

Week 11 Worksheet

Bouncing Ball

Jacob Erlikhman

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Exercise 1. Griffiths 9.6 Quantum Bouncing Ball. A ball of mass m bounces elastically on the floor.

- a) What is the potential as a function of the height x above the floor?

Hint: If the ball can't go below $x = 0$, what is the potential in that region?

- b) Solve the Schrödinger equation. You don't need to normalize your solution.

Hint: You should get Airy's differential equation, $\psi''(z) - z\psi(z) = 0$. One way to manipulate the Schrödinger equation into such a form is to notice that for $\psi''(x) - \alpha^3 x \psi(x) = 0$, the substitution $z = \alpha x$ does the trick. The solutions of this equation are the Airy functions, $\text{Ai}(z)$ and $\text{Bi}(z)$. The graphs of these functions are below.

- c) Calculate (approximately) the first 4 energies, using $g = 10 \text{ m/s}^2$ and $m = 0.100 \text{ kg}$.

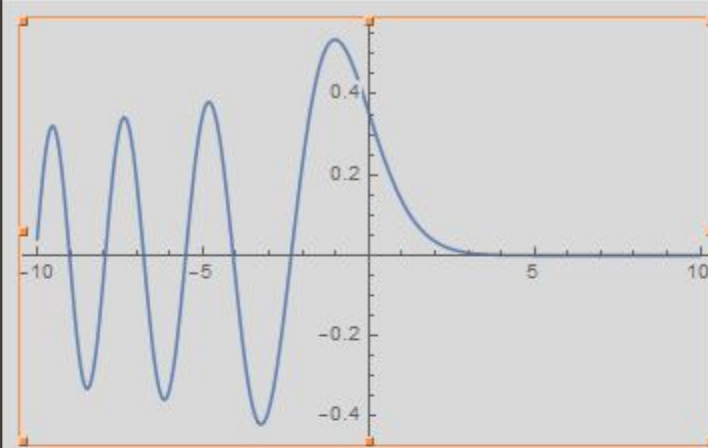
- d) Now, analyze this problem using the WKB approximation. Find the allowed energies E_n in terms of m , g , and \hbar .

Hint: The connecting WKB wavefunctions are

$$\psi(x) = \begin{cases} \frac{2D}{\sqrt{p(x)}} \sin\left(\frac{1}{\hbar} \int_x^{x_2} p(x') dx' + \frac{\pi}{4}\right), & x < x_2 \\ \frac{D}{\sqrt{p(x)}} \exp\left(-\frac{1}{\hbar} \int_{x_2}^x |p(x')| dx'\right), & x > x_2 \end{cases}.$$

- e) Plug in the values from (c), and compare the WKB calculation to the exact one for the first four energies.
- f) How large would n have to be to give the ball an average height of 1 meter above the ground?

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Plot[AiryAi[z], {z, -10, 10}]
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Plot[AiryBi[z], {z, -10, 10},  
PlotRange -> {{-10, 10}, {-0.5, 2}}]
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