Hw 9

3.9.1 a) $[A|I_m]$ RREF $\rightarrow [B|P] \rightarrow PA=B$

Because RREF keeps row relationships, $A \overset{row}{\approx} B$. row equivalency only exists if and only if $PA=B$ with P being a non singular matrix.

c) $PA = E_A$

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 7 \\ 1 & 2 & 3 & 6 \end{pmatrix} \begin{matrix} R_2 - 2R_1 \to R_2 \\ R_3 - R_1 \to R_3 \end{matrix} \begin{matrix} 1st \\ 2nd \end{matrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{matrix} R_1 + 4R_2 \to R_1 \\ -R_2 \to R_2 \\ 2R_2 + R_3 \to R_3 \end{matrix} \begin{matrix} 1st & 1\,4\,0 \\ 2nd & 0\,{-1}\,0 \\ 2nd & 0\,2\,1 \end{matrix} = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = E_A$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -5 & 2 & 1 \end{pmatrix} = P$$

3.9.2 b) $$A = \begin{pmatrix} 2 & 2 & 0 & -1 \\ 3 & -1 & 4 & 0 \\ 0 & -8 & 8 & 3 \end{pmatrix} \text{ RREF} \rightarrow \begin{pmatrix} 1 & 1 & 0 & -1/2 \\ 0 & 1 & -1 & -3/8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\neq$

$$B = \begin{pmatrix} 2 & -6 & 8 & 2 \\ 5 & 1 & 4 & -1 \\ 3 & -9 & 12 & 3 \end{pmatrix} \text{ RREF} \rightarrow \begin{pmatrix} 1 & 0 & 1 & -1/8 \\ 0 & 1 & -1 & -3/8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

No they are not

3.9.3 If $A \overset{row}{\sim} B$, then $RREF(A) = RREF(B) \rightarrow$ basic columns of A are the same spots as basic columns of B.

3.10.1 a) $A = LU$

$$\begin{bmatrix} \boxed{U_{11}} & \boxed{U_{12}} & \boxed{U_{13}} \\ \boxed{L_{21}}U_{11} & L_{21}U_{12}+\boxed{U_{22}} & L_{21}U_{13}+\boxed{U_{23}} \\ \boxed{L_{31}}U_{11} & L_{31}U_{12}+\boxed{L_{32}}U_{22} & L_{31}U_{13}+L_{32}U_{23}+\boxed{U_{33}} \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 18 & 26 \\ 3 & 16 & 30 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1} & \boxed{4} & \boxed{5} \\ \boxed{4} \cdot 1 & 4 \cdot 4+\boxed{2} & 4 \cdot 5+\boxed{6} \\ \boxed{3} \cdot 1 & 3 \cdot 4+\boxed{2} \cdot 2 & 3 \cdot 5+2 \cdot 6+\boxed{3} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$