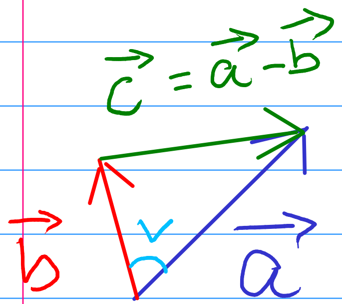
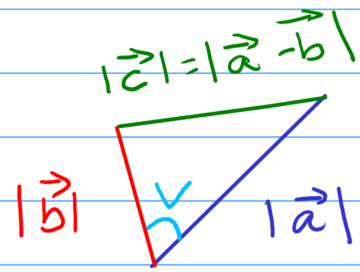
Som vektorer



Som en trekant



Anvender cosinusrelationen på trekanten

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix} \quad \vec{c} = \begin{pmatrix} a_x - b_x \\ a_y - b_y \end{pmatrix}$$

Cos-rel: $\boxed{\cos(C) = \frac{a^2 + b^2 - c^2}{2 \cdot a \cdot b}}$

$$\cos(v) = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2 \cdot |\vec{a}| \cdot |\vec{b}|}$$

$$|\vec{a}|^2 = \left(\sqrt{a_x^2 + a_y^2}\right)^2$$

$$\cos(v) = \frac{a_x^2 + a_y^2 + b_x^2 + b_y^2 - \left((a_x - b_x)^2 + (a_y - b_y)^2\right)}{2 \cdot |\vec{a}| \cdot |\vec{b}|}$$

$$\cos(v) = \frac{a_x^2 + a_y^2 + b_x^2 + b_y^2 - \left(a_x^2 + b_x^2 - 2 \cdot a_x b_x + a_y^2 + b_y^2 - 2 \cdot a_y b_y\right)}{2 \cdot |\vec{a}| \cdot |\vec{b}|}$$

$$\cos(v) = \frac{\cancel{a_x^2} + \cancel{a_y^2} + \cancel{b_x^2} + \cancel{b_y^2} - \cancel{a_x^2} - \cancel{b_x^2} + 2 \cdot a_x \cdot b_x - \cancel{a_y^2} - \cancel{b_y^2} + 2 \cdot a_y \cdot b_y}{2 \cdot |\vec{a}| \cdot |\vec{b}|}$$

$$\cos(v) = \frac{\cancel{2}(\cdot a_x \cdot b_x + \cdot a_y \cdot b_y)}{\cancel{2} \cdot |\vec{a}| \cdot |\vec{b}|}$$

$$\boxed{|\vec{a}| \cdot |\vec{b}| \cdot \cos(v) = \underbrace{a_x \cdot b_x + a_y \cdot b_y}_{\vec{a} \cdot \vec{b}}}$$