(1) $F=-kx \qquad U=\frac{1}{2}kx^2$

$$E=\frac{1}{2}kx_1^2+\frac{1}{2}mv_1^2=\frac{1}{2}kx_2^2+\frac{1}{2}mv_2^2$$

$$k\left(\cancel{\frac{1}{2}}x_1^2-\cancel{\frac{1}{2}}x_2^2\right)=\cancel{\frac{1}{2}}mv_2^2-\cancel{\frac{1}{2}}mv_1^2$$

$$k=\frac{mv_2^2-mv_1^2}{x_1^2-x_2^2}$$

$$\omega=\sqrt{\frac{k}{m}}$$

$$\boxed{\omega=\sqrt{\frac{v_2^2-v_1^2}{x_1^2-x_2^2}}}$$

$$\boxed{A=\sqrt{\frac{x_2^2v_1^2-x_1^2v_2^2}{v_1^2-v_2^2}}}$$

$$E=\frac{1}{2}kA^2$$

$$\cancel{\frac{1}{2}}kA^2=\cancel{\frac{1}{2}}kx_2^2+\cancel{\frac{1}{2}}mv_2^2$$

$$A=\mathrm{Sqrt}\left[x_2^2+\frac{mv_2^2}{k}\right]$$

$$\frac{\cancel{m}v_2^2}{1}\cdot\frac{x_1^2-x_2^2}{mv_2^2-\cancel{m}v_1^2}=\frac{v_2^2(x_1^2-x_2^2)}{v_2^2-v_1^2}$$

$$\frac{x_2^2(v_2^2-v_1^2)}{v_2^2-v_1^2}+\frac{v_2^2(x_1^2-x_2^2)}{v_2^2-v_1^2}$$

$$=\frac{v_2^2x_1^2-v_1^2x_2^2}{v_2^2-v_1^2}$$

$$=\frac{x_2^2v_1^2-x_1^2v_2^2}{v_1^2-v_2^2}$$

(2) $U(r) = U_0\left(\frac{r}{R} + \lambda^2 \frac{R}{r}\right)$

Equilibrium found when $\frac{dU}{dr} = 0$, no force.

$$\frac{dU}{dr} = \frac{U_0}{R} - \frac{U_0 \lambda^2 R}{r^2} = 0$$

$$\frac{\cancel{U_0}}{R} = \frac{\cancel{U_0}\lambda^2 R}{r^2} \qquad \frac{r^2}{R} = \lambda^2 R$$

$$r^2 = \lambda^2 R^2$$

$$\boxed{r_0 = \lambda R}$$

$x = r - r_0$ = distance from Equilibrium

For small x, taylor expansion about $x = 0$

$$U(x) = U_0\left(\frac{x+\lambda R}{R} + \frac{\lambda^2 R}{x+\lambda R}\right)$$

$$U(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \qquad x_0 = 0$$

$$U(x) = \frac{U_0 x + U_0 \lambda R}{R} + \frac{U_0 \lambda^2 R}{x+\lambda R}$$

$$U'(x) = \frac{U_0}{R} - \frac{U_0 \lambda^2 R}{(x+\lambda R)^2} \qquad U''(x) = \frac{U_0 \lambda^2 R \cdot 2}{(x+\lambda R)^3}$$

$$U(x) = U(0) + U'(0)x + \frac{1}{2}U''(0)x^2$$

$$U(0) = U_0(\lambda + \lambda) = 2U_0\lambda = \text{const.}$$

$$U'(0) = \frac{U_0}{R} - \frac{U_0}{R} = 0$$

$$U''(0) = \frac{2U_0\lambda^2 R}{\lambda^3 R^3} = \frac{U_0 2}{\lambda R^2}$$

(2) continued...

$$U(x) = 2\lambda U_0 + \frac{U_0 2}{\lambda R^2} x^2$$

$$\boxed{U(x) = \text{const.} + \frac{1}{2}\underbrace{\left(\frac{4U_0}{\lambda R^2}\right)}_{\text{"spring" constant}} x^2}$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$\omega = \sqrt{\frac{\frac{4U_0}{\lambda R^2}}{\frac{m}{1}}} = \sqrt{\frac{4U_0}{\lambda R^2} \cdot \frac{1}{m}} = \boxed{\sqrt{\frac{4U_0}{m\lambda R^2}} = \omega}$$