Week 2 Worksheet
Math Review

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Exercise 0. Warm up. a) Write down the divergence theorem.

b) Write down Stokes' theorem.

c) Suppose in the divergence theorem I let the volume I was integrating over be given by the *open* ball:

$$B = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | {x_1}^2 + {x_2}^2 + {x_3}^2 < 1\}.$$

What does the divergence theorem say in this case? Does it make sense? Why or why not?

Exercise 1. a) What does the gradient tell you about a function? Why?
Hint: If $\nabla f(\mathbf{x}) = \mathbf{w}$, argue or show that

$$D_{\mathbf{v}} f(\mathbf{x}) = \mathbf{v} \cdot \mathbf{w},$$

where $D_{\mathbf{v}} f(\mathbf{x})$ is the directional derivative of f at $\mathbf{x}$ in the direction $\mathbf{v}$.

Remark. Notice that this result holds in *any dimension* $n \in \mathbb{N}$.

b) What does the curl tell you about a vector field? Why?
Hint: Draw and calculate the curls of some example vector fields, like $-y\hat{x} + x\hat{y}$ or $x\hat{y}$. Now, try the vector fields $x\hat{x} + y\hat{y} + z\hat{z}$, $\hat{z}$, and $z\hat{z}$.

c) Use (a) and (b) to give an intuitive explanation of why the curl of a gradient is always 0.

d) Show that $\nabla \times \nabla f = 0$ directly.

Exercise 2. *Griffiths 1.13*. Let $\mathbf{d}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and let d be its length. Show that

a) $\nabla(d^2) = 2\mathbf{d}$,

b) $\nabla(1/d) = -\hat{d}/d^2$.

c) What is the general formula for $\nabla(d^n)$?

d) You computed these formulas in cartesian coordinates. Do they hold in other coordinate systems? Why or why not?

Remark. To prove this would require more technology than we currently have at our disposal. However, you should be able to come up with an intuitive argument.