HW4

Monday, March 10, 2025 2:37 PM

1. Consider the following problem:

$$\begin{array}{lrcrcl} \min & 2x_1 & + & x_2 & & \\ s.t. & x_1 & + & 2x_2 & \geq & 4 \\ & x_1 & - & x_2 & \leq & 1 \\ & x_1, & & x_2 & \geq & 0. \end{array}$$

(a) Rewrite it in standard form.

(b) Consider the basic solution obtained using x_1 and x_4 as basic variables. What is the corresponding value of $\boldsymbol{x}_B = B^{-1}\boldsymbol{b}$ and $\boldsymbol{x}_N$? Is this a feasible solution?

(c) Consider another basic solution that has x_1 and x_2 as basic variables. What is the corresponding value of $\boldsymbol{x}_B$ and $\boldsymbol{x}_N$? Is this a feasible solution?

(d) Consider the basic feasible solution (BFS) you found in either step 2 or 3. What is its objective function value?

(e) Compute a feasible direction $\boldsymbol{d}$ by choosing a suitable non-basic variable x_j.

(f) What is the reduced cost $\bar{c}_j$?

(g) Can you obtain a new BFS if you move along direction $\boldsymbol{d}$? If so, what is that BFS and what is its objective function value?

a) $\min \begin{bmatrix} 2, 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

st $\begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$x \geq 0$

b) $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$x_B = B^{-1}b = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$x_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-3 < 0$ so not feasible

```
B:matrix([1,0],[1,1]);
B1:invert(B);
b:transpose(matrix([4,1]));
```

$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

```
B1.b;
```

$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

c) $B = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$x_B = B^{-1}b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $x_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_B \geq 0$ so feasible

```
B:matrix([1,2],[1,-1]);
B1:invert(B);
b:transpose(matrix([4,1]));
```

$\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$

$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\left(\frac{1}{3}\right) \end{pmatrix}$

$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

```
B1.b;
```

$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

d) $2x_1 + x_2 = 5$ for $x_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

e) We know $Ad = 0$ so consider

e) We know $Ad=0$ so consider

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Trivially $d=0$.

Else

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} d_1 + 2d_2 = 1 \\ d_1 - d_2 = 0 \end{matrix} \Rightarrow d = \begin{bmatrix} 1/3 \\ 1/3 \\ 1 \\ 0 \end{bmatrix}$$ which won't help since $d \geq 0$

Else

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} d_1 + 2d_2 = 0 \\ d_1 - d_2 + 1 = 0 \end{matrix} \Rightarrow \begin{matrix} 3d_2 = 1 \\ d_1 = d_2 - 1 \end{matrix} \Rightarrow d = \begin{bmatrix} -2/3 \\ 1/3 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2/3 \\ 1/3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ So feasible

f) $\bar{C}_j = C_j - C_B^T B^{-1} A_j$

$B = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$ $C_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $A_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C_j = 0$

$\bar{C}_j = -1$

```
B:matrix([1,2],[1,-1]);
B1:invert(B);
c:transpose(matrix([2,1]));
a:transpose(matrix([0,1]));
```

$$\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$$
$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\left(\frac{1}{3}\right) \end{pmatrix}$$
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

```
-transpose(c).B1.a;
```

-1

g) Step size $\frac{2}{-(-\frac{2}{3})} = 3$

$$X_{new} = X_{old} + 3d = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix} \geq 0$$

$[2 \ 1 \ 0 \ 0] X_{new} = 2$

Exercise 3.4 Consider the problem of minimizing $\mathbf{c}'\mathbf{x}$ over the set $P = \{\mathbf{x} \in \Re^n \mid \mathbf{A}\mathbf{x} = \mathbf{b},\ \mathbf{D}\mathbf{x} \le \mathbf{f},\ \mathbf{E}\mathbf{x} \le \mathbf{g}\}$. Let $\mathbf{x}^*$ be an element of P that satisfies $\mathbf{D}\mathbf{x}^* = \mathbf{f}$, $\mathbf{E}\mathbf{x}^* < \mathbf{g}$. Show that the set of feasible directions at the point $\mathbf{x}^*$ is the set

$$\{\mathbf{d} \in \Re^n \mid \mathbf{A}\mathbf{d} = \mathbf{0},\ \mathbf{D}\mathbf{d} \le \mathbf{0}\}.$$

$d \in \mathbb{R}^n$ or else it could not be a feasible direction.

We know then $\forall \varepsilon > 0$, $x^* + \varepsilon d \in P$

$\underline{Ad=0}$ Consider $A(x^* + \varepsilon d) = b \Rightarrow Ad = 0$ because $Ax^* = b$ and $\varepsilon \ne 0$.

$\underline{Dd \le 0}$ Since $x^* + \varepsilon d \in P$, $D(x^* + \varepsilon d) \le f$

$$f + \varepsilon Dd \le f \quad (Dx^* = f)$$
$$\varepsilon Dd \le 0 \quad (\varepsilon \ne 0)$$
$$Dd \le 0$$

$\underline{Ed}$ Since $Ex^* < g$, we can always choose ε small enough so that $E(x^* + \varepsilon d) \le g$. Thus we do not need to restrict Ed.

Thus feasible directions are $\{d \in \mathbb{R}^n \mid Ad = 0, Dd \le 0\}$

Exercise 3.5 Let $P = \{\mathbf{x} \in \Re^3 \mid x_1 + x_2 + x_3 = 1,\ \mathbf{x} \ge \mathbf{0}\}$ and consider the vector $\mathbf{x} = (0, 0, 1)$. Find the set of feasible directions at $\mathbf{x}$.

Remember that the set of feasible solutions is infinite.

We know feasible directions satisfy $x + \varepsilon d \in P$ $\forall \varepsilon > 0$.

Thus $x_1 + x_2 + x_3 + \varepsilon(d_1 + d_2 + d_3) = 1$
$\Rightarrow d_1 + d_2 + d_3 = 0$

We also know
$$\cancel{x_1}^{0} + \varepsilon d_1 \ge 0$$
$$\cancel{x_2}^{0} + \varepsilon d_2 \ge 0$$
$$\cancel{x_3}^{1} + \varepsilon d_3 \ge 0$$

So $d_1 \ge 0$, $d_2 \ge 0$ and $d_3 \ge -\frac{1}{\varepsilon}$.

Since $d_1 + d_2 + d_3 = 1$, $d_3 = -d_1 - d_2$

Thus the set of feasible directions is

$$\mathbb{D} := \{d \in \mathbb{R}^3 \mid d_1 + d_2 + d_3 = 0,\ d_1 \ge 0,\ d_2 \ge 0\}.$$

4. Solve the following problem (after an appropriate transformation) using the simplex method. Use dictionaries as seen in class or tableaux. Feel free to write B, B^{-1}, N, $\mathbf{d}_B$ if that helps you, but write all dictionaries/tableaux until the optimal one.

$$\begin{array}{lll} \min & 2x_1 - x_2 & \\ s.t. & 3x_1 + x_2 & \leq 9 \\ & x_1 + 2x_2 & \leq 8 \\ & x_1, x_2 & \geq 0. \end{array}$$

Use $\mathcal{B} = \{3, 4\}$ as the first basis. For this and the following problems, you might find the following URL useful, especially if you hate arithmetic: `http://www.princeton.edu/~rvdb/JAVA/pivot/simple.html`. There is a link to a tool for minimization problems at the bottom of that page.

$$\min \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{st} \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$$

$$x \geq 0$$

$$\boxed{\begin{aligned} x_3 &= 9 - 3x_1 - x_2 \\ x_4 &= 8 - x_1 - 2x_2 \\ z &= 2x_1 - x_2 \end{aligned}}$$

x_2 enters since most negative coefficient

$x_3: \frac{9}{1} = 9$ $\quad x_4: \frac{8}{2} = 4$ $\quad x_4$ leaves

$$x_2 = 4 - \tfrac{1}{2}x_1 - \tfrac{1}{2}x_4$$

$$z = 2x_1 - (4 - \tfrac{1}{2}x_1 - \tfrac{1}{2}x_4) = -4 + \tfrac{5}{2}x_1 + \tfrac{1}{2}x_4$$

$$x_3 = 9 - 3x_1 - (4 - \tfrac{1}{2}x_1 - \tfrac{1}{2}x_4) = 5 - \tfrac{5}{2}x_1 + \tfrac{1}{2}x_4$$

$$\boxed{\begin{aligned} x_2 &= 4 - \tfrac{1}{2}x_1 - \tfrac{1}{2}x_4 \\ x_3 &= 5 - \tfrac{5}{2}x_1 + \tfrac{1}{2}x_4 \\ z &= -4 + \tfrac{5}{2}x_1 + \tfrac{1}{2}x_4 \end{aligned}}$$

All coefficients are positive So $x = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ with the optimum of -4

$0 + 4 \leq 9$
$0 + 2(4) = 8 \leq 8$
$0, 4 \geq 0$

Thus feasible

Consider the problem

$$\begin{array}{lrcrl} \text{minimize} & -2x_1 & - & x_2 & \\ \text{subject to} & x_1 & - & x_2 & \leq 2 \\ & x_1 & + & x_2 & \leq 6 \\ & & x_1, x_2 \geq 0 & & \end{array}$$

(a) *Convert the problem into standard form and construct a basic feasible solution at which* $(x_1, x_2) = (0, 0)$.

(b) *Carry out the full tableau implementation of the simplex method, starting with the basic feasible solution of part (a).*

(c) *Draw a graphical representation of the problem in terms of the original variables* x_1, x_2, *and* *solution of part (a).*

(c) *Draw a graphical representation of the problem in terms of the original variables x_1, x_2, and indicate the path taken by the simplex algorithm.*

a) $\min \begin{bmatrix} -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

st
$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$x \geq 0$

$x = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 6 \end{bmatrix}$ with $B = \{3, 4\}$

b)

	x_1	x_2	x_3	x_4
0	-2	-1	0	0
$x_3 = 2$	1	-1	1	0
$x_4 = 6$	1	1	0	1

x_1 is most negative

$\frac{2}{1} < \frac{6}{1}$ so switch out x_3

	x_1	x_2	x_3	x_4
$z = 4$	0	-3	2	0
$x_1 = 2$	1	-1	1	0
$x_4 = 4$	0	2	-1	1

x_2 is most negative

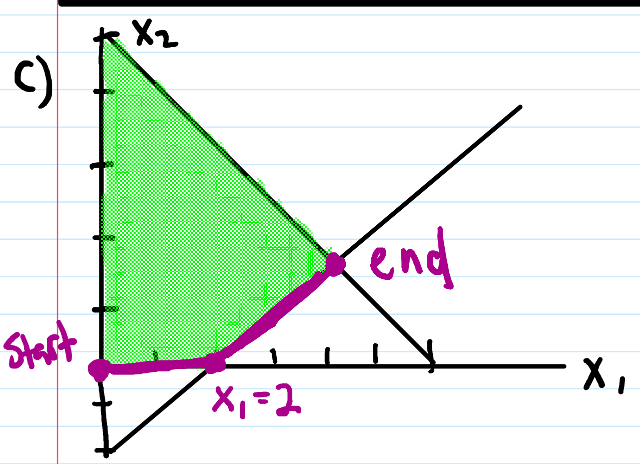
$\frac{2}{-1} < \frac{4}{2}$ so switch out x_4

	x_1	x_2	x_3	x_4
$z = 10$	0	0	1/2	1/2
$x_1 = 4$	1	1	1/2	1/2
$x_2 = 2$	0	1	-1/2	1/2

1/2, 1/2 are both positive so found optimum

$x = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ with $z = 10$

c)



6. Solve the following problem, after reducing it to standard form, using the simplex method (with dictionaries):

$$\begin{array}{rrcrcrcl} \min & -2x_1 & + & x_2 & + & 2x_3 & & \\ s.t. & x_1 & + & 2x_2 & + & x_3 & \leq & 4 \\ & x_1 & - & x_2 & + & x_3 & \leq & 1 \\ & x_1, & & x_2, & & x_3 & \geq & 0. \end{array}$$

Start with the slack basis $\mathcal{B} = \{4, 5\}$.

$\min -2x_1+x_2+2x_3$

st

$x_1+2x_2+x_3+x_4=4$

$x_1-x_2+x_3+x_5=1$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

$$\min \begin{bmatrix} -2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

st

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$x \geq 0$

$$\boxed{\begin{aligned} x_4 &= 4 - x_1 - 2x_2 - x_3 \\ x_5 &= 1 - x_1 + x_2 - x_3 \\ z &= -2x_1 + x_2 + 2x_3 \end{aligned}}$$

x_1 enter since most negative coefficient

$x_4: \frac{4}{1} = 4 \quad x_5: \frac{1}{1} = 1 \quad x_5$ leaves

$z = -2(1+x_2-x_3-x_5)+x_2+2x_3 = -2-x_2+4x_3+2x_5$

$x_4 = 4-(1+x_2-x_3-x_5)-2x_2-x_3 = 3-3x_2+x_5$

$$\boxed{\begin{aligned} x_1 &= 1+x_2-x_3-x_5 \\ x_4 &= 3-3x_2+x_5 \\ z &= -2-x_2+4x_3+2x_5 \end{aligned}}$$

x_2 leaves since most negative coefficient

$x_1: \frac{1}{1}=1 \qquad x_4: \frac{3}{3}=1$ so we can pick. Choose x_4

$x_2 = 1-\frac{1}{3}x_4+\frac{1}{3}x_5$

$z = -2-(1-\frac{1}{3}x_4+\frac{1}{3}x_5)+4x_3+2x_5 = -3+4x_3+\frac{1}{3}x_4+\frac{5}{3}x_5$

$x_1 = 1+1-\frac{1}{3}x_4+\frac{1}{3}x_5-x_3-x_5 = 2-x_3-\frac{1}{3}x_4-\frac{2}{3}x_5$

$$\boxed{\begin{aligned} x_2 &= 1-\tfrac{1}{3}x_4+\tfrac{1}{3}x_5 \\ x_1 &= 2-x_3-\tfrac{1}{3}x_4-\tfrac{2}{3}x_5 \\ z &= -3+4x_3+\tfrac{1}{3}x_4+\tfrac{5}{3}x_5 \end{aligned}}$$

All coefficients are positive

Thus $x = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and the optimum of the problem is -3

$2+2(1) = 4 \leq 4$

$2-1=1 \leq 1$

$2, 1, 0 \geq 0$ Thus feasible

While solving a standard form problem, we arrive at the following tableau, with x_3, x_4, and x_5 being the basic variables:

$$\begin{array}{r|rrrrr} -10 & \delta & -2 & 0 & 0 & 0 \\ \hline 4 & -1 & \eta & 1 & 0 & 0 \\ 1 & \alpha & -4 & 0 & 1 & 0 \\ \beta & \gamma & 3 & 0 & 0 & 1 \end{array}$$

The entries $\alpha, \beta, \gamma, \delta, \eta$ in the tableau are unknown parameters. For each one of the following statements, find some parameter values that will make the statement true.

(a) *The current solution is optimal and there are multiple optimal solutions.*

(b) *The optimal cost is $-\infty$.*

(c) *The current solution is feasible but not optimal.*

a)

$$\boxed{\begin{array}{cccccc} -10 & 0 & -2 & 0 & 0 & 0 \\ 4 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -4 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}}$$

This cannot be degenerate so $\beta = 0$

$\delta = 0$ since $-2 < 0$.

We want $\gamma \geq 0$ and η shouldn't change anything

b)

$$\boxed{\begin{array}{cccccc} -10 & -1 & -2 & 0 & 0 & 0 \\ 4 & -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & -4 & 0 & 1 & 0 \\ 1 & -1 & 3 & 0 & 0 & 1 \end{array}}$$

$\delta < 0, \alpha < 0, \gamma < 0$ so that it is unbdd.

$\beta > 0$ so it is still feasible

$\eta = 0$ for fun

c)

$$\boxed{\begin{array}{cccccc} -10 & 1 & -2 & 0 & 0 & 0 \\ 4 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -4 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}}$$

$\beta > 0$ so it is feasible

$d > 0$ to force x_2 out

$\eta = 1$ $\alpha = 1$ $\gamma = 1$

8. Let

$$B = \begin{bmatrix} 4 & 1 & 4 & 4 \\ 3 & 4 & 4 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 4 & 1 & 2 \end{bmatrix} \text{ and } B^{-1} = \frac{1}{125} \begin{bmatrix} 26 & 24 & -57 & -12 \\ -19 & 4 & 8 & 28 \\ -4 & 14 & 28 & -27 \\ 14 & -49 & 27 & 32 \end{bmatrix}$$

Use the product-form update from the revised simplex method to compute the inverse of the new matrix

$$\bar{B} = \begin{bmatrix} 4 & 1 & 2 & 4 \\ 3 & 4 & 4 & 1 \\ 1 & 2 & 2 & 2 \\ 2 & 4 & 2 & 2 \end{bmatrix}$$

$$\bar{B}^{-1} = B^{-1} - \frac{(v - e_3)e_3^T B^{-1}}{v_3} \qquad v = B^{-1}\bar{b}_3$$

```
B1-(v-e3).transpose(e3).B1/v[3,1];
```

$$\begin{pmatrix} \frac{6}{25} & \frac{4}{25} & -\left(\frac{17}{25}\right) & \frac{3}{25} \\ -\left(\frac{3}{25}\right) & -\left(\frac{2}{25}\right) & -\left(\frac{4}{25}\right) & \frac{11}{25} \\ -\left(\frac{2}{25}\right) & \frac{7}{25} & \frac{14}{25} & -\left(\frac{27}{50}\right) \\ \frac{2}{25} & -\left(\frac{7}{25}\right) & \frac{11}{25} & \frac{1}{25} \end{pmatrix} \qquad v \quad \begin{pmatrix} \frac{2}{5} \\ \frac{2}{5} \\ \frac{2}{5} \\ -\left(\frac{2}{5}\right) \end{pmatrix}$$

```
B:matrix([4,1,4,4],[3,4,4,1],[1,2,4,2],[2,4,1,2]);
B1:invert(B);
b3:transpose([4,4,4,1]);
bh3:transpose([2,4,2,2]);
e3:transpose([0,0,1,0]);
v:B1.bh3;
```

9. One application of linear programming is to "blending problems." Here, several ingredients are to be blended to produce a final product. The ingredients have different levels of component features, and the constraints may require certain levels of features as percentages of the total amount of final product. Such a constraint can be modeled as a ratio. For example:

$$\frac{\sum_j a_j x_j}{\sum_j x_j} \geq p$$

where a_j is the amount of feature per unit in ingredient j, x_j is the total amount of ingredient j used, and p is the fraction of feature j required in the final product. The constraint can be converted to a linear inequality by multiplying both sides by the denominator, which represents the total amount produced:

$$\sum_j a_j x_j \geq p \left(\sum_j x_j \right).$$

One needs to be careful that the denominator does not end up being zero.

Use this idea to formulate the following LP. Solve your LP using AMPL. A .CSV file is provided with the data table.

The *risk index* of an investment can be obtained from return on investment (ROI) by taking the percentage change in the value of the investment (in absolute terms) for each year and averaging them.

Suppose you are trying to determine what percentage of your money should be invested in T-bills, gold, and stocks. In the table below and the file `riskindex.csv` you are given the annual returns (change in value) for these investments for the years 2004–2024. Let the risk index of a portfolio be the weighted (according to the fraction of your money assigned to each investment) average of the risk index of each individual investment. Suppose that the amount of each investment must be between 20% and 50% of the total invested. You would like the risk index of your portfolio to equal 0.15 and your goal is to maximize the expected return on your portfolio. Formulate an LP whose solution will maximize the expected return on your portfolio, subject to the given constraints. Use the average return earned by each investment during the years 2004–2024 as your estimate of expected return.

Year	Stocks	Gold	T-bills
2004	11	11	5
2005	−9	8	7
2006	4	−14	7
2007	14	14	4
2008	19	44	4
2009	−15	66	7
2010	−27	64	8
2011	37	0	6
2012	24	−22	5
2013	−7	18	5
2014	7	31	7
2015	19	59	10
2016	33	99	11
2017	−5	−25	15
2018	22	4	11
2019	23	−11	9
2020	6	−15	10
2021	32	−12	8
2022	19	16	6
2023	5	22	5
2024	17	−2	6

Write clear and readable AMPL models with descriptive identifiers, well structured layout and white space, and helpful comments. Submit your `.mod` and `.dat` files as a zip file to the Homework 4 Code assignment on Canvas. Name your zip file `math-8100-<userid>-hw<hwnum>.zip`, where `<userid>` is your Clemson userid and `<hwnum>` is the homework number. For example, `math-8100-mjs-hw5.zip`. Also include your model and data listings in the PDF you submit to Gradescope.

```
ampl: include HW4.run;
CPLEX 22.1.1:              CPLEX 22.1.1: optimal solution; objective 10.93
1 simplex iterations
Z = 10.934

x [*] :=
1  0.205446
2  0.294554
3  0.5
```

```
reset; #clean slate

model HW4.mod; #choose model type
data HW4.dat; #choose data for model

option solver cplex; #choose solver

solve; #solve command

#Display results
display Z;
display x;
```

```
#Matrix from file provided
param YoY_returns:
     1   2   3 :=
2004  11  11  5
2005  -9  8   7
2006  4   -14 7
2007  14  14  4
2008  19  44  4
2009  -15 66  7
2010  -27 64  8
2011  37  0   6
2012  24 -22  5
2013 -7   18  5
2014 7    31  7
2015 19   59  10
2016 33   99  11
2017 -5   -25 15
2018 22   4   11
2019 23   -11 9
2020 6    -15 10
2021 32   -12 8
2022 19   16  6
2023 5    22  5
2024 17   -2  6;
```

```
#Create sets
set years = 2004..2024;
set cats = 1..3;

#Import data
param YoY_returns {years,cats};

#Calculate average return and the percentage change in the value of the
#investment (in absolute terms) for each year and averaging them
param Avg_returns {j in cats}:=(sum{i in years} YoY_returns[i,j])/card(years);
param risk {j in cats}:=(sum{i in years} abs(YoY_returns[i,j]))/card(years)/100;

#Set variable bounds
var x {i in cats} <= 0.50 >= 0.20;

#Maximize profits with risk tolerance and sum of |x|_1=1
maximize Z: sum{i in cats} Avg_returns[i]*x[i];
subject to F: sum{i in cats} risk[i]*x[i] = 0.15;
subject to E: sum{i in cats} x[i]=1;
```