MATH 8660 Sec.1: Homework No.9

Due: Tuesday, October 29

No.1. (3 pts.) Let G denote a closed subspace of a Hilbert space H. Show that $G^{\perp}$ is a closed subspace of H.

No.2. (3 pts.) Suppose $H = \mathbb{R}^3$, endowed with the usual vector dot product, and $G = span\{[1,2,3]^T\}$. Give an explicit representation for $G^{\perp}$.

No.3. (3 pts.) Give the Lagrangian, cubic basis functions on the reference interval, i.e. $[0,1] \subset \mathbb{R}$, for the nodal points 0, 1/3, 2/3, 1.

No.4. (5 pts.) Continuous piecewise cubic basis functions on $\widehat{\mathcal{T}}$.
There are ten piecewise cubic basis functions on $\widehat{\mathcal{T}}$: three nodal basis functions, six edge basis function (two per side), and one *bubble function.*
The *bubble function* is given by: $\widehat{\phi}_{10}(\xi,\eta) \ = \ 27\,\xi\,\eta\,(1-\xi-\eta)$.
Note that the bubble function is zero on all the edges of $\widehat{\mathcal{T}}$ and is equal to 1 at $(\xi,\eta) = (1/3\,,\,1/3)$.
Determine the other nine basis functions, which are Lagrangian.
(The nodal points are equally spaced along each edge of $\widehat{\mathcal{T}}$.

No.5. (4 pts.) Let H denote a Hilbert space, $\mathcal{L} : H \longrightarrow H'$ a linear operator, and $\{G_n\}_{n=1}^{\infty}$ a Cauchy sequence in $Range(\mathcal{L}) \subset H'$. Show that there exists a Cauchy sequence $\{g_n\} \in H$ such that $G_n(v) \ = \ \langle g_n, v\rangle$, for all $v \in H$.
