MATH 8030 Homework 6 Hints

1. (a) Use the definition of conditional probability. For each probability in the resulting ratio, condition on the chain up to and including time $n+1$, and then condition on the chain up to and including time $n-1$ (or the other way around). Note that the Markov property says that X_n and $(X_0, \ldots, X_{n-2})$ are *conditionally* independent given X_{n-1}. Also, conditionally on X_n, the values X_{n-1} and X_{n+1} are independent. In general, given X_{n-1}, the values of X_n and X_{n+1} are correlated.

2. Use a computer.

3. (a) Use either the dissection principle or the regular Markov property by writing out $\{\eta_i = n\} = \{X_1 \neq i, \ldots, X_{n-1} \neq i, X_n = i\}$. To show $f_{20} = f_{21} f_{10}$, sum $f_{20}^{(n)}$ over n and switch the order of summation. (b) and (c) are outlined in the Midterm solutions. (Note that when $X_0 = i$ and $j \neq i$, then $\eta_j = \tau_j$.)