1. **Reverse-engineering a Boolean model**. Consider a Boolean model $f = (f_1, f_2, f_3)$ whose (synchronous) phase spaces consists of the following:

$$(0,0,1) \xrightarrow{f} (1,0,1) \xrightarrow{f} (1,1,1) \xrightarrow{f} (1,1,0) \xrightarrow{f} (0,1,0) \xrightarrow{f} (0,0,0).$$

   In this problem, you will reverse-engineer the wiring diagram and then the models space.

   (a) Do the following steps for each $k = 1, 2, 3$.

      i. Write down the corresponding set of *data*

$$\mathcal{D}_k := \big\{(\boldsymbol{s}_1, t_{1k}), \dots, (\boldsymbol{s}_5, t_{5k})\big\}$$

      that arises from the k^{th} coordinate of this time-series.

      ii. Find the monomial ideal $I_{\Delta_k^c}$ of non-disposable sets, and compute its primary decomposition to find the min-sets.

      iii. Find the pseudomonomial ideal $J_{\Delta_k^c}$, and compute its primary decomposition to find the signed min-sets.

   (b) Find all min-sets and signed min-sets of the original Boolean model.

   (c) Use Macaulay2 to compute the vanishing ideal $I(\mathcal{D})$. How big is it?

   (d) Find the model space $\mathrm{Mod}(\mathcal{D}) = \prod_{i=1}^{3} \big[f + I(\mathcal{D})\big]$. Use Lagrange interpolation to find $f = (f_1, f_2, f_3)$, and Macaulay2 to reduce them modulo $I(\mathcal{D})$.

2. **Reverse-engineering a ternary model**. Repeat the previous problem but for the following *time series* of a 3-node algebraic model over $\mathbb{F}_3$:

$$(1,1,1) \xrightarrow{f} (2,0,1) \xrightarrow{f} (2,0,0) \xrightarrow{f} (0,2,2) \xrightarrow{f} (0,2,2).$$