HW4

Tuesday, October 8, 2024 7:49 AM

1. **Reduction**. Consider the following Boolean network model of the *lac* operon.

$$
\begin{aligned}
f_1 &= x_4 \wedge \overline{x_5} \wedge \overline{x_6} & f_8 &= x_9 \vee x_{10} \\
f_2 &= x_1 & f_9 &= x_3 \wedge \overline{G_e} \wedge L_e \\
f_3 &= x_1 & f_{10} &= \overline{G_e} \wedge \left(L_e \vee (x_3 \wedge L_{em})\right) \\
f_4 &= \overline{G_e} & f_{G_e} &= G_e \\
f_5 &= \overline{x_7} \wedge \overline{x_8} & f_{L_e} &= L_e \\
f_6 &= x_5 \vee (\overline{x_7} \wedge \overline{x_8}) & f_{L_{em}} &= L_{em} \\
f_7 &= x_2 \wedge x_9
\end{aligned}
$$

Here, the variables represent

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}) = (M, B, P, C, R, R_m, A, A_m, L, L_m).$$

(a) Use Macaulay2 to reduce this Boolean network, as much as possible.

(b) Draw the wiring diagram of the reduced network. Find its fixed point(s) and use these to determine the fixed point(s) of the original network by back-substitution.

$X_1 = X_2 = X_3$ $X_5 = X_6$ $X_7 = X_9$ $X_8 = X_{10}$

$X_4 = \overline{Ge}$

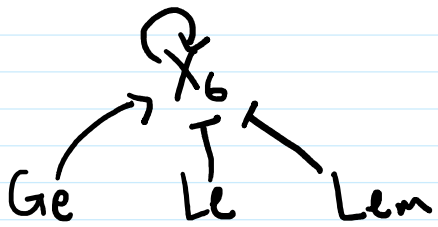
The Macaulay2 at bottom gb => $X_1 = \overline{X_6}$

$X_1 = \overline{Ge} \wedge \overline{X_6}$

$X_6 = (Ge \vee \overline{Le}) \wedge (Ge \vee X_6 \vee \overline{Lem})$

$X_7 = \overline{Ge} \wedge \overline{X_6} \wedge Le$

$X_8 = (\overline{Ge} \wedge \overline{X_6} \wedge Lem) \vee (\overline{Ge} \wedge Le)$

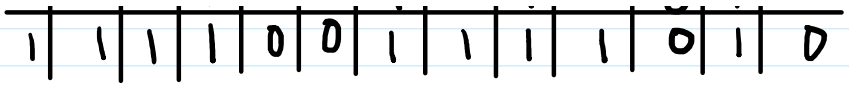


X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	Ge	Le	Lem
0	0	0	0	1	1	0	0	0	0	1	0	0
0	0	0	0	1	1	0	0	0	0	1	0	1
0	0	0	0	1	1	0	0	0	0	1	1	1
0	0	0	0	1	1	0	0	0	0	1	1	0
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	1
1	1	1	1	0	0	0	1	0	1	0	0	1
1	1	1	1	0	0	1	1	1	1	0	1	1
1	1	1	1	0	0	1	1	1	1	0	1	0

Checking with cyclone

Le Lem Ge

```
COMPONENT #1:
  component size: 1024
  fixed point: [0 0 0 1 1 1 0 0 0 0 0 0 0]

COMPONENT #2:
  component size: 1024
  fixed point: [0 0 0 0 1 1 0 0 0 0 0 0 1]

COMPONENT #3:
  component size: 1006
  fixed point: [0 0 0 1 1 1 0 0 0 0 0 1 0]

COMPONENT #4:
  component size: 1024
  fixed point: [0 0 0 0 1 1 0 0 0 0 0 1 1]

COMPONENT #5:
  component size: 1024
  fixed point: [1 1 1 1 0 0 1 1 1 1 1 0 0]

COMPONENT #6:
  component size: 1024
  fixed point: [0 0 0 0 1 1 0 0 0 0 1 0 1]
```



```
COMPONENT #6:
  component size: 1024
  fixed point: [0 0 0 0 1 1 0 0 0 0 1 0 1]

COMPONENT #7:
  component size: 1024
  fixed point: [1 1 1 1 0 0 1 1 1 1 1 1 0]

COMPONENT #8:
  component size: 1024
  fixed point: [0 0 0 0 1 1 0 0 0 0 1 1 1]

COMPONENT #9:
  component size: 18
  fixed point: [1 1 1 1 0 0 0 1 0 1 0 1 0]
```

```
i19 :  f1=sub(f1,{x4=>f4})
```

$$o19 = x5\,x6\,Ge + x5\,x6 + x5\,Ge + x6\,Ge + x5 + x6 + Ge + 1$$

$$o19 : Q$$

```
i20 :  f1=sub(f1,{x5=>x6})
```

$$o20 = x6\,Ge + x6 + Ge + 1$$

$$o20 : Q$$

```
i21 :  f6=sub(f6,{x5=>x5})
```

$$o21 = x5\,x7\,x8 + x5\,x7 + x5\,x8 + x7\,x8 + x7 + x8 + 1$$

$$o21 : Q$$

```
i22 :  f6=sub(f6,{x5=>f5})
```

$$o22 = x7\,x8 + x7 + x8 + 1$$

$$o22 : Q$$

```
i23 :  f2=sub(f2,{x1=>f1})
```

$$o23 = x6\,Ge + x6 + Ge + 1$$

$$o23 : Q$$

```
i24 :  f3=sub(f3,{x1=>f1})
```

$$o24 = x6\,Ge + x6 + Ge + 1$$

```
i25 :  f7=sub(f7,{x2=>f2})
```

$$o25 = x6\,x9\,Ge + x6\,x9 + x9\,Ge + x9$$

$$o25 : Q$$

```
i26 :  f9=sub(f9,{x3=>f3})
```

$$o26 = x6\,Le\,Ge + x6\,Le + Le\,Ge + Le$$

$$o26 : Q$$

```
i27 :  f10=sub(f10,{x3=>f3})
```

$$o27 = x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem$$

$$o27 : Q$$

```
i28 :  f7sub(f7,{x9=>f9})
stdio:39:6:(3): error: no method for adjacent objects:
 • f7sub (of class Symbol)
 • (x6 x9 Ge + x6 x9 + x9 Ge + x9, {x9 ⇒ x6 Le Ge + x6 Le + Le Ge + Le}) (of class Sequence)

stdio:39:1-39:6: here is the first use of f7sub
```

```
i29 :  f7=sub(f7,{x9=>f9})
```

$$o29 = x6\,Le\,Ge + x6\,Le + Le\,Ge + Le$$

$$o29 : Q$$

```
i30 : f8
```

$$o30 = x9\,x10 + x9 + x10$$

```
i31 :  f8=sub(f8,{x9=>f9})
```

$$o31 = x6\,x10\,Le\,Ge + x6\,x10\,Le + x6\,Le\,Ge + x10\,Le\,Ge + x6\,Le + x10\,Le + Le\,Ge + x10 + Le$$

$$o31 : Q$$

```
i32 :  f8=sub(f8,{x10=>f10})
```

$$o32 = x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem$$

$$o32 : Q$$

```
i33 : f6
```

$$o33 = x7\,x8 + x7 + x8 + 1$$

$$o33 : Q$$

```
i34 :  f6=sub(f6,{x7=>f7})
```

$$o34 = x6\,x8\,Le\,Ge + x6\,x8\,Le + x6\,Le\,Ge + x8\,Le\,Ge + x6\,Le + x8\,Le + Le\,Ge + x8 + Le + 1$$

$$o34 : Q$$

```
i35 :  f6=sub(f6,{x8=>f8})
```

$$o35 = x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem + 1$$

$$o35 : Q$$

```
i36 : f1
```

$$o36 = x6\,Ge + x6 + Ge + 1$$

```
i37 :  gens gb ideal(f1+x1, f6+x6)
```

$$o37 = \begin{pmatrix} x1 + x6 + 1 & x6\,Ge + Ge & x6\,Le + Le\,Ge & Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + x6 + Le + Lem + 1 \end{pmatrix}$$

$$o37 : \text{Matrix } Q^1 \longleftarrow Q^4$$

```
i38 : (f1,f6)
```

$$o38 = (x6\,Ge + x6 + Ge + 1,\ x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem + 1)$$

$$o38 : \text{Sequence}$$

```
i39 :  f1=sub(f1,{x6=>f6})
```

$$o39 = x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem$$

$$o39 : Q$$

```
i40 : Ge|((Le+1)&(x6|(Lem+1)))
```

$$o40 = x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem + 1$$

$$o40 : Q$$

```
i41 : (Ge|(Le+1))&(Ge|x6|(Lem+1))
```

$$o41 = x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem + 1$$

$$o41 : Q$$

```
i42 : f6
```

$$o42 = x6\,Le\,Lem\,Ge + x6\,Le\,Lem + x6\,Lem\,Ge + Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + Lem\,Ge + Le + Lem + 1$$

```
i43 :  gens gb ideal(f1+x1, f6+x6)
```

$$o43 = \begin{pmatrix} x1 + x6 + 1 & x6\,Ge + Ge & x6\,Le + Le\,Ge & Le\,Lem\,Ge + x6\,Lem + Le\,Lem + Le\,Ge + x6 + Le + Lem + 1 \end{pmatrix}$$

$$o43 : \text{Matrix } Q^1 \longleftarrow Q^4$$

$f_1 = X_4 \wedge \overline{X_5} \wedge \overline{X_6}$

$f_2 = f_1 \qquad X_1 = X_2 = X_3$

$f_3 = f_1$

$f_4 = \overline{Ge}$

$f_5 = \overline{X_7} \wedge \overline{X_8}$

$f_6 = X_5 \vee (\overline{X_7} \wedge \overline{X_8}) \qquad X_5 = X_6$

$f_7 = f_1 \wedge X_9$

$f_8 = X_9 \vee X_{10}$

$f_9 = f_1 \wedge \overline{Ge} \wedge Le$

$f_{10} = \overline{Ge} \wedge (Le \vee (f_1 \wedge Lem))$

$f_6 \to f_6, X_5 \to f_5$

$X_1 = \overline{X_6}$

$X_7 = X_9$

$X_8 = X_{10}$

$f_6 = \overline{X_7} \wedge \overline{X_8} \qquad X_5 = X_6$

$f_1 = \overline{Ge} \wedge \overline{X_6}$ ↙

~~$f_7 = \overline{Ge} \wedge \overline{X_6} \wedge X_9$~~

$f_9 = \overline{Ge} \wedge \overline{X_6} \wedge Le$

$f_{10} = \overline{Ge} \wedge (Le \vee (\overline{Ge} \wedge \overline{X_6} \wedge Lem))$

$= (\overline{Ge} \wedge Le) \vee (\overline{Ge} \wedge \overline{X_6} \wedge Lem)$

$f_7 = \overline{Ge} \wedge \overline{X_6} \wedge Le$

$f_8 = (\overline{Ge} \wedge \overline{X_6} \wedge Le) \vee ((\overline{Ge} \wedge Le) \vee (\overline{Ge} \wedge X_6 \wedge Lem))$

$= (\overline{Ge} \wedge Le) \vee (\overline{Ge} \wedge \overline{X_6} \wedge Lem) \vee (\overline{Ge} \wedge X_6 \wedge Le)$

$= (\overline{Ge} \wedge \overline{X_6} \wedge Lem) \vee ([\overline{Ge} \wedge Le] \vee \overline{Ge}) \wedge [\overline{Ge} \wedge Le] \vee \overline{X_6}] \wedge [[\overline{Ge} \wedge Le] \vee Le])$

$= (\overline{Ge} \wedge \overline{X_6} \wedge Lem) \vee ([\overline{Ge} \wedge Le] \wedge (\overline{X_6} \vee \overline{Ge}) \wedge (\overline{X_6} \vee Le))$

$= (\overline{Ge} \wedge \overline{X_6} \wedge Lem) \vee (\overline{Ge} \wedge Le)$

$f_6 = \overline{(\overline{Ge} \wedge \overline{X_6} \wedge Le)} \wedge \overline{(\overline{Ge} \wedge \overline{X_6} \wedge Lem) \vee (\overline{Ge} \wedge Le)}$

$= (Ge \vee X_6 \vee \overline{Le}) \wedge (\overline{Ge} \wedge \overline{X_6} \wedge Lem) \wedge \overline{(\overline{Ge} \wedge Le)}$

$= (Ge \vee X_6 \vee \overline{Le}) \wedge (Ge \vee X_6 \vee \overline{Lem}) \wedge (Ge \vee \overline{Le})$

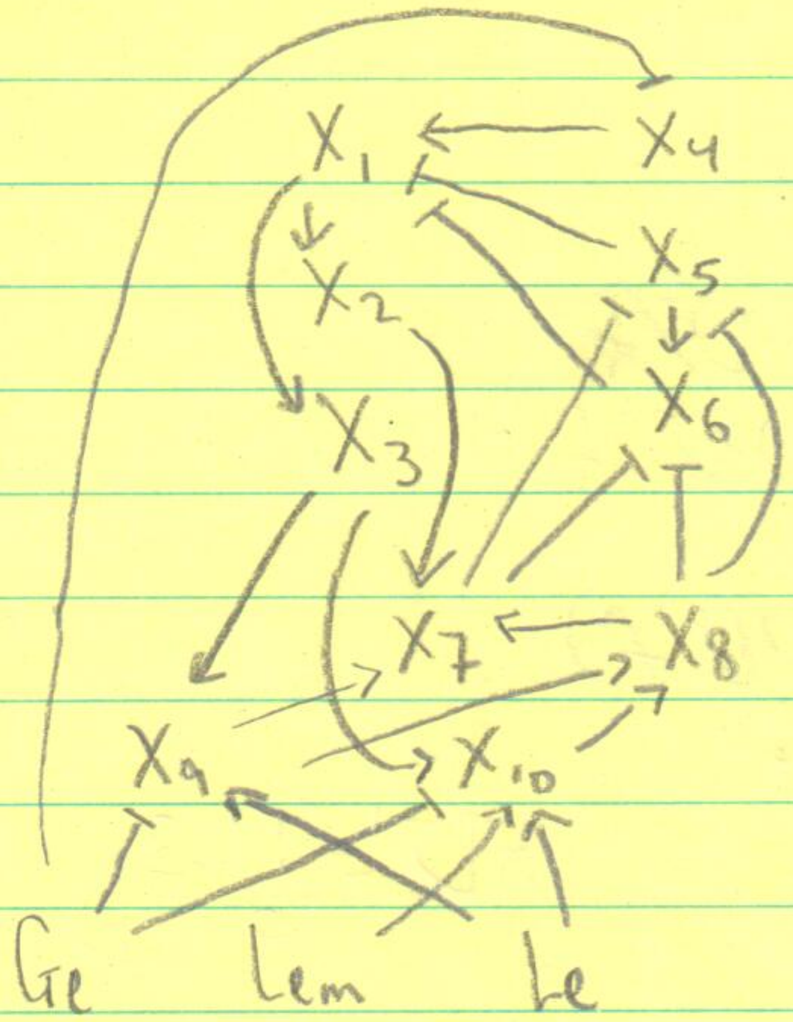
$= (Ge \vee \overline{Le}) \wedge (Ge \vee X_6 \vee \overline{Lem})$

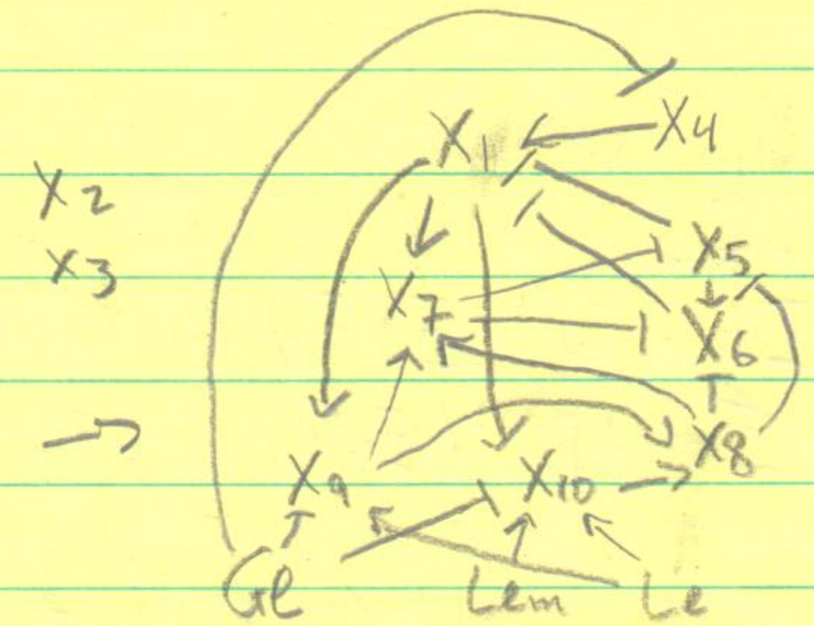
$= ((Ge \vee \overline{Le}) \wedge Ge) \vee (((Ge \vee \overline{Le}) \wedge X_6) \vee ((Ge \vee \overline{Le}) \wedge \overline{Lem}))$

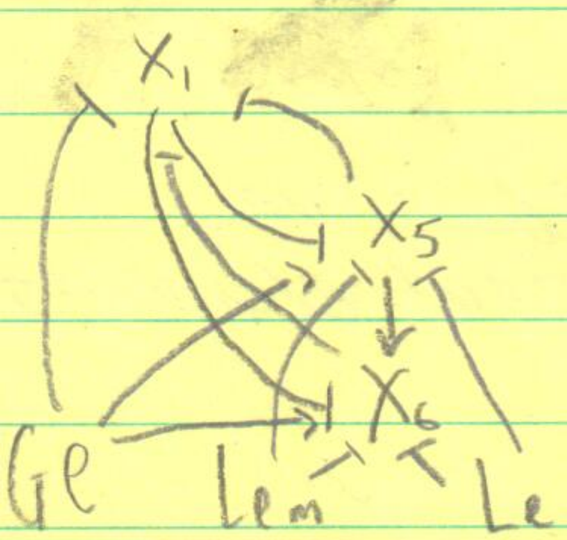
$= Ge \vee (\cancel{X_6 \wedge Ge}) \vee (X_6 \wedge \overline{Le}) \vee (\cancel{Ge \wedge \overline{Lem}}) \vee (\overline{Le} \wedge \overline{Lem})$

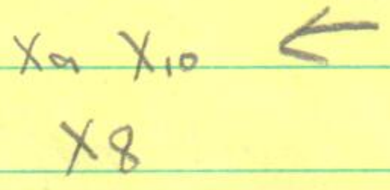
$= Ge \vee (X_6 \wedge \overline{Le}) \vee (\overline{Le} \wedge \overline{Lem})$

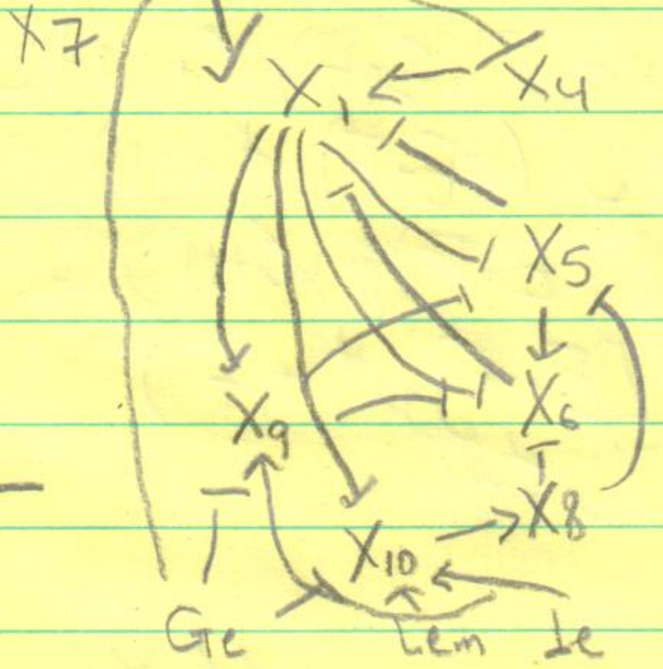
$= Ge \vee (\overline{Le} \wedge (X_6 \vee \overline{Lem}))$

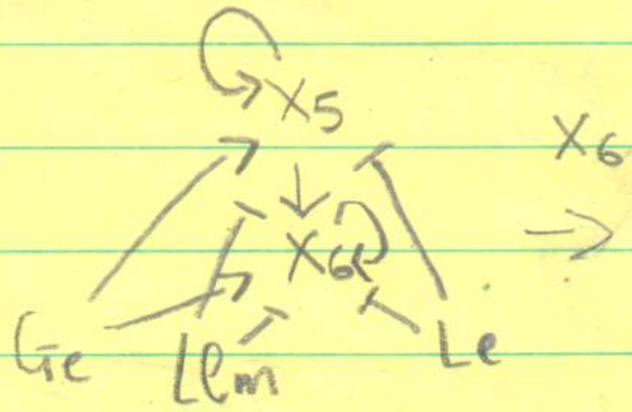

X1
X4
X2
X5
X3
X6
X7
X8
X9
X10
Ge
Lem
Le

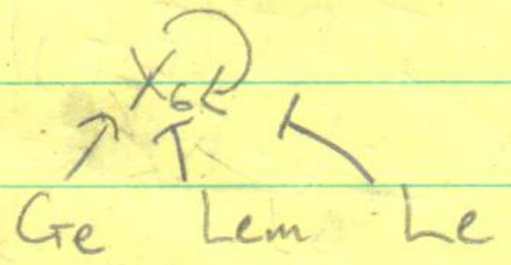

X2
X3
X1
X4
X5
X7
X6
X8
X9
X10
Ge
Lem
Le


X1
X5
X6
Ge
Lem
Le


Xm X10
X8


X7
X1
X4
X5
X9
X6
X8
X10
Ge
Lem
Le


X5
X6
X6
Ge
Lem
Le


X6
Ge
Lem
Le