Jacob Manning HW4

2. 5.10 $\quad x_1^2 - x_2^2 = 0 \qquad x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\qquad 2x_1 x_2 - 1 = 0$

$J = \begin{bmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{bmatrix}$

$J(x_0)s_0 = -F(x_0)$

$\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} s_0^1 \\ s_0^2 \end{bmatrix} = -\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$s_0 = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} \qquad x_1 = x_0 + s_0 = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

1. 5.6

By MVT $|f(x) - f(x^*)| = f'(c)|x - x^*|$ for some c

If $|f'(c)| < 1$ This is a contraction mapping $\therefore$ By Banach fixed point theorem there will exist a unique fixed point.

a) $|g_1'(3)| = |1-6| = |-5| \nless 1$ so no fixed point

b) $|g_2'(3)| = |1 - \frac{6}{9}| = |\frac{1}{3}| < 1$ so fixed point

c) $g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2}{2x} = x - \frac{x}{2} = \frac{x}{2}$

5.3b f is a square root function so quadratic converge[nce]

x_0 has 4 bits of accuracy $e_0 < 2^{-4}$

How many Newton iterations until 24bit and 53bit

$e_n < 2^{-2^{n+2}} = 2^{-24} \qquad e_n < 2^{-2^{n+2}} = 2^{-53}$

$2^{n+2} = 24 \qquad 2^{n+2} = 53$

$n = \left\lceil \frac{\ln 24}{\ln 2} - 2 \right\rceil = 3 \qquad n = \left\lceil \frac{\ln 53}{\ln 2} - 2 \right\rceil = 4$

3. 5.12

$x_{k+1} = x_k - \frac{f(x_k)}{d}$

a) If $|d|<1$ then by the MVT and BFPT, there will be a fixed point

b) In general the converge rate will be linear, it is possible that it will be superlinear. Quadratic only if $d=f'$

c) Yes when $d=f'$

4. 5.8 Smallest positive root of $\cos x + \frac{1}{1+e^{-2x}} = 0$

$x_0 = 3$

a) $x_{k+1} = \cos^{-1}\left(\frac{-1}{1+e^{-2x_k}}\right)$

Check $|g'(x)| = \left|\frac{2e^{-2x}}{(e^{2x}+1)\sqrt{2e^{2x}+1}}\right|$

$|g'(3)| = \frac{2e^{-6}}{(e^{6}+1)\sqrt{2e^{6}+1}} \approx 4.313 \cdot 10^{-7} < 1$

This should converge

no r

b) $x_{k+1} = \frac{1}{2}\log\left(\frac{-1}{1+\frac{1}{\cos x_k}}\right)$

$|g'(x)| = \left|\frac{-\sin(x)}{2\cos x(\cos x+1)}\right|$ $|g'(3)| = \left|\frac{-\sin 3}{2\cos 3(\cos 3+1)}\right| \approx 7.122$

This should not converge $r \approx 1$ check code

c) $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ $|f'(x)| = \left|\frac{2e^{-2x}}{(e^{-2x}+1)^2} - \sin(x)\right|$

$|f'(3)| = \left|\frac{2e^{-6}}{(e^{-6}+1)^2} - \sin 3\right| \approx 0.136$

This should converge $r \approx 2$ check code