Jacob Manning HW6

1. BE

$y' = -y^2 \quad y_0 = 1 \quad h = 0.1$

a) $y_{k+1} = y_k + h(-y_{k+1}^2) \Rightarrow y_1 = 1 + 0.1(-y_1^2)$

$$\boxed{\begin{aligned} g(y) &= 0.1y^2 + y - 1 \\ g'(y) &= 0.2y + 1 \end{aligned}}$$

b) $y_{k+1} = y_k - \frac{g(y_k)}{g'(y_k)} \Rightarrow \boxed{y_1 = 1 - \frac{0.1y^2 + y - 1}{0.2y + 1}}$

c) $y_{k+1} = y_k - 0.1y_k^2 \Rightarrow 1 - 0.1 \cdot 1^2 = \boxed{0.9 = y_1}$

d) $y_1 = 0.9 - \frac{0.9^2 \cdot 0.1 + 0.9 - 1}{0.2 \cdot 0.9 + 1} \approx \boxed{0.916}$

2. $y' = \frac{2}{x}y + x^2e^x \quad y(1) = 0 \quad \text{exact} \Rightarrow y(x) = x^2(e^x - e)$

$Er = |1.1^2(e^{1.1} - e) - y_1| \quad x_k = 1 + kh$

Euler

a) $y_{k+1} = \overset{0}{y_k} + h\left(\frac{2}{x_k}\overset{0}{y_k} + x_k^2 e^{x_k}\right)$

$y_1 = 0.1 \cdot e = \boxed{\frac{e}{10}}$

$\boxed{Er \approx 0.074}$

B.E.

b) $y_{k+1} = y_k + h\left(\frac{2}{x_{k+1}}y_{k+1} + x_{k+1}^2 e^{x_{k+1}}\right)$

$10y_1 - \frac{2}{1.1}y_1 = 1.1^2 e^{1.1}$

$\boxed{y_1 \approx 0.444}$

$\boxed{Er \approx 0.098}$

Heun's

c) $y_{k+1} = y_k + \frac{h}{2}\left(f(x_k, y_k) + f(x_k + h, y_k + hf(x_k, y_k))\right)$

$f(x_k, y_k) = e \qquad y_0 + hf(x_k, y_k) = 0.1e$

$y_1 = \frac{0.1}{2}\left(e + \frac{2e}{1.1} \cdot 0.1 + 1.1^2 e^{1.1}\right) \approx \boxed{0.342}$

$\boxed{Er \approx 0.004}$

Taylor 2nd

2d) $y_{k+1} = y_k + hf(x_k, y_k) + \frac{h^2}{2}\left(f_x(x_k, y_k) + f_y(x_k, y_k) f(x_k, y_k)\right)$

$f_x(x_k, y_k) = (x_k^2 + 2x_k)e^{x_k} - \frac{2y_k}{x_k^2}$ $f_y(x_k, y_k) = \frac{2}{x_k}$

$y_1 = 0.1 \cdot e + \frac{0.1^2}{2}[(1+2)e + 2e] = 0.1e + \frac{0.1^2}{2} \cdot 5e \approx \boxed{0.340}$

$\boxed{Er \approx 0.142}$

3. $y' = -5y$ $y_0 = 1$ $h = 0.5$

a) $|1 + \lambda h| = |1 - \frac{5}{2}| = \frac{3}{2} > 1$ $\boxed{\text{Not stable}}$ pg 396

b) $\left|\frac{1}{1-\lambda h}\right| = \left|\frac{1}{1+\frac{5}{2}}\right| = \frac{2}{7} < 1$ $\boxed{\text{Stable}}$ pg 399

4. $x^3 y''' - x^2 y'' + 3xy' - 4y = 5x^3 \ln x + 9x^3$ $1 \le x \le 2$

$y(1) = 0$ $y'(1) = 1$ $y''(1) = 3$

a) $y''' = y'' x^{-1} - 3x^{-2} y' + 4x^{-3} y + 5\ln x + 9$ $u_1 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

$u = \begin{bmatrix} y \\ y' \\ y'' \end{bmatrix} \Rightarrow \begin{bmatrix} u_2 \\ u_3 \\ y''' \end{bmatrix} = u'$

$u' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4x^{-3} & -3x^{-2} & x^{-1} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 5\ln x + 9 \end{bmatrix} \Rightarrow \boxed{u' = Au + b}$

b) $h = \frac{b-a}{n} \Rightarrow n = 10$

c) See code pdfs

(I don't know how to show b so I printed the list of values over the interval)