Week 4 Worksheet
Electrostatics

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Exercise 1. Using Dirac delta functions in the appropriate coordinates if necessary, express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{r})$.

a) In spherical coordinates, a charge Q uniformly distributed over a spherical shell of radius R.

b) In cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b.

c) In cylindrical coordinates, a charge Q spread uniformly over a flat circular disc of negligible thickness and radius R.

d) Same as (c), but in spherical coordinates.

Exercise 2. Two infinite parallel plates carry equal and opposite uniform charge densities $\pm\sigma$. Find the electric field in each of three regions: To the left of both, in between them, and to the right of both.

Exercise 3. a) What property of the electric field allows us to define the electric potential?

b) The potential at a point $\mathbf{r}$ is defined as

$$V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell},$$

where $\mathcal{O}$ is some reference point. Explain why this is well-defined (i.e. unambiguous, up to the choice of $\mathcal{O}$).

c) An infinite plate carries a uniform charge density σ. Using your result from Exercise 2, find the potential everywhere.
Hint: Where would you put your reference point $\mathcal{O}$?

Exercise 4. Consider a uniformly charged spherical shell of radius R and charge Q.

a) Find the electric field everywhere using Gauss' law.

b) Find the potential everywhere by direct integration (without using Gauss' law).
Hint: Consider a single point a distance z from the center of the sphere, and use *cylindrical* symmetry.

c) Set up the integral to find the electric field at a point a distance z from the center of the sphere (without using Gauss' law). Consider separately the cases $z < R$ and $z > R$.

Exercise 5. Repeat Exercise 4, but this time for a solid sphere of radius R and charge Q.
Hint: For part (c), use your result from Exercise 4(c).