Recitation 3

1) $2n^3 + 35n - 46 \in O(n^3)$

$2n^3 + 35n - 46 \leq 2n^3 + 35n - 0 = 2n^3 + 34n$

$\leq 2n^3 + 35n^2 \leq 2n^3 + 35n^3$

$c = 37$ $\quad cg(n)$

$n_0 = 1$ $\quad g(n) = n^3$

Formal definition of Big O

$0 \leq f(n) \leq c \cdot g(n)$

positive constant

For some 'c' $\forall n \geq n_0$

$f(n) \leq c \cdot O(g(n))$

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2) $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_n$

$f(x) = O(x^n)$

$|a+b| \leq |a| + |b|$

$|f(x)| = |a_n x^n + a_{n-1} x^{n-1} + \dots + a_0|$

$\leq |a_n x^n| + |a_{n-1} x^{n-1}| + \dots + |a_0|$

so we're solving for 'c' in g(x)' $f(x) = c \cdot g$

$= |a_n| x^n + |a_{n-1}| x^{n-1} + \dots + |a_0|$

$= x^n \left(|a_n| + \frac{|a_{n-1}|}{x} + \dots + \frac{|a_0|}{x^n} \right)$

$\leq x^n \left(|a_n| + |a_{n-1}| + \dots |a_0| \right)$

$f(x) = cx^n$

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3) $f(n) = n!$

To prove $\rightarrow O(n^n)$

$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \dots 1 \leq n^n$

$= 1 \cdot 2 \cdot 3 \cdot 4 \dots n$

$\leq n \cdot n \cdot n \cdot n \dots n$

$= n^n$

$n! = O(n^n)$

$f_1(n) \in O(g(n))$
$f_2(n) \in O(g_2(n))$
To prove,
$f_1(n) + f_2(n) \in O(g_1(n) + g_2(n))$

$f_1(n) \leq c \cdot g_1(n)$
$f_2(n) \leq c \cdot g_2(n)$
$f_1(n) + f_2(n) \leq c \cdot g_1(n) + c \cdot g_2(n)$
$f_1(n) + f_2(n) \leq c \cdot (g_1(n) + g_2(n))$
$f_1(n) + f_2(n) \in O(g_1(n) + g_2(n))$