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After this activity, you should know: • *Tell when Gauss's Law is useful and when it is not.* • *Use Gauss's Law in situations with spherical symmetry.*

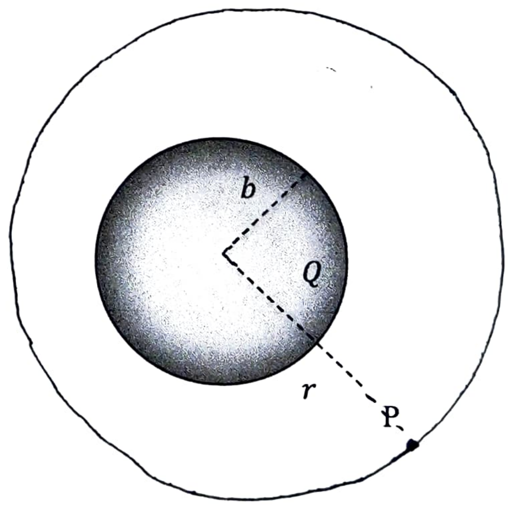
1. What is the surface area and volume of a sphere of radius r?

area = $4\pi r^2$ volume = $\frac{4}{3}\pi r^3$

Using Gauss's law in situations with high symmetry: The general strategy to find the electric field for situations for high symmetry is as follows:

- Use symmetry to determine the direction of $\vec{E}$ around the object and what variables the magnitude $\|\vec{E}\|$ can depend on. For example, does E only depend on r? If there is not enough symmetry, Gauss's law, though still true, will not be useful for finding the electric field.
- Draw a Gaussian (closed) surface such that $\vec{E} \cdot \hat{n} = E\cos\theta$ must be constant on the surface.
- Use the definition of the electric flux $\Phi = \oint \vec{E} \cdot d\vec{A}$ to write the flux in terms of the unknown electric field E.
- Determine the charge enclosed by the Gaussian surface and use Gauss's law $\Phi = q_{enc}/\epsilon_o$ to write a second expression for the electric flux.
- Set the two expressions for the electric flux equal and solve for the unknown E.

2. **Spherical Symmetry (outside sphere):** A sphere of radius b has charge Q distributed uniformly over its volume. We want to find the electric field at a point P a distance r from the center of the sphere. Here we assume that $r > b$ so that P is outside the sphere.



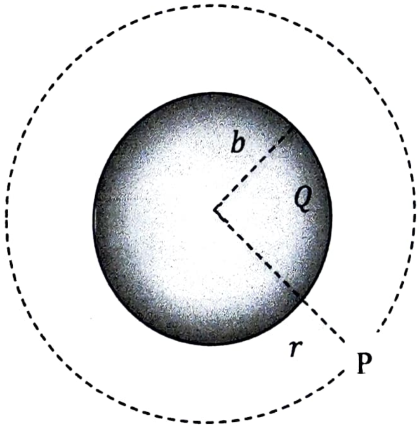
a. From symmetry, which statement below must be true regarding the direction of the electric field? **Choose one!**
   i. The electric field points upwards everywhere.
   ii. The E field is tangent to the surface of the sphere everywhere.
   iii. The E field is radial everywhere. (Radial means away or towards the center of the sphere.)

b. From the symmetry, which statement is true regarding the magnitude $E = \|\vec{E}\|$ due to the sphere? **Choose one!**
   i. E depends on the polar angle θ only.
   ii. E depends on the distance r from the center only.
   iii. E depends on both θ and r.

c. Draw the Gaussian surface you need to determine the electric field at point *P*. Check that your surface that has **ALL** the following properties:
   - The surface must be closed since Gauss's law only holds for closed surfaces.
   - The surface must go through the point P since we want to know the electric field at that point.
   - The surface must have $\vec{E} \cdot \hat{n} = E\cos\theta$ constant on the surface in order to make the flux integral easy.

d. Use the definition of electric flux $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint \vec{E} \cdot \hat{n}\, dA$ to write the flux in terms of the unknown E and the radii r and/or b. (You may not need all these symbols.) This integral is easy since you chose the suface so that $\vec{E} \cdot \hat{n}$ is constant on the surface. Check that your expression for the area has the correct dimensions.

$$\Phi_E = E\int dA = 4E\pi r^2$$



e. The Gaussian surface you should have drawn on the previous page is represented by the dashed line. *Remember that the Gaussian surface is actually the surface of a three dimensional sphere.*

   i. What is q_{enc} for this case? Your answer should contain only Q, r and/or b. *You can write down the answer just by looking at the picture **without any calculations!***

   Q

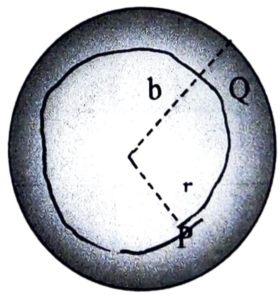
   i. Use Gauss' law and (i) to write the electric flux through the surface in terms of $Q, r,$ and/or b and physical constants.

   $$\phi = \frac{Q}{\epsilon_0}$$

f. We now have two different ways to write the electric flux: (i) in terms of the unknown E as given in part (d) and in terms of the charge in part (e.ii). Set the two expressions for the flux equal and solve for the unknown E in terms of $Q, b,$ and/or r. Your answer should look very familiar.

$$4E\pi r^2 = \frac{Q}{\epsilon_0} \qquad \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

3. **Spherical Symmetry (inside sphere):** We will now repeat the steps above for a point P inside the sphere.



   a. Draw the Gaussian surface you need to determine the $\vec{E}$ at point P where $r < b$.

   b. Use the definition of electric flux to write the flux in terms of the unknown E and the radii r and/or b. *Your expression should look familiar.*

   $$\phi = E\int dA = E\, 4\pi r^2$$

   c. Determine the charge enclosed by your Gaussian surface in terms of $Q, r,$ and/or b.

   $$\frac{Q}{\frac{4}{3}\cancel{\pi} b^3}\cdot\frac{\cancel{4}}{\cancel{3}}\cancel{\pi} r^3 \qquad q_{enc} = \frac{Qr^3}{b^3}$$

   d. Use your answer for (c) and Gauss's law to write the electric flux in terms of of $Q, r,$ and/or b and physical constants.

   $$\phi = \frac{q_{enc}}{\epsilon_0} = \frac{Qr^3}{\epsilon_0 b^3}$$

e. You now have two expressions for the electric flux. Set these two expressions equal and solve for your unknown E for $r < b$.

$$4E\pi \cancel{r^2} = \frac{Q r^{\cancel{3}}}{\epsilon b^3} \qquad E = \frac{Qr}{4\pi\epsilon_0 b^3}$$

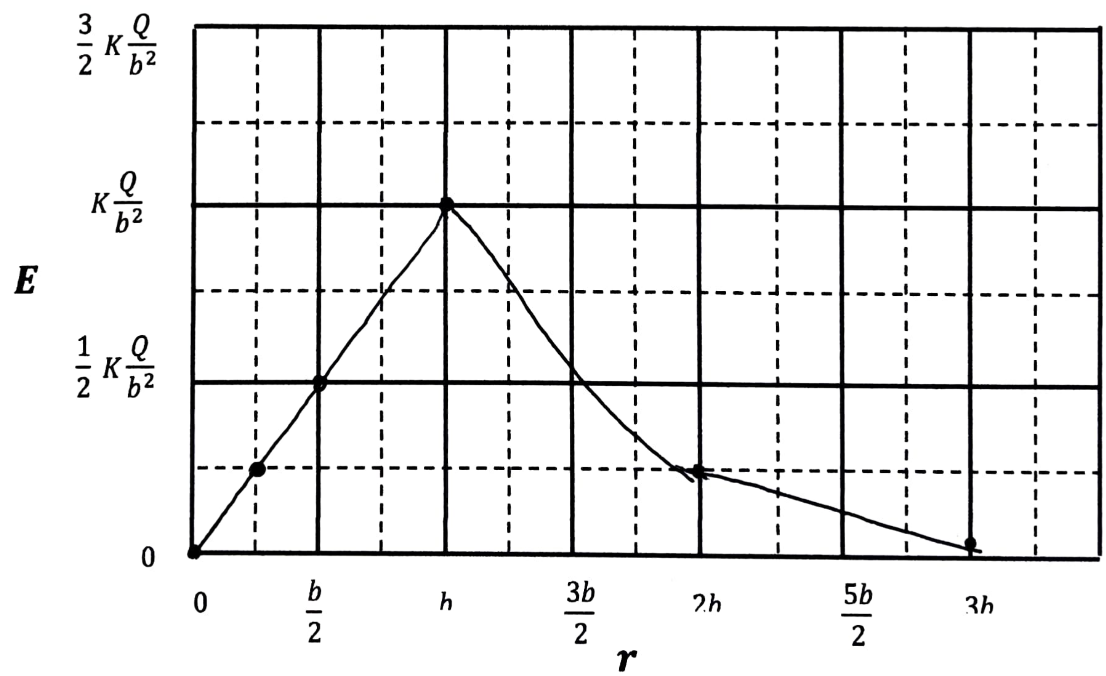
4. **Grraph of $E(r)$:** You should have found that the electric field inside and outside the sphere is:

$$E = \begin{cases} K\dfrac{Qr}{b^3} & r \le b \\ K\dfrac{Q}{r^2} & r \ge b \end{cases}$$

where $K = 1/4\pi\epsilon_o$. Evaluate the electric field at the following distances from the center r and complete the table below. Give answers in terms of Q and b which we assume are constants. Assume Q is positive.

	$r = 0$	$r = b/4$	$r = b/2$	$r = b$	$r = 2b$	$r = 3b$
E	$K\frac{Q(0)}{b^3} = 0$	$K\frac{Q}{4b^2}$	$K\frac{Q}{2b^2}$	$K\frac{Q}{b^2}$	$K\frac{Q}{4b^2}$	$K\frac{Q}{9b^2}$

Use the table above to help complete the graph of E vs. r. Treat Q and b as constants.

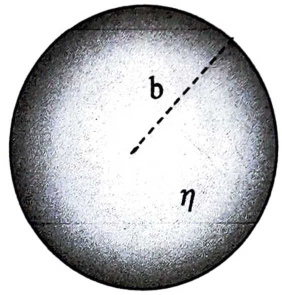


From Gauss's Law, we can get a general result for any spherically symmetric charge distribution; the electric field at a distance r from the center of a spherical charge distribution is:

$$E = \frac{q_{enc}}{4\pi\epsilon_o r^2}$$

where q_{enc} is the total charge inside a sphere of radius r. This is similar to Newton's Shell Theorem from Physics 211.

5. A sphere of radius b in which all the charge is spread out uniformly on the outer surface of the sphere. The charge per area on the outer surface is η (units = $Coul/m^2$). *η is the Greeek letter "eta".*



a. What is the total charge of the sphere? Answer in terms of η, r and/or b.

$Q = \eta \cdot 4\pi b^2$

b. What is the charge enclosed and electric field if $r < b$, i.e., inside the sphere? Answer in terms of η, r and/or b. Use ϵ_o in your answer.

$4\eta\pi b^2 \cdot \frac{r^3}{b^3}$ $\qquad$ $\frac{4\eta\pi r^3}{b} \cdot \frac{1}{4\pi\epsilon_0 r^2}$

$q_{enc} =$ $\frac{4\eta\pi r^3}{b}$ $\qquad$ $E =$ $\frac{\eta r}{\epsilon_0 b}$

c. What is the charge enclosed and the electric field if $r > b$, i.e., outside the sphere? Answer in terms of η, r and/or b. Check that your expressions have the correct dimensions.

$\frac{1}{4\pi\epsilon_0} \cdot \frac{4\eta\pi b^2}{r^2}$

$q_{enc} = 4\eta\pi b^2$ $\qquad$ $E =$ $\frac{\eta b^2}{\epsilon_0 r^2}$

d. Make a graph of the electric field for this case. Treat η and b as constants.