

## MATH 8030 Homework 10

*Directions: Complete all problems. Upload solutions to [Problems 2 and 3](#) by 11:59 pm Thursday, April 10.*

*When not otherwise stated,  $\{N(t) : t \geq 0\}$  is a Poisson process with rate  $\lambda > 0$  and event times  $\{T_n : n \geq 1\}$ .*

1. Fix an increasing sequence  $\{t_n\}_{n \geq 0} \subset [0, \infty)$  such that  $t_n \uparrow \infty$ , and define  $X_n = N(t_n)$  for each  $n \geq 0$ . Is  $\{X_n : n \geq 0\}$  a Markov chain? If so, specify its state space and compute its transition probabilities.
2. Find the conditional distribution  $T_1, \dots, T_n | T_{n+1} = t$ .
3. Busloads of customers arrive at an infinite server queue according to a Poisson process with rate  $\lambda$ . Each customer is served independently with common service time distribution having distribution function  $G$ . A bus contains  $j$  customers with probability  $\alpha_j$ ,  $j = 1, 2, 3, \dots$ . Let  $X(t)$  denote the number of customers that have has service completed by time  $t$ .
  - (a) Compute  $E[X(t)]$ .
  - (b) Does  $X(t)$  have a Poisson distribution?
4. [Postponed until Homework 11](#). Suppose cars enter a one-way infinite highway, represented by  $[0, \infty)$ , according to  $\{N(t)\}$  (i.e., one car enters at location 0 at each event time  $T_i$ ). The  $i$ th car to enter chooses a velocity  $V_i$  and travels at this velocity. Assume that  $\{V_i\}$  are i.i.d. positive random variables with distribution function  $F$ .
  - (a) For  $(a, b) \subset [0, \infty)$  and  $t > 0$ , find the distribution of the number of cars located in  $(a, b)$  at time  $t$ .
  - (b) Suppose the speed limit on this highway is  $v > 0$ . Find the joint distribution of the number of cars going above the speed limit and the number going below the speed limit located in a given interval  $(a, b) \subset [0, \infty)$  at time  $t$ .