Togo, Africa: On Track for over 3°C of Warming by 2100

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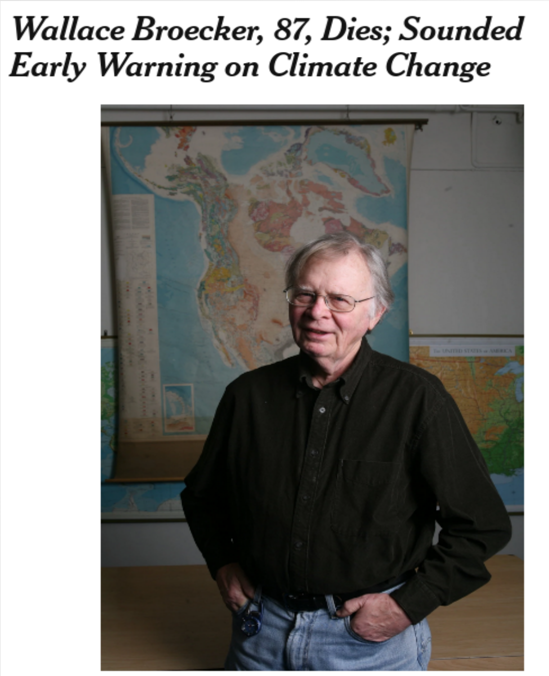
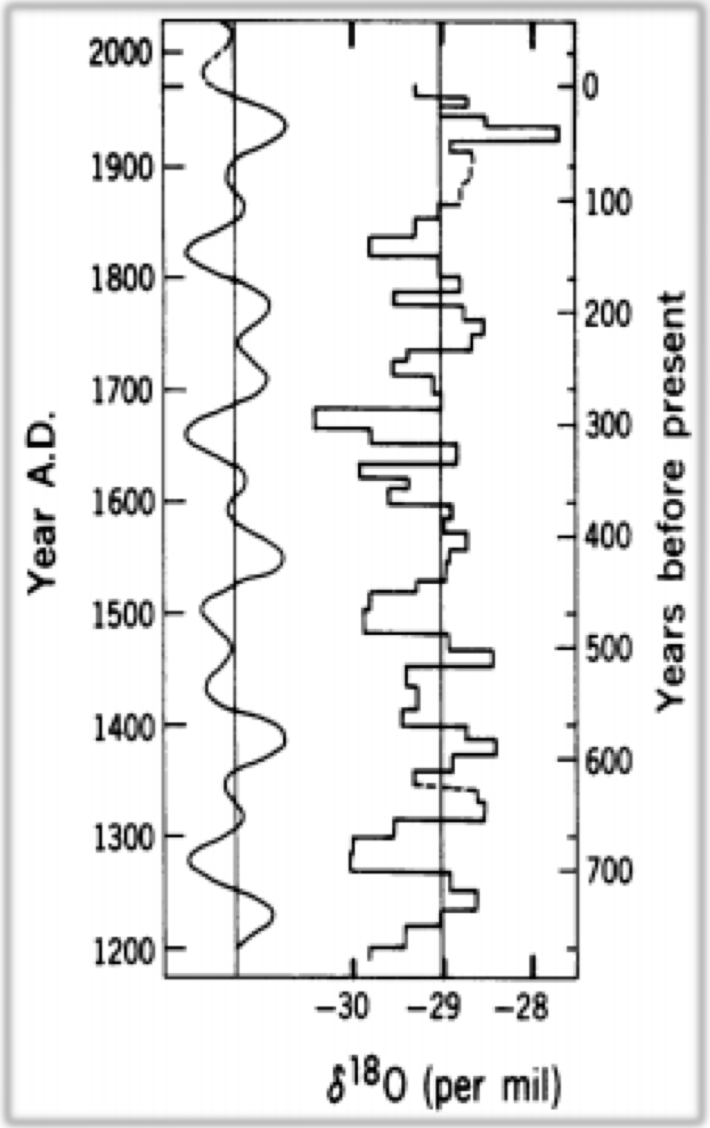
Dr. Wally Broecker died this year:



- In 1975 he wrote a paper with a provocative title: *Climatic Change: Are We on the Brink of a Pronounced Global Warming?* – due to the end of a natural cooling cycle, and the rapid escalation of fossil fuel use worldwide.

- Wally based his prediction on the curve at right, showing the superposition of an 80 year and180 year cycles on ice core data from Greenland (obtained "by power spectral techniques").

(Oxygen isotope ratio is a proxy for temperature.)



Togo Temperature Project Creation Story

- At the start of my 2018 math modeling course, I told my students that my brother-in-law works in the Togolese Meteorological Service, and could get us historical temperature data from Togo.
- I also said that I was pretty sure (like really, really sure) that the data would suggest increasing temperatures. Our job: study and model it: e.g.

$$
\begin{aligned}
&MaxTemp(year, longitude, latitude, elevation) = \\
&\qquad \text{temporalTrend}(year) + \\
&\qquad \sum_{i=1}^{nP} \left(a_i \sin\left(\frac{2\pi\ year}{P_i}\right) + b_i \cos\left(\frac{2\pi\ year}{P_i}\right)\right) + \\
&\qquad \text{geographical}(longitude, latitude, elevation)
\end{aligned}
$$

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Our focus: temporal trend, and periods (cycles), which we would hope to tie to climate cycles (e.g. El Nino).

Singular Spectrum Analysis (SSA)

- Period detection tool
- Implemented with the RSSA package in R
- Two steps:
  1. Decomposition – SVD with a special matrix *A*
  2. Reconstruction – Eigentriple grouping and diagonal averaging

The i^{th} eigentriple is denoted: $(\sigma_i, \boldsymbol{u}_i, \boldsymbol{v}_i)$

$$A = \begin{bmatrix} y_1 & y_2 & \cdots & y_K \\ y_2 & y_3 & \cdots & y_{K+1} \\ y_3 & y_4 & \cdots & y_{K+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_L & y_{L+1} & \cdots & y_N \end{bmatrix} = \underset{L \times L}{U} \; \underset{L \times K}{\Sigma} \; \underset{K \times K}{V^T} = \begin{bmatrix} u_{11} & u_{21} & \cdots & u_{L1} \\ u_{12} & u_{22} & \cdots & u_{L2} \\ u_{13} & u_{23} & \cdots & u_{L3} \\ \vdots & \vdots & \ddots & \vdots \\ u_{1L} & u_{2L} & \cdots & u_{LL} \end{bmatrix} \begin{bmatrix} \sigma_1 & & & 0 & \\ & \sigma_2 & & & 0 \\ & & \ddots & & \\ 0 & & & \sigma_r & \\ & 0 & & & 0 \end{bmatrix} \begin{bmatrix} v_{11} & v_{21} & \cdots & v_{K1} \\ v_{12} & v_{22} & \cdots & v_{K2} \\ v_{13} & v_{23} & \cdots & v_{K3} \\ \vdots & \vdots & \ddots & \vdots \\ v_{1K} & v_{2K} & \cdots & v_{KK} \end{bmatrix}^T$$

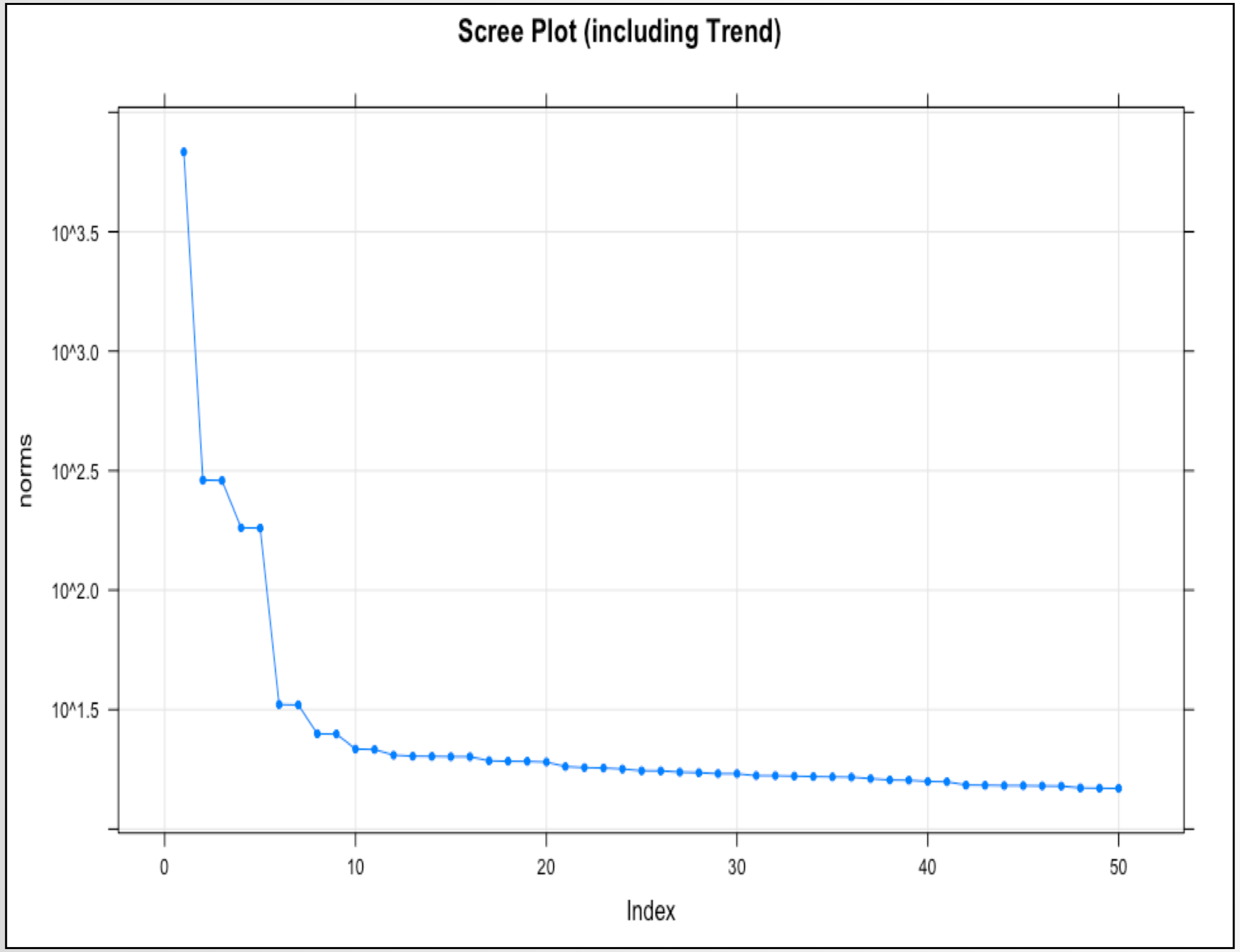
Case Study: Sokodé (Minima)

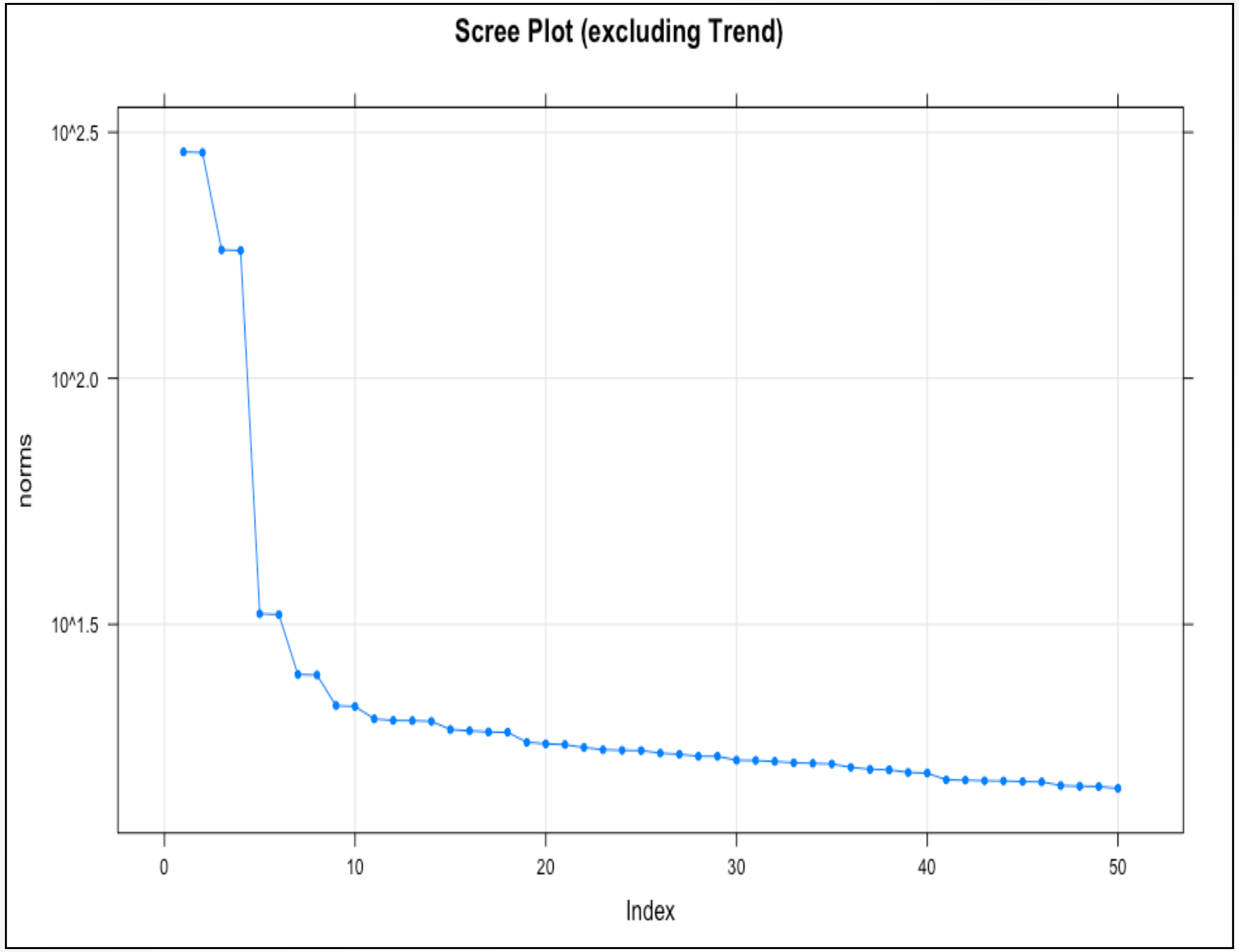
$$A = \begin{bmatrix} T_{1/1961} & T_{2/1961} & \cdots & T_{7/1988} \\ T_{2/1961} & T_{3/1961} & \cdots & T_{8/1988} \\ T_{3/1961} & T_{4/1961} & \cdots & T_{9/1988} \\ \vdots & \vdots & \ddots & \vdots \\ T_{6/1988} & T_{7/1988} & \cdots & T_{12/2015} \end{bmatrix}$$

1. Apply SVD
2. Group eigentriples with similar singular values
3. Average the grouped matrices to obtain the new time series

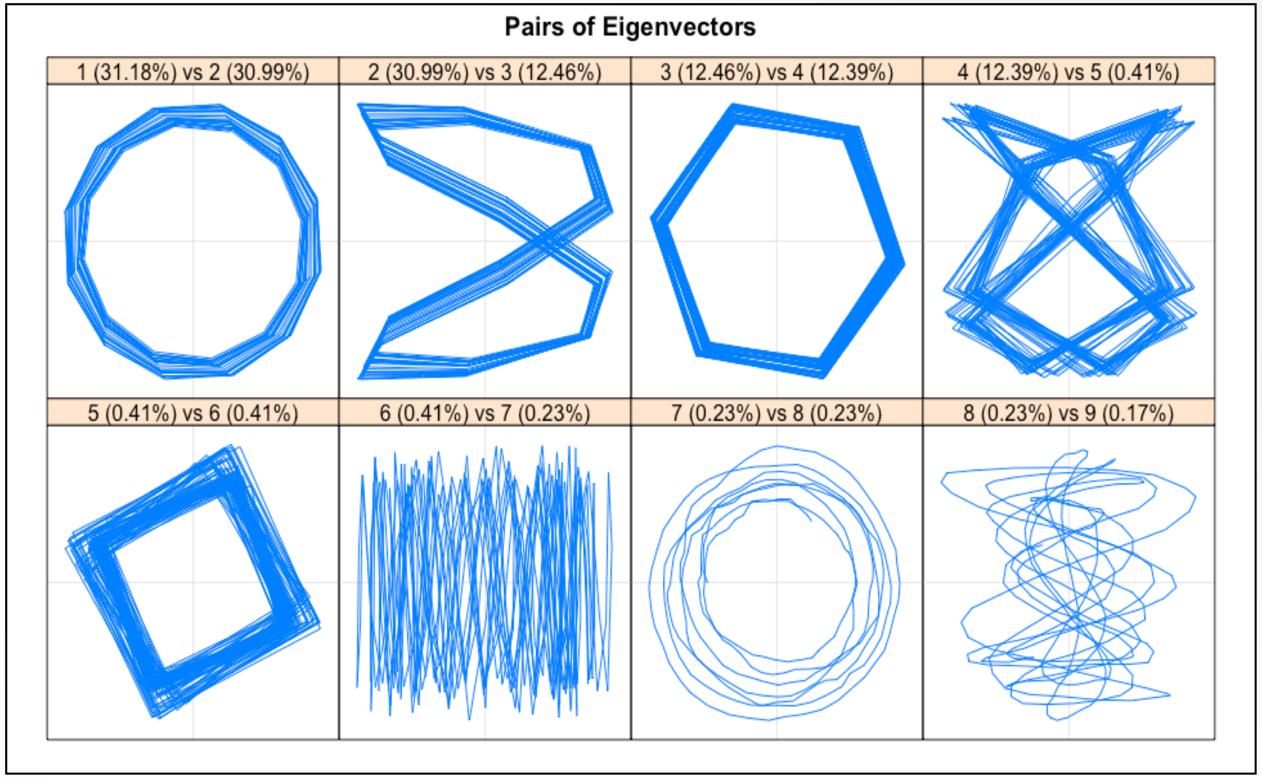
$$Y_t = \sum_{k=1}^{m} \tilde{y}_t^{(k)}$$

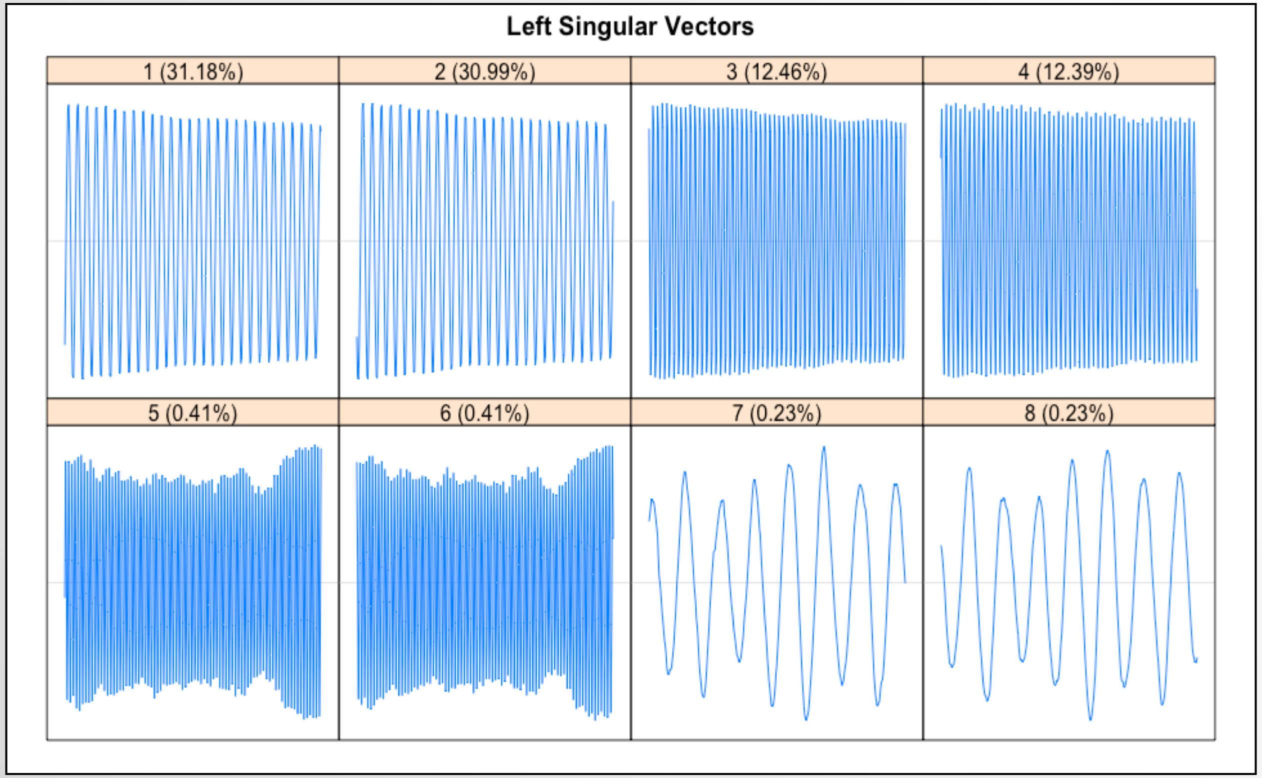
Case Study: Sokodé





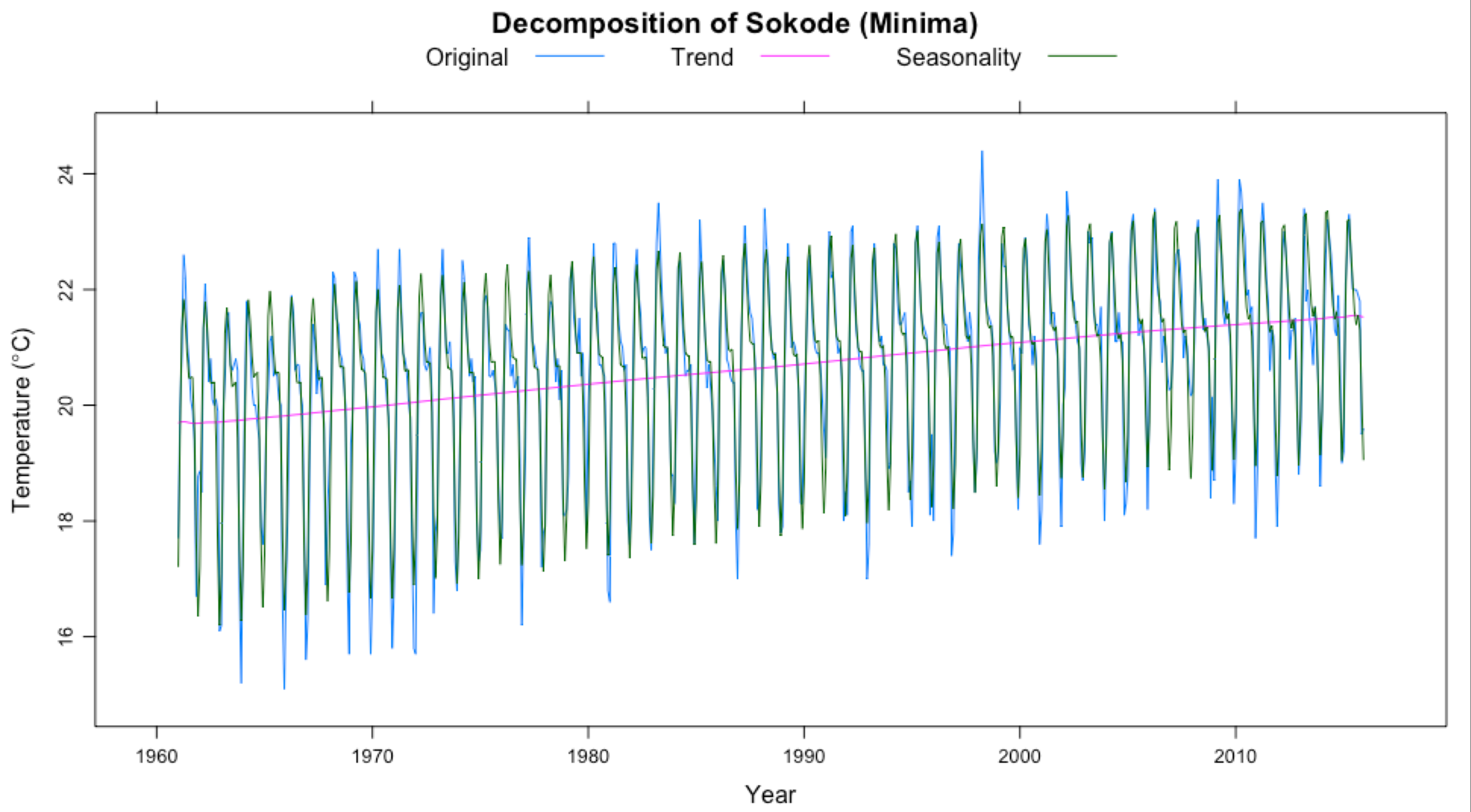
Period Detection in Detrended Sokodé Minima





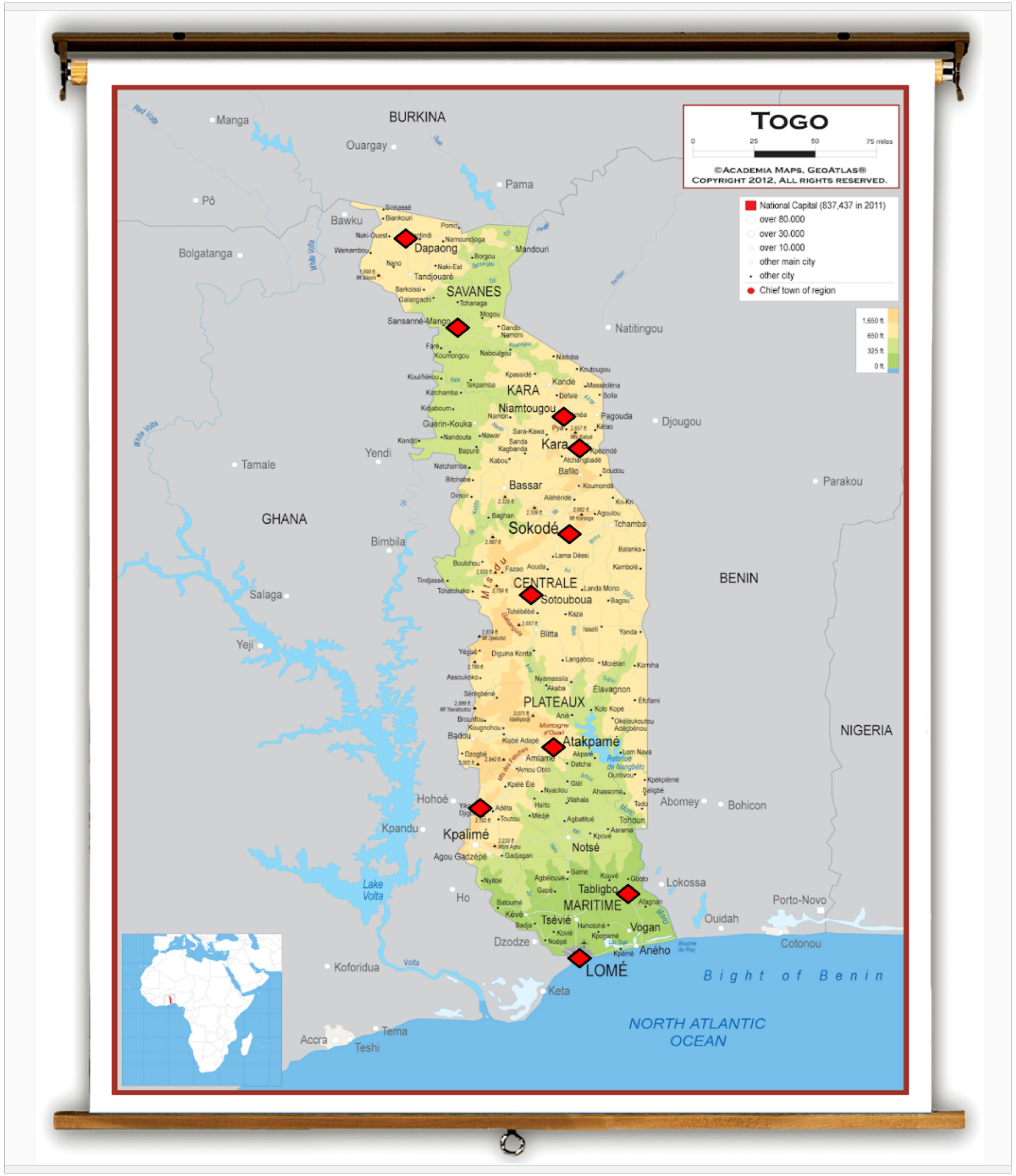
Periods Detected:

- 12 month
- 6 month
- 4 month
- ~9 year (see Pacific Decadal Oscillation)


Decomposition of Sokode (Minima)
Original
Trend
Seasonality
Temperature (°C)
24
22
20
18
16
1960
1970
1980
1990
2000
2010
Year

From North to South:

1. Dapaong
2. Mango
3. Niamtougou
4. Kara
5. Sokodé
6. Sotouboua
7. Atakpamé
8. Kouma-Konda
9. Tabligbo
10. Lomé



Summary of Modeling

- SSA repeated 20x (10 cities x 2 temperature sets)
- Periods detected implemented into a pair of global regression models

Linearization of Cosine:

$$C + A\cos(ax + b)$$

$$C + A\left[\cos(ax)\cos(b) - \sin(ax)\sin(b)\right]$$

$$C + \alpha\cos(ax) + \beta\sin(ax)$$

	Minimum Estimate	Maximum Estimate	Known Period
Intercept	-21.531***	-31.558***	
Year	0.031***	0.018***	
3 Month Period (s)		-0.066**	
3 Month Period (c)		-0.195***	
4 Month Period (s)	-0.114***	-0.081***	
4 Month Period (c)	-0.228***	-0.391***	
6 Month Period (s)	-0.085***	-0.540***	
6 Month Period (c)	-0.891***	-0.658***	
12 Month Period (s)	1.061***	2.104***	
12 Month Period (c)	-0.759***	2.176***	
3.75 Year Period (s)	0.023		ENSO
3.75 Year Period (c)	0.124***		ENSO
4.67 Year Period (s)	-0.052**		ENSO
4.67 Year Period (c)	-0.088***		ENSO
9.83 Year Period (s)		0.047**	PDO
9.83 Year Period (c)		0.017	PDO
13 Year Period (s)		-0.123***	PDO
13 Year Period (c)		-0.006	PDO
21 Year Period (s)	0.027		IPO, HC
21 Year Period (c)	-0.090***		IPO, HC
25.42 Year Period (s)		0.022	IPO
25.42 Year Period (c)		0.043*	IPO
Longitude	15.133***	-11.063***	
Latitude	-6.104***	7.824***	
Elevation	-0.002***	-0.009***	
Latitude^2	0.411***	-0.447***	
Longitude^2	-3.688***	3.045***	
Latitude*Longitude	-0.891***	0.622***	

***$p<0.001$. **$p<0.01$. *$p<0.10$.
ENSO: El Niño Southern Oscillation.
PDO: Pacific Decadal Oscillation.
IPO: Interdecadal Pacific Oscillation.
HC: Hale Cycle.

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Wally Broecker, in 2017: "it was dumb luck..."

– "As other records including those for ice cores from southern and central Greenland appeared, none showed Dansgaard's 180- and 80-year cycles."

– "I place [the NAO index] side by side with Dansgaard's Greenland ^{18}O record and that for the NAO index. While by no means identical to the Dansgaard record, the NAO record has humps and valleys of similar duration."

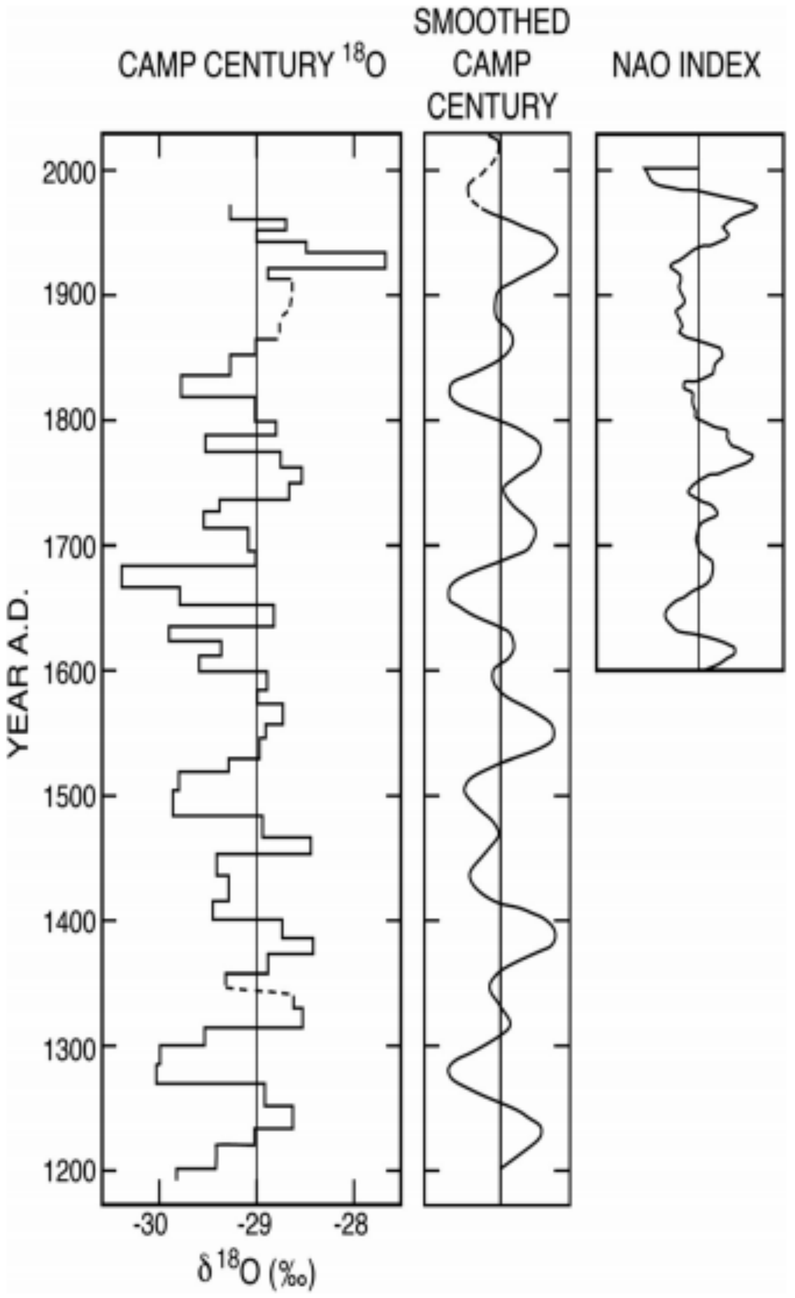


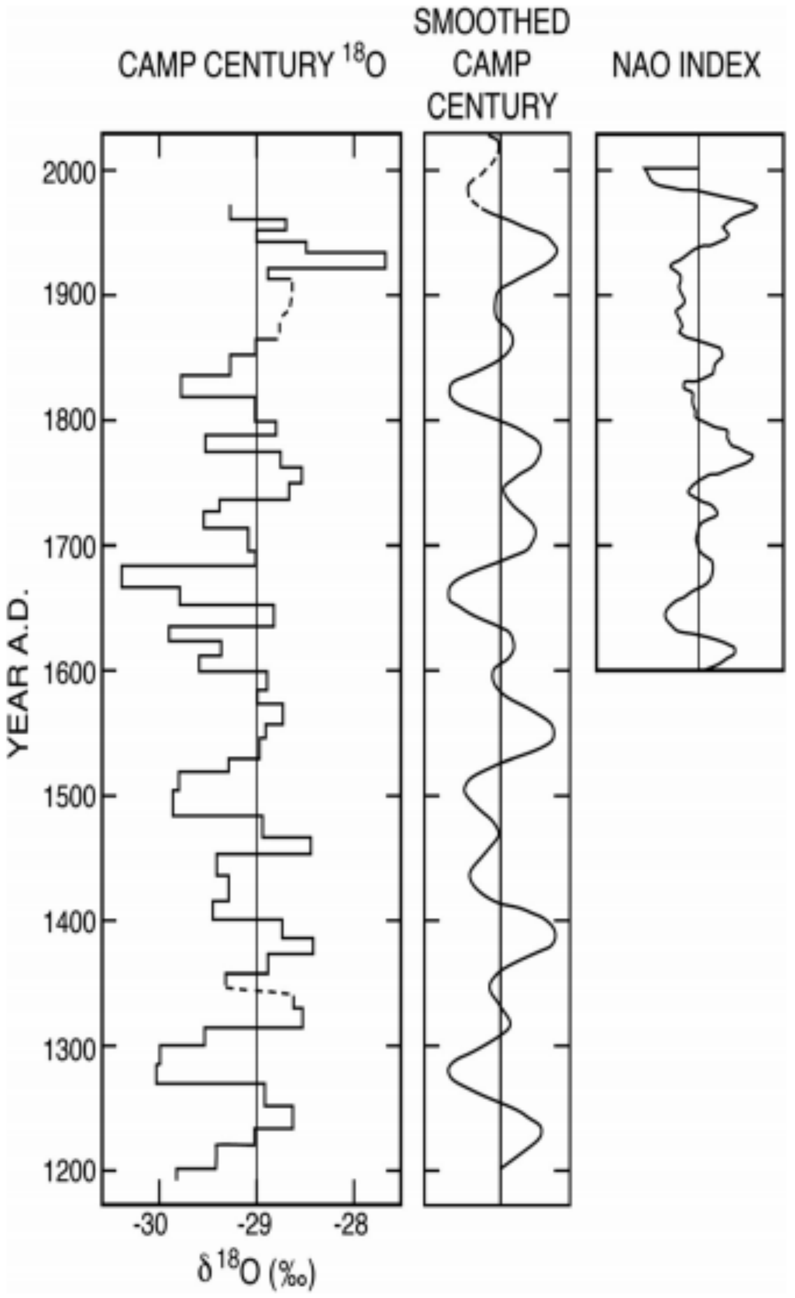
Fig. 2 The Camp Century Greenland ^{18}O record for the last 800 years (Dansgaard et al. 1971) compared with the record of the North Atlantic Oscillation for the last 400 years (Trouet et al. 2009). Also shown is Dansgaard's demonstration that 80- and 180-year cycles explain a sizable portion of the variance in his ^{18}O record

Wally Broecker, in 2017: "it was dumb luck…"

– "Vinther et al. (2003) had shown that the ^{18}O record in Greenland ice resembled that for the NAO. If so, it certainly cannot be used, as I did, as a proxy for global temperature."

Wally assumed that a pattern – an oscillatory pattern – in his data was a proxy for global temperature, and he extrapolated from that.

A cautionary tale!



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Thank you for your attention!

Andy says:
"It's gettin' hot –
let's have a drink!"

Questions or Comments?

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