MATH 512 - Project 2

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Question 1 (a)

We use the *Kolmogorov-Smirnov* test to test for the uniformity of the random numbers generated by the LCG. The test statistic is given by

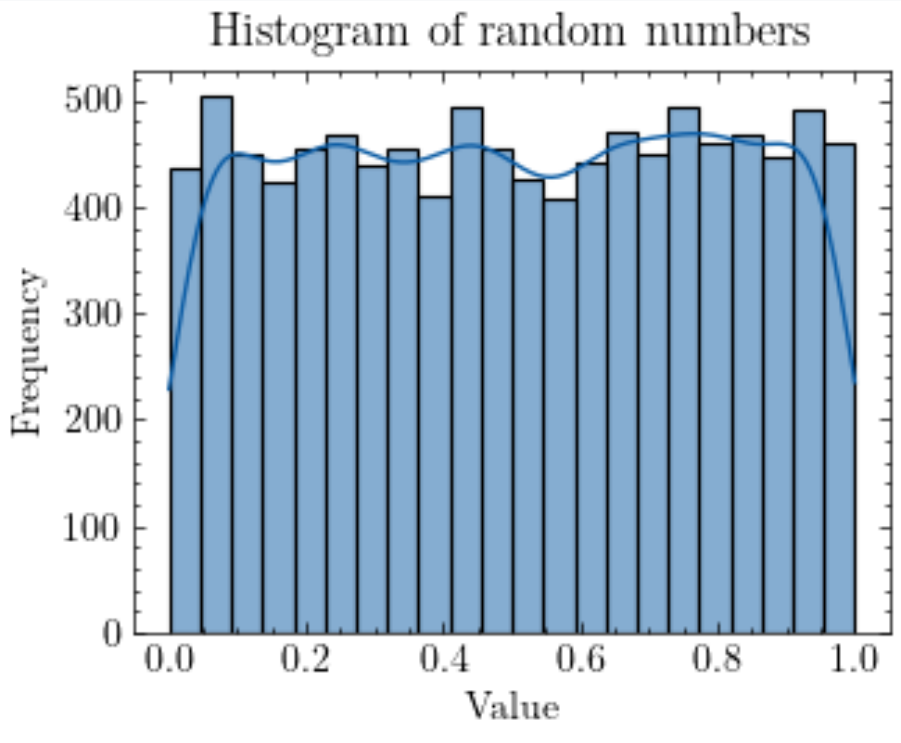
$$D_n = \max_{1 \leqslant i \leqslant n} \left(\frac{i}{n} - U_{(i)} \right) \vee \max_{1 \leqslant i \leqslant n} \left(U_{(i)} - \frac{i-1}{n} \right)$$

where $U_{(i)}$ is the i-th order statistic of the U_i's.

- $H_\circ$: the random numbers are uniformly distributed.

We find that $D_n = 0.0069$ and a p-value of 0.708. This means that we fail to reject $H_\circ$ at the 5% significance level and conclude that the random numbers are uniformly distributed.

Question 1 (a)


Histogram of random numbers
Frequency
Value
0
100
200
300
400
500
0.0
0.2
0.4
0.6
0.8
1.0

Question 1 (b)

Parameters: $a = 6, m = 11, x_0 = 3, c = 1$ and $n = 10$

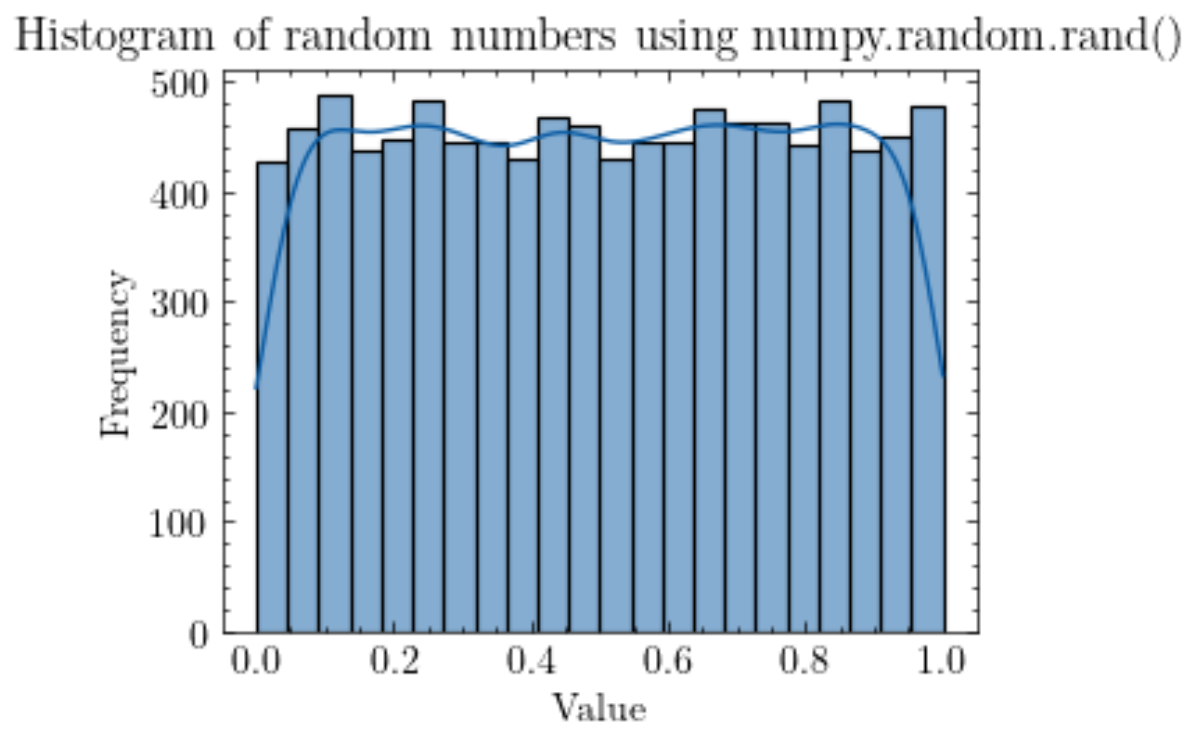
- The sequence is $\{3, 7, 9, 10, 5, 8, 4, 2, 1, 6\}$
- The period is 1

Parameters: $a = 6, m = 10, x_0 = 3, c = 1$ and $n = 10$

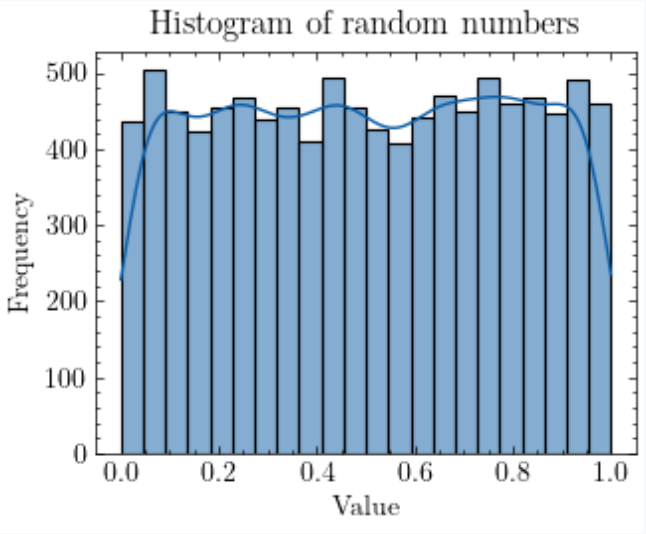
- The sequence is $\{3, 8, 8, 8, 8, 8, 8, 8, 8, 8\}$
- The period is 2

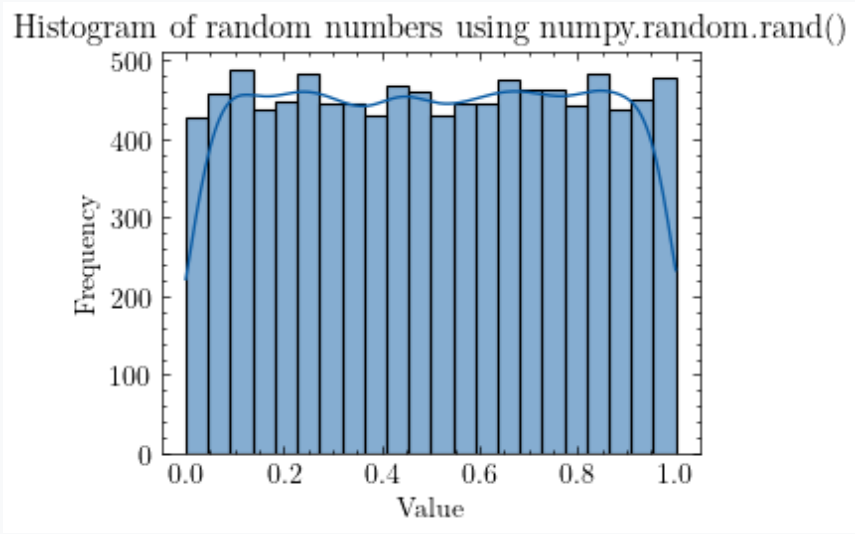
We notice that even a *small* change in the parameters results in a seemingly non-random sample.

Question 1 (c)


Histogram of random numbers using numpy.random.rand()
Frequency
Value
0
100
200
300
400
500
0.0
0.2
0.4
0.6
0.8
1.0

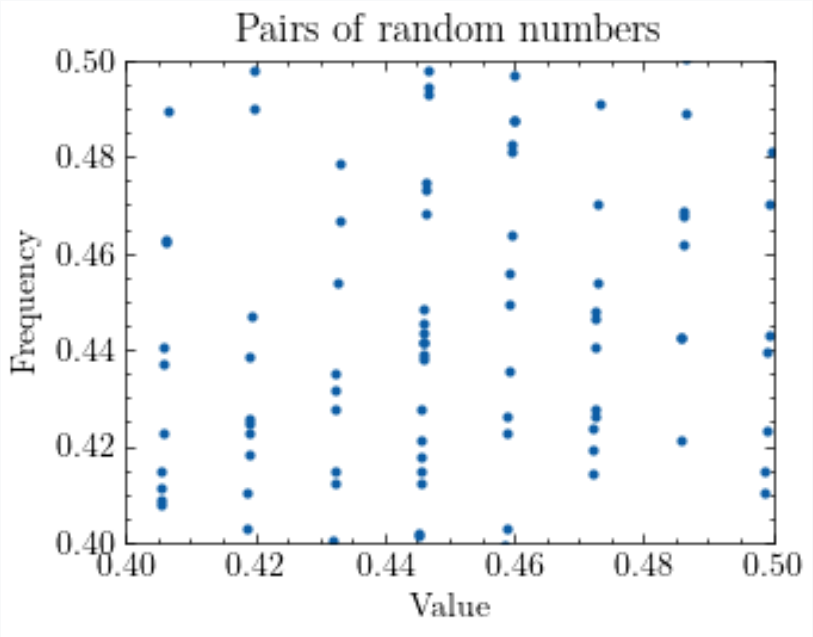
Question 1 (d)





- The two histograms look relatively similar, meaning both look relatively uniform.

Question 1 (e)

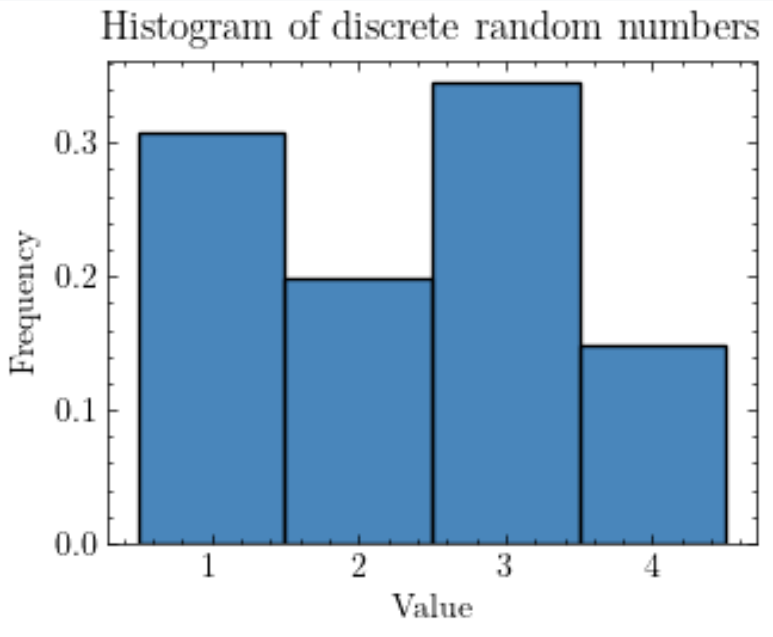


- I'm not seeing much of a pattern...

Question 1 (f)

Disadvantages of LCG:

- It can appear random with the right set of parameters, but as we saw, it can get "stuck" in a loop.
- The randomness depends on the bit position of the seed.
- The randomness depends on the choice of parameters.


Histogram of discrete random numbers
Frequency
0.0
0.1
0.2
0.3
1
2
3
4
Value

Questions?