

Section	Group	Name	Signature
Grade		Jacob H	Jacob H

After this activity, you should know: • Be able to determine direction of cross product using right-hand rule • Be able to determine magnitude of cross product using geometry • Be able to determine cross product using components.

Cross products (also known as vector products) are important for magnetic fields and magnetic forces. Here we review some properties of cross products.

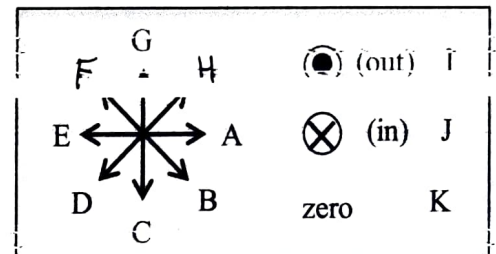
1. Use the legend (A-K) to answer the next questions.

- a. The cross product  $\vec{W} \times \vec{V}$  is perpendicular to both vectors  $\vec{W}$  and  $\vec{V}$  involved in the cross product. Which two directions are perpendicular to both vectors?

I J

- b. What is the direction of  $\vec{W} \times \vec{V}$ ?

J



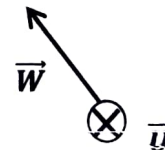
2. Consider the vectors  $\vec{W}$  and  $\vec{U}$ .

- a. Which two directions are perpendicular to both  $\vec{W}$  and  $\vec{U}$ ?

D H

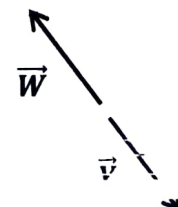
- b. What is the direction of  $\vec{W} \times \vec{U}$ ?

D

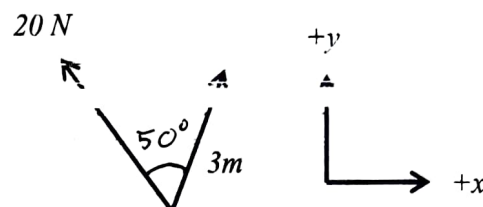


- d. What is the direction of  $\vec{W} \times \vec{V}$ ?

0



3. The torque due to a force  $\vec{F}$  acting at a displacement  $\vec{r}$  from the axis of rotation is given by  $\vec{\tau} = \vec{r} \times \vec{F}$ . The force is oriented  $110^\circ$  ccw from  $+x$  and  $\vec{r}$  is oriented at  $60^\circ$  from  $+x$ . Determine the magnitude and direction of the torque.



$$= 20 \times 3$$

$$= 20 \cdot 3 \cdot \sin(50) = 45.96$$

4. The cross product can also be determined if we know the components of the two vectors involved. The easiest way to do this is to write the cross product the determinant of a  $3 \times 3$  matrix. The first row of the matrix is the unit vectors  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$ . The 2<sup>nd</sup> row is the components of the first vector and the 3<sup>rd</sup> row is the components of the second vector. The determinant of the  $3 \times 3$  matrix can be written in terms of determinants of  $2 \times 2$  matrices:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

The determinant of a  $2 \times 2$  matrix is the product of the diagonal terms minus the product of the off-diagonal terms. Therefore

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

- a. Let  $\vec{r} = (2\hat{i} - 3\hat{k}) \text{ m}$  and  $\vec{F} = (5\hat{i} - 4\hat{j} - 10\hat{k}) \text{ N}$ . Determine the torque in component vector form. Don't forget to include units.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -3 \\ 5 & -4 & -10 \end{vmatrix} = \langle 12\hat{i} + 25\hat{j} + -8\hat{k} \rangle \text{ Nm}$$

- b. What is the magnitude of the torque?

$$\sqrt{12^2 + 25^2 + 8^2} \\ = \sqrt{833} \text{ Nm}$$