

# Week 12 Worksheet

## Scattering

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### Exercise 1. Warm up.

- a) How do the phase shifts  $\delta_\ell$  appear in partial wave scattering, and what is their physical significance?
- b) What is the fundamental assumption on the form of the wavefunctions in the Born approximation?  
*Hint:* If the scattering potential is weak, what approximation can we make?
- c) Starting from the Lippmann-Schwinger equation,

$$\psi(\mathbf{x}) = \varphi_{\mathbf{k}}(\mathbf{x}) + \int d^3x' G_0(\mathbf{x}, \mathbf{x}', E) V(\mathbf{x}') \psi(\mathbf{x}'),$$

where  $G_0$  is the free particle, time-independent Green's function and

$$\begin{aligned} \varphi_{\mathbf{k}}(\mathbf{x}) &= \langle \mathbf{x} | \mathbf{k} \rangle \\ &= \frac{e^{i\mathbf{k} \cdot \mathbf{x}}}{(2\pi)^{3/2}}, \end{aligned}$$

explain how you would derive the Born approximation.

*Hints:* The Green's function is (note that  $E = \hbar^2 k^2 / 2m$ )

$$G_0(\mathbf{x}, \mathbf{x}', E) = \frac{e^{ik|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x} - \mathbf{x}'|}.$$

**Exercise 2. Spin-spin Interaction.** Consider two spin-1/2 particles that interact in a potential of the form

$$V(r) = V_0(r) + V_s(r) \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}.$$

Suppose that both the orbital and spin interactions are short range in the interparticle separation  $r$  (i.e. vanish faster than  $1/r$  as  $r \rightarrow \infty$ ).

- a) The first Born approximation for the scattering amplitude is given by

$$f(\mathbf{k}, \mathbf{k}') = -\frac{4\pi^2 m}{\hbar^2} \langle \mathbf{k}' | V | \mathbf{k} \rangle.$$

Use a Fourier transform to express the scattering amplitude in terms of

$$\int e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_0} V_0(r_0) d^3r_0,$$

and a similar expression for  $V_s(r_0)$ .

*Hint:* Don't forget to account for the incoming and outgoing spins!

- b) You computed on midterm 1 that the eigenvectors of  $\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}$  are the singlet and triplet states, with eigenvalues  $-3$  and  $1$ , respectively. If the incoming particles have parallel spins, is a spin flip possible? Why or why not? Explain why the scattering is elastic or inelastic in this case, and then calculate it.
- c) Calculate the scattering amplitude for incident particles with opposite spins. You should be able to split it into two channels: an elastic one and an inelastic one.