Hw 14

5.5.1: a)

$$S = \text{span}\left\{ x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix},\ x_2 = \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix},\ x_3 = \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \right\}$$

a) $v_1 = x_1$, $\|v_1\| = 2$, $\langle v_1, v_1 \rangle = 4$

$$v_2 = x_2 - \frac{\langle v_1, x_2 \rangle}{\langle v_1, v_1 \rangle} v_1 = \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ -1/4 \end{pmatrix} = \begin{pmatrix} 9/4 \\ -3/4 \\ -3/4 \\ 3/4 \end{pmatrix}$$

$$v_3 = x_3 - \frac{\langle v_1, x_3 \rangle}{\langle v_1, v_1 \rangle} v_1 - \frac{\langle v_2, x_3 \rangle}{\langle v_2, v_2 \rangle} v_2$$

$$= \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} + \frac{15}{27}\begin{pmatrix} 9/4 \\ -3/4 \\ -3/4 \\ 3/4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

the corresponding orthonormal set is:

$$v_1 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix},\ v_2 = \frac{2}{3\sqrt{3}}\begin{pmatrix} 9/4 \\ -3/4 \\ -3/4 \\ 3/4 \end{pmatrix},\ v_3 = \frac{1}{\sqrt{6}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

b) $v_1 \cdot v_2 = v_1 \cdot v_3 = v_2 \cdot v_3 = 0$

calculator

$0 = 0 = 0 = 0$ ✓

6.1.1 a)

$$A = \begin{pmatrix} 3 & -2 & 1 \\ -5 & 4 & 0 \\ 2 & 1 & 6 \end{pmatrix}$$

$$|A| = 3\overset{72}{[4(6)-0]} + 2\overset{-60}{[-30-0]} + 1\overset{-13}{[-5-8]}$$

$$= -1$$

b)

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 6 & 2 & 1 \\ -2 & 2 & 1 \end{pmatrix}$$

$$|A| = 2[2-2] - 1\overset{-8}{(6+2)} + 1\overset{16}{(12+4)} = 8$$

c)

$$A = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & \beta & 0 \\ \gamma & 0 & 0 \end{pmatrix}$$

$$|A| = 0\cdot(\sim) + 0(\sim) + \alpha(0+\beta\cdot\gamma)$$

$$= \alpha\beta\gamma$$

6.1.3 a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 1 & 4 & 4 \end{pmatrix} \xrightarrow[\text{5 steps}]{\text{REF}} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 5/2 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad |A| = -1^5 \cdot (1(1-0))$

$= \boxed{-1}$

b) $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 4 & 2 \\ 3 & -2 & 4 \end{pmatrix} \xrightarrow[\text{4 steps}]{\text{REF}} \begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad |A| = -1^4 \cdot (1(0-0))$

$= \boxed{0}$

d) $A = \begin{pmatrix} 0 & 0 & -2 & 3 \\ 1 & 0 & 1 & 2 \\ -1 & 1 & 2 & 1 \\ 0 & 2 & -3 & 0 \end{pmatrix} \xrightarrow[\text{7 steps}]{\text{REF}}$ $\quad |A| = -1^7 \cdot (\qquad)$

$0(\sim) - 0(\sim) + {+2}\left(\begin{vmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 0 & 2 & 0 \end{vmatrix}\right) - 3\left(\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & -3 \end{vmatrix}\right)$

$= -\big(2(1(0-2) - 0(\sim) + 2(-2-0)) - 3(1(-3-4) - 0(\sim) - 1(-2-0))\big)$

$\quad -2(-2 + 4) + 3(-7+1)$

$= \boxed{+14}$

e) $A = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \xrightarrow[\text{5 steps}]{\text{REF}} \begin{pmatrix} 1 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & 1 & -\frac{4}{5} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ $\quad |A| =$

$-1^5 \, (1(1(1(1(1) - 0\tfrac{4}{5})) = \boxed{-1}$

6.1.6 $\det(AB) = \det(A)\det(B)$

Let $B = A^{-1}$

$$\det(A \cdot A^{-1}) = \det(A)\det(A^{-1})$$

$$\det(I) = 1 \quad \Rightarrow \quad \det(A)\det(A^{-1}) = 1$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

6.1.8