Finance Essentials

Calculating Historical Return Statistics

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First draft: February 17, 2003
This revision: November 21, 2009

This document shows how to calculate commonly used statistics—average returns, standard deviations of returns, and covariances between returns—for historical data. In this document I have used 60 monthly returns on IBM (denoted as r_1) and LUV (denoted as r_2).

The Data

The following table lists historical monthly returns on the two stocks. Returns from 07/2005 to 09/2009 are omitted from this table to conserve space. The bottom two rows show the number of elements (count) and the sum of the elements in the column.

Month	r_1	r_2
11/2004	5.20%	-0.26%
12/2004	4.60%	3.53%
01/2005	-5.23%	-11.02%
02/2005	-0.71%	-4.38%
03/2005	-1.30%	2.91%
…	…	…
10/2009	2.57%	-6.77%
Count	60	60
Total	0.5073	-0.3429

For calculations, we also need the sum of squares and cross-products of the two returns:

Month	r_1^2	r_2^2	$r_1 r_2$
11/2004	0.0027	0.0000	-0.0001
12/2004	0.0021	0.0012	0.0016
01/2005	0.0027	0.0121	0.0058
02/2005	0.0001	0.0019	0.0003
03/2005	0.0002	0.0008	-0.0004
…	…	…	…
10/2009	0.0007	0.0046	-0.0017
Count	60	60	60
Total	0.2408	0.4103	0.0971

The Calculation Details

Let us define the following notation:

r_{1t}	=	Return on asset 1 during time period (month) t
r_{2t}	=	Return on asset 2 during time period (month) t
T	=	Number of time periods (months)
$\bar{r}_1$	=	Average return on asset 1
$\bar{r}_2$	=	Average return on asset 2
σ_1	=	Standard deviation of returns on asset 1
σ_2	=	Standard deviation of returns on asset 2
$\text{Cov}(r_1, r_2)$	=	Covariance between returns on assets 1 and 2
ρ_{12}	=	Correlation between returns on assets 1 and 2

Now we can use the following formulas to calculate the various statistics. Consult any standard probability and statistics text book for the explanation of these formula.

$$\bar{r}_1 = \frac{\sum r_{1t}}{T} = \frac{0.5073}{60} = 0.008455396 \approx 0.85\%$$

$$\bar{r}_2 = \frac{\sum r_{2t}}{T} = \frac{-0.3429}{60} = -0.00572000 \approx -0.57\%$$

$$\sigma_1^2 = \frac{\sum r_{1t}^2 - \frac{(\sum r_{1t})^2}{T}}{T-1}$$

$$= \frac{0.2408 - \frac{0.5073^2}{60}}{60-1}$$

$$= 0.004009355$$

$$\sigma_1 = \sqrt{0.004009355}$$

$$= 0.063319467 \approx 6.33\%$$

$$\sigma_2^2 = \frac{\sum r_{2t}^2 - \frac{(\sum r_{2t})^2}{t}}{t-1}$$

$$= \frac{0.4103 - \frac{-0.3429^2}{60}}{60-1}$$

$$= 0.006921573$$

$$\sigma_2 = \sqrt{0.006921573}$$

$$= 0.08319599 \approx 8.32\%$$

$$\text{Cov}(r_1, r_2) = \frac{\sum r_{1t} r_{2t} - \frac{(\sum r_{1t} \sum r_{2t})}{T}}{T-1}$$

$$= \frac{0.0971 - \frac{0.5073 \times -0.3429}{60}}{60-1}$$

$$= 0.001695375$$

$$\rho_{12} = \frac{\text{Cov}(r_1, r_2)}{\sigma_1 \sigma_2}$$

$$= \frac{0.001695375}{0.063319467 \times 0.08319599}$$

$$= 0.321829753 \approx 0.32$$

The Statistics

The statistics are usually expressed in a tabular format shown below:

				Correlation	
	T	Avg Ret	Std Dev	1	2
1	60	0.85%	6.33%	1.00	0.32
2	60	-0.57%	8.32%	0.32	1.00

These statistics can also be obtained by utilizing the spreadsheet functions `AVERAGE`, `STDEV`, `COVAR`, and `CORREL`. These calculations and more are shown in the spreadsheet `Calculating_Historical_Return_Statistics.xlsx`.

Keep in mind that these statistics describe historical behavior, not the prospective behavior.