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After this activity, you should know: • *Use Gauss's Law in situations with cylindrical symmetry.*

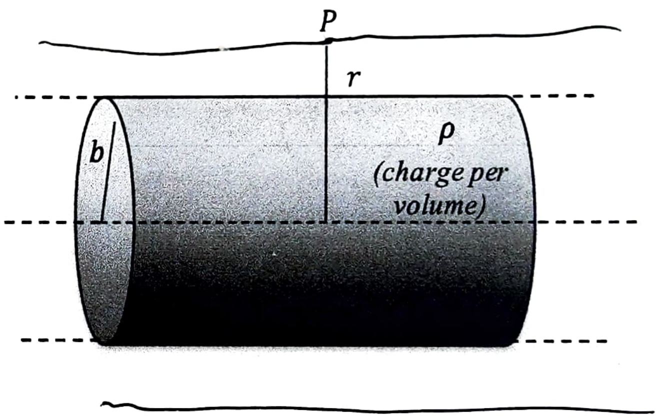
1. What is the volume of a cylinder of radius r and length L? What is the surface area of the curved portion of surface? What is the surface area of one of the endcaps?

   volume = $\cancel{4}\pi r^2 L$ area (curved section) = $2\pi r L$ area (one end) πr^2

2. A very long (treat as infinite) solid cylindrical tube of radius b has a uniform charge per volume ρ (units: $Coul/m^3$). We want to determine the electric field at point P a distance r from the central axis. Assume $r > b$.

   a. From symmetry, the electric field will be radial (either away or toward the central axis) and the magnitude of the electric field depends only on the distance r from the central axis.

   On the diagram, draw the cylindrical Gaussian surface you would use to find the electric field at point *P*. *To make this a closed surface, introduce a finite length L for your Gaussian cylinder.*



   b. What is the flux through the endcaps of the Gaussian cylinder? Give answer in terms of the unknown electric field E and lengths L, r and/or b.

   Hint: pick a point on the endcap. What is the angle between the unit normal and the electric field at that point? Remember the tube is infinite in length.

   $$E\cos(\cancel{0}\,90)\int dA = EA = E\pi b^2$$
   $$= 0$$

   c. What is the flux through the curved portion of your Gaussian cylinder? Give answer in terms of the unknown electric field E and the lengths L, r and/or b. *Make sure your expression for the area has the correct dimensions.*

   $2E\pi \cancel{b}^{\,r} L\cos(\theta)$ $\cancel{4}2E\pi\cancel{b}^{\,r}L$

   d. Use Gauss's law and your result from (c) to write the electric field in terms of q_{enc}, L, b and/or r.

   $$2E\pi r L = \frac{q_{enc}}{\epsilon_0} \qquad E = \frac{q_{enc}}{2\pi\epsilon_0 \cancel{b}\, r L}$$

   e. Since L was not a given we want to eliminate it in the final answer. Rewrite your answer in (d) in terms of the the charge per length enclosed, $\lambda_{enc} = q_{enc}/L$ and b and/or r.

   $$E = \frac{\lambda_{enc}}{2\pi\epsilon_0 \cancel{b} r}$$

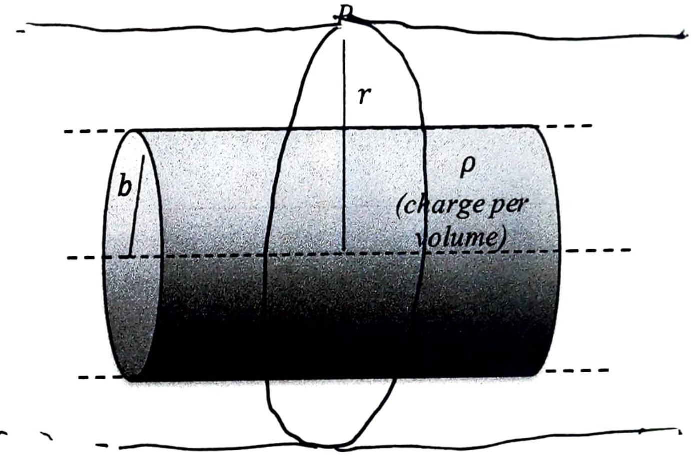
The expression is the same as E for an infinite line of charge except the charge per length λ is replaced by charge per length enclosed λ_{enc}:

$$E = \frac{\lambda_{enc}}{2\pi\epsilon_0 r}$$

Therefore the electric field due to a cylindrical charge distribution is the same as if all the charge inside r is at the central axis while the E field from the cylinder outside r cancels out.

f. We start by assuming point P is outside the cylinder so that $r > b$.

i. Draw the Gaussian surface you would use to find E at point P. *This is the same surface as on first page.*



ii. What is the charge enclosed and charge per length enclosed by the Gaussian surface in this case? Write your answers in terms of the charge per volume ρ and lengths r, b, and/or L (the length of your imaginary Gaussian cylinder). Check your units.

$q_{enc} =$ $\rho\pi b^2 L$

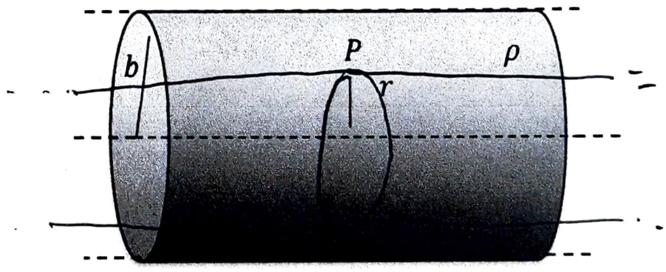
$\lambda_{enc} =$ $\frac{\rho\pi b^2 \cancel{L}}{\cancel{L}} = \rho\pi b^2$

iii. What is the electric field for $r > b$?

$$E = \frac{\rho \cancel{\pi} b^2}{2\cancel{\pi}\epsilon_0 r} = \frac{\rho b^2}{2\epsilon_0 r} = \frac{\lambda_{enc}}{2\pi\epsilon_0 r}$$

g. Now we assume that point P is inside the cylinder so that $r < b$.

i. Draw the Gaussian surface you would use to find E at point P.



ii. What is the charge per length enclosed by the Gaussian surface in this case? Answer in terms of ρ, r and/or b. Check your units.

$q_{enc} =$ $\rho\pi r^2 L$

$\lambda_{enc} =$ $\frac{\rho\pi r^2 \cancel{L}}{\cancel{L}} = \rho\pi r^2$

iii. What is the electric field for $r < b$?

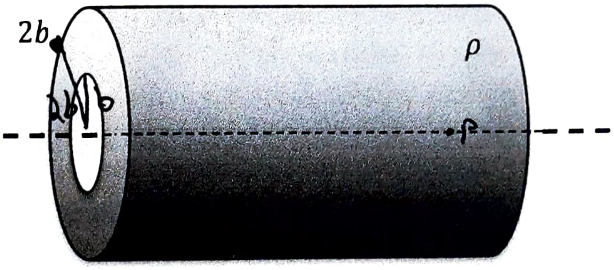
$$\frac{\rho\cancel{\pi} r^{\cancel{2}}}{2\cancel{\pi}\epsilon_0 \cancel{r}} \qquad E_{r<b} = \frac{\rho r}{2\epsilon_0}$$

3. A very long cylinder of radius $2b$ with a co-axial cylindrical hole of radius b. (Co-axial means the two cylinders have the same axis.) The charge per volume ρ is constant in the solid shell $b < r < 2b$.

   a. What is the charge per length enclosed and the electric field for point P inside the hole ($r < b$)? Answer in terms of ρ, b and/or r.

$4\pi r^2 L \sim \pi r^2 L$

$V = 3\pi r^2 L$



$\cancel{3\pi r^2 L\rho}$ 0

$\lambda_{enc} =$ __________

$E =$ ____ 0 ____

   b. What is the charge per length enclosed and electric field for a point P inside the solid region ($b < r < 2b$)?

$E = \dfrac{\lambda_{enc}}{2\pi\epsilon_0 r}$

$q_{enc} = \rho\left(\pi r^2 L - \pi b^2 L\right)$

$\lambda_{enc} = \dfrac{\rho\pi\cancel{L}(r^2-b^2)}{\cancel{L}} = \rho\pi(r^2-b^2)$

$E = \dfrac{\rho\cancel{\pi}(r^2-b^2)}{2\cancel{\pi}\epsilon_0 r}$

   c. What is the charge per length enclosed and electric field for a point P outside the cylinder ($r > 2b$)?

$q_{enc} = 3\pi b^2 L\rho$

$\lambda_{enc} = \dfrac{3\pi b^2 L\rho}{L} = 3\pi b^2\rho$

$E = \dfrac{3\cancel{\pi} b^2\rho}{2\epsilon_0 r}$