Final Review
COSC 320: Advanced Data Structures and Algorithm Analysis

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1. All homeworks, labs, projects, reviews, exams, and lecture notes.
2. Consider the problem of, given a graph $G = (V, E)$, finding the shortest path between $u, v \in V$ such that $u \neq v$. Argue that an optimal solution to this problem exhibits the *optimal substructure*: for a vertex w along the shortest path $u \rightsquigarrow v$, the subpaths $u \rightsquigarrow w$ and $w \rightsquigarrow v$ are both as short as possible. Does this property also hold for finding a longest path $u \rightsquigarrow v$? Prove why, or give an example why not.
3. Explain why the running time of DFS and BFS are expressed as $O(|V| + |E|)$, rather than in terms of only $n = |V|$. Give examples of different graphs which would demonstrate the various reasons for the specificity.
4. Give an example of a connected, undirected, graph with eight vertices.
   (a) Demonstrate the execution of BFS on the graph, illustrating the distances, parents, colors, as well as the resulting BFS tree.
   (b) Demonstrate the execution of DFS on the graph, illustrating discovery time, finish time, colors, parents, and the resulting DFS tree.
5. Give an example of a connected, directed, and acyclic graph on eight vertices. Show a valid topological ordering of the vertices, and state why the graph must be acyclic for this to be computed.
6. Give an example of a connected, directed, and cyclic graph on eight vertices. Show the execution of the Strongly-Connected-Components algorithm on the graph.
7. Show how to determine how to determine if a directed graph G contains a *universal sink* – a vertex with in-degree $|V| - 1$ and out-degree 0 – in time $O(|V|)$ given then adjacency matrix of G. (**Hint:** try some examples that do and do not have a universal sink. What patterns form in the matrix?)
8. Show that one can determine whether an undirected graph contains a cycle using time at most $O(|V|)$, independent of $|E|$.