Math 312
Worksheet for September 1

Jacob Harkins
jah6863@psu.edu

May 20, 2024

Exercise 1) Let x and y be two real numbers. Prove that $|xy| = |x||y|$

Proof by cases:

Case 1: $x \geq 0 \wedge y \geq 0$

$$\begin{aligned} |xy| &= xy \\ |x| = x \wedge |y| &= y \\ |xy| = |x||y| &= xy \end{aligned}$$

Case 2: $x < 0 \wedge y \geq 0$

$$\begin{aligned} & xy \leq 0 \\ \implies & |xy| = -(xy) \\ & |x| = -x \wedge |y| = y \\ \implies & |x||y| = -xy = |xy| \end{aligned}$$

Case 3: $x < 0 \wedge y < 0$

$$\begin{aligned} & |x| = -x \wedge |y| = -y \\ \implies & |x||y| = xy = |xy| \end{aligned}$$

□

Exercise 2) Find the required formula for the minimum of x and y and prove it is correct.

Formula = $\frac{x+y-|x-y|}{2}$

Let $x < y$ such that $y = x + L$ with $L > 0$

$$\begin{aligned} \frac{x+y-|x-y|}{2} &= \frac{x+x+L-|x-x+L|}{2} \\ &= \frac{2x+L-L}{2} \\ &= x = min(x, y) \end{aligned}$$

□

Exercise 3) Prove the statement "If the equality $|x+y| = |x|+|y|$ holds true, then x and y have the same sign."

Proof by contradiction:

Without loss of generality, let $x < 0 \wedge y > 0$

$$\begin{aligned} |-x+y| &= |-x| + |y| \\ |-x+y| &= x+y \\ -x+y = -x-y &\vee -x+y = x+y \end{aligned}$$

□

Exercise 4) Let x, y, L be three real numbers. Given that $|x-L| \leq 1, |y-L| \leq 3, |l| \leq 5$, obtain a numerical estimate for $|xy - L^2|$

$$\begin{aligned} |x-L| \times |y-L| &= |xy + xL - xL - L^2| \\ &\leq |x(y+L) - L(x+L)| \\ &= |-x(y-L) + L(x-L)| \\ &= 3|x| + 5 \\ &= 23 \end{aligned}$$

□
