

# Week 4 Worksheet

## More Electrostatics

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### Exercise 1.

- a) The potential at a point  $\mathbf{r}$  is defined as

$$V(\mathbf{r}) = - \int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\ell,$$

where  $\mathcal{O}$  is some reference point. Explain why this is well-defined (i.e. unambiguous, up to the choice of  $\mathcal{O}$ ).

*Hint:* Use Stokes' theorem.

- b) An infinite plate carries a uniform charge density  $\sigma$ . Find the potential everywhere.

*Hints:* Recall that the electric field is  $E = \sigma/2\varepsilon_0$ . Where would you put your reference point  $\mathcal{O}$ ?

### Exercise 2.

Consider a uniformly charged spherical shell of radius  $R$  and charge  $Q$ .

- a) Find the electric field everywhere using Gauss' law.

- b) Find the potential everywhere by direct integration, i.e. using

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{r}')}{z} d\tau',$$

where the integral is taken over the shell.

*Hints:* Consider a single point a distance  $z$  from the center of the sphere, and use *cylindrical* symmetry (but spherical coordinates). Also, you can figure out what  $z'$  is using the cosine law. Lastly, **be very careful to take the positive square root: Consider the separate cases  $z < R$  and  $z > R$ .**

- c) Set up the integral to find the electric field at a point a distance  $z$  from the center of the sphere (without using Gauss' law). Compute all the integrals except the  $\theta'$  integral, so that your final answer is of the form

$$\mathbf{E} = \int \text{stuff} d\cos\theta' \hat{r}$$

(an integral over  $\theta'$  is fine too).

*Hint:* The  $\hat{z}$  that appears in the original integral does not point in  $\hat{r}$ , but  $\mathbf{E}$  does. What happens to the electric field in the non-radial directions?