MATH 8100
Mathematical Programming
Spring 2025
Homework 6

Due April 21, 2025

1. Consider the LP problem

$$\begin{array}{lrcl} \min & x_1 + 2x_2 & & \\ \text{s.t.} & 3x_1 + x_2 & \leq & 9 \\ & x_1 + x_2 & \geq & 5 \\ & x_1, x_2 & \geq & 0. \end{array}$$

   (a) Put it in standard form.
   (b) Verify that the optimal basis is $\mathcal{B} = \{1, 2\}$ by computing $B^{-1}\boldsymbol{b}$ and the reduced costs of the nonbasic variables.
   (c) Suppose a variable x_5, unrestricted in sign, is added to the original problem, with objective coefficient α and column $A_5 = (1, 1)^\top$. How does the standard form change?
   (d) Determine the values of α such that the optimal solution of the new problem has the same entries for x_1 and x_2.
   (e) Determine the values of α such that the problem is bounded.

2. Consider the LP problem:

$$\begin{array}{lrcl} \min & 2x_1 + x_2 & & \\ \text{s.t.} & 3x_1 + x_2 & \geq & 6 \\ & x_1 + x_2 & \geq & 4 \\ & x_1, x_2 & \geq & 0. \end{array}$$

   (a) After putting it in standard form, verify that the basis $\mathcal{B} = \{1, 2\}$ is optimal.
   (b) Add the constraint $x_1 + 3x_2 = \alpha$. For what value of α is the problem feasible?
   (c) Choose an α such that the problem is feasible and perform the necessary operations to find the new optimal solution.

3. Consider the LP problem:

$$\begin{array}{ll} \min & (1+\alpha)x_1 + (2-\alpha)x_2 \\ \text{s.t.} & 3x_1 + x_2 \geq 9 \\ & x_1 + x_2 \geq 5 \\ & x_1, x_2 \geq 0. \end{array}$$

   Compute the values of α such that the problem is infeasible, unbounded, or has a finite optimal solution.

4. B&T 7.1
5. B&T 7.9
6. B&T 7.10
7. B&T 7.22
8. B&T 7.31