HW10

4.1.6: a) $\{(1, 1\ 1)\}$ rank $= 1$, does not span $\mathbb{R}^3$

b) $\{(1\ 0\ 0), (0\ 0\ 1)\}$ rank $= 2$, does not span $\mathbb{R}^3$

c) $\{(0\ 1\ 0), (1\ 1\ 0), (0\ 0\ 1)\}$ rank $= 3$, spans $\mathbb{R}^3$

d) $\{(1\ 2\ 1), (2\ 0\ -1), (4\ 4\ 1)\}$

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 0 & 4 \\ 1 & -1 & 1 \end{pmatrix} \xrightarrow{rref} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$ rank $= \cancel{3}\ 2$, ~~spans $\mathbb{R}^3$~~ does not span $\mathbb{R}^3$

e) $\{(1\ 2\ 1), (2\ 0\ -1), (4\ 4\ 0)\}$

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 0 & 4 \\ 1 & -1 & 0 \end{pmatrix} \xrightarrow{rref} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ rank $= 3$, spans $\mathbb{R}^3$

4.1.1 a) $\{X \mid x_i \geq 0\}$ $\rightarrow$ does not hold true for scalar multiplication, since $x_i \in X \geq 0$, Let $\alpha = -1$, $\alpha x_i \notin X$.

e) $\{X \mid \sum_{j=1}^{n} x_j = 1\}$ $\rightarrow$ does not hold true for scalar multiplication, since $x_j \in X \doteq 1$, Let $\alpha = -1$, $\alpha x_j \notin X$.

3: $$A = \begin{pmatrix} 1 & 2 & 1 & 1 & 5 \\ -2 & -4 & 0 & 4 & -2 \\ 1 & 2 & 2 & 4 & 9 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 2 & 0 & -2 & 1 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Since A_{*2}, A_{*4}, A_{*5} are linear combinations of A_{*1}, A_{*3}, they are in the span of A_{*1}, A_{*3}.

$$R(A) = \text{span}\left(\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right)$$

4:

$$A = \begin{pmatrix} 1 & 2 & 1 & 1 & 5 \\ -2 & -4 & 0 & 4 & -2 \\ 1 & 2 & 2 & 4 & 9 \end{pmatrix} \xrightarrow{RREF} \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{pmatrix} 1 & 2 & 0 & -2 & 1 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

from our reduced matrix:

$$\cancel{x+2y+0z-2w+v=0} \qquad x_1 + 2x_2 + 0x_3 - 2x_4 + x_5 = 0$$

$$\cancel{z+3w+4v=0} \qquad x_3 + 3x_4 + 4x_5 = 0$$

$$x_1 = -2x_2 + 2x_4 - x_5$$

$$x_3 = -3x_4 - 4x_5$$

$$N(A) = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 2 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix} x_4 + \begin{pmatrix} -1 \\ 0 \\ -4 \\ 0 \\ 1 \end{pmatrix} x_5$$