

Jacob Wright  
CS 312 Project 2

```
convex_hull.py > ConvexHullSolver > merge_hulls
You, 15 hours ago | 2 authors (You and others)
26 class ConvexHullSolver(QObject):
27
28     # Class constructor
29     def __init__(self):
30         super().__init__()
31         self.pause = False
32
33     # Some helper methods that make calls to the GUI, allowing us to send updates
34     # to be displayed.
35     def showTangent(self, line, color):
36         self.view.addLines(line, color)
37         if self.pause:
38             time.sleep(PAUSE)
39
40     def eraseTangent(self, line):
41         self.view.clearLines(line)
42
43     def blinkTangent(self, line, color):
44         self.showTangent(line, color)
45         self.eraseTangent(line)
46
47     def showHull(self, polygon, color):
48         self.view.addLines(polygon, color)
49         if self.pause:
50             time.sleep(PAUSE)
51
52     def eraseHull(self, polygon):
53         self.view.clearLines(polygon)
54
55     def showText(self, text):
56         self.view.displayStatusText(text)
57
58
59     def compute_hull(self, points, pause, view):
60         self.pause = pause
61         self.view = view
62         assert type(points) == list and type(points[0]) == QPointF
63
64         t1 = time.time()
65
66         # sort points by x coordinate
67         points.sort(key=lambda point:point.x())
68
69         t2 = time.time()
70         t3 = time.time()
71
72         hull = self.convex_hull_helper(points)
```

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58
59     def compute_hull(self, points, pause, view):
60         self.pause = pause
61         self.view = view
62         assert type(points) == list and type(points[0]) == QPointF
63
64         t1 = time.time()
65
66         # sort points by x coordinate
67         points.sort(key=lambda point:point.x())
68
69         t2 = time.time()
70         t3 = time.time()
71
72         hull = self.convex_hull_helper(points)
73
74         polygon = [QLineF(hull[i], hull[(i + 1) % len(hull)]) for i in range(0, len(hull))]
75
76         t4 = time.time()
77
78         # when passing lines to the display, pass a list of QLineF objects. Each QLineF
79         # object can be created with two QPointF objects corresponding to the endpoints
80         self.showHull(polygon, RED)
81         self.showText("Time Elapsed (Convex Hull): {:.3f} sec".format(t4 - t3))
82
83
84     # recursive helper function for computing the convex hull
85     def convex_hull_helper(self, points):
86         # base case
87         if len(points) <= 2:
88             return points
89
90         # get left half of points and right half of points
91         left_points = points[:len(points) // 2]
92         right_points = points[len(points) // 2:]
93
94         # compute left and right hulls recursively
95         left_hull = self.convex_hull_helper(left_points)
96         right_hull = self.convex_hull_helper(right_points)
97
98         # merge the two hulls
99         return self.merge_hulls(left_hull, right_hull)
100
101
102     # merge two given hulls and return the merged hull
103     def merge_hulls(self, left_hull, right_hull):
104         # get leftmost and rightmost points
105         leftmost = 0
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102     # merge two given hulls and return the merged hull
103     def merge_hulls(self, left_hull, right_hull):
104         # get leftmost and rightmost points
105         leftmost = 0
106         rightmost = left_hull.index(max(left_hull, key=lambda point:point.x()))
107
108         # get points from upper and lower tangents
109         upper_right, upper_left = self.get_upper_tangent(leftmost, rightmost, left_hull, right_hull)
110         lower_right, lower_left = self.get_lower_tangent(leftmost, rightmost, left_hull, right_hull)
111
112         # create new hull to return
113         merged_hull = []
114
115         # add points from in left_hull from x=0 to upper left point
116         for x in range(0, upper_left):
117             merged_hull.append(left_hull[x])
118         merged_hull.append(left_hull[upper_left])
119
120         i = 0      You, 17 hours ago • FEAT: works, but is keeping all lines? ...
121         curr = upper_right
122
123         # add points from right_hull from upper right point to lower right point
124         while i < len(right_hull):
125             merged_hull.append(right_hull[curr])
126             if curr == lower_right:
127                 break
128             i += 1
129             curr = (curr + 1) % len(right_hull)
130
131         # add points from in left_hull from lower left point to x=0
132         if lower_left != upper_left and lower_left != 0:
133             for x in range(lower_left, len(left_hull)):
134                 merged_hull.append(left_hull[x])
135         return merged_hull
136
137
138     # get upper tangent between two hulls and return two points, one from each hull
139     def get_upper_tangent(self, leftmost, rightmost, left_hull, right_hull):
140         # start upper tangent points at leftmost and rightmost
141         upper_left = leftmost
142         upper_right = rightmost
143
144         # From pseudocode from slides
145         done = False
146         while not done:
147             next_left = self.max_slope_clockwise(left_hull[upper_right], upper_left, right_hull)
148             next_right = self.max_slope_counter_clockwise(right_hull[next_left], upper_right, left_hull)
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138     # get upper tangent between two hulls and return two points, one from each hull
139     def get_upper_tangent(self, leftmost, rightmost, left_hull, right_hull):
140         # start upper tangent points at leftmost and rightmost
141         upper_left = leftmost
142         upper_right = rightmost
143
144         # From pseudocode from slides
145         done = False
146         while not done:
147             next_left = self.max_slope_clockwise(left_hull[upper_right], upper_left, right_hull)
148             next_right = self.max_slope_counter_clockwise(right_hull[next_left], upper_right, left_hull)
149
150             # if the next left and right points are the same as the current left and right points
151             if next_left == upper_left and next_right == upper_right:
152                 done = True
153             else:
154                 upper_left = next_left
155                 upper_right = next_right
156
157         return upper_left, upper_right
158
159
160     # get lower tangent between two hulls and return two points, one from each hull
161     def get_lower_tangent(self, leftmost, rightmost, left_hull, right_hull):
162         # start lower tangent points at leftmost and rightmost
163         lower_left = leftmost
164         lower_right = rightmost
165
166         # From pseudocode from slides
167         done = False
168         while not done:
169             next_left = self.max_slope_counter_clockwise(left_hull[lower_right], lower_left, right_hull)
170             next_right = self.max_slope_clockwise(right_hull[next_left], lower_right, left_hull)
171
172             # if the next left and right points are the same as the current left and right points
173             if next_left == lower_left and next_right == lower_right:
174                 done = True
175             else:
176                 lower_left = next_left
177                 lower_right = next_right
178
179         return lower_left, lower_right
180
181
182     # return the index of the point in hull that has the maximum slope with respect to starting point
183     def max_slope_clockwise(self, right_best_point, start, hull):
184         best_slope_index = start
185         for i in range(1, len(hull)):
186             slope = self.slope(hull[start], hull[i])
187             best_slope_index = i if slope > self.slope(hull[start], hull[best_slope_index]) else best_slope_index
188         return best_slope_index
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180
181
182     # return the index of the point in hull that has the maximum slope with respect to starting point
183     def max_slope_clockwise(self, right_best_point, start, hull):
184         best_slope_index = start
185         prev_slope = None
186         curr = start
187
188         while True:
189             curr_slope = (hull[curr].y() - right_best_point.y()) / (hull[curr].x() - right_best_point.x())
190             if prev_slope == None or curr_slope > prev_slope:
191                 prev_slope = curr_slope
192                 best_slope_index = curr
193             else:
194                 best_slope_index = (curr - 1) % len(hull)
195                 break
196             curr = (curr + 1) % len(hull)
197         return best_slope_index
198
199
200     # return the index of the point in hull that has the maximum slope with respect to starting point
201     def max_slope_counter_clockwise(self, left_best_point, start, hull):
202         best_slope_index = start
203         prev_slope = None
204         curr = start
205
206         while True:
207             curr_slope = (hull[curr].y() - left_best_point.y()) / (hull[curr].x() - left_best_point.x())
208             if prev_slope == None or curr_slope < prev_slope:
209                 prev_slope = curr_slope
210                 best_slope_index = curr
211             else:
212                 best_slope_index = (curr + 1) % len(hull)
213                 break
214             curr = (curr - 1) % len(hull)
215         return best_slope_index
```

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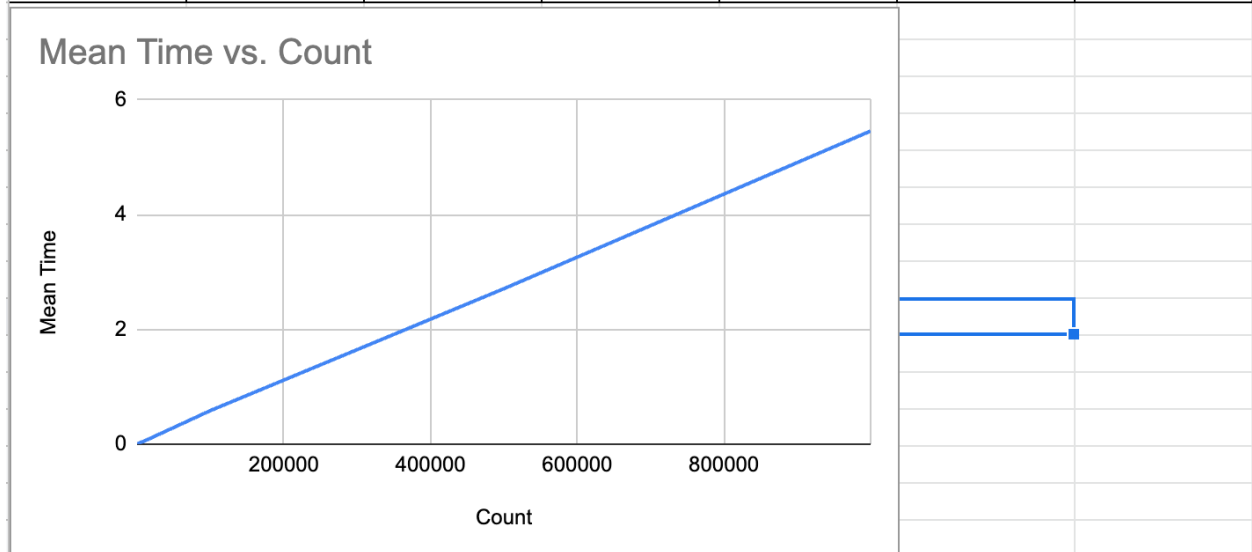
2. My solution for convex hull can be broken into the following pieces:

- `convex_hull_helper`: recursive function call to compute hull
  - Constant time, runs  $\log(n)$  times
    - Calls `merge_hulls`
  - $O(\log(n))$  time complexity
- `merge_hulls`
  - Calls `get_upper_tangent` & `get_lower_tangent`
  - Worst case, iterates over  $n$  points in left & right hulls to add to convex hull
  - $O(n)$  time complexity
- `get_upper_tangent` and `get_lower_tangent`
  - iterates over points in left and right hull and finds upper/lower tangent respectively
  - calls `max_slope`
    - gets max/best slope from given starting point for the tangent line
  - Worst case, iterates over  $N$  points in left or right hull and calls `max_slope` each iteration
  - Runs  $O(n)$  times and calls `max_slope` which at worst case has a time complexity of  $O(n)$
  - Thus, `get_upper_tangent` and `get_lower_tangent` run in  $O(n^2)$  worst case (very very rare).
- `Max_slope` (clockwise and counter clockwise)
  - Iterates over points in given hull and finds best slope (highest or lowest) for respective tangent using a given point
  - Absolute worst case, runs in  $O(N)$  time complexity if it iterated over every single point in the hull. (however, since we are using the divide and conquer algorithm we end up iterating over just a few nodes, making it much less than  $O(n)$  in practice.

Overall, using the D&C algorithm, we get  $O(n \log(n))$ , however, using the smaller numbers we are using (7 digits and less), we can expect nearly linear times.

3.

Count	Time 1	Time 2	Time 3	Time 4	Time 5	Mean Time
10	0.001	0	0	0	0	0.0002
100	0.002	0.002	0.002	0.002	0.002	0.002
1000	0.011	0.011	0.011	0.011	0.011	0.011
10000	0.055	0.054	0.055	0.055	0.055	0.0548
100000	0.585	0.587	0.582	0.591	0.583	0.5856
500000	2.707	2.698	2.718	2.709	2.714	2.7092
1000000	5.46	5.449	5.463	5.458	5.457	5.4574



I did not use a logarithmic graph, but if I did then it would make the linear relationship look like a logarithmic relationship. However, the best relationship is linear. This can be seen by inspection, and also calculated the ratio from 500,000  $\rightarrow$  100,000 = 2.7s  $\rightarrow$  5.4s (both growth 2x).

4. As mentioned above, there are some discrepancies from the theoretical and empirical analysis. Theoretically, it should be  $O(n \log(n))$ , however, in practice, with numbers 7 digits and less, it is essentially linear.

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5. (Green dots cause it's a lot easier to see than blue LOL)

