Exercise in *Localization, Navigation, and Mapping*
GPS Signals: Code Phase and Doppler Shift

Sivan Toledo

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A note: this exercise guides you towards an implementation in Matlab. It may be possible to implement the code in another environment, but it will probably require a bit of research to find equivalent functionality.

This problem shows how to find the code phase of GPS signals and how to decode the data that they encode.

1. We start by reading a file, `rtlsdr-1575_42.wav`, containing complex samples of GPS signals. The following code snippet reads the samples, which consist of 32ms worth of data, from the file. The first two lines specify the sampling rate and the frequency to which the GPS L1 frequency, 1575.42 MHz, was shifted (zero in this case). Both parameters depend on how the samples were recorded.

```
sampleRate = 2048000;
foffset = 0;
[samples,Fs] = audioread('rtlsdr-1575_42.wav');
ms = 32
s = samples(1:ms*sampleRate/1000,1) ...
    + sqrt(-1)*samples(1:ms*sampleRate/1000,2);
```

2. Create a replica of 1 ms of the signal transmitted by one particular satellite, satellite number 30 whose signal does appear in our recording. We create the replica in three steps. First, we call a function to produce the 1023-chip pseudo-random Gold code c transmitted by the satellite. The `cacode` function is not built into Matlab but it is provided with this exercise. The function returns a 0/1 vector and we need a ±1 vector, so we scale and shift the output.

```
SV = 30; % index of space vehicle (satellite)
code = (2*cacode(SV)-1)';
```

Next, implement a function `waveformBinary(code,chipRate,sampleRate)` that creates a 1 ms real signal $g(t) = c_{\lfloor t \times 1{,}023{,}000) \rfloor}$ that is sampled at the same sample rate. The constant 1,023,000 is the chip rate in GPS C/A signals. Finally, use the provided modulator function to create a sampled PSK signal with carrier frequency `foffset` plus the Doppler shift that is caused by the movement of the satellite.

```
doppler=-1000;
replica = modulator('p',g,sampleRate,foffset+doppler);
```

3. Correlate the replica with the samples using `xcorr` and plot the absolute value of the cross-correlation. The results should resemble the results shown in the left side of Figure 1. Repeat the computation, but with the replica of satellite 29, whose signal is not present in our samples, and with satellite 30 but without the Doppler shift. The results should look like the graph on the right side of Figure 1.

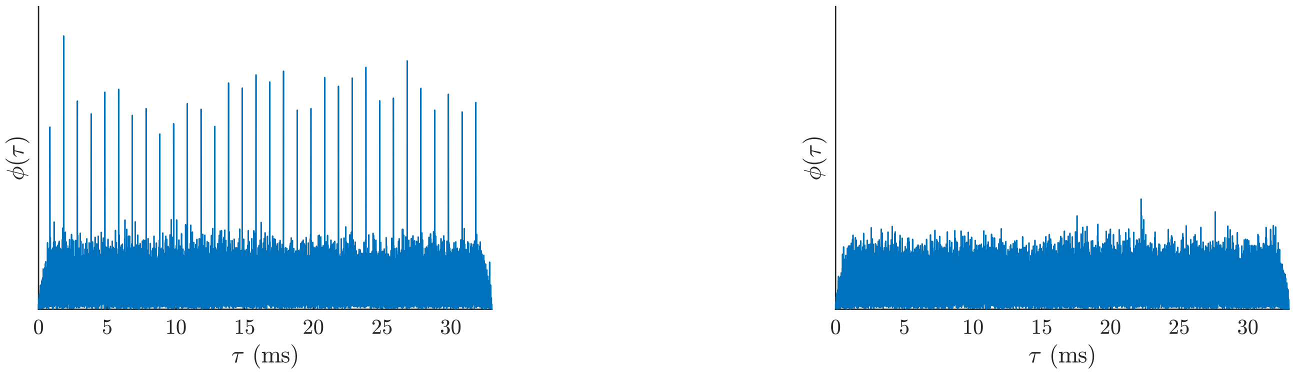

Figure 1: The cross-correlation of 32 ms GPS signal with a replica of the code of a particular satellite. On the left the replica is of satellite 30, whose signal is represented in the samples. The replica was shifted by −1 kHz to compensate for Doppler shift. On the left the replica is of satellite 29, whose signal is not present in the samples.

Figure 2: Doppler-shift search for satellite 30 (left) and 29 (right).

4. Implement a Doppler search. Use a loop to cross-correlate replicas of the signal of satellite 30 with Doppler shifts ranging from −7500 Hz to 7500 Hz in steps of 250 Hz. (In practice, steps of 500 Hz are sufficiently small; a range of ±5 kHz is sufficient for stationary receivers and ±10 kHz is sufficient even for high-speed receivers.) Save the maximum of the absolute value of the correlator at each frequency, plot this value as a function of frequency, and plot the output of the correlator at the frequency at which the absolute value is highest. This graph should look like the one in Figure 2.

5. Iterate over satellites 1 through 32, estimate the Doppler shift for each one, correlate the samples with the satellite's code at that shift, and plot the absolute value of the cross correlation. The function `drawnow` can be used to force the graphs to show while the script is still running, and the function `pause` can ensure that they stay on the screen long enough for you to inspect them. Determine by visual inspection the set of satellites whose signal is clearly present in the samples. (When you do not see clear peaks in the cross correlation every 1ms, the signal of the satellite might not be present, or it might be too weak to create clear peaks in the cross correlation vector.)

6. Develop a criterion to decide whether the signal of a satellite is present or not in a vector of samples. Test your criterion on the samples. Does it identify the same set of satellites that you identified visually?