Let's walk before we run

Suppose management wants to achieve a targeted value at risk on new contracts

- Value at risk (VaR) is the α quantile of portfolio value where α ("alpha") is the organization's tolerance for risk.
- VaR is the maximum amount of tolerable loss. But more loss is possible.

Now suppose

- Management is considering a \$1 billion contract.
- This contract will be two Iberian companies and one United Kingdom-based (UK) company working in Spain.
- Only \$100 million in reserves are available to cover any losses.
- The Board wants some comfort that no more than a 10 % age loss (the average "return" μ) would occur.
- The Board has set the organization's tolerance for risk at 5%: that is, a maximum of 5% of the time could losses exceed 10%.
- To keep things simple: losses are normally distributed. . .

Let perform a "back of the envelope" analysis. We let R stand for returns, so that $-R$ is a loss. Our management team wants

$$Prob(R < -0.10) = 0.05,$$

that is, the probability of a loss worse than 10 % is no more than 5 %.

Let's now do some algebra

- Let w be the "weight" invested in the risky contract.
- The rest, $1 - w$ in high quality collateral assets like treasury bonds.
- The weight w can take on values from 0% (0.0) to 100% (1.0).
- No collateral means $w = 1$; no contract means $w = 0$.

The average return, μ, on the contract is

$$\mu = w(0.1) + (1 - w)(0.02).$$

- This is the weighted average return of the contract at 10% and collateral at 2%.
- The average level of risk in this model is given by the standard deviation of this combination of risky contract and default-free collateral.

Management currently believes that a 25% standard deviation, "sigma" or σ, is reasonable.

$$\sigma = w^2(0.25)^2 + (1-w)^2(0.0).$$

Collateral is not "risky" in this scenario.

- We now try to figure out what w.
- More precisely, the percentage of total assets as investment in this contract, that will make losses happen no greater than 5% of the time.

We form a normalizing "z-score" to help us:

$$z = \frac{-0.1 - \mu}{\sigma}.$$

- This is the ratio of potential deviation of loss from the mean maximum loss per unit of risk.
- Our job is to find w such that the score under the normal distribution cannot exceed 5%.

$$Prob(R < -0.10) = Normal(z(w)) = 0.05,$$

where *Normal* is the cumulative normal distribution (you might know this as =Norm.S.Dist() in Excel or `qnorm()` in `R`).

Using our models of μ and σ we get

$$z = \frac{-0.1 - 0.1w - 0.02(1-w)}{0.25w},$$

or, better after combining constant terms and terms in w and putting this into the target probability:

$$z = Normal\left[\frac{-0.12 - 0.12w}{0.25w}\right] = 0.05.$$

FINALLY, we solve for w in a few more steps.

1. Takes the inverse of the normal distribution on both sides of the equation. On the left hand side this means that we are left with the z score as a function of w, the percentage of all wealth in the risk contract.

$$NormalInverse\left[Normal\left(\frac{-0.12 - 0.12w}{0.25w}\right)\right] = NormalInverse(0.05)$$

*** We can calculate $NormalInverse(0.05)$ using R

```
qnorm(0.05)
```

```
## [1] -1.644854
```

or in Excel with `=norm.s.inv(0.05)`.

2. This means that loss cannot exceed 1.64 times the portfolio standard deviation in the direction of loss ("negative" or less than the mean). Plugging this value in

$$\left(\frac{-0.12 - 0.12w}{0.25w}\right) = NormalInverse(0.05) = -1.64$$

multiplying each side by $0.25w$, combining terms in w and dividing by the coefficient of that last combined w we get

In R:

```r
-0.12/(0.25 * (-1.64) + 0.12)
```

```
## [1] 0.4137931
```

Implication?

- Objective: To fund the new contract at this tolerance for risk and with these reserves.
- Risky contract + collateral = portfolio value.
- 42% of portfolio value = risky contract value.
- Portfolio value = \$1 billion / 0.42 = \$2.38 billion.
- Collateral value = \$2.38 billion - \$1 billion = \$1.38 billion or 68% of portfolio value.

- We just found the notorious "tangency" portfolio.
- This portfolio, when combined with a risk-free (really "default-free" asset), will yield the best mix of risky and risk-free assets.
- "Best" here is in the sense of not violating the organization's risk tolerance policy.

How?

1. Find the optimal combination of risky assets, the tangency portfolio.
2. Then find the optimal mix of tangency assets and the risk-free asset.
3. Working capital's "risk-free" asset is the cash account and the process of getting there is the cash-conversion cycle.

All in

- Now that we have our basic procedure, let's complicate this problem with many risky assets.
- The basic solution will be choosing weights to minimize the portfolio risk given risk-adjusted return targets. This is the Markowitz (1952) portfolio solution.
- For this task we need to define a matrix version of the portfolio allocation problem.
- Our three risky "assets" will be the euro/USD and GBP/USD exchange rates and Brent crude.

This is all about the normally distributed universe.

First, the return matrix R for N assets across T sample periods and the subscript indicates the row (observation) and column (asset):

$$\left[\begin{array}{ccc} R_{11} & ... & R_{1N} \\ ... & ... & ... \\ R_{1T} & ... & R_{TN} \end{array} \right]$$

Then, the mean return vector μ is the arithmetic average of each column of R

$$\left[\begin{array}{c} \mu_1 \\ ... \\ \mu_N \end{array} \right],$$

after exchanging rows for columns (transpose).

Now try this ...

Try this computation

- Be sure the qrmdata package is installed.
- Call this package and the daily data in it.
- Look up the apply function to see if you can compute row averages.

```
require(qrmdata)
require(xts)
# The exchange rate data was obtained
# from OANDA (http://www.oanda.com/)
# on 2016-01-03
data("EUR_USD")
data("GBP_USD")
# The Brent data was obtained from
# Federal Reserve Economic Data
# (FRED) via Quandl on 2016-01-03
data("OIL_Brent")
data.1 <- na.omit(merge(EUR_USD, GBP_USD,
    OIL_Brent))
R <- na.omit(diff(log(data.1)) * 100)
names.R <- c("EUR.USD", "GBP.USD", "OIL.Brent")
colnames(R) <- names.R
```

```r
(mean.R <- apply(R, 2, mean))
```

```
##      EUR.USD      GBP.USD    OIL.Brent
##  0.001538585 -0.002283062  0.010774203
```

Some questions

1. Look at a `summary` of a few columns. Notice anything odd or curious?
2. What does the 2 indicate in the `apply` function.
3. What is Brent crude's annualized mean "return"?

Thinking...

Results

For question 1:

```
summary(R)
```

```
##      Index                  EUR.USD              GBP.USD
##  Min.   :2000-01-05   Min.   :-2.522684   Min.   :-4.648461
##  1st Qu.:2003-12-18   1st Qu.:-0.308317   1st Qu.:-0.277715
##  Median :2007-12-05   Median : 0.013886   Median : 0.006097
##  Mean   :2007-12-19   Mean   : 0.001539   Mean   :-0.002283
##  3rd Qu.:2011-12-19   3rd Qu.: 0.322014   3rd Qu.: 0.286959
##  Max.   :2015-12-28   Max.   : 3.463777   Max.   : 3.140629
##    OIL.Brent
##  Min.   :-19.89065
##  1st Qu.: -1.15322
##  Median :  0.03604
##  Mean   :  0.01077
##  3rd Qu.:  1.24927
##  Max.   : 18.12974
```

- Means are much less than medians.
- Huge max and min returns.
- We can also look at `acf` and `ccf`, absolute returns, run GARCH models, and so on...

But save that for another day.

For question 2:
Look up `??apply` and read that the 2 indicates that we are calculating the mean for the second dimension of the data matrix, namely, the assets.

For question 3:
Brent crude's annualized mean return is calculated on a 252 average days traded in a year basis as:

```
(1 + mean.R[3]/100)^252 - 1
```

```
##  OIL.Brent
## 0.02752144
```

Some folks use 253 days. But this is all a back of the envelope computation.

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