MATH 8660 Sec. 1: Homework No.7/8

Numerical Approximation of the 1-D Convection–Diffusion Equation

(20 pts.) **Part A: Due Thursday October 17 – Continuous Piecewise Linear Approximation**

No.1. Write a MATLAB code to approximate

$$-(k(x)u')' + b(x)u' + e(x)u = f(x)\,, \ \alpha < x < \beta\,, \tag{1}$$

$$u(\alpha) = u_0\,, u(\beta) = u_1\,. \tag{2}$$

From Canvas download the following flies: `Driver_1d.m, kfun.m, d1_quad_3, d1_quad_5, d1_CtsLin.m, d1_ip_Dten0_Dten0.m, d1_ip_ten0_ten0.m, d1_ip_ten0.m, OneScalFun.m, uFun.m, d1_CalcErr.m`

Your program should

(a) implement a continuous piecewise linear approximation,

(b) (i) implement a 3-point Gaussian quadrature rule for evaluation of the integrals in the assembly of the coefficient matrix and the rhs vector,

(ii) implement a 5-point Gaussian quadrature rule for evaluation of the integrals in the computation of the error,

(c) calculate the L^2 error for u_h and u_h'. (If the true solution is unknown compute the L^2 norms for u_h and u_h'.

No.2. Let $k(x) = 2x + 1$, $b(x) = x^2$, $e(x) = 5x + 2$, $\alpha = 0$, $\beta = 2$.

(a) For 4 subintervals, write out (and turn in) the coefficient matrix your implementation generates.

(b) Verify your implementation by

(i) Choosing a solution in the test space, (use $u(x) = 3x - 3$).

(ii) Verifying the experimental convergence rate, (use $u(x) = x^4 + 1$).

Turn in a discussion of why you believe your program is correct. Include in your discussion a table showing your program's approximation converging as expected to the true solution.

You will need to write the following files/routines:
`d1_ip_ten0_Dten0.m, bfun.m, efun.m, rhsfun.m, DuFun.m, uTrue.m, DuTrue.m`

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(10 pts.) **Part B: Due Tuesday October 22 – Continuous Piecewise Quadratic Approximation**

No.1. Implement a continuous piecewise quadratic approximation to (1) and (2).

No.2. Let $k(x) = 2x + 1$, $b(x) = x^2$, $e(x) = 5x + 2$, $\alpha = 0$, $\beta = 2$.
(a) For 3 subintervals, write out (and turn in) the coefficient matrix your implementation generates.
(b) Verify your implementation by
 (i) Choosing a solution in the test space, (use $u(x) = 3x^2 - 3$).
 (ii) Verifying the experimental convergence rate, (use $u(x) = x^4 + 1$).

Turn in a discussion of why you believe your program is correct. Include in your discussion a table showing your program's approximation converging as expected to the true solution.

You will need to write the following files/routines: `d1_CtsQuad.m`