Midterm Review Session Problems

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Exercise 1. When we solve the hydrogen atom, we assume that the nucleus is a point charge. In this problem, we will compute the approximate change to the energy levels due to the finite size of the nucleus. This is called the **volume effect**. Model the nucleus as a uniform sphere of radius $r_0 A^{1/3}$, where $A^{1/3}$ is the number of nucleons (so this works for e.g. deuterium) and $r_0 = 1.3 \cdot 10^{-13}$ cm.

a) What is the potential $V(r)$?
Hint: Outside the nucleus, $V(r)$ is just the Coulomb potential. Inside the nucleus, use Gauss' law to determine $V(r)$.

b) What is H', where H^0 is the hydrogen atom hamiltonian?

c) Argue that the $\ell = 0$ states are only slightly affected by this perturbation.
Hint: Think about the small r behavior of the wavefunctions for s-states vs. $\ell > 0$ states.

d) Calculate the correction to the energy levels for all states with $\ell = 0$. Note that

$$R_{n0}(0) = \frac{2}{(na_0)^{3/2}},$$

where $a_0 = \hbar^2/me^2$.

e) For hydrogen, calculate the correction to the $n = 1$ and $n = 2$ states in eV.

f) Fine structure is of order $\alpha^4 mc^2$. Compare the magnitude of the volume effect to that of fine structure.

Exercise 2. ***Griffiths 7.45*****. Stark Effect in Hydrogen.** When an atom is placed in a uniform electric field $\mathbf{E}_{\text{ext}}$, the energy levels are shifted. This is known as the **Stark effect**. You'll analyze the Stark effect for the $n = 1$ and $n = 2$ states of hydrogen. Suppose $\mathbf{E}_{\text{ext}} = E_{\text{ext}}\hat{z}$, so that

$$H^1 = eE_{\text{ext}} r \cos\theta$$

is the perturbation of the hamiltonian for the electron, where $H^0 = \frac{p^2}{2m} - \frac{e^2}{r}$.

a) Show that the ground state energy is unchanged at first order.

b) How much degeneracy does the first excited state have? List the degenerate states.

c) Determine the first-order corrections to the energy. Into how many levels does E_2 split?

Hint: All W_{ij} are 0 except for two, and you can avoid doing all of the zero integrals in this problem by using symmetry and selection rules. Since selection rules weren't covered in this course, the specific one you'll need is that

$$\langle n'\ell'm'|\mathbf{x}|n\ell m\rangle = \mathbf{0}$$

if $\ell + \ell'$ is even. You'll also need the following

$$\begin{aligned}\psi_{210} =& \frac{1}{2\sqrt{6}} a^{-3/2} \frac{r}{a} e^{-r/2a} \sqrt{\frac{3}{4\pi}} \cos\theta \\ \psi_{200} =& \frac{1}{\sqrt{2}} a^{-3/2} \left(1 - \frac{r}{2a}\right) e^{-r/2a} \frac{1}{2\sqrt{\pi}}.\end{aligned}$$

d) What are the "good" wavefunctions for (b)? Find the expectation value of the electric dipole moment in each of these states.

Exercise 3. Explain the physical origins and give the relative magnitudes of

a) fine structure

b) Lamb shift

c) hyperfine structure.

Exercise 4. ***Griffiths 8.19.*** Find the lowest bound on the ground state of hydrogen using the variational principle and an exponential trial wavefunction,

$$\psi(\mathbf{r}) = Ae^{-br^2},$$

where A is determined by normalization and b is a variational parameter. Express your answer in eV.

Exercise 5. Use the variational principle to get an approximation for the ground state energy in the **Yukawa potential**

$$V(r) = e^{-\alpha r} \frac{e^2}{r},$$

using the trial function

$$\psi\,(r) = \sqrt{\frac{b^3}{\pi}} e^{-br}.$$

Show that when $\alpha = 0$, the trial function saturates the bound; why? Comment on the accuracy of the bound you obtain as α increases. Note that

$$\nabla^2 f(r) = \frac{1}{r^2}\partial_r(r^2 \partial_r f(r)).$$

Hint: You won't get a closed form solution for b when $\alpha \neq 0$, but you can still give a physical explanation of what's going on as α increases.