MATH 8660 Sec.1: Homework No.6

Due: Tuesday, October 1

Part A.

No.1. (10 pts.) Of interest is

$$\int_a^b func(x)\,dx\,. \tag{0.1}$$

(a) Write a MATLAB file that, given the interval $[a, b]$, and the number of subintervals, Nsub, successively calls the MATLAB function

d1_ip_fun(ab, quadrule, func)

for each subinterval to approximate (0.1).
(From the MATLAB file you need to use d1_ip_fun(ab, @quadrule, @func) when calling/passing the function names to the function d1_ip_fun.)
(b) For $[a, b] = [0, 1]$, $func(x) = fun1(x) = \sin(\pi x) + (\cos(\pi x))^2$, and Nsub = 4, 8, 16, 32, approximate (0.1) using quadrature rule d1_quad_3.m.
(c) Construct a table containing Nsub, the error in your approximation, and the numerical convergence rate α, where $error \sim Ch^\alpha$, for $h = (b - a)/Nsub$.
(d) Discuss your numerical rate of convergence. Is it what you expected?

No.2. (2 pts.) Write a MATLAB file d1_quad_5.m (similar to d1_quad_3.m) that contains the quadrature points and weights for the 3 point Gaussian quadrature rule (order 5).

No.3. (5 pts.) For $func(x) = fun2(x) = x^{1/2}$ and quadrature rule d1_quad_5.m repeat No.1. (b), (c), (d).
(Remember to recalculate the value for the integral.)

Part B.

No.1. (5 pts.) Of interest is $\int_a^b f_1(x)\,dx$ and $\int_a^b f_2(x)\,dx$.

We want to compute both integrals simultaneously by computing $\int_a^b \mathbf{F}(x)\,dx$ where $\mathbf{F}(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}$ is a vector function.
(a) Adapt d1_ip_fun(ab, quadrule, func) so that func can be a vector function.
(b) Test your program using $fun3(x) = \begin{bmatrix} \sin(\pi x) + (\cos(\pi x))^2 \\ x^{1/2} \end{bmatrix}$.

Turn in your adapted d1_ip_fun(ab, quadrule, func), your function $\mathbf{F}(x)$ ($\equiv fun3(x)$), and evidence that your code is working correctly.

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Part C.

No.1. (6 pts.) Of interest is $\int_a^b f_j(x)\, g_i(x) dx$, for $j = 1, 2, \ldots, N_j$, and $i = 1, 2, \ldots, N_i$, i.e.,

$$\int_a^b \mathbf{F}(x)\, \mathbf{G}(x)^T\, dx \quad \text{where} \quad \mathbf{F}(x) \;=\; \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_{N_j} \end{bmatrix}, \; \mathbf{G}(x) \;=\; \begin{bmatrix} g_1(x) \\ g_2(x) \\ \vdots \\ g_{N_i} \end{bmatrix}. \tag{0.2}$$

(a) Adapted d1_ip_fun(ab, quadrule, func) to d1_ip_fun(ab, quadrule, func1, func2) in order to compute (0.2).
(b) Test your program using fun3(x) (from Part B.) and $g_i(x) = x^{i-1}$, $i = 1, 2, 3$.

Turn in your d1_ip_fun(ab, quadrule, func1, func2), your function $\mathbf{G}(x)$, and evidence that your code is working correctly.
