1. Let X, Y be normed linear spaces. If $T_1 : X \to Y$ is a closed linear operator and $T_2 \in \mathcal{B}(X, Y)$, show that $T_1 + T_2$ is a closed linear operator.

2. Let X, X_1, X_2 be Banach spaces and $T_1 : \mathcal{D}(T_1) \to X_1$, $T_2 : \mathcal{D}(T_2) \to X_2$ be closed linear operators with $\mathcal{D}(T_1) \subset \mathcal{D}(T_2) \subset X$. Show that there exists $M > 0$ such that
$$\|T_2 x\|_{X_2} \leqslant M \left(\|x\|_X + \|T_1 x\|_{X_1}\right), \ \forall x \in \mathcal{D}(T_1).$$

3. Let $A \subset C[0,1]$ be a closed subspace that only consists of $C^1[0,1]$ functions. Show that A is finite dimensional.

4. (a) Let $T : C[0,1] \to C[0,1]$ be defined by $Tx = vx$, where $v \in C[0,1]$ is fixed. Find the spectrum $\sigma(T)$.
   (b) Find a linear operator $T : C[0,1] \to C[0,1]$ whose spectrum $\sigma(T)$ is a given interval $[a,b]$, where $a, b \in \mathbb{R}$.

5. Let T be the left shift operator on l^p sequences. Namely, $T : l^p \to l^p$ is defined as
$$T\left(\{x_1, x_2, x_3, \cdots\}\right) = \{x_2, x_3, x_4, \cdots\}, \ \forall x = \{x_1, x_2, x_3, \cdots\} \in l^p, \ 1 \leqslant p \leqslant \infty.$$
   (a) For $p = \infty$. If $|\lambda| > 1$ show that $\lambda \in \rho(T)$; and if $|\lambda| \leqslant 1$, show that λ is an eigenvalue and find the corresponding eigenspace.
   (b) For $1 \leqslant p < \infty$. If $|\lambda| = 1$, is λ an eigenvalue of T?

6. Let $T : l^2 \to l^2$ be the right shift operator, namely,
$$T(x) = \{0, x_1, x_2, \cdots\}, \ \forall x = \{x_1, x_2, \cdots\} \in l^2.$$
Find $\sigma_p(T)$, $\sigma_c(T)$, and $\sigma_r(T)$.

7. (*Extra credits*) Show that the Closed Graph Theorem implies the Open Mapping Theorem.