MATH 8100
Mathematical Programming
Spring 2025
Homework 3

Due May 7, 2025

1. Solve the following problem using the graphical method:

$$\begin{array}{rlcrcl}
\min & 5x_1 & + & x_2 & & \\
s.t. & x_1 & + & 3x_2 & \leq & 6 \\
 & x_1 & + & 6x_2 & \geq & 6 \\
 & x_1 & - & x_2 & \leq & 2 \\
 & x_1 & & & \geq & 0.
\end{array}$$

   Then write the problem in standard form. Do not attempt to solve the problem in standard form with the graphical method.

2. In class, we discussed the following "tracks on the ceiling" problems:

   > An engineering project needs to place three sensors in the ceiling, each connected to a central reporting system. A map looking like a coordinate system describes where the sensors are to be placed. If the "origin" is in the southwest corner of the room, then the three sensors are located at (10, 3), (5, 15), and (20, 25). Our job is to place the central unit. Due to the design of the ceiling, th e wiring between the sensors can run only along horizontal or vertical tracks in th e ceiling. Design two convex models to complete the following tasks:
   >
   > - Minimize the total amount of wire used;
   > - Minimize the maximum wire length from a sensor to the central unit.

   Convex models were introduced in class with respect to the Manhattan distance. Solve the models using AMPL and CPLEX or Gurobi. Your AMPL formulation should use descriptive variable names, informative comments, legible layout, and appropriate index sets and data tables so that your model can be used with different data without editing the model file.

3. Solve Exercise 2.4 in B&T.

4. Find an extreme point of each of the following polyhedra, or a line contained in it. If you find an extreme point $\boldsymbol{x}^\star$, write the active linearly independent constraints at $\boldsymbol{x}^\star$.

(a) $P = \{\boldsymbol{x} \in \mathbb{R}^n : A\boldsymbol{x} \geq \boldsymbol{b}\}$, where A is a square matrix with n linearly independent columns;

(b) $P = \{\boldsymbol{x} \in \mathbb{R}^n : \boldsymbol{e}^\top \boldsymbol{x} \geq 0\}$ with $\boldsymbol{e}^\top = (1\ 1\ \cdots\ 1)$.

5. Solve Exercise 2.10 in B&T.

6. Solve Exercise 2.13 in B&T.

7. Suppose that a free variable x_i is replaced by $x_i^+ - x_i^-$ $(x_i^+, x_i^- \geq 0)$ when a linear program is converted to standard form. Show that, in a basic s olution of the resulting standard form, at least one of the variables x_i^+ and x_i^- has value 0.