

# Week 7 Worksheet

## Boundary Value Problems and the Multipole Expansion

Jacob Erlikhman

**Exercise 1. Griffiths 3.13.** Two infinite grounded metal plates lie parallel to the  $(x, z)$ -plane. One is at  $y = 0$  and the other at  $y = a$ . The left end, at  $x = 0$ , is closed off with an infinite strip insulated from the two plates and maintained at a potential

$$V_0(y) = \begin{cases} V_0, & y \in \left(0, \frac{a}{2}\right) \\ -V_0, & y \in \left(\frac{a}{2}, a\right). \end{cases}$$

- a) What are the boundary conditions for this problem?
- b) Argue that the situation is independent of  $z$ , so that we can use the Laplace equation in the  $x$  and  $y$  coordinates only.
- c) Qualitatively describe the behavior of the potential as a function of  $x$  for  $x \gg 0$ .
- d) Write down Laplace's equation, and separate variables.
- e) After obtaining something of the form

$$\frac{X''}{X} + \frac{Y''}{Y} = 0,$$

argue that each term must be individually constant.

*Hint:* The second term is a function of  $y$  only, so, fixing  $y = y_0$ , it must remain constant as we vary  $x$ . What happens to the first term as we do this?

- f) Write down the general form of the solutions for  $X$  and  $Y$ .
- g) Enforce the boundary conditions on your solutions, and solve for the potential inside the slot.

**Exercise 2.** A half-infinite dipole string with linear dipole moment density  $\mathbf{p}_l = p_l \hat{x}$  is placed along the negative  $z$ -axis.

- a) The potential due to a dipole *at the origin* is given by

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}.$$

Generalize this to find the potential  $V(x, y, z)$  due to the string. This is probably easier if you use cylindrical coordinates.

- b) Investigate the behavior of the potential from (a) as you approach the  $z$ -axis in the regions  $z < 0$  and  $z > 0$ .