

MATH 8030 Homework 8

Directions: Complete all problems. Upload solutions to Problem 2 by 11:59 pm Thursday, March 27.

1. Is it possible for no states of a finite-state Markov chain to be positive recurrent?
2. The distribution of a random variable X is said to be *memoryless* if

$$P(X > s + t | X > s) = P(X > t)$$

holds for all $s, t \in \mathbb{R}$. For this problem, the $\text{Geom}(p)$ distribution is the one with values $k \in \{1, 2, 3, \dots\}$ and mass function $P(X = k) = (1 - p)^{k-1}p$.

- (a) Verify that the $\text{Geom}(p)$ and $\text{Exp}(\lambda)$ distributions are memoryless.
- (b) Consider a random variable $X \geq 0$ with a memoryless distribution. Show that if X is continuous then it has an exponential distribution, and if X is $\{1, 2, \dots\}$ -valued then it has a geometric distribution.