1. Let X and Y be Banach spaces and $\{T_n\} \subset \mathcal{B}(X,Y)$. Show that the following are equivalent:

   (a) $\{\|T_n\|\}$ is bounded.

   (b) $\{\|T_n x\|\}$ is bounded for all $x \in X$.

   (c) $\{|\varphi(T_n x)|\}$ is bounded for all $x \in X$ and all $\varphi \in Y'$.

2. Let $y = \{y_n\} \subset \mathbb{C}$ be such that $\sum_{n=1}^{\infty} \overline{x_n} y_n$ converges for every $x = \{x_n\} \in c_0$, where $c_0 \subset l^\infty$ denotes the space of all (complex) sequences that converge to zero. Show that
$$\sum_{n=1}^{\infty} |y_n| < \infty.$$

3. Let X be a Banach space, Y be a normed linear space and $\{T_n\} \subset \mathcal{B}(X,Y)$. Suppose $\{T_n\}$ satisfies the property that for any $\{x_n\} \subset X$ with $\|x_n\| \to 0$, then $\|T_n(x_n)\| \to 0$. Show that
$$\sup_{n \in \mathbb{N}} \|T_n\| < \infty.$$

4. Let H be a Hilbert space and sequences $\{x_n\}, \{y_n\} \subset H$ such that $x_n \xrightarrow{w} x \in H$ and $y_n \to y \in H$, as $n \to \infty$. Show that
$$\langle x_n, y_n \rangle \to \langle x, y \rangle,\ n \to \infty.$$

5. A *weak Cauchy sequence* in a normed linear space X is a sequence $\{x_n\} \subset X$ such that for every $\varphi \in X'$ the sequence $\{\varphi(x_n)\}$ is Cauchy in $\mathbb{C}$. Show that a weak Cauchy sequence is bounded.

6. A normed linear space X is called *weakly complete* if each weak Cauchy sequence in X converges weakly in X. Show that if X is reflexive, then X is weakly complete.

7. Let X be Banach. Show that a sequence $\{\varphi_n\} \subset X'$ is weak* convergent if and only if

   (a) The sequence $\{\varphi_n\}$ is bounded.

   (b) The sequence $\{\varphi_n(x)\}$ is Cauchy in $\mathbb{C}$ for every x in a dense subspace A of X.

8. (*Extra credits*) Show that in l^1 strong and weak convergence are equivalent.