpha}x)}$$

$$\text{w/ } \alpha = \frac{q^2B^2}{m^2}$$

B) If $\vec{E}$ is along z axis,

$$\vec{E} = \langle E_x, E_y, E_z \rangle = \langle 0, 0, E_z \rangle$$

$$|\vec{E}| = E_z$$

$$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$$

$$\langle E_x + E_y + E_z \rangle + \begin{matrix} V_x & V_y & V_z \\ B_x & B_y & B_z \end{matrix} = \langle E_x + V_y B_z - V_z B_y, \; E_y + V_z B_x - V_x B_z, \; E_z + V_x B_y - V_y B_x \rangle$$

$$\vec{B} = \langle 0, 0, B_z \rangle$$

$$|\vec{B}| = B_z$$

$$\rightarrow \langle \underbrace{0 + V_y B}_{\text{unchanged}}, \underbrace{0 - V_x B}_{\text{unchanged}}, \underbrace{E + 0 - 0}_{\text{new}} \rangle$$

Thus

$$\boxed{\begin{aligned} m\dot{V}_x &= qBV_y \\ m\dot{V}_y &= -qBV_x \\ &\text{and} \\ m\dot{V}_z &= qE \end{aligned}}$$

with a kick in the xy plane, the particle will interact w/ the magnetic field and receive a force on it that is centripetal, giving it circular motion in the x-y plane. The particle also will have a force on it directed along the +z direction due to the Electric field, giving it a path of an increasingly elongated helical "upwards".

solutions to the differentials ...

$$V_x = A\cos(\sqrt{\alpha}\,x) + C\sin(\sqrt{\alpha}\,x)$$

$$V_y = -A\sin(\sqrt{\alpha}\,x) + C\cdot\cos(\sqrt{\alpha}\,x)$$

where $\alpha = \frac{q^2B^2}{m^2}$

$$m\dot{V}_z = qE$$

$$\dot{V}_z = \frac{qE}{m} \qquad \int \dot{V}_z = \int \frac{qE}{m}$$

$$V_z = C_1 + \frac{qE}{m}z$$