Finance Essentials

Converting Monthly Return Statistics to Annual

Ravi Shukla

First draft: November 2, 2008
This revision: June 5, 2012

Often we calculate return statistics—average and standard deviation—using monthly data, but need to express them in the annual units. Monthly statistics are converted to annual units using the following formulas:

$$\text{avg}(r_a) = 12\,\text{avg}(r_m) \tag{1}$$

$$\sigma(r_a) = \sqrt{12}\,\sigma(r_m) \tag{2}$$

where subscripts a and m denote annual and monthly, respectively.

These conversion formulas are based on the assumption that the monthly returns over the sample period have identical and independent probability distributions (i.i.d.). Imagine drawing 12 returns, one for each month of the year, from this probability distribution. Suppose the draws are r_{m_1}, r_{m_2}, …, $r_{m_{12}}$. Using these returns and the principles of compound interest we can calculate the annual return, r_a, as:

$$1 + r_a = (1 + r_{m_1})(1 + r_{m_2}) \cdots (1 + r_{m_{12}}) \tag{3}$$

Expanding the right hand side of equation (3) we get:

$$r_a = r_{m_1} + r_{m_2} + \cdots + r_{m_{12}} + \text{sum of cross-product terms} \tag{4}$$

where cross-product terms are products of 2, 3, 4, …, 12 combinations of monthly returns such as $r_{m_1}r_{m_2}$, $r_{m_1}r_{m_3}$, …, $r_{m_1}r_{m_2}r_{m_3}$, $r_{m_1}r_{m_2}r_{m_4}$, …, $r_{m_1}r_{m_2}r_{m_3}\cdots r_{m_{12}}$. Taking expectation of both sides of equation (4) we get:

$$E(r_a) = E(r_{m_1}) + E(r_{m_2}) + \cdots + E(r_{m_{12}}) + E(\text{sum of cross-product terms}) \tag{5}$$

Since the successive draws of monthly returns are assumed to be independent, the expected values of the cross-product terms are zeros. Furthermore, due to the assumption of identical probability distributions, $E(r_{m_1}) = E(r_{m_2}) = \cdots = E(r_{m_{12}}) = E(r_m)$. Therefore we get:

$$E(r_a) = 12E(r_m) \tag{6}$$

To calculate the annualized variance, write equation (4) dropping the cross product terms since their contribution to the variance would be zero because of the independence assumption.

$$r_a = r_{m_1} + r_{m_2} + \cdots + r_{m_{12}} \tag{7}$$

Taking the variance of both the sides and recognizing that the variances of all the monthly returns are identical, equal to $\text{Var}(r_m)$, and the covariances between the various pairs of monthly returns are zero because of the independence assumption, we get

$$\text{Var}(r_a) = 12\,\text{Var}(r_m) \tag{8}$$

Taking the square root on both the sides, we get equation (2).

It may be useful to know that with the i.i.d. assumption, the corresponding conversion rules for the covariance and correlation between the returns of stocks A and B are:

$$\text{Covar}(r_A, r_B)_a = \text{Covar}(r_A, r_B)_m \tag{9}$$

$$\text{Correl}(r_A, r_B)_a = \text{Correl}(r_A, r_B)_m \tag{10}$$

The results given by equations (1) and (2) would only be as good as the validity of the i.i.d. assumption.

The equations derived above assume that r's are effective returns calculated as $(p_2 - p_1)/p_1$. If r's are continuously compounded returns calculated as $\ln(p_2/p_1)$, then we would write:

$$e^{r_a} = e^{r_{m_1}} e^{r_{m_2}} \cdots e^{r_{m_{12}}} = e^{[r_{m_1} + r_{m_2} + \cdots + r_{m_{12}}]} \tag{11}$$

$$\Rightarrow \qquad r_a = r_{m_1} + r_{m_2} + \cdots + r_{m_{12}} \tag{12}$$

which is similar to equation (4) except that it is without the cross-product terms.

Taking expectation of equation (12) gives us equation (1) without requiring the independence assumption to make the expected value of cross product terms equal to zero. So, equation (1) will apply exactly to continuously compounded returns while it will only be an approximation for the effective returns. The approximation will be better, the greater the independence of successive returns (i.e., the lower the serial correlation). The equality of standard deviation of annual returns and annualized standard deviation of monthly returns for continuously compounded returns would still require the independence assumption but not as critically as the effective returns.

To demonstrate the conversion formulas, I calculated effective and continuously compounded versions of monthly and annual returns for VFINX from December 1987 to December 2011 and then calculated their statistics. Table 1 shows the summary of results.

Table 1: Statistics of Monthly and Annual returns for VFINX, December 1987–December 2011

Panel A. Effective returns

			Annual/Monthly Ratio	
	Monthly	Annual	Observed	Expected
N	288	24		
Average	0.84%	10.98%	13.12	12
Std Dev	4.31%	18.58%	4.31	3.46

Panel B. Continuously compounded returns

			Annual/Monthly Ratio	
	Monthly	Annual	Observed	Expected
N	288	24		
Average	0.74%	8.89%	12.00	12
Std Dev	4.34%	18.57%	4.27	3.46

The ratio of annual average to monthly average for continuously compounded returns is exactly 12 as expected. The ratio is not 12 for effective returns indicating the impact of non-independence of returns.[1]

The ratio of standard deviations is expected to be $\sqrt{12} = 3.46$ assuming independence of returns. The observed ratio is 4.27 for continuously compounded returns and 4.31 for the effective returns indicating the effects of serial correlation among returns in the sample.

[1] Lag 1, 2 and 3 serial correlations for the monthly returns over this sample period were 0.052, −0.022 and 0.085, respectively.