MATH 8660 Sec.1: Homework No.1

Due: Tuesday, August 27

Is a "pointwise solution" the physically meaningful solution?

Mathematically the 1-d Heat Equation is

$$\frac{\partial u(x,t)}{\partial t} \; - \; \kappa \, \frac{\partial^2 u(x,t)}{\partial x^2} \; = \; f(x,t) \, , \quad \text{for } 0 < x < L, \; t > 0 \, . \tag{0.1}$$

No.1. (2 pts.) Using principles from Calculus I, explain (12.1.1). (This argument should also apply to $\int_x^{x+\delta x} \widetilde{f}(x,t) \, \rho \, A \, d\xi$.)

No.2. (4 pts.)Turn in your derivation of (0.1).

Note that the derivation of the "pointwise equation", (0.1), actually represents a local average about a point.

No.3. (6 pts.) Let

$$\Omega \; = \; [0,1] \times [-1,1], \quad \mathbf{b}(\mathbf{x}) \; = \; \begin{bmatrix} 1 \\ x_1 x_2 \end{bmatrix}, \quad u(\mathbf{x}) \; = \; \sin(2x_1) \, .$$

Verify that

$$\int_\Omega \mathbf{b}(\mathbf{x}) \cdot \nabla u(\mathbf{x}) \, d\Omega \; = \; \int_{\partial\Omega} \mathbf{b}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}) \; u(\mathbf{x}) \, ds \; - \; \int_\Omega \nabla \cdot \mathbf{b}(\mathbf{x}) \; u(\mathbf{x}) \, d\Omega \, ,$$

where $\mathbf{n}(\mathbf{x})$ denotes the unit outer normal to $\partial\Omega$.