MBC 638

LIVE SESSION WEEK 9

Agenda

Topic	Time	Sunday Section	Wednesday Section
Introduction	5 min	6:30-6:35	9:00 - 9:05
Highlights from Week 9 Video	30 min	6:35-7:05	9:05 - 9:35
Start on Final Review	25 min	7:05-7:30	9:35 - 10:00
Breakout on Regression	20 min	7:30 – 7:50	10:00 – 10:20
Review of Upcoming Assignments and Open Question	10 min	7:50-8:00	10:20 - 10:30

9.2 Introduction to Time Series

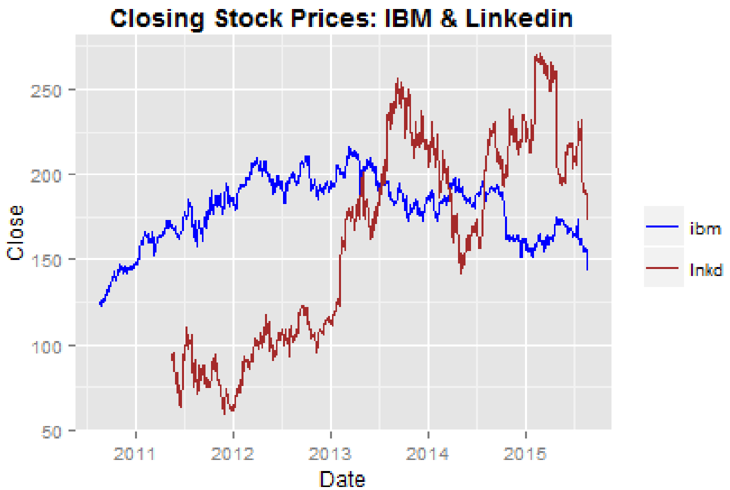
Time series analysis comprises methods for analyzing **time series** data in order to extract meaningful statistics and other characteristics of the data.

Data is collected at regular intervals over a given time period.

Time series forecasting is the use of a model to predict future values based on previously observed values. (a bit like looking in the rear view mirror to predict the future)

Time = input, Time Series =output

Y = f(y), doesn't really consider x's driving the output



http://blog.revolutionanalytics.com/2015/08/plotting-time-series-in-r.html

9.2 Introduction to Time Series

Potential Components of Variation

1. Trend
   - Long-term rise and fall

2. Calendar cycles
   - Seasonality

3. Business cycles
   - Affected by American politics

4. Autoregressive behavior - A stochastic process used in statistical calculations in which future values are estimated based on a weighted sum of past values. **An autoregressive process operates under the premise that past values have an effect on current values.** A process considered AR(1) is the first order process, meaning that the current value is based on the immediately preceding value. An AR(2) process has the current value based on the previous two values. http://www.investopedia.com/terms/a/autoregressive.asp#ixzz3cTF1ZCQm

5. Random variation

9.2 Introduction to Time Series

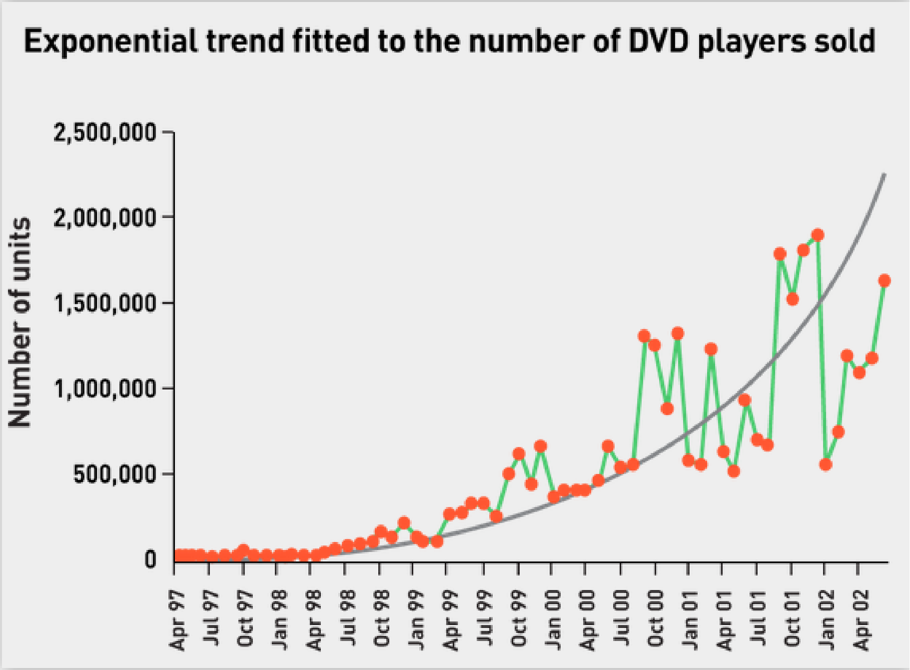
Time Series Analysis

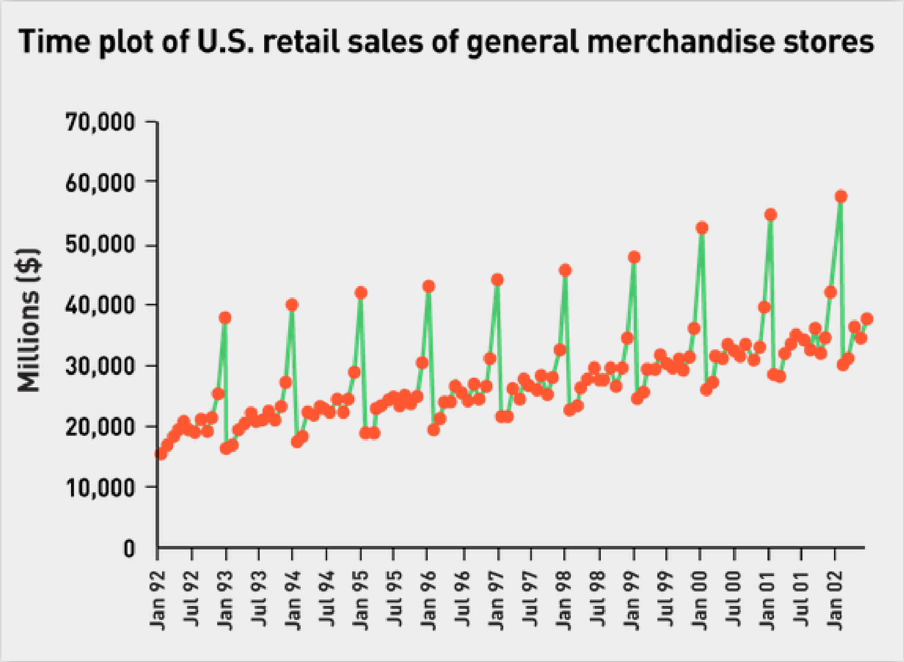
1. Time plot: tool to study/visualize time series
2. Model patterns: trends and seasonality
3. Forecast: predict future values of time series
4. Remember practical, graphical, analytical

9.2 Introduction to Time Series

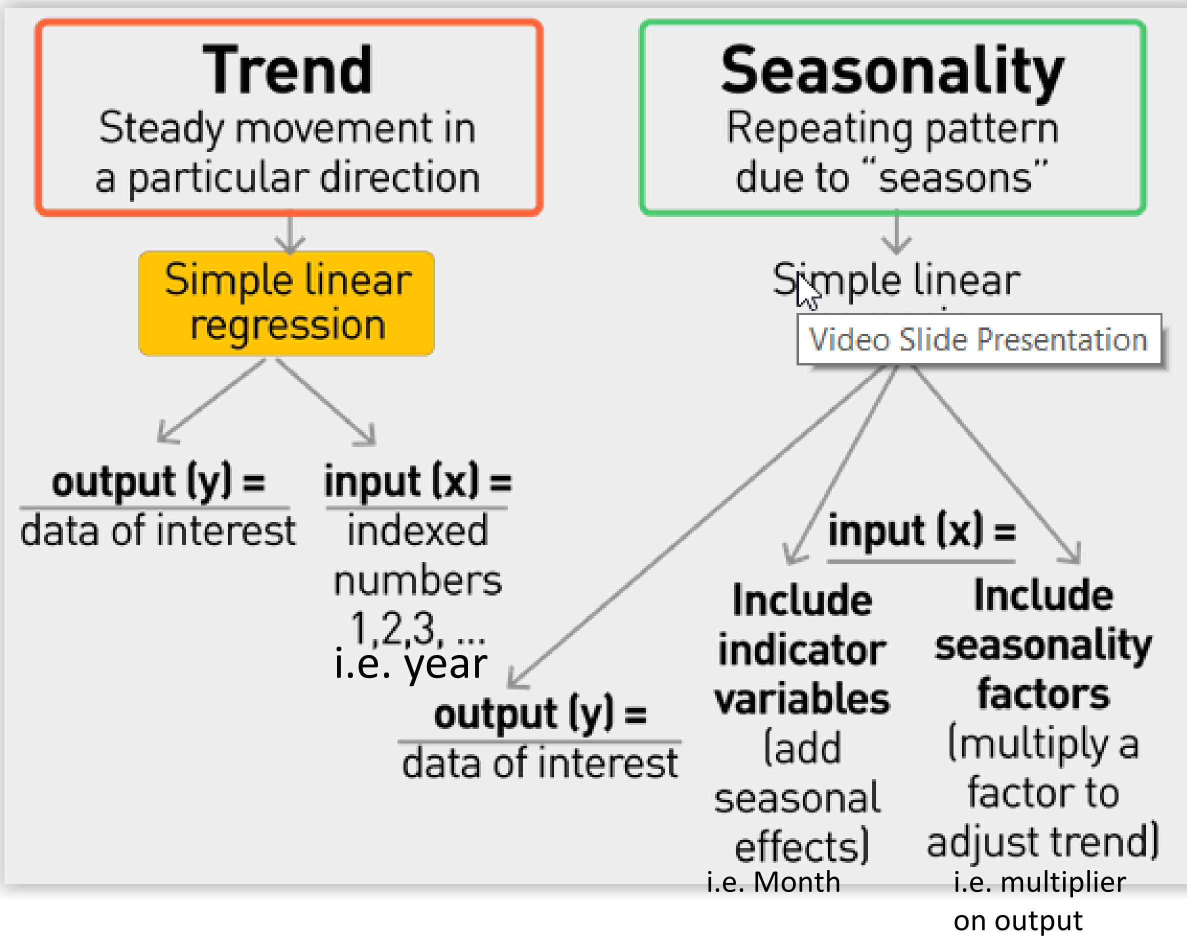
Looking for Systematic Patterns

- Trends
- Steady movement in a particular direction
- Seasonality (repeated pattern)

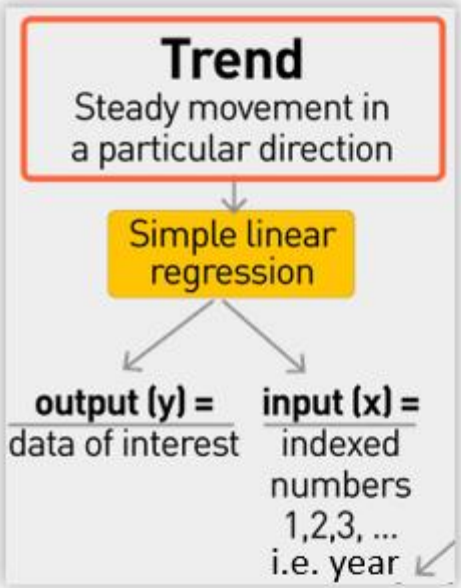




9.2 Introduction to Time Series



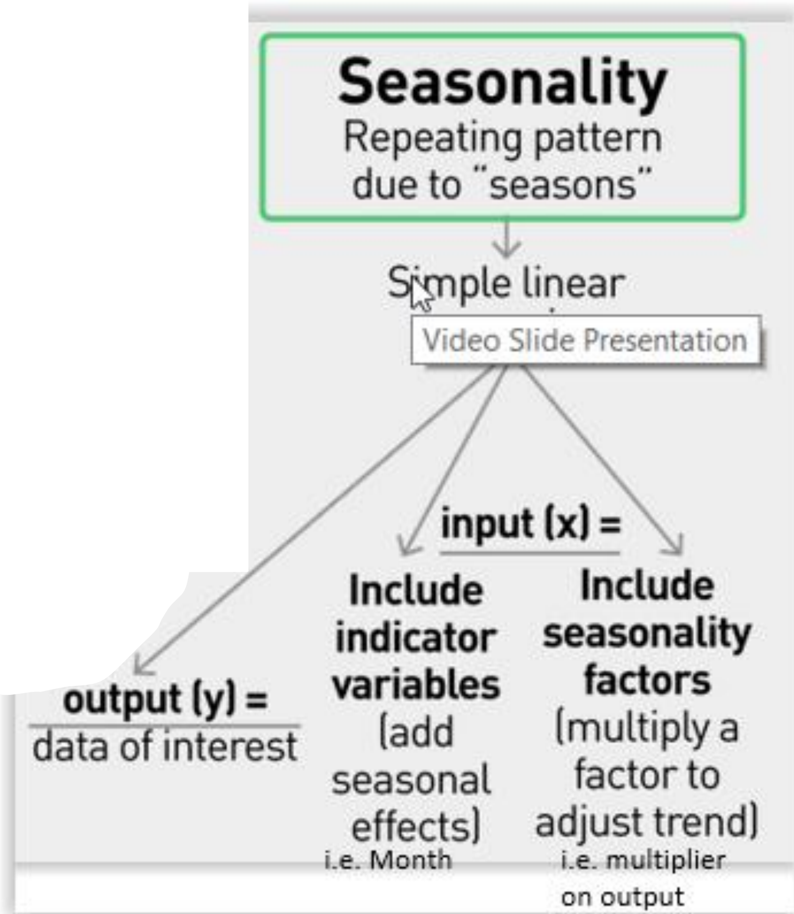
9.2 Introduction to Time Series



Trend

- Simple linear regression
  - Input (x) = indexed numbers, e.g., 1979, 1980, 1981
  - Output (y) = data of interest, e.g., budget information

9.2 Introduction to Time Series



Seasonality

- Simple linear regression
  - Input (x) =
    - Indicator variables
    - Seasonality factors
  - Output (y) = data of interest

Indicator Variables

- Months as indicator variable
  - Use x_1–x_{11} (K – 1)
- Trend + season model:
  $\hat{y} = \beta_0 + \beta_1 x + \beta_2 x_1 + \beta_3 x_2 + \ldots + \beta_{12} x_{11}$
  - Trend: $\beta_0 + \beta_1 x$
  - Seasonality: $\beta_2 x_1 + \beta_3 x_2 + \ldots + \beta_{12} x_{11}$

Seasonality Factors

- Calculate adjustments, multiply by regression equation
- For each data point in time series, calculate ratio

$$\frac{\text{Actual } y}{\text{Predicted } y} = \text{Seasonality Factor (SF)}$$

- Average SF by month → 12 SFs
- Multiply regression equation by a given month's SF to account for seasonality in a trend model.
- Trend + season model: $\hat{y} = (\beta_0 + \beta_1 x) \times \text{SF}$

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9.3 Autocorrelation

Regression on Time Series Data

Modeling trend and seasonal components may not generate random residuals

Residual plots help assess the fit of a regression line

Autocorrelation: Definition

- Relationships between neighboring points
  - E.g., January data affects February, February affects March, etc.
  - Can cause lack of randomness in data
- **Autocorrelation**: correlation between successive values

DEFINITION of 'Autocorrelation'

A mathematical representation of the degree of similarity between a given time series and a lagged version of itself over successive time intervals. It is the same as calculating the correlation between two different time series, except that the same time series is used twice - once in its original form and once lagged one

http://www.investopedia.com/terms/a/autocorrelation.asp#ixzz3cU65w3O0

3 Autocorrelation

Autocorrelation refers to the correlation of a time series with its own past and future values. Autocorrelation is also sometimes called *"lagged correlation"* or *"serial correlation"*, which refers to the correlation between members of a series of numbers arranged in time. Positive autocorrelation might be considered a specific form of *"persistence"*, a tendency for a system to remain in the same state from one observation to the next. For example, the likelihood of tomorrow being rainy is greater if today is rainy than if today is dry. Geophysical time series are

www.ltrr.arizona.edu/.../notes_3.pdf

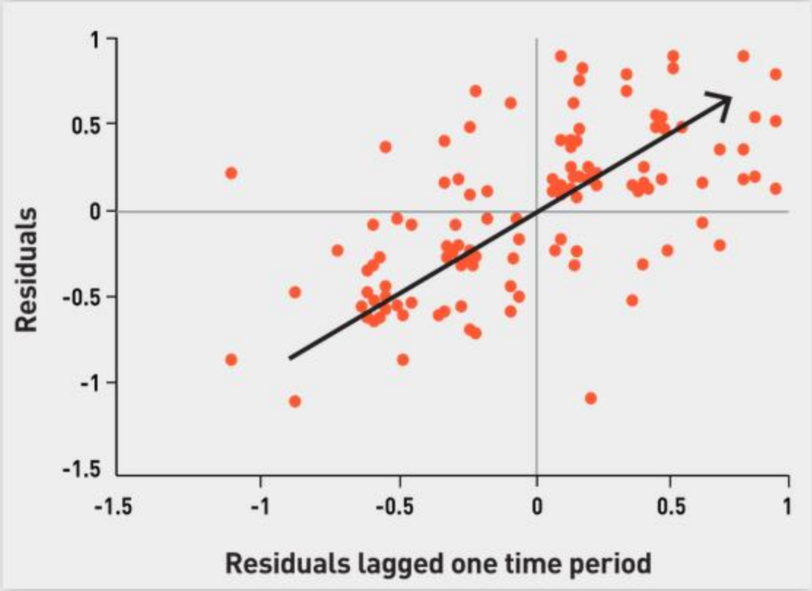
9.3 Autocorrelation

Autocorrelation: Why Do We Care?

- Can't use regression model if data violates assumption of independent residuals
- Autocorrelation in residuals indicates opportunity to improve fit
  - Add elements to model to increase predictive power

Autocorrelation: How Can We Tell?

- Test residuals by lagging, moving one time period
  - Residual = e = $y_{actual} - y_{predicted}$
  - Lagged residual plot = $(e_1, e_2), (e_2, e_3), (e_3, e_4)...(e_{n-1}, e_n)$
- Plot residuals, look for pattern



9.3 Autocorrelation

Summary

- We manipulate the y as input variable
- Work with limited existing data to better predict future through manipulation
- Look for autocorrelation, which indicates manipulation is required
  - Autocorrelation: relationship between neighboring points

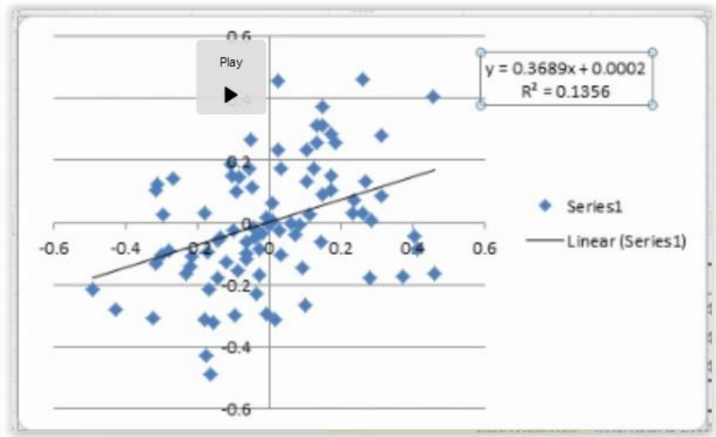
9.4 Is Autocorrelation Present?

Step 1: generate X by using lagged Y

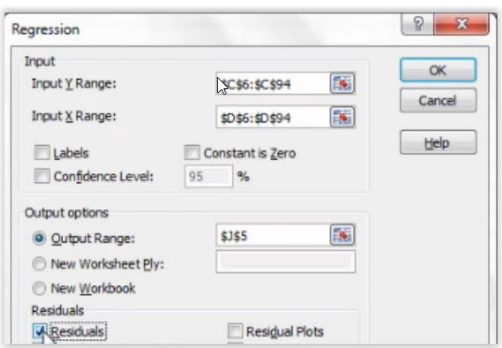
A	B	C	D
	Great Lakes Water Level		
			(this is y_{t-1})
		output (y)	(x)
	Year	**Lake Level**	**Lagged output**
1	1918	176.887	
2	1919	176.745	176.887
3	1920	176.625	176.745
4	1921	176.488	176.625
5	1922	176.445	176.488
6	1923	176.264	176.445
7	1924	176.187	176.264
8	1925	175.919	176.187
9	1926	175.885	175.919
10	1927	176.148	175.885

Step 3: Create a residual and lagged residual plot: Is there a pattern?
Y = residual, X=lagged residual

x input lagged residu	y output Residual
	-0.06271085
-0.06271085	-0.06305948
-0.06305948	-0.09837441
-0.09837441	-0.02627936
-0.02627936	-0.17051346
-0.17051346	-0.09528201
-0.09528201	-0.29732568
-0.29732568	-0.10556241
-0.10556241	0.18662801
0.186628012	0.25921724
0.259217242	0.46256087
0.462560873	-0.16461966
-0.16461966	-0.49019318
-0.49019318	-0.21794482



Step 2: Run a regression on your X and Y above, skipping the first value



	Coefficients	*Standard Error*
Intercept	27.4096104	10.19938281
X Variable 1	0.844597862	0.057804166

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*
1	176.8077108	-0.062710846
2	176.6880595	-0.063059482

Step 4: Conclusion R^2 is small, so not a strong relationship, so it is ok to use regression

9.5 Three Time Series Models

Time Series Models

1. First-order autoregressive model, a.k.a. AR(1)
2. Moving average forecast model
3. Exponential smoothing model

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series
- **First-order autoregressive model**
  - Linear regression equation: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$
  - y_t = output at time t
    - Example: March; $t = 3$
    - $y_3 = \beta_0 + \beta_1 y_{3-1} + \varepsilon_3$
    - I.e., predicting March data by using February data

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9.6 Forecast the Next Month: First Order Autoregressive Model

Forecast 2008:: using first order autoregressive model

Run Regression
plug 2008 input value into the equation to estimate y.

Assume we do have autocorrelation, assumption for regression is independence of errors this model, autoregressive, gives us permission to use regression anyways.

Total	88	12.42291	
	Coefficients	Standard Err	
Intercept	27.40961	10.19938	2
X Variable 1	0.8445979	0.057804	14.611

: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$

0.84459x + 27.4096 = y predicated = y hat
forecast 2008
use 2007 data as the input for x
175.943

Great Lakes Water Level

	Year	output (y) Lake Level	(this is y_{t-1}) (x) Lagged output
1	1918	176.887	
2	1919	176.745	176.887
3	1920	176.625	176.745
4	1921	176.488	176.625
5	1922	176.445	176.488
6	1923	176.264	176.445
7	1924	176.187	176.264
8	1925	175.919	176.187
9	1926	175.885	175.919
10	1927	176.148	175.885

85	2002	176.118	175.951
86	2003	175.892	176.118
87	2004	176.111	175.892
88	2005	176.090	176.111
89	2006	176.016	176.090
90	2007	175.943	176.016
91	2008		175.943

=0.84459*(175.943)+27.4096

176.0092984 meters 2008

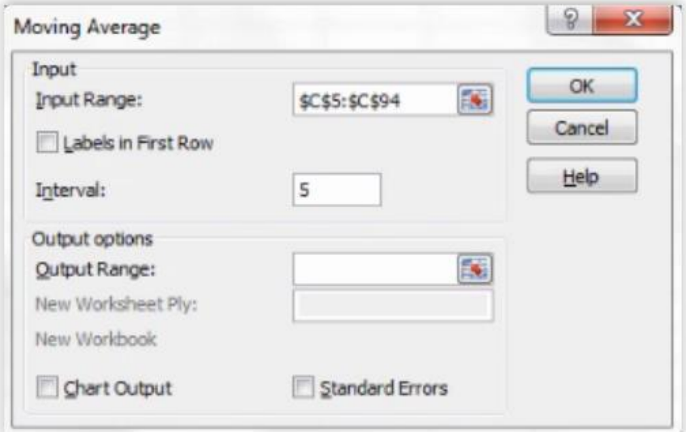
9.5 Three Time Series Models

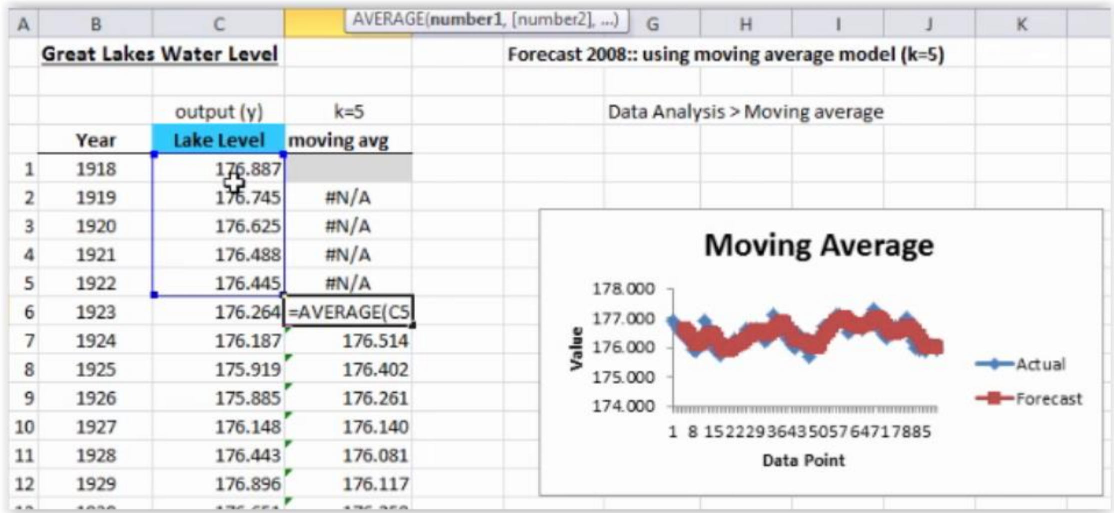
Moving Average Model

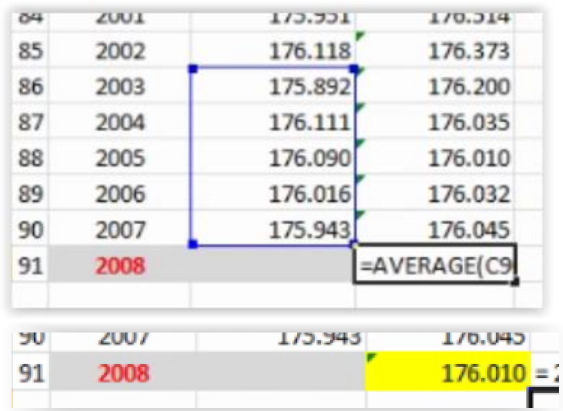
A.k.a. "rolling average method"; smooths out short-term fluctuations

- Uses average of last several values of time series to forecast next value; k= number of values in span
  - Example: Monthly data, span (k) = 3
    - I.e., use average of values from January, February, and March to predict April value
- Can look back more than one time period
- Disadvantage: If, say, n = 100 and k = 5, forecast overlooks 95% of available data

9.7 Forecast the Next Month: Moving Average Model







Smaller your K, the bumpier your forecast the closer to the actual data.

9.5 Three Time Series Models

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally
- Forecasting equation: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
  - $\hat{y}_t$ = estimate of y at time period t
    - Example: t = February
  - w= smoothing constant
    - Your choice, pick any number between 0 and 1
    - Example: w = 80%

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
- February forecast = $\hat{y}_t$
- 80% of January value = wy_{t-1} = 0.8 y_{t-1}
- 20% of January forecast = $(1 - w)$ $\hat{y}_{t-1}$

1 − 0.8)

$\hat{y}_{t-1}$

).2

$\hat{y}_{t-1}$

- January forecast = $\hat{y}_{t-1} = \hat{y}_{2-1}$
  - January forecast incorporates data from December and prior months
- Final equation: $\hat{y}_2 = 0.8\ y_{2-1} + (1 - 0.8)\ \hat{y}_{2-1}$

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Forecast Feb = 80% of Actual January + 20% of Forecasted January

Here w=80%, if it were lower than you are using less of actual January

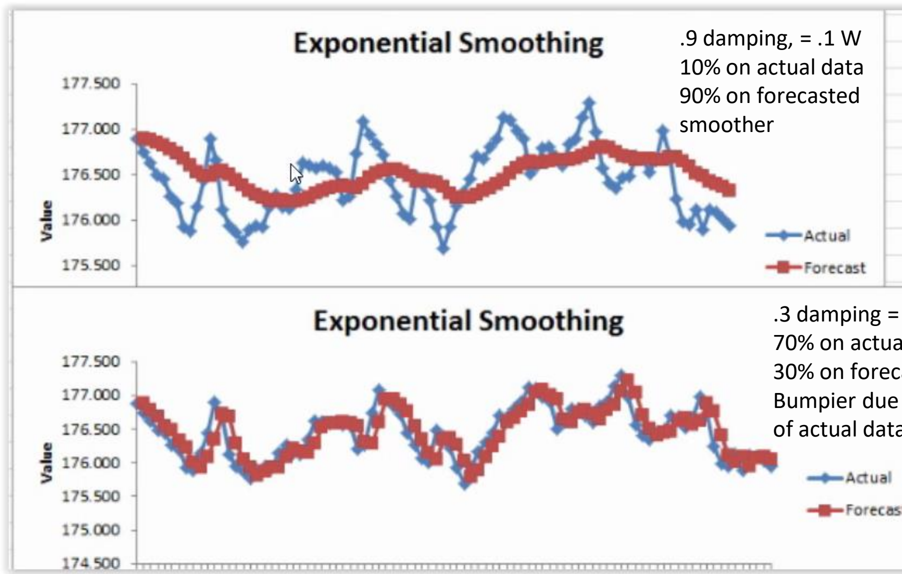
Exponential Smoothing Notes

- The smaller the w, the greater its smoothing effect
  - Smoothing constant always between 0 and 1
  - Larger smoothing constant → more fluctuation → closer to actual data
- Excel uses "damping constant": $(1 - w)$

9.8 Forecast the Next Month: Exponential Smoothing

A	B	C	D
	Great Lakes Water Level		
		output (y)	smoother
	Year	Lake Level	dampfact=0.9
1	1918	176.887	#N/A
2	1919	176.745	176.887
3	1920	176.625	176.8725
4	1921	176.488	176.84775
5	1922	176.445	176.8118083
6	1923	176.264	176.7751275
7	1924	176.187	176.7240314
20	1937	175.923	176.2436688
21	1938	176.141	176.2115519

Damping Factor: 1-W, so if damping factor is lower, more emphasis on current data, less smooth



.3 damping = .7 W
70% on actual data
30% on forecasted data
Bumpier due to greater influence of actual data

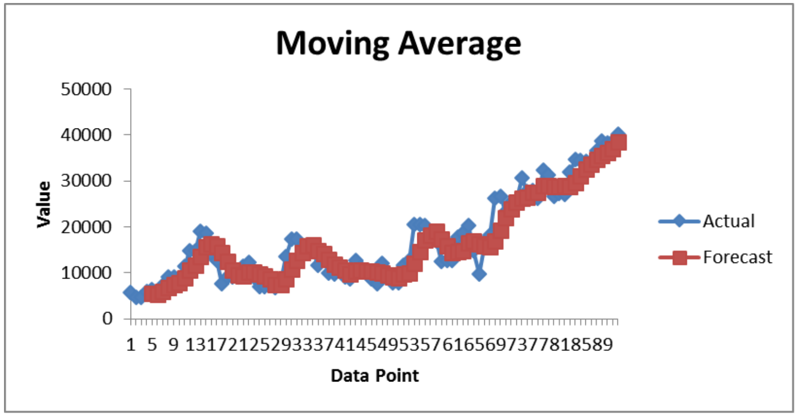
90	2007	175.943	176.3226931	176.0355809	
91	2008		176.2847571	175.9710076	= forecast 2008 damp factor 0.3
			forecast 2008 with 0.9 damp		

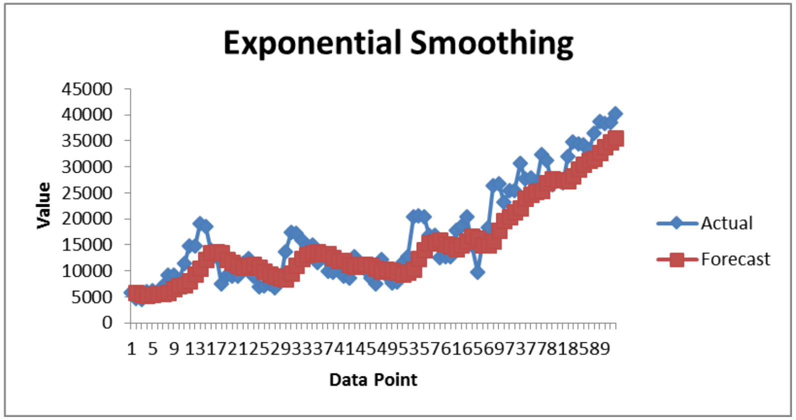
How might you determine which overall forecast was "better"?

9.9 Test Your Knowledge: Time Series Models

Forecast year 2008 attendance using these 3 methods: autoregressive AR(1), m

Year	Attendance	AR (1) data	Moving average data (use k=5)	Exponential smoothing (use 1-w=0.8)
1916	5743			#N/A
1917	4678	5743	#N/A	5743
1918	4558	4678	#N/A	5530
1919	5978	4558	#N/A	5335.6
1920	6244	5978	#N/A	5464.08
1921	5396	6244	5440	5620.064
1922	7135	5396	5371	5575.2512
1923	9139	7135	5862	5887.20096
1924	9191	9139	6778	6537.560768
1925	8086	9191	7421	7068.248614
1936	9083	8995	10494	11103.34224
2000	34438	34739	29562	29608.1323
2001	34314	34438	31121	30574.10584
2002	33248	34314	32504	31322.08467
2003	36576	33248	33746	31707.26774
2004	38660	36576	34663	32681.01419
2005	38272	38660	35447	33876.81135
2006	38558	38272	36214	34755.84908
2007	40154	38558	37063	35516.27927
2008		40093	38444	36444
		subsitute in formula	drag down	drag down





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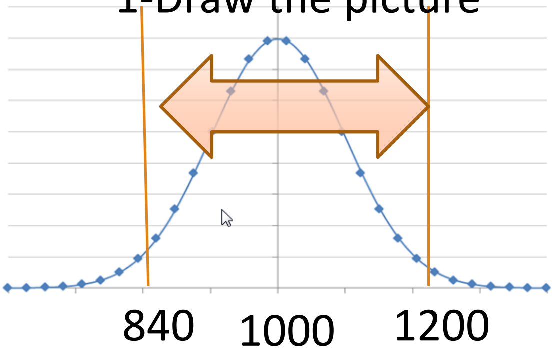
Final Review

- **Regression**
  - **How do you write down the formula from Excel Output?**
  - **How to use a regression equation to predict the output(y)?**
  - **How do you tell which variables are useful to have in your regression?**
- **Correlation Coefficient R vs Coefficient of Determination R2 – what do they represent?**
- **When is data not appropriate for regression? What are residuals?**
- **Causation vs Correlation**
- **Z calculation for the probability of a value falling between A and B**
- **Time Series - Autocorrelation and R2**
- **List of Statistical tools: Correlation, regression, hypothesis testing, scatter plots, process control charts, chi-square testing, etc etc.**
- **Basic ways to describe data and Calculate: mean, median, mode, range, standard deviation, variance**
- **Sample size formula and manipulation impacts**
- **Margin of error and confidence intervals**
- **Process Control charts**
- **How can you determine if your measurement system is repeatable and reproducible?**
- **Hypothesis testing – at what alpha do you reject, at what p-value do you reject**

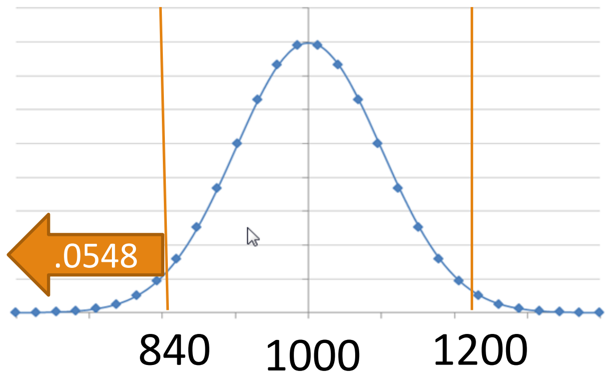
Quiz 2 Prep Question 1: Practice with Z calculations

The distribution of weekly incomes of supervisors at the ABC Company follows the normal distribution, with a mean of \$1000 and a standard deviation of \$100.
What percent of the supervisors have a weekly income between \$840 and \$1200?

1-Draw the picture



2-Think about what you are calculating related to the picture



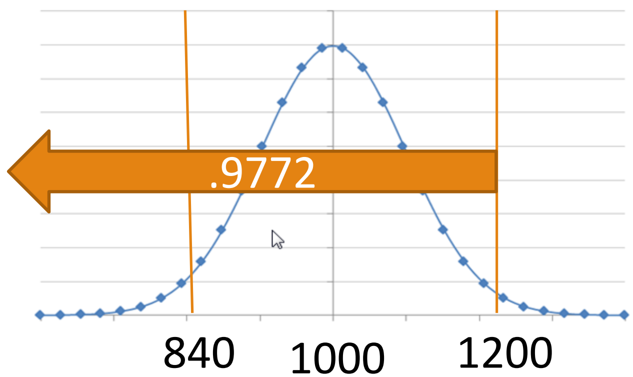
$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{840 - 1000}{100} = -1.6$$

Look up in tables, p = .0548

Or in Excel

=NORM.DIST(840,1000,100,TRUE)



$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{1200 - 1000}{100} = 2$$

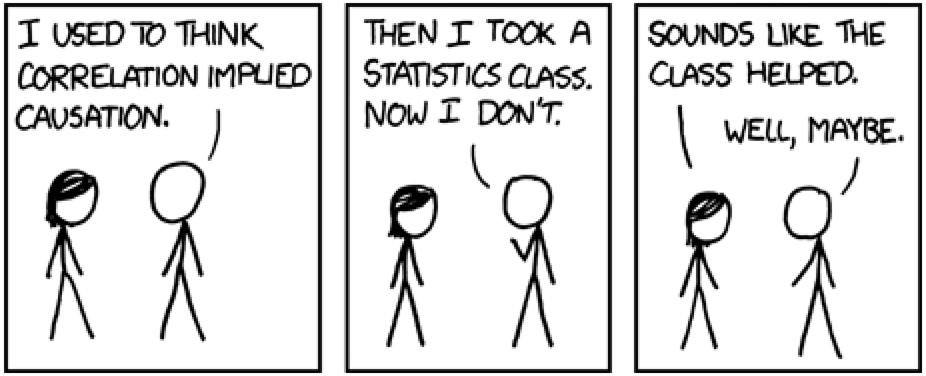
Look up in tables, p = .9772

Or in Excel

=NORM.DIST(1200,1000,100,TRUE)

.9772-.0548 = .9224, so 92.24% have a weekly income between \$840 and \$1200

Correlation vs Causation



From xkcd, a comic by Randall Munroe

Correlation refers to the degree in which two measurements tend to vary together.

Take the correlation between ice cream sales and drowning deaths. As ice cream sales increase, so do drowning deaths. Does that mean selling ice cream causes people to drown? Probably not. More likely is that people swim more and eat more ice cream the hotter it gets, so both are driven by the outside temperature.

Strong correlation doesn't mean cause and effect relationship….

Correlation has different causes

- the first caused the second
- the second caused the first
- Confounding factor– interference by a third variable distorts the association being studied between two other variables, because of a strong relationship with both of the other variables
- Common Cause – like the ice cream example
- Coincidence

http://blog.fastfedora.com/2011/07/8-things-to-know-about-correlation.html

Highlights: Video Segment 6.7:Correlation

Two Indices

1. Correlation coefficient (r)
2. Coefficient of determination (r^2)

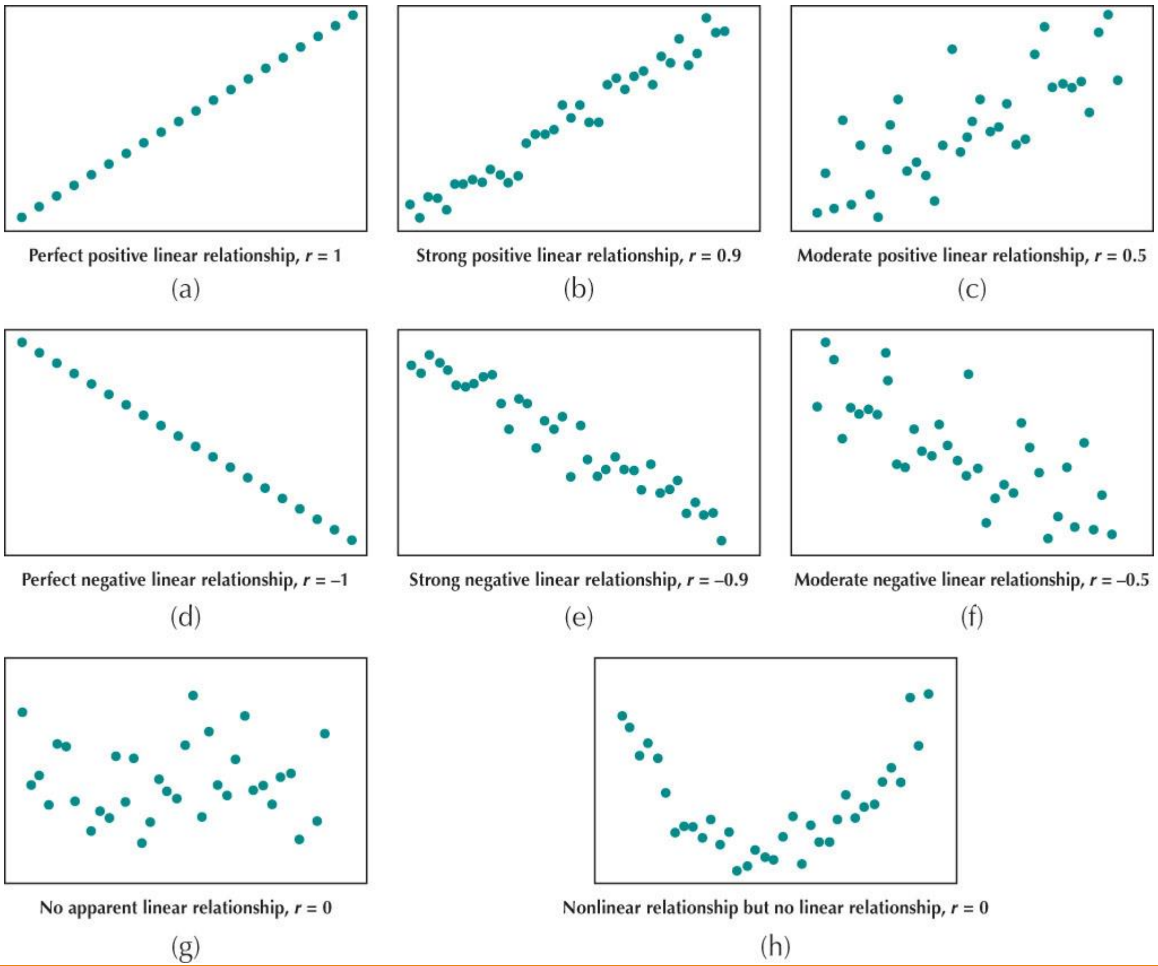
Correlation Coefficient (r)

- $-1 < r$
- -1 = perfect negative correlation
- 1 = perfect positive correlation
- 0 = no relationship
- Rule of thumb: r value of ~ ± 0.7 desired
  - Indicates meaningful relationship

Scatterplots provide a visual description of the relationship between two quantitative variables. The *correlation coefficient* is a numerical measure for quantifying the linear relationship between two quantitative variables.

If the variability decreases, what does your correlation coefficient get closer to?
What does a correlation coefficient r=-.72 mean?

Properties of *r*



Perfect positive linear relationship, $r = 1$
(a)

Strong positive linear relationship, $r = 0.9$
(b)

Moderate positive linear relationship, $r = 0.5$
(c)

Perfect negative linear relationship, $r = -1$
(d)

Strong negative linear relationship, $r = -0.9$
(e)

Moderate negative linear relationship, $r = -0.5$
(f)

No apparent linear relationship, $r = 0$
(g)

Nonlinear relationship but no linear relationship, $r = 0$
(h)

Answer: 1 or -1
Answer: There is a moderate negative correlation between two factors

What if you performed a linear regression analysis on successive values of a time series analysis and you see autocorrelation.....what might your r^2 ?

Highlights: Video Segment 6.7:Correlation

Two Indices

1. Correlation coefficient (r)
2. Coefficient of determination (r^2)

Coefficient of Determination (r^2)

- Correlation coefficient squared
- Measure of the percentage of variability in y that can be accounted for by x
  - Trying to find an input x that is influencing our output y
  - x will not explain all of y
  - Recall: There is variability in everything we do.
- Metric for whether input x is really contributing to output

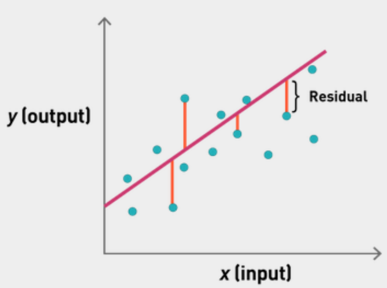
Measures the goodness of fit of the regression equation to the data. We interpret r^2 as the proportion of the variability in y that is accounted for by the linear relationship between y and x. The values that r^2 can take are $0 \leq r^2 \leq 1$.

Answer: correlation, r would be closer to 1 or -1, which would mean r2 would be close to 1

Highlights: Video Segment 6.9:Residuals and Other Warnings

What Is a Residual?

- Synonymous to error; should be random
- The distance between actual data point and the line determined by linear equation
- Determined by the difference between observed and predicted values of y
  - Ideally, points fall on regression line (i.e., perfect model)
  - Error would then be zero (rare).
- When plotted, a random series of points around a zero reference with no evidence of a pattern



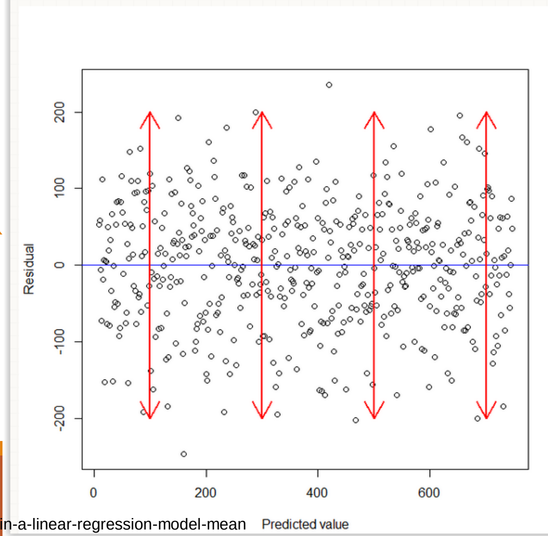
$$e_i = y_i - \hat{y}_i$$

Residual = Observed response (actual) − Predicted response (formula)

Assumptions of Regression

1. Residuals are independent.
2. Residuals are normally distributed with a mean of zero.
   - The regression line will sometimes be high or low (i.e., over- or underpredicting).
3. There are equal variances (σ^2) of y.

It means that when you plot the individual error against the predicted value, the variance of the error predicted value should be constant. See the red arrows in the picture below, the length of the red lines (a proxy of its variance) are the same.

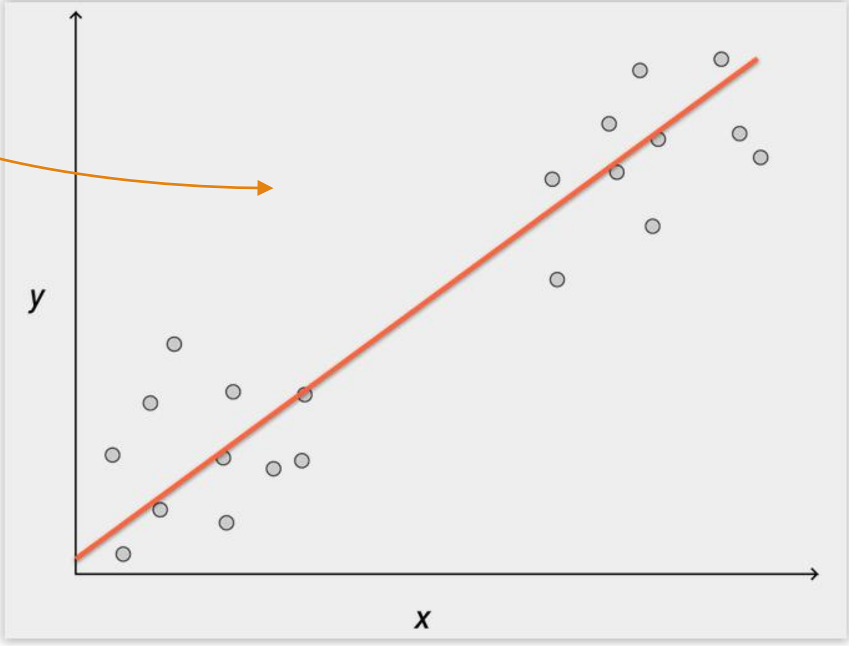


http://stats.stackexchange.com/questions/52089/what-does-having-constant-variance-in-a-linear-regression-model-mean

Highlights: Video Segment 6.9:Residuals and Other Warnings

Other Points of Interest

- Certain data is inappropriate for a regression analysis:
  - Residuals form a pattern.
  - Large outliers are present.
  - "Clumped" data appears linear.
- Avoid extrapolating outside data.
- Beware of lurking variables, or Simpson's paradox.
- A strong correlation does not mean causation.



Agenda

Topic	Time	Sunday Section	Wednesday Section
Introduction	5 min	6:30-6:35	9:00 - 9:05
Highlights from Week 9 Video	30 min	6:35-7:05	9:05 - 9:35
Start on Final Review	25 min	7:05-7:30	9:35 - 10:00
Breakout on Regression	20 min	7:30 – 7:50	10:00 – 10:20
Review of Upcoming Assignments and Open Question	10 min	7:50-8:00	10:20 - 10:30

Breakouts on Regression

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Review of Upcoming Assignments: Wednesday Section

1. Homework #6, due Saturday, March 18th, Midnight EST in Learning Management System based on a file you will download in the assignments section for homework #6. **You will upload 1 excel file**.

2. Data Collection Paper due 3/25 – 1 FILE

3. Story Board due 3/27

4. Final Exam due 3/29

- Password: AnalysisExam3
- Time limit: 90 mins
- No partial credit

March 2017

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Week #9	12	13	14	15	16	17	18
				Live Class #9			***Homework #6 Due:*** 1. Time Series Problem posted in Excel
Week #10	19	20	21	22	23	24	25
				Live Class #10			***Data collection and Analysis Paper DUE***
	26	27	28	29	30	31	1
		Project Storyboard DUE		***Final Exam DUE***			

Homework #6

	A	B
1	**The next tab of this Excel spreadsheet contains the NFL raw data for these problems.**	
2		
3	**In the National Football League, the philosophy for winning (rushing, passing, defense) seems to go through cycles. Consider a time series of the average number of rushing yards in the NFL per regular season from 1980 to 2008.**	
4		
5	*1) Make a time series plot. Is there evidence that the average rushing yards is trending in one direction? Describe the general movement of the series.*	
6		
7	*2) Fit a **first order autoregressive model** [**AR(1)**] using y(t) as the response variable and y (t-1) as the input variable. Record the regression equation.*	
8		
9	*3) Based on the **AR(1) model**, forecast the average number of rushing yards in the NFL for the 2009 regular season.*	
10		
11	*4) Calculate the **exponential smoothing models** using Excel damping factors 0.8 and 0.2 For each of the exponential smoothing models forecast the average number of rushing yards in the NFL for the 2009 season.*	
12		
13	*5) Calculate a **moving average model** using k=5 (Excel interval). Forecast the average number of rushing yards in the NFL for the 2009 season.*	
14		