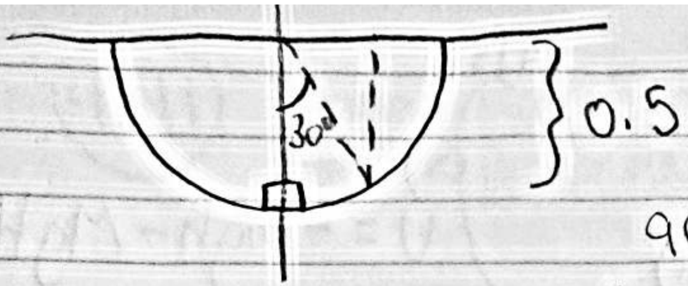


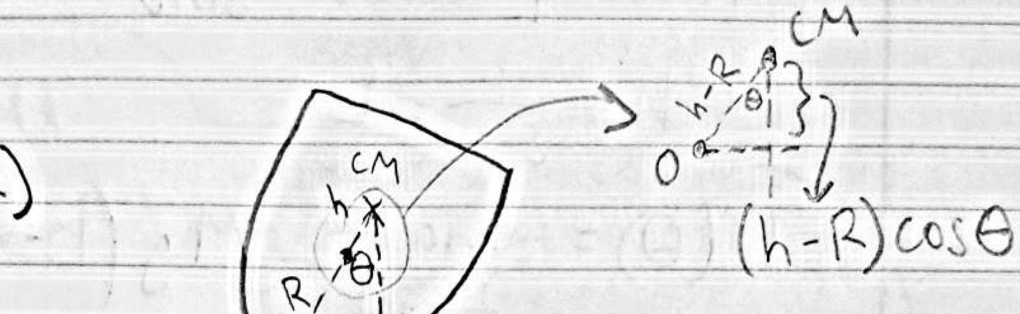
$$90° = \frac{\pi}{2}$$

$$0.5 - \sin\left(\frac{\pi}{2} - \theta\right) \cdot 0.5$$

$$\omega = \frac{v}{r}$$

4.30

a)



The center point does not change height as the toy tilts

The CM has same height as O at $\theta = \pi/2$

CM starts w/ height h, or $R+(h-R)$
ends w/ $R+(h-h)$

$PE = m_{cm} g h$

$$\boxed{PE = m_{cm} g \left(R + (h-R)\cos\theta\right)}$$

b) $\frac{dU}{d\theta} = 0$ for stability, and $\frac{d^2U}{d\theta^2}$ is positive

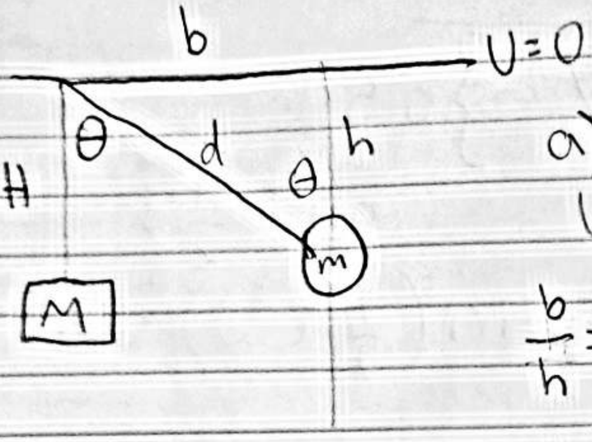
$$\frac{dU}{d\theta} = -gm(h-R)\sin(\theta)$$

which is always 0 at $\theta = 0$.

$$\frac{d^2U}{d\theta^2} = -gm(h-R)\cos(\theta) \quad \text{at } \theta = 0 \rightarrow -gm(h-R)$$

The toy is stable at $\theta = 0$ when $\boxed{R > h}$

4.36



a)

$$U = -mgh - MgH$$

$$\frac{b}{h} = \tan\theta \qquad h = \frac{b}{\tan\theta}$$

$$H + d = L \qquad d = \frac{b}{\sin\theta}$$

$$H + \frac{b}{\sin\theta} = L$$

$$U(\theta) = -\frac{mgb}{\tan\theta} - Mg\left(L - \frac{b}{\sin\theta}\right)$$

where m, g, b, M, L are constants

$$\frac{dU}{d\theta} = -bgM\cot(\theta)\csc(\theta) + bgm\csc^2(\theta)$$

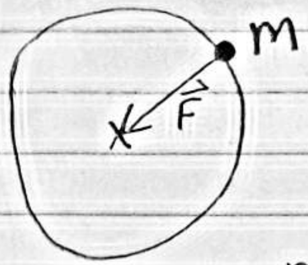
$$\text{theta} \to \arctan^{-1}\left[\frac{m}{M}, -\frac{\sqrt{-m^2+M^2}}{M}\right] + 2\pi n$$ where $n \in \mathbb{Z}$

It appears, according to mathematica, that m must be smaller than M for an equilibrium solution.

I don't think we can have stability in this situation.

3

4.41



$$E = U + T \qquad E = kr^n + T$$

$$\frac{dU}{dr} + \frac{dT}{dr} = 0 \qquad nkr^{n-1} + \frac{dT}{dr} = 0$$

$$T = \frac{1}{2} m r^2 \omega^2$$

$$F = -\nabla U$$

$$-nkr^{n-1} = \frac{mv^2}{r} \qquad \frac{nkr^n}{m} = v^2$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} nkr^n = \frac{1}{2} U n$$