Week 11 Worksheet
Scattering

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Exercise 1. Warm up.

a) How do the phase shifts δ_ℓ appear in partial wave scattering, and what is their physical significance?

b) What is the fundamental assumption on the form of the wavefunctions in the Born approximation? *Hint*: If the scattering potential is weak, what approximation can we make?

c) Starting from the Lippmann-Schwinger equation,

$$\psi(\mathbf{x}) = \varphi_{\mathbf{k}}(\mathbf{x}) + \int \mathrm{d}^3x'\, G_0(\mathbf{x}, \mathbf{x}', E) V(\mathbf{x}')\psi(\mathbf{x}'),$$

where G_0 is the free particle, time-independent Green's function and

$$\begin{aligned}\varphi_{\mathbf{k}}(\mathbf{x}) &= \langle \mathbf{x} | \mathbf{k} \rangle \\ &= \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}},\end{aligned}$$

explain how you would derive the Born approximation. (Just list the steps, no need to work them out.)
Hint: The Green's function is (note that $E = \hbar^2k^2/2m$)

$$G_0(\mathbf{x}, \mathbf{x}', E) = \frac{e^{ik|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|}.$$

Exercise 2. Spin-spin Interaction. Consider two spin-1/2 particles that interact in a potential of the form

$$V(r) = V_{\mathrm{o}}(r) + V_{\mathrm{s}}(r)\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}.$$

Suppose that both the orbital and spin interactions are short range in the interparticle separation r (i.e. vanish faster than $1/r$ as $r \to \infty$).

a) The first Born approximation for the scattering amplitude is given by

$$f(\mathbf{k},\mathbf{k}') = -\frac{4\pi^2 m}{\hbar^2}\left\langle \mathbf{k}'|V|\mathbf{k}\right\rangle.$$

Use a Fourier transform to express the scattering amplitude in terms of

$$\int e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_0} V_{\mathrm{o}}(r_0)\,\mathrm{d}^3 r_0,$$

and a similar expression for $V_{\mathrm{s}}(r_0)$.
Hints: Don't forget to account for the initial and final spins! Note that

$$\left\langle \mathbf{x}|\mathbf{k}\right\rangle = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}}.$$

b) Show that the eigenvalues of $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$ are the singlet and triplet states, with eigenvalues -3 and 1, respectively.
Hint: This is easiest to do if you write $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$ in terms of operators for which the singlet and triplet are eigenstates.

c) If the incoming particles have parallel spins, is a spin flip possible? Why or why not? Explain why the scattering is elastic or inelastic in this case, and then calculate the scattering amplitude.

d) Calculate the scattering amplitude for incident particles with opposite spins. You should be able to split it into two channels: an elastic one and an inelastic one (why?).