HW 8 $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \frac{1}{ad-bc}$

3.7.1 b) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ inverse $= \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \cdot \frac{1}{4-4} = \frac{1}{0}$ $\boxed{DNE}$

A

d) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $\begin{array}{c} R_2 - 4R_1 \Rightarrow R_2 \\ R_3 - 7R_1 \Rightarrow R_3 \end{array}$ $\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -7 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -13 \end{pmatrix}$ $R_3 - 2R_2 \Rightarrow R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -7 & -2 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -1 \end{pmatrix}$ $\begin{array}{c} R_2 \cdot -\frac{1}{3} \Rightarrow R_2 \\ R_3 \cdot -1 \Rightarrow R_3 \end{array}$ $\begin{pmatrix} 1 & 0 & 0 \\ +\frac{4}{3} & -\frac{1}{3} & 0 \\ 7 & 2 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ $\begin{array}{c} R_1 - 3R_3 \Rightarrow R_1 \\ R_2 - 2R_3 \Rightarrow R_2 \end{array}$ $\begin{pmatrix} -20 & -6 & 3 \\ -\frac{1}{3} & \frac{1}{3} & -6\frac{1}{3} & 2 \\ 7 & 2 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $R_1 - 2R_2 \Rightarrow R_1$ $\begin{pmatrix} 5\frac{1}{3} & 6\frac{2}{3} & -1 \\ -12\frac{2}{3} & -6\frac{1}{3} & 2 \\ 7 & 2 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$ $\begin{pmatrix} 5\frac{1}{3} & 6\frac{2}{3} & -1 \\ -12\frac{2}{3} & -6\frac{1}{3} & 2 \\ 7 & 2 & -1 \end{pmatrix} = A^{-1}$

e)
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix} \quad \begin{matrix} R_2 - R_1 \Rightarrow R_2 & \text{third} \\ R_3 - R_2 \Rightarrow R_3 & 2^{nd} \\ R_4 - R_3 \Rightarrow R_4 & 1^{st} \end{matrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{matrix} R_1 - R_2 \Rightarrow R_1 & 1^{st} \\ R_2 - R_3 \Rightarrow R_2 & 2^{nd} \\ R_3 - R_4 \Rightarrow R_3 & \text{third} \end{matrix} \quad \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = I \qquad \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} = A^{-1}$$

3.7.2: $X = AX + B$ $\qquad$ $X - AX = B$ $\qquad$ $X(I-A) = B$ $\qquad$ $X = (I-A)^{-1} B$

$$A = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{pmatrix}$$

$$I - A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(I-A) \cdot B = \underset{3\times3}{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}} \underset{3\times2}{\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{pmatrix}} = \underset{3\times2}{\begin{pmatrix} 3 & 3 \\ 5 & 4 \\ 3 & 3 \end{pmatrix}} = X$$

3.7.10: $A_{r \times r}$, $B_{s \times s}$, $C_{r \times s}$

a) $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{pmatrix}$

$\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \cdot \begin{pmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{pmatrix} = \begin{pmatrix} A \cdot A^{-1} & 0 \\ 0 & B \cdot B^{-1} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$ ✓

b) $\begin{pmatrix} A & C \\ 0 & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & A^{-1}CB^{-1} \\ C & B^{-1} \end{pmatrix}$

$\begin{pmatrix} A & C \\ 0 & B \end{pmatrix} \cdot \begin{pmatrix} A^{-1} & A^{-1}CB^{-1} \\ 0 & B^{-1} \end{pmatrix} = \begin{pmatrix} A \cdot A^{-1} & C \cdot 0 \\ 0 \cdot A^{-1}CB^{-1} & B \cdot B^{-1} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$ ✓

3.9.2 i $A = \begin{pmatrix} 2 & 2 & 0 & -1 \\ 3 & -1 & 4 & 0 \\ 0 & -8 & 8 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -6 & 8 & 2 \\ 5 & 1 & 4 & -1 \\ 3 & -19 & 12 & 3 \end{pmatrix}$

RREF ↓   RREF ↓

$\begin{pmatrix} 1 & 0 & 1 & -\frac{1}{8} \\ 0 & 1 & -1 & -3/8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 1 & -\frac{1}{8} \\ 0 & 1 & -1 & -3/8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

a) They are equivalent.

b) They are row equivalent

c) They are column equivalent.