Jacob H...

Hw 2:

1.3.1   the augmented matrix is:

$$\left(\begin{array}{ccc|c} 0 & 4 & -3 & 3 \\ -1 & 7 & -5 & 4 \\ -1 & 8 & -6 & 5 \end{array}\right)$$ $R_2 \longleftrightarrow R_1$

$$\left(\begin{array}{ccc|c} -1 & 7 & -5 & 4 \\ 0 & 4 & -3 & 3 \\ -1 & 8 & -6 & 5 \end{array}\right)$$ $R_1 \cdot -1 \to R_1$

$$\left(\begin{array}{ccc|c} -1 & 7 & -5 & 4 \\ 0 & 4 & -3 & 3 \\ -1 & 8 & -6 & 5 \end{array}\right)$$ $R_3 + R_1 \to R_3$

$$\left(\begin{array}{ccc|c} -1 & 7 & -5 & 4 \\ 0 & 4 & -3 & 3 \\ 0 & 1 & -1 & 1 \end{array}\right)$$ $R_2 - 3R_3 \to R_2$

$$\left(\begin{array}{ccc|c} -1 & 7 & -5 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right)$$ $R_3 - R_2 \to R_3$

$$\left(\begin{array}{ccc|c} -1 & 7 & -5 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right)$$ $R_3 \cdot -1 \to R_3$

$$\left(\begin{array}{ccc|c} -1 & 7 & -5 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right)$$ $R_1 - 7R_2 \to R_1$

$$\left(\begin{array}{ccc|c} -1 & 0 & -5 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right)$$ $R_1 + 5R_3 \to R_1$

$$\begin{pmatrix} -1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \qquad R_1 \cdot -1 \Rightarrow R_1$$

$$\begin{pmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \qquad \begin{array}{l} x = 1 \\ y = 0 \\ z = -1 \end{array}$$

1.3.2: the augmented matrix is:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & | & 1 \\ 1 & 2 & 2 & 2 & | & 0 \\ 1 & 2 & 3 & 3 & | & 0 \\ 1 & 2 & 3 & 4 & | & 0 \end{pmatrix} \qquad \begin{array}{l} R_2 - R_1 \Rightarrow R_2 \\ R_3 - R_1 \Rightarrow R_3 \\ R_4 - R_1 \Rightarrow R_4 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & | & 1 \\ 0 & 1 & 1 & 1 & | & -1 \\ 0 & 1 & 2 & 2 & | & -1 \\ 0 & 1 & 2 & 3 & | & -1 \end{pmatrix} \qquad \begin{array}{l} R_1 - R_2 \Rightarrow R_1 \\ \\ R_3 - R_2 \Rightarrow R_3 \\ R_4 - R_2 \Rightarrow R_4 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & | & 2 \\ 0 & 1 & 1 & 1 & | & -1 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & 2 & | & 0 \end{pmatrix} \qquad \begin{array}{l} R_2 - R_3 \Rightarrow R_2 \\ \\ R_4 - R_3 \Rightarrow R_4 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & 0 & | & -1 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \end{pmatrix} \qquad R_3 - R_4 \Rightarrow R_3$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & 0 & | & -1 \\ 0 & 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \end{pmatrix}$$

$x = 2$
$y = -1$
$z = 0$
$n = 0$

2.1.1 a):
$$\begin{pmatrix} 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 9 \\ 2 & 6 & 7 & 6 \end{pmatrix}$$

$R_2 - 2R_1 \Rightarrow R_2$
$R_3 - 2R_1 \Rightarrow R_3$

$$\begin{pmatrix} 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 1 & 0 \end{pmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{pmatrix} 1 & 2 & 3 & 3 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$R_2 \cdot \frac{1}{2} \Rightarrow R_2$
$R_3 \cdot \frac{1}{3} \Rightarrow R_3$

$$\begin{pmatrix} \boxed{1} & 2 & 3 & 3 \\ 0 & \boxed{1} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \boxed{1} \end{pmatrix}$$

rank $= 3$

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Basic Columns

b):
$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 8 \\ 2 & 6 & 0 \\ 1 & 2 & 5 \\ 3 & 8 & 6 \end{pmatrix}$$

$R_2 - 2R_1 \Rightarrow R_2$
$R_3 - 2R_1 \Rightarrow R_3$
$R_4 - R_1 \Rightarrow R_4$
$R_5 - 3R_1 \Rightarrow R_5$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 2 & -6 \\ 0 & 0 & 2 \\ 0 & 2 & -3 \end{pmatrix}$$

$R_3 - R_2 \Rightarrow R_3$

$R_5 - R_2 \Rightarrow R_5$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & -8 \\ 0 & 0 & 2 \\ 0 & 0 & -5 \end{pmatrix}$$

$R_3 \leftrightarrow R_4 \quad ; \quad R_3 \cdot \frac{1}{2}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & -8 \\ 0 & 0 & -5 \end{pmatrix}$$

$R_4 + 8R_3$

$R_5 + 5R_3$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

rank = 3

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Basic Columns

Basic Columns.

c)
$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{2} & 0 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

rank = 3

2.1.2: a) not REF
b) not REF
c) REF
d) REF