# MBC 638

LIVE SESSION WEEK 6

# Agenda

Topic	Time	Sunday Section	Wednesday Section
Introduction	5 min	6:30-6:35	9:00-9:05
Quiz 2 Prep	40 min	6:35-7:15	9:05-9:45
Highlights from Week 6 Video	40 min	7:15-7:55	9:45-10:25
Review of Upcoming Assignments and Open Question	5 min	7:55-8:00	10:25-10:30

 A random sample of 100 vehicles traveling on a rural section of highway had a mean speed of 62 miles per hour. Assume the population standard deviation is 7 miles per hour. Find a 95% confidence interval for vehicle speed of all cars traveling on this section of highway.

**√a.** a) 60.6–63.4

Correct. This is a 95% confidence interval.

Text Reference - pages 429-432

Section 8.1: Z Interval for the Population Mean

**b.** b) 60.8–63.2

Incorrect. This is the result found if  $\alpha$  is not divided by 2.

Text Reference - pages 429-432

Section 8.1: Z Interval for the Population Mean

c. c) 50.5-73.5

Incorrect. This is the result found when the population standard deviation ( $\sigma$ ) is used instead of

the sampling distribution standard deviation  $(\sigma/\sqrt{n})$ .

Text Reference - pages 429-432

Section 8.1: Z Interval for the Population Mean

d. d) 62

Incorrect. This is the point estimate, not a confidence interval.

Text Reference - pages 429-432

Section 8.1: Z Interval for the Population Mean

1.	loon -
	62=2
	0 = 7 O Istrain
	95% CI 12
	y + 2 * 5
-	
	62 ± 1.96 7/100
	V150 \
	ZX = 1.96 62± 1.96 (7
	.05 in 2 tails
	62 ± 1.96(.7)
	62± 1.372
	(60.628, 63.372) ANSWER A
	(Assessed Augustus A

- 2. A beekeeper examines a sample of 300 honeybees from one of his colonies and determines that 90 of them are carrying *Varroa destructor* parasites. Find a 99% confidence interval for the proportion of honeybees with the parasite in this colony.
  - **a.** a) 0.24–0.36

Incorrect. This is the result found if  $\alpha$  is not divided by 2. Text Reference – pages 464-465

Section 8.1: Z Interval for the Population Proportion

**b.** b) 0.23-0.37

Correct. This is a 99% confidence interval.

Text Reference – pages 464-465

Section 8.1: Z Interval for the Population Proportion

c. c) 0.26-0.34

Incorrect. This is a 90% confidence interval. Text Reference – pages 464-465 Section 8.1: Z Interval for the Population Proportion

**d.** d) 0.30

Incorrect. This is the point estimate, not a confidence interval. Text Reference – pages 464-465 Section 8.1: *Z* Interval for the Population Proportion

2. 1=300	
$\rho = 90 = .3$	P+ 7* P(1-P)
9990 CI	1 / n
	· 3 ± 2576 [-3(13)
(1,232,	₹ 360
10	·3 ± 2.574 \[ -21 \] 360
enswer 5	13-2,576 (.02646)
	- 3± .068

3. The mean time spent at the gas pump for a sample of 10 customers at a filling station was 245 seconds, with a sample standard deviation of 35 seconds. The population is known to be normally distributed. Find a 90% confidence interval rounded to whole seconds for time spent at the gas pump.

**a.** a) 230–260

Incorrect. This is the result found if  $\alpha$  is not divided by 2.

Text Reference – pages 451-452

Section 8.2: t Interval for the Population Proportion

**b.** b) 225-265

Correct. This is a 90% confidence interval using the *t* interval because the population standard deviation is not known.

Text Reference - pages 451-452

Section 8.2: t Interval for the Population Proportion

**c.** c) 223–267

Incorrect. This is a 90% confidence interval, but it uses the Z interval instead of the t interval. Text Reference – pages 451-452

Section 8.2: t Interval for the Population Proportion

**d.** d) 226–264

Incorrect. This is the result found when the degree of freedom for the t interval is set to 10 rather than 9.

Text Reference – pages 451-452

Section 8.2: t Interval for the Population Proportion

3. X = 245 seconds	
	sample
	on stddeu
S = 35 Unkna	on steller
90% CI	
x + + 5/n	off=n-1
-	df=10-1=9
x ± t 5/m 245± 1.833 35/	d+=10-1-7
124 - 1 000 35-	
1245-11033	
710	
1 1 1 2 (35)	
245 + 1,833 (35)	
245± 1,833 (11.08)	
+ 00 23	
245 = 20.30	
(22.116 2162)	ANSWER B
(224,69, 265,3)	HUDREL
(	

- 4. A restaurant that delivers pizza wants to study its delivery time performance. The manager collects a sample of 100 deliveries and finds a 90% confidence interval for delivery times to be 24–29 minutes. Assuming that the population standard deviation was used in calculating the confidence interval, what is this standard deviation?
  - a. a) 19.5 minutes

Incorrect. It appears that you based your calculation on 10% in each tail. Be sure that you only use half of the  $\sigma$  in each tail.

Text Reference - page 433

Section 8.1: Z Interval for the Population Mean

**b.** b) 15.20 minutes

Correct.

Text Reference - page 433

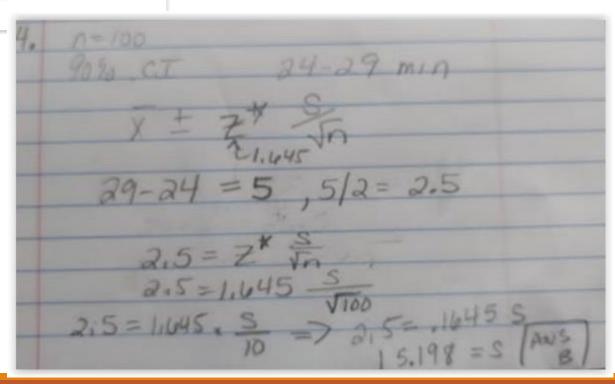
Section 8.1: Z Interval for the Population Mean

c. c) 15.06 minutes

Incorrect. It appears that you used the t distribution. The assumption is that the population standard deviation was used, so the Z distribution should be used.

Text Reference - page 433

Section 8.1: Z Interval for the Population Mean



- 5. Consider a 95% confidence interval for μ based on the *t* distribution. Which of the following would be TRUE if the sample size were increased by an unspecified value?
  - a. a) The degrees of freedom would decrease.

Incorrect. The degrees of freedom will increase if n increases. Text Reference – pages 451-454 Section 8.2: t Interval for the Population Mean

Section 8.2. t interval for the reputation freeze

 $\mathbf{b}$ . b) The margin of error E would decrease.

Correct.

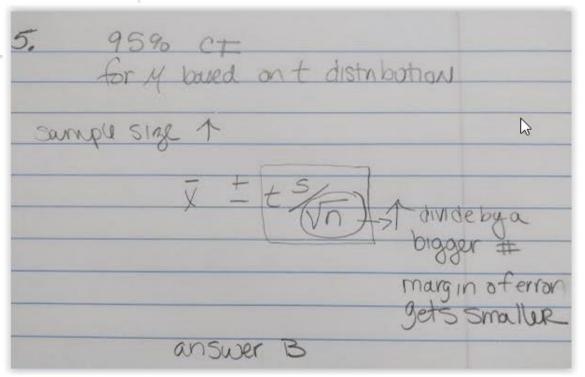
Text Reference – pages 451-454 Section 8.2: *t* Interval for the Population Mean

c. c) The width of the confidence interval would increase.

Incorrect. The width of the confidence interval will decrease if the sample size increases. Text Reference – pages 451-454
Section 8.2: t Interval for the Population Mean

d. d) None of the answers is correct.

Incorrect. Choice b) is correct: the margin of error would decrease. Text Reference – pages 451-454
Section 8.2: *t* Interval for the Population Mean





### Quiz 2 Prep Question #3:Chi Square Test for Independence

Q3:Three work shifts producing the same product sorted the finished product into 4 categories based on its quality level and displayed the results in the following table. Determine whether there is dependence between the shift and the quality of the product? (Does product quality depend on the shift that produces it?) Assume alpha= 0.05

	1st Shift	2 <sup>nd</sup> Shift	3 <sup>rd</sup> Shift	
Perfect product	185	175	170	Ho: Quality and Shift are independent
Acceptable product	55	60	65	Ha: Quality and Shift are NOT independen
<b>Defective product</b>	15	15	15	ria. Quality and Silit are NOT independen
Reworked product	10	15	15	

Observed				Expected							
1st shift	2nd shift	3rd shift	Total		1st shift	2nd shift	3rd shift	Total			
185	175	170	530	Perfect product	176.67	176.667	176.67	530			
55	60	65	180	Acceptable product	60	60	60	180			
15	15	15	45	Defective product	15	15	15	45			
10	15	15	40	Reworked product	13.333	13.3333	13.333	40			
265	265	265	795		265	265	265	795			
=CHISQ.TEST(I	H21:J22,N21:P	22)									
0.8403											
	1st shift 185 55 15 10 265 =CHISQ.TEST(H	1st shift     2nd shift       185     175       55     60       15     15       10     15       265     265	1st shift     2nd shift     3rd shift       185     175     170       55     60     65       15     15     15       10     15     15       265     265     265       =CHISQ.TEST(H21:J22,N21:P22)	1st shift     2nd shift     3rd shift     Total       185     175     170     530       55     60     65     180       15     15     15     45       10     15     15     40       265     265     265     795   =CHISQ.TEST(H21:J22,N21:P22)	1st shift       2nd shift       3rd shift       Total         185       175       170       530       Perfect product         55       60       65       180       Acceptable product         15       15       45       Defective product         10       15       15       40       Reworked product         265       265       795    =CHISQ.TEST(H21:J22,N21:P22)	1st shift         2nd shift         3rd shift         Total         1st shift           185         175         170         530         Perfect product         176.67           55         60         65         180         Acceptable product         60           15         15         45         Defective product         15           10         15         15         40         Reworked product         13.333           265         265         265         795         265	1st shift         2nd shift         3rd shift         Total         1st shift         2nd shift         2nd shift           185         175         170         530         Perfect product         176.67         176.667           55         60         65         180         Acceptable product         60         60           15         15         45         Defective product         15         15           10         15         15         40         Reworked product         13.333         13.3333           265         265         265         795         265         265         265	1st shift         2nd shift         3rd shift         Total         1st shift         2nd shift         3rd shift           185         175         170         530         Perfect product         176.67         176.667         176.67           55         60         65         180         Acceptable product         60         60         60           15         15         45         Defective product         15         15         15           10         15         15         40         Reworked product         13.333         13.3333         13.333           265         265         265         795         265         265         265	1st shift         2nd shift         3rd shift         Total         1st shift         2nd shift         3rd shift         Total           185         175         170         530         Perfect product         176.67         176.667         176.67         530           55         60         65         180         Acceptable product         60         60         60         180           15         15         45         Defective product         15         15         45           10         15         15         40         Reworked product         13.333         13.333         13.333         40           265         265         265         795         265         265         265         795	1st shift       2nd shift       3rd shift       Total       1st shift       2nd shift       3rd shift       Total         185       175       170       530       Perfect product       176.67       176.667       176.67       530         55       60       65       180       Acceptable product       60       60       60       180         15       15       15       45       Defective product       15       15       15       45         10       15       15       40       Reworked product       13.333       13.3333       13.3333       40         265       265       265       795       265       265       265       795	1st shift       2nd shift       3rd shift       Total       1st shift       2nd shift       3rd shift       Total         185       175       170       530       Perfect product       176.67       176.667       176.67       530         55       60       65       180       Acceptable product       60       60       60       180         15       15       45       Defective product       15       15       45         10       15       15       40       Reworked product       13.333       13.3333       13.3333       40         265       265       265       795       265       265       265       795

If P is low, Ho must go. P is .84 not lower than alpha = .05, so can NOT reject Ho — must accept Ho which says Quality and Shift ARE Independent.

At what alpha would you reject Ho....anything larger than .84, so alpha = .85 you would reject Ho.

## Quiz 2 Prep Question #4: Hypothesis Testing with Z statistics

4) A bullet manufacturer claims to have produced a projectile having a mean muzzle velocity of more than 3000 feet per second. From a random sample of 60 bullets he calculates a sample mean of 3012 feet per second and a sample standard deviation of 112 feet per second. Does the

3000 3012

data from the sample support his claim?

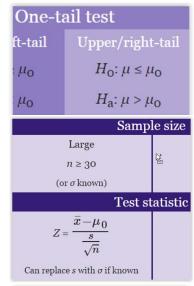
Solution:

Ha: 
$$\mu > 3000$$
 1 sample test = purple chart

$$z = \frac{x \ bar - \mu}{\frac{S}{\sqrt{n}}} \qquad z = \frac{3012 - 3000}{\frac{112}{\sqrt{60}}} = \frac{3012 - 3000}{\frac{112}{7.75}} = \frac{3012 - 3000}{14.45} = \frac{12}{14.45} = .830$$

Look up .830 in Z tables, =.7967 so 1-.7967=.2033 Or =NORMDIST(3012, 3000,14.459,TRUE) =.796712, 1-.796712=.203288

If p is low then how must go, but in this case alpha is not provided...however p is not lower than the typical alpha of .05, if this is what you assumed then You can't reject Ho which means that the sample does NOT support his claim.



Upper/right-tail p = area right of Z or t

$\boldsymbol{z}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8623
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.944
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750]	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0 3.1 3.2 3.3 3.4	0.9987 0.9990 0.9993 0.9995 0.9997	0.9987 0.9991 0.9993 0.9995 0.9997	0.9987 0.9991 0.9994 0.9995 0.9997	0.9988 0.9991 0.9994 0.9996 0.9997	0.9988 0.9992 0.9994 0.9996 0.9997	0.9989 0.9992 0.9994 0.9996 0.9997	0.9989 0.9992 0.9994 0.9996 0.9997	0.9989 0.9992 0.9995 0.9996 0.9997	0.9990 0.9993 0.9995 0.9996 0.9997	0.999 0.999 0.999 0.999

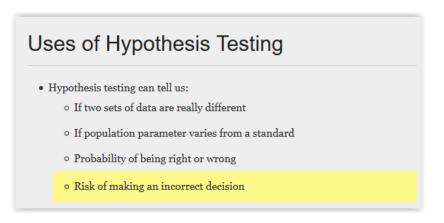
# Highlights: Video Segment 3.7: Hypothesis Testing

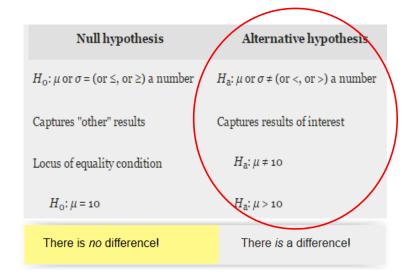
Use sample data from a population to confirm or reject a statement that we make about the population.

Hypothesis Statements can tell us if two sets of data are really different from each other, or from a standard value.

Also provides the probability of being right or wrong and the risk of making the wrong decision.

Only pertains to population parameters – mean and standard deviation.





The alternative: Is what you want the result to be....we want to have improved our process, reduced the cycle time, made it lower.

### Quiz 2 Prep Additional Question: Hypothesis Testing with T statistics

The gas industry has issued data that the standard cost of gas is \$2.14 per gallon at stand alone gas stations. You believe the cost of gas at supermarket gas stations is greater than the cost of gas at stand alone gas stations.

You collect data from 10 signs for super market gas stations. Assume an alpha of .05.

The gas prices at the super market gas stations are (\$/gallon): 2.04, 2.05, 2.05, 2.09, 2.09, 2.07, 2.10, 2.12, 2.05, 2.06

Is the cost of gas at supermarket gas stations greater than the gas at stand alone gas stations?

Ho: super market gas prices <= standard gas price at stand alone stations Ha: super market gas prices > standard gas price at stand alone stations

a: super market gas prices > standard gas price at stand alone station 
$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{2.072 - 2.14}{\frac{.027}{\sqrt{10}}} = \frac{2.072 - 2.14}{\frac{.027}{\sqrt{10}}} = \frac{-.068}{\frac{.027}{3.16}} = \frac{-.068}{.0085} = -.8$$



Lookup in the t table, -8 at df=9, note there is no negative in the table the probabilities are symmetrical, so < than .005

Or = T.DIST(-8, 9, TRUE) in excel = 1.10674E-05, so almost 0

We want the area to the right, so 1-.00001 = almost 1

df = n - 1

Upper/right-tail p = area right of Z or t

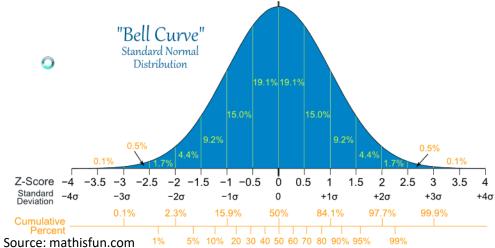
If p is low, Ho must go, P of almost 1 is NOT lower than alpha =.05, so can NOT reject Ho, you must accept that super market gas prices are lower than the stand alone gas station prices.

You can not conclude that super markets have higher prices than stand alone gas stations.

	80%	90%	Confidence level 95%	98%	99%
			Area in one tail		
	0.10	0.05	0.025	0.01	0.00
	0.20	0.10	Area in two tails	0.02	0.01
10 1	0.20	0.10	0.05	0.02	0.01
df 1 2	3.078 1.886	6.314 2.920	12.706 4.303	31.821 6.965	63.65 9.92
3	1.638	2.353	3.182	4.541	5.84
4	1.533	2.132	2.776	3.747	4.60
5	1.476	2.015	2.571	3.365	4.03
6	1.440	1.943	2.447	3.143	3.70
7	1.415	1.895	2.365	2.998	3.49
8	1.397	1.860	2.306	2.896	3.35
9	1.383	1.833	2.262	2.821	3.25
N10	1.372	1.812	2.228	2.764	3.16
\210 11	1.363	1.796	2.201	2.718	3.10
12	1.356	1.782	2.179	2.681	3.05
13	1.350	1.771	2.160	2.650	3.0
14	1.345	1.761	2.145	2.624	2.97
15	1.341	1.753	2.131	2.602	2.94
16	1.337	1.746	2.120	2.583	2.92
17	1.333	1.740	2.110	2.567	2.89
18	1.330	1.734	2.101	2.552	2.87
19 20	1.328 1.325	1.729 1.725	2.093 2.086	2.539 2.528	2.86
					2.84
21	1.323	1.721	2.080	2.518	2.83
22 23	1.321 1.319	1.717 1.714	2.074 2.069	2.508 2.500	2.80
24	1.318	1.711	2.064	2.492	2.79
25	1.316	1.708	2.060	2.485	2.78
26	1.315	1.706	2.056	2.479	2.77
27	1.314	1.703	2.052	2.473	2.77
28	1.313	1.701	2.048	2.467	2.76
29	1.311	1.699	2.045	2.462	2.75
30	1.310	1.697	2.042	2.457	2.75
31	1.309	1.696	2.040	2.453	2.74
32	1.309	1.694	2.037	2.449	2.73
33	1.308	1.692	2.035	2.445	2.73
34	1.307	1.691	2.032	2.441	2.72
35	1.306	1.690	2.030	2.438	2.72
36	1.306	1.688	2.028	2.435	2.7
37	1.305	1.687	2.026	2.431	2.7
38	1.304	1.686	2.024	2.429	2.7
39 40	1.304 1.303	1.685 1.684	2.023 2.021	2.426 2.423	2.70 2.70
50	1.299	1.676	2.009	2.403	2.67
60	1.296	1.671	2.000	2.390	2.60
70 80	1.294 1.292	1.667 1.664	1.994 1.990	2.381 2.374	2.64
90	1.292	1.662	1.987	2.368	2.63
100	1.290	1.660	1.984	2.364	2.62
1000	1.282	1.646	1.962	2.330	2.62
z	1.282	1.645	1.960	2.326	2.57

# Highlights: Video Segment 3.4: Normal(Continuous Data)

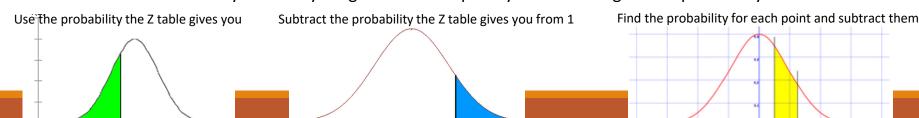
Total Area under the curve = 1 or 100% of the opportunities are under the curve, since they are probabilities



For Normal, In order to solve for the probability for continuous data....

- 1. Calculate the standard value for Z in order to look up the probability
- $z = \frac{x-\mu}{\sigma}$ ,  $\mu$  = average of the sample,  $\sigma$  =standard deviation, x = the point you are interested in finding the probability for
- 2. Solve for Z, then look it up in the table to convert to a probability.

Remember: The Z value is always for everything to left of the point you are looking for the probability of....



# Highlights: Video Segment 3.4: Normal(Continuous Data)

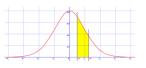
Using Excel to solve for probabilities and Z values using the normal distribution

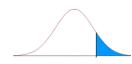
1 – If you didn't calculate a Z value first

=norm.dist(60,50,5,True) = .97725 or 97.725%

Means give me the probability if I want to know what probability there is a child measures shorter than 60 inches, if the average is 50, and the standard deviation is 5, true means I want everything to the left of the curve.

DRAW the PICTURE – so you know what probability you are looking for.







2 – If you already have or calculated the Z value

=norm.s.dist(2,true) = .97725 or 97.725%

Means give me the probability for a Z test statistic of 2, and true means that I want everything to the left of the curve.

3 – If you have a probability and want a z value

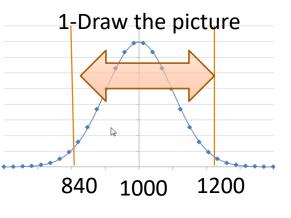
=norm.s.inv(.97725)

Means give me the Z value associated with the probability of .97725.

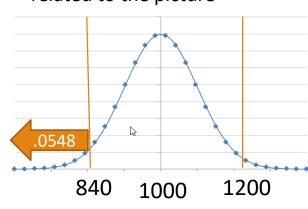
### Quiz 2 Prep Question 1: Practice with Z calculations

The distribution of weekly incomes of supervisors at the ABC Company follows the normal distribution, with a mean of \$1000 and a standard deviation of \$100.

What percent of the supervisors have a weekly income between \$840 and \$1200?



2-Think about what you are calculating related to the picture



$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{840 - 1000}{100} = -1.6$$

Look up in tables, p = .0548

Or in Excel

=NORM.DIST(840,1000,100,TRUE)

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{1200 - 1000}{100} = 2$$

Look up in tables, p = .9772

Or in Excel

=NORM.DIST(1200,1000,100,TRUE)

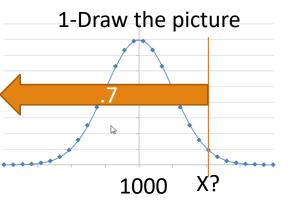
.9772-.0548 = .9224, so 92.24% have a weekly income between \$840 and \$1200

$\boldsymbol{z}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.535
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.575
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.614
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.651
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.687
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.725
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.754
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.785
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.815
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.835
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.86
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.88
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.90
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.91
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.93
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.94
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.95
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.96
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.97
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.97
2.0 2.1 2.2 2.3 2.4	0.9772 0.9821 0.9861 0.9893 0.9918	0.9778 0.9826 0.9864 0.9896 0.9920	0.9783 0.9830 0.9868 0.9898 0.9922	0.9788 0.9834 0.9871 0.9901 0.9925	0.9793 0.9838 0.9875 0.9904 0.9927	0.9798 0.9842 0.9878 0.9906 0.9929	0.9803 0.9846 0.9881 0.9909 0.9931	0.9808 0.9850 0.9884 0.9911 0.9932	0.9812 0.9854 0.9887 0.9913 0.9934	0.98 0.98 0.98 0.99
2.5 2.6 2.7 2.8 2.9	0.9938 0.9953 0.9965 0.9974 0.9981	0.9940 0.9955 0.9966 0.9975 0.9982	0.9941 0.9956 0.9967 0.9976 0.9982	0.9943 0.9957 0.9968 0.9977 0.9983	0.9945 0.9959 0.9969 0.9977 0.9984	0.9946 0.9960 0.9970 0.9978 0.9984	0.9948 0.9961 0.9971 0.9979 0.9985	0.9949 0.9962 0.9972 0.9979 0.9985	0.9951 0.9963 0.9973 0.9980 0.9986	0.999 0.999 0.999 0.999
3.0 3.1 3.2 3.3 3.4	0.9987 0.9990 0.9993 0.9995 0.9997	0.9987 0.9991 0.9993 0.9995 0.9997	0.9987 0.9991 0.9994 0.9995 0.9997	0.9988 0.9991 0.9994 0.9996 0.9997	0.9988 0.9992 0.9994 0.9996 0.9997	0.9989 0.9992 0.9994 0.9996 0.9997	0.9989 0.9992 0.9994 0.9996 0.9997	0.9989 0.9992 0.9995 0.9996 0.9997	0.9990 0.9993 0.9995 0.9996 0.9997	0.99 0.99 0.99 0.99

## Quiz 2 Prep Question 1 Extended: Practice with Z calculations

The distribution of weekly incomes of supervisors at the ABC Company follows the normal distribution, with a mean of \$1000 and a standard deviation of \$100.

Management wants to give bonuses to those supervisors within the top 30% of weekly incomes. What is the weekly income cut off, the lowest weekly income a supervisor can have and still receive the bonus?



2-Think about what you are calculating related to the picture

$$Z = \frac{X - \mu}{\sigma}$$
  $Z = \frac{x - 1000}{100} = ?$ 

We know p = .7, what Z goes with that p value...look it up from the table = .52 or in Excel=NORM.S.INV(0.7)=.52

$$.52 = \frac{x - 1000}{100}$$

$$.52 * 100 = X - 1000$$

$$52 = X - 1000$$

$$52 + 1000 = X$$

$$1052 = X$$

\$1052 is the lowest weekly income a supervisor can have and still receive a bonus

### Quiz 2 Review

# Highlights: Video Segment 3.5: Binomial(Discrete Data)

No set shape

Mean:  $\mu$ =np, n = number of trials, p=probability of success

Variance:  $\sigma^2 = n \times p(1-p)$ 



A binomial experiment is an experiment which satisfies these four conditions:

A fixed number of trials

Each trial is independent of the others

There are only two outcomes

The probability of each outcome remains constant from trial to trial.

You're taking a quiz with five true/false questions. You didn't study and plan to guess. What's the probability you get three questions correct?

Find P(X = 3), the probability that the number of successes is equal to three.

- n = 5
- p = 0.5

			p	(probabil	ity of	a succes	s)	
n	X	0.10	0.15	0.20		0.40	0.45	0.50
÷	÷							
4	0	0.6561	0.5220	0.4096		0.1296	0.0915	0.0625
	1	0.2916	0.3685	0.4096		0.3456	0.2995	0.2500
	2	0.0486	0.0975	0.1536		0.3456	0.3675	0.3750
	3	0.0036	0.0115	0.0256		0.1536	0.2005	0.2500
	4	0.0001	0.0005	0.0016		0.0256	0.0410	0.0625
5	0	0.5905	0.4437	0.3277		0.0778	0.0503	0.0312
	1	0.3280	0.3915	0.4096		0.2592	0.2059	0.1562
	2	0.0729	0.1382	0.2048		0.3456	0.3369	0.3125
	3	0.0081	0.0244	0.0512		0.2304	0.2757	0.3125

Use the tables in the back of the book or Excel to calculate probabilities for a binomial distribution for discrete data or use Excel: =binom.dist(3,5,.5,False) = .3125

Means give me the probability that I get 3 successes, out of 5 trials, when each has a probability of success of .5, and False means exactly 3 successes(True would mean all those probabilities up to and including 3 successes:1,2,3 successes all added together)

### Quiz 2 Review

# Highlights: Video Segment 3.5: Binomial(Discrete Data)

#### Another Example:

For a multiple choice test that you are guessing on, you want to know the probability you get at least 3 correct, on a test that has 5 multiple choice questions, and each has 4 choices.



n=5 test questions, p=.25(chance of answering correctly on each problem(1/4)), x>=3

=BINOM.DIST(3,5,0.25,FALSE)

This formula means, the probability that I get 3 questions correct, with 5 questions on the test, and 4 answers for each question so a  $\frac{1}{4}$ =.25 chance of getting each correct, and false means I don't want the cumulative percent because I won't pass the test if I get 1 or 2 correct. This will give me probability of getting exactly 3 correct, then I would do the same with the probability at 4 and 5 correct and add the three probabilities together – because I want to know the probably of getting at least 3 correct, which means 3 or more correct.

```
=BINOM.DIST(3,5,0.25,FALSE)= .088
```

.104 or 10.4% probability that you get at least 3 correct

Or =BINOM.DIST(2,5,0.25,TRUE) means probability I get 1 or 2 correct, then subtract from 1 to get probability of 3,4,5 correct

=.896, so 1-.896=.1035 or .104, so 10.4% probability that you get at least 3 correct

<sup>=</sup>BINOM.DIST(4,5,0.25,FALSE)= .015

<sup>=</sup>BINOM.DIST(5,5,0.25,FALSE)= .0010

Table B Binomial distribution

						p				
n	X	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.1800	0.2550	0.3200	0.3750	0.4200	0.4550	0.4800	0.4950	0.500
	2	0.0100	0.0225	0.0400	0.0625	0.0900	0.1225	0.1600	0.2025	0.2500
3	0	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.125
	1	0.2430	0.3251	0.3840	0.4219	0.4410	0.4436	0.4320	0.4084	0.375
	2	0.0270	0.0574	0.0960	0.1406	0.1890	0.2389	0.2880	0.3341	0.375
	3	0.0010	0.0034	0.0080	0.0156	0.0270	0.0429	0.0640	0.0911	0.125
4	0	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.062
	1	0.2916	0.3685	0.4096	0.4219	0.4116	0.3845	0.3456	0.2995	0.250
	2	0.0486	0.0975	0.1536	0.2109	0.2646	0.3105	0.3456	0.3675	0.375
	3	0.0036	0.0115	0.0256	0.0469	0.0756	0.1115	0.1536	0.2005	0.250
	4	0.0001	0.0005	0.0016	0.0039	0.0081	0.0150	0.0256	0.0410	0.062
5	0	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.031
	1	0.3280	0.3915	0.4096	0.3955	0.3602	0.3124	0.2592	0.2059	0.156
	2	0.0729	0.1382	0.2048	0.2637	0.3087	0.3364	0.3456	0.3369	0.312
	3	0.0081	0.0244	0.0512	0.0879	0.1323	0.1811	0.2304	0.2757	0.312
	4	0.0004	0.0022	0.0064	0.0146	0.0284	0.0488	0.0768	0.1128	0.156
	5		0.0001	0.0003	0.0010	0.0024	0.0053	0.0102	0.0185	0.031
6	0	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.015
	1	0.3543	0.3993	0.3932	0.3560	0.3025	0.2437	0.1866	0.1359	0.093
	2	0.0984	0.1762	0.2458	0.2966	0.3241	0.3280	0.3110	0.2780	0.234
	3	0.0146	0.0415	0.0819	0.1318	0.1852	0.2355	0.2765	( <sup>cm</sup> )32	0.312
	4	0.0012	0.0055	0.0154	0.0330	0.0595	0.0951	0.1382	0.1861	0.234
	5	0.0001	0.0004	0.0015	0.0044	0.0102 0.0007	0.0205	0.0369 0.0041	0.0609	0.093
,	0	0.4792	0.2206							
7		0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.007
	1	0.3720	0.3960	0.3670	0.3115	0.2471	0.1848	0.1306	0.0872	0.054
	2	0.1240 0.0230	0.2097 0.0617	0.2753	0.3115	0.3177 0.2269	0.2985	0.2613	0.2140 0.2918	0.164
	4		0.0017	0.1147 0.0287	0.1730 0.0577	0.0972	0.1442	0.1935	0.2388	0.273
	5	0.0026	0.00109	0.0287	0.0377	0.0972	0.1442	0.1933	0.2388	0.273
	6	0.0002	0.0012	0.00043	0.0013	0.0236	0.0084	0.0172	0.0320	0.104
	7		0.0001	0.0004	0.0001	0.0002	0.0006	0.0016	0.0037	0.007
8	0	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.003
	1	0.3826	0.3847	0.3355	0.2670	0.1977	0.1373	0.0896	0.0548	0.031
	2	0.1488	0.2376	0.2936	0.3115	0.2965	0.2587	0.2090	0.1569	0.109
	3	0.0331	0.0839	0.1468	0.2076	0.2541	0.2786	0.2787	0.2568	0.218
	4	0.0046	0.0185	0.0459	0.0865	0.1361	0.1875	0.2322	0.2627	0.273
	5	0.0004	0.0026	0.0092	0.0231	0.0467	0.0808	0.1239	0.1719	0.218
	6		0.0002	0.0011	0.0038	0.0100	0.0217	0.0413	0.0703	0.109
	7		_	0.0001	0.0004	0.0012	0.0033	0.0079	0.0164	0.031
	8					0.0001	0.0002	0.0007	0.0017	0.003

### Quiz 2 Prep Question

Twenty percent of the employees of ABC Company use direct deposit and have their wages sent directly to the bank. Assume we random sample five employees. What is the probability that all five employees use direct deposit?

#### Solution:

```
n=5, trials is 5
p=.2, 20% chance they use direct deposit(yes or no)
X=5, we want to know the probability 5 employees used direct deposit
=Table B in your book, n=5,x=5,.2 column = .0003 is the probability all 5 employees use direct deposit
```

Or in Excel =BINOM.DIST(5,5,0.2,FALSE)
BINOM.DIST(successes, trials, probability, cumulative), false because we want exactly 5 successes not cumulative)

# Highlights: Video Segment 5.4 Sample Size for Continuous

# Sample Size Formula for Continuous Data

$$n = \left(\frac{z * \hat{\sigma}}{E}\right)^2$$

Look at sample size formula:

higher error = smaller sample size

Higher z value(higher confidence level) = larger sample size Higher sigma(std deviation) = larger sample size

#### **Example: Time to Complete Job**

- Suppose you have collected a simple random sample of data and found the standard deviation to be three minutes.
- How many samples are needed to detect a change in job completion time after a process improvement project is implemented?
  - · You are okay with a margin of error of two minutes.
  - Assume you want 95% confidence.

### Example: Time to Complete Job (cont.)

$$n = \left(\frac{1.96(3)}{2}\right)^2$$
$$= 8.6 \approx 9$$

- $z^*$  at 95% confidence = 1.96
- $\hat{\sigma}=3$ 
  - o Estimated population standard deviation
  - $\circ~$  Equivalent to sample standard deviation, s
- E = 2

Video

# Highlights: Video Segment 1.4 Types of Data





Discrete: the number of or proportion that fit into a category 789

Examples: Eye color, martial status, good/bad, boy girl, grade, objects that come in whole units(people, cars, animals, etc.)





Examples: Weight, height, distance, money, time, temperature, length



# Highlights: Video Segment: 1.7:Describing Data

#### 3 Ways to Measure the Center of the Data

- 1. <u>Mean</u>(average): To find the mean of the values in a data set, simply add up all the numbers and divide by how many numbers you have. Sensitive to outliers.
- 2. <u>Mode</u>(most frequent value): The mode of a data set is the data value that occurs with the greatest frequency.
- 3. <u>Median</u>(middle point of the data): The median of a data set is the *middle data value* when the data are put into ascending order. Half of the data values lie below the median, and half lie above. If the sample size *n* is odd, then the median is the middle value. If the sample size *n* is even, then the median is the mean of the two middle data values. **Less sensitive to outliers than mean.**

#### 3 Ways to Measure the Dispersion(or Spread) of the Data

- 1. <u>Range</u>: The range of a data set is the difference between the largest value and the smallest value in the data set:
- 2. <u>Standard Deviation</u>: may be interpreted as the typical difference between a data value and the sample mean for a given data set. Measures the spread of the data.  $s = \sqrt{s^2} = \sqrt{\frac{\Sigma(x-x)^2}{n-1}}$
- 3. <u>Variance</u>: is approximately the mean of the squared deviations in the sample given by the formula, standard deviation squared  $\frac{\nabla u}{s^2 = \Sigma (x \bar{x})^2}$

# Agenda

Topic	Time	Sunday Section	Wednesday Section
Introduction	5 min	6:30-6:35	9:00-9:05
Quiz 2 Prep	40 min	6:35-7:15	9:05-9:45
Highlights from Week 6 Video	40 min	7:15-7:55	9:45-10:25
Review of Upcoming Assignments and Open Question	5 min	7:55-8:00	10:25-10:30

# Highlights: Video Segment 6.3: Causation Video

It might be useful to explain that "causes" is an asymmetric relation (X causes Y is different from Y causes X), whereas "is correlated with" is a symmetric relation.

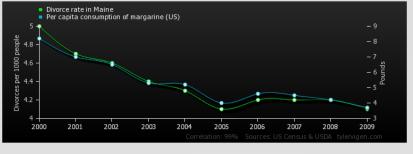
For instance, homeless population and crime rate might be correlated, in that both tend to be high or low in the same locations. It is equally valid to say that homelesss population is correlated with crime rate, or crime rate is correlated with homeless population. To say that crime causes homelessness, or homeless populations cause crime are different statements. And correlation does not imply that either is true. For instance, the underlying cause could be a 3rd variable such as drug abuse, or unemployment.

The mathematics of statistics is not good at identifying underlying causes, which requires some other form of judgement.

# Causality is also a relationship between two things, but it is not mathematical, it is physical (or philosophical).

Something causes something else if there is a chain of events between the first thing and the second thing, each of which causes the next thing in the chain to happen. Causality involves time; the first thing happens, and then later the second thing happens as a result. We say the first thing is the cause, and the second thing is the effect. Note that unlike correlation, the relationship is unsymmetrical.

# Divorce rate in Maine correlates with Per capita consumption of margarine (US)



	2000	2001	2002	2003	2004	2005	2006	2007	2008	<u>2009</u>
Divorce rate in Maine Divorces per 1000 people (US Census)	5	4.7	4.6	4.4	4.3	4.1	4.2	4.2	4.2	4.1
Per capita consumption of margarine (US) Pounds (USDA)	8.2	7	6.5	5.3	5.2	4	4.6	4.5	4.2	3.7
Correlation: 0.992558										

Permalink - Not interesting

http://www.businessinsider.com/spurious-correlations-by-tyler-vigen-2014-5

Correlation is the mathematical relationship between two things which are measured. It is given as a value between 0 and 1. A correlation of 0 means the two things are unrelated; given the first value, there is no way to predict the second. A correlation of 1 means the two things are completely related, the first thing always predicts the second. As an example, let's say you measure the heights and weights of a group of people. These have a high correlation, somewhere around .8; height is a good predictor of weight, and vice-versa. Now say you took the same group and measured eye color. There is a low correlation between eye color and height, pretty close to 0. They are basically independent, knowing one doesn't tell you anything about the other.

http://www.w-uh.com/posts/030302a\_correlation\_vs\_ca.html

### Highlights: Video Segment 6.3: Causation Video

#### **Acting on Correlation**

Correlation may not indicate causation, but it can lead to action. Consider the marketing analyst that finds a strong correlation between site searches and revenue. To increase revenue, do we drive visitors to the search page on our site? Maybe, but what if visitors go to the site search page because they are frustrated and can't find the product they want to purchase? Do we want to increase frustration?

So what good is correlation if you are telling me I can't assume causation? Correlation gives you a strong starting point for further testing and optimization. Since we know there is a relationship between site search and revenue, we can now look at why.

Through additional analysis, A/B tests, and customer surveys, we can better understand the relationship, context, and impact that our decisions will have on other factors.

#### So What?

Before diving into analysis, ask yourself "Why?" What is the business question and what value can be derived from my analysis? An important follow-up task is to consider the ability to take action and realize the value. As we get closer to understanding our target audience and what causes desired outcomes, remember the work is not finished until we act.

http://blogs.adobe.com/digitalmarketing/analytics/hey-did-i-do-that-the-difference-between-correlation-and-causation/

### Highlights: Video Segment 6.4:Regression Intro

### How Is Regression Useful?

- · A way to see whether an input variable and output variable are related
- A statistical tool that can determine the strength of that relationship
  - $\circ~$  Gives numeric value for strength of relationship where  $\chi^2$  test only tests for presence of relationship
- A model that predicts relationship between the two variables

### Example of Use in Sales

A tool to increase sales efficiency:

- Predicting when a customer might cancel order/not purchase product
- Anticipating customers' growth (production volume)
- · Understanding which customers to call and when

When you think of regression, think prediction. A regression uses the historical relationship between an independent and a dependent variable to predict the future values of the dependent variable. A simple linear regression uses only one independent variable, and it describes the relationship between the independent variable and dependent variable as a straight line.

https://www0.gsb.columbia.edu/premba/analytical/s7/s7\_6.cfm

# Regression Thought Process

- 1. Practical: Are these two variables practically linked?
- 2. Graphical: Plot the data!
  - What does the scatter plot look like?
- 3. Statistical: Now perform statistical analysis.

# **Scatterplots**

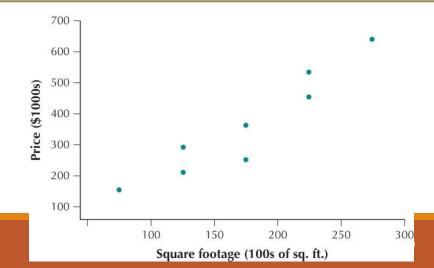
Whenever you are examining the relationship between two quantitative variables, your best bet is to start with a scatterplot. A **scatterplot** is used to summarize the relationship between two quantitative variables that have ben measured on the same element.

A **scatterplot** is a graph of points (x,y), each of which represents one observation from the data set.

One of the variables is measured along the horizontal axis and is called the *x variable*.

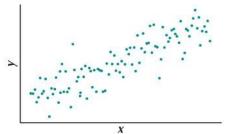
The other variable is measured along the vertical axis and is called the *y* variable.

Lot	x=square footage (100s of sq ft)	y=sales price (\$1000s)
Harding St	75	155
Newton Ave	125	210
Stacy Ct	125	290
Eastern Ave	175	360
Second St	175	250
Sunnybrook Rd	225	450
Ahlstrand Rd	225	530
Eastern Ave	275	635

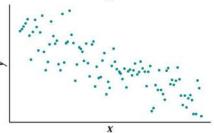


# **Scatterplots**

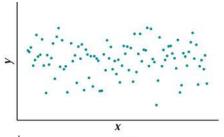
The relationship between two quantitative variables can take many different forms. Four of the most common are:



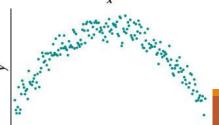
**Positive linear relationship:** As *x* increases, *y* also tends to increase.



**Negative linear relationship:** As *x* increases, *y* tends to decrease.



**No apparent relationship:** As *x* increases, *y* tends to remain unchanged.



**Nonlinear relationship:** The *x* and *y* variable are related, but not in a way that can be approximated using a straight line.

### Highlights: Video Segment 6.4:Regression Intro

# Simple Linear Regression: Equation

- Tries to create best-fitting line through plot
- Describes the relationship between two variables

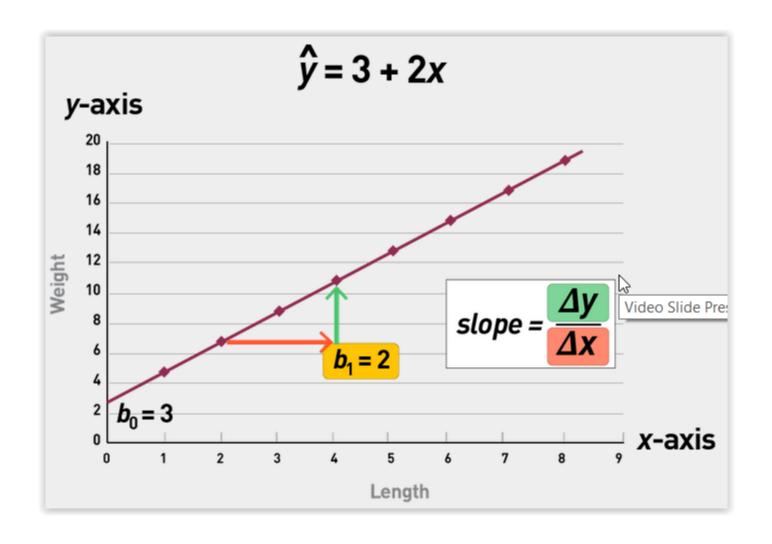
$$\hat{y} = b_0 + b_1 x$$

Output variable (predicted response)

y-intercept Slope Input variable

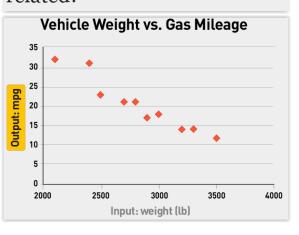
- *y*-intercept: where the line crosses the *y*-axis
- **Slope:** how slanted the line is

Highlights: Video Segment 6.4:Regression Intro

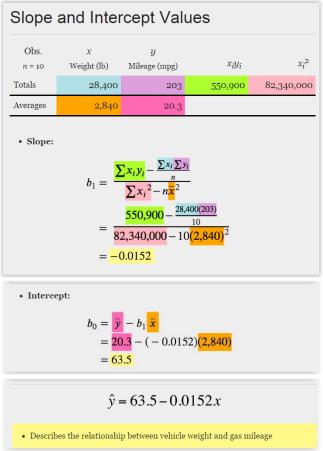


# Highlights: Video Segment 6.5: Regression Example By Hand

Example problem: How are the variables "vehicle weight" and "gas mileage" related?

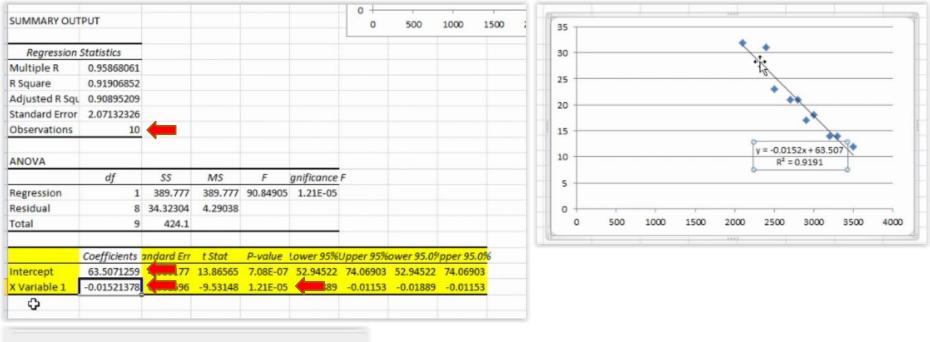


Obs.	X	y		
(i)	Weight (lb)	Mileage (mpg)	$x_i y_i$	$x_i^2$
1	3,000	18	54,000	9,000,000
2	2,800	21	58,800	7,840,000
3	2,100	32	67,200	4,410,000
4	2,900	17	49,300	8,410,000
5	2,400	31	74,400	5,760,000
6	3,300	14	46,200	10,890,000
7	2,700	21	56,700	7,290,000
8	3,500	12	42,000	12,250,000
9	2,500	23	57,500	6,250,000
10	3,200	14	44,800	10,240,000
Totals	28,400	203	550,900	82,340,000
Averages	2,840	20.3	·	



This formula can be used to predict gas mileage of any weight vehicle.

# Highlights: Video Segment 6.6:Regression in Excel (p value and R square)



 $\hat{y} = 63.5 - 0.0152x$ 

Describes the relationship between vehicle weight and gas mileage

P-value = t distribution two-tailed probability, If one divides this by 2, it is the probability that the true value of the coefficient has the opposite sign to that found. You want this probability to be small in order to be sure that this variable really influences y, certainly less than 0.1 (10%). http://people.clarkson.edu/~wwilcox/ES100/regrint.htm

These are the probabilities that the coefficients are not statistically significant. <a href="http://www.udel.edu/johnmack/frec834/regression\_intro.htm">http://www.udel.edu/johnmack/frec834/regression\_intro.htm</a>

Correlation Coefficient(r) is a measure of strength and direction and has values between -1 and 1.

### Highlights: Video Segment 6.7:Correlation

# Two Indices 1. Correlation coefficient (r)2. Coefficient of determination $(r^2)$

# Correlation Coefficient (r)

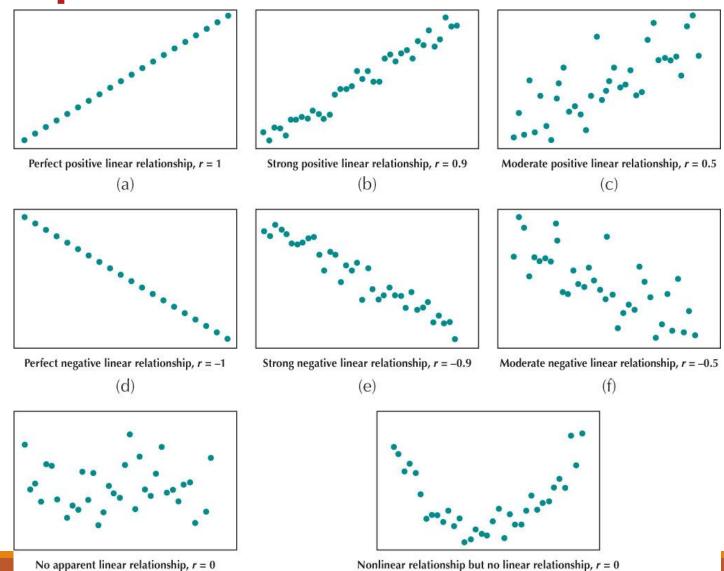
- -1 < r
- −1 = perfect negative correlation
- 1 = perfect positive correlation
- o = no relationship
- Rule of thumb: rvalue of  $\sim \pm 0.7$  desired
  - Indicates meaningful relationship

Scatterplots provide a visual description of the relationship between two quantitative variables. The *correlation coefficient* is a numerical measure for quantifying the linear relationship between two quantitative variables.

# Properties of r

- 1. The correlation coefficient r is always  $-1 \le r \le 1$ .
- 2. When r = +1, a perfect positive relationship exists between x and y.
- 3. Values of r near +1 indicate a positive relationship between x and y.
  - The closer *r* gets to +1, the stronger the evidence for a positive relationship.
  - The variables are said to be positively associated.
  - As *x* increases, *y* tends to increase.
- 4. When r = -1, a perfect negative relationship exists between x and y.
- 5. Values of *r* near -1 indicate a negative relationship between *x* and *y*.
  - The closer *r* gets to -1, the stronger the evidence for a negative relationship.
  - The variables are said to be negatively associated.
  - As *x* increases, *y* tends to decrease.
- 6. Values of r near 0 indicate there is no linear relationship between x and y.
  - The closer r gets to 0, the weaker the evidence for a linear relationship.
  - The variables are not linearly associated.
  - A nonlinear relationship may exist between x and y.

# Properties of r



(h)

### Highlights: Video Segment 6.7:Correlation

### Two Indices

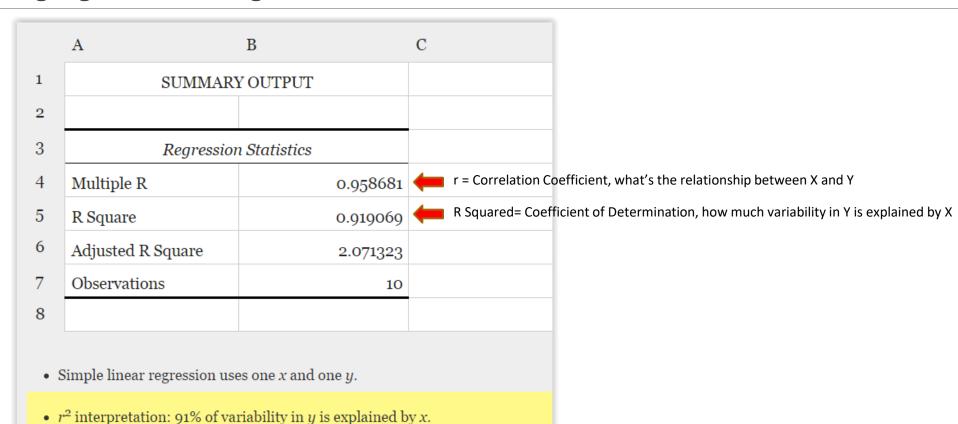
- 1. Correlation coefficient (r)
- 2. Coefficient of determination  $(r^2)$

# Coefficient of Determination $(r^2)$

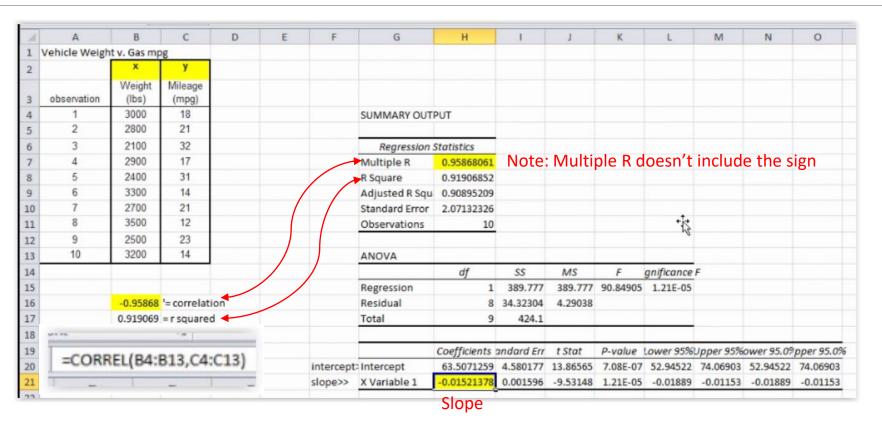
- · Correlation coefficient squared
- Measure of the percentage of variability in y that can be accounted for by x
  - $\circ$  Trying to find an input x that is influencing our output y
  - x will not explain all of y
  - Recall: There is variability in everything we do.
- Metric for whether input x is really contributing to output

Measures the goodness of fit of the regression equation to the data. We interpret  $r^2$  as the proportion of the variability in y that is accounted for by the linear relationship between y and x. The values that  $r^2$  can take are  $0 \le r^2 \le 1$ .

# Highlights: Video Segment 6.7:Correlation



# Highlights: Video Segment 6.8:Using Excel



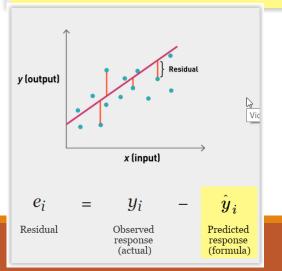
Multiple R = r, Correlation: get the sign from the X variable or separate correlation calc

# Highlights: Video Segment 6.9:Residuals and Other Warnings

#### What Is a Residual?

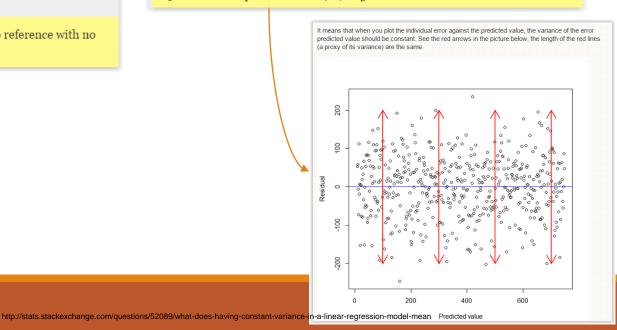
- Synonymous to error; should be random
- The distance between actual data point and the line determined by linear equation
- Determined by the difference between observed and predicted values of  $\boldsymbol{y}$ 
  - o Ideally, points fall on regression line (i.e., perfect model)
  - o Error would then be zero (rare).

• When plotted, a random series of points around a zero reference with no evidence of a pattern



### Assumptions of Regression

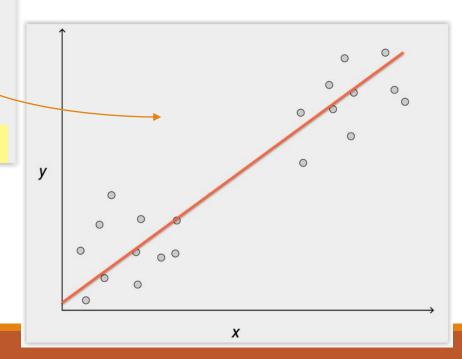
- 1. Residuals are independent.
- 2. Residuals are normally distributed with a mean of zero.
  - The regression line will sometimes be high or low (i.e., over- or underpredicting).
- 3. There are equal variances ( $\sigma^2$ ) of y.



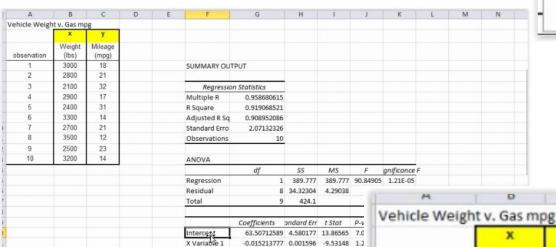
# Highlights: Video Segment 6.9:Residuals and Other Warnings

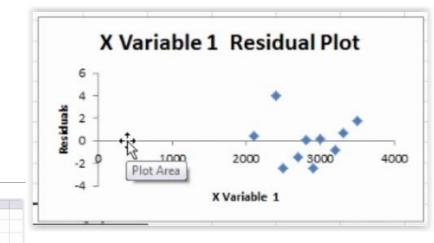
### Other Points of Interest

- Certain data is inappropriate for a regression analysis:
  - o Residuals form a pattern.
  - · Large outliers are present.
  - o "Clumped" data appears linear.
- Avoid extrapolating outside data.
- Beware of lurking variables, or Simpson's paradox.
- A strong correlation does not mean causation.



# Highlights: Video Segment 6.10:Residuals and Other Warnings Using Excel



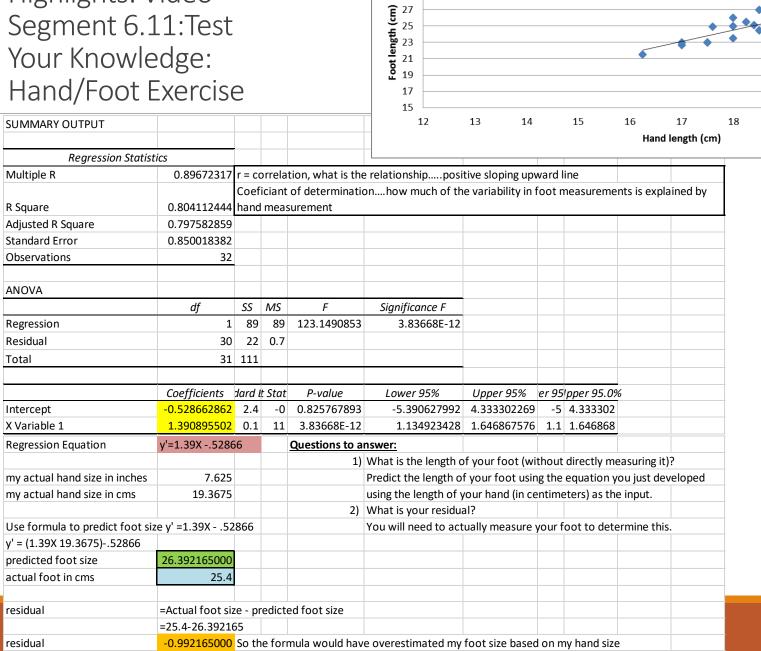


**RESIDUAL OUTPUT** 

	х	У	
observation	Weight (lbs)	Mileage (mpg)	
1	3000	18	
2	2800	21	
3	2100	32	
4	2900	17	
5	2400	31	
6	3300	14	
7	2700	21	
8	3500	12	
9	2500	23	
10	3200	14	

Predicted Y	Residuals
17.86579572	0.134204
20.90855107	0.091449
31.55819477	0.441805
19.3871734	-2.38717
26.99406176	4.005938
13.30166271	0.698337
22.42992874	-1.42993
10.25890736	1.741093
25.47268409	-2.47268
14.82304038	-0.82304
	17.86579572 20.90855107 31.55819477 19.3871734 26.99406176 13.30166271 22.42992874 10.25890736

# Highlights: Video Segment 6.11:Test Your Knowledge:



31

29

**Hand / Foot Relationship** 

y = 1.3909x - 0.5287

 $R^2 = 0.8041$ 

19

20

21

22

# Agenda

Topic	Time	Sunday Section	Wednesday Section
Introduction	5 min	6:30-6:35	9:00-9:05
Quiz 2 Prep	40 min	6:35-7:15	9:05-9:45
Highlights from Week 6 Video	40 min	7:15-7:55	9:45-10:25
Review of Upcoming Assignments and Open Question	5 min	7:55-8:00	10:25-10:30

# Review of Upcoming Assignments: Wednesday

- 1. Quiz #2 is due, Saturday, February 25th, midnight EST in the Learning Management System.
  - There are 5 calculation questions and 1, 10 part definitional question
  - DO NOT LEAVE ANY BLANK
  - You can not start and stop the Quiz.
  - There is a 2 hr. time limit. There is no timer, you must keep track of your own time.
  - Password for the Quiz is: TestTime101
  - At the end must click, NEXT at the bottom of the questions, then click Submit all and Finish, then click AGAIN Submit all and Finish in dialog box



- 2. Optional Learning: 7.9 Relate Regression to Your Project
- 3. HMWK #4 isn't due until 3/4 however it is a Learning Curve on Chapter 4: Correlation and Regression so you may want to do it after you are done with your Quiz #2 just to get it out of the way
- 4. Understanding Variation Book start reading it before Live Class #8, Wednesday, 3/8
- 5. Projects Analyze should be in full swing moving into Improve