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Zipline Optimization Problem

In short, the objective of this assignment is to determine the wire rope tension and vertical drop for a zipline such that it will maximize the speed attained by a 70 kg rider along the ride, while ensuring that this rider and riders of other sizes reach the end of the zipline at a safe speed.

This entails selecting the wire rope tension and vertical drop such that the following conditions and constraints are met:

- The vertical drop that the riders experience while traversing a total of 500 m horizontally cannot be less than 10 m, nor greater than 40 m;
- The wire rope tension cannot exceed the safety limit of 50 kN (and if it were zero or negative, would be physically impossible);
- A 30 kg, 70 kg, and 140 kg rider must each traverse the entire length of the zipline, and reach the end at a speed no greater than the safety limit of 5 m/s;
- And the 70 kg rider's speed, at any point except the zipline exit, should reach the highest value that it can while all of the above conditions are also met.

By making use of the equations and background context discussed in Mungan and Lipscombe's paper [1] on the trajectory of a rider traversing a zipline, one can use computational software such as MATLAB to solve systems of these equations for the paths taken by each of the three riders, then take the derivative of their paths and solve for the velocities that they experience along the way, by passing permissible combinations of the wire rope tension and the vertical drop into an optimization algorithm.

My approach to solving this problem was to compute the paths and squared velocities for all three riders simultaneously, by employing for loops in MATLAB, in order to ensure that all the velocity constraint was met for all three before obtaining an optimal solution. In a master script file, I called fmincon on a function file that computed those paths and squared velocities, including by passing in lower and upper bounds consisting of the minimum and maximum permissible values, respectively, for the wire rope tension and vertical drop guesses that fmincon was performing. The function file outputs the negative of the maximum velocity achieved by the 70 kg rider, after passing through penalty statements that subtract a quantity proportional to the disparity between the other riders' exit velocities and the constraint of 0-5 m/s (in cases where they do not meet these constraints), allowing fmincon to "minimize" this quantity and thus obtain the wire rope tension and vertical drop that maximize that speed.

After obtaining the optimal values, I then passed in them into a function file partially identical to the other function file, that neglected the velocity constraints and simply computed the paths and squared velocities, but plotted these as requested by the assignment (Figures 1 and 2), and outputted the maximum and exit velocities experienced by all riders to be printed back in the master script to verify that the velocity constraints were still met. Finally, the master script plots a surface of the maximum velocities attained by the 70 kg rider for numerous combinations of wire rope tensions and vertical drops, to prove that there are no maxima in the permissible solution space larger than the optimal value identified by running fmincon (Figure 3).

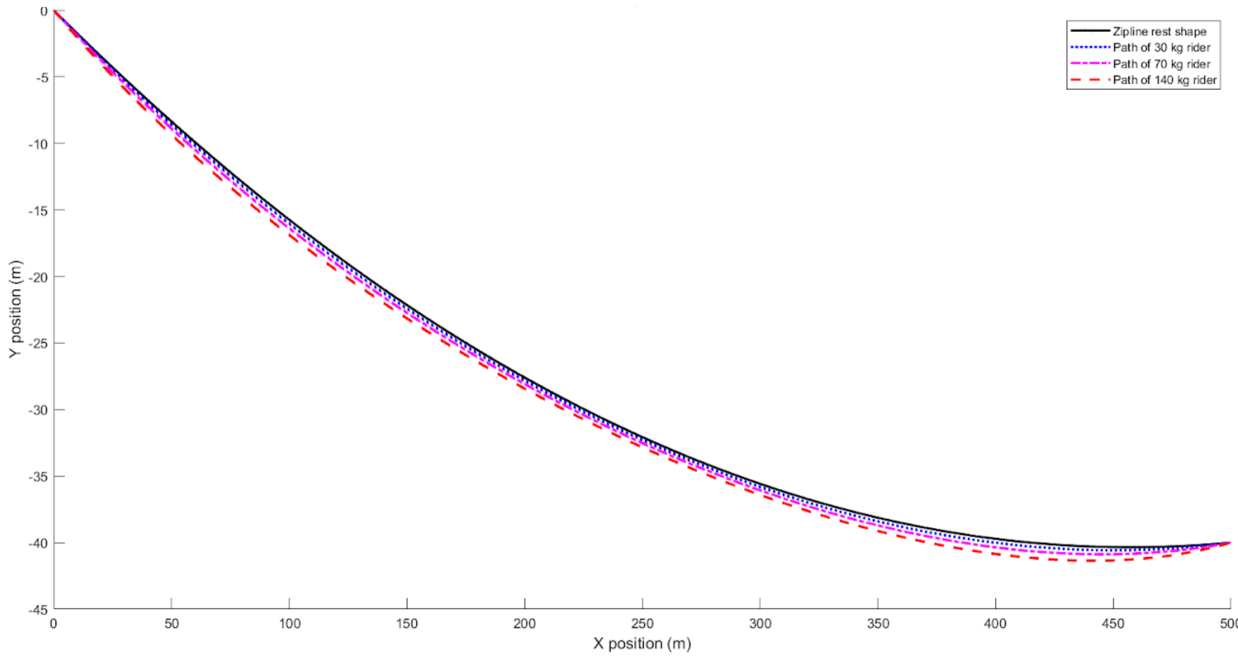

Figure 1. Rider paths at optimal parameters, with comparison to riderless zipline's shape.

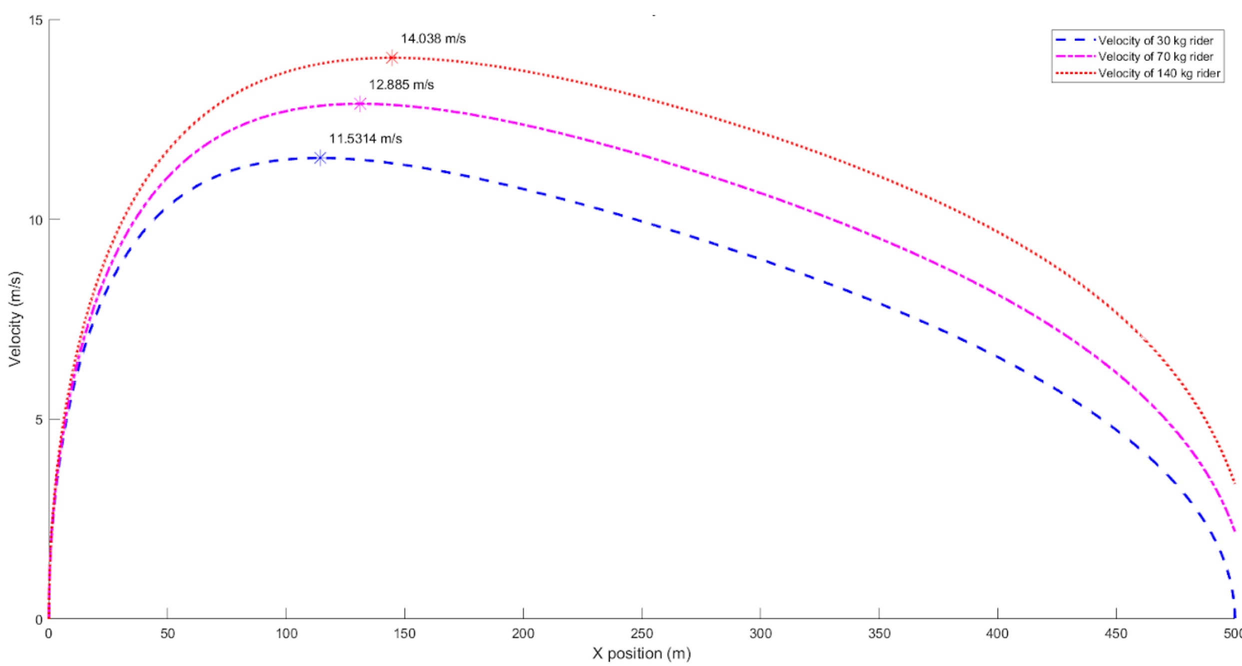

Figure 2. Rider velocities at optimal parameters, with maximum velocities highlighted.

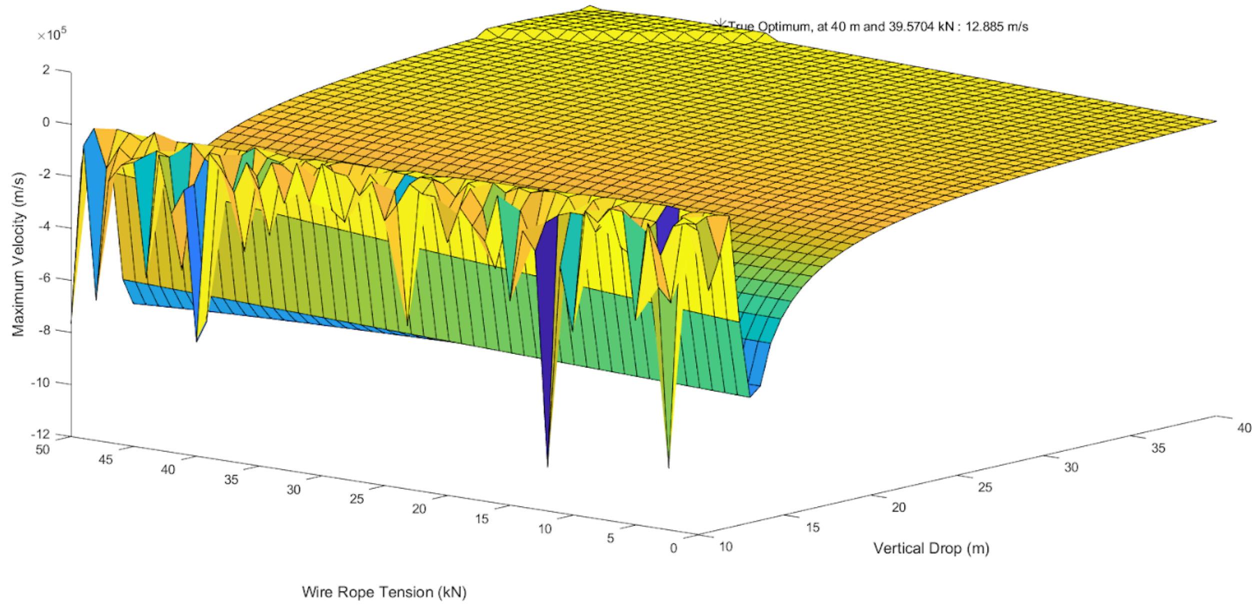

Figure 3. Maximum velocity attained by 70 kg rider for various permissible tensions and drops. Note that the optimal values determined by fmincon achieve a velocity just above the highest point on the feasible portion of the plane (where the riders reach the end at acceptable speeds).

So, as in Figure 4 we find that the optimal conditions are a vertical drop of 40 m and a wire rope tension of 39570.4454 kN, and the 70 kg rider attains a maximum velocity of 12.885 m/s, while all three riders reach the end at a speed between 0 and 5 m/s. I am confident that these are indeed the optimal values, because I performed the minimization procedure with several different starting guesses spanning most of the feasible solution space (i.e. 15 m/15 kN, 25 m/25 kN, 35 m/35 kN, 40 m/45 kN), and each time the values converged on 40 m and 39.57 kN, to at least 6 significant figures. Furthermore, while it is difficult to see without panning around the surface plot depicted in Figure 4, the true maximum velocity lies just above and next to the nearest test point, 40 m and 40 kN, which is itself the highest point on the elevated plateau in the top center of Figure 3. The small inflection point at exactly 40 m and 50 kN is an edge case where the riders exceed the permissible exit velocity of 5 m/s. The large flat area that begins to curve down and then spike up (due to the penalty behavior) at small vertical drops demonstrates that this is a region where the maximum velocity of the 70 kg rider was artificially decreased, meaning at least one of the riders did not reach the end of the zipline. Finally, that the exit velocity of the 30 kg rider reaches just negligibly above zero (and is only nonzero due to rounding errors) shows that this optimal solution is truly at the boundaries of possible solutions.

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For a horizontal span of 500 m and wire rope weight per length of 15 N/m,
and with an optimum vertical drop of 40 m and wire rope tension of 39570.4454 kN:
A 30 kg rider experiences a maximum velocity of 11.5314 m/s and an exit velocity of 8.3464e-05 m/s.
A 70 kg rider experiences a maximum velocity of 12.885 m/s and an exit velocity of 2.1837 m/s.
A 140 kg rider experiences a maximum velocity of 14.038 m/s and an exit velocity of 3.3788 m/s.
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Figure 4. Print statement showing maximum and exit velocities achieved by each rider.

References

[1] C. E. Mungan and T. C. Lipscombe. “Traveling along a zipline,” *Lat. Am. J. Phys. Educ. Vol 5, No. 1, 0DUFK 2011*. 2011. Available: http://www.lajpe.or. ISSN: 1870-9095. [Accessed April 24, 2024].