Midterm Review Session Problems

Jacob Erlikhman

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Exercise 1. When we solve the hydrogen atom, we assume that the nucleus is a point charge. In this problem, we will compute the approximate change to the energy levels due to the finite size of the nucleus. This is called the **volume effect**. Model the nucleus as a uniform sphere of radius $r_0 A^{1/3}$, where $A^{1/3}$ is the number of nucleons (so this works for e.g. deuterium) and $r_0 = 1.3 \cdot 10^{-13}$ cm.

a) What is the potential $V(r)$?
Hint: Outside the nucleus, $V(r)$ is just the Coulomb potential. Inside the nucleus, use Gauss' law to determine $V(r)$.

b) What is H', where H^0 is the hydrogen atom hamiltonian?

c) Argue that the $\ell = 0$ states are only slightly affected by this perturbation.
Hint: Think about the small r behavior of the wavefunctions for s-states vs. $\ell > 0$ states.

d) Calculate the correction to the energy levels for all states with $\ell = 0$. Note that

$$R_{n0}(0) = \frac{2}{(na_0)^{3/2}},$$

where $a_0 = \hbar^2/me^2$.

e) For hydrogen, calculate the correction to the $n = 1$ and $n = 2$ states in eV.

f) Fine structure is of order $\alpha^4 mc^2$. Compare the magnitude of the volume effect to that of fine structure.

Exercise 2. ***Griffiths 7.45*****. Stark Effect in Hydrogen.** When an atom is placed in a uniform electric field $\mathbf{E}_{\text{ext}}$, the energy levels are shifted. This is known as the **Stark effect**. You'll analyze the Stark effect for the $n = 1$ and $n = 2$ states of hydrogen. Suppose $\mathbf{E}_{\text{ext}} = E_{\text{ext}}\hat{z}$, so that

$$H' = eE_{\text{ext}} r \cos\theta$$

is the perturbation of the hamiltonian for the electron, where $H^0 = \frac{p^2}{2m} - \frac{e^2}{r}$.

a) Show that the ground state energy is unchanged at first order.

b) How much degeneracy does the first excited state have? List the degenerate states.

c) Determine the first-order corrections to the energy. Into how many levels does E_2 split?

Hint: All W_{ij} are 0 except for two, and you can avoid doing all of the zero integrals in this problem by using symmetry and selection rules. You'll need the following

$$\begin{aligned}\psi_{210} =& \frac{1}{2\sqrt{6}} a^{-3/2} \frac{r}{a} e^{-r/2a} \sqrt{\frac{3}{4\pi}} \cos\theta \\ \psi_{200} =& \frac{1}{\sqrt{2}} a^{-3/2} \left(1 - \frac{r}{2a}\right) e^{-r/2a} \frac{1}{2\sqrt{\pi}}.\end{aligned}$$

d) What are the "good" wavefunctions for (b)? Find the expectation value of the electric dipole moment in each of these states.

Exercise 3. Explain the physical origins of

a) fine structure

b) Lamb shift

c) hyperfine structure.

Exercise 4. Zeeman Effect. In this problem you will do a relaistic calculation of the effect of magnetic fields on the $n = 2$ states of hydrogen. We will allow the magnetic field to *take on any value*, so that we won't assume that the Zeeman term is necessarily small or large in comparison to the fine structure terms. It will be helpful to use units where $m = \hbar = c = 1$, called **atomic units**. It will also be helpful to use the dimensionless variable

$$x = \frac{B}{B_1},$$

where B is the external magnetic field and B_1 is α times the strength $B_0 = 1$ magnetic field in atomic units,

$$\begin{aligned}B_1 =& \frac{e^7 m^2}{\hbar^5 c} \\ =& \alpha B_0 \\ =& \alpha \frac{m^2 e^5}{\hbar^4}.\end{aligned}$$

Note that I am using gaussian units for the formulas above.

For this problem, the hamiltonian H^0 is the hydrogen atom hamiltonian and

$$H^1 = H_r + H_{\text{SO}} + H_Z.$$

a) Make a table which shows which of the operators $L^2, L_x, L_y, L_z, S^2, S_x, S_y, S_z, J^2, J_x, J_y, J_z$ commute with H^0 and which commute with H^1.

b) Use the table from (a) to find a basis in the 8-dimensional subspace of the $n = 2$ degenerate energy levels of H^0 for which the perturbing hamiltonian will be as diagonal as possible.
Hint: After choosing this basis, there should be only 16 off-diagonal matrix elements which you need to calculate.

c) Find all eight levels for the $n = 2$ states of hydrogen perturbed by H^1.

d) Expand the results of (c) for small x, and show that they agree with the energy levels for the weak field Zeeman effect. Do the same for large x, and show that they agree with the energy levels for the strong field Zeeman effect.