

(HW #26) CID 6265

① centrifugal force is directed behind me, pulling me straight backwards. The coriolis force will push me to my right if I lean forward.

② we have $\left(\frac{d^2 \vec{r}}{dt^2}\right)_{s_0} = \left(\frac{d}{dt}\right)_s \left[\left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega} \times \vec{r} \right] + \vec{\Omega} \times \left[\left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega} \times \vec{r} \right]$

extra term to find: $m \vec{r} \times \dot{\vec{\Omega}}$

Newton's 2nd Law: $m \left(\frac{d^2 \vec{r}}{dt^2}\right)_{s_0} = \vec{F}$

$$\left(\frac{d^2 \vec{r}}{dt^2}\right)_{s_0} = \left(\frac{d^2 \vec{r}}{dt^2}\right)_s + \underbrace{\left(\frac{d}{dt}\right)_s \vec{\Omega} \times \vec{r}}_{\dot{\vec{\Omega}} \times \vec{r}} + \underbrace{\vec{\Omega} \times \left(\frac{d}{dt}\right)_s \vec{r}}_{\vec{\Omega} \times \dot{\vec{r}}} + \underbrace{\vec{\Omega} \times \left[\left(\frac{d\vec{r}}{dt}\right)_s + \vec{\Omega} \times \vec{r} \right]}_{\vec{\Omega} \times \dot{\vec{r}}}$$

$$\left(\frac{d^2 \vec{r}}{dt^2}\right)_{s_0} = \left(\frac{d^2 \vec{r}}{dt^2}\right)_s + 2 \vec{\Omega} \times \dot{\vec{r}} + \underbrace{\dot{\vec{\Omega}} \times \vec{r}}_{\text{circumferential}} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

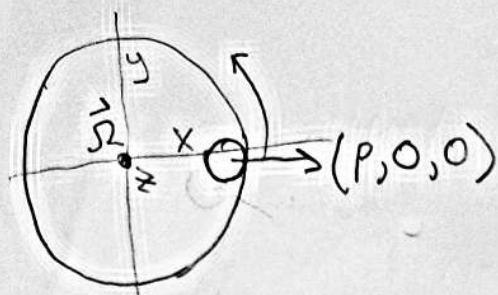
Plug into $m \left(\frac{d^2 \vec{r}}{dt^2}\right)_{s_0} = \vec{F}$

$$m \ddot{\vec{r}} = \vec{F} + 2m \dot{\vec{r}} \times \vec{\Omega} + m(\dot{\vec{\Omega}} \times \vec{r}) \times \vec{\Omega} + m \vec{r} \times \dot{\vec{\Omega}}$$

when subbed in and moved to right.

where $\ddot{\vec{r}} = \left(\frac{d^2 \vec{r}}{dt^2}\right)_s$

3



3x3 3x1

$$A) \quad \vec{r}_s = R^{-1} \vec{r}_{s_0} = \begin{bmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{\vec{r}}_s = \begin{bmatrix} -\Omega p \sin \Omega t \\ \Omega p \cos \Omega t \\ 0 \end{bmatrix} = \begin{bmatrix} p \cos \Omega t \\ -p \sin \Omega t \\ 0 \end{bmatrix} = \vec{r}_s$$

$$\ddot{\vec{r}}_s = \begin{bmatrix} -\Omega^2 p \cos \Omega t \\ \Omega^2 p \sin \Omega t \\ 0 \end{bmatrix}$$

$$\vec{\Omega} = \langle 0, 0, \Omega \rangle$$

B) Both!

$$C) \quad F_{\text{cor}} = 2m \dot{\vec{r}} \times \vec{\Omega} = 2m \begin{vmatrix} -\Omega p \sin \Omega t & \Omega p \cos \Omega t & 0 \\ 0 & 0 & \Omega \end{vmatrix}$$

$$= 2m \langle -p \Omega^2 \cos \Omega t, +p \Omega^2 \sin \Omega t, 0 \rangle = F_{\text{cor}}$$

$$F_{\text{cent}} = m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} = m \begin{vmatrix} 0 & 0 & \Omega \\ p \cos \Omega t & -p \sin \Omega t & 0 \end{vmatrix} \times \vec{\Omega}$$

$$= m \begin{vmatrix} p \Omega \sin \Omega t & \Omega p \cos \Omega t & 0 \\ 0 & 0 & \Omega \end{vmatrix}$$

$$= m \langle \Omega^2 p \cos \Omega t, -p \Omega^2 \sin \Omega t, 0 \rangle = F_{\text{cent.}}$$

$$\begin{bmatrix} 2m(-p\Omega^2 \cos \Omega t) \\ 2m(p\Omega^2 \sin \Omega t) \\ 0 \end{bmatrix} + \begin{bmatrix} m(+p\Omega^2 \cos \Omega t) \\ m(-p\Omega^2 \sin \Omega t) \\ 0 \end{bmatrix} = \begin{bmatrix} -m(p\Omega^2 \cos \Omega t) \\ m(p\Omega^2 \sin \Omega t) \\ 0 \end{bmatrix}$$

$$= m \begin{bmatrix} -(p\Omega^2 \cos \Omega t) \\ (p\Omega^2 \sin \Omega t) \\ 0 \end{bmatrix} = m \ddot{\vec{r}}_s = F_{\text{cent}} + F_{\text{cor}}$$