

Week 8 Worksheet

Variational Principle

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Exercise 1. Prove the variational principle,

$$E_{\text{gs}} \leq \langle \psi | H | \psi \rangle.$$

Exercise 2. Use the variational principle to get an approximation for the ground state energy in the **Yukawa potential**

$$V(r) = e^{-\alpha r} \frac{e^2}{r},$$

using the trial function

$$\psi(r) = \sqrt{\frac{b^3}{\pi}} e^{-br}.$$

Show that when $\alpha = 0$, the trial function saturates the bound; why? Comment on the accuracy of the bound you obtain as α increases. Note that

$$\nabla^2 f(r) = \frac{1}{r^2} \partial_r (r^2 \partial_r f(r)).$$

Hint: You won't get a closed form solution for b when $\alpha \neq 0$, but you can still give a physical explanation of what's going on as α increases.