Finance Essentials

Rate of Return

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Investment Trade-Offs

Investing provides a *trade-off* between consumption forgone today and additional wealth available for consumption in the future.

> Shares of stock $\mathcal{A}$ are selling for \$10 each. One year from now, shares of $\mathcal{A}$ will sell for \$15. (Let us assume perfect certainty for now; we'll incorporate uncertainty a little bit later.) You forgo consumption of \$10 today and buy a share of $\mathcal{A}$ instead. One year from now, you will sell the share and consume \$15.
>
> Shares of stock $\mathcal{B}$ are also selling for \$10 each, and one year from now will sell for \$18.

If we have to choose between $\mathcal{A}$ and $\mathcal{B}$, we should choose $\mathcal{B}$ since it provides a higher future payoff for the same initial investment.

Returns

In the examples above, it was easy to compare $\mathcal{A}$ and $\mathcal{B}$ since both had the same current stock price: \$10. To make the comparison possible between stocks that are not priced the same, we quantify the trade-off between the consumption forgone and future consumption using the metric known as *return*:

$$r_1 = \frac{p_1 - p_0}{p_0} \tag{1}$$

where p_0 and p_1 are the prices today and in the future, respectively. r_1 is the return during period 1, i.e., between time 0 and time 1. It measures profits per dollar invested. For now I am assuming that there will not be any intermediate cash flows such as dividends or interest. We'll introduce intermediate cash flows shortly. It may be helpful to note that return can also be written as:

$$r_1 = \frac{p_1}{p_0} - 1$$

> Returns on stocks $\mathcal{A}$ and $\mathcal{B}$:

$$r_{1_\mathcal{A}} = \frac{\$15 - \$10}{\$10} = 0.5 = 50\%$$
$$r_{1_\mathcal{B}} = \frac{\$18 - \$10}{\$10} = 0.8 = 80\%$$

While I used *stocks* in the examples so far, I could have as easily used bonds or other investments in the example.

Bond $\mathcal{Q}$ is selling for \$9,782. One year from now its price will be \$10,000. The return on the bond investment is:

$$r_{1_{\mathcal{Q}}} = \frac{\$10{,}000 - \$9{,}782}{\$9{,}782} = 0.022285831 \approx 2.23\%$$

Returns with Intervening Cash Flows

If the investment provides some cash flow such as dividend or interest, the return would be calculated as:

$$r_1 = \frac{d_1 + p_1 - p_0}{p_0} \tag{2}$$

where d_1 is the dividend, interest, or some other cash flow. This equation is based on the assumption that the cash flow d_1 happens at the end of the year, a split second before the end-of-the-year price p_1 is recorded. Note that $d_1 + p_1$ represents the value of the investment at the end of the year. This equation can also be written as:

$$r_1 = \frac{d_1 + p_1}{p_0} - 1$$

Shares of stock $\mathcal{C}$ are selling for \$20 each. One year from now, the stock will pay a dividend of \$1 per share. Immediately after the stock pays the dividend, the price of the shares will be \$124. The return is calculated as:

$$r_1 = \frac{\$1 + \$24}{\$20} - 1 = 25\%$$

As mentioned above, the calculations above are based on the assumption that the cash flow (dividend) occurs one year from now, a moment before the price p_1 is recorded. If the cash flow occurs at some time between $t = 0$ and $t = 1$, we have to use its value at the end of the year from reinvesting in the stock.

Shares of stock $\mathcal{D}$ are selling for \$100 each. Six months from now, the stock will pay a dividend of \$2 per share. The price of the stock immediately after the dividend will be \$106. The price of the stock at the end of the year will be \$105. The rate of return can be calculated using equation (2) but d_1 needs to be the value of the dividend from reinvesting in the stock. With \$2 dividend, $\$2/\$106$ shares of the stock can be purchased. The value of these shares purchased using the dividend will be $\$2/\$106 \times \$105 = \$1.981132075$ at the end of the year. So, the return will be:

$$r_1 = \frac{\$1.981132075 + \$105 - \$100}{\$100} = 0.069811321 \approx 6.98\%$$

If information is not available to calculate the reinvested value, we need to calculate the internal rate of return using the cash flows.

A stock is priced at \$100. Six months from now it will pay a dividend of \$2 per share. A year from now, the stock will be priced at \$106. Since we don't know the ex-dividend price six months from now, we can't calculate the reinvested value of the dividend. So, we have to resort to the internal rate of return. The rate of return is the IRR, i.e., the discount rate that makes the present value of future cash flows equal to the current price:

$$\$100 = \frac{\$2}{(1+r)^{0.5}} + \frac{\$106}{1+r}$$

We can solve for r using trial-and-error to get 7.07% as the return.

Rates of Returns

Returns do not provide a complete information about the investments. For example, two investments may provide a return of 11%. However, one of these investments may provide this return over a six month period while the other one may do so over a one year period. To avoid confusion in such circumstances, returns are standardized by associating the time unit with the return and are expressed as % per year or % per quarter. Returns per unit of time are known as rates of return. Rates of return allow us to compare the trade-offs inherent in investments of different lengths or time horizons.

Sometimes rates of return are expressed in different time units. To compare them, we need to convert the rates to a common time unit using principles of compound interest.

- Because of compounding 1% per month is equivalent to 12.6825% per year, not 12% per year.
- In general,

$$(1 + r_{\mathrm{annual}}) = (1 + r_{\mathrm{monthly}})^{12} \tag{3}$$

because there are 12 months in a year. Similar relationships can be written for other time intervals.

Stock $\mathcal{M}$ has a rate of return of 4.2% per quarter while stock $\mathcal{N}$ has a rate of return of 17.5% per year. To find out which stock has a higher rate of return we can either convert $\mathcal{M}$'s rate of return to annual unit or $\mathcal{N}$'s rate of return to quarterly unit. The relationship between a return expressed in quarterly units (r_q) and the same return expressed in annual unit (r_a) is:

$$(1 + r_{\mathsf{annual}}) = (1 + r_{\mathsf{quarterly}})^4 \tag{4}$$

Let's try both alternatives:

- Convert $\mathcal{M}$'s return to annual unit:

$$(1 + r_{\mathsf{annual}}) = (1 + 4.2\%)^4 \quad\Longrightarrow\quad r_{\mathsf{annual}} = 0.178883464 \approx 17.89\%$$

So, the rate of return of $\mathcal{M}$, 17.89% per year, is higher than the rate of return of $\mathcal{N}$, 17.5% per year.

- Convert $\mathcal{N}$'s return to quarterly unit:

$$(1 + 17.5\%) = (1 + r_{\mathsf{quarterly}})^4 \quad\Longrightarrow\quad r_{\mathsf{quarterly}} = 0.041140802 \approx 4.11\%$$

So, the rate of return of $\mathcal{M}$, 4.2% per quarter, is higher than the rate of return of $\mathcal{N}$, 4.11% per quarter.

Expected Rates of Returns

In our calculations so far, we pretended that we know the future cash flows and prices with perfect certainty. In real life we never have certainty about the future cash flows. so, we use expected values of future cash flows, $E(d_1)$ and $E(p_1)$ to calculate the expected return, $E(r)$ using a slight variation of equation (2):

$$E(r_1) = \frac{E(d_1) + E(p_1) - p_0}{p_0} \tag{5}$$