Jacob Manning HW4

1.

Time	0	0.5	1.0	1.5	2.0
Product	0	0.19	0.26	0.29	0.31

a. Find $P_4(x)$ w/Vandemonde matrix system

$$\begin{bmatrix} T^{0T} & T^{T} & T^{2T} & T^{3T} & T^{4T} \end{bmatrix} X = P^T$$

$X = T^{-1}P^T$ Used as a symbol, but not calculating T^{-1}

$X = [0, 0.57833, -0.49166, 0.2066, -0.033]^T$

$x(t) = 0.57833t - 0.49166t^2 + 0.2066t^3 - 0.033t^3$

b. $x(0.7) = 0.2268$

c. Check code pdfs

2. Let $P_3(x)$ be interpolating polynomial w/Newton's Divided Difference Hint $6 = \alpha_3$

0	0	$\frac{4-0}{1/2} = 2y$	$\frac{6-2y-2y}{1-0} = 6-4y$	$\frac{\frac{-14+4y}{3} - \left(\frac{18-12y}{3}\right)}{2-0} = \frac{-16+8y}{3}$
0.5	y	$\frac{3-y}{1/2} = 6-2y$	$\frac{-1-6+2y}{3/2} = \frac{2(-7+2y)}{3}$	
1	3	$\frac{2-3}{2-1} = -1$		
2	2			

$P_3(x) = 0 + 2xy + (6-4y)(x^2 - \frac{x}{2}) + (\frac{-16+8y}{3})(x^2 - \frac{x}{2})(x-1)$

$\frac{-16+8y}{3} x^3 = 6x^3$

$-16+8y = 18$

$8y = 34$

$y = \frac{17}{4}$

3. Construct Lagrange Polynomial for $f(x) = \cos x + \sin x$ at $x_0 = 0$ $x_1 = 1/4$ $x_2 = 1/2$ $x_3 = 1$

$$P(x) = f(0)\frac{(x-1/4)(x-1/2)(x-1)}{(-1/4)(-1/2)(-1)} + f(\tfrac{1}{4})\frac{x(x-1/2)(x-1)}{(1/4-1/2)(1/4)(1/4-1)} + f(\tfrac{1}{2})\frac{x(x-1/4)(x-1)}{(1/2)(1/2-1/4)(1/2-1)}$$

$$+ f(1)\frac{x(x-1/4)(x-1/2)}{(1/2)(1-1/4)}$$

$$= f(0)[-8x^3+14x^2-17x+1] + f(\tfrac{1}{4})[\tfrac{64x^3-96x^2+32x}{3}] + f(\tfrac{1}{2})[16x^3-16x^2+4x] + f(1)[\tfrac{8x^3-6x^2+x}{3}]$$

$f(x_i)$ is just notation

2cont. $E(x) = \frac{f^{(n+1)}(c)}{(n+1)!}\prod_{j=0}^{n}(x-x_j) \le \frac{\max f^{(n+1)}(c)}{(4+1)}(x-0)(x-\frac{1}{4})(x-\frac{1}{2})(x-1)$

Note $|x-x_j| \le (j+1)h$, $0 \le j \le n$

$= \frac{\max f^{(5)}(c)}{5!} x(x-\frac{1}{4})(x-\frac{1}{2})(x-1)$

$f^{(5)}(c) = f'(c)$

≈ 0.00004

4. $f(x) = \frac{1}{1+x}$

a.

0	1	$\frac{\frac{1}{2}-1}{1} = -\frac{1}{2}$	$\frac{-\frac{1}{6}+\frac{1}{2}}{2-0} = \frac{1}{6}$
1	$\frac{1}{2}$	$\frac{\frac{1}{3}-\frac{1}{2}}{2-1} = -\frac{1}{6}$	$\frac{\frac{1}{12}+\frac{1}{6}}{3-1} = \frac{1}{24}$
2	$\frac{1}{3}$	$\frac{\frac{1}{4}-\frac{1}{3}}{3-2} = -\frac{1}{12}$	
3	$\frac{1}{4}$		

$\frac{\frac{1}{24}-\frac{1}{6}}{3-0} = -\frac{1}{24}$

b. $P_3(x) = 1 - \frac{1}{2}x + \frac{1}{6}x(x-1) - \frac{1}{24}x(x-1)(x-2)$

$= \frac{-x^3}{24} + \frac{7x^2}{24} - \frac{5x}{4} + 1$

c. Check code pdf

5a. $E(x) = \frac{f^{(n+1)}(c)}{(n+1)!}\prod_{j=0}^{n}(x-x_j) \le \max e^c h^{n+1} = (\frac{1}{2})^4 = \frac{1}{16}$

$\max_{c\in[0,1]} e^c = 1$

b.

0	1
$\frac{1}{2}$	$e^{1/2}$
1	e

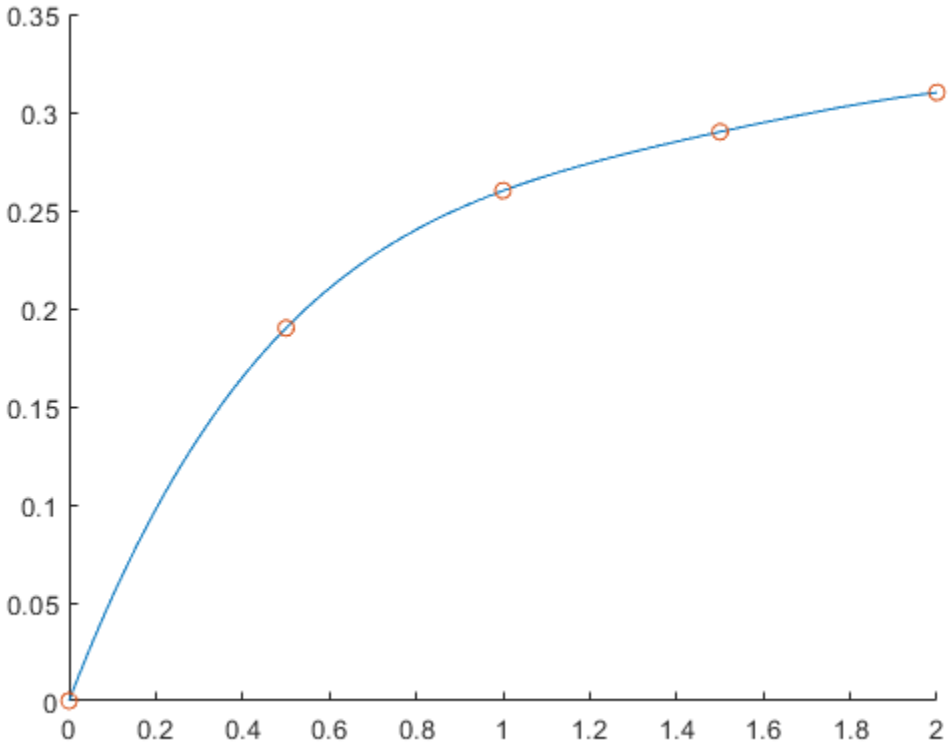
$P_2(x) = \frac{(x-\frac{1}{2})(x-1)}{(-\frac{1}{2})(-1)} + e^{\frac{1}{2}}\frac{x(x-1)}{(\frac{1}{2}-1)(\frac{1}{2})} + e\frac{x(x-\frac{1}{2})}{(1)(\frac{1}{2})}$

$P_2(x) = 1 + (-3 + 4e^{\frac{1}{2}} - e)x + (2 - 4e^{\frac{1}{2}} + 2e)x^2$

c. d. Check code pdfs

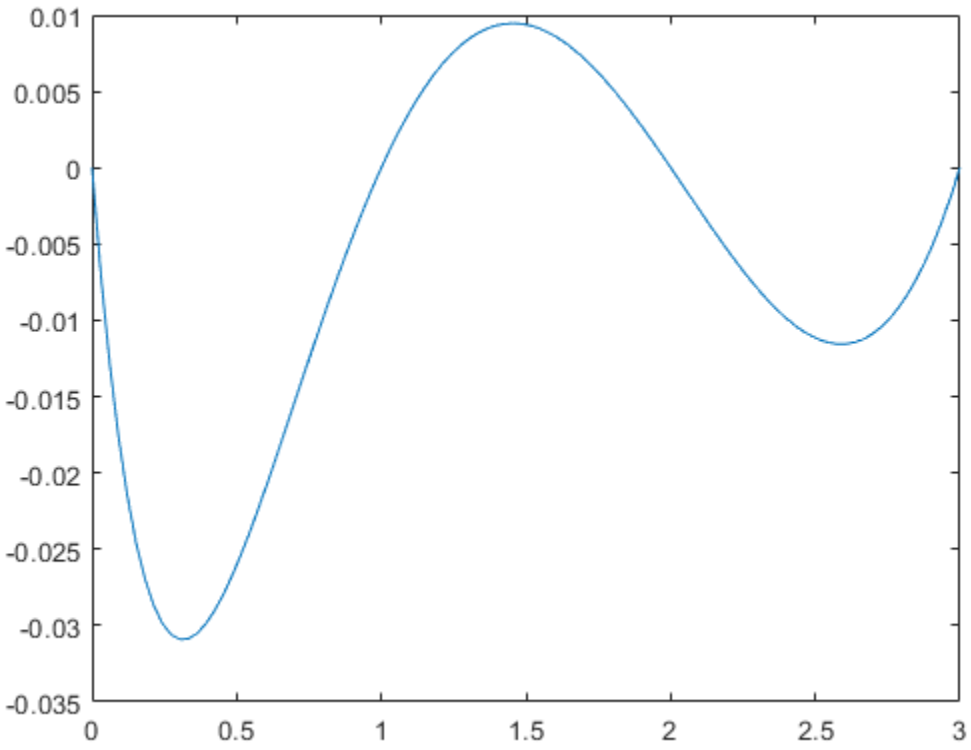
→ Checked max error and it is less than part a

```
%HW4P1

format long

t=[0 0.5 1 1.5 2];
p=[0 0.19 0.26 0.29 0.31];

t0=t.^0;
t2=t.^2;
t3=t.^3;
t4=t.^4;

tm=[t0' t' t2' t3' t4'];

yp=tm\p';
xp=linspace(0,2,100);

a=pf(yp,0.7);

hold on
plot(xp,pf(yp,xp));
scatter(t,p);
hold off

function px=pf(y,t)
    px=y(1)+y(2)*t+y(3)*t.^2+y(4)*t.^3+y(5)*t.^4;
end
```



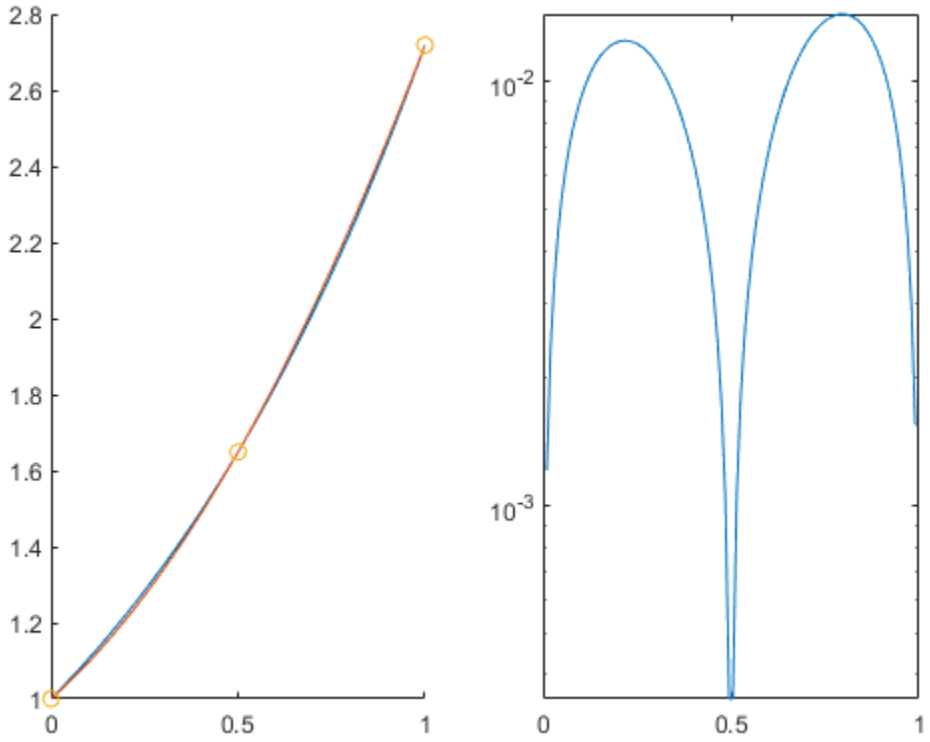
Published with MATLAB® R2023b

```
%HW4P4

format long

t=[0 1 2 3];
p=[1 1/2 1/3 1/4];

xp=linspace(0,3,100);

plot(xp,f(xp)-pf(xp));

function px=pf(t)
    px = 1- t./2+t.*(t-1)./6-t.*(t-1).*(t-2)./24;
end

function ft = f(t)
    ft = (1+t).^-1;
end
```



Published with MATLAB® R2023b

```
%HW4P5

format long

t=[0 1/2 1];
p=[1 exp(1/2) exp(1)];

xp=linspace(0,1,100);

figure(1);
subplot(1,2,1)
hold on
plot(xp,exp(xp));
plot(xp,pf(xp));
scatter(t,p);
hold off
subplot(1,2,2)
semilogy(xp,abs(exp(xp)-pf(xp)))

m=max(abs(exp(xp)-pf(xp)));

function px=pf(t)
    px = 1+(-3+ 4*exp(1/2)-exp(1))*t+(2-4*exp(1/2)+2*exp(1))*t.^2;
end
```



Published with MATLAB® R2023b