Jacob Manning HW 5

1. $T(f) = 5$ $\quad \int_0^1 f(x)dx$

$M(f) = 4$

$$S(f) \approx \frac{2}{3}M(f) + \frac{1}{3}T(f)$$

$$\approx \frac{2}{3}\cdot 4 + \frac{1}{3}\cdot 5 = \frac{13}{3}$$

2. $M_k(f) = h\sum_{j=1}^{k} f\left(\frac{x_{j-1}+x_j}{2}\right)$ $\quad \int_0^1 x^3 dx$

$h = \frac{b-a}{k} \Rightarrow k = \frac{b-a}{h}$ $\quad h = 0.5$ & $h = 1$

a) $k = 2$ $\quad k = 1$

$M_2(f) = \frac{1}{2}\left[\left(\frac{0+\frac{1}{2}}{2}\right)^3 + \left(\frac{\frac{1}{2}+1}{2}\right)^3\right] = 0.219$

$M_1(f) = \left(\frac{0+1}{2}\right)^3 = 0.125$

b) $\frac{4F(h/2) - F(h)}{3} = \frac{4(0.219) - 0.125}{3} = 0.25$

pg 356 says midpoint $\sim O(h^2)$ and with Richardson adds another order so $\sim O(h^3)$ therefore it is a perfect solution

3a. Nodes at roots of Chebyshev polynomial

$p(t) = 4t^3 - 3t = 0$ $\quad t = \frac{\pm\sqrt{3}}{2}, 0$

$\omega \approx \frac{\pi}{n} = \frac{\pi}{3}$

$\int_{-1}^{1} f(x)dx \approx \frac{\pi}{3}\left[f\left(-\frac{\sqrt{3}}{2}\right) + f(0) + f\left(\frac{\sqrt{3}}{2}\right)\right]$

b. $2n-1 = 5$ so each polynomial upto degree 5 will be perfect

4. $f(0)=1$ $f(\frac{1}{2})=5/2$ $f(1)=2$ $f(\frac{1}{4})=f(\frac{3}{4})=\alpha$

$k=4$

$T_k(f)=1.75$ $h=\frac{b-a}{k} \Rightarrow h=\frac{1}{4}$

$h\left(\frac{f(0)+f(1)}{2}+f(\frac{1}{2})+f(\frac{1}{4})+f(\frac{3}{4})\right)=1.75$

$\frac{1}{4}\left(\frac{1+2}{2}+\frac{5}{2}+2\alpha\right)=1.75$

$\alpha=1.5$

5. $\int_{-1}^{1} f(x) = af(-1)+bf(1)+cf'(-1)+df'(1)$

$a+b=1-{-1}=2$

$-a+b+c+d=\frac{1^2-(-1)^2}{2}=0$

$a+b-2c+2d=\frac{1^3-(-1)^3}{3}=2/3$

$-a+b+3c+3d=\frac{1^4-(-1)^4}{4}=0$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 2 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -2 & 2 & 2/3 \\ -1 & 1 & 3 & 3 & 0 \end{array}\right]$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1/3 \\ -1/3 \end{bmatrix}$

6. $\int_0^2 e^{2x}\sin 3x\,dx$ $\|E\|_\infty \le 10^{-4}$

Trap $\Rightarrow$ $E=\frac{-1}{12}h^2(b-a)f''(c)$ $h=\frac{b-a}{n}$

$=-\frac{(b-a)^3}{12n^2}f''(c)$

$\|E\|_\infty=\frac{(b-a)^3}{12n^2}\|f''(c)\|_\infty<10^{-4}$

$n>\sqrt{\frac{(b-a)^3}{12}\cdot 10^4\|f''(c)\|_\infty}=\sqrt{\frac{8\cdot 10^4}{12}\|f''(c)\|_\infty}$

Max values are found with extreme value theorem and first derivative test

$\|f''(c)\|_\infty=705.36$ note $\|f''''(c)\|_\infty=4475.41$

Trap $n>2168.5$

Simp $\Rightarrow$ $\|E\|_\infty=\frac{(b-a)^5}{2880n^4}\|f''''(c)\|_\infty<10^{-4}$

$\Rightarrow \sqrt[4]{\frac{2^5\cdot 10^4}{2880}\|f''''(c)\|_\infty}<n$

$n>26.5$

See code pdfs