MATH 8600 Scientific Computing

HW #4 (Due: Tuesday, Oct. 31)

1. (10pts.) (Computing problem) Given the data

| Time | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 |
|---|---|---|---|---|---|
| Product | 0.0 | 0.19 | 0.26 | 0.29 | 0.31 |

   a. Find the interpolating polynomial p(x) of degree 4 by solving a Vandemonde matrix system.

   b. Calculate p(0.7).

   c. Plot your polynomial and the data points in the same window.

2. (4pts.) Let $P_3(x)$ be the interpolating polynomial for the data $(0,0)$, $(0.5, y)$, $(1, 3)$ and $(2, 2)$. Find $y$ if the coefficient of $x^3$ is 6. (Hint: Use Newton's divided differences.)

3. (6pts.) Construct the Lagrange interpolating polynomial for the function $f(x) = cos\,x + sin\,x$ at $x_0 = 0$, $x_0 = 1/4$, $x_2 = 1/2$ and $x_3 = 1$, and find a bound for the absolute error on the interval $[x_0, x_3]$.

4. (10pts.) Let $f(x) = 1/(1 + x)$. Choose $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, and let $f_i = f(x_i)$ for $i = 0, 1, 2, 3$.

   a. Calculate the divided differences $f[x_0, x_1]$, $f[x_0, x_1, x_2]$ and $f[x_0, x_1, x_2, x_3]$.

   b. Using the divided differences in (a), find the cubic polynomial $P_3(x)$ that interpolates $f(x)$ at the given node points $\{x_0, x_1, x_2, x_3\}$.

   c. Graph the error $f(x) - P_3(x)$ on the interval $[0, 3]$ using Matlab.

5. (12pts.) Suppose we want to approximate the function $e^x$ on the interval $[0, 1]$ by polynomial interpolation with $x_0 = 0$, $x_1 = 1/2$, and $x_2 = 1$. Let $P_2(x)$ denote the interpolating polynomial.

   a. Find an upper bound for the error $|e^x - P_2(x)|$

   b. Find the interpolating polynomial, $P_2(x)$, using one method of your choice.

   c. Plot $e^x$ and $P_2(x)$ on the same window.

   d. Plot the error $|e^x - P_2(x)|$ using log-scale ("semilogy" in Matlab) and verify by inspection that it is below the bound found in (a).