Hw 11

4.3.1 a) $\left\{ \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}1\\5\\9\end{pmatrix} \right\} \to \begin{pmatrix}1&2&1\\2&1&5\\3&0&9\end{pmatrix} \xrightarrow{RREF} \begin{pmatrix}1&0&3\\0&1&-1\\0&0&0\end{pmatrix}$

So $\begin{pmatrix}1\\5\\9\end{pmatrix} = 3\begin{pmatrix}1\\2\\3\end{pmatrix} + -1\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}$

b) $\{(2\ 2\ 2\ 2), (2\ 2\ 0\ 2), (2\ 0\ 2\ 2)\} \to \begin{pmatrix}2&2&2\\2&2&0\\2&0&2\\2&2&2\end{pmatrix}$

$\downarrow RREF$

l.i. $\Leftarrow \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{pmatrix}$

c) $\left\{ \begin{pmatrix}3\\2\\1\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix} \right\} \to \begin{pmatrix}3&1&2\\2&0&1\\1&0&0\end{pmatrix} \xrightarrow{RREF} \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$\Downarrow$

l.i.

e) $\left\{ \begin{pmatrix}1\\2\\0\\4\\0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\4\\1\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\1\\4\\0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\4\\0\\3\\1\end{pmatrix} \right\} \to \begin{pmatrix}1&0&0&0\\2&2&2&2\\0&0&1&2\\4&4&4&4\\0&1&0&0\\3&3&3&3\\0&0&0&1\end{pmatrix}$

RREF

l.i. $\Leftarrow \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$

4.3.2 a) $A = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 6 & 3 & 2 & 2 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & \frac{1}{2} & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$R(A) = \left\{ \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\}$

b) $\{ \{A_{*1}, A_{*3}\}, \{A_{*1}, A_{*4}\}, \{A_{*2}, A_{*3}\}, \{A_{*2}, A_{*4}\}, \{A_{*3}, A_{*}$

$\{A_{*2}, A_{*3}, A_{*4}\}, \{A_{*1}\}, \{A_{*2}\}, \{A_{*3}\}, \{A_{*4}\}$

4.3.5 a) because S_{*1} can be written as αS_{*1} where $\alpha \in \mathbb{R}$

b) because any member of the set times the zero vector = zero vector, which means the zero vector is always a combination of itself and.

4.3.7 $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$

$x_1' + x_2' + x_3' + x_4'$

$\begin{pmatrix} x_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} x_2 & x_2 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} x_3 & x_3 \\ x_3 & 0 \end{pmatrix} + \begin{pmatrix} x_4 & x_4 \\ x_4 & x_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} x_1 + x_2 + x_3 + x_4 & x_2 + x_3 + x_4 \\ x_3 + x_4 & x_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow{RREF} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \text{l.i.}$$

4.4.3

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 8 \\ -4 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 0 \\ 6 \end{pmatrix} \right\}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 1 & 2 & 1 & 3 \\ 2 & 0 & 8 & 1 & 3 \\ -1 & 0 & -4 & 1 & 0 \\ 3 & 2 & 8 & 1 & 6 \end{pmatrix} \xrightarrow{RREF} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

minimum spanning set : $\left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

So $\dim(S) = 3$