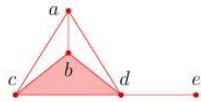


# HW6

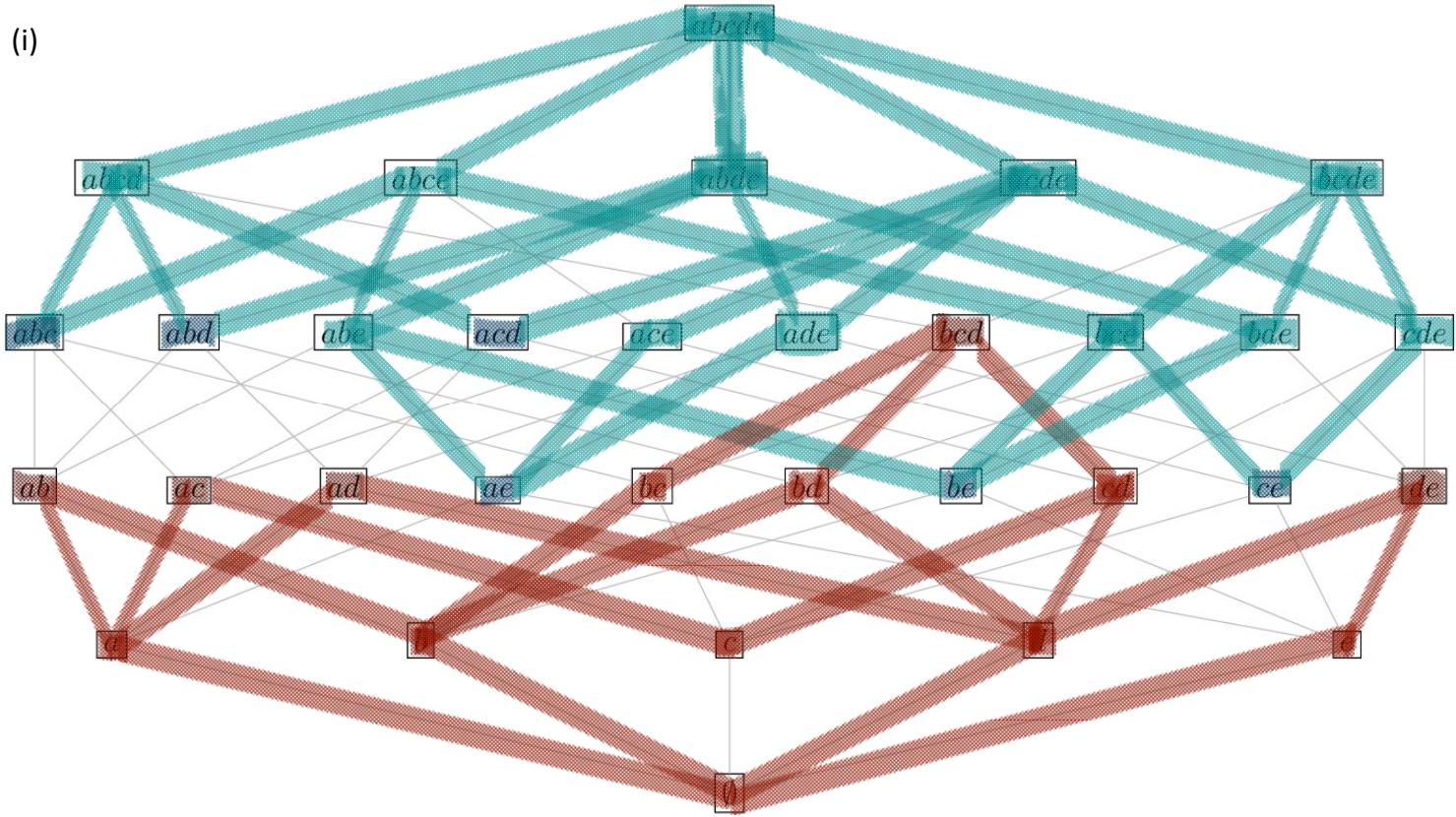
Thursday, October 31, 2024 7:03 PM

1. **Alexander duality.** Consider the simplicial complex  $\Delta$  over  $X = \{a, b, c, d, e\}$  shown below:



The 5-dimensional Boolean lattice  $2^X$  also appears below.

- (i) In the Boolean lattice, color the faces red, and the nonfaces blue.
- (ii) Find the maximal faces, and the minimal generators of the Stanley-Reisner ideal  $I_{\Delta^c}$ .
- (iii) Compute the primary decomposition of  $I_{\Delta^c}$ .



(ii)

$$\{de, bcd, ac, ab, ad\}$$

$$\{abc, abd, acd, ae, be, ce\}$$

(iii)

$$I_{\Delta^c} = \langle a, b, c \rangle \cap \langle a, c \rangle \cap \langle b, d, e \rangle \cap \langle c, d, e \rangle \cap \langle b, c, e \rangle$$