SI 2/5

$f(n)$ is in $\omega(g(n))$ if and only if $g(n)$ is in $o(f(n))$

1) $8n^2 - 13n + 89 \in O(n^2)$

$8n^2 + 13n^2 + 89n^2 \leq cn^2$

$21n^2 + 89n^2$

$110n^2 \leq cn^2$

$110 = c \qquad n_0 = 1$

2) $2n^3 + 3n \in \omega(n)$

$2n^3 + 3n^3 > c \cdot n$

$5n^3 > cn$

$5n^2 > c$

$n_0 = 1 \qquad c = 4$

3) $2n^2 + 3n \in O(n^2)$ By contradiction,

$2n^2 + 3n^2 \leq c \cdot n^2$

$5n^2 \leq c \cdot n^2$

$c = 1 \Rightarrow$ False By proof by contradiction, $2n^2 + 3n \notin O(n^2)$

4) $4\log n \in o(n \log n)$

$4 \log n < c \cdot n \log n$

$4 < c \cdot n$

$n_0 = 1$

5) $2\log n + n \in \Omega(\log n)$

$2\log n + n \geq c \cdot \log n$

$n \geq c \cdot \log n - 2\log n$

$c = 2 \quad n_0 = 1$

we know this because $2\log(n) \geq 1 + \log(n) \;\; \forall n \geq 1$

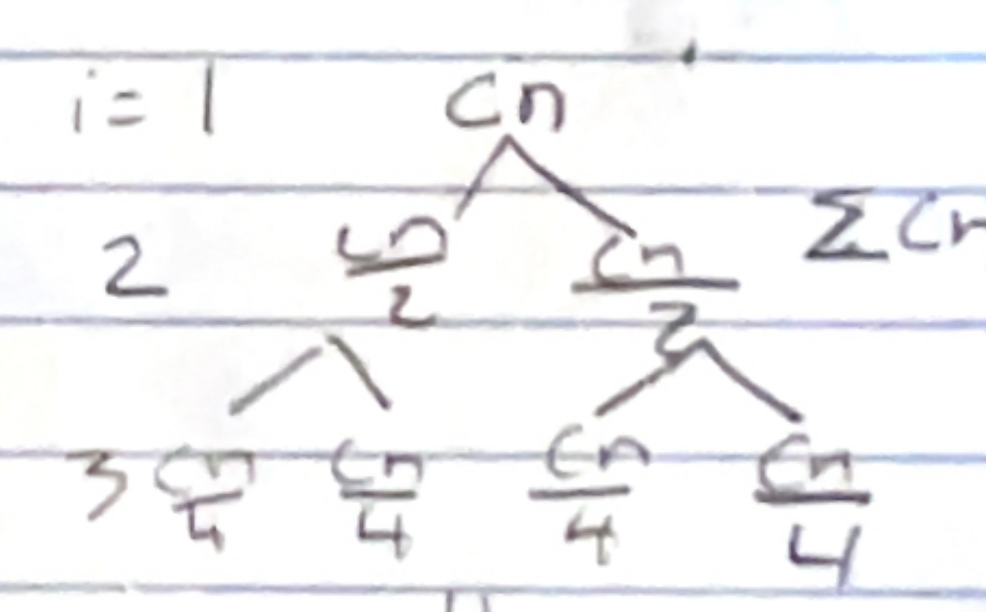
and $n \geq \log n \;\; \forall n \geq 1$, Thus $2\log n + n \geq \max(2\log n$

Prove that the run time for $T(n) = 2T(n/2) + cn$ where $T(2) = c$ is $O(n \log n)$

$T(n) = 2T(n/2) + cn$; $T(2) = c$

$T(2) = c$

$T(3) = \quad T(n) = T(\frac{n}{2}) + cn$



$cn(1+1+1+\ldots+1) \quad 2^{k} \quad 2^{k-\log} \quad 2^{k-1} = cn \log n$

Proof

Base: w/ $n_0 > 0$, $\log(n) > 0$ implying $n\log(n) > 0$ since $T(2)$ is constant

Base: $T(2) = c * n\log(n)$ and since $\log(n) > 0$ and $n > 0$,

$c \leq cn\log(n)$ thus the base case holds.

Induction Assume that this holds for all values less than n, then we need to prove that

$T(n) \leq c\log(n)$ for all $n > n_0$

$$T(n) \leq 2\left(c\left(\tfrac{n}{2}\right)\cdot\log\left(\tfrac{n}{2}\right)\right) + c$$
$$\leq cn\log\left(\tfrac{n}{2}\right) + c = cn\log(n) - cn\log(2) + c$$
$$\leq cn\log(n) \text{; QED}$$