Section	Group	Name	Signature
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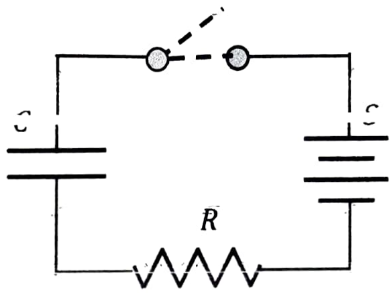
After this activity you should know: • *the time dependence of the voltage across the capacitor for charging and discharging capacitors* • *The time constant for RC circuits and how to obtain it.*

1. The video https://psu.mediaspace.kaltura.com/media/1_u0czbj79 shows four experiments in which a capacitor C is charged by a EMF $\mathcal{E}$ through a resistor R. The experiments show graphs of the voltage across the capacitor versus time for (a) $\mathcal{E} = 10V$, $R = 5\Omega$, and $C = 0.05\ F$ (b) $\mathcal{E} = 20V$, $R = 5\Omega$, and $C = 0.05\ F$, (c) $\mathcal{E} = 10V$, $R = 20\ \Omega$, and $C = 0.05\ F$ and (d) $\mathcal{E} = 10V$, $R = 5\Omega$, and $C = 0.2\ F$.[1]



For each case below, determine how making the change described affects (i) the final value of the voltage across the capacitor and (ii) how long it takes to charge the capacitor. You just need to give a qualitative answer (not numbers).

a. What happens when you increase the EMF while keeping the resistance and capacitance at their original values?

Voltage increases

time stays same

b. What happens when you increase the resistance while keeping the capacitance and EMF at their original values?

Voltage stays same

time increases

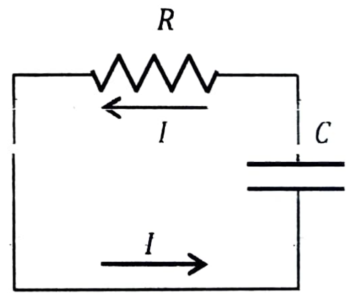
c. What happens when you increase the capacitance while keeping the resistance and EMF at their original values?

Voltage increases

time increases

Discharging Capacitor: Kirchoff's loop rule applied to the discharging capacitor circuit gives

$$\sum_{loop} \Delta V_i = IR - \frac{Q}{C} = 0$$



The current is leaving the positive plate so $I = -dQ/dt = -C\, d\Delta V_c/dt$. Substituting this into loop rule gives a differential equation for $Q(t)$:

$$RC\frac{dV_c}{dt} + V_c = 0$$

This is the simplest differential equation you will learn in your differential equations class. The solution for the differential equation for $V_c(t)$ is an exponential:

$$V_c(t) = V_o e^{-t/(RC)} \qquad \textit{(discharging capacitor)}$$

where ΔV_o is the voltage across the capacitor when discharge begins ($t = 0$). The quantity $\tau_c = RC$ has dimensions of time and is called the time constant of the circuit. It describes how long it takes for the capacitor to charge or discharge.

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[1] You may also use the PhET simulation "*PhET simulation "Circuit Construction Kit (AC+DC)*" to do the experiments yourself.

2. Show that a $ohm \cdot farad$ is the same as a second. Show steps.

$$\Omega \cdot F = \frac{kg \cdot m^2}{A^2 \cdot s^3} \cdot \frac{s^4 A^2}{m^2 \cdot kg} = s$$

3. A 3 μF capacitor initially has a voltage of 1.5 V when it is discharged through a resistance R. The voltage *3 ms* after it has started to discharge is 0.25 Volts.

   a. Determine the time constant τ_C.

$\tau_c = RC = 3\mu R$

$0.25 = 1.5 \cdot e^{-t/\tau_c} \equiv \frac{1}{6} = e^{-t/\tau_c}$

$\ln\left|\frac{1}{6}\right| = \frac{-t}{\tau_c}$

$\tau_c = -0.003/\ln\left|\frac{1}{6}\right|$

   b. Determine the resistance R.

$\tau_c = -0.003/\ln\left(\frac{1}{6}\right) = RC = R \cdot 3 \cdot 10^{-6}$

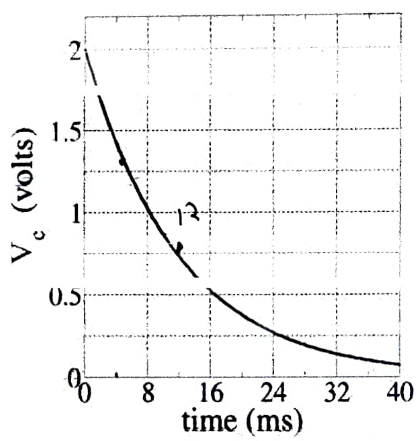
$= -1000/\ln\left(\frac{1}{6}\right)$

4. You can easily find the time constant if you are given a graph of voltage across a discharging capacitor as a function of time. When $t = \tau_c = RC$, the voltage across the capacitor is

$$V_c(t = \tau_c) = V_o\, e^{-1} \approx 0.37\, \Delta V_o$$

Therefore the time constant is just how long it takes for $\Delta V_c(t)$ to reach 37% of its initial value.

The graph shows the voltage across a discharging 0.2 μF capacitor as a function of time. Use the following steps to determine the resistance.



   a. What is 37% of the initial voltage?

$2 \cdot 0.37 = 0.74V$

   b. How long does it take (after discharge starts) for the voltage to reach 37% of the initial voltage? This time is equal to the time constant.

$V_c = 0.74$

$\tau \approx 12$

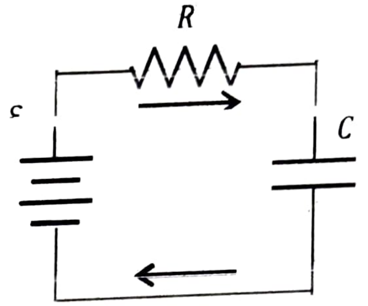
   c. Use your time constant from (b) to determine the resistance.

$12 = 0.2 \times 10^{-6} \cdot R$

$R = 6 \cdot 10^{4}$

Charging Capacitor: For a charging capacitor the Kirchoff's Loop Rule gives

$$\mathcal{E} - IR - \frac{Q}{C} = 0$$



In this case the current is entering the positive plate so $I = dQ/dt = C d\Delta V_c/dt$ and we get

$$\mathcal{E} - RC\frac{dV_c}{dt} - V_c = 0$$

The solution to the differential equation is

$$V_c(t) = \mathcal{E}\,(1 - e^{-t/(RC)}) \qquad \textit{(charging capacitor)}$$

Notice that $V_c(0) = 0$ and $V_c(\infty) = \mathcal{E}$ as we expect for a charging capacitor.

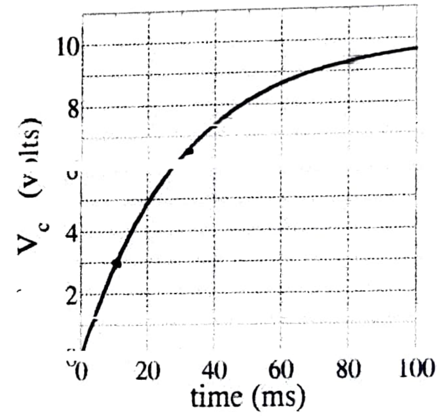
5. You can easily find the time constant if you are given a graph of voltage across a charging capacitor as a function of time. When $t = RC$, the voltage across the capacitor is

$$V_c(t = RC) = \mathcal{E}(1 - e^{-1}) \approx 0.63\,\mathcal{E}.$$

Therefore the time constant is just how long it takes for $\Delta V_c(t)$ to reach 63% of the EMF.

The graph shows the voltage across a charging capacitor as a function of time. The resistance of the circuit is $7.5\ k\Omega$.

a. Determine the capacitance of the capacitor.



$R = 7.5\ K\Omega \cdot F = 30\ ms$

$$C = \frac{30\ ms}{7.5\ K\Omega} = \boxed{4\ F}$$

b. What is the current at $t = 10\ ms$? *Hint: the easiest way to do this is to use the loop rule.*

$V = IR$

$I = \frac{V}{R}$

$I = \frac{3}{7500}$ $\boxed{I = 0.4\ mA}$