Math 311W
Worksheet for March 23 -24

Jacob Harkins
jah6863@psu.edu

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Exercise 1) Let A and B be two subsets of U. Prove that: $A \subseteq B \iff A \cap B = A$.
First we will show $A \cap B = A$ by double containment, assuming $A \subseteq B$:

$$\text{Let } x \in (A \cap B)$$
$$\implies x \in A$$
$$\text{Therefore, } A \cap B \subseteq A$$

$$\text{Let } x \in A$$
$$\implies x \in B$$
$$\implies x \in (A \cap B)$$
$$\text{Therefore, } A \subseteq A \cap B$$

Now, we will show $A \subseteq B$, assuming $A \cap B = A$:

$$\text{Let } x \in A$$
$$\implies x \in (A \cap B)$$
$$\implies x \in B$$
$$\text{Therefore, } A \subseteq B$$

$\square$

Exercise 2) Let A and B be two subsets of U. Prove: If $B' \subseteq A'$, then $A \subseteq B$.

$$\text{Let } x \in A$$
$$\implies x \notin A'$$
$$\implies x \notin B'$$
$$\implies x \in B$$
$$\text{Therefore, } A \subseteq B$$

$\square$

Exercise 3) Let A = {3,5,7}, B = {1,2,3,5,9}, C = {1,2,7,9} and U = {1,2,3,4,5,6,7,8,9,10}.

(a) Find $A \cup B \cup C$

$$A \cup B \cup C = \{1, 2, 3, 5, 7, 9\}$$

(b) Find $(A \cup B \cup C)'$

$$(A \cup B \cup C)' = \{4, 6, 8, 10\}$$

(c) Find A', B', C'

$$A' = \{1, 2, 4, 6, 8, 9, 10\}$$
$$B' = \{4, 6, 7, 8, 10\}$$
$$C' = \{3, 4, 5, 6, 8, 10\}$$

(d) Is $(A \cup B \cup C)' = A' \cap B' \cap C'$?

$$\text{Yes}$$

Exercise 4) Let A = {a,b}. Find all of its subsets.

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

Exercise 5) Let A = {a,b,c,d}. Find all of its subsets.

$$\mathcal{P}(A) = \Big\{ \emptyset,$$
$$\{a\}, \{b\}, \{c\}, \{d\},$$
$$\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$$
$$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\},$$
$$\{a, b, c, d\} \Big\}$$

Exercise 6) Let A = {1,3,5} and B = {2,4,6}. Find A x B and B x A.

$$A \times B = \big\{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)\big\}$$
$$B \times A = \big\{(2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)\big\}$$

Exercise 7) Let A be a set. Prove that $\emptyset \times A = \emptyset$.

$$\emptyset \times A = \{(x, y) | x \in \emptyset \wedge y \in A\}$$
$$= \emptyset$$

$\square$

Exercise 8) If B $\subseteq$ D, then A x B $\subseteq$ A x D

$$\text{Let } (a, b) \in (A \times B)$$
$$\implies b \in B$$
$$b \in B \wedge B \subseteq D \implies b \in D$$
$$\implies (a, b) \in (A \times D)$$

$\square$