Finance Essentials

Single Period Return and IRR

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This document shows that single period return formula is a special case of the multiple period average return calculated as the internal rate of return (IRR) for a set of cash flows.

Single Period

In a single period setting, where you buy a security at time $t = 0$ for some price P_0, hold it for one period, receive a cash flow (dividend or coupon) C_1 and then sell it for P_1, we calculate the return as:

$$r = \frac{C_1 + P_1 - P_0}{P_0} = \frac{C_1 + P_1}{P_0} - 1 \tag{1}$$

If there is no cash flow at $t = 1$, i.e., $C_1 = 0$, we get:

$$r = \frac{P_1 - P_0}{P_0} = \frac{P_1}{P_0} - 1 \tag{2}$$

Multiple Period

In a multiple period setting, you buy the stock at $t = 0$ for P_0 and receive cash flows C_1, C_2, C_3, etc. at $t = 1$, $t = 2$, $t = 3$, and sell the stock for P_n at $t = n$. In that case, the *average* rate of return over the holding period is calculated by solving for the discount rate r that makes the NPV of the cash flows equal to zero:[1]

$$-P_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots + \frac{C_n + P_n}{(1+r)^n} = 0 \tag{3}$$

This process of calculating r given P_0, C_1, C_2, $\cdots$, C_n and P_n is known as calculating internal rate of return (IRR). The IRR is typically calculated using trial and error because the equation doesn't have a closed form solution.

[1] To generalize the process so the cash flows are not evenly spaced but may occur at any time points, we can have cash flows at t_1, t_2, t_3, etc., and the equation will be:

$$-P_0 + \frac{C_1}{(1+r)^{t_1}} + \frac{C_2}{(1+r)^{t_2}} + \frac{C_3}{(1+r)^{t_3}} + \cdots + \frac{C_n + P_n}{(1+r)^{t_n}} = 0$$

Proof that Single Period Return is an IRR

To demonstrate that the single period return is actually an IRR, write the IRR equation for a single period and do some rearranging:

$$\begin{aligned} -P_0 + \frac{C_1 + P_1}{1+r} &= 0 \\ \frac{C_1 + P_1}{1+r} &= P_0 \\ C_1 + P_1 &= P_0(1+r) \\ C_1 + P_1 &= P_0 + P_0 r \\ C_1 + P_1 - P_0 &= P_0 r \\ \Rightarrow \quad r &= \frac{C_1 + P_1 - P_0}{P_0} \end{aligned}$$

which is identical to the single period return equation (1).

Alternatively, start with equation (1) and rearrange:

$$\begin{aligned} r &= \frac{C_1 + P_1 - P_0}{P_0} \\ rP_0 &= C_1 + P_1 - P_0 \\ P_0 + rP_0 &= C_1 + P_1 \\ (1+r)P_0 &= C_1 + P_1 \\ P_0 &= \frac{C_1 + P_1}{(1+r)} \\ -P_0 + \frac{C_1 + P_1}{(1+r)} &= 0 \end{aligned}$$

which is the IRR setup for a single period situation.