

# Week 8 Worksheet

## (Nondegenerate) Perturbation Theory

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October 13, 2024

**Exercise 1.** Let  $H(\lambda) = H^0 + \lambda H'$  be a perturbed hamiltonian. Suppose we know

$$H^0 \psi_n^0 = E_n^0 \psi_n^0,$$

where the  $\psi_n^0$  are unperturbed, orthonormal, nondegenerate eigenfunctions.

- Write the  $\psi_n$  and  $E_n$  as power series in  $\lambda$ .
- Write the Schrödinger equation for  $H(\lambda)$  in terms of the above power series.
- Truncate the above equation to first order, and derive the first order corrections to the energies. You should get

$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle.$$

- Along the way to solving (c), you should have come up with the equation

$$H^0 \psi_n^1 + H' \psi_n^0 = E_n^1 \psi_n^0 + E_n^0 \psi_n^1.$$

Rewrite this as an inhomogeneous differential equation for  $\psi_n^1$ , and solve it via the power series method, thus obtaining the first order corrections to the wavefunctions.

- Derive the second order corrections to the energies,  $E_n^2$ .

**Exercise 2.** Suppose you want to calculate the expectation value of some observable  $A$  in the  $n^{\text{th}}$  energy eigenstate of a system perturbed by  $H'$ ,

$$\langle A \rangle = \langle \psi_n | A | \psi_n \rangle.$$

Suppose further that all eigenstates are nondegenerate.

- Replace  $\psi_n$  by its perturbation expansion, and write down the formula for the first order correction to  $\langle A \rangle$ .

b) Use the first order corrections to the wavefunctions,

$$\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \psi_m^0, \quad (0.1)$$

to rewrite  $\langle A \rangle^1$  in terms of the unperturbed eigenstates.

c) If  $A = H'$ , what does the result of (b) tell you? Explain why this is consistent with Equation 1.