HW #29 CID: 6265

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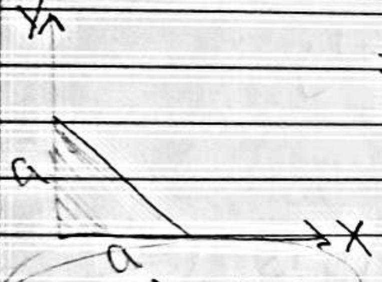


$$\vec{I} = \frac{ma^2}{12}\begin{pmatrix}2 & -1 & 0\\ -1 & 2 & 0\\ 0 & 0 & 4\end{pmatrix}$$

$$\vec{I} - \vec{1}\lambda = \begin{pmatrix}2\mu-\lambda & -\mu & 0\\ -\mu & 2\mu-\lambda & 0\\ 0 & 0 & 4\mu-\lambda\end{pmatrix}$$

Say $\mu = \frac{ma^2}{12}$

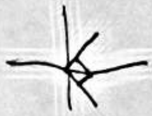
$$\det(\vec{I} - \vec{1}\lambda) = 0$$

$$= -\lambda^3 + 8\lambda^2\mu - 19\lambda\mu^2 + 12\mu^3 = 0$$

$$\lambda_1 = \mu \quad \lambda_2 = 3\mu \quad \lambda_3 = 4\mu$$

$$(2\mu-\lambda)\left[(2\mu-\lambda)(4\mu-\lambda)\right] + \mu\left[\left[\lambda\mu - 4\mu^2\right]\right]$$

$$(\vec{I} - \lambda\vec{1})\vec{\omega} = 0$$



λ_1 : $\left[\mu\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & 0\\ 0 & 0 & 4\end{bmatrix} - \begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_1 & 0\\ 0 & 0 & \lambda_1\end{bmatrix}\right]\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} = 0$

$= \cancel{\mu}\begin{bmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = 0$

$\omega_x - \omega_y = 0 \qquad \omega_x = \omega_y$

$-\omega_x + \omega_y = 0$

$3\omega_z = 0 \qquad \omega_z = 0$

Thus the principal axis is along $\langle 1, 1, 0\rangle$ with principal moment $\lambda_1 = \mu = \frac{ma^2}{12}$

λ_2 : $\mu\begin{bmatrix}-1 & -1 & 0\\ -1 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = 0$

$-\omega_x - \omega_y = 0 \qquad \omega_x = -\omega_y$

$-\omega_x - \omega_y = 0$

$\omega_z = 0 \qquad \omega_z = 0$

Thus the 2nd principal axis is along $\langle -1, 1, 0\rangle$ with principal moment $\lambda_2 = 3\mu = \frac{ma^2}{4}$

λ_3 : $\mu\begin{bmatrix}-2 & -1 & 0\\ -1 & -2 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} = 0$

$-2\omega_1 - \omega_2 = 0 \qquad \omega_3 = t$

$-\omega_1 - 2\omega_2 = 0 \qquad \omega_1 = 0 = \omega_2$

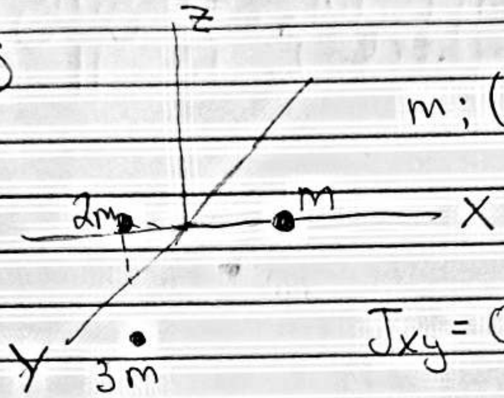
$\vec{\omega} = t\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$

Thus the 3rd principal axis is along $\langle 0, 0, 1\rangle$ w/

eigenvalue (principal moment) $\lambda_3 = 4\mu = \frac{ma^2}{3}$

thus $\vec{I}' = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \frac{ma^2}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

2 10.35



m; $(a,0,0)$ 2m: $(0,a,a)$
3m: $(0,a,-a)$

$J_{xy} = 0$ $J_{xz} = 0$

$J_{yz} = 2ma^2 - 3ma^2 = -ma^2$

$J_{xx} = ma^2$ $J_{yy} = 2ma^2 + 3ma^2 = 5ma^2$
$J_{zz} = 2ma^2 + 3ma^2 = 5ma^2$

$\vec{I} = \begin{bmatrix} 10ma^2 & 0 & 0 \\ 0 & 6ma^2 & ma^2 \\ 0 & ma^2 & 6ma^2 \end{bmatrix} = \mu \begin{bmatrix} 10 & 0 & 0 \\ 0 & 6 & 1 \\ 0 & 1 & 6 \end{bmatrix}$

use mathematica to find eigensystem

we get $\lambda_1 = 10\mu$ $\lambda_2 = 7\mu$ $\lambda_3 = 5\mu$

$\downarrow$ $\downarrow$ $\downarrow$

$\langle 1,0,0 \rangle$ $\langle 0,1,1 \rangle$ $\langle 0,-1,1 \rangle$

As our 3 principle moments and axes.
(which are orthogonal)

3) These are the axes which permit $\vec{L}$ to be parallel to $\vec{\omega}$ if we set any one of them as our rotational axis. we have seen that in general $\vec{L}$ is not parallel to $\vec{\omega}$, so principal axes are useful for studying the unique case where $\vec{L}$ is $\parallel$ to $\vec{\omega}$.