Practice Set #1

solution revised 1/21/2017

Purpose, Process, Product

These practice sets will repeat various `R` features in this chapter. Specifically we will practice defining vectors, matrices (arrays), and data frames and their use in present value, growth, future value calculations, We will build on this basic practice with the computation of ordinary lease squares coefficients and plots using `ggplot2`. We will summarize our findings in debrief documented with an `R markdown` file and output.

`R Markdown` set up

1. Open a new `R Markdown` pdf document file and save it with file name `MYName-FIN654-PS01` to your working directory. The `Rmd` file extension will automatically be appended to the file name. Create a new folder called `data` in this working directory and deposit the `.csv` file for practice set #2 to this directory.
2. Modify the `YAML` header in the `Rmd` file to reflect the name of this practice set, your name, and date.
3. Replace the `R Markdown` example in the new file with the following script.

```
# Practice set 1: present value
(INSERT results here)
# Practice set 2: regression
(Insert results here)
```

4. Click `knit` in the `Rstudio` command bar to produce the `pdf` document.

Warmups

In a very few lines of `R` code

1. Calculate the present value of receiving \$1 in perpetuity at a yield of 10% per annum when growth rates per annum might take on values of 0%, 3%, or 5%, each in perpetuity.

```r
cashflow <- 1
rate <- 0.1
growth <- c(0, 0.03, 0.05)
(pv.perpetuity <- cashflow/(rate - growth))
```

```
## [1] 10.00000 14.28571 20.00000
```

2. Calculate the present value of receiving \$1 for each of 5 years at yields of 5%, 10%, and 15% per annum, each for the 5 year term of the present value.

```r
cashflow <- 1
rate <- c(0.05, 0.1, 0.15)
term <- 5
(pv.perpetuity <- cashflow * (1/rate -
    1/(rate * (1 + rate)^term)))
```

```
## [1] 4.329477 3.790787 3.352155
```

3. Calculate the cumulative sum of working capital across 5 years, when the starting value of working capital is 100, and when working capital growth rates might take on values of 0%, 3%, or 5% per annum, for each year of the 5 year projection.

Set A

Problem

We work for a mutual fund that is legally required to fair value the stock of unlisted companies it owns. Your fund is about to purchase shares of InUrCorner, a U.S. based company, that provides internet-of-things legal services.

- We sampled several companies with business plans similar to InUrCorner and find that the average weighted average cost of capital is 18%.
- InUrCorner sales is $80 million and projected to growth at 50% per year for the next 3 years and 15% per year thereafter.
- Cost of services provided as a percent of sales is currently 70% and projected to be flat for the foreseeable future.
- Depreciation is also constant at 5% of net fixed assets (gross fixed asset minus accumulated depreciation), as are taxes (all-in) at 25% of taxable profits.
- Discussions with InUrCorner management indicate that the company will need an increase in working capital at the rate of 15% each year and an increase in fixed assets at the rate of 5% of sales each year. Currently working capital is $10, net fixed assets is $90, and accumulated depreciation is $15.

Solutions

1. Let's project `sales`, `cost`, increments to net fixed assets `NFA`, increments to working capital `WC`, `depreciation`, `tax`, and free cash flow `FCF` for the next 4 years. We will use a table to report the projection.

```
growth <- rep(0.5, 4)  # vector of 4 growth ratios:
growth[4] <- 0.15  # replace 4 year growth value
sales0 <- 80  # constant
WC0 <- 10  # constant
NFA0 <- 90  # constant
DEP.accum <- 15  # constant
time <- 1:4  # time index
year0 <- 2016  # base (valuation) year
year <- year0 + time  # projection years
sales <- sales0 * (1 + growth)^time  # sales projection
sales[4] <- sales[3] * (1 + growth[4])  # correct last year's forecast for change in growth
cost.sales <- 0.7  # constant ratio: cost / sales
cost <- cost.sales * sales  # cost projection
WC.incr.sales <- 0.1  # constant ratio: incrWC / sales
NFA.incr.sales <- 0.05  # constant ratio: incrNFA / sales
WC.incr <- WC.incr.sales * sales  # working capital increment projection
NFA.incr <- NFA.incr.sales * sales  # net fixed assets increment projection
WC <- cumsum(c(WC0, WC.incr))[-1]  # working capital projection
NFA <- cumsum(c(NFA0, NFA.incr))[-1]  # net fixed assets projection
depreciation.NFA <- 0.05  # constant ratio: depreciation / net fixed assets
depreciation <- depreciation.NFA * NFA  # depreciation projection
```

```
tax.rate <- 0.25  # tax rate constant:
tax <- (sales - cost - depreciation) *
    tax.rate
# tax projection
FCF <- sales - cost - depreciation -
    tax - WC.incr - NFA.incr
# free cash flow projection
```

Let's use this code to build and display a table.

```
# Form table of results
table.names <- c("Sales", "Cost", "Working Capital (incr.)",
    "Net Fixed Assets (incr.)", "Free Cash Flow")
# Assign projection labels
table.year <- year  # Assign projection years
table.data <- rbind(sales, cost, WC.incr,
    NFA.incr, FCF)  # Layer projections
rownames(table.data) <- table.names  # Replace rows with projection labels
colnames(table.data) <- table.year  # Replace columns with projection years
knitr::kable(table.data)  # Display a readable table
```

	2017	2018	2019	2020
Sales	120.0	180.0000	270.00000	310.50000
Cost	84.0	126.0000	189.00000	217.35000
Working Capital (incr.)	12.0	18.0000	27.00000	31.05000
Net Fixed Assets (incr.)	6.0	9.0000	13.50000	15.52500
Free Cash Flow	5.4	9.5625	15.80625	18.26156

2. Compute the present value of the cash flows assuming that year three (2019) is the "terminal" year. This can be interpreted such that year four (2020) is the assumed perpetual cash flow with growth rate 15% in perpetuity. Total present value is composed of the present value of free cash flows from year 1 through year 4 earning the weighted average cost of capital plus the present value of the lump-sum terminal value of free cash flows from years four in perpetuity. Assuming that the riskiness of the entity does not appreciably change from year 1 into perpetuity, then the same weighted average cost of capital may be used for terminal value as for the valuation of years one through three. We can then express the total present value as:

$$PV = \sum_{t=1}^{3} \frac{FCF_t}{(1+WACC)^t} + \frac{1}{(1+WACC)^3}\left(\frac{FCF_4}{WACC-g}\right)$$

where, FCF_t is free cash flow at year t, $WACC$ is the weighted average cost of capital, and g is the perpetual growth rate.

- Compute the present value of free cash flows from years 2017 through 2019 inclusive.
- Compute the present value of terminal value cash flows.
- Compute the total present value of the entity.
- Construct a table to report the results.

```
WACC <- 0.18
g <- 0.15
t <- 1:3
(pv.1to3 <- sum(FCF[1:3]/(1 + WACC)^t))
```

```
## [1] 21.06408
```

```
(pv.terminal <- (1/(1 + WACC)^4) * FCF[4]/(WACC -
    g))
```

```
## [1] 313.9704
```

```
(pv.total <- pv.1to3 + pv.terminal)
```

```
## [1] 335.0344
```

```
table.col <- "Present Value"
table.row <- c("2017-2019", "2020+",
    "Total")
pv.table <- matrix(c(pv.1to3, pv.terminal,
    pv.total), ncol = 1)
colnames(pv.table) <- table.col
rownames(pv.table) <- table.row
knitr::kable(pv.table)
```

	Present Value
2017-2019	21.06408
2020+	313.97036
Total	335.03444

Set B

Problem

We work for a healthcare insurer and our management is interested in understanding the relationship between input admission and outpatient rates as drivers of expenses, payroll, and employment. We gathered a sample of 200 hospitals in a test market in this data set.

```
x.data <- read.csv("data/hospitals.csv")
```

Solutions

1. Build a table that explores this data set variable by variable and relationships among variables.

```
head(x.data)
```

```
##   hospital beds admissions outpatients births expense payroll  fte
## 1        1  210       7713       86982    312   56831   22061  792
## 2        2  347      16065      149222   1077  127223   55799 1762
## 3        3  511      23028      222565   1027  157093   61326 2310
## 4        4  142       4338       36710    355   24462   10503  328
## 5        5   40        905       13350    168   13730    6368  181
## 6        6  220      15563       88721   3810   93257   33920 1077
```

```
tail(x.data)
```

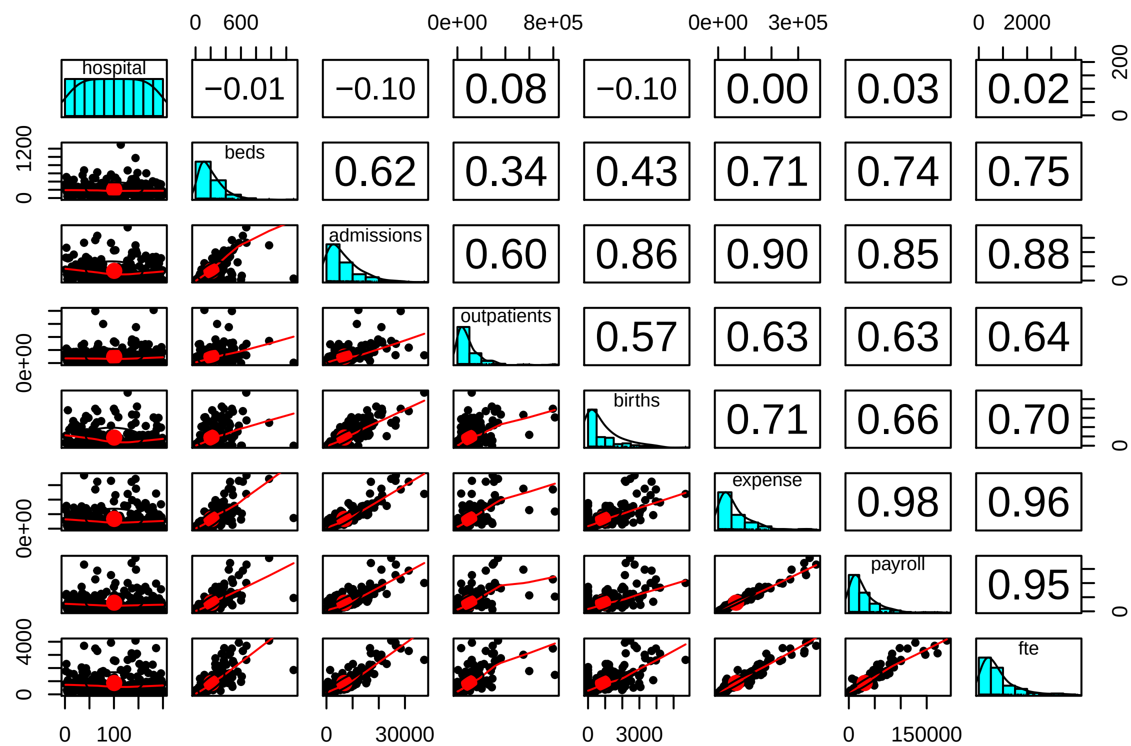
```
##     hospital beds admissions outpatients births expense payroll  fte
## 195      195   70       2089       24369    387   17257    7425  216
## 196      196  334      15696      102641   1946  168045   78118 1593
## 197      197  190       6395      244254    545   79859   33639 1055
## 198      198  122        441           0      0   15321    8878  399
```

```
## 199       199  170        7244       167454     838    58247    25018   834
## 200       200   73         352         9714      51     4565     2228   104
```

```r
summary(x.data)
```

```
##     hospital           beds          admissions     outpatients    
##  Min.   :  1.00   Min.   :   7.0   Min.   :  111   Min.   :     0  
##  1st Qu.: 50.75   1st Qu.:  84.5   1st Qu.: 1615   1st Qu.: 27316  
##  Median :100.50   Median : 160.0   Median : 4777   Median : 65329  
##  Mean   :100.50   Mean   : 209.9   Mean   : 6832   Mean   : 98225  
##  3rd Qu.:150.25   3rd Qu.: 270.0   3rd Qu.: 9766   3rd Qu.:123263  
##  Max.   :200.00   Max.   :1297.0   Max.   :37375   Max.   :813369  
##      births         expense          payroll            fte        
##  Min.   :   0   Min.   :  2082   Min.   :  1053   Min.   :  50.0  
##  1st Qu.:   0   1st Qu.: 20544   1st Qu.:  8693   1st Qu.: 314.0  
##  Median : 480   Median : 43365   Median : 20740   Median : 589.5  
##  Mean   : 874   Mean   : 67140   Mean   : 30501   Mean   : 861.5  
##  3rd Qu.:1309   3rd Qu.: 89899   3rd Qu.: 40275   3rd Qu.:1095.2  
##  Max.   :5699   Max.   :367706   Max.   :188865   Max.   :4087.0  
```

```r
require(psych)
pairs.panels(x.data)
```



2. Investigate the influence of admission and outpatient rates on expenses and payroll. First, form these arrays.

```r
y <- as.vector(x.data[, "expense"])
X <- as.matrix(cbind(1, x.data[, c("admissions",
```

```r
    "outpatients")]))
head(y)
```

```
## [1]  56831 127223 157093  24462  13730  93257
```

```r
tail(y, n = 3)
```

```
## [1] 15321 58247  4565
```

```r
head(X)
```

```
##      1 admissions outpatients
## [1,] 1       7713       86982
## [2,] 1      16065      149222
## [3,] 1      23028      222565
## [4,] 1       4338       36710
## [5,] 1        905       13350
## [6,] 1      15563       88721
```

Next, compute the regression coefficients.

```r
XTX.inverse <- solve(t(X) %*% X)
(beta.hat <- XTX.inverse %*% t(X) %*%
    y)
```

```
##                     [,1]
## 1           -118.9178095
## admissions     8.6994845
## outpatients    0.0796671
```

Finally, compute the regression statistics.

```r
e <- y - X %*% beta.hat
e <- y - X %*% beta.hat
(e.sse <- t(e) %*% e)
```

```
##              [,1]
## [1,] 171528065201
```

```r
(n <- dim(X)[1])
```

```
## [1] 200
```

```r
(k <- nrow(beta.hat))
```

```
## [1] 3
```

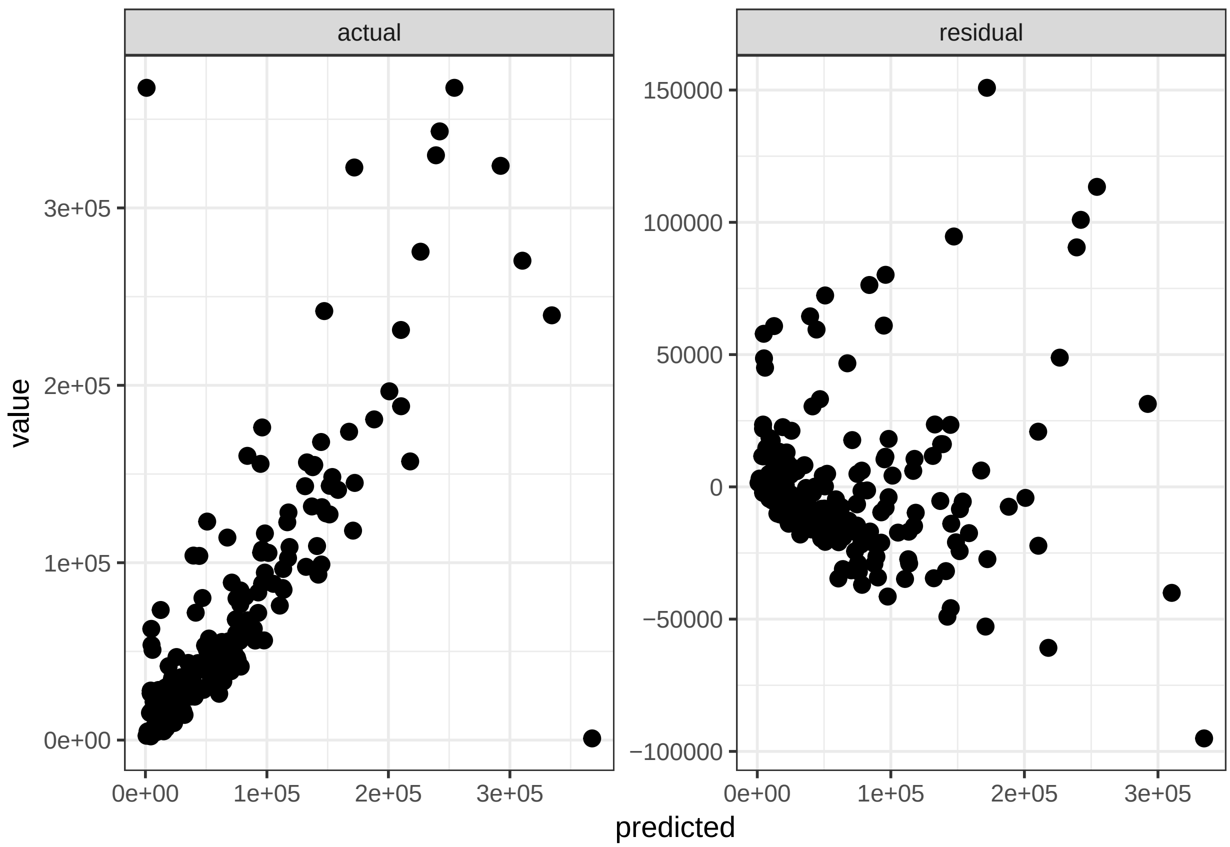
```r
(e.se <- (e.sse/(n - k))^0.5)
```

```
##          [,1]
## [1,] 29507.64
```

3. Use this code to investigate further the relationship among predicted expenses and the drivers, admissions and outpatients.

```r
require(reshape2)
require(ggplot2)
actual <- y
predicted <- X %*% beta.hat
residual <- actual - predicted
results <- data.frame(actual = actual,
```

```r
    predicted = predicted, residual = residual)
# Insert comment here
min_xy <- min(min(results$actual), min(results$predicted))
max_xy <- max(max(results$actual), max(results$predicted))
# Insert comment here
plot.melt <- melt(results, id.vars = "predicted")
# Insert comment here
plot.data <- rbind(plot.melt, data.frame(predicted = c(min_xy,
    max_xy), variable = c("actual", "actual"),
    value = c(max_xy, min_xy)))
# Insert comment here
p <- ggplot(plot.data, aes(x = predicted,
    y = value)) + geom_point(size = 2.5,
    colours = "blue") + theme_bw()
p <- p + facet_wrap(~variable, scales = "free")
p
```



Practice Set Debrief

1. List the R skills needed to complete these practice labs.
2. Explain each of the packages used to compute and graph results.
3. Discuss how well did the results begin to answer the business questions posed at the beginning of each practice lab.