Hw 13

5.1.2 $u = \begin{pmatrix} 2 \\ 1 \\ -4 \\ -2 \end{pmatrix} \quad v = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$

a) $= \|u-v\| = \sqrt{(2-1)^2+(1+1)^2+(-4-1)^2+(-2+1)^2}$
$= \sqrt{31}$

b) $\|u\| = \sqrt{25}$
$\|v\| = \sqrt{4}$
$\|u+v\| = \sqrt{(2+1)^2+(1-1)^2+(-4+1)^2+(-2+1)^2} = \sqrt{19}$

$\sqrt{19} \leq \sqrt{25} + \sqrt{4}$

c) $|\langle u|v\rangle|^2 \leq \langle u|u\rangle \cdot \langle v|v\rangle$
$|\langle u|v\rangle| \leq \|u\| \cdot \|v\|$
$0 \leq 10$ true

5.4.1 $\langle x,y\rangle = 0$

a) $\langle x,y\rangle = -8+8 = 0$ ✓

b) $\langle x,y\rangle = 0 + 2i + -4 - 2i = -4-2i$ ✗

c) $\langle x,y\rangle = \cancel{4+-4} + -3 + 4 = 1$ ✗

d) $\langle x,y\rangle = 1+1+-3+1 = 0$ ✓

e) $\langle x,y\rangle = 0+0+\ldots+0 = 0$ ✓

5.4.3 $\left\{ x_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, x_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \\ 0 \end{pmatrix} \right\}$

a) $\langle x_1 | x_2 \rangle = 1 - 1 = 0$

$\langle x_1 | x_3 \rangle = -1 + 1 = 0$

$\langle x_2 | x_3 \rangle = -1 - 1 + 2 = 0$

b) augmented matrix

$$\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 1 & 1 & 0 \\ -1 & -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Downarrow$ RREF

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$a_1 + a_4 = 0$

$a_2 - a_4 = 0$

$a_3 = 0$

$a_4 = 1 \Rightarrow a_1 = -1$

$a_2 = 1$

$a_3 = 0$

$$x_4 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

c) $\|x_1\| = \sqrt{6}$

$\|x_2\| = \sqrt{3}$

$\|x_3\| = \sqrt{6}$

$\|x_4\| = \sqrt{3}$

$$\text{basis} = \left\{ \begin{pmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ 0 \\ \frac{2}{\sqrt{6}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \end{pmatrix} \right\}$$

5.4.4 $\xi_1 = \langle u_1 | x \rangle = \frac{1}{\sqrt{2}}$

$\xi = \langle u_2 | x \rangle = -\frac{1}{\sqrt{3}}$

$\xi = \langle u_3 | x \rangle = -\frac{5}{\sqrt{6}}$

$$x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} + -\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} + -\frac{5}{\sqrt{6}}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$$

5.4.6: $\theta = \arccos\left(\frac{u \cdot v}{\|u\|\|v\|}\right) = \arccos\left(\frac{3}{\sqrt{6}\cdot\sqrt{6}}\right) = \arccos\left(\frac{1}{2}\right) = 60°$

5.4.10 Trace $(A^T A) =$

$\overset{4}{\text{trace}(\langle B|B\rangle)}$, $\overset{2}{\text{trace}(\langle I|I\rangle)}$

a) $\text{trace}\left(\begin{pmatrix}1&1\\1&1\end{pmatrix}^T\begin{pmatrix}2&0\\0&1\end{pmatrix}\right) = 2 = \|B\|\|I\|\cos(\theta)$ $\quad \theta = \arccos\left(\frac{2}{8}\right) = 75°$

b) $\arccos\left(\frac{\text{trace}\left(\begin{pmatrix}2&2\\-2&0\end{pmatrix}\begin{pmatrix}1&3\\2&4\end{pmatrix}\right)}{\text{trace}\left(\begin{pmatrix}2&2\\-2&0\end{pmatrix}\begin{pmatrix}2&-2\\3&0\end{pmatrix}\right)\cdot\text{trace}\left(\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}1&3\\2&4\end{pmatrix}\right)}\right) = \arccos\left(\frac{0}{12\cdot 30}\right) = 90° = \theta$