Math 311W

Worksheet for Wednesday, March18

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Exercise 1) Let A = {1,3,5,7,9}, B = {1,2,3,4,5}, C = {5,6,7,8,9} and U = {1,2,3,4,5,6,7,8,9,10}

(a) $A \cup (B \cup C)$

$$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

(b) $A \cup (B \cap C)$

$$A \cup (B \cap C) = A$$

(c) $(A \cup B) \cap (A \cup C)$

$$(A \cup B) \cap (A \cup C) = A \cup (B \cap C) = A$$

(d) $A \cap (B \cup C)$

$$A \cap (B \cup C) = A$$

(e) A'

$$A' = \{2, 4, 6, 8, 10\}$$

(f) B'

$$B' = \{6, 7, 8, 9, 10\}$$

(g) $A' \cup B'$

$$A' \cup B' = \{6, 8, 10\}$$

(h) $(A \cap B)'$

$$(A \cap B)' = \{2, 4, 6, 7, 8, 9, 10\}$$

Exercise 2) Prove the commutative property of intersection (that is $A \cap B = B \cap A$), using the double inclusion method.

$$\begin{aligned} &\text{Let } x \in A \cap B \\ &\Longrightarrow x \in A \text{ and } x \in B \\ &\Longrightarrow x \in B \cap A \\ \text{Therefore, } &A \cap B \subseteq B \cap A \\ &\text{Let } n \in B \cap A \\ &\Longrightarrow n \in B \text{ and } n \in A \\ &\Longrightarrow n \in A \cap B \\ \text{Therefore, } &B \cap A \subseteq A \cap B \end{aligned}$$

□

Exercise 3) Prove the distributive property of union over intersection $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
First we'll show $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$:

$$\begin{aligned} &\text{Let } x \in A \cup (B \cap C) \\ &\Longrightarrow x \in A \vee x \in B \cap C \end{aligned}$$

If $x \in A$

$$\begin{aligned} &x \in (A \cup B) \wedge x \in (A \cup C) \\ \Longrightarrow &x \in (A \cup B) \cap (A \cup C) \end{aligned}$$

If $x \in B \cap C$

$$\begin{aligned} &x \in B \wedge x \in C \\ \Longrightarrow &x \in (A \cup B) \wedge x \in (A \cup C) \\ \Longrightarrow &x \in (A \cup B) \cap (A \cup C). \end{aligned}$$

So, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$
Now we'll show $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$:

$$\begin{aligned} &\text{Let } x \in (A \cup B) \cap (A \cup C) \\ &\Longrightarrow x \in (A \cup B) \wedge x \in (A \cup C) \\ &\Longrightarrow x \in A \vee (x \in B \wedge x \in C). \end{aligned}$$

If $x \in A$

$$x \in A \cup (B \cap C)$$

If $x \in B \wedge x \in C$

$$\begin{aligned} &x \in (B \cap C) \\ \Longrightarrow &x \in A \cup (B \cap C). \end{aligned}$$

So $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$
Therefore, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ □