MATH 8030 Homework 5 Hints

3. (a) This is similar to the construction of a Markov chain in terms of i.i.d. uniform random variables from class. (b) Write down X_n in terms of A_n and X_{n-1}.

4. (a) When i and j communicate, there is $k \geq 0$ so that $p_{ij}^{(k)} > 0$. Since C is finite, there are only finitely many such $p_{ij}^{(k)}$ to consider. (b) Part (a) says that the probability of going between any two states in C within n steps is at least ε. In other words, starting from $i \in C$, the probability of *not* hitting state $j \in C$ within n steps is no more than $1 - \varepsilon$. The event $\{\eta_j > kn\}$ means that $X_1 \neq j, \ldots, X_{(k-1)n} \neq j$, and then the chain does not go between the states $X_{(k-1)n}$ and j within the next n steps. So, first condition on the value of $X_{(k-1)n}$ to get

$$P_i(\eta_j > kn) \leq (1 - \varepsilon)P_i(\eta_j > (k-1)n),$$

and then iterate. (c) Recall the tail sum formula for the mean of $\{0, 1, 2, \ldots\}$-valued random variables.