Week 2 Worksheet
Math Review

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Exercise 0. Warm up. a) Write down the divergence theorem.

b) Write down Stokes' theorem.

c) Suppose in the divergence theorem I let the volume I was integrating over be given by the *open* ball:

$$B = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | {x_1}^2 + {x_2}^2 + {x_3}^2 < 1\}.$$

What does the divergence theorem say in this case? Does it make sense? Why or why not?

Exercise 1. a) What does the gradient tell you about a function? Why?
Hint: If $\nabla f(\mathbf{x}) = \mathbf{w}$, argue or show that

$$D_{\mathbf{v}} f(\mathbf{x}) = \mathbf{v} \cdot \mathbf{w},$$

where $D_{\mathbf{v}} f(\mathbf{x})$ is the directional derivative of f at $\mathbf{x}$ in the direction $\mathbf{v}$.

Remark. Notice that this result holds in *any dimension* $n \in \mathbb{N}$.

b) What does the curl tell you about a vector field? Why?
Hint: Draw and calculate the curls of some example vector fields, like $-y\hat{x} + x\hat{y}$ or $x\hat{y}$. Now, try the vector fields $x\hat{x} + y\hat{y} + z\hat{z}$, $\hat{z}$, and $z\hat{z}$.

c) Use (a) and (b) to give an intuitive explanation of why the curl of a gradient is always 0.

d) Show that $\nabla \times \nabla f = 0$ directly.

Exercise 2. *Griffiths 1.13.* Let $\mathbf{d}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and let d be its length. Show that

a) $\nabla(d^2) = 2\mathbf{d}$,

b) $\nabla(1/d) = -\hat{d}/d^2$.

c) What is the general formula for $\nabla(d^n)$?

d) You computed these formulas in cartesian coordinates. Do they hold in other coordinate systems? Why or why not?

Remark. To prove this would require quite a bit of work or more tools than we have at our disposal. However, you should be able to come up with an intuitive argument.