MATH 8600 Scientific Computing

HW4 (Due: Thur. Oct. 19)

1. (6pts.) 5.6 on p249 and 5.3 (b) on p248
2. (4pts.) 5.10 on p249
3. (5pts.) 5.12 on p249
4. (10pts.) 5.8 on p250 (written and computing problem)
5. (6pts.) (computing problem) Consider the system of nonlinear equations

$$\begin{aligned}(x_1+3)(x_2^3-7)+18 &= 0\\ sin(x_2 e^{x_1}-1) &= 0\end{aligned}$$

   Use Newton's method to solve the system with the starting point $x_0 = [-0.5\ 1.4]^T$, $M = 50$ and $TOL = 10^{-6}$. Compute the convergence rate of the method using the error ($\|x^k - x^*\|_2$) at each iteration, given that the exact solution is $x^* = [0\ 1]^T$.

6. (10pts.) (computing problem) Use Newton's method to approximate the root of (a) $f(x) = x^2 - 4sinx$ using $x^{(0)} = 4$ and (b) $g(x) = x^2 - 2x + 1$ using $x^{(0)} = 2$. Use stopping criteria of a maximum number of iterations 30 along with the successive error less than $0.5 \times 10^{-6}$. The desired roots are $f(1.93375376282702) = 0$ and $g(1.0) = 0$. Tabulate the following

| $k$ | $x^{(k)}$ | $e_k$ | rate |
|---|---|---|---|

   Discuss your results and give the numerical rate of convergence. Also, solve the equations using the Matlab command **fzero** and compare with your solutions.

**Calculating a rate of convergence**

An iteration is said to converge to $x^*$ at a rate $r$ if

$$\lim_{k\to\infty} \frac{e_{k+1}}{e_k^r} = C$$

where the error is defined as $e_k = |x^{(k)} - x^*|$. For example, the iteration is said to converge *linearly* if $e_{k+1} \approx Ce_k$ and quadratically if $e_{k+1} \approx Ce_k^2$. However, this is an asymptotic definition, i.e., it holds as $k \to \infty$. To determine the numerical rate of convergence suppose we have $e_k$, $e_{k+1}$, and $e_{k+2}$ and assume that

$$e_{k+1} = Ce_k^r \qquad\qquad e_{k+2} = Ce_{k+1}^r$$

Now we don't know $r$ or $C$ but we can eliminate $C$ from both equations to obtain

$$C = \frac{e_{k+1}}{e_k^r} = \frac{e_{k+2}}{e_{k+1}^r}$$

Solving for $r$ we get

$$\left(\frac{e_k}{e_{k+1}}\right)^r = \frac{e_{k+1}}{e_{k+2}}$$

which implies

$$r\, ln\left(\frac{e_k}{e_{k+1}}\right) = ln\left(\frac{e_{k+1}}{e_{k+2}}\right)$$

As $k \to \infty$, this value, $r$, should approach the numerical rate of convergence.