Math 8260 Partial Differential Equations HW2

Assigned on Jan 30th, due on Feb 10th by the beginning of the lecture

1. (20 points) (a) Find the solution of the initial boundary value problem for the one dimensional wave equation with homogeneous Neumann boundary condition

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 & 0 < x < L, t > 0 \\ u(x,0) = g(x),\ u_t(x,0) = h(x) & 0 < x < L \\ u_x(0,t) = u_x(L,t) = 0 & t > 0. \end{cases}$$

   Please include the details as we did for the homogeneous Dirichlet boundary condition during the lecture.

   (b) Consider the initial boundary value problem

$$\begin{cases} u_{tt} - u_{xx} = 0 & 0 < x < \pi, t > 0 \\ u(x,0) = x^2,\ u_t(x,0) = 0 & 0 < x < \pi \\ u_x(0,t) = u_x(\pi,t) = 0 & t > 0. \end{cases}$$

   Find a Fourier series solution.

2. (20 points) Find the solution of the initial initial boundary value problem for the one dimensional homogeneous wave equation with nonhomogeneous Dirichlet boundary condition

$$\begin{cases} u_{tt} - u_{xx} = 0 & 0 < x < \pi, t > 0 \\ u(x,0) = 0,\ u_t(x,0) = 0 & 0 < x < \pi \\ u(0,t) = u(\pi,t) = 2t^2 - 3 & t > 0. \end{cases}$$

   (Hint: As we did during the lecture, first convert it into a nonhomogeneous wave equation with homogeneous Dirichlet boundary condition. Then you can decompose the problem into a homogeneous equation with homogeneous Dirichelet boundary condition, for which we already have the solution from the lecture, and another part of a nonhomogeneous equation with zero initial condition, for which you can use Duhamel's principle. When you solve the "w" part in Duhamel's principle, you will find that w satisfies a homogeneous equation with homogeneous Dirichelet boundary condition, for which we again have solution.)

3. (20 points) Find the solution of the following initial value problems. You can use the Kirchhoff's formula directly. (Hint: you can use the spherical coordinator. You should be able to compute the integral explicitly.)

   (a)

$$\begin{cases} u_{tt} - \Delta u = 0 & x = (x_1, x_2, x_3) \in \mathbb{R}^3, t > 0 \\ u(x,0) = 0,\ u_t(x,0) = x_1^2 + x_2^2 + x_3^2 & x = (x_1, x_2, x_3) \in \mathbb{R}^3 \end{cases}$$

   (b)

$$\begin{cases} u_{tt} - \Delta u = 0 & x = (x_1, x_2, x_3) \in \mathbb{R}^3, t > 0 \\ u(x,0) = x_1^2 + x_2^2,\ u_t(x,0) = 0 & x = (x_1, x_2, x_3) \in \mathbb{R}^3 \end{cases}$$