

MATH 8600 Scientific Computing

HW #2 (Due on Sep. 21)

1. (8pts.) Prove that
  - a. the product of two upper triangular matrices is upper triangular.
  - b. the inverse of a nonsingular upper triangular matrix is upper triangular.

2. (8pts.) For the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}$$

find the permutation matrix  $P$ , the unit lower triangular  $L$  and the upper triangular  $U$  such that  $PA = LU$ . Use the decomposition in solving  $Ax = b$ , where  $b = [-2 \ 10 \ 14]^T$ .

3. (12pts.) An  $n \times n$  matrix  $A$  is said to be in upper Hessenberg form if all its elements below the first subdiagonal are zero, so that  $a_{ij} = 0, i > j+1$ . Assume  $A$  is nonsingular and no pivoting is needed.
  - a. Provide an efficient algorithm for  $LU$  decomposition.
  - b. What is the sparsity structure of the resulting  $L$ , i.e., where are its nonzeros?
  - c. How many operations (to a leading order) does it take to solve a linear system  $Ax = b$ , where  $A$  is upper Hessenberg?
4. (6pts.) Let  $A$  be an  $n \times n$  nonsingular matrix. Consider the following algorithm.
  1. Scan column 1 through  $n$  of  $A$  in succession, and permute rows, if necessary, so that the diagonal entry is the largest entry in magnitude on or below the diagonal in each column. The result is  $PA$  for some permutation matrix  $P$ .
  2. Now carry out Gauss elimination without pivoting to compute the  $LU$  factorization of  $PA$ .

Is this algorithm numerically stable? if so, explain why. If not, give a counterexample to illustrate.

5. (6pts.) How would you solve a partitioned linear system of the form

$$\begin{bmatrix} L_1 & 0 \\ B & L_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix},$$

where  $L_1$  and  $L_2$  are nonsingular lower triangular matrices, and the solution and right hand side vectors are partitioned accordingly? Show the specific steps you would perform in terms of the given submatrices and vectors.