Week 9 - Portfolio Analytics

Let's keep moving on...

So, what is the context?

- We have working capital with three main drivers of risk and return: two exchange rates and a commodity price.
- Over time how do these factors act and interact to produce EBITDA returns on Assets? [EBITDA = Earnings Before Interest and Tax adding back in non-cash Depreciation and Amortization].
- Given risk and performance in the market-place (as ONE place we would look...remembering there are many other, much more qualitative factors in play as well) what is the least risky for the return combination of these factors in our working capital?
- Then, how does that combination compare with today's reality and especially answer the CFO's question of what to do about the millstone of working capital around the neck of EBITDA?

Given this context, and the data we found earlier on, next in line is the calculation of the variance-covariance matrix. The diagonals of this matrix are the variances, so that the square root of the diagonal will yield standard deviations. The off-diagonals can be converted to correlations.

```
(mean.R <- apply(R, 2, mean))
```

```
##      EUR.USD      GBP.USD    OIL.Brent
##  0.001538585 -0.002283062  0.010774203
```

```
(cov.R <- cov(R))
```

```
##             EUR.USD   GBP.USD OIL.Brent
## EUR.USD   0.3341046 0.1939273 0.1630795
## GBP.USD   0.1939273 0.2538908 0.1809121
## OIL.Brent 0.1630795 0.1809121 5.0572328
```

```
(sd.R <- sqrt(diag(cov.R)))  # remember these are in daily percentages
```

```
##   EUR.USD   GBP.USD OIL.Brent
## 0.5780178 0.5038758 2.2488292
```

Now for some programming (quadratic that is...)

In a mathematical nutshell we are formally (more tractable version to follow...) solving the problem of minimizing working capital factors risk, subject to target returns and a budget that says it all has to add up to our working capital position. We define weights as percentages of the total working capital position. Thus the weights need to add up to one.

$$
\begin{gathered}
min_w w_T \Sigma w \\
subject\ to \\
1^T w = 1 \\
w^T \mu = \mu_0
\end{gathered}
$$

where

- w are the weights in each instrument.
- Σ is the variance-covariance matrix we just estimated, `cov.R`.
- 1 is a vector of ones's with length equal to the number of instruments.
- μ are the mean returns we just estimated, `mean.R`.
- μ_0 is the target portfolio return.
- T is the matrix transpose.
- min_w means to find weights w that minimizes portfolio risk.

(ENGLISH version to follow...)

(Well some English here...) The expression $w_T \Sigma w$ is our measure of portfolio risk and is a quadratic form that looks like this for two instruments:

$$\begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Multiplied out we get the following quadratic formula for portfolio variance:

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_1 w_2 \sigma_{12} + w_2 w_1 \sigma_{21}$$

and because $\sigma_{12} = \sigma_{21}$ this reduces a bit to

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12}$$

Tedious? Definitely. But useful to explain the components of portfolio risk

1. Two dashes of own asset risk $w_1^2\sigma_1^2 + w_2^2\sigma_2^2$, and
2. Two dashes of relational risk $2w_1w_2\sigma_{12}$

When $\sigma_{12} < 1$ we have *diversification*.

Try this

Suppose you have two commodities (New York Harbor No. 2 Oil and Henry Hub Natural Gas) feeding a production process (Electricity Generation).

Big Allis LIC, NY

These are the weights in this process:

$$w = \{w_{oil} = -.5, w_{ng} = -.5, w_{ele} = 1.0\}$$

The percentage changes in terms of the prices of these commodities are:

$$\mu = \{\mu_{oil} = 0.12, \mu_{ng} = -0.09, \mu_{ele} = 0.15\}.$$

Standard deviations are

$$\sigma = \{\sigma_{oil} = 0.20, \sigma_{ng} = 0.15, \sigma_{ele} = 0.40\}$$

The correlation matrix is

$$\rho = \begin{bmatrix} 1.0 & 0.2 & 0.6 \\ 0.2 & 1.0 & 0.4 \\ 0.6 & 0.4 & 1.0 \end{bmatrix}$$

1. Using the formula

$$\Sigma = (\sigma\sigma^T)\rho$$

   Calculate the variance-covariance matrix Σ using your `R` knowledge of arrays. [Hint: `t()` is the transpose of an array so that σ^T is `t(sigma)`.]
2. Calculate the portfolio mean return.
3. Calculate the portfolio standard deviation.

Thinking...

Results

1. Use this R code:

```
sigma <- c(0.2, 0.15, 0.4)
rho = c(1, 0.2, 0.6, 0.2, 1, 0.4, 0.6,
    0.4, 1)
(rho <- matrix(rho, nrow = 3, ncol = 3))
```

```
##      [,1] [,2] [,3]
## [1,]  1.0  0.2  0.6
## [2,]  0.2  1.0  0.4
## [3,]  0.6  0.4  1.0
```

```
(Sigma <- (sigma %*% t(sigma)) * rho)
```

```
##       [,1]   [,2]  [,3]
## [1,] 0.040 0.0060 0.048
## [2,] 0.006 0.0225 0.024
## [3,] 0.048 0.0240 0.160
```

The diagonals are the squared standard deviations.

2. Use this R code:

```
w <- c(-0.5, -0.5, 1)
mu <- c(0.12, -0.09, 0.15)
(mu.P <- t(w) %*% mu)
```

```
##       [,1]
## [1,] 0.135
```

2. Next the portfolio average level of “risk”:

```
rho = c(1, 0.2, 0.6, 0.2, 1, 0.4, 0.6,
    0.4, 1)
(rho <- matrix(rho, nrow = 3, ncol = 3))
```

```
##      [,1] [,2] [,3]
## [1,]  1.0  0.2  0.6
## [2,]  0.2  1.0  0.4
## [3,]  0.6  0.4  1.0
```

```
(Sigma2 <- (sigma %*% t(sigma)) * rho)
```

```
##       [,1]   [,2]  [,3]
## [1,] 0.040 0.0060 0.048
## [2,] 0.006 0.0225 0.024
## [3,] 0.048 0.0240 0.160
```

```
(Sigma.P <- (t(w) %*% Sigma2 %*% w))^0.5
```

```
##           [,1]
## [1,] 0.3265348
```

3. What does this mean?

- Running a power-generating plant (or refinery, or distribution chain, ...) over time financially really means generating a spark spread: the margin between costs of inputs natural gas and oil (the negative or short position) and revenue from the output of electricity (the positive or long position).
- The average spark spread for this plant is 10.67%.
- The average standard deviation of the spark spread is 32.65%.

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Our next job is to use these mechanics about portfolio means, standard deviations, and correlations to find the best set of weights that minimizes portfolio risk while attempting to achieve a target level of return.

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