CSCE 625 Homework #2

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November 20, 2017

1)

- $\forall x.tomato(x) \implies fruit(x) \vee vegetable(x)$
- $\exists x.mushroom(x) \wedge poisonous(x)$
- $\forall x.sides(x, 3) \wedge sumOfAngles(x, 180) \implies triangle(x)$
- $\forall x.plant(x) \wedge polinated(x) \implies produce(x, seeds)$
- $\forall x.movie(x) \wedge made(StephenKing, x) \wedge \neg is(x, Cujo) \implies favorite(John, x)$
- $\forall x, y, z.game(z, football) \wedge greater(points(x), points(y)) \wedge IsTeam(x) \wedge IsTeam(y) \implies winner(x)$
- $\forall x, g.FordExplorer(x) \wedge GasTank(g, x) \wedge leq(PercentFuelLeft(g), 0.1) \implies WarningLight(x)$
- $\exists x, y.computer(x) \wedge computer(y) \wedge bought(Al, x) \wedge bought(Bob, y) \implies SameManufacturer(x, y)$
- $\forall x.laptop(x) \wedge sold(Dell, x, 2012) \implies geq(MemoryInGigs(x), 4)$

2)

1. $\forall x.P(x) \implies \big[\forall y.P(y) \implies P(f(x, y))\big] \wedge \big[\neg\forall y.Q(x, y) \implies P(y)\big]$
2. $\forall x.\neg P(x) \vee \big[\forall y.\neg P(y) \vee P(f(x, y))\big] \wedge \big[\neg\forall y.\neg Q(x, y) \vee P(y)\big]$ (Remove implications)
3. $\forall x.\neg P(x) \vee \big[\forall y.\neg P(y) \vee P(f(x, y))\big] \wedge \big[\exists y.Q(x, y) \wedge \neg P(y)\big]$ (Negate universal quantifier)
4. $\forall x.\neg P(x) \vee \big[\forall y.\neg P(y) \vee P(f(x, y))\big] \wedge \big[Q(x, f(x)) \wedge \neg P(f(x))\big]$ (Skolemize y)
5. $(\neg P(x) \vee (\neg P(y) \vee P(f(x, y)))) \wedge (\neg P(x) \vee (Q(x, (f(x)) \wedge \neg P(f(x)))))$ (Remove universal quantifier and distribute terms)
6. $(\neg P(x) \vee (\neg P(y) \vee P(f(x, y)))) \wedge ((\neg P(x) \vee Q(x, (f(x)) \wedge (\neg P(x) \vee \neg P(f(x))))))$ (Distribute the $\vee$ over the $\wedge$)

3)

a)

1. $pompeian(Marcus)$
2. $\forall x.pompeian(x) \implies roman(x)$
3. $ruler(Caesar)$
4. $\forall x.roman(x) \implies (loyal(x, Caesar) \wedge \neg hate(x, Caesar)) \vee (\neg loyal(x, Caesar) \wedge hate(x, Caesar))$

5. $\forall x.\exists y.loyal(x, y)$
6. $\forall x, y.person(x) \land AttemptAssassination(x, y) \land ruler(y) \implies \neg loyal(x, y)$
7. $AttemptAssassination(Marcus, Caesar)$
8. $person(Marcus)$

b)

9. $Roman(Marcus)$ $\big[GMP|1, 2, \theta = x/Marcus\big]$
10. $\neg loyal(Marcus, Caesar)$ $\big[GMP|3, 6, 7, 8\theta = x/Marcus, y/Caesar\big]$
11. $(loyal(Marcus, Caesar) \land \neg hate(Marcus, Caesar)) \lor (\neg loyal(marcus, Caesar) \land hate(marcus, Caesar))$
    $\big[GMP|4, 8, \theta = x/Marcus\big]$
12. (a) $loyal(Marcus, Caesar) \lor (\neg loyal(Marcus, Caesar) \land hate(Marcus, Caesar)) \big[Distributive : 10\big]$
    (b) $\neg hate(Marcus, Caesar) \lor (\neg loyal(Marcus, Caesar) \land hate(Marcus, Caesar)) \big[Distributive : 10\big]$
13. We can distribute the $\lor$ further in both 11a and 11b and then split the clauses up to end with the following:
    (a) $loyal(Marcus, Caesar) \lor \neg loyal(Marcus, Caesar)$
    (b) $loyal(Marcus, Caesar) \lor hate(Marcus, Caesar)$
    (c) $\neg hate(Marcus, Caesar) \lor \neg loyal(Marcus, Caesar)$
    (d) $\neg hate(Marcus, Caesar) \lor hate(Marcus, Caesar)$

    For all of the clauses in 12 to be true, $hate(Marcus, Caesar)$ must be true since we know $\neg loyal(Marcus, Caesar)$ is true.
14. $hate(Marcus, Caesar)$ [From the above reason.]

4)

a)

1. $obs(1, Y)$
2. $obs(2, W)$
3. $obs(3, Y)$
4. $lab(1, W)$
5. $lab(2, Y)$
6. $lab(3, B)$
7. $\forall x.lab(1, x) \implies \neg Cont(1, x)$
8. $\forall x.lab(2, x) \implies \neg Cont(2, x)$
9. $\forall x.lab(3, x) \implies \neg Cont(3, x)$
10. $\forall x.obs(1, x) \implies (cont(1, B) \lor cont(1, x))$
11. $\forall x.obs(2, x) \implies (cont(2, B) \lor cont(2, x))$
12. $\forall x.obs(3, x) \implies (cont(3, B) \lor cont(3, x))$

13. $\forall x.cont(x, Y) \vee cont(x, W) \vee cont(x, B)$
14. $\forall x.cont(1, x) \implies \neg cont(2, x) \wedge \neg cont(3, x)$
15. $\forall x.cont(2, x) \implies \neg cont(1, x) \wedge \neg cont(3, x)$
16. $\forall x.cont(3, x) \implies \neg cont(1, x) \wedge \neg cont(2, x)$

b)

17. $cont(2, B) \vee cont(2, W)$ [GMP — 2, 11, $\Theta = \{x/W\}$]
18. $\neg cont(3, B)$ [GMP — 6, 9, $\Theta = \{x/B\}$]
19. $cont(3, B) \vee cont(3, Y)$ [GMP — 3, 12, $Theta = \{x/Y\}$]
20. $cont(3, Y)$ [Res. on 18, 19]
21. (a) $\neg cont(1, Y)$ [GMP — 16, 20, $\Theta = \{x/Y\}$]
    (b) $\neg cont(2, Y)$ [GMP — 16, 20, $\Theta = \{x/Y\}$]
22. $cont(1, B) \vee cont(1, Y)$ [GMP — 1, 10, $\Theta = \{x/Y\}$]
23. $cont(1, B)$ [Res. on 21a, 22]
24. (a) $\neg cont(2, B)$ [GMP — 14, 23, $\Theta = \{x/B\}$]
    (b) $\neg cont(3, B)$ [GMP — 14, 23, $\Theta = \{x/B\}$]
25. $cont(2, W)$ (Res. on 17, 24a)

5)

The following is an FOL knowledge base:

1. $\forall i, j.TowInARow(O, i) \wedge b(i, j) \implies CanWin(O, i, j)$
2. $\forall i, j.TowInACol(O, j) \wedge b(i, j) \implies CanWin(O, i, j)$
3. $\forall i, j.TowinADiag(O, i, j) \wedge b(i, j) \implies CanWin(O, i, j)$
4. $\forall i, j.TwoInARow(X, i) \wedge b(i, j) \implies CanWin(X, i, j)$
5. $\forall i, j.TowInACol(X, j) \wedge b(i, j) \implies CanWin(X, i, j)$
6. $\forall i, j.TowInADiag(X, i, j) \wedge b(i, j) \implies CanWin(X, i, j)$
7. $\forall i, j.CanWin(X, i, j) \implies Move(X, i, j)$
8. $\forall i, j.\neg CanWin(X, i, j) \implies CannotWin(X, i, j)$
9. $\forall i, j.CanWin(O, i, j) \wedge CannotWin(X, i, j) \implies ForcedMove(X, i, j)$
10. $\forall i, j.ForcedMove(X, i, j) \implies Move(X, i, j)$