$$\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix}$$

$$\hat{a} = \begin{pmatrix} -a_y \\ a_x \end{pmatrix} \qquad \hat{b} = \begin{pmatrix} -b_y \\ b_x \end{pmatrix}$$

$$\vec{a} \cdot \hat{b} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} \cdot \begin{pmatrix} -b_y \\ b_x \end{pmatrix} = a_x \cdot (-b_y) + a_y \cdot b_x$$

$$\vec{a} \cdot \hat{b} = \boxed{a_y \cdot b_x - a_x \cdot b_y}$$

$$\det(\vec{b}, \vec{a}) = \begin{vmatrix} b_x & a_x \\ b_y & a_y \end{vmatrix} = \boxed{b_x a_y - a_x \cdot b_y}$$

$$\vec{a} \cdot \hat{b} = \det(\vec{b}, \vec{a})$$

$$0 = \det(\vec{b}, \vec{a}) = -\det(\vec{a}, \vec{b})$$

$$b_x \cdot a_y - a_x \cdot b_y = 0$$

$$a_x \cdot b_y - a_y \cdot b_x = 0$$