Week 11 Worksheet
Bouncing Ball

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Exercise 1. ***Griffiths 9.6*** **The Quantum Bouncing Ball.** A ball of mass m bounces elastically on the floor.

a) What is the potential as a function of the height x above the floor?
Hints: We assume the ball has nowhere to "tunnel" to, since the ground extends to $x = -\infty$. So what is the potential when $x < 0$?

b) Solve the Schrödinger equation. You don't need to normalize your solution.

Hint: You should get Airy's differential equation, $\psi''(z) - z\psi(z) = 0$. One way to manipulate the Schrödinger equation into such a form is to notice that for $\psi''(x) - \alpha^3 x\psi(x) = 0$, $z = \alpha x$ works. You should use a slight modification of this. The solutions of this equation are the Airy functions, $\mathrm{Ai}(z)$ and $\mathrm{Bi}(z)$. The graphs of these functions are below.

c) Calculate (approximately) the first 4 energies, using $g = 10\ \mathrm{m/s^2}$ and $m = 0.100$ kg.
Possible Hint: $0.2^{1/3} \approx 0.58$.

d) Now, analyze this problem using the WKB approximation. Find the allowed energies E_n in terms of m, g, and $\hbar$.

Hint: The connecting WKB wavefunctions are

$$\psi(x) = \begin{cases} \dfrac{2D}{\sqrt{p(x)}} \sin\left(\dfrac{1}{\hbar}\displaystyle\int_x^{x_2} p(x')\,\mathrm{d}x' + \dfrac{\pi}{4}\right), & x < x_2 \\ \dfrac{D}{\sqrt{p(x)}} \exp\left(-\dfrac{1}{\hbar}\displaystyle\int_{x_2}^{x} |p(x')|\,\mathrm{d}x'\right), & x > x_2 \end{cases}.$$

e) Plug in the values from (c), and compare the WKB calculation to the "exact" one for the first four energies. It is OK to use a calculator in this part and the next.

f) How large would n have to be to give the ball an average height of 1 meter above the ground?

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Plot[AiryAi[z], {z, -10, 10}]
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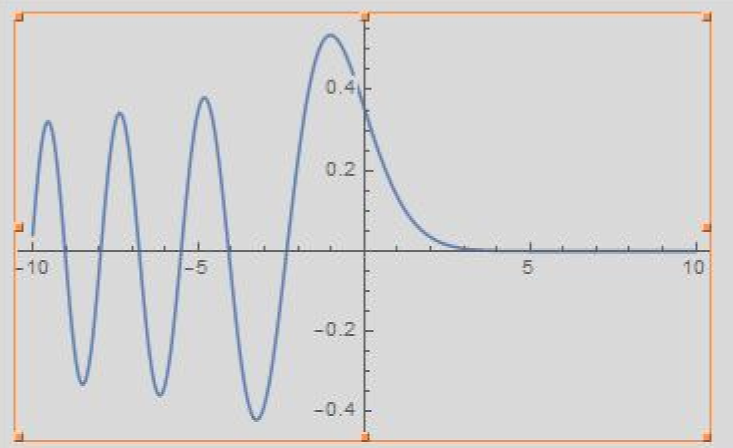


```
Plot[AiryBi[z], {z, -10, 10},
 PlotRange → {{-10, 10}, {-0.5, 2}}]
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