MATH 8610 (SPRING 2024) HOMEWORK 1

Assigned 01/23/24, due 01/30/24 (Tuesday) 11:59pm.

Instructor: Dr. Fei Xue, Martin O-203, fxue@clemson.edu.

1. **[Q1] (10 pts)** (a) Find the absolute and relative condition numbers of $f(x) = e^{-2x}$ and $f(x) = \ln^3 x$. For what values of x are these functions sensitive to perturbations? (b) Let $x_1, x_2 \in \mathbb{R}^+$, and $f(x_1, x_2) = x_1^{x_2}$. Find the relative condition number of $f(x)$, and for what range of values of x_1 and x_2 is the problem ill-conditioned.

2. **[Q2] (10 pts)** Consider the recurrence $x_{k+1} = 111 - \frac{1130 - \frac{3000}{x_{k-1}}}{x_k}$, whose general solution is $x_k = \frac{100^{k+1}a + 6^{k+1}b + 5^{k+1}c}{100^k a + 6^k b + 5^k c}$, where a, b and c depend on the initial values. Given $x_0 = \frac{11}{2}$ and $x_1 = \frac{61}{11}$, we have $a = 0$, $b = c = 1$.
   (a) Show that this gives a monotonically increasing sequence to the limit of value 6.
   (b) Implement this recurrence on MATLAB, plot $\{x_k\}$, compare with the exact solution. Explain any major discrepancies you see. What is the condition number of the limit of this particular sequence as a function of x_0 and x_1?

3. **[Q3] (10 pts)** Let $p_{24}(x) = (x-1)(x-2)\cdots(x-24) = x^{24} + a_{23}x^{23} + \cdots + a_1 x + a_0$, where $a_{14} = 9.2447 \times 10^{16}$, $a_{15} = -5.7006 \times 10^{15}$, $a_{16} \approx 2.9089 \times 10^{14}$, $a_{17} \approx -1.2191 \times 10^{13}$, $a_{18} \approx 4.1491 \times 10^{11}$.

   Evaluate the relative condition number of the k-th root $x_k = k$ subject to the perturbation of a_k for $k = 14, 15, \ldots, 18$ and find the root that is most sensitive to the perturbation of the corresponding coefficient. Use the attached MATLAB data file `wilk24mc.mat` containing the coefficients $a_{24}, a_{23}, \ldots, a_1, a_0$, and use MATLAB's `roots` to find the roots. Compare with the true roots and comment on what you see.