Week 5 Worksheet
More Perturbation Theory

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September 24, 2025

Exercise 1. ***Griffiths 7.54.*** Last week, you derived the first order correction to the expectation value of an observable A in the n^{th} energy eigenstate of a system perturbed by H^1. You found

$$\langle A\rangle^1 = 2\text{Re} \sum_{m\neq n} \frac{\left\langle n^0\middle|A\middle|m^0\right\rangle\left\langle m^0\middle|H^1\middle|n^0\right\rangle}{E_n^0 - E_m^0}.$$

Suppose we have a particle of charge q in a weak electric field $\mathbf{E} = E_{\text{ext}}\hat{x}$, so that $H^1 = -qE_{\text{ext}}x$. This induces a dipole moment $p_e = qx$ in the "atom." The expectation value of p_e is proportional to the applied field, and the proportionality factor is called the **polarizability**, α. Show that

$$\alpha = -2q^2 \sum_{m\neq n} \frac{|\langle n^0|x|m^0\rangle|^2}{E_n^0 - E_m^0}.$$

Find α for the ground state of a 1-D harmonic oscillator, and compare the classical answer.

Hint: Recall that x can be written in terms of creation and annihilation operators. Given

$$H^0 = \frac{1}{2m}\left[p^2 + (m\omega x)^2\right],$$

you can derive what a and $a^\dagger$ should be in terms of x and p by using the sum of squares formula. To get the "usual" form, rescale each of them by $a \to \frac{1}{\sqrt{\hbar\omega}}a$ (so that the hamiltonian can be written $\frac{H^0}{\hbar\omega} = a^\dagger a + 1/2$).