MATH 512 - Project 3

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Question 1 Overview

- Let W_t be a standard Wiener process, with drift parameter zero and variance parameter $\sigma^2 = 1$.
- We divide the interval $[0, 2]$ into L subintervals $[t_i, t_{i+1}]$, where $t_i = i\delta t$ and $\delta t = 2/L$.
- Let $W_i = W(t_i)$ and $\delta W_i = W_{i+1} - W_i$.
- We verify numerically that:
  - $\sum_{i=0}^{L-1} |\delta W_i|$ is unbounded as δt goes to zero.
  - $\sum_{i=0}^{L-1} \delta W_i^2$ converges to 2 in probability as δt goes to zero.

Question 1 Response

Refer to Figure 1

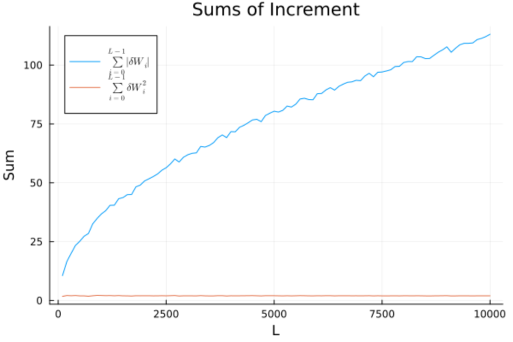

Figure: Stochastic Plots

Notice that as the L parameter increases, the $|\delta W_i|$ term is unbounded while δW_i^2 converges to 2 in probability.

Question 2a

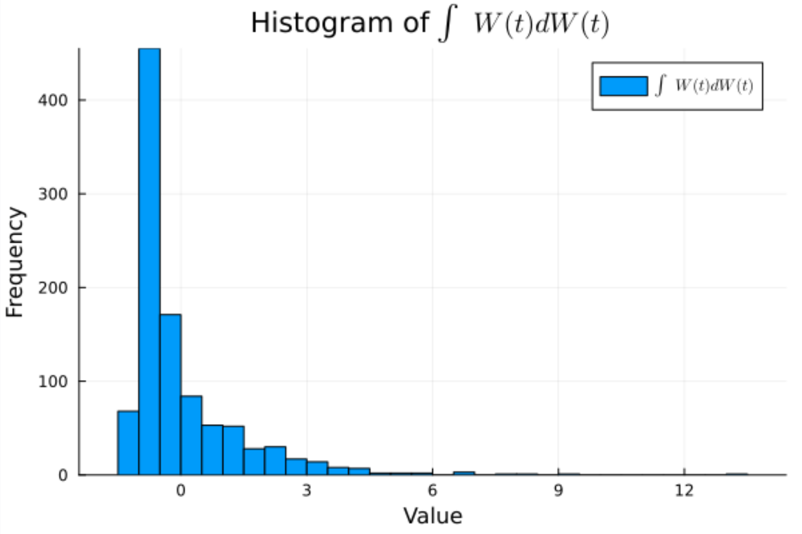

Figure: 2a

Question 2b

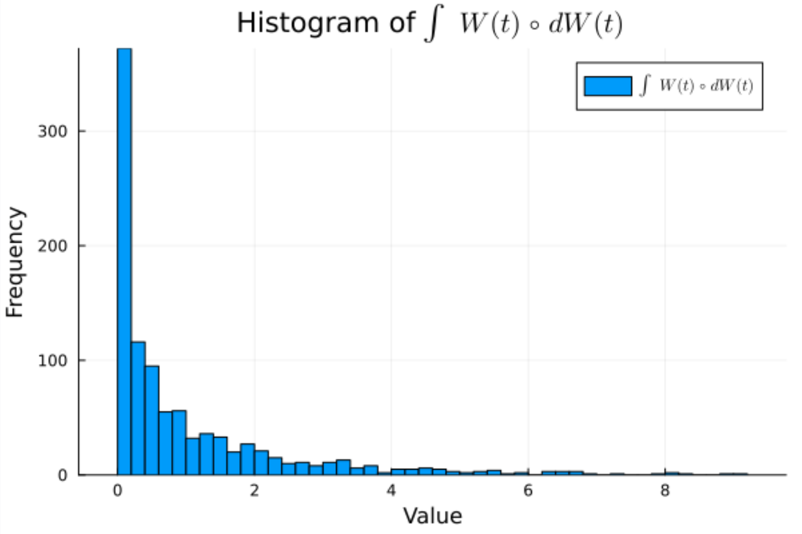

Figure: 2b

Question 2c

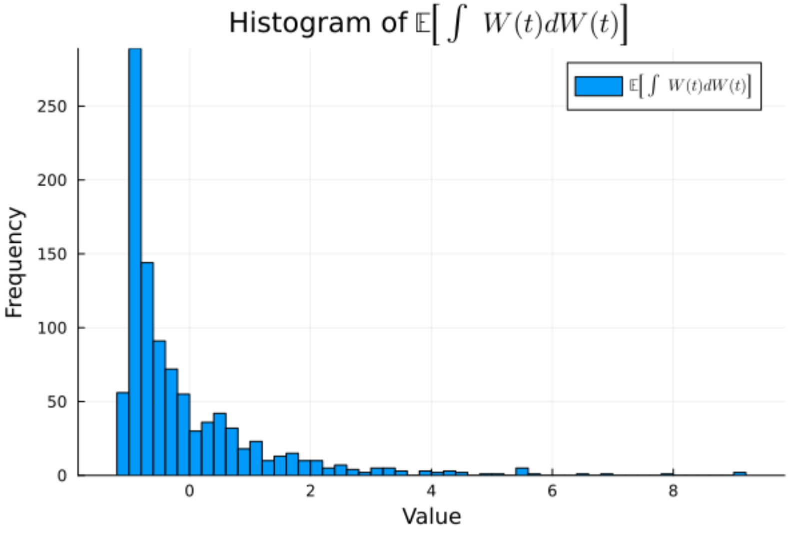

Figure: 2c

Question 2d

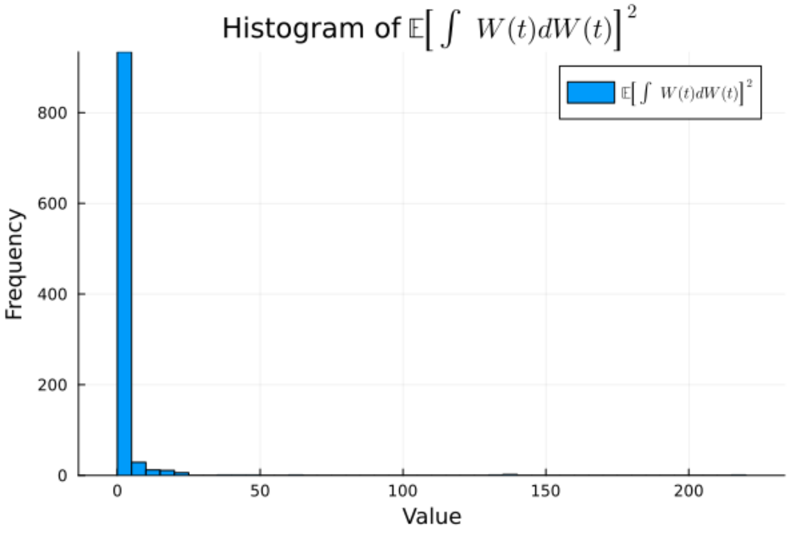

Figure: 2d

Question 2e

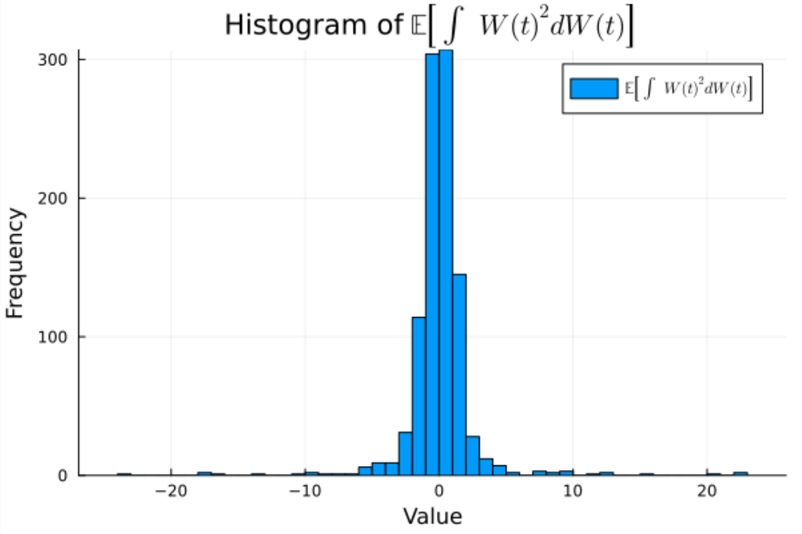

Figure: 2e

Question 2f

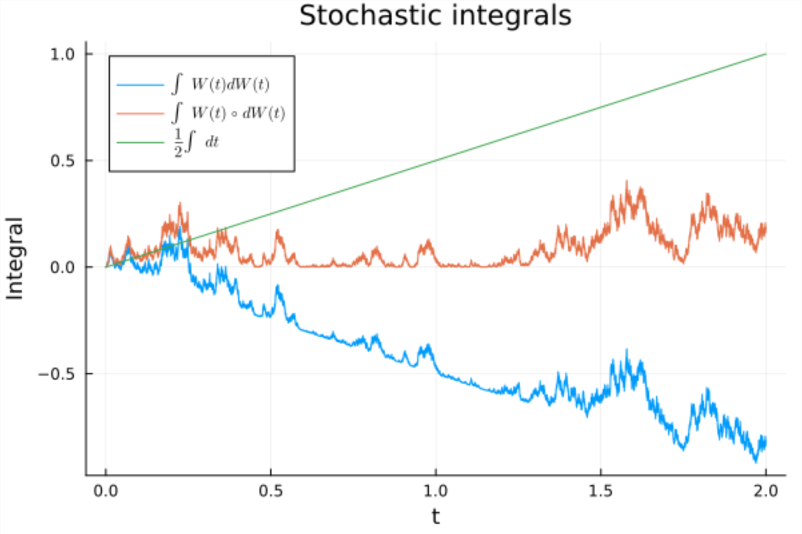

Figure: 2f

Question 3

Question 4

Question 5

Algorithm for computing the mean exit time function $\tau(x)$ for $x \in [0.5, 3]$:

for $i = 1$ to 1000 **do**
  $dw \leftarrow \sqrt{dt} \times \text{randn}()$
  $x \leftarrow x_0$
  for $j = 1$ to n **do**
    $x \leftarrow \text{euler\_maruyama}(x, dt, dw)$
    $dw \leftarrow \sqrt{dt} \times \text{randn}()$
    if $x < a$ or $x > b$ **then**
      $\text{exit\_times}[i] \leftarrow j \times dt$
      break
    end if
  end for
end for

Question 5 Continued

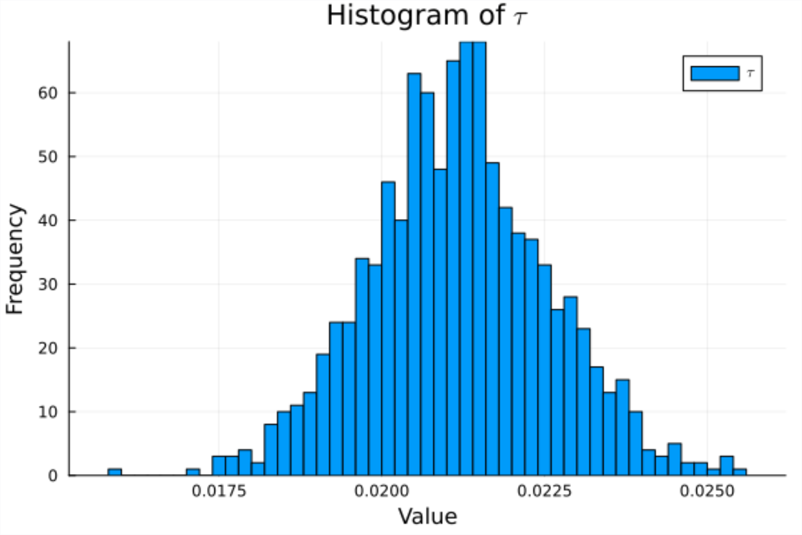

Figure: 2e

Questions?