Week 8 Worksheet
Boundary Value Problems and the Multipole Expansion

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Exercise 1. ***Griffiths 3.13.*** Two infinite grounded metal plates lie parallel to the (x,z)-plane. One is at $y=0$ and the other at $y=a$. The left end, at $x=0$, is closed off with an infinite strip insulated from the two plates and maintained at a potential

$$V_0(y) = \begin{cases} V_0, & y \in \left(0, \frac{a}{2}\right) \\ -V_0, & y \in \left(\frac{a}{2}, a\right). \end{cases}$$

a) What are the boundary conditions for this problem?

b) Argue that the situation is independent of z, so that we can use the Laplace equation in the x and y coordinates only.

c) Qualitatively describe the behavior of the potential as a function of x for $x \gg 0$.

d) Write down Laplace's equation, and separate variables.

e) After obtaining something of the form

$$\frac{X''}{X} + \frac{Y''}{Y} = 0,$$

argue that each term must be individually constant.
Hint: The second term is a function of y only, so, fixing $y = y_0$, it must remain constant as we vary x. What happens to the first term as we do this?

f) Write down the general form of the solutions for X and Y.

g) Enforce the boundary conditions on your solutions, and solve for the potential inside the slot.

Exercise 2. A half-infinite dipole string with linear dipole moment density $\mathbf{p}_l = p_l\hat{x}$ is placed along the negative z-axis.

a) The potential due to a dipole *at the origin* is given by

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{r}}{r^2}.$$

Generalize this to find the potential $V(x,y,z)$ due to the string. This is probably easier if you use cylindrical coordinates.

b) Investigate the behavior of the potential from (a) as you approach the z-axis in the regions $z<0$ and $z>0$.