

# Week 8 Worksheet

## More Perturbation Theory

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March 4, 2024

**Exercise 1. Griffiths 7.54.** Last week, you derived the first order correction to the expectation value of an observable  $A$  in the  $n^{\text{th}}$  energy eigenstate of a system perturbed by  $H'$ . You found

$$\langle A \rangle^1 = 2 \sum_{m \neq n} \frac{\langle \psi_n^0 | A | \psi_m^0 \rangle \langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0}.$$

Suppose we have a particle of charge  $q$  in a weak electric field  $\mathbf{E} = E_{\text{ext}} \hat{x}$ , so that  $H' = -qE_{\text{ext}}x$ . This induces a dipole moment  $p_e = qx$  in the “atom.” The expectation value of  $p_e$  is proportional to the applied field, and the proportionality factor is called the **polarizability**,  $\alpha$ . Show that

$$\alpha = -2q^2 \sum_{m \neq n} \frac{|\langle \psi_n^0 | x | \psi_m^0 \rangle|^2}{E_n^0 - E_m^0}.$$

Find  $\alpha$  for the ground state of a 1-D harmonic oscillator, and compare the classical answer.

*Hint:* Recall that  $x$  can be written in terms of creation and annihilation operators. Given

$$H^0 = \frac{1}{2m} [p^2 + (m\omega x)^2],$$

you can derive what  $a$  and  $a^\dagger$  should be in terms of  $x$  and  $p$  by using the sum of squares formula. To get the “usual” form, rescale each of them by  $a \rightarrow \frac{1}{\sqrt{\hbar\omega}}a$  (so that the hamiltonian can be written  $H^0 = a^\dagger a + 1/2$ ).

**Exercise 2. Griffiths 7.45. Stark Effect in Hydrogen.** When an atom is placed in a uniform electric field  $\mathbf{E}_{\text{ext}}$ , the energy levels are shifted. This is known as the **Stark effect**. You’ll analyze the Stark effect for the  $n = 1$  and  $n = 2$  states of hydrogen. Suppose  $\mathbf{E}_{\text{ext}} = E_{\text{ext}} \hat{z}$ , so that

$$H' = eE_{\text{ext}}r \cos \theta$$

is the perturbation of the hamiltonian for the electron, where  $H^0 = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}$ .

a) Show that the ground state energy is unchanged at first order.

- b) How much degeneracy does the first excited state have? List the degenerate states.
- c) Determine the first-order corrections to the energy. Into how many levels does  $E_2$  split?

*Hint:* All  $W_{ij}$  are 0 except for two, and you can avoid doing all of the zero integrals in this problem by using symmetry and selection rules. You'll need the following

$$\psi_{210} = \frac{1}{2\sqrt{6}} a^{-3/2} \frac{r}{a} e^{-r/2a} \sqrt{\frac{3}{4\pi}} \cos \theta$$
$$\psi_{200} = \frac{1}{\sqrt{2}} a^{-3/2} \left(1 - \frac{r}{2a}\right) e^{-r/2a} \frac{1}{2\sqrt{\pi}}.$$

- d) What are the “good” wavefunctions for (b)? Find the expectation value of the electric dipole moment in each of these states.