

Week 4 Worksheet

(Nondegenerate) Perturbation Theory

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Exercise 1. Suppose you want to calculate the expectation value of some observable A in the n^{th} energy eigenstate of a system perturbed by H^1 ,

$$\langle A \rangle = \langle n | A | n \rangle.$$

Suppose further that all eigenstates are nondegenerate.

- a) Replace $|n\rangle$ by its perturbation expansion, and write down the formula for the first order correction to $\langle A \rangle$, $\langle A \rangle^1$.
- b) Use the first order corrections to the states,

$$|n^1\rangle = \sum_{m \neq n} \frac{\langle m^0 | H^1 | n^0 \rangle}{E_n^0 - E_m^0} |m^0\rangle,$$

to rewrite $\langle A \rangle^1$ in terms of the unperturbed eigenstates.

- c) If $A = H^1$, what does the result of (b) tell you? Explain why this is consistent with the second order corrections to the energies,

$$E_n^2 = \sum_{m \neq n} \frac{|\langle m^0 | H^1 | n^0 \rangle|^2}{E_n^0 - E_m^0}.$$