Week 13 Worksheet
Gravitational Radiation

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Exercise 1. No Dipole Radiation. In this problem, you will show that there is no dipole term in the multipole expansion for gravitational radiation; hence, the quadrupole term derived in class is the leading contribution to gravitational radiation.

a) Recall that in electromagnetism, the dipole moment of two charged particles of equal charge q separated by a distance $\mathbf{x}$ is given by

$$\mathbf{p} = q\mathbf{x}.$$

Generalize this to i) n particles of charges q_i with separations $\mathbf{x}_{ij}$ and ii) a continuous charge distribution with charge density $\rho(\mathbf{x})$. What is the electric dipole density, or polarization per unit volume, $\mathbf{P}$ (where $\mathbf{p} = \int \mathbf{P} d^3x$)?
Hint: For (ii), you need to expand the scalar potential

$$V(\mathbf{x}) = \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

to first order to obtain the "dipole term"

$$\frac{\mathbf{p} \cdot \hat{x}}{x^2}.$$

b) Do part (a) for gravitational masses instead of electrically charged particles.

c) Physically, what is the first rate of change $\dot{\mathbf{p}}$ of the gravitational dipole moment?
Hint: It may be easier to first solve this problem with the discrete form for $\mathbf{p}$, i.e. the one with a finite number of particles.

d) Argue that $\ddot{\mathbf{p}} = 0$; therefore, there is no mass dipole radiation.

e) In electromagnetism, the next strongest form of radiation is due to the magnetic dipole moment: Given a current density $\mathbf{J}(\mathbf{x})$, the magnetization is

$$\mathbf{M} = \frac{1}{2}\mathbf{x} \times \mathbf{J},$$

so that the magnetic dipole moment is[1]

$$\mathbf{m} = \int \mathbf{M} d^3 x.$$

Write down the specialization of this general formula to n charged particles of charges q_i moving with velocities $\mathbf{v}_i$.

f) Do part (e) for gravitational masses instead of electrically charged particles.

g) For the gravitational analog of the magnetic dipole moment, show that $\dot{\mathbf{m}} = \mathbf{0}$; hence, there is no gravitational dipole radiation *at all*.

[1]This follows from performing the multipole expansion of the vector potential **A**.