Week 2 Worksheet
Identical Particles

Jacob Erlikhman

9/8/22

Exercise Warm up. Suppose you had three particles, and three distinct one-particle states are available, ψ_a, ψ_b, and ψ_c. How many different three-particle states can be constructed if they are (a) distinguishable particles, (b) identical bosons, or (c) identical fermions? (The particles don't have to be in *different* states—if they are distinguishable, we could have $\psi_a(x_1)\psi_a(x_2)\psi_a(x_3)$ as an allowed state, for example).

Exercise 1.

Write down the hamiltonian for two noninteracting identical particles in the infinite square well. Write down the ground states for the three cases: distinguishable, fermions, bosons. Recall that the one-particle wavefunctions are

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right),$$

with energies $E_n = n^2\pi^2\hbar^2/2ma^2$.

Find the first three excited states and their energies for each of the three cases (distinguishable, fermions, bosons).

Exercise 2. In Exercise 1, we ignored spin (or at least supposed that the particles are in the same spin state).

Do it now for particles of spin 1/2. Construct the four lowest-energy configurations, and specify their energies and degeneracies.

Do the same for spin 1 (you will need the Clebsch-Gordan table from bCourses).

Exercise 3. Suppose now we have 2 noninteracting particles (mass m) in the infinite square well. If one is in the state ψ_n and the other is in the state ψ_l with $l \neq n$, calculate $\langle (x_1 - x_2)^2 \rangle$, assuming that they are (a) distinguishable, (b) identical bosons, or (c) identical fermions.