Finance Essentials

Risky Loan and Risk Premium

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Note: Calculations shown in this document are performed in the spreadsheet Risky Loan and Risk Premium.

The Scenario

Suppose a lender, possibly a banker, charges interest rate of 5% per year on one-year loans to risk-free borrowers. Risk-free borrowers will pay back the loan plus interest on time as per the loan contract. Therefore, \$1,000,000 loaned to a group of risk-free borrowers will result in a cash inflow of $\$1{,}000{,}000 \times (1 + 5\%) = \$1{,}050{,}000$ one year later.

Now consider a group of risky borrowers. These borrowers are risky because *some* of them will not pay back the loan as required by the contract, i.e., will default on the loan. The lender is unable to differentiate between the potential defaulters and non-defaulters among the borrowers in the risky group.[1] If the lender could differentiate, he/she would include the non-defaulters in the risk-free group and not make loans to the defaulters.

In the real world, there are many different risky groups with different characteristics. Credit scores and credit ratings are used to differentiate among the various risky groups. For the sake of simplicity, I am considering just one risky group.

Suppose 10% of the borrowers in the risky group will default on their payments while the remaining 90% will pay back the loan as per the contract. When a borrower defaults, the lender will have to use other means to collect the loan. From some defaulted borrowers, the lender will not get back anything while from others it will collect a partial amount. Let's say that, on an average, net of the collection costs, the lender will collect 60 cents of each dollar loaned to the borrowers who default, i.e., will lose 40 cents of every dollar loaned.

The problem the lender faces is what interest rate it should charge the borrowers in the risky group. Let us denote this rate by i. The *expected* cash flow from a \$1,000,000 loaned to the risky group will be:

$$\$1{,}000{,}000 \times (1 - 10\%) \times (1 + i) + \$1{,}000{,}000 \times 10\% \times 60\%$$

[1] It may not be possible for anyone, even the borrowers, to know with certainty in advance whether an individual borrower in the risky group will default or not.

The first term in the expression above shows the cash flow from the borrowers who will not default. The second term shows the cash flow from the borrowers who will default. The expression can be simplified as:

$$\$1{,}000{,}000\,[(1-10\%)\times(1+i)+10\%\times 60\%]$$

Risk-Neutral Lender

To find the interest rate i that the lender should charge, let us start with the case where the lender is *risk-neutral*, and therefore will be indifferent between making the loan to the risk-free and risky groups as long as the *expected* future cash flow from the risky group equals the future cash flow from the risk-free group:

$$\$1{,}000{,}000\,[(1-10\%)\%\times(1+i)+10\%\times 60\%]=\$1{,}050{,}000$$

which gives us

$$i=10\%$$

The general expression for the interest rate is:

$$i=\frac{(1+r_f)-(p_D\times c)}{1-p_D}-1 \tag{1}$$

where r_f is the interest rate charged on loans to the risk-free group, p_D is the percent of risky borrowers that will default, and c is the percent of original loaned amount ("cents per dollar") recovered from the defaulted risky borrowers.

The expected return on investment from the loan made to risky group of borrowers is the weighted average of the interest rate from the risky borrowers who don't default and the loss from those that do default:

$$(1-10\%)\times 10\%+10\%\times(60\%-1)=5\%$$

Not surprisingly, the expected return from the group of risky borrowers is equal to the interest rate charged from the risk-free borrowers. This is a direct result of the specification that the expected future cash flow from risky borrowers is equal to the future cash flow from the risk-free borrowers.

The general expression for the expected return is:

$$E(r)=(1-p_D)\times i+p_D\times(c-1) \tag{2}$$

It is important to understand the difference between i and $E(r)$: i is the contractual interest rate between the lender and the borrower while $E(r)$ is the expected return on the lender's capital. If all borrowers make payments as per the contract, the expected return will be equal to the contractual interest rate. Since some borrowers will default and the lender will collect only a fraction of the amount loaned from the defaulted borrowers, the expected return on the amount loaned will be less than the contractual interest rate.

Risk-Averse Lender

The calculations in the previous section are based on the assumption that the lender is risk-neutral since he/she is indifferent between some rate of return on a risk-free investment (loan to the risk-free borrowers) and the same amount the same amount as expected return on a risky investment.

Real lenders are not risk averse. They would ask: Why should I take the chance by making loans to risky borrowers when I can earn the same return by making the loan to risk-free borrowers? What if my *sample* of risky borrowers doesn't turn out exactly as expected? What if more than 10% of the borrowers in my group of risky borrowers default? What if only 30 cents of each dollar loaned is recovered instead of 60 cents? These thing can happen because the experience of a particular lender during a particular lending period may be different than the characteristics of the population.

Risk averse lenders will seek some premium, extra expected return, to make the loan to the risky group. Suppose the lender wants to earn a 2% premium on his investment to justify taking this risk, i.e., a *risk premium* of 2%. Incorporating the risk premium, the expected return on the capital loaned to the risky borrowers is $5\% + 2\% = 7\%$. The interest rate charged from the borrowers in the risky group can now be calculated using equation (2):

$$\begin{aligned} 7\% &= (1 - 10\%) \times i + 10\% \times (60\% - 1) \\ i &= \frac{7\% - 10\% \times (60\% - 1)}{1 - 10\%} = 12.22\% \end{aligned}$$

The extra interest rate paid by the borrowers in the risky group, therefore is $12.22\% - 5\% = 7.22\%$.

Some remarks:

1. I assumed that the higher interest rate will not change the behavior of the risky borrowers. This will not be the case in reality. A higher interest rate itself may lead to a greater percent of borrowers defaulting.

2. I made up risk premium required by the lender to be 2%. In reality, the required risk premium will depend on the investor's risk-aversion and the market conditions.

3. Since borrowers in the risky group are paying 12.22% on their loans compared to 5% being paid by the risk-free borrowers, some borrowers who find themselves in the risky group though they believe that they should be classified as risk-free, or at least less risky than some of the other borrowers in the group. They will be highly motivated to provide evidence of their being risk-free or lower risk. This will lead to the creation of several risk pools.

4. The ideas developed in this note can be applied, with modifications, to investing, insurance and other fields.

Conclusion

Investors need risk premium to entice them to invest in risky ventures. The value of risk premium would depend on the riskiness of the venture and the risk-aversion of the investor. Risk premium may be measured as the difference between the expected return demanded on the risky venture minus the risk-free rate of return:

$$\text{risk premium} = \text{expected return} - \text{risk-free rate}$$