# MATH 8030 Homework 9 Hints

*When not otherwise stated,* $\{N(t) : t \geq 0\}$ *is a Poisson process with rate* $\lambda > 0$ *and event times* $\{T_n : n \geq 1\}$.

1. (c) Recall that for any integer $n \geq 0$ and any $t \in [0, \infty)$, $\{T_n \leq t\} = \{N(t) \geq n\}$. Also note that $T_{N(t)} \leq t$ for any $t$. So, for any $r \in [0, t]$, $\{T_{N(t)} \leq r\} = \{N(r) \geq N(t)\}$. But $N(r) \leq N(t)$ when $r \leq t$, so $\{T_{N(t)} \leq r\} = \{N(r) = N(t)\} = \{N(r, t] = 0\}$.

2. (a) Try using generating functions. You want to show that $E[z^{N(t)-N(s)}] = E[z^{N(t-s)}]$. Use the explicit form of the Poisson generating function and the independence of $N(t) - N(s)$ and $N(s)$.

3. (a) The answer is a certain binomial distribution. (b) You don't need to know the conditional distribution of $T_k$ given $\{N(t) = n\}$. The mean can be computed with the tail integral formula
$$E[T_k | N(t) = n] = \int_0^\infty P(T_k > s | N(t) = n)\, ds.$$
The $k \leq n$ and $k > n$ cases will be different. For example, if $k > n$ and $s > t$ then
$$\begin{aligned} P(T_k > s | N(t) = n) &= P(N(s) < k | N(t) = n) \\ &= P(N(s) - N(t) < k - n | N(t) = n) \\ &= P(N(s) - N(t) < k - n) \\ &= P(N(s - t) < k - n) = P(T_{k-n} > s - t) = P(t + T_{k-n} > s). \end{aligned}$$
In this case this says that the distribution of $T_k$ given $\{N(t) = n\}$ is the same as the distribution of $t + T_{k-n}$ (why does this make sense?).

4. (c) You essentially did part of this problem already on Homework 2.