

COSC 320 - Advanced Data Structures and Algorithm Analysis

Homework 3

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Due: 1 November 2019

1. In the Hire-Assistant algorithm, assuming all n candidates are presented in a uniformly random order (i.e. all orders are equally likely), what is the probability that you hire exactly one time? What is the probability that you hire exactly n times?
2. Compute the expected value of the sum of n dice.
3. Use indicator random variables to solve the following problem, which is known as the hat-check problem. Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?
4. During the execution of Randomized-Quicksort, how many calls are made to the pseudo-random number generator in the worst case? In the best case? Give your answers with Θ -notation, and provide justification.
5. Give a non-recursive algorithm that performs an in-order binary tree walk. (Hint: try using a stack as an auxiliary data structure).
6. Write an algorithm to find the *predecessor* of a node in a binary search tree.
7. An algorithm to sort a given array of integers is to repeatedly call Tree-Insert and insert them all into a binary tree, then return an array given by the output of an in-order walk on that tree. What is the running time of this sorting algorithm in the worst and best case?
8. Is the operation of deletion “commutative” in the sense that deleting x and then y from a binary search tree leaves the same tree as deleting y and then x ? Argue why it is or give a counterexample.
9. What is the largest possible number of internal nodes in a red-black tree whose root has black-height k ? What is the smallest possible number?
10. Describe a red-black tree on n keys that realizes the largest possible ratio of red internal nodes to black internal nodes. What is this ratio? What tree has the smallest possible ratio, and what is the ratio?