Qubits And Quantum Computers

QuTe - Copenhagen

Erasmus+

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Ready, set, go!

Grab your *Qubits and Quantum Computers* notes and follow along!

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Bits and Qubits

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Bits

- Two state system

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Bits

- Two state system
- heads or tails

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Bits

- Two state system
- heads or tails
- up or down

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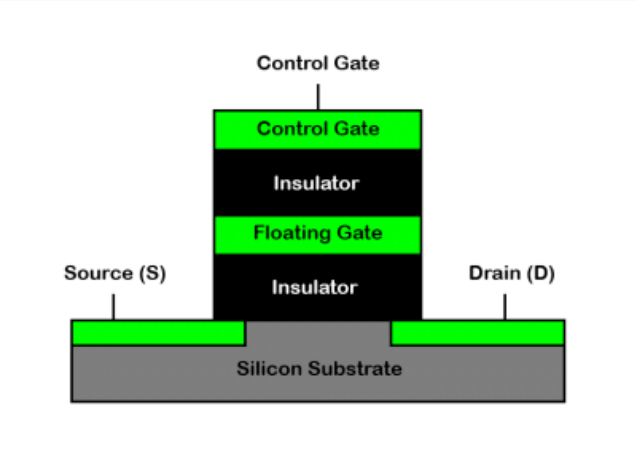
Bits

- Two state system
- heads or tails
- up or down
- charge or no charge



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Bits

- Two state system
- heads or tails
- up or down
- charge or no charge
- 0 or 1

0 1

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Bits

Exercise 1 p. 2 in the notes

How many possible combinations are possible with 4 bits? Same question but with 10 bits and with n bits?

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A classical computer

How many of the numbers are greater than 4?

8	2	0	3	1	4	5	8	2	1	7	2	7	3	8	5	5	2	9	0	6	3	1	6	4
0	8	7	3	3	1	9	7	5	2	5	7	6	9	8	0	3	6	2	5	1	2	7	5	2
2	3	3	8	6	1	4	2	4	0	2	6	1	8	9	5	2	6	9	8	3	4	0	1	0
4	7	5	5	6	3	0	7	7	1	9	1	6	1	7	4	1	7	1	3	7	9	3	3	7
1	9	3	9	5	3	4	9	5	5	2	7	5	8	0	3	4	8	8	1	2	7	5	3	4
2	8	7	8	1	4	1	4	9	4	2	4	1	5	2	9	4	6	2	1	5	2	8	1	9
8	4	8	5	1	3	9	6	6	0	7	2	1	9	0	2	0	6	7	0	6	0	1	3	0
0	3	8	8	4	7	5	1	5	1	7	3	4	5	2	0	7	4	7	9	6	6	7	7	4
3	5	3	1	9	3	7	4	9	5	0	2	0	1	4	6	2	5	4	5	8	5	0	9	2
3	4	5	9	5	2	7	9	8	9	0	5	5	8	5	1	7	7	3	5	5	4	7	7	2
4	1	5	3	0	9	1	3	7	2	5	8	7	7	1	3	6	3	9	7	8	7	9	1	7
7	2	9	5	6	7	8	5	4	5	3	4	5	4	1	9	8	6	7	5	7	9	3	1	8
5	9	2	8	9	8	6	4	4	1	5	3	7	7	0	8	0	2	5	6	0	6	1	2	0
1	3	3	3	9	0	5	2	8	7	4	0	9	0	3	7	3	1	7	9	4	5	5	2	8
4	6	0	1	0	8	6	2	1	0	0	5	0	3	1	5	4	9	0	3	7	4	7	0	1
7	7	0	6	6	3	2	8	8	5	8	9	5	6	4	0	5	9	1	8	0	5	4	9	4
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1	7	5	9	0	0	2	0	5	6	5	8	5	1	9	5	3	3	7	4	0	5	8	2	4
0	3	9	6	9	4	7	3	5	7	0	6	5	4	7	1	1	8	5	3	2	8	0	9	8

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8	4	8	5	1	3	9	6	6	0	7	2	1	9	0	2	0	6	7	0	6	0	1	3	0
0	3	8	8	4	7	5	1	5	1	7	3	4	5	2	0	7	4	7	9	6	6	7	7	4
3	5	3	1	9	3	7	4	9	5	0	2	0	1	4	6	2	5	4	5	8	5	0	9	2
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4	1	5	3	0	9	1	3	7	2	5	8	7	7	1	3	6	3	9	7	8	7	9	1	7
7	2	9	5	6	7	8	5	4	5	3	4	5	4	1	9	8	6	7	5	7	9	3	1	8
5	9	2	8	9	8	6	4	4	1	5	3	7	7	0	8	0	2	5	6	0	6	1	2	0
1	3	3	3	9	0	5	2	8	7	4	0	9	0	3	7	3	1	7	9	4	5	5	2	8
4	6	0	1	0	8	6	2	1	0	0	5	0	3	1	5	4	9	0	3	7	4	7	0	1
7	7	0	6	6	3	2	8	8	5	8	9	5	6	4	0	5	9	1	8	0	5	4	9	4
3	3	8	5	7	5	7	4	3	4	5	7	9	6	9	5	0	7	7	6	6	8	8	5	9
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1	7	5	9	0	0	2	0	5	6	5	8	5	1	9	5	3	3	7	4	0	5	8	2	4
0	3	9	6	9	4	7	3	5	7	0	6	5	4	7	1	1	8	5	3	2	8	0	9	8

Perfect for a classical computer: One thing at a time, very fast!

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Unlock the bike

Exercise 2 p. 2 in the notes

The code cylinder on a bicycle lock is made up of 5 numbers between 0 and 9: How many possible combinations (i.e. how many codes) does the lock have?

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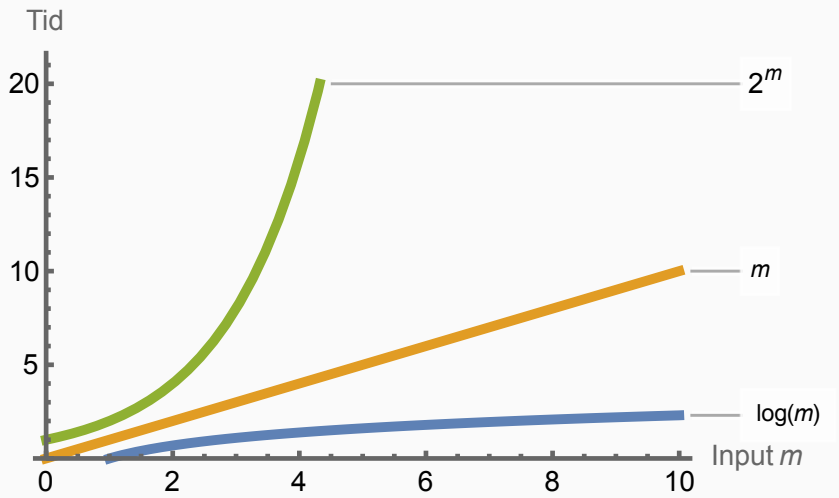
Complexity

Definition 1

Complexity of a calculation: the number of operations needed to find a solution to a given problem when the size of the input is m.



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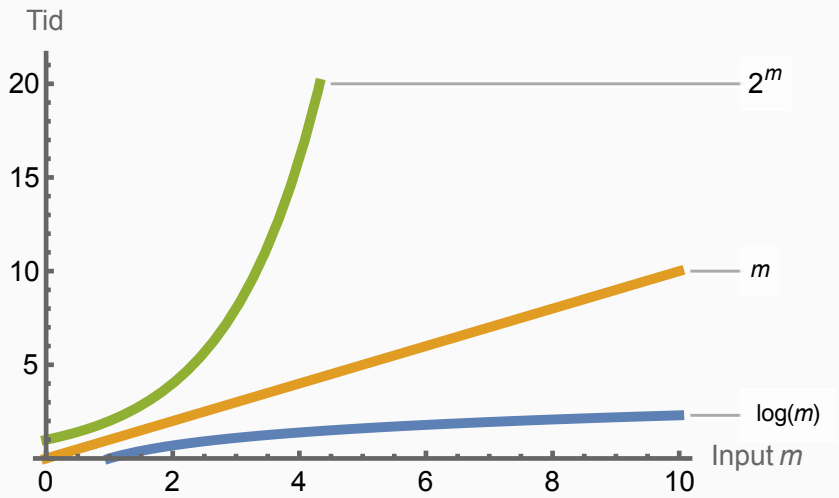

Often the problems increases *exponentially*.

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Exercises

Exercise 3 p. 3 in the notes

A classical supercomputer can perform 10^{17} operation each second. Vi use it to break a code lock like the one before, except with m digits instead of 5. Assume that the classical supercomputer can check 10^{17} different combinations on the lock each second. How large can m be if the classical supercomputer must be able to complete the task in a year?

Exercise 4 p. 3 in the notes

Estimate the time it would take you to try all of the possible combinations of the clothes (including accessories!) you choose between each morning.

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Qubits

- Qubits are *quantum systems*

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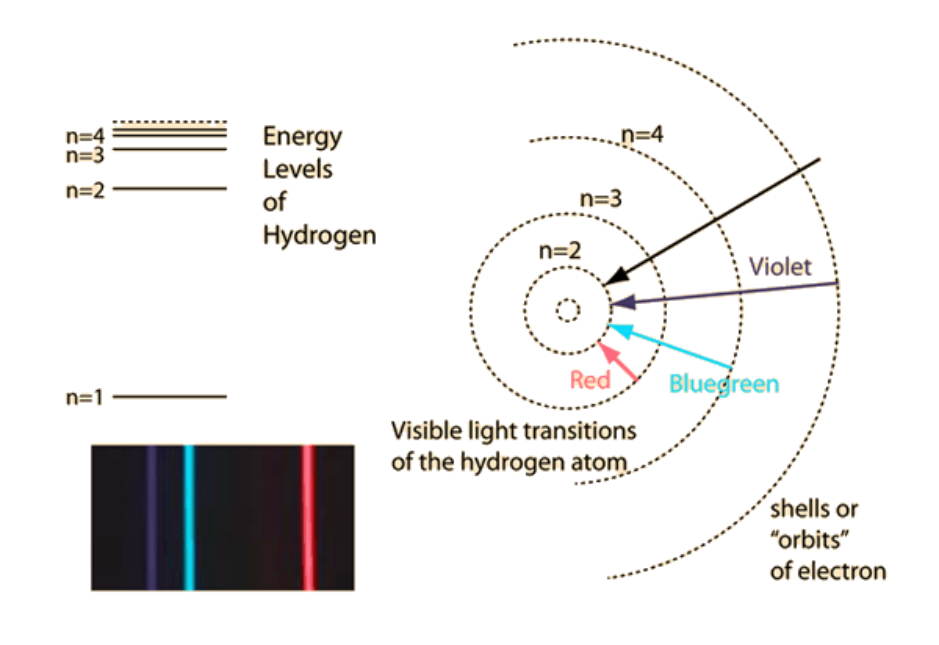
Qubits

- Qubits are *quantum systems*
- Like Bohr's atomic model



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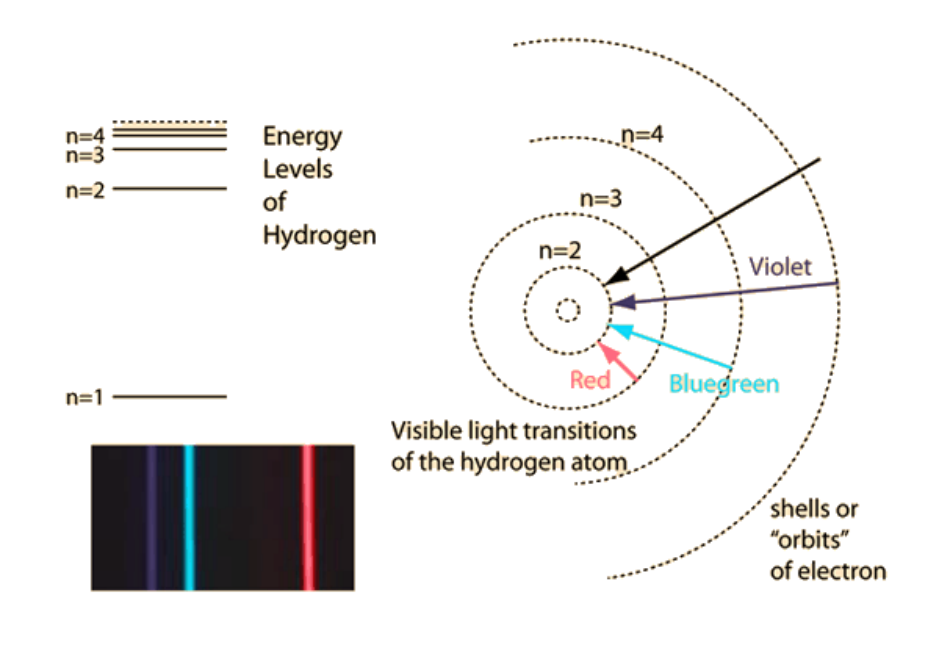
Qubits

- Qubits are *quantum systems*
- Like Bohr's atomic model
- Qubits are "simple". *Only 2 states*



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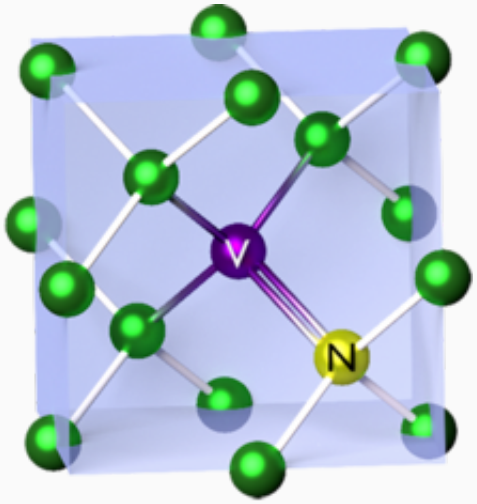
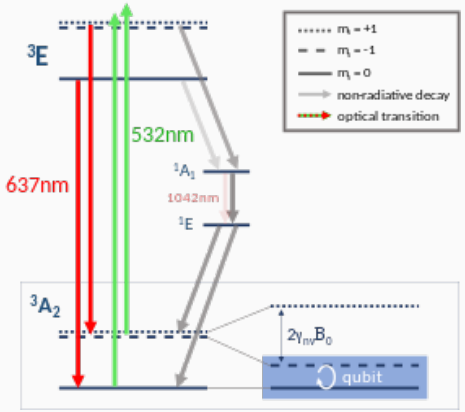
Qubits

- Qubits are *quantum systems*
- Like Bohr's atomic model
- Qubits are "simple". *Only 2 states*
- You know, "**real simple**", like NV Centers...





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Bits and Qubits

Bits

- Denoted 0 and 1
- If a bit is in state 0 we are sure to measure 0
- If a bit is in state 1 we are sure to measure 1
- Can *only* be in either state 0 or 1

Qubits

- Denoted $|0\rangle$ and $|1\rangle$
- If a bit is in state $|0\rangle$ we are sure to measure 0
- If a bit is in state $|1\rangle$ we are sure to measure 1
- Can be in either state $|0\rangle$, $|1\rangle$ **or in a superposition of them.**

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Bits and Qubits

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Funny notation

A quantum system i described by a state which we denote $|\ldots\rangle$.

The symbol $|\ldots\rangle$ is called a *ket*.

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Lingual oddities

- A ket, $|\ldots\rangle$, comes together with a

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Lingual oddities

- A ket, $|\ldots\rangle$, comes together with a
- *bra*, $\langle\ldots|$

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- A ket, $|\ldots\rangle$, comes together with a
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- Combining them we get a *bra(c)ket*

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Lingual oddities

- A ket, $|\ldots\rangle$, comes together with a
- *bra*, $\langle\ldots|$
- Combining them we get a *bra(c)ket*
- or bra(c)ket, $\langle\ldots|\ldots\rangle$, like in *parenthesis*

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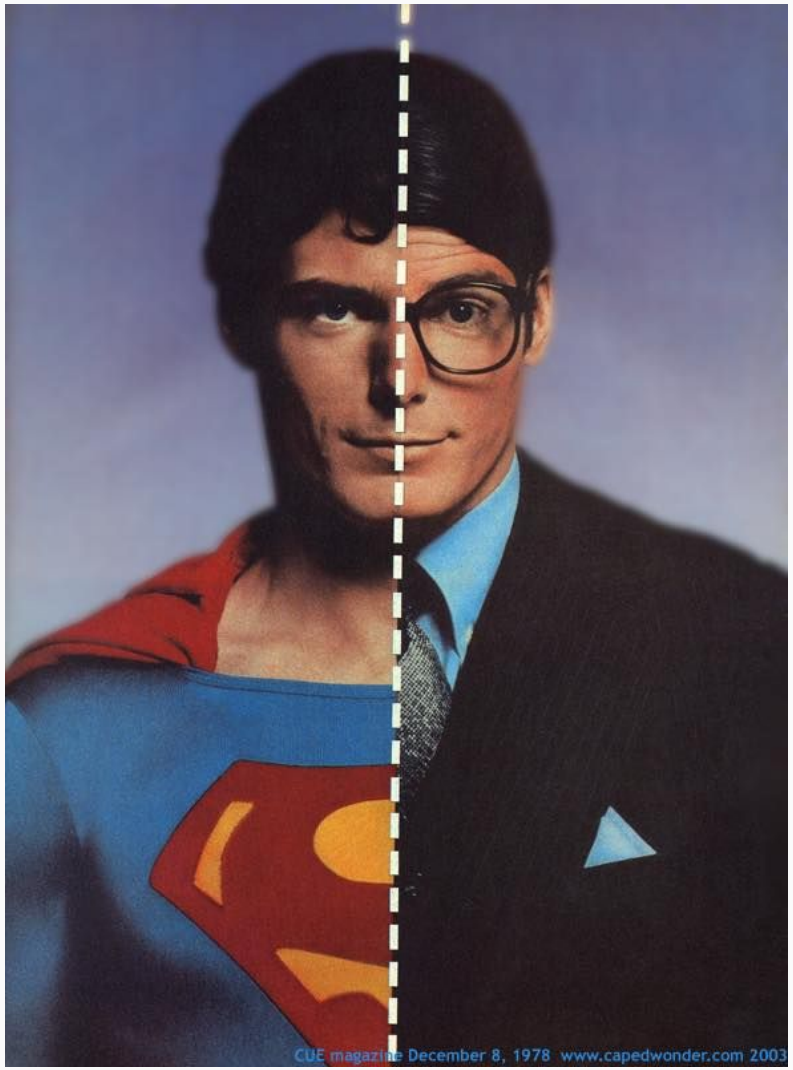
Superposition



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Single Qubit Superposition

A single qubit can be in either:

- state $|0\rangle$
- state $|1\rangle$

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A single qubit can be in either:

- state $|0\rangle$
- state $|1\rangle$
- or in a **superposition** $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

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Single Qubit Superposition

Or with a coin analogy

- state $|0\rangle$
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Superposition and normalisation

Definition 2: Superposition

$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

α and β can be choosen arbitrarily as long it is **normalised**.

Definition 3: Normalisation

The qubit state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ is normalised if $\alpha^2 + \beta^2 = 1$.

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The qubit state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ is normalised if $\alpha^2 + \beta^2 = 1$.

Read through the example and do exercise 5 and 6 on page 4 in the notes.

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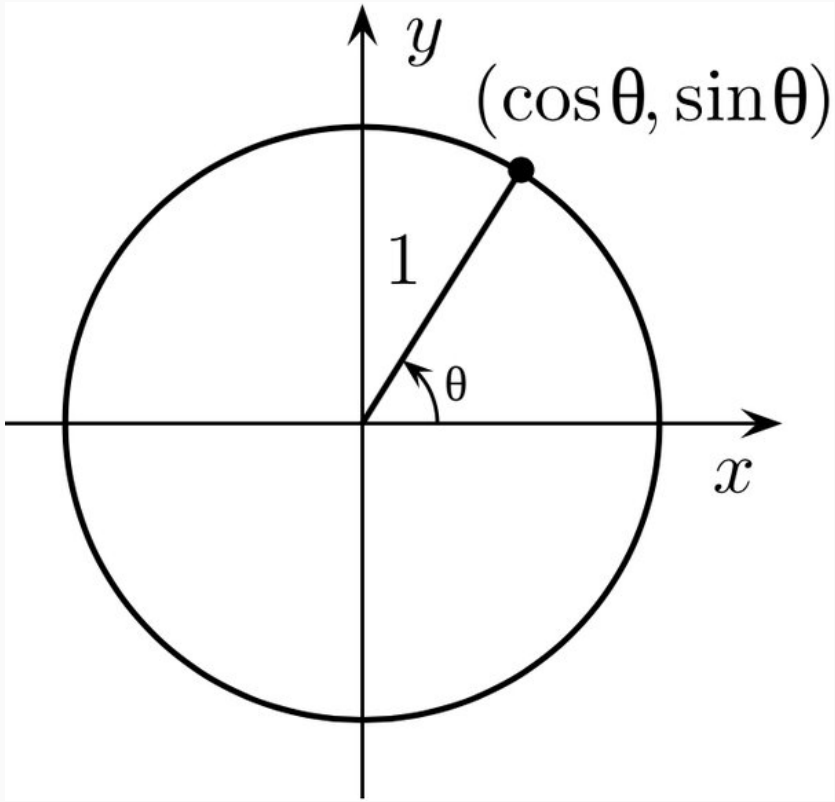
A unit vector

You can think of the kets as unit vectors.

- $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Unit vector along x-axis.
- $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ Unit vector along y-axis.
- $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ Unit vector pointing from (0,0) to (α, β) on the unit circle.



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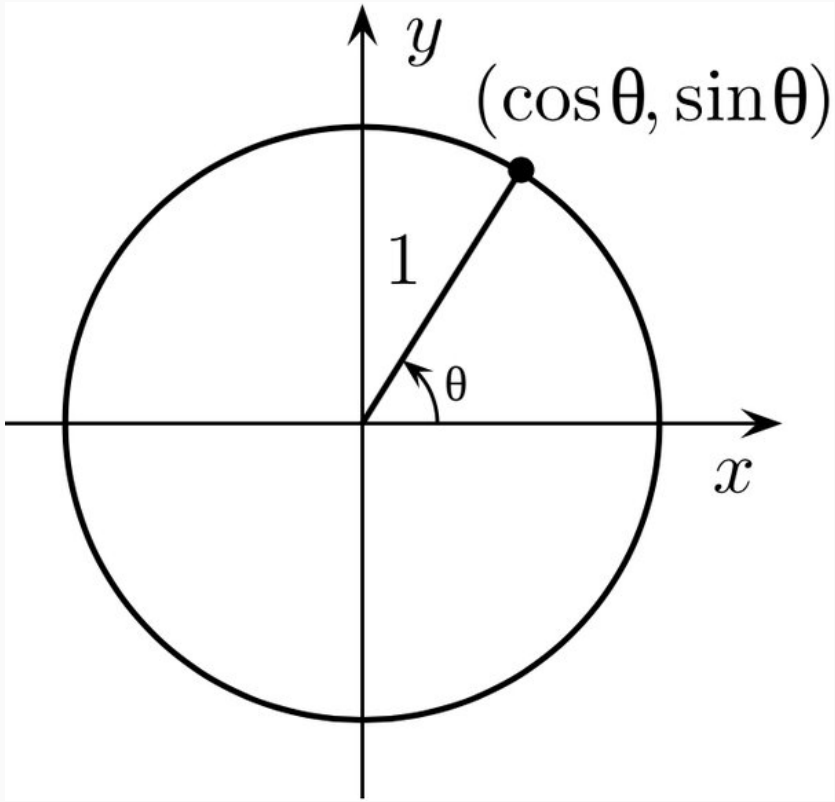
Do exercise 7 on page 4 in the notes.



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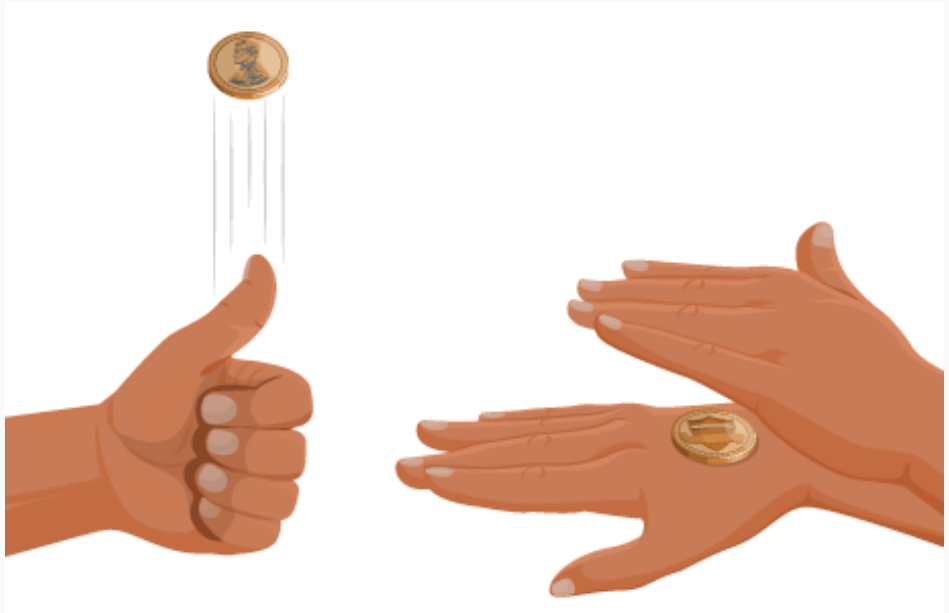
Measurements and Probabilities



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Measuring on $|\psi\rangle = |0\rangle$

- If the qubit is in the state $|\psi\rangle = |0\rangle$.

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Measuring on $|\psi\rangle = |0\rangle$

- If the qubit is in the state $|\psi\rangle = |0\rangle$.
- and the quantum computer performs a measurement.

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Measuring on $|\psi\rangle = |0\rangle$

- If the qubit is in the state $|\psi\rangle = |0\rangle$.
- and the quantum computer performs a measurement.
- we get 0 with 100% probability.

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Measuring on $|\psi\rangle = |1\rangle$

- If the qubit is in the state $|\psi\rangle = |1\rangle$.

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Measuring on $|\psi\rangle = |1\rangle$

- If the qubit is in the state $|\psi\rangle = |1\rangle$.
- and the quantum computer performs a measurement.

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Measuring on $|\psi\rangle = |1\rangle$

- If the qubit is in the state $|\psi\rangle = |1\rangle$.
- and the quantum computer performs a measurement.
- we get 1 with 100% probability.

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Measuring $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- If the qubit now is in the state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.

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Measuring $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- If the qubit now is in the state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.
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Measuring $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- If the qubit now is in the state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.
- and the quantum computer performs a measurement.
- we get **either** 0 or 1 and **never** something in between.

or

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Measuring $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- If the qubit now is in the state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.
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- we get **either** 0 or 1 and **never** something in between.
- We cannot predict the outcome. Only calculate the probabilities!

or

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Measuring $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- If the qubit now is in the state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.
- and the quantum computer performs a measurement.
- we get **either** 0 or 1 and **never** something in between.
- We cannot predict the outcome. Only calculate the probabilities!
- We have to abandon **determinism**.

or

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Probabilities

Definition 4: Probability

If our qubit is in the state $|\psi\rangle$ then the probability of measuring 0 is given by $P_0 = (\langle 0|\psi\rangle)^2$ and the probability of measuring 1 is given by $P_1 = (\langle 1|\psi\rangle)^2$.

Definition 5: Rules

$$\langle 0|0\rangle = \langle 1|1\rangle = 1 \quad \text{and} \quad \langle 1\rangle = \langle 1|0\rangle = 0$$

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$$\langle 0|0\rangle = \langle 1|1\rangle = 1 \quad \text{and} \quad \langle 1\rangle = \langle 1|0\rangle = 0$$

If you are into vectors, our rules above correspond to taking the dot product of two vectors. The rules $\langle 0|0\rangle = \langle 1|1\rangle = 1$ mean that the vectors $|0\rangle$ and $|1\rangle$ both have length 1, and the rules $\langle 0|1\rangle = \langle 1|0\rangle = 0$ mean that $|1\rangle$ and $|0\rangle$ are perpendicular.

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Time to work

Read through the examples, remarks and do the exercises on page 7 and 8 in the notes.

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Time to work

Read through the examples, remarks and do the exercises on page 7 and 8 in the notes.

When you are done, you should realise that:

Definition 6: Alternative to definition 4

Probability: If our qubit is in the state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ then the probability of measuring 0 is given by $P_0 = \alpha^2$ and the probability of measuring 1 by $P_1 = \beta^2$.

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The measurement affects the state

- A qubit is in a state $|\psi\rangle$.

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The measurement affects the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.

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The measurement affects the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.

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The measurement affects the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.
- We do the same all over again.

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The measurement affects the state

- A qubit is in a state $|\psi\rangle$.
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The measurement affects the state

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- and get 0.
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- We do a measurement.
- and get 1.

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The measurement affects the state

- A qubit is in a state $|\psi\rangle$.
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- and get 0.
- We do the same all over again.
- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 1.
- What can we say about the state $|\psi\rangle$?

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The measurement affects the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.
- We do the same all over again.
- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 1.
- What can we say about the state $|\psi\rangle$?
- Only that the probabilities P_0 and P_1 are non zero!

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.
- We do a measurement again.

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.
- We do a measurement again.
- and get 0.

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
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- We do a measurement again.
- and get 0.
- and again.
- and again.

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.
- We do a measurement again.
- and get 0.
- and again.
- and again.
- The initial state is collapsed.

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Collapse of the state

- A qubit is in a state $|\psi\rangle$.
- We do a measurement.
- and get 0.
- We do a measurement again.
- and get 0.
- and again.
- and again.
- The initial state is collapsed.

Definition 7

If we measure and the result is 0, then the state of the qubit immediately after the measurement is the state $|0\rangle$. Likewise, if the measurement result is 1, then the state of the qubits immediately after the measurement is the state $|1\rangle$.

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Time for some exercise

Do exercise 10 on page 8 in the notes.

Exercise 10

Show that the state $\frac{1}{5}(3\,|0\rangle + 4\,|1\rangle)$ is normalised and determine the probability, P_1, of measuring 1 if the qubit is initially in the state $|\psi\rangle = \frac{1}{5}(3\,|0\rangle + 4\,|1\rangle)$.

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Operators and Diagrams

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Single Qubit operators

We want to do something with the qubits! We do that with **operators**.

Definition 9: Operator

An operator acts on a state and returns a state.

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Doing nothing

$$|0\rangle \text{ ———— } |0\rangle$$

- A single qubit is drawn as a horizontal line.
- Each end indicate the state of the qubit.
- Read from left to right.

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The Z operator

Definition 10: The operator Z

$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

$$|0\rangle \;\text{——}\; \boxed{Z} \;\text{——}\; |0\rangle$$

$$|1\rangle \;\text{——}\; \boxed{Z} \;\text{——}\; -|1\rangle$$

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- An operator acts on a state from the *left*.

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- An operator acts on a state from the *left*.
- i.e. $Z|\psi\rangle = (|0\rangle\langle 0| - |1\rangle\langle 1|)|\psi\rangle$

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$$|1\rangle \;\text{——}\; \boxed{Z} \;\text{——}\; -|1\rangle$$

- An operator acts on a state from the *left*.
- i.e. $Z|\psi\rangle = (|0\rangle\langle 0| - |1\rangle\langle 1|)|\psi\rangle$
- What does Z do?

Exercise 15 in the notes.

Use the definition of Z, i.e. that $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$, and let it act on $|0\rangle$ and $|1\rangle$. Show that the result is the same as depicted in the figure.

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The X operator

Definition 11: The operator X

$X = |1\rangle\langle 0| + |0\rangle\langle 1|$

$$|0\rangle \;\text{——}\; \boxed{X} \;\text{——}\; |1\rangle$$

$$|1\rangle \;\text{——}\; \boxed{X} \;\text{——}\; |0\rangle$$

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The X operator

Definition 11: The operator X

$X = |1\rangle\langle 0| + |0\rangle\langle 1|$

$$|0\rangle \;\text{——}\; \boxed{X} \;\text{——}\; |1\rangle$$

$$|1\rangle \;\text{——}\; \boxed{X} \;\text{——}\; |0\rangle$$

What does the X operator do?

Exercise 16 in the notes.

Show what happen when X acts on the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. If you have studied Appendix 2, then note that you have just shown that $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is an eigenstate of X with eigenvalue 1!

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The Hadamard operator, H

Definition 12: The operator H

$$H = \frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)$$

$$|0\rangle \;\text{——}\boxed{H}\text{——}\quad \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|1\rangle \;\text{——}\boxed{H}\text{——}\quad \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

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The Hadamard operator, H

Definition 12: The operator H

$H = \frac{1}{\sqrt{2}} \left(|0\rangle \langle 0| + |0\rangle \langle 1| + |1\rangle \langle 0| - |1\rangle \langle 1| \right)$

$$|0\rangle \;\text{—}\boxed{H}\text{—}\; \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|1\rangle \;\text{—}\boxed{H}\text{—}\; \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

- Work through the exercises 17, 18, 19 on page 14 and 15 in the notes. Consult the surrounding examples if in doubt or if you need a hint.

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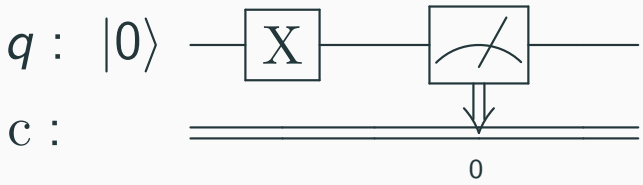
Diagrams with measurements



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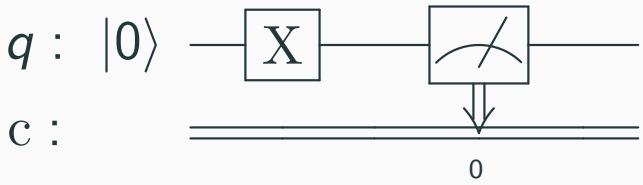
Diagrams with measurements



1. q for **q**ubit and c for **c**lassical bit.

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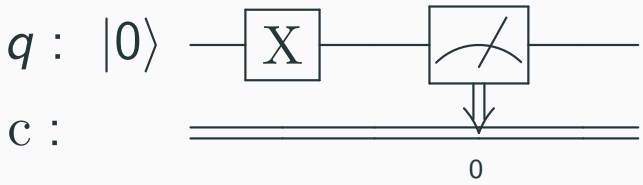
Diagrams with measurements



1. q for **q**ubit and c for **c**lassical bit.
2. qubit starts in state $|0\rangle$

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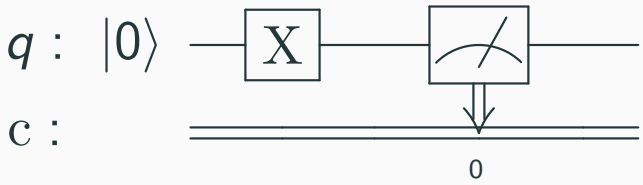
Diagrams with measurements



1. q for **q**ubit and c for **c**lassical bit.
2. qubit starts in state $|0\rangle$
3. Operator X acts on the state.

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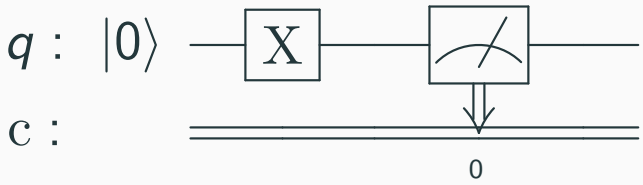
Diagrams with measurements



1. q for **q**ubit and c for **c**lassical bit.
2. qubit starts in state $|0\rangle$
3. Operator X acts on the state.
4. The new state is measured (acted on by the measuring operator)

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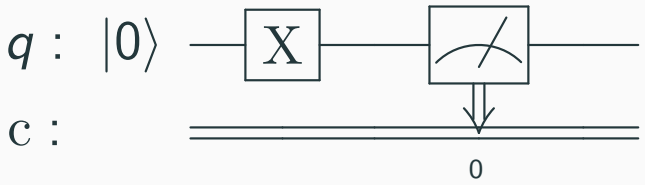
Diagrams with measurements



1. q for **q**ubit and c for **c**lassical bit.
2. qubit starts in state $|0\rangle$
3. Operator X acts on the state.
4. The new state is measured (acted on by the measuring operator)
5. The outcome of the measurement is stored in classical bit number 0 (counting from 0 instead of 1).

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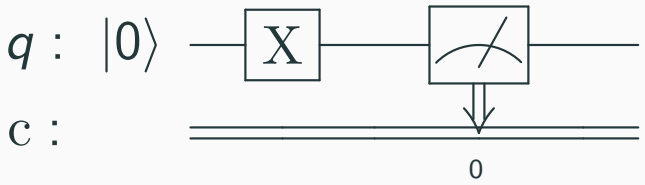
Diagrams with measurements



1. q for **q**ubit and c for **c**lassical bit.
2. qubit starts in state $|0\rangle$
3. Operator X acts on the state.
4. The new state is measured (acted on by the measuring operator)
5. The outcome of the measurement is stored in classical bit number 0 (counting from 0 instead of 1).
6. The diagram before the measurement translates to the following calculation:

$$X|0\rangle = (|1\rangle\langle 0| + |0\rangle\langle 1|)|0\rangle$$

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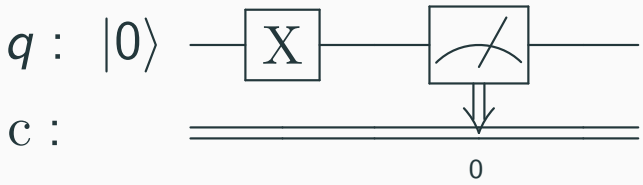
Diagrams with measurements



1. q for **q**ubit and c for **c**lassical bit.
2. qubit starts in state $|0\rangle$
3. Operator X acts on the state.
4. The new state is measured (acted on by the measuring operator)
5. The outcome of the measurement is stored in classical bit number 0 (counting from 0 instead of 1).
6. The diagram before the measurement translates to the following calculation:

$$X|0\rangle = (|1\rangle\langle 0| + |0\rangle\langle 1|)|0\rangle$$

7. Predict the outcome of the measurement.

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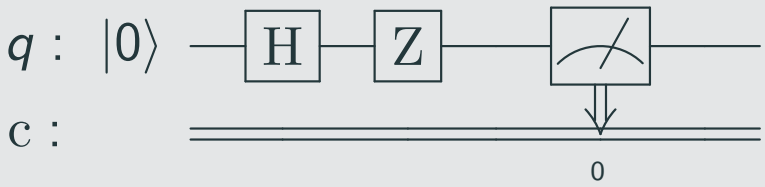
Example and exercise

Read through the example concerning the Hadamard operator starting at the buttom of 16 continuing to page 17 in the notes. Then do exercise 21 on page 17 in the notes.

Exercise 21



Our qubit start out in the state $|0\rangle$. Explain what the state of our qubit is after H and then Z have acted on it, i.e. calculate $ZH|0\rangle$. Then determine what the probability is of measurement giving 0 and 1, respectively.

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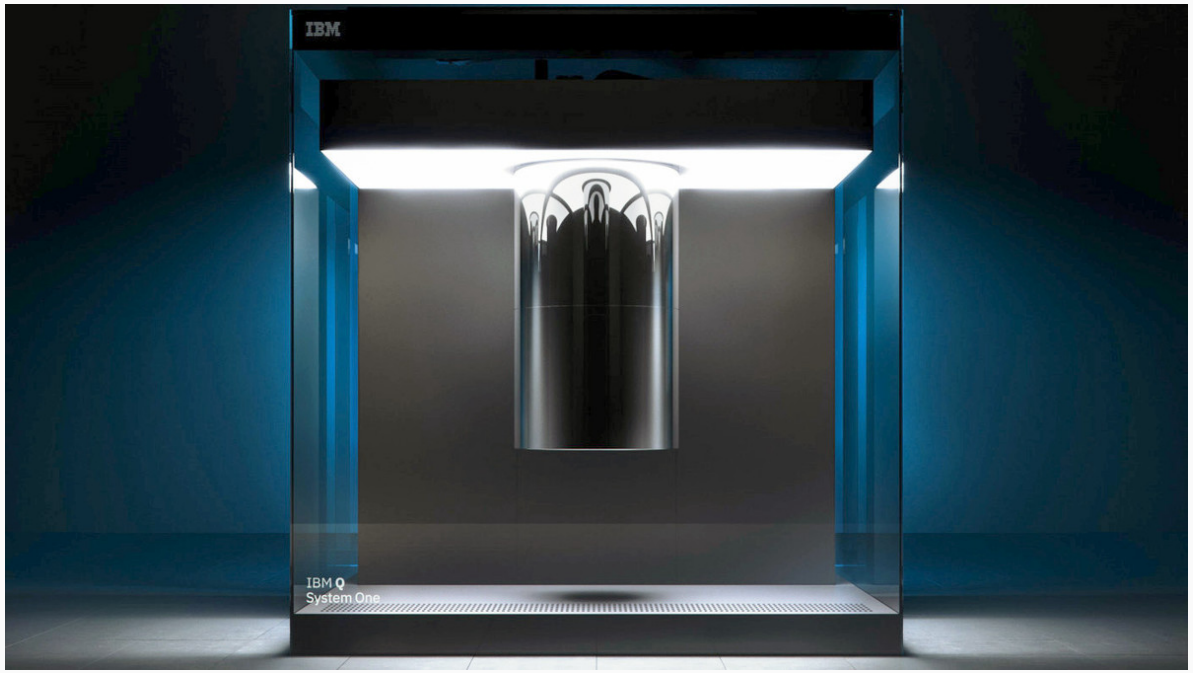
IBM Q



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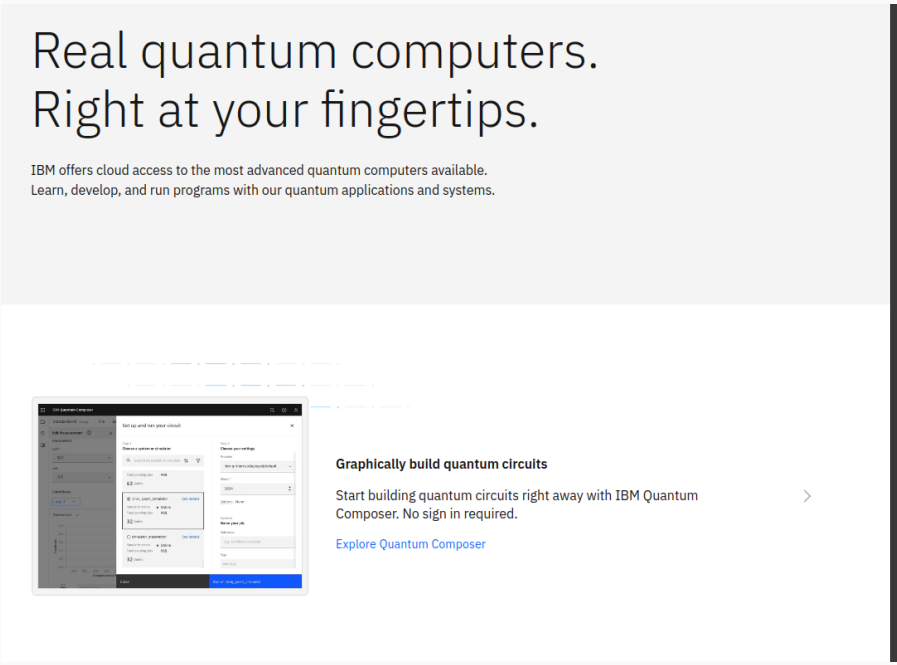
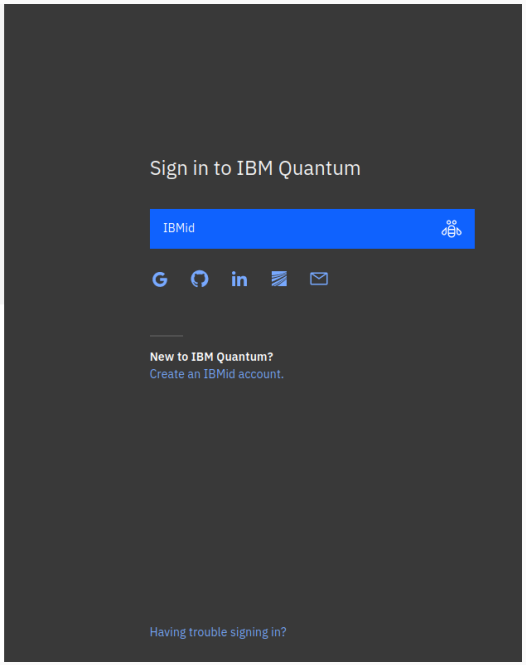
Access IBM Q

- Visit https://quantum-computing.ibm.com/
- Create an IBM account or sign in with one of the other options.
- Go to “Lab”.





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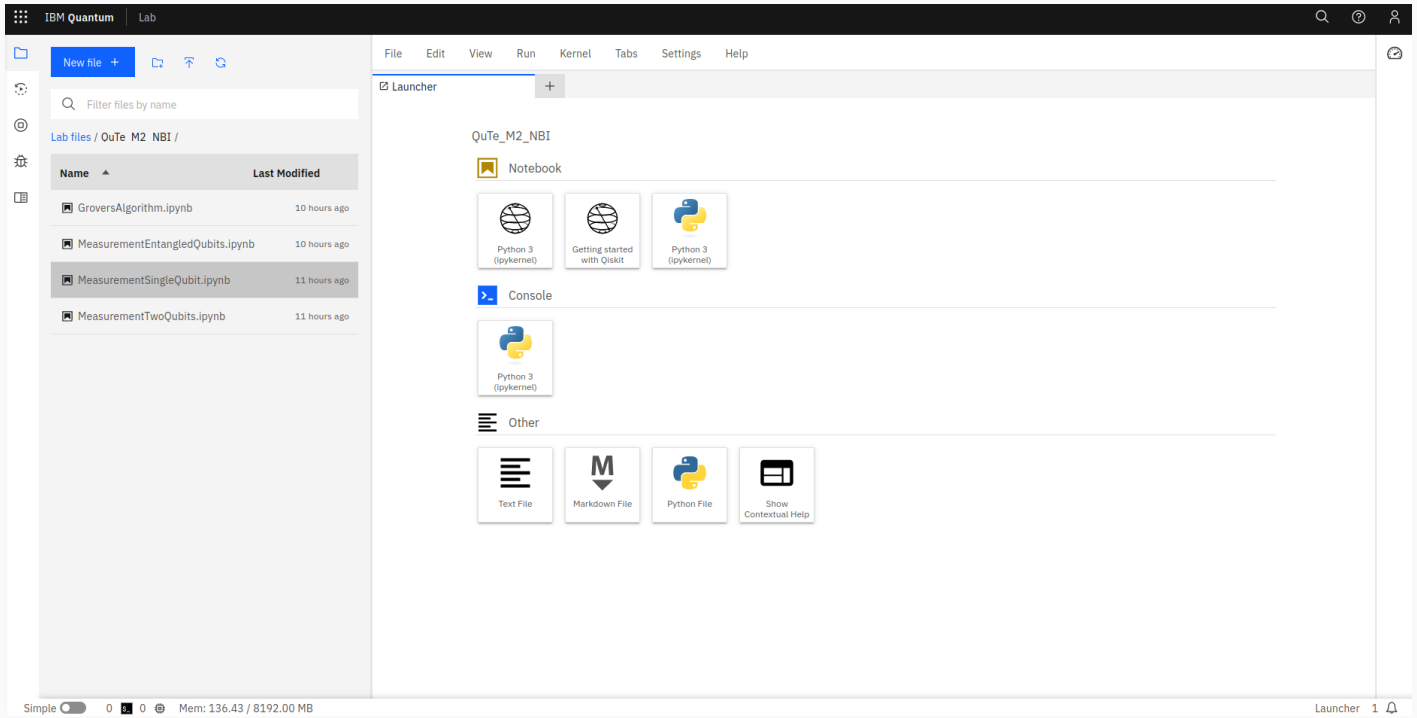
Upload the IBM Q exercises

Upload the following exercises to the lab file browser. (You can use drag and drop).

- MeasurementSingleQubit.ipynb
- MeasurementTwoQubits.ipynb
- MeasurementEntangledQubits.ipynb
- GroversAlgorithm.ipynb



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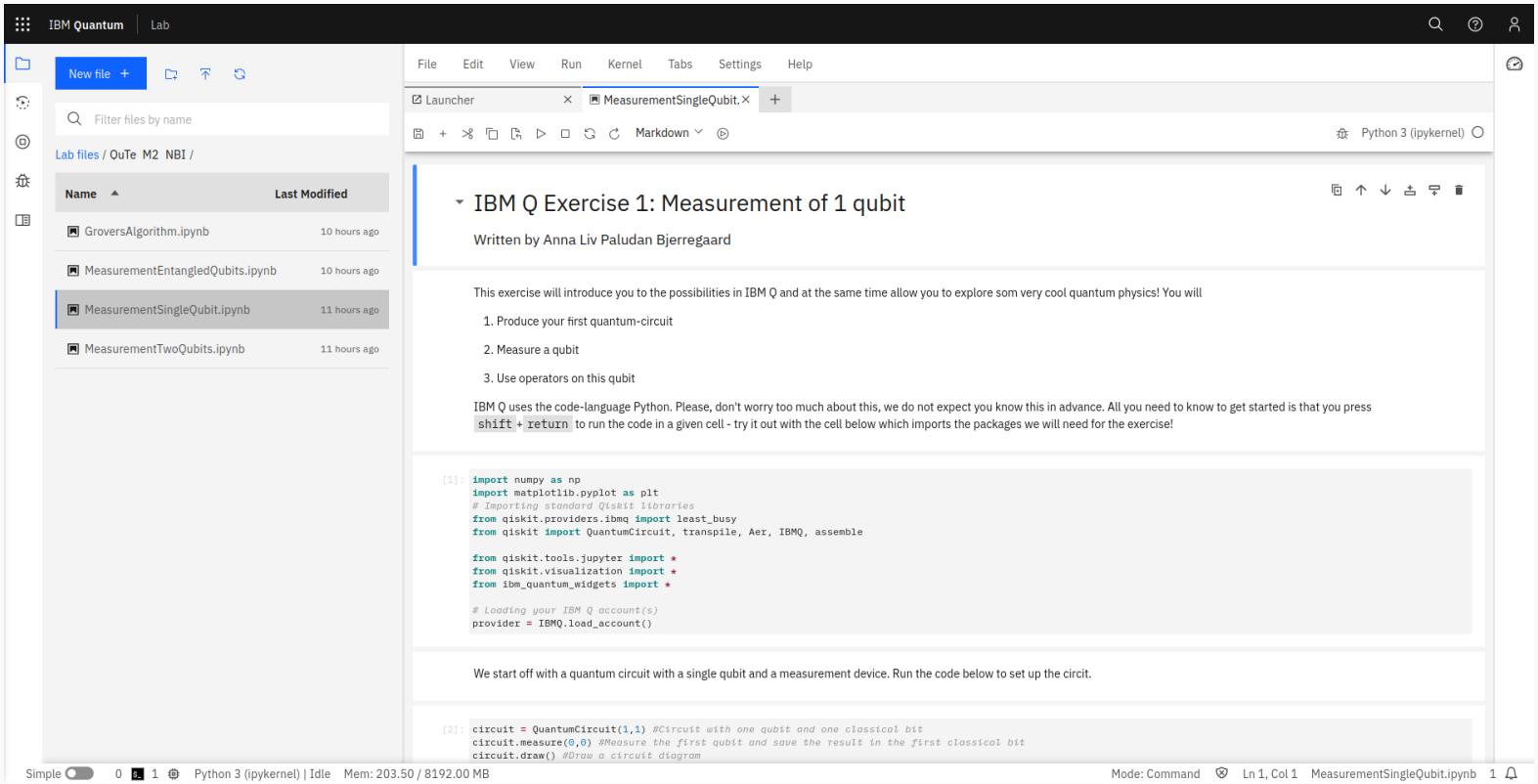
Single Qubit Measurements

Run the *Measurement SingleQubit.ipynb* notebook and follow the instructions. You are now able to perform **real** work on a real **quantum computer**! (If you set the variable sym=False)



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Pun intended!

Did you get the pun?

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Pun intended!

Did you get the pun?

Monty Python → IBM Q *is* related → IBM Q uses *Python* → Python is named after Monty Python,

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Pun intended!

Did you get the pun?

Monty Python → IBM Q *is* related → IBM Q uses *Python* → Python is named after Monty Python, because programming should be fun!

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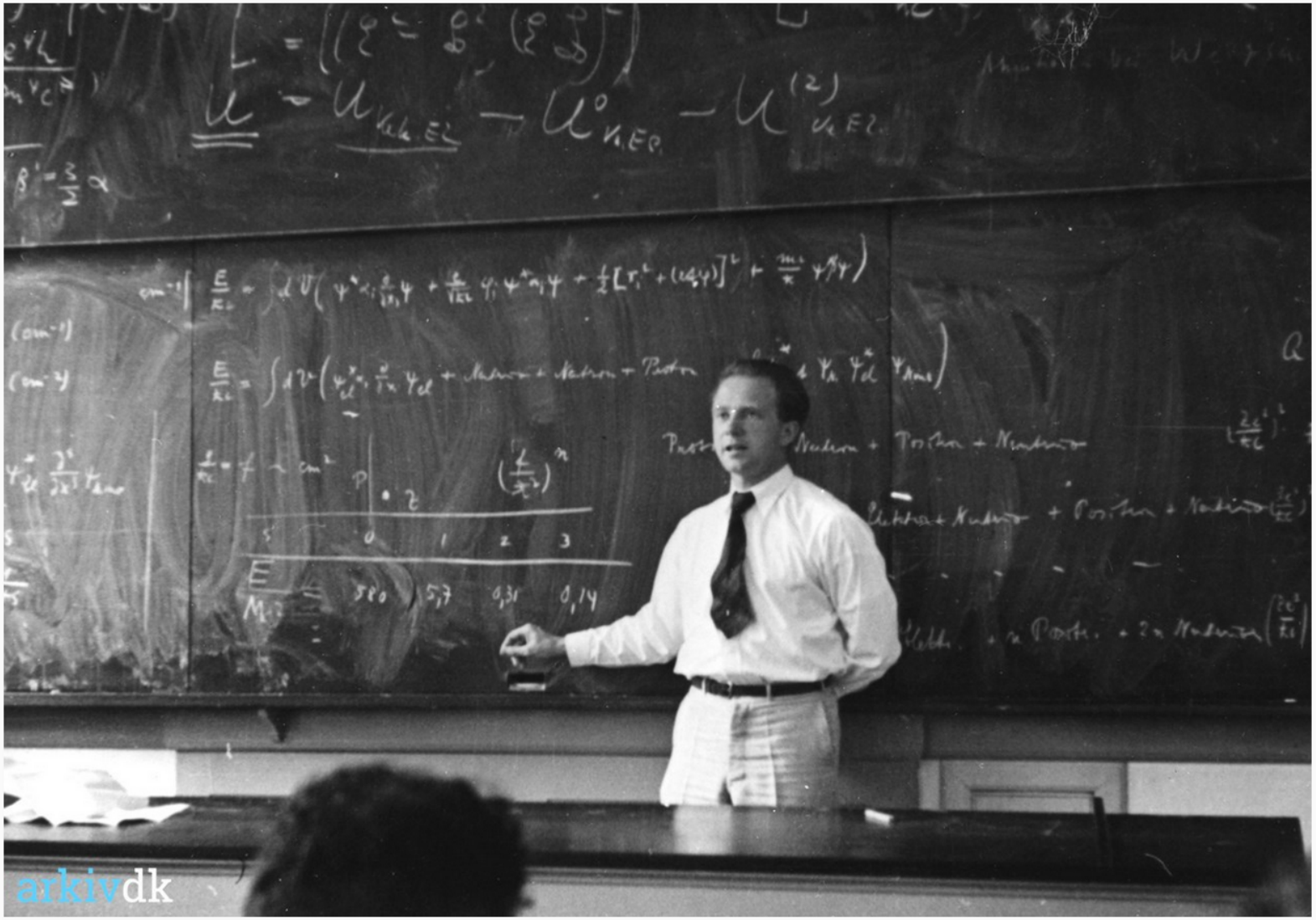
Back to school!



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One, Many

How many is *many?*

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One, Many

How many is *many?*

- One

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One, Many

How many is *many?*

- One, two

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One, Many

How many is *many?*

- One, two, many

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the European Union

One, Many

How many is *many?*

- ~~One, two, many~~

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One, Many

How many is *many?*

- ~~One, two, many~~

This time it is only

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One, Many

How many is *many?*

- ~~One, two, many~~

This time it is only

- One

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the European Union

One, Many

How many is *many?*

- ~~One, two, many~~

This time it is only

- One, many

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the European Union

One, Many

How many is *many?*

- ~~One, two, many~~

This time it is only

- One, many

This time around we are going to work with **two** qubit systems.

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One, Many

How many is *many?*

- ~~One, two, many~~

This time it is only

- One, many

This time around we are going to work with **two** qubit systems.

With two or more qubits we can do...

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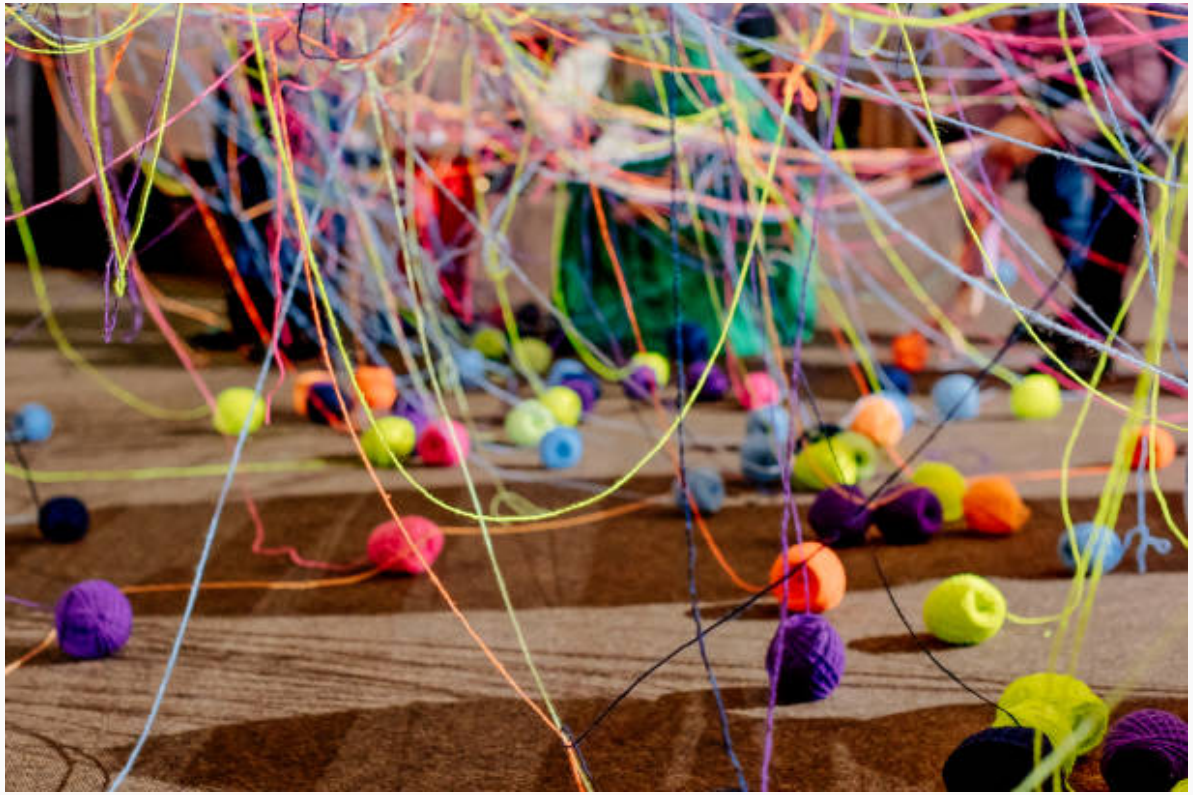
Entanglement



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Two-qubit entanglement

Definition 8: Entanglement

Two qubits are entangled if the state of one qubit depends on the other. When two qubits are entangled it is *impossible* to write the total state as a product of a state of the first qubits and a state of the second.

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Two-qubit entanglement

Definition 8: Entanglement

Two qubits are entangled if the state of one qubit depends on the other. When two qubits are entangled it is *impossible* to write the total state as a product of a state of the first qubits and a state of the second.

Let us see some examples. (Read the examples in Chapter V section 10 in the notes, page 10 and 11)

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None entangled two qubit systems

Two qubits not entangled

If qubit 1 is in the state $|0\rangle$ and qubit 2 is in the state $|1\rangle$, then the total state of the qubits is $|1\rangle|0\rangle = |10\rangle$

Another two qubits not entangled

If the state of qubit 1 is $|0\rangle$ and the state of qubit 2 is $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, then the total state of the two qubits is
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$.

A third example

Two *not entangled* qubits: The two qubits in the state
$\frac{1}{5}(3|01\rangle + 4|11\rangle)$ are not entangled, since we can write the state as
$\frac{1}{5}(3|01\rangle + 4|11\rangle) = \frac{1}{5}(3|0\rangle + 4|1\rangle)|1\rangle$.

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Entangled two qubit systems

Example: Entangled qubits

Two qubits are in the state $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$. They are *entangled* because the state of the first qubit depends on the state of the second: If the first qubit is 1 then the other is 0, and vice versa (if the first qubit is 0 then the second i 1). To put it differently, you can not write the total state $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$ as a product of two states $|\ldots\rangle\,|\ldots\rangle$.

Another example

The two qubits in the state $\frac{1}{5}(3\,|00\rangle + 4\,|11\rangle)$ are entangled.

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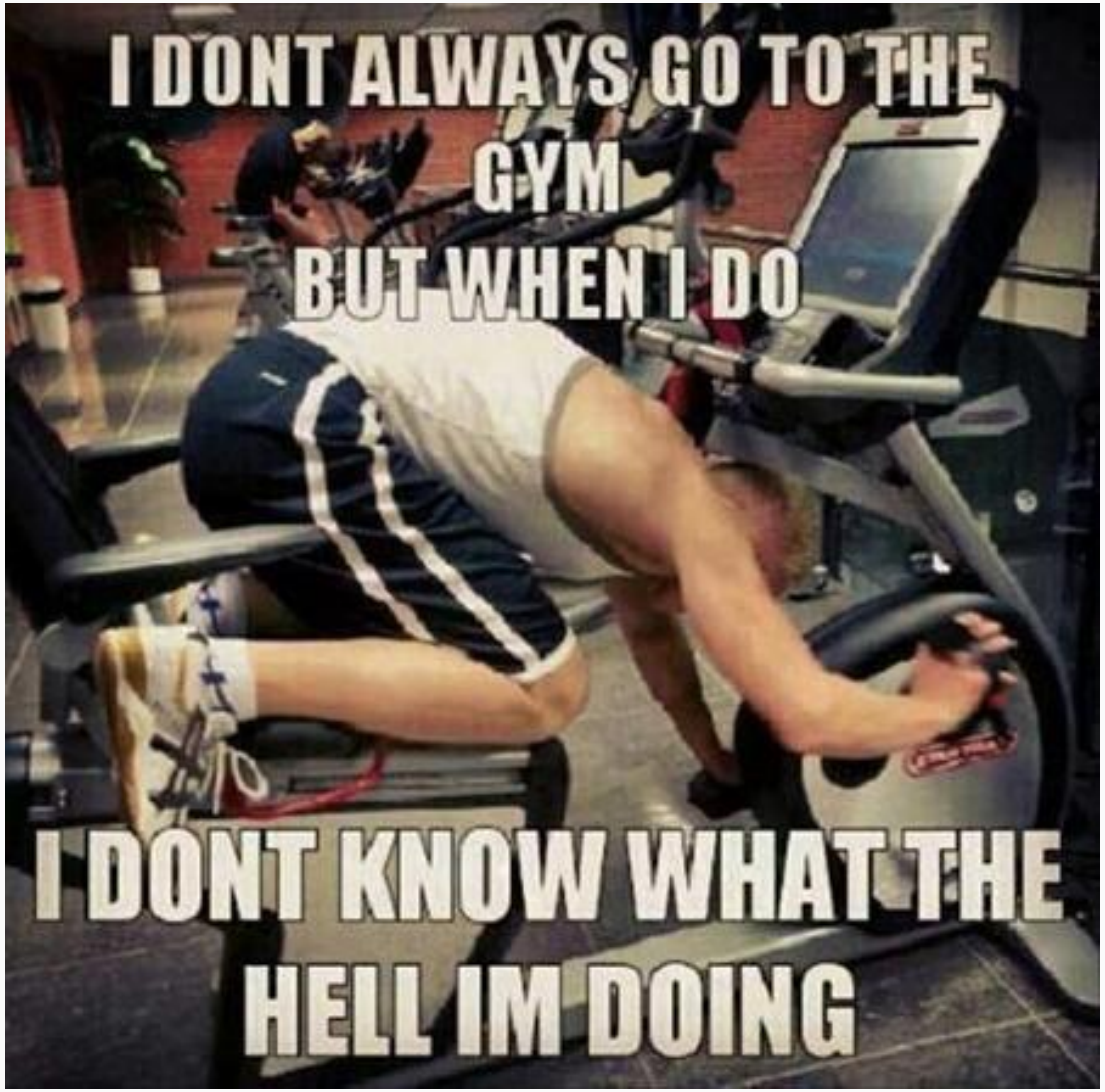
Time for some exercise



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Exercises

Exercise 11 page 10 in the notes.

Two qubits are in the state $|01\rangle$. What is the state of qubit 1? what about qubit 2?

Exercise 12 page 11 in the notes.

Determine which of the following two-qubit states are entangled:

1. $|11\rangle$
2. $\frac{1}{\sqrt{2}}(|11\rangle + |00\rangle$
3. $\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle$

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But what is the difference between entangled and none entangled states?

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But what is the difference between entangled and none entangled states?

It's all about the measurements.

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None entangled qubits

Two qubits are in the state $|01\rangle$(read from the right).

- Qubit 2 is in state $|0\rangle$.

- Qubit 1 is in state $|1\rangle$.

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None entangled qubits

Two qubits are in the state $|01\rangle$(read from the right).

- Qubit 2 is in state $|0\rangle$.
- Qubit 1 is in state $|1\rangle$.

Measuring on any of the qubits

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None entangled qubits

Two qubits are in the state $|01\rangle$(read from the right).

- Qubit 2 is in state $|0\rangle$.
- Qubit 1 is in state $|1\rangle$.

does not alter the other.

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None entangled qubits

Two qubits are in the state $|01\rangle$(read from the right).

- Qubit 2 is in state $|0\rangle$.
- Qubit 1 is in state $|1\rangle$.

The state remains unaltered.

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Entangled qubits

Two qubits are in the entangled state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Qubit 2

Qubit 1

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Entangled qubits

Two qubits are in the entangled state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Qubit 2

Qubit 1

We measure qubit 1.

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Entangled qubits

Two qubits are in the entangled state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Qubit 2

Qubit 1

and get 0.

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Entangled qubits

Two qubits are in the entangled state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Qubit 2

Qubit 1

Qubit 1 is now in state $|0\rangle$ forcing the total state to be $|10\rangle$

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Entangled qubits

Two qubits are in the entangled state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Qubit 2

Qubit 1

The measurement on qubit 1 affects qubit 2 instantaneously.

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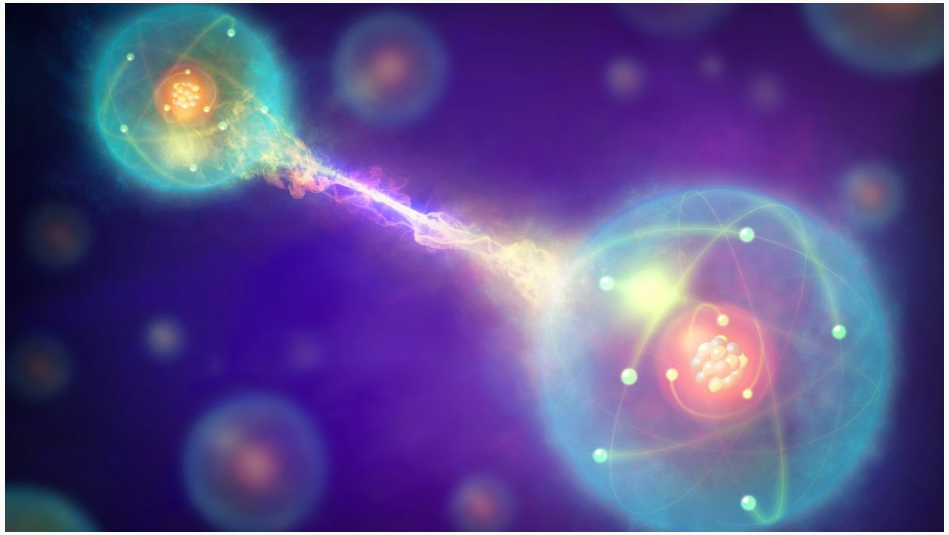
Albert Einstein

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from **spooky action at a distance**.



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Untangle these exercises

Exercise 13 page 11 in the notes.

Two qubits are in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. What is the probability of a measurement on the first qubit giving 0 and 1, respectively. If we measure the first qubit, and the outcome is 1, what is the total state of the two qubits immediately after the measurement?

Exercise 14 page 12 in the notes.

Two qubits are in the state $\frac{1}{5}(3\,|01\rangle - 4\,|10\rangle$. Will a measurement on the second qubit affect the state of the first qubit?

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Operating on two qubits

The Controlled NOT gate

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The CNOT gate

Definition 13: The CNOT operator

First qubit is the control bit

$CNOT = |00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|$

Second qubit is the control bit

$CNOT = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

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The CNOT gate

Definition 13: The CNOT operator

First qubit is the control bit

$CNOT = |00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|$

Second qubit is the control bit

$CNOT = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

But what does it do?

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The CNOT gate

Definition 13: The CNOT operator

First qubit is the control bit

$CNOT = |00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|$

Second qubit is the control bit

$CNOT = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

But what does it do?

If the control qubit is 0, it does nothing.

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Second qubit is the control bit

$CNOT = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

But what does it do?

If the control qubit is 0, it does nothing.

If the control qubit is 1, it changes the state of the other qubit (by applying the X operator in the other qubit.)

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Let us check

Example: $CNOT\,|01\rangle$ with first qubit as control

$$
\begin{aligned}
CNOT\,|01\rangle &= (|00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|)\,|01\rangle \\
&= |00\rangle\langle 00|01\rangle + |11\rangle\langle 01|01\rangle + |10\rangle\langle 10|01\rangle + |01\rangle\langle 11|01\rangle \\
&= |00\rangle\,0 + |11\rangle\,1 + |10\rangle\,0 + |10\rangle\,0 \\
&= |11\rangle
\end{aligned}
$$

As you can see the first qubit remains unchanged while the second qubit changes from state $|0\rangle$ to $|1\rangle$.

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Let us check

Example: $CNOT\,|01\rangle$ with first qubit as control

$$
\begin{aligned}
CNOT\,|01\rangle &= (|00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|)\,|01\rangle \\
&= |00\rangle\langle 00|01\rangle + |11\rangle\langle 01|01\rangle + |10\rangle\langle 10|01\rangle + |01\rangle\langle 11|01\rangle \\
&= |00\rangle\,0 + |11\rangle\,1 + |10\rangle\,0 + |10\rangle\,0 \\
&= |11\rangle
\end{aligned}
$$

As you can see the first qubit remains unchanged while the second qubit changes from state $|0\rangle$ to $|1\rangle$.

Do a quick check of the other combinations of $CNOT$ and the states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.

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Creating an entangled state

Example: $CNOT$ on $\frac{1}{\sqrt{2}}(|01\rangle + |00\rangle)$ with qubit 1 as control

$$
\begin{aligned}
CNOT\frac{1}{\sqrt{2}}(|01\rangle + |00\rangle) &= (|00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|)\frac{1}{\sqrt{2}}(|01\rangle + |00\rangle) \\
&= \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)
\end{aligned}
$$

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Creating an entangled state

Example: $CNOT$ on $\frac{1}{\sqrt{2}}(|01\rangle + |00\rangle)$ with qubit 1 as control

$$
\begin{aligned}
CNOT \frac{1}{\sqrt{2}}(|01\rangle + |00\rangle) &= (|00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|)\frac{1}{\sqrt{2}}(|01\rangle + |00\rangle) \\
&= \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)
\end{aligned}
$$

We went from an none entangled state to an entangled state!

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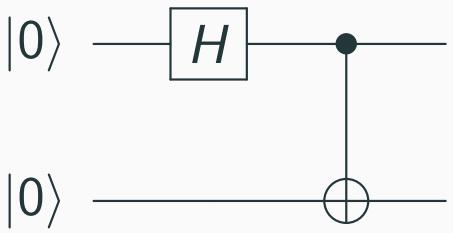
Creating an entangled state from scratch



The diagram shows how we, using a Hadamard and a *CNOT*-operator, can produce the entangled state $\frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$. The dot in the *CNOT*-operator indicates the control qubit and the other end, which looks like a sight, points to the qubit which the CNOT operators does something to (the target qubit).

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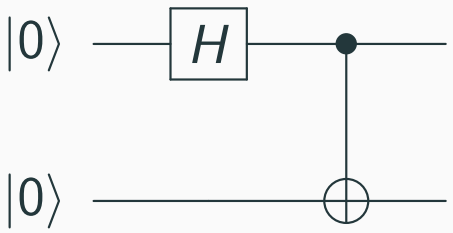
Creating an entangled state from scratch



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The state goes

from $|0\rangle\,|0\rangle$

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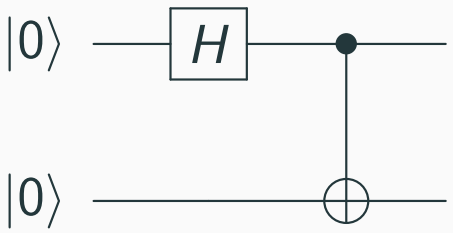
Creating an entangled state from scratch



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The state goes

from $|0\rangle\,|0\rangle$

to $|0\rangle\,\frac{1}{\sqrt{1}}\,(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\,(|00\rangle + |01\rangle)$

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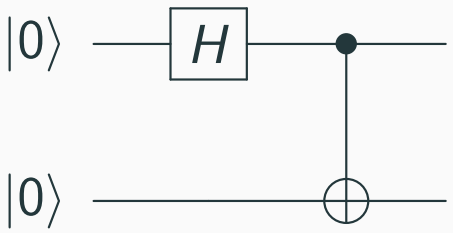
Creating an entangled state from scratch



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The state goes

from $|0\rangle\,|0\rangle$

to $|0\rangle\,\frac{1}{\sqrt{1}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\left(|00\rangle + |01\rangle\right)$

to $CNOT\,\frac{1}{\sqrt{2}}\left(|00\rangle + |01\rangle\right)$

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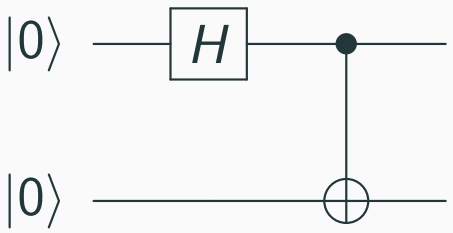
Creating an entangled state from scratch



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The state goes

from $|0\rangle\,|0\rangle$

to $|0\rangle\,\frac{1}{\sqrt{1}}\,(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\,(|00\rangle + |01\rangle)$

to $CNOT\,\frac{1}{\sqrt{2}}\,(|00\rangle + |01\rangle)$

to the final *entangled* state $\frac{1}{\sqrt{2}}\,(|11\rangle + |00\rangle)$

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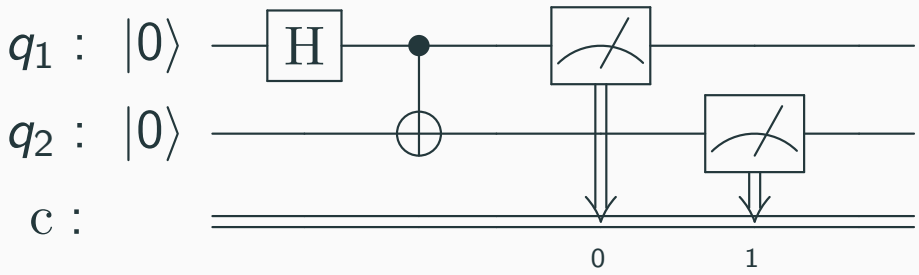
Measuring entangled states



In the figure we perform measurements on each of the two qubits which are in the state $\frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$ before the measurement. The measurement on the first qubit will give 0 with 50% probability. If the measurement gives 0, the state of the first qubit is $|0\rangle$ after the measurement. Therefore, the total state of the two qubits is $|00\rangle$ after we have measured 0 for the first qubit. We now measure the second qubit and get 0 with 100% probability, since the second qubit is in the state $|0\rangle$.

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Back to IBM Q

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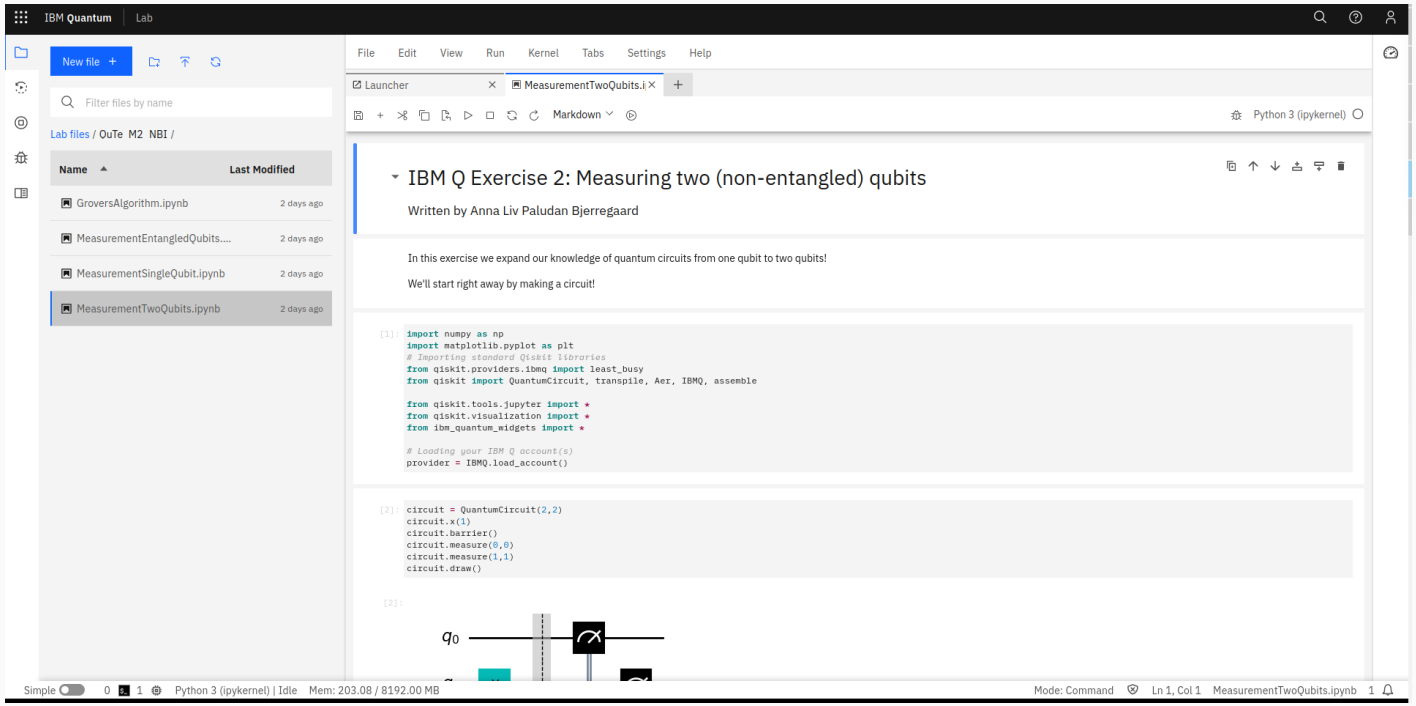
Measuring two (non-entangled) qubits

Log in to https://lab.quantum-computing.ibm.com and fire up exercise *MeasurementTwoQubits.pynb*



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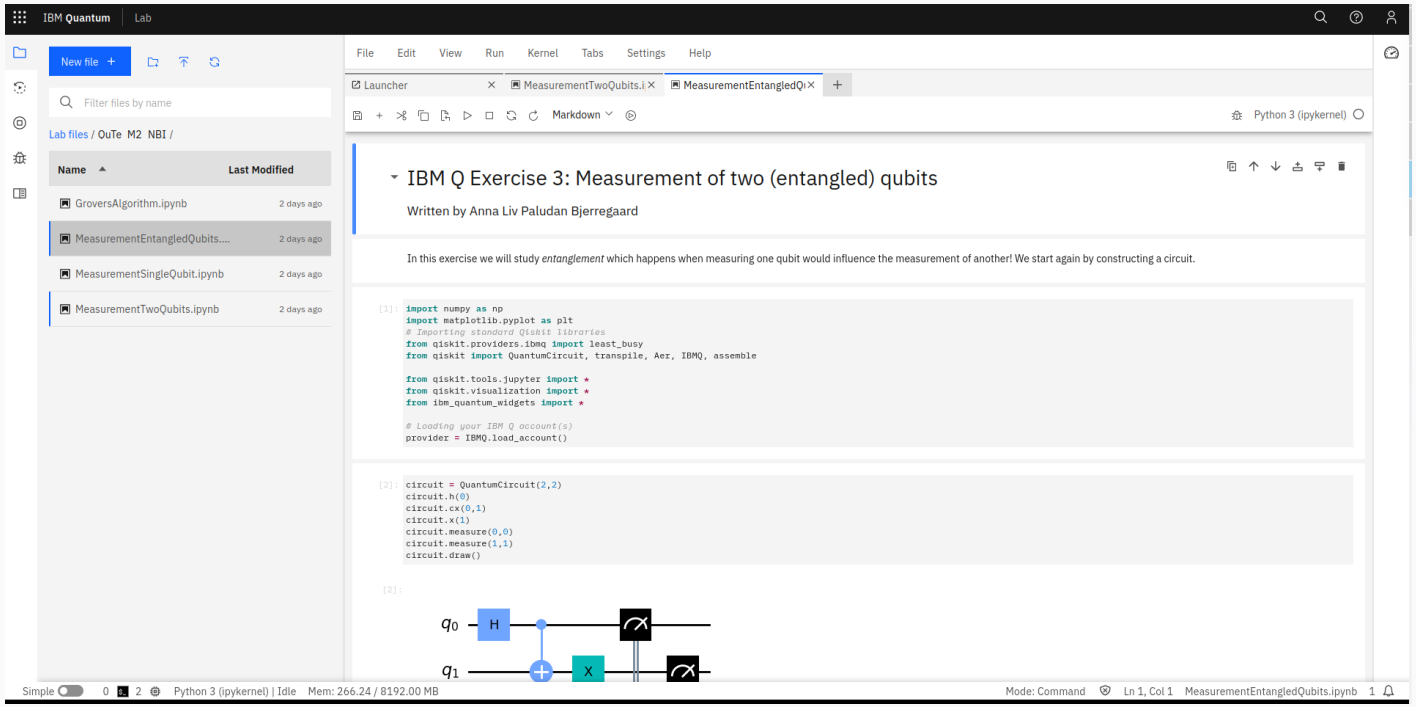
Measurement of two (entangled) qubits

While still at https://lab.quantum-computing.ibm.com work on exercise *MeasurementEntangledQubits.pynb*



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NO, NO!

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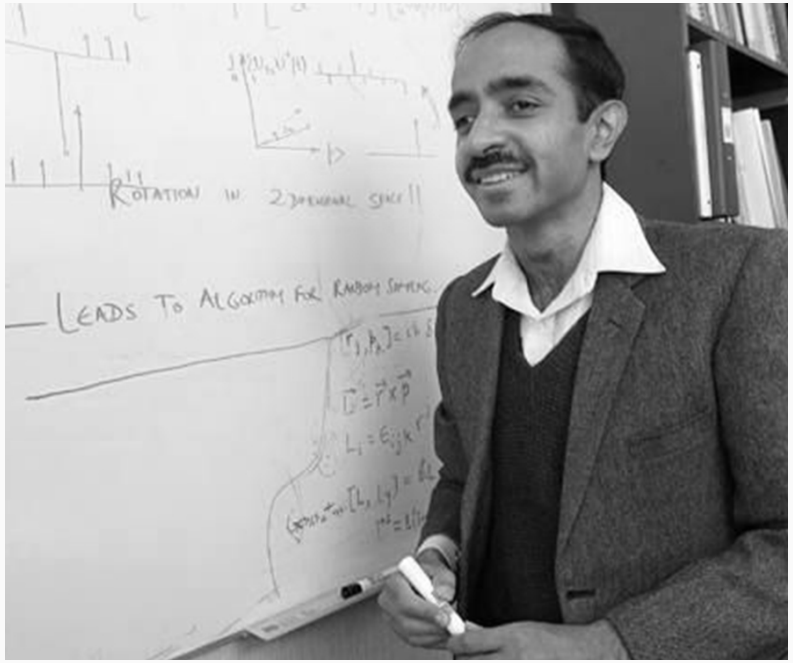
Grover Baby! - Yeah!



Grover's algorithm

- You have put your smart phone in the cabinet.

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Grover's algorithm

- You have put your smart phone in the cabinet.
- but you forgot the exact drawer!

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Grover's algorithm

- You have put your smart phone in the cabinet.
- but you forgot the exact drawer!
- A classical computer would on average use $\frac{N}{2}$ searches.

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Grover's algorithm

- You have put your smart phone in the cabinet.
- but you forgot the exact drawer!
- A classical computer would on average use $\frac{N}{2}$ searches.
- A quantum computer would on average use $\sqrt{N}$ searches using *Grover's algorithm*.

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Grover's algorithm

- You have put your smart phone in the cabinet.
- but you forgot the exact drawer!
- A classical computer would on average use $\frac{N}{2}$ searches.
- A quantum computer would on average use $\sqrt{N}$ searches using *Grover's algorithm*.
- Let us retrieve the phone using Grover's algorithm.

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Grover's algorithm with 4 drawers

- 4 drawers can be represented by $n = 2$ qubits in the special states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.
- 00, 01, 10, 11 in binary are simply 0, 1, 2, 3 in decimal numbers

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Grover's algorithm with 4 drawers

- 4 drawers can be represented by $n = 2$ qubits in the special states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.
- 00, 01, 10, 11 in binary are simply 0, 1, 2, 3 in decimal numbers if you recall.

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Grover's algorithm with 4 drawers

- We assume that the phone is in drawer 11 (4)

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Grover's algorithm with 4 drawers

- We assume that the phone is in drawer 11 (4)
- First step - Prepare the state: $|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$

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Grover's algorithm with 4 drawers

- We assume that the phone is in drawer 11 (4)
- First step - Prepare the state: $|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$
- Second step - act with the operator: $O = -|11\rangle\langle 11|$

$$
\begin{aligned}
O|\psi\rangle &= (-|11\rangle\langle 11|)\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
&= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)
\end{aligned}
$$

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Grover's algorithm with 4 drawers

- Third step - act with the operator:$(2\left|\psi\right\rangle\left\langle\psi\right| - I)$

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Grover's algorithm with 4 drawers

- Third step - act with the operator:$(2\left|\psi\right\rangle\left\langle\psi\right| - I)$
- The combined operator is called $G = (2\left|\psi\right\rangle\left\langle\psi\right| - I)\,O$

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Grover's algorithm with 4 drawers

- Third step - act with the operator:$(2\left|\psi\right\rangle\left\langle\psi\right| - I)$
- The combined operator is called $G = (2\left|\psi\right\rangle\left\langle\psi\right| - I)\,O$
- $G\left|\psi\right\rangle = (2\left|\psi\right\rangle\left\langle\psi\right| - I)\,O\left|\psi\right\rangle$

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Grover's algorithm with 4 drawers

- Third step - act with the operator:$(2\left|\psi\right\rangle\left\langle\psi\right| - I)$
- The combined operator is called $G = (2\left|\psi\right\rangle\left\langle\psi\right| - I)\,O$
- $G\left|\psi\right\rangle = (2\left|\psi\right\rangle\left\langle\psi\right| - I)\,O\left|\psi\right\rangle$
- Fancy trick!

$$
\begin{aligned}
O\left|\psi\right\rangle &= \frac{1}{2}\left(\left|00\right\rangle + \left|01\right\rangle + \left|10\right\rangle - \left|11\right\rangle\right) \\
&= \frac{1}{2}\left(\left|00\right\rangle + \left|01\right\rangle + \left|10\right\rangle + \left|11\right\rangle\right) - \left|11\right\rangle \\
&= \left|\psi\right\rangle - \left|11\right\rangle
\end{aligned}
$$

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Grover's algorithm with 4 drawers

- All combined:

$$
\begin{aligned}
G\,|\psi\rangle &= (2\,|\psi\rangle\,\langle\psi| - I)\,O\,|\psi\rangle \\
&= (2\,|\psi\rangle\,\langle\psi| - I)\,(|\psi\rangle - |11\rangle) \\
&= 2\,|\psi\rangle\,\langle\psi|\psi\rangle - I\,|\psi\rangle - 2\,|\psi\rangle\,\langle\psi|11\rangle + I\,|11\rangle \\
&= 2\,|\psi\rangle - |\psi\rangle - 2\,|\psi\rangle\,\frac{1}{2} + |11\rangle \\
&= |11\rangle
\end{aligned}
$$

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Grover's algorithm with 4 drawers

- All combined:

$$
\begin{aligned}
G\left|\psi\right\rangle &= \left(2\left|\psi\right\rangle\left\langle\psi\right| - I\right) O\left|\psi\right\rangle \\
&= \left(2\left|\psi\right\rangle\left\langle\psi\right| - I\right)\left(\left|\psi\right\rangle - \left|11\right\rangle\right) \\
&= 2\left|\psi\right\rangle\left\langle\psi|\psi\right\rangle - I\left|\psi\right\rangle - 2\left|\psi\right\rangle\left\langle\psi|11\right\rangle + I\left|11\right\rangle \\
&= 2\left|\psi\right\rangle - \left|\psi\right\rangle - 2\left|\psi\right\rangle\frac{1}{2} + \left|11\right\rangle \\
&= \left|11\right\rangle
\end{aligned}
$$

- If we measure now we will get 11 with 100% probability.

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Grover's algorithm with 4 drawers

- All combined:

$$
\begin{aligned}
G\left|\psi\right\rangle &= \left(2\left|\psi\right\rangle\left\langle\psi\right| - I\right) O\left|\psi\right\rangle \\
&= \left(2\left|\psi\right\rangle\left\langle\psi\right| - I\right)\left(\left|\psi\right\rangle - \left|11\right\rangle\right) \\
&= 2\left|\psi\right\rangle\left\langle\psi|\psi\right\rangle - I\left|\psi\right\rangle - 2\left|\psi\right\rangle\left\langle\psi|11\right\rangle + I\left|11\right\rangle \\
&= 2\left|\psi\right\rangle - \left|\psi\right\rangle - 2\left|\psi\right\rangle\frac{1}{2} + \left|11\right\rangle \\
&= \left|11\right\rangle
\end{aligned}
$$

- If we measure now we will get 11 with 100% probability.
- But it's like cheating! We told the *Oracle* (O operator) where the phone was from the beginning.

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Grover's algorithm with 4 drawers

- All combined:

$$
\begin{aligned}
G\left|\psi\right\rangle &= \left(2\left|\psi\right\rangle\left\langle\psi\right| - I\right) O\left|\psi\right\rangle \\
&= \left(2\left|\psi\right\rangle\left\langle\psi\right| - I\right)\left(\left|\psi\right\rangle - \left|11\right\rangle\right) \\
&= 2\left|\psi\right\rangle\left\langle\psi|\psi\right\rangle - I\left|\psi\right\rangle - 2\left|\psi\right\rangle\left\langle\psi|11\right\rangle + I\left|11\right\rangle \\
&= 2\left|\psi\right\rangle - \left|\psi\right\rangle - 2\left|\psi\right\rangle\frac{1}{2} + \left|11\right\rangle \\
&= \left|11\right\rangle
\end{aligned}
$$

- If we measure now we will get 11 with 100% probability.
- But it's like cheating! We told the *Oracle* (O operator) where the phone was from the beginning.
- Grover's algorithm needs to be part of a complete algorithm.

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Grover's algorithm with 8 drawers

Read the example about Grover's algorithm for $N = 8$ on page 22 and 23 in the notes.

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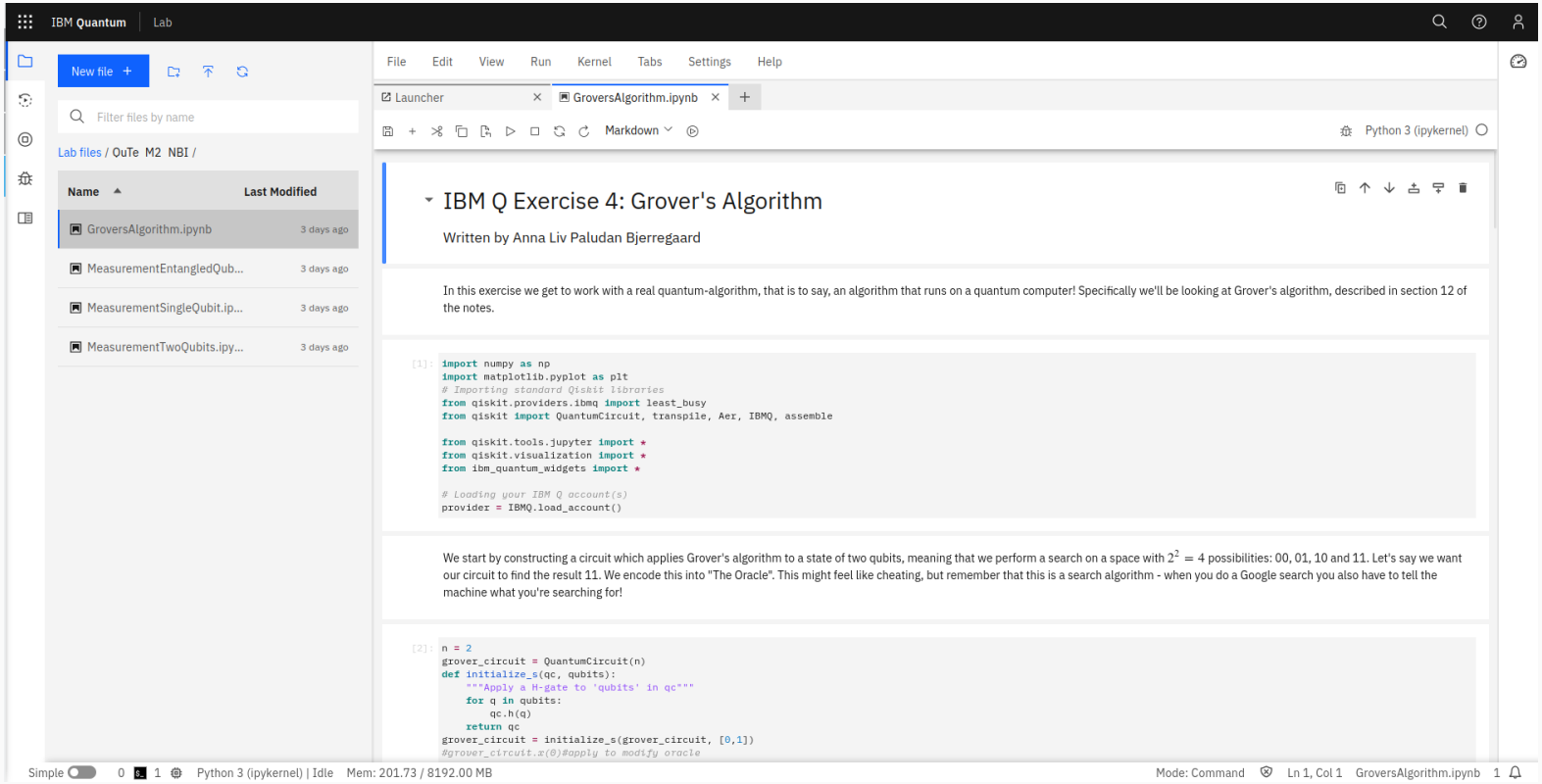
Grover's algorithm on IBM Q

Now finally open and work on the exercise *GroversAlgorithm.ipynb* on https://lab.quantum-computing.ibm.com



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