

Week 5 Worksheet

More Electrostatics

Jacob Erlikhman

Exercise 1. a) The potential at a point \mathbf{r} is defined as

$$V(\mathbf{r}) = - \int_{\mathbb{O}}^{\mathbf{r}} \mathbf{E} \cdot d\boldsymbol{\ell},$$

where \mathbb{O} is some reference point. Explain why this is well-defined (i.e. unambiguous, up to the choice of \mathbb{O}).

Hint: Use Stokes' theorem.

b) An infinite plate carries a uniform charge density σ . Using your result from Exercise 2, find the potential everywhere.

Hint: Where would you put your reference point \mathbb{O} ?

Exercise 2. Consider a uniformly charged spherical shell of radius R and charge Q .

a) Find the electric field everywhere using Gauss' law.

b) Find the potential everywhere by direct integration, i.e. using

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{z} d\tau',$$

where the integral is taken over the shell.

Hints: Consider a single point a distance z from the center of the sphere, and use *cylindrical* symmetry (but spherical coordinates). Also, you can figure out what z is using the cosine law. Lastly, **be very careful to take the positive square root: Consider the separate cases $z < R$ and $z > R$.**

c) Set up the integral to find the electric field at a point a distance z from the center of the sphere (without using Gauss' law). Compute all the integrals except the θ' integral, so that your final answer is of the form

$$\mathbf{E} = \int \text{stuff} d\cos\theta' \hat{r}$$

(an integral over θ' is fine too).

Hint: The \hat{z} that appears in the original integral does not point in \hat{r} , but \mathbf{E} does. What happens to the electric field in the non-radial directions?