

MATH 8030 Homework 10

Directions: Complete all problems. Upload solutions to Problems 3 and 4 by 11:59 pm Thursday, April 10.

When not otherwise stated, $\{N(t) : t \geq 0\}$ is a Poisson process with rate $\lambda > 0$ and event times $\{T_n : n \geq 1\}$.

1. Fix an increasing sequence $\{t_n\}_{n \geq 0} \subset [0, \infty)$ such that $t_n \uparrow \infty$, and define $X_n = N(t_n)$ for each $n \geq 0$. Is $\{X_n : n \geq 0\}$ a Markov chain? If so, specify its state space and compute its transition probabilities.
2. Find the conditional distribution $T_1, \dots, T_n | T_{n+1} = t$.
3. Busloads of customers arrive at an infinite server queue according to a Poisson process with rate λ . Each customer is served independently with common service time distribution having distribution function G . A bus contains j customers with probability α_j , $j = 1, 2, 3, \dots$. Let $X(t)$ denote the number of customers that have has service completed by time t .
 - (a) Compute $E[X(t)]$.
 - (b) Does $X(t)$ have a Poisson distribution?
4. Suppose cars enter a one-way infinite highway, represented by $[0, \infty)$, according to $\{N(t)\}$ (i.e., one car enters at location 0 at each event time T_i). The i th car to enter chooses a velocity V_i and travels at this velocity. Assume that $\{V_i\}$ are i.i.d. positive random variables with distribution function F .
 - (a) For $(a, b) \subset [0, \infty)$ and $t > 0$, find the distribution of the number of cars located in (a, b) at time t .
 - (b) Suppose the speed limit on this highway is $v > 0$. Find the joint distribution of the number of cars going above the speed limit and the number going below the speed limit located in a given interval $(a, b) \subset [0, \infty)$ at time t .