Recitation 2-7

Recursion tree method

Factorial :

```
f(n) {
    if f(n==1) return 1; ← Base condition
    return n*f(n-1); ← recursion step
}
```

$f(5)$

$5 \times f(4)$

$4 \times f(3)$

$3 \times f(2)$

$2 \times f(1)$

$T(n) = T(n-1) + 1$

$T(n-1) \to 1$

$\vdots$

$1 \to 1$

$T(n) = O(n)$

Example) $T(n) = 2T(n-1) + 1$

$T(n) = O(2^n - 1)$
$= O(2^n)$

$T(n)$ — 1

$T(n-1)$ $T(n-1)$ — 2

$\vdots$ n

$T(1) \cdot 2^n$

$1 + 2 + 4 + 8$

$2^0 + 2^1 + 2^2 + 2^3 \ldots 2^n$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(2^n - 1)}{2 - 1}$

$2^n - 1$

Recursion tree method

$T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + n$

?? Asymmetric tree

$T(n) + n$

$T\left(\frac{n}{3}\right)$ $T\left(\frac{2n}{3}\right)$ $+ n$

$T\left(\frac{n}{3^2}\right)$ $T\left(\frac{2n}{3^2}\right)$ $T\left(\frac{2n}{3^2}\right)$ $T\left(\frac{2^2 n}{3^2}\right)$ $+ n$

$T(1)$ $T(1)$ $T(1)$ $T(1)$

$n, \frac{2n}{3}, \frac{4n}{9}, \frac{8n}{27}, \frac{2^k n}{3^k}$

$n = \left(\frac{3}{2}\right)^k = \log_{\frac{3}{2}} n = \log_{\frac{3}{2}} \left(\frac{3}{2}\right)^k = k$

$n \times \log_{\frac{3}{2}} n = O(n \log_{\frac{3}{2}} n)$

Master Theorem: $T(n) = aT(\frac{n}{b}) + f(n)$

Case 1:

If there exists a constant $\epsilon > 0$

Such that $f(n) = O(n^{\log_b a - \epsilon})$

Then, $T(n) = \Theta(n^{\log_b a})$

e.g. $T(n) = 9T(\frac{n}{3}) + n$

$a = 9$

$b = 3$ $\quad n^{\log_b a} = n^{\log_3 9} = n^2$

$f(n) = n$ $\quad O(n^{2-\epsilon})$ $\quad 0 \le \epsilon \le 1$

$T(n) = \Theta(n^2)$

Case 2:

If there exists constant $k \geq 0$

Such that $f(n) = \Theta(n^{\log_b a} \log^k n)$, then

$T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$

$T(n) = T(\frac{2n}{3}) + 1$

$a = 1$ $\quad g(n) = 1$

$b = \frac{3}{2}$