MATH512 - Project 4

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April 2024

Question 1

Refer to figure **??** and **??**.
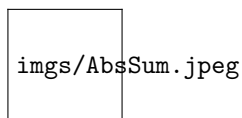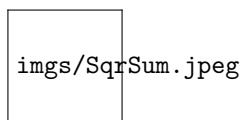

Figure 1: $|\delta W_i|$

Figure 2: δW_i^2

Notice that as the L parameter increases, the $|\delta W_i|$ term is unbounded while δW_i^2 converges to 2 in probability.
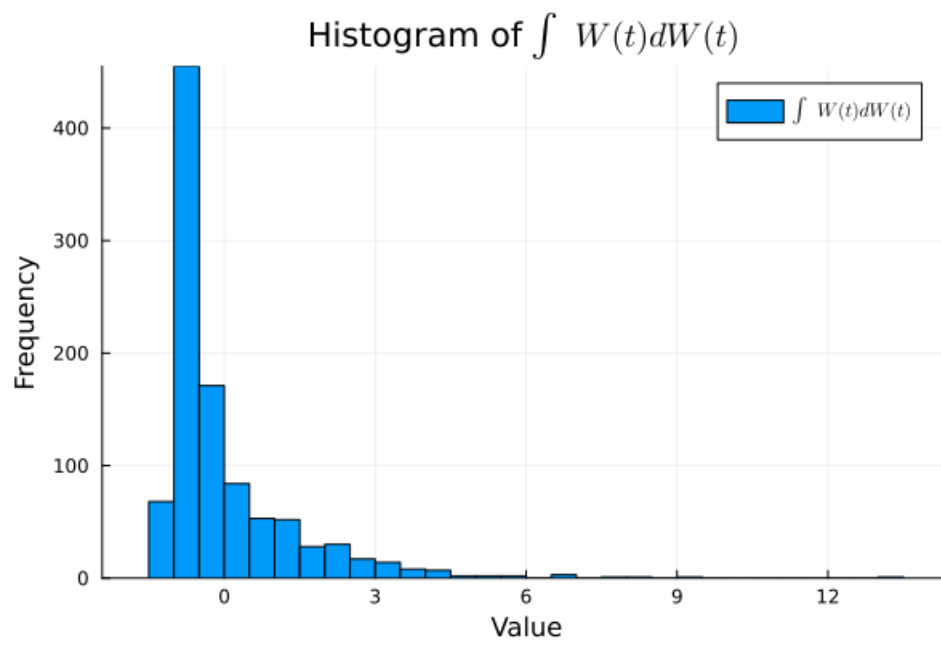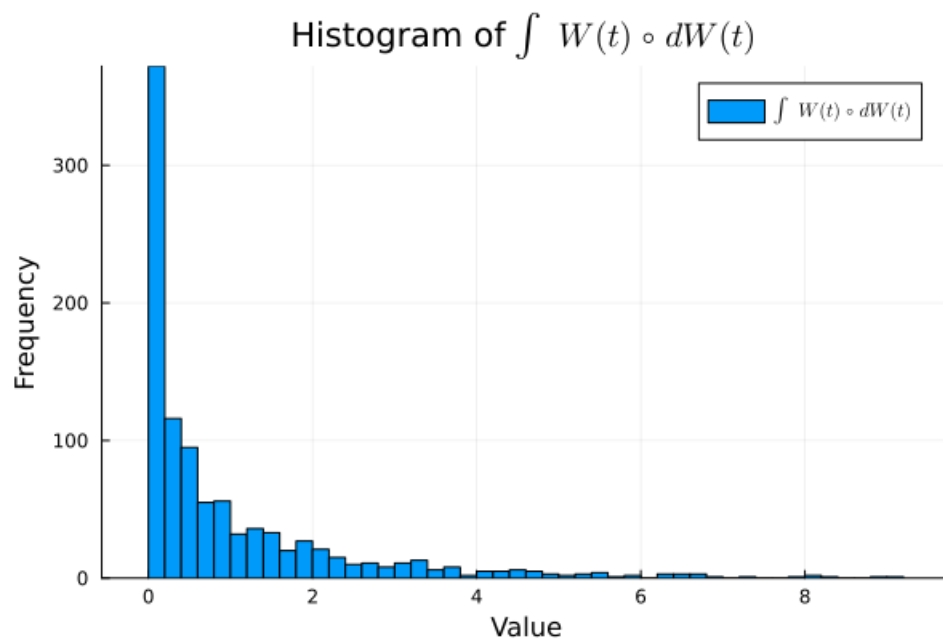
Question 2
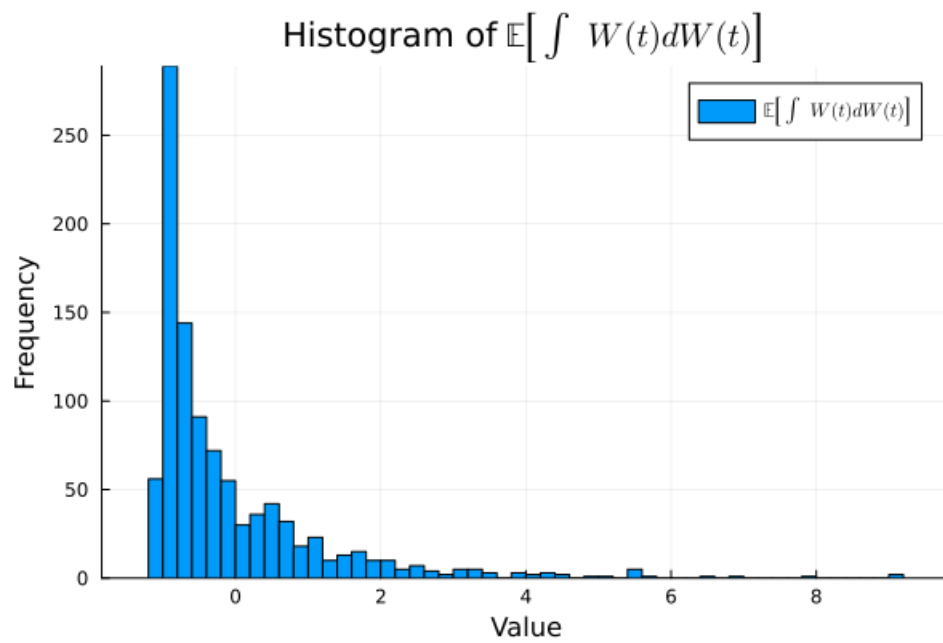

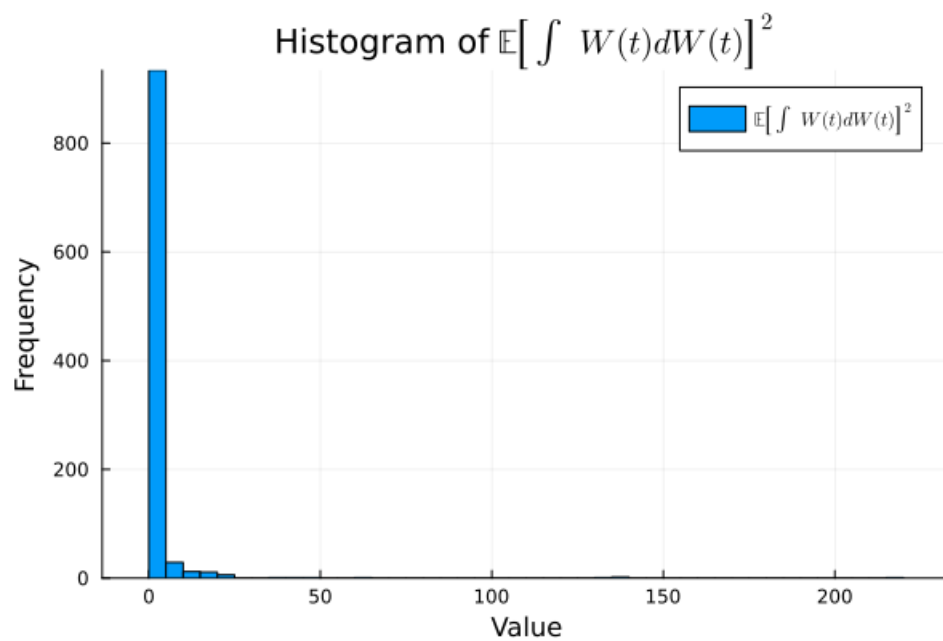Figure 3: 2a

1.

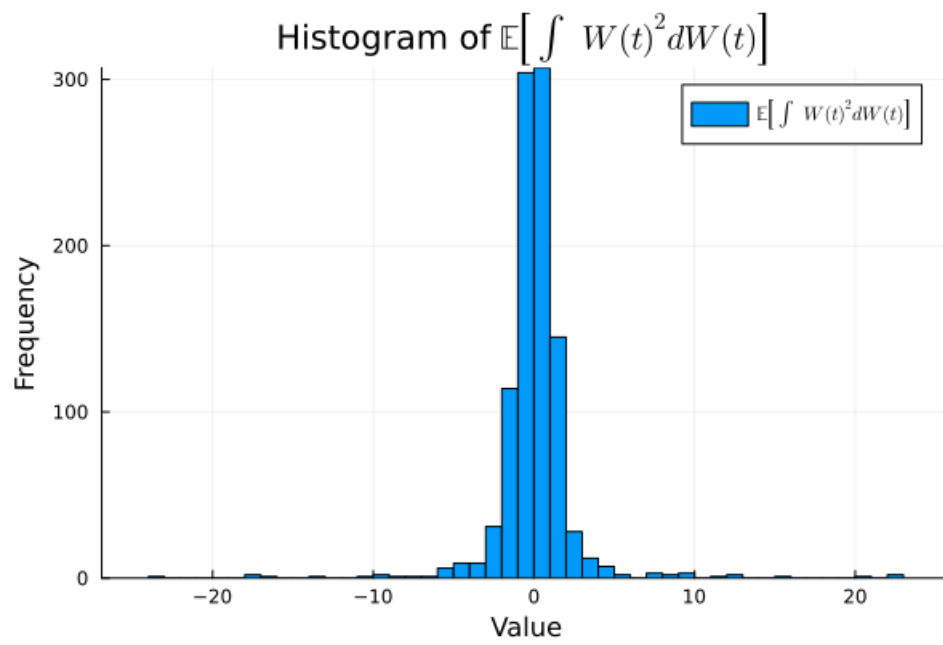Figure 4: 2b

2.

Figure 5: 2c

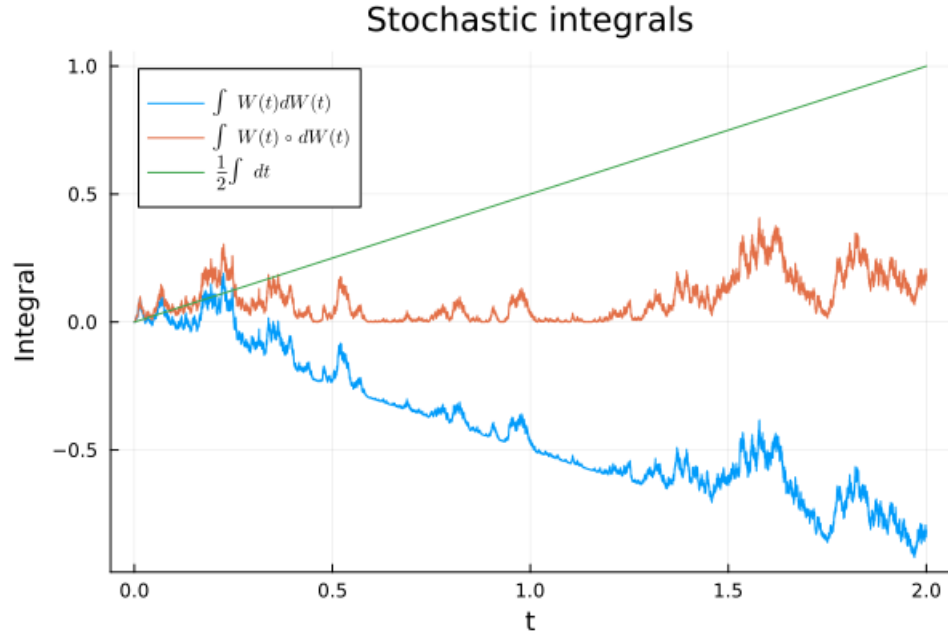3.

Figure 6: 2d

4.

Figure 7: 2e

5.

Figure 8: 2f

6.

Question 3

Objective: Analyze the weak and strong order of convergence of the EM method for solving the given SDE:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t), \quad X(0) = 3, \quad \mu = 2, \quad \sigma = 0.10$$

- **Weak order of convergence equal to 1:** to show that

$$|E[X_1] - E[X(1)]| = C\Delta t$$

- **Strong order of convergence equal to 0.5:** to show that

$$E|X_1 - X(1)| = C\Delta t^{0.5}$$

where X_1 stands for the exact solution and X(1) stands for the estimated solution.

Weak Order of Convergence focuses on the **expected values** of the numerical solution compared to the exact solution

$$|E[X_1] - E[X(1)]| = C\Delta t$$

- A plot of the error versus Δt for the weak order of convergence of 1.

imgs/fig1.png

Strong Order of Convergence to 0.5 is concerned with the **pathwise accuracy**, evaluating how closely the numerical solution follows individual realizations of the exact solution.

$$E|X_1 - X(1)| = C\Delta t^{0.5}$$

- A plot of the error versus $\Delta t^{0.5}$ for the strong order of convergence of 0.5.
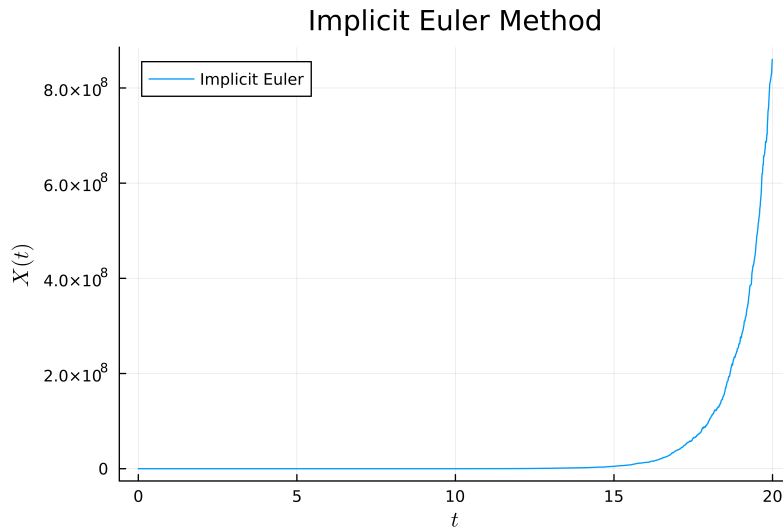
imgs/fig2.png

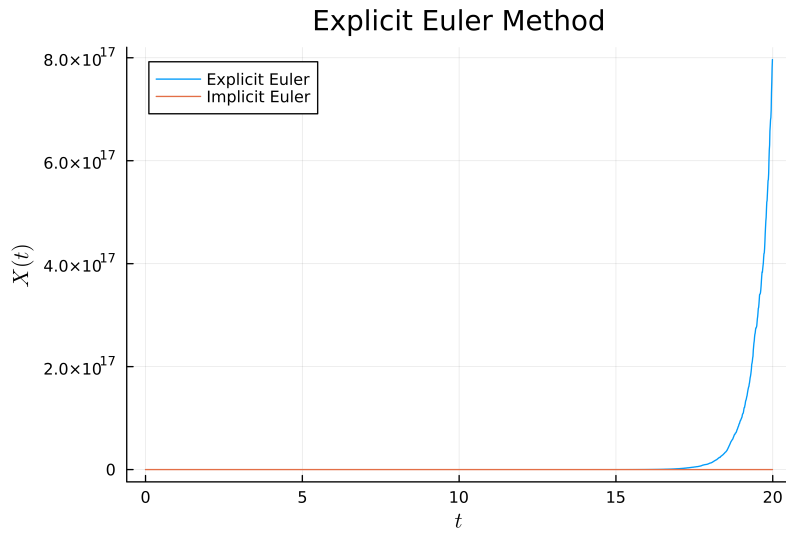# Question 4

Figure 9: Implicit Euler
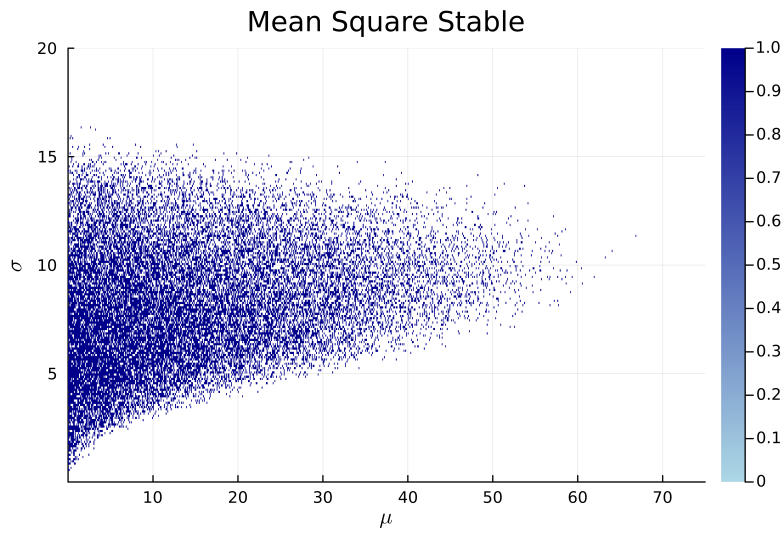
Figure 10: Explicit Euler

Figure 11: Mean Square Stable Values

Question 5

An algorithm to simulate exit times for the SDE:

1: Choose a step size Δt
2: Choose some paths, M

3: **for** $s = 1$ to M **do**
4:      Set $t_n = 0$ and $X_n = X_0$
5:      **while** $X_n > a$ and $X_n < b$ **do**
6:          Compute a $N(0,1)$ sample ξ_n
7:          Replace X_n by $X_n + \mu X_n \Delta t + \sigma X_n \xi_n$
8:          Replace t_n by $t_n + \Delta t$
9:      **end while**
10:     Set $T_s^{exit} = t_n - 1/2 \Delta t$
11: **end for**
12: Set $a_M = \frac{1}{M} \sum_{s=1}^{M} T_s^{exit}$
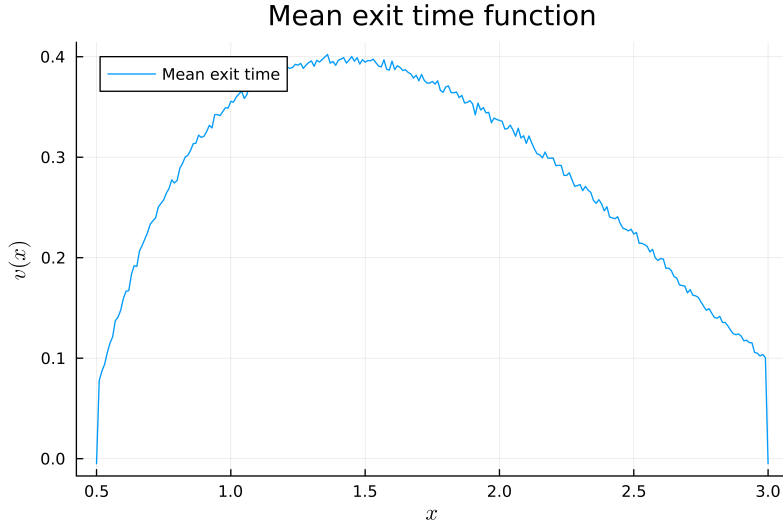13: Set $b_M^2 = \frac{1}{M-1} \sum_{s=1}^{M} (T_s^{exit} - a_M)^2$

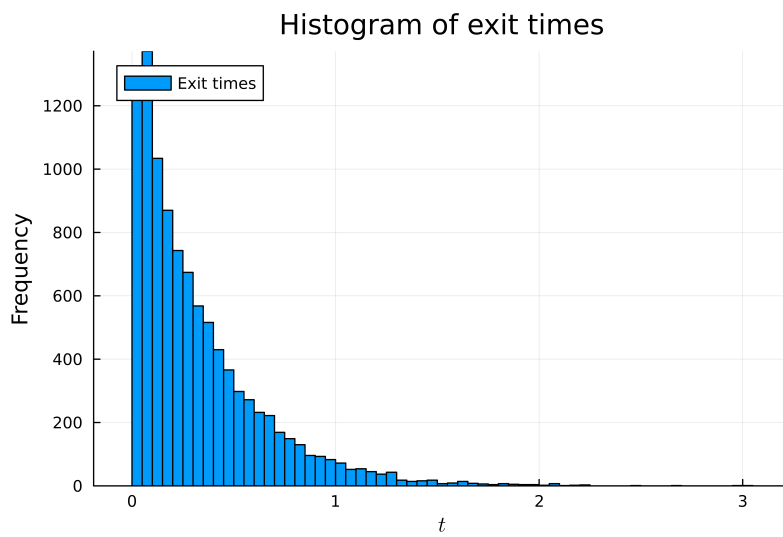## Mean exit time function

Figure 12: 5

Figure 13: 5