HW 7

2: $T: R^3 \to R^2$ $\quad T(e_1) = (1,3)$ $\quad T(e_2) = (4,-7)$ $\quad T(e_3) = (-5,4)$

where $e_1 = (1,0,0)$ $\quad e_2 = (0,1,0)$ $\quad e_3 = (0,0,1)$

$$T = (T(e_1) \quad T(e_2) \quad T(e_3))$$

$$T = \begin{pmatrix} 1 & 4 & -5 \\ 3 & -7 & 4 \end{pmatrix}$$

2×2

4: $T: R^2 \to R^2$ $\quad \theta = -\pi/4$ $\quad T(e_1) = (1/\sqrt{2}, -1/\sqrt{2})$

$$T(e_1) = T\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} \cos x \\ -\sin x \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$T(e_2) = T\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} \sin x \\ \cos x \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$T = (T(e_1) \quad T(e_2))$$

$$T = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

6: $T: R^2 \to R^2$ $\quad e_1 \to e_1$ $\quad e_2 \to e_2 + 3e_1$

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

8:

8: $T: R^2 \to R^2$ (2×1)

$\begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}1\\0\end{bmatrix} \wedge \begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}0\\-1\end{bmatrix}$

x_1-axis ↑ ↙ $x_2 = x_1$

$\begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}0\\1\end{bmatrix} \wedge \begin{bmatrix}0\\-1\end{bmatrix} \to \begin{bmatrix}-1\\0\end{bmatrix}$

$T(e_1) = \begin{bmatrix}0\\1\end{bmatrix}$

$T(e_2) = \begin{bmatrix}-1\\0\end{bmatrix}$

$T = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

10 $T: R^2 \to R^2$ (2×2)

$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}+1\\0\end{pmatrix} \wedge \begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix}$ ↗ $\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}-1\\0\end{pmatrix} \wedge \begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\1\end{pmatrix}$

x_2-axis $x = \pi/2$

$T(e_1) = \begin{pmatrix}0\\-1\end{pmatrix}$

$T(e_2) = \begin{pmatrix}-1\\0\end{pmatrix}$

$T = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$