HW5

Deep Patel
Jacob Hurst

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1 What

The overall goal of this homework is to gain experience doing numerical modeling with ODEs. We modeled and simulated the flocking behavior of birds in a two-dimensional space. This problem has actual applications, e.g., for the dynamics of diseases, magnets, populations, etc

2 How

Objective: we derive a set of ODEs, as a mathematical model, that describes the flocking behavior of a flock of N birds located at B_k, where k refers to the current bird.

The leader bird, with location B_1, does not care very much about the other birds, and is primarily interested in eating some tasty food being handed out by a bird feeder at a certain location C.

There are four primary forces that act on the birds at any instant are:

1. At any instance of time, B_1 is trying to reduce its distance to C by Food attraction force:

$$F_1^{food}(t) = \gamma_1(C(t) - B_1(t))$$

2. The rest of the birds try to stay close to the leader B_1 and are hence governed by the follow force:

$$F_k^{follow}(t) = \gamma_2(B_1(t) - B_k(t))$$

for birds k=2,3,...N.

3. In order to increase safety, the follower birds want to be as close to the center of flock as possible.

$$F_k^{flock}(k) = \kappa(\bar{B}(t) - B_k(k))$$

where $\bar{B}(t) = \sum B_k(t)/N$

4. The repulsion of to avoid collision due to closest five neighbors $[l_1, l_2, l_3, l_4, l_5]$ is:

$$F_k^{rep}(t) = \sum_{i=1}^{5} \rho \frac{(B_k(t) - B_{l_i^k}(t))}{(B_k(t) - B_{l_i^k}(t))^2 + \delta}$$

for birds k=2,3,...N.

2.1 Effects of the forces

for forces on leader bird (k=1):

$$f(t, \bar{y}(t)) = F_1^{food}(t) = \gamma_1(C(t) - B_1(t))$$

for follower birds (k=2,3,...N):

$$f(t, \bar{y}(t)) = F_k^{follow}(t) + F_k^{flock}(k) + F_k^{rep}(t)$$

$$f(t, \bar{y}(t) = \gamma_2(B_1(t) - B_k(t)) + \kappa(\bar{B}(t) - B_k(k)) + \sum_{i=1}^{5} \rho \frac{(B_k(t) - B_{l_i^k}(t))}{(B_k(t) - B_{l_i^k}(t))^2 + \delta}$$

For $x_c > x_1$:

1. The bird 1 (leader) will try to minimize the distance to food (x_c). The more the γ_1, the more the attractive force to the feeder: the taster the food, the higher the γ_1.

2. The other (following) birds, will try to minimize the distance to the leader at rate proportional to γ_2, which is proportional to the charisma of the leader.

3. Each bird in the flock will feel an attractive force towards the center of the flock that is proportional to κ.

4. Each bird in the flock will feel a repulsive force from the five closest numbers.

For $x_c = x_1$:The bird 1 will not feel any force due to no distance between leader bird and the food. The following birds will only feel forces of attraction of feeder and center of flock.

3 Parameter Impacts on The Flock

The flock behavior is governed by several parameters. The parameters impacting the flock are food-flag, alpha, gamma-1, gamma-2, kappa, rho, delta.

The food flag simply governs which function $C(t)$ the path of the food traces, the choices being $C(t) = (0, 0)$ if food flag is 0 $C(t) = (sin(\alpha * t), cos(\alpha * t))$ if food flag is 1 (Note t represents time and the components of C represent x and y location. The remaining parameters can take any values and will the impacts of various forces.

Alpha governs the period of the $C(t) = (sin(\alpha * t), cos(\alpha * t))$, specifically, how fast the food travels around its path. Smaller values of alpha result in slower movement of the food, while larger values result in faster movement of the food (as well as a smaller radius of path). Values tested were alpha = 0.2, 0.4, 8.0.

Gamma-1 governs the level of attraction the leader bird has to the food. For larger values of gamma-1, we noticed the leader bird almost exactly following the path of the food. For smaller values, we noticed the leader bird did not so closely follow the path of the food. Additionally, we tested setting a negative gamma-1 value for the food and saw the leader bird being repulsed by the food and flying away from it. Values tested were gamma-1 = -1.0, 2.0, 5.0.

Gamma-2 governs the charisma of the leader bird, or how attracted the rest of the flock is to the leader bird. For larger values of gamma-2, the flock would almost disregard other forces and center in exactly where the leader bird was. We also note that as gamma-2 passed a certain threshold (gamma-2 = 9.625), the flock of the birds seem to be polarized and then separate from their leader bird. For smaller values of gamma-2, we observed the diameter of the overall flock being larger as they were less interested in being close to their leader. Values tested were gamma-2 = 4.0-10.0.

Kappa governs the safety of the flock, namely, how close the flock stays to its center of mass. For larger values of kappa, we noticed a smaller flock diameter as the birds were primarily governed by the location of one another, slightly following the leader and slight repulsed from one another. However, past a certain threshold (kappa = 8) the birds appeared to split in 4 directions. For smaller values of kappa, we again noticed a larger flock diameter but not necessarily centered around the leader. Values tested were kappa = 2.0, 4.0, 8.0.

Rho governs the magnitude of the repelling force between the birds of

the flock. For larger values of rho, we noticed a larger flock diameter with erratic movement of the birds. For smaller values of rho we noticed the flock converging directly to the leader bird resulting in a small flock diameter. Values tested were rho = 1.0, 2.0, 4.0.

Delta also governs the magnitude of the repelling force between the birds of the flock. However, for larger values of delta, we noticed a smaller flock diameter as force would be smaller, resulting in less repulsion between the birds. For smaller values of delta we noticed again noticed a larger flock diameter with erratic movement of the birds. Values tested were 0.25, 0.5, 1.0.

4 Flock Diameter

Our flock diameter is approximated by the distance between the leader and farthest bird from the leader. We found the farthest bird from the flock by subtracting bird locations from leader bird location and summing in absolute value then sorting and taking the maximum. Next we computed the distance between the farthest bird and the leader bird by the distance formula.

Our default parameter results in an initial convergence of flock diameter towards 0.02 followed by some oscillations between 0.06 and 0.12.

Flock diameter is also impacted by the charisma of the leader, center of mass, and repelling force between the birds. A more charismatic leader resulted in a smaller flock diameter with time. A less charismatic leader resulted in a larger flock diameter. Similarly, a stronger attraction to the center of mass resulted in a smaller flock diameter with time. A weaker attraction to the center of mass resulted in a larger flock diameter with time. A larger repelling force between the birds resulted in a larger flock diameter. A smaller repelling force between the birds resulted in a smaller flock diameter.

Figures describing the flock diameter can be found below.

2.

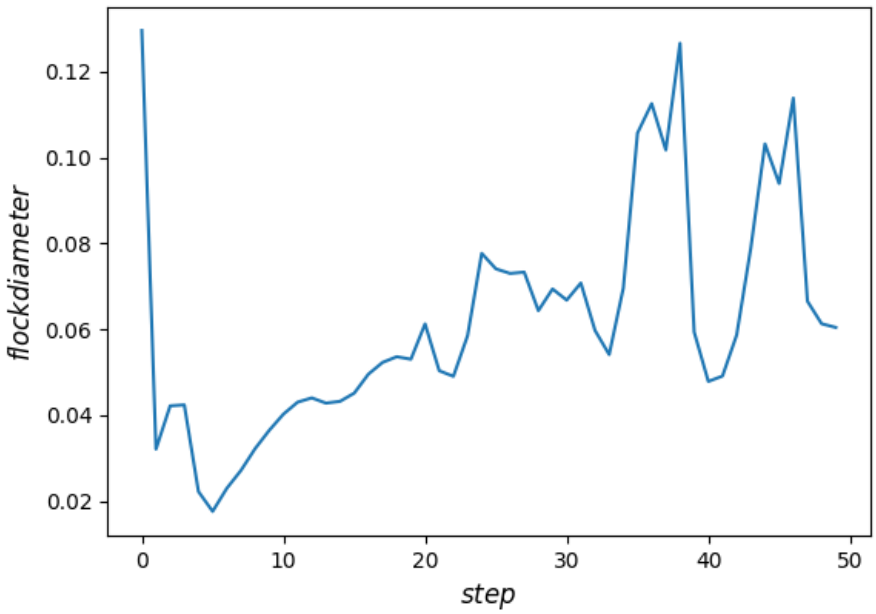

Figure 1: This figure shows flock diameter with respect to time (default).

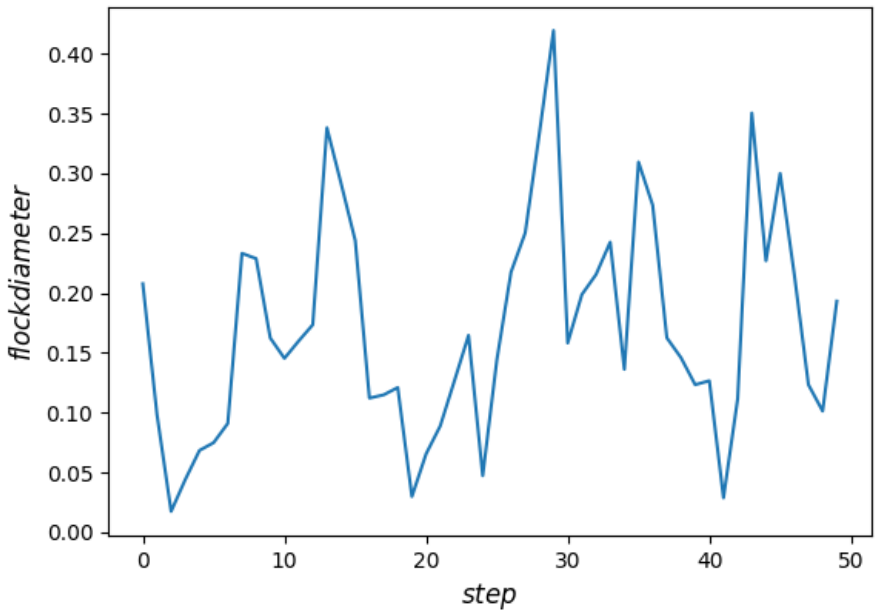

Figure 2: This figure shows flock diameter with respect to time (delta=0.25).

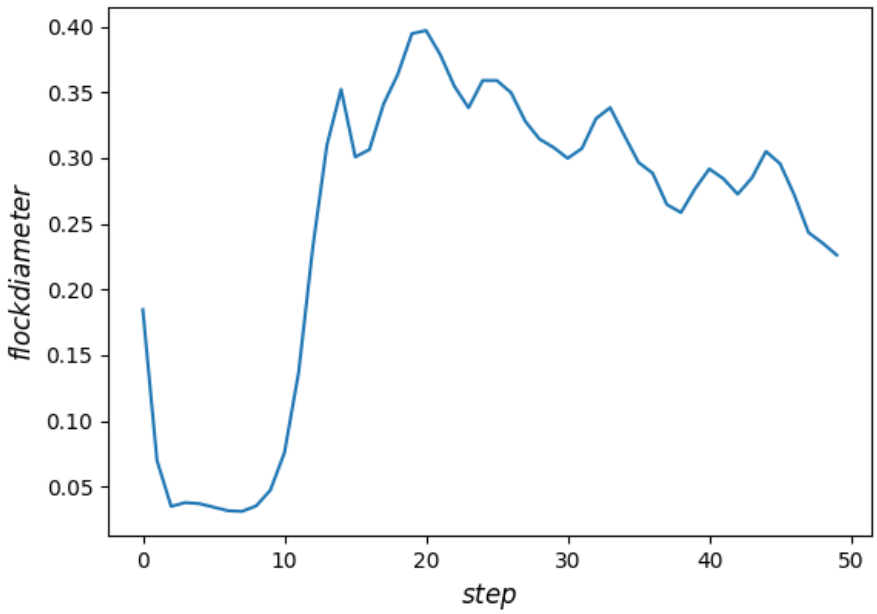

Figure 3: This figure shows flock diameter with respect to time (gamma-1=5).

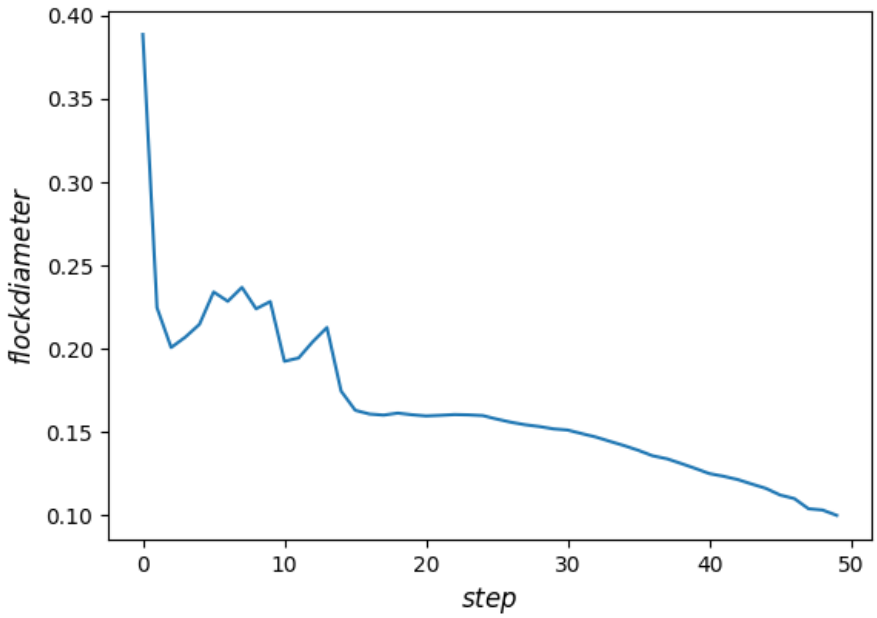

Figure 4: This figure shows flock diameter with respect to time (gamma-2=9.625 near the threshold).

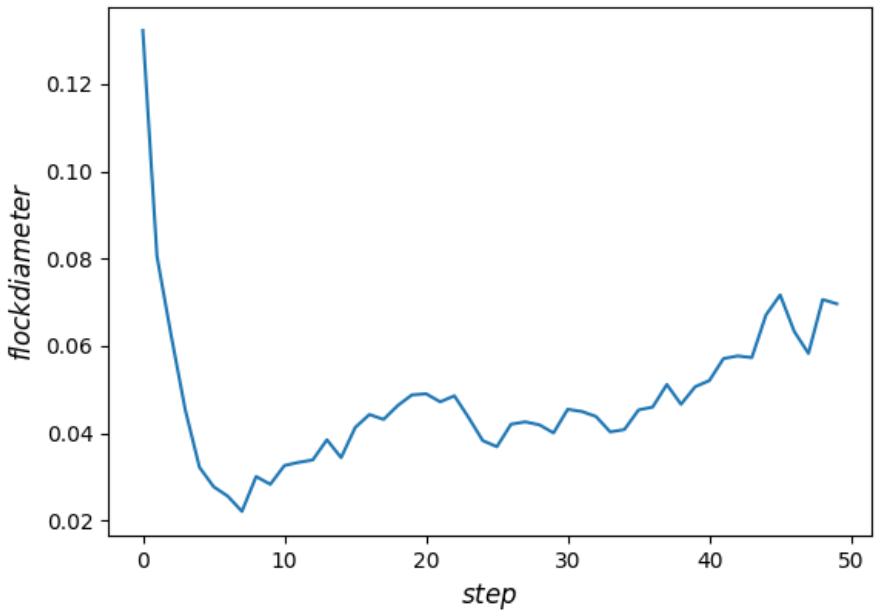

Figure 5: This figure shows flock diameter with respect to time (kappa=2.0).

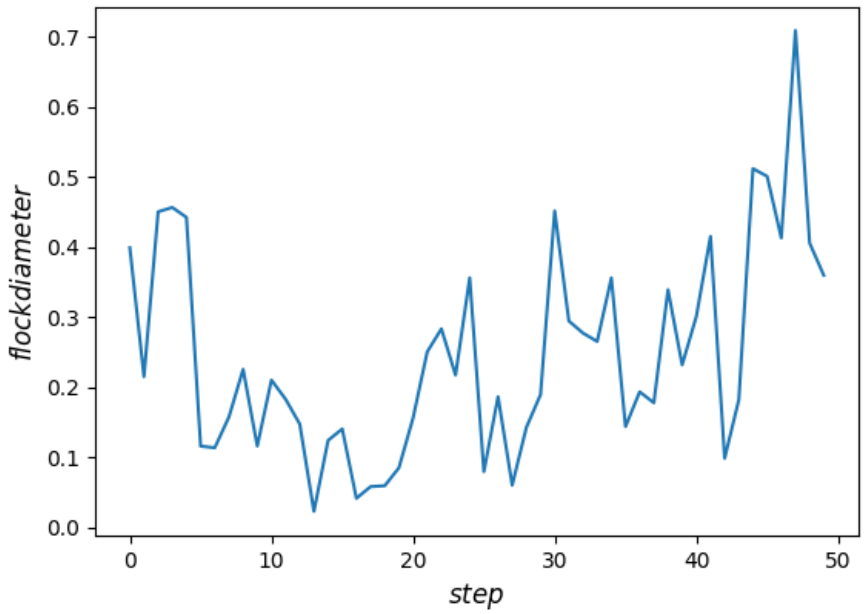

Figure 6: This figure shows flock diameter with respect to time (rho=4.0).

Additional figures for all the various tests can be found within the report subdirectory of the course repository.

5 Addition of the Smelly Bird

Adding the smelly bird to the center of the flock reduced the intensity of the flocking force which is observed by increase of the flock diameter due to higher dispersion of the flocking birds. The smelly birds is shown as a yellow cross on the video. The smelly bird is governed by the parameter beta. Movies regarding the smelly bird may be found in the report subdirectory of the course repository.

6 Addition of the Predator

Adding the predator to the system resulted in interesting movements of the flock. For a stationary predator, the flock gravitated towards the food while keeping it's distance from the center (where the predator was sitting). For the moving predator, we implemented a parametric equation of the lemniscate function (figure eight movement). $C(t) = (cos(\alpha * t)/(1 + sin(\alpha * t)^2), cos(\alpha * t) * sin(\alpha * t)/(1 + sin(\alpha * t)^2))$. For the moving predator we observed the flock attempting to maximize distance from the predator and minimize distance from the food. The predator bird is governed by the parameter gamma-3. Movies regarding the predator may be found in the report subdirectory of the course repository.

7 Instructions to find videos

- The report, movies, and flock diameter plots for this homework are located in the *report* sub directory (*hw4/report*). There are three sub directories: *figures*, *movies*, *tests*.
- The *hw4/report/movies* subdirectory contains selected movies of the of bird dynamics sampling various cases: predator, smelly bird, etc.
- The *hw4/report/figures* subdirectory contains the plots of flock diameter against steps for cases corresponding to those in *movies* subdirectory.
- The *hw4/report/tests* subdirectory contains movies and flock diameter plots for various experimentally varied parameter from the default. The naming convention for files is parameter name on the LHS and parameter assignment to RHS. For example, the name $*movie_2gamma9.mp4*$ means the movie of when parameter *gamma2* was assigned the value of *9*.
- The file titled *HW_5_report.pdf* is our final report.