Project 3 Mathematics 512

Instructor: Ricardo Mancera Spring 2024

Due date: Wednesday March 27th

1.

Estimate the following expected value: $E(W_3^2 + \sin(W_3) + 2e^{W_3})$
Where W_t is a standard Wiener process, that is the drift parameter is zero and the Variance parameter $\sigma^2 = 1$.

2.

Let S_t be a Geometric Brownian Motion process: $S_t = S_0 e^{\left(\sigma W_t + \left(r - \frac{\sigma^2}{2}\right)t\right)}$ where $r = 0.05$, $\sigma = 0.20$, $S_0 = 90$ and W_t is a standard Wiener process. Estimate $E(S_3)$.

3.

Evaluate the following expected value and probability:

$$E(X_2^{0.6}) \quad , \quad P(X_2 > 2)$$

Where the Ito's processes X evolve according to the following SDE:

$$dX_t = \left(\frac{1}{4} + \frac{1}{3}X_t\right)dt + \frac{3}{5}dW_t\,, \;\; X_0 = 2$$

and W is a standard Wiener process.

4.

Consider the following SDE:

$$dX_t = aX_t dt + bX_t dW_t\,, \;\; X_0 = 100, \qquad a = 0.07\,, \qquad b = 0.12$$

a) Simulate this stochastic process using the discretization schemes of Euler-Maruyama

b) Compare with the analytical solution.