MATH 8660 Sec.1: Homework No.10

Numerical Approximation of the 2-D Convection–Diffusion Equation

Due: Tuesday, November 5 at 11:59am.

No.1. (15 pts.). Write a MATLAB code to approximate

$$-\nabla \cdot (kfun(x)\nabla u) \; + \; Vfun(x) \cdot \nabla u \; + \; qfun(x)u \;\; = \;\; ffun(x)\,, \; x \in \Omega\,, \qquad (1)$$

$$u(x) \;\; = \;\; gfun(x)\,, \quad x \text{ on } \partial\Omega\,. \qquad (2)$$

Your program should

(a) (i) implement a continuous piecewise linear approximation,

(ii) implement a continuous piecewise quadratic approximation,

(iii) implement a higher order quadrature rule,

(b) calculate the L^2 error for u_h, ∇u_h, and H^1 error for u_h if the true solution is known, otherwise compute the L^2 norms for u_h, ∇u_h, and H^1 norm for u_h.

(Specifically, you need to write:
(i) inner_prod_ten0_Grad_ten0_Vec.m
(ii) inner_prod_ten0.m
(iii) CtsQuad.m
(iv) quad_75.m
(v) ffun.m)

Turn in your MATLAB files: inner_prod_ten0_Grad_ten0_Vec.m , inner_prod_ten0.m , CtsQuad.m , quad_75.m , ffun.m

For Problems **No.2**–**No.3** use $\Omega = [-1, 1] \times [0, 1]$.

No.2. (5 pts.). Take $kfun = 1.0$, $Vfun = [-x \quad y]^T$, $qfun = 2$, $gfun\ (= utrue) = 2x^2 + \pi xy + 7$ with $ffun$ determined by these choices.
Investigate the L^2 rates of convergence for the both the piecewise linear and piecewise quadratic approximations.
Turn in tables of the L^2 and H^1 errors along with experimental convergence rates. Discuss if you think your program is working correctly.

No.3. (6 pts.). L^2– Convergence of the Approximation – Smooth Solution
For $kfun = 1.0$, $Vfun = [3 \quad 2]^T$, $qfun = 2$, $gfun\ (= utrue) = x\sin(2\pi xy) + x$ with $ffun$ determined by these choices,

(a) Investigate the L^2 rates of convergence for the both the piecewise linear and piecewise quadratic approximations.

(b) Investigate if the different quadrature formulas effect the rate of convergence for the piecewise quadratic approximations.

Turn in tables of the L^2 and H^1 errors along with experimental convergence rates. Discuss the experimental convergence and if the different quadrature formulas made a difference.