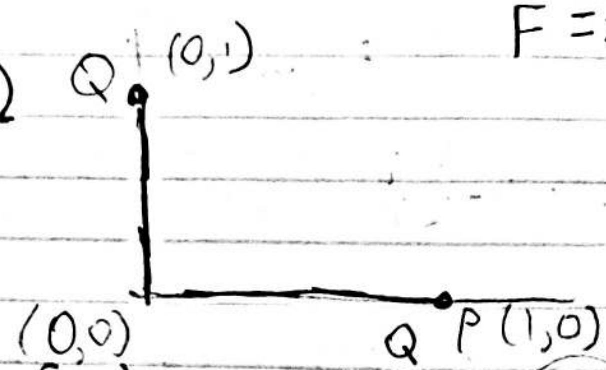
HW 7

(4.3)

$\vec{F} = \langle -y, x \rangle$

a)



$$W = \int_P^{Q} \vec{F} \cdot d\vec{r} + \int_{(0,0)}^{Q} \vec{F} \cdot d\vec{r} \rightarrow d\vec{r} = \langle 0, dy, 0 \rangle$$

$$d\vec{r} = \langle dx, 0, 0 \rangle$$

$$\int_P^{(0,0)} \langle -y, x, 0 \rangle \cdot \langle dx, 0, 0 \rangle = \int_{(0,1)}^{(0,0)} -y \, dx$$

$$= -yx \Big|_{(0,1)}^{(0,0)} = 0 + 0 = \underline{0}$$

$$\int_{(0,0)}^{(1,0)} \langle -y, x \rangle \cdot \langle 0, dy \rangle = \int_{(0,0)}^{(1,0)} x \, dy = xy \Big|_{(0,0)}^{(1,0)}$$

$$= \underline{0 - 0 = 0}$$

$$\boxed{W = 0}$$

b) $y = 1 - x \rightarrow \frac{dy}{dx} = \frac{d}{dx}(1-x) \rightarrow dy = -dx$

$$\int_{+1}^{0} \langle -y, x\rangle \cdot \langle dx, dy\rangle = \int_{0}^{0} -y\,dx + x\,dy$$

$$\int_{1}^{0} -y\,dx - x\,dx = \int_{1}^{0} (1-x)dx - x\,dx$$

$$= \int_{1}^{0} -1\,dx = -x \Big|_{1}^{0} = 0 + 1 = \boxed{1}$$

c) $\vec{r} = \langle \cos\theta, \sin\theta \rangle \quad \vec{F} = \langle -\sin\theta, \cos\theta \rangle$

and $d\vec{r} = \langle -\sin\theta, \cos\theta \rangle d\theta$

so

$$W = \int_{0}^{\pi/2} \langle -\sin\theta, \cos\theta \rangle \cdot \langle -\sin\theta, \cos\theta \rangle d\theta$$

$$= \int_{0}^{\pi/2} (\sin^2\theta + \cos^2\theta)\, d\theta$$

$$= \int_{0}^{\pi/2} d\theta = \boxed{\pi/2}$$

no torque, since force is directed down $\vec{r}$. Thus momentum is conserved.

4.4

a) $L_1 = L_2 \quad I_1\omega_1 = I_2\omega_2$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\cancel{m}r_1^2\omega_1}{\cancel{m}r_2^2}$$

$$\boxed{\omega_2 = \left(\frac{r_1}{r_2}\right)^2\omega_1}$$

b) work is done down to increase kinetic energy.

$$W = \int_{r_1}^{r_2} \vec{F}_T \cdot d\vec{r} \qquad \vec{F}_T = \frac{mv^2}{r}$$

$$v = r\omega$$

$$\vec{F}_T = \frac{m\left(r\cdot\left(\frac{r_1}{r}\right)^2\right)^2\omega_1^2}{r}$$

$$= \frac{\cancel{r}mr_1^4\omega_1^2}{r^4}$$

$$W = -\int_{r_1}^{r_2} \frac{mr_1^4\omega_1^2}{r^3}dr = \left.\frac{mr_1^4\omega_1^2}{2r^2}\right|_{r_1}^{r_2}$$

since $\vec{F}$ is antiparallel to $d\vec{r}$

$$\boxed{\frac{mr_1^4\omega_1^2}{2r_2^2} - \frac{mr_1^2\omega_1^2}{2}}$$

c) $\frac{1}{2}I_1\omega_1^2 \rightarrow \frac{1}{2}I_2\omega_2^2$

$$\frac{1}{2}m_1r_2^2\omega_2^2 - \frac{1}{2}m_1r_1^2\omega_1^2 = \Delta KE$$

$$\frac{1}{2}m_1r_2^2\left(\frac{r_1}{r_2}\right)^4\omega_1^2 = \frac{m_1r_1^4\omega_1^2}{2r_2^2}$$

Thus $\boxed{\frac{m_1r_1^4\omega_1^2}{2r_2^2} - \frac{m_1r_1^2\omega_1^2}{2}}$

Which is the same answer as the work done.

4.8 $PE_0 = mgh$

$h = R$

$$PE = mgR\cos\theta$$

$$E = mgR = \frac{1}{2}mv^2 + MgR\cos\theta$$

$$mgR - MgR\cos\theta = \frac{1}{2}mv^2$$

$\vec{F} = m\vec{a}$ $\quad \vec{F} = \frac{mv^2}{R}$

The puck has Normal force and gravitational force

$$\vec{F} = \frac{mv^2}{R} = mg\cos\theta - N$$

$$mv^2 = Rmg\cos\theta - N\cdot R$$

$$2mg\cancel{R} - 2mg\cancel{R}\cos\theta = mg\cancel{R}\cos\theta - N\cdot\cancel{R}$$

$$N = mg(3\cos\theta - 2)$$

4.8 continued...

$$N = mg(3\cos\theta - 2)$$

we are looking for when N becomes zero, (where the Puck leaves the Sphere).

$$3\cos\theta - 2 = 0 \qquad \cos\theta = \frac{2}{3} \qquad \boxed{\theta = 48.19^\circ}$$

(measured from the top of sphere)