Hw 3

2.2.1 a)

$$A = \begin{pmatrix} 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 9 \\ 2 & 6 & 7 & 6 \end{pmatrix}$$

plug into RREF calculator

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A_{*1}, A_{*2}, A_{*4} are basic columns

$A_{*1} \cdot 2 + A_{*2} = A_{*3}$

b)

$$A = \begin{pmatrix} 2 & 1 & 1 & 3 & 0 & 4 & 1 \\ 4 & 2 & 4 & 4 & 1 & 5 & 5 \\ 2 & 1 & 3 & 1 & 0 & 4 & 3 \\ 6 & 3 & 4 & 8 & 1 & 9 & 5 \\ 0 & 0 & 3 & -3 & 0 & 0 & 3 \\ 8 & 4 & 2 & 14 & 1 & 13 & 3 \end{pmatrix}$$

plug into RREF calculator

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A_{*1}, A_{*3}, A_{*5} are basic columns

$A_{*1} \cdot \frac{1}{2} = A_{*2}$

$A_{*1} \cdot 2 - A_{*3} = A_{*4}$

$A_{*1} \cdot 2 - A_{*5} \cdot 3 = A_{*6}$

$A_{*3} + A_{*5} = A_{*7}$

2.2.3 because one of the columns is a linear combination of another, there can be at most $n-1$ pivots.
This implies there is at most $n-1$ basic columns in A.
which implies $\text{rank}(A) \le n-1$ or $\text{rank}(A) < n$.

2.2.5
$$E_{*3} = -E_{*1} - 2E_{*2}$$
$$E_{*1} = -2E_{*2} - E_{*3}$$
$$E_{*2} = \frac{1}{2}(-E_{*1} - E_{*3})$$

4) since $RREF(A) = E$, column relationships hold true.

$$E_{*3} = -E_{*1} - 2E_{*2} \longrightarrow A_{*3} = -A_{*1} - 2A_{*2}$$

$$-\begin{pmatrix}1\\4\\7\end{pmatrix} - 2\begin{pmatrix}2\\5\\8\end{pmatrix} = \begin{pmatrix}-5\\-14\\-23\end{pmatrix}$$

so
$$A = \begin{pmatrix}1 & 2 & -5\\4 & 5 & -14\\7 & 8 & -23\end{pmatrix}$$

2.3.1: d) the augmented matrix is:

$$A = \begin{pmatrix}1 & -1 & 1 & 1\\1 & -1 & -1 & 2\\1 & 1 & -1 & 3\\1 & 1 & 1 & 2\end{pmatrix} \quad RREF(A) = \begin{pmatrix}1 & 0 & 0 & 2\\0 & 1 & 0 & \frac{1}{2}\\0 & 0 & 1 & -\frac{1}{2}\\0 & 0 & 0 & 0\end{pmatrix}$$

so the rank of $A = 3$

the coefficient matrix is:

$$B = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix} \quad RREF(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

the rank of $B = 3$

since rank (A) = rank (B) the system is consistent.

e) the augmented matrix is:

$$A = \begin{pmatrix} 2 & 1 & 3 & 5 & 1 \\ 4 & 0 & 4 & 8 & 0 \\ 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \quad RREF(A) = \begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

So the rank of $A = 3$

The coefficient matrix is:

$$B = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & 0 & 4 & 8 \\ 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \end{pmatrix} \quad RREF(B) = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

the rank of $B = 2$

since rank (A) $\neq$ rank (B),
the system is not consistent

f: the coefficient matrix is:

$$A = \begin{pmatrix} 2 & 1 & 3 & 5 & 7 \\ 4 & 0 & 4 & 8 & 8 \\ 1 & 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 & 3 \end{pmatrix} \quad RREF(A) = \begin{pmatrix} 1 & 0 & 1 & 2 & 2 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

So the rank of $A = 2$

the coefficient matrix is:

$$B = \begin{pmatrix} 2 & 1 & 3 & 5 & \cancel{7} \\ 4 & 0 & 4 & 8 \\ 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \end{pmatrix} \quad RREF(B) = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

the rank of $B = 2$

Since rank (A) = rank (B)
the system is consistent