Togo, Africa: On track for over 3 °C of warming by 2100

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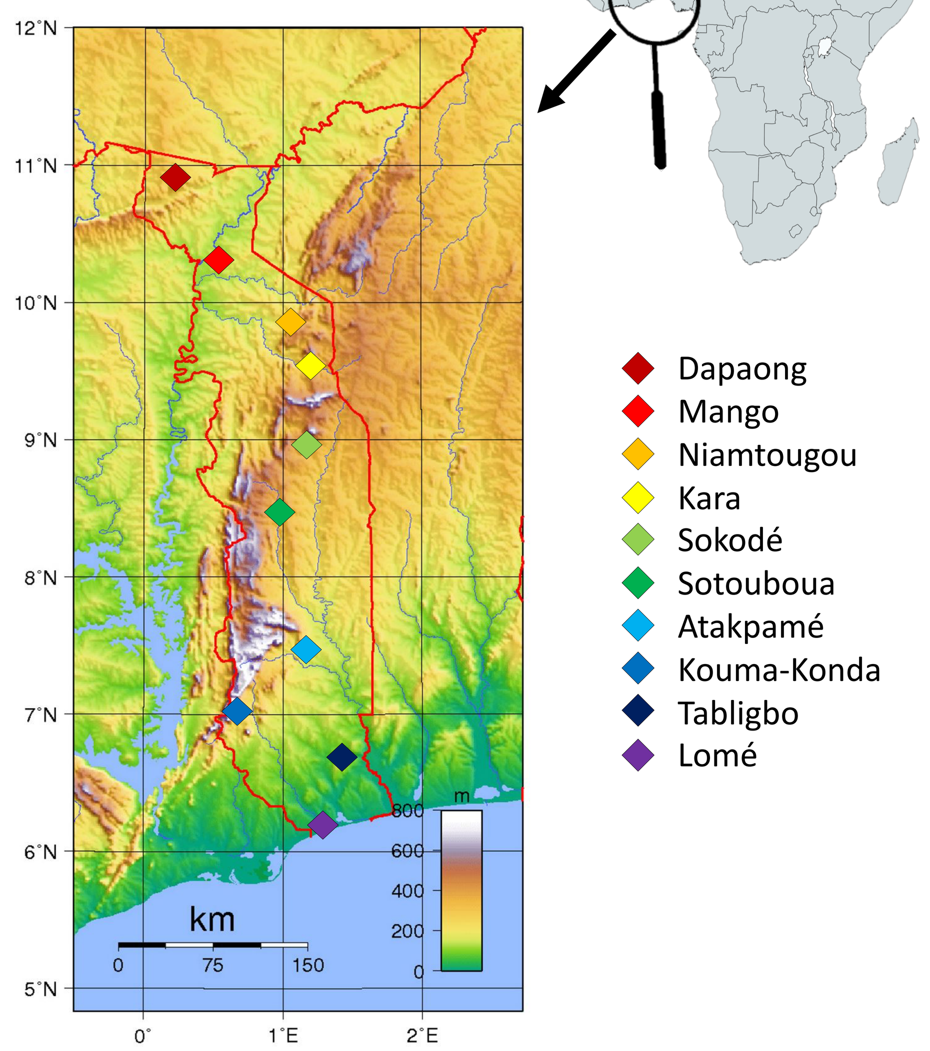
Abstract

The West African nation of Togo has experienced rapidly rising temperatures over the course of the past six decades. Continually rising temperatures have disastrous implications for the agriculture, public health, and economy of Togo. We have constructed a regression model that captures the annual increase in temperature for all of Togo, based on monthly temperature data from ten different cities in the various geographic regions of Togo over the course of nearly 60 years. Singular Spectrum Analysis (SSA), an extension of the Singular Value Decomposition (SVD), is used to identify and thus incorporate seasonal fluctuations that align with known climatological phenomena into the model, permitting us to isolate the underlying increasing trend in temperature - a trend that suggests over 3 °C of continued warming from the start of the industrial age to the year 2100.

Introduction

The Togolese Institute de Meteorologié provided data including average monthly minimum and maximum temperatures (°C) of ten representative cities in Togo going back as far as January 1960. The country has a diverse geography characterized by the Togo Mountains, the Sahara desert to the north, and the Gulf of Guinea to the south. For this reason, different regions in Togo can experience very distinct climate and weather patterns. The cities observed are considered to be representative of all of Togo.



Singular Spectrum Analysis

Decomposition

A time series Y_t is embedded into an L – trajectory matrix where L represents the window length. The columns of this matrix are formed by the lagged time-series of length N.

$$A = \begin{bmatrix} y_1 & y_2 & \cdots & y_K \\ y_2 & y_3 & \cdots & y_{K+1} \\ y_3 & y_4 & \cdots & y_{K+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_L & y_{L+1} & \cdots & y_N \end{bmatrix} = \underset{L\,x\,L}{U} \; \underset{L\,x\,K}{\Sigma} \; \underset{K\,x\,K}{V^T} = \begin{bmatrix} u_{11} & u_{21} & \cdots & u_{L1} \\ u_{12} & u_{22} & \cdots & u_{L2} \\ u_{13} & u_{23} & \cdots & u_{L3} \\ \vdots & \vdots & \ddots & \vdots \\ u_{1L} & u_{2L} & \cdots & u_{LL} \end{bmatrix} \begin{bmatrix} \sigma_1 & & & 0 & \\ & \sigma_2 & & & 0 \\ & & \ddots & & \\ 0 & & & \sigma_r & \\ & 0 & & & 0 \end{bmatrix} \begin{bmatrix} v_{11} & v_{21} & \cdots & v_{K1} \\ v_{12} & v_{22} & \cdots & v_{K2} \\ v_{13} & v_{23} & \cdots & v_{K3} \\ \vdots & \vdots & \ddots & \vdots \\ v_{1K} & v_{2K} & \cdots & v_{KK} \end{bmatrix}^T$$

Using SVD, A is rewritten as the product of three matrices U, Σ, *and* V^T. Σ is a diagonal matrix composed of the descending singular values of A, which are the square roots of the eigenvalues of A^TA. The corresponding eigenvectors form the columns of V (right singular vectors of A). The columns of U (left singular vectors) form an orthogonal set of decreasingly important patterns which reproduce the columns of A.

Reconstruction

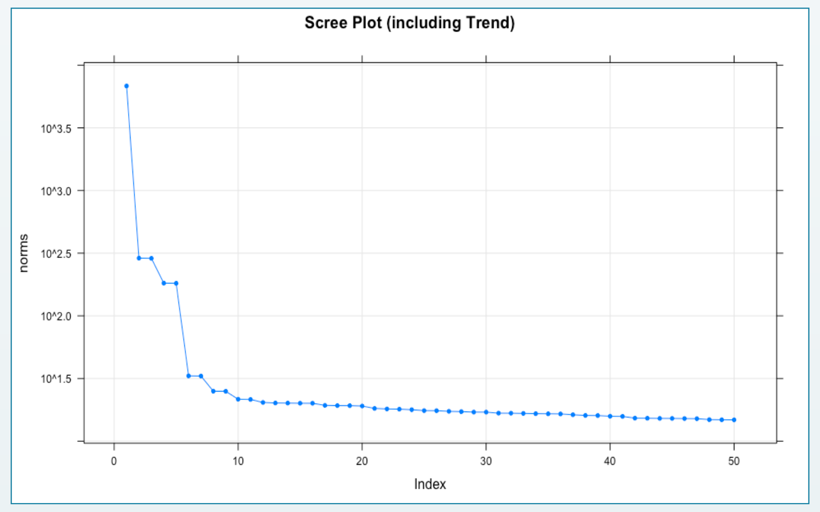
A can now be rewritten as $\sum_{i=1}^{r} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T$, where $(\sigma_i, \boldsymbol{u}_i, \boldsymbol{v}_i)$ is known as the i^th eigentriple of the decomposition. Through a process called eigentriple grouping, triples with comparable σ_i's are combined. This is where the seasonal detection occurs. Diagonal averaging is then used to transform each grouped matrix into a new time series $\tilde{y}_t$, each of length N and corresponding to a seasonal trend with a distinct length.

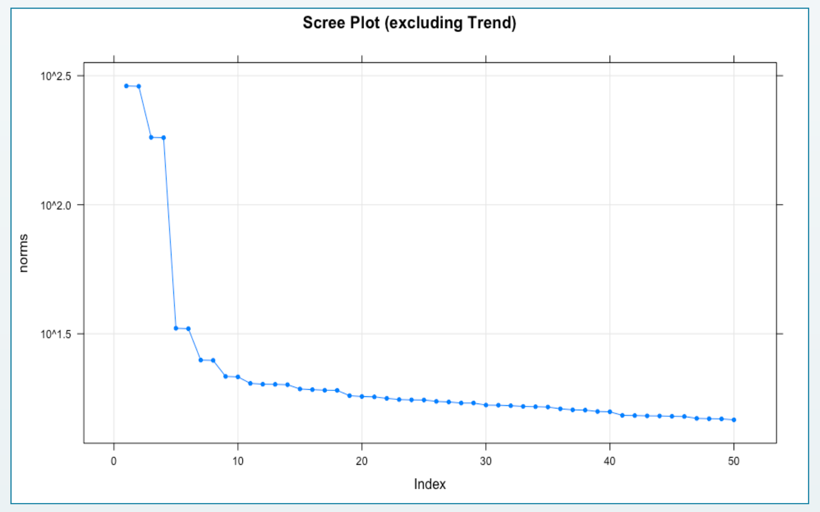
$$Y_t = \sum_{k=1}^{m} \tilde{y}_t^{(k)}$$

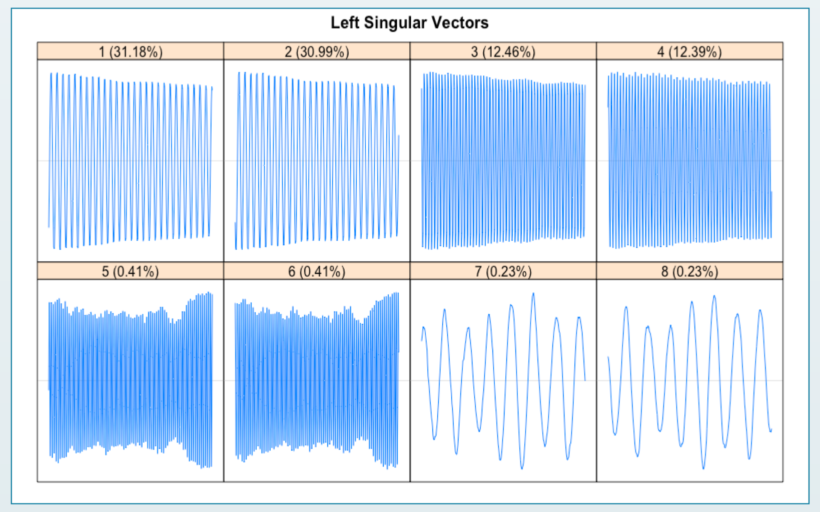
Results

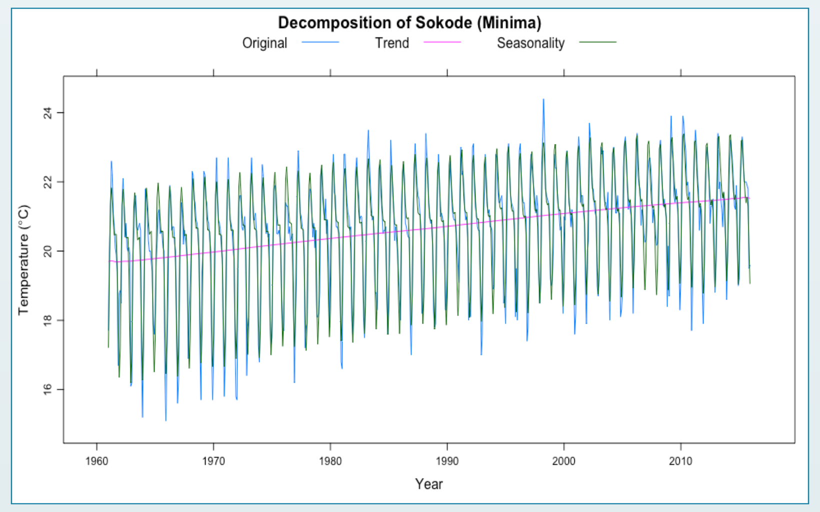
Case Study: Sokodé

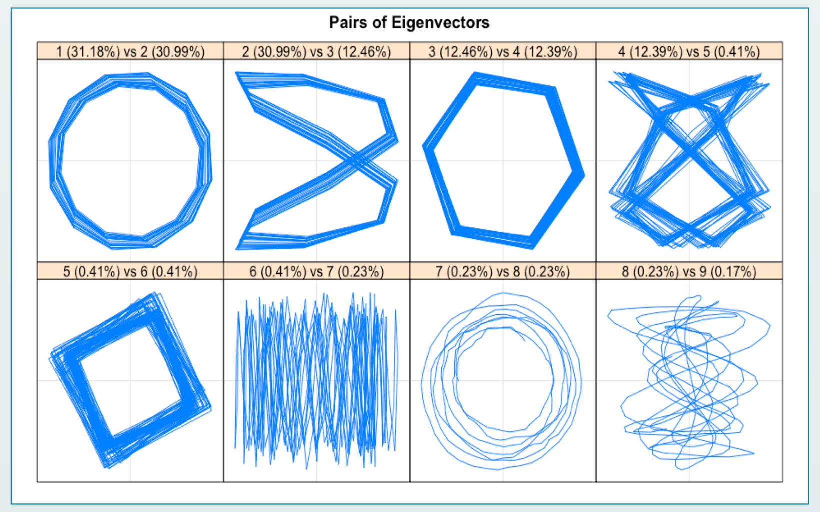
Minimum Temperatures



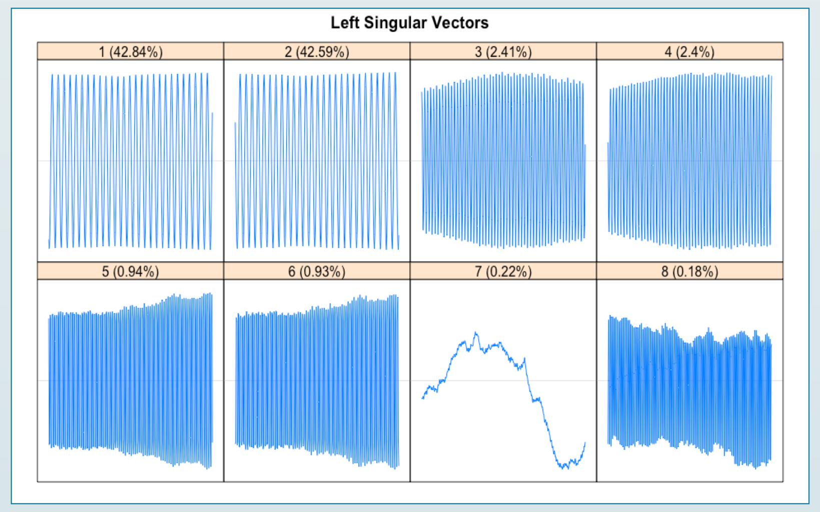


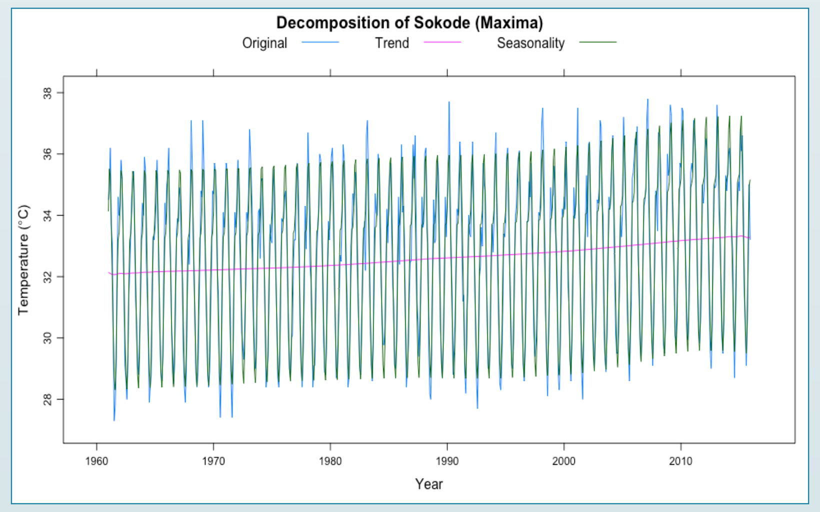


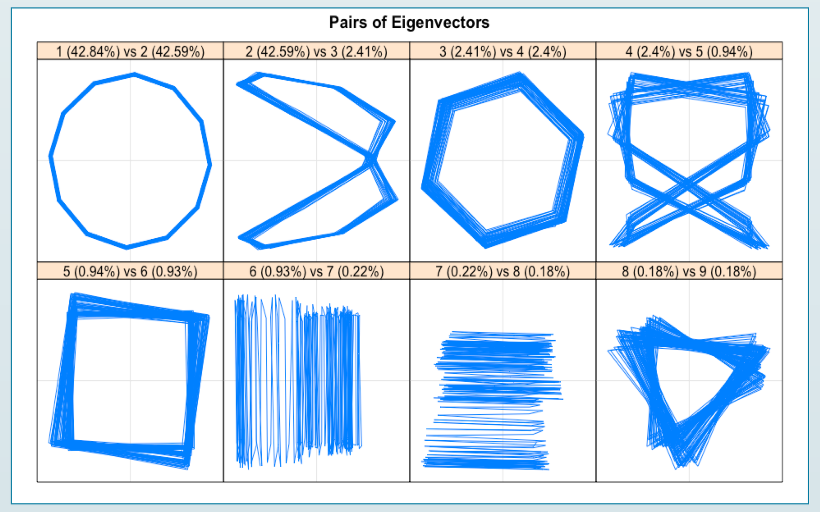




Maximum Temperatures







Global Models

	Minimum Estimate	Maximum Estimate	Known Period
Intercept	−21.531***	−31.558***	
Year	0.031***	0.018***	
3 Month Period (s)		−0.066**	
3 Month Period (c)		−0.195***	
4 Month Period (s)	−0.114***	−0.081***	
4 Month Period (c)	−0.228***	−0.391***	
6 Month Period (s)	−0.085***	−0.540***	
6 Month Period (c)	−0.891***	−0.658***	
12 Month Period (s)	1.061***	2.104***	
12 Month Period (c)	−0.759***	2.176***	
3.75 Year Period (s)	0.023		ENSO
3.75 Year Period (c)	0.124***		ENSO
4.67 Year Period (s)	−0.052**		ENSO
4.67 Year Period (c)	−0.088***		ENSO
9.83 Year Period (s)		0.047**	PDO
9.83 Year Period (c)		0.017	PDO
13 Year Period (s)		−0.123***	PDO
13 Year Period (c)		−0.006	PDO
21 Year Period (s)	0.027		IPO, HC
21 Year Period (c)	−0.090***		IPO, HC
25.42 Year Period (s)		0.022	IPO
25.42 Year Period (c)		0.043*	IPO
Longitude	15.133***	−11.063***	
Latitude	−6.104***	7.824***	
Elevation	−0.002***	−0.009***	
Latitude^2	0.411***	−0.447***	
Longitude^2	−3.688***	3.045***	
Latitude*Longitude	−0.891***	0.622***	

***p<0.001. **p<0.01. *p<0.10.
ENSO: El Niño Southern Oscillation.
PDO: Pacific Decadal Oscillation.
IPO: Interdecadal Pacific Oscillation.
HC: Hale Cycle.

Implications

- Our global regression models indicate annual warming of ~ 0.031°C in the minima (0.018°C in the maxima). Assuming this trend continues, we estimate that in the year 2100, the average minimum temperature will have increased by 2.635 °C and the average maximum temperature will have increased by 1.53 °C.
- Public health in Togo will continue to decline as higher temperatures have been linked with increases in heat-related illnesses such as dehydration, certain vector-borne diseases such as malaria, and respiratory illness due to higher levels of particulates in the air.
- The Togolese economy, heavily reliant upon rainfall-dependent agriculture, will have to adapt to new rainfall patterns and increasingly frequent droughts and floods.

Acknowledgements

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