

Week 8 Worksheet

(Nondegenerate) Perturbation Theory

Jacob Erlikhman

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Exercise 1. Let $H = H^0 + H^1$ be a perturbed hamiltonian. Suppose we know

$$H^0 |n^0\rangle = E_n^0 |n^0\rangle,$$

where the $|n^0\rangle$ are the unperturbed, orthonormal, nondegenerate eigenstates.

- Expand the exact solutions for H , $|n\rangle$ and E_n , in perturbation expansions.
- Write the Schrödinger equation for H in terms of the above expansions.
- Study the first order part of the equation from (b), and derive the first order corrections to the energies. You should get

$$E_n^1 = \langle n^0 | H^1 | n^0 \rangle.$$

- Along the way to solving (c), you should have come up with the equation

$$H^0 |n^1\rangle + H^1 |n^0\rangle = E_n^1 |n^0\rangle + E_n^0 |n^1\rangle.$$

Using this equation, find the expansion of $|n^1\rangle$ in the eigenbasis of H^0 .

Hint: In order to find the component of $|n^1\rangle$ which is parallel to $|n^0\rangle$, enforce normalization of $|n\rangle$ to first order, i.e. $|n^0\rangle + |n^1\rangle$ should have norm 1.

- Derive the second order corrections to the energies, E_n^2 .

Exercise 2. Suppose you want to calculate the expectation value of some observable A in the n^{th} energy eigenstate of a system perturbed by H^1 ,

$$\langle A \rangle = \langle n | A | n \rangle.$$

Suppose further that all eigenstates are nondegenerate.

- Replace n by its perturbation expansion, and write down the formula for the first order correction to $\langle A \rangle$.

b) Use the first order corrections to the states,

$$|n^1\rangle = \sum_{m \neq n} \frac{\langle m^0 | H^1 | n^0 \rangle}{E_n^0 - E_m^0} |m\rangle, \quad (1)$$

to rewrite $\langle A \rangle^1$ in terms of the unperturbed eigenstates.

c) If $A = H^1$, what does the result of (b) tell you? Explain why this is consistent with Equation 1.