| Section | Group | Name | Signature |
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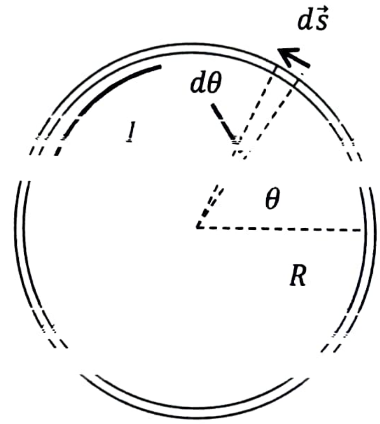
| Grade | | Jacob Harkins | |

After this activity you should know: • the magnetic field at the center of a circular current loop or a circular arc of current. • the magnetic field due to combinations of straight segments and arcs

1. Consider a circular loop of radius $R$ carrying current $I$. We want to determine the magnetic field generated by the current at the center of the circle.




$$d\vec{\boldsymbol{B}} = \frac{\mu_o I \; d\vec{s} \times \hat{\boldsymbol{r}}}{4\pi r^2},$$

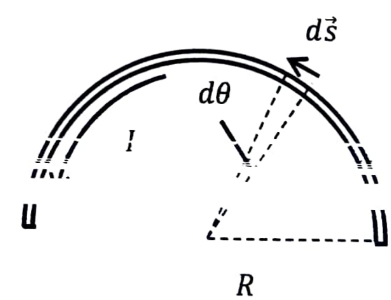
to find the magnitude $dB$ of the magnetic field at the center due to the small segment $ds$. Write $dB$ in terms of $R, I, d\theta$ and/or $\theta$. The arc length formula $ds$ = $R\,d\theta$ may be useful.

$= \frac{\mu_0 I}{4\pi R^2} \cdot \int ds$

b. Usually we need to break up $d\vec{\boldsymbol{B}}$ into components before integrating but note that all the contributions $d\vec{\boldsymbol{B}}$ point the same way so we can just integrate the magnitude $dB$. What is the total magnetic field (magnitude and direction) at the center of the circular loop? *Note the integral is really easy!*

$\frac{\mu_0 I}{2R}$

c. Now consider the half loop shown. What is the magnitude and direction of the magnetic field at the center due to the half loop? You should be able to figure this out easily from (b).



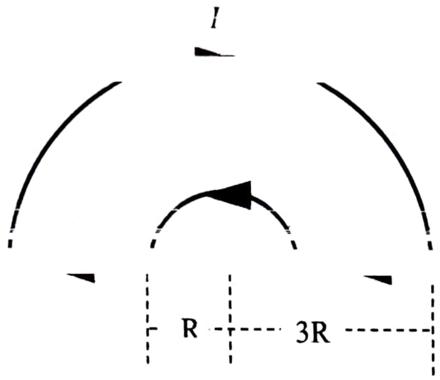

$\frac{\mu_0 I}{4R}$

d. What, if anything, would change for your answers to (b) and (c) if the current was clockwise instead of counterclockwise?

The sign

2. For each case, determine the magnitude and direction at the center of the half circles.

a.




$$\frac{+\mu_0 I}{4R} - \frac{\mu_0 I}{12R}$$

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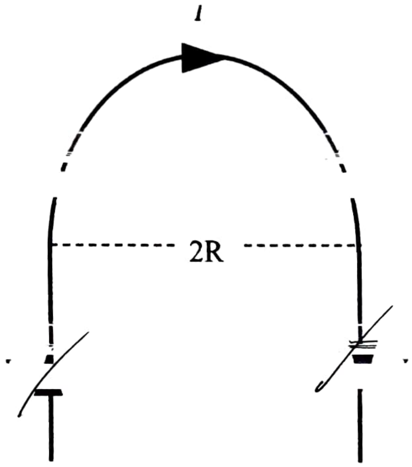
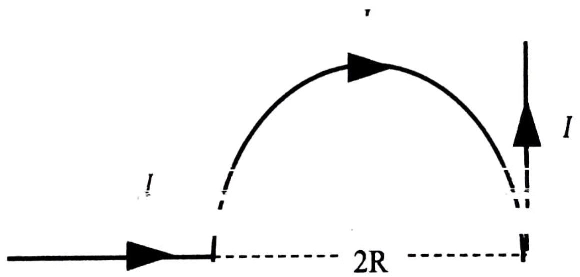
Assume the straight segments of current below are semi-infinite (i.e., very long compared to $R$).

b.



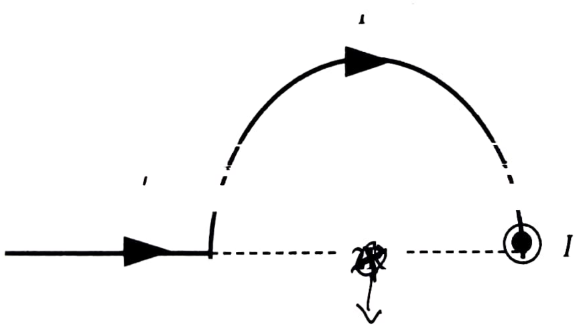

$$\frac{-\mu_0 I}{2R}$$

c.



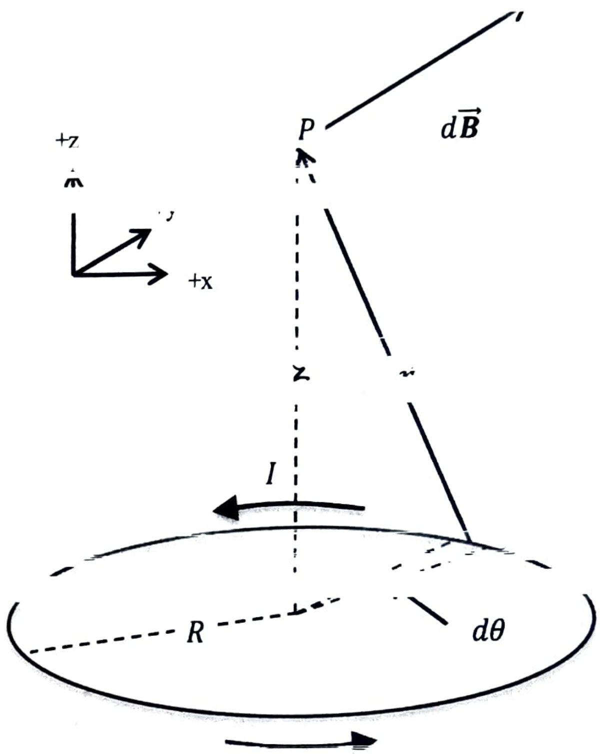

$$\frac{\mu_0 I}{4\pi R} - \frac{\mu_0 I}{2R}$$

d. For direction, give the angle relative to the plane of the paper.



3. **Bonus: (4 – you must finish the rest of the worksheet first to receive any credit for bonus):** We want to find the magnetic field at point *P* a distance z on the axis from the center of a circular current loop of current *I* and radius *R*.

a) What is the magnitude of the small magnetic field $dB$ at point P due to the current in the small segment of angle $d\theta$ of the ring? Answer in terms of $I, R, z, d\theta$ and/or $\theta$.




$$\frac{\mu_0}{4\pi} \quad \frac{I\,d\theta}{R^2}$$

$dB =$

b. From symmetry, only the net magnetic field at P will be in the z direction. What is $dB_z$, the z component of $d\vec{B}$? Answer in terms of $I, R, z, d\theta$ and/or $\theta$. *Be careful, there are three different angles in the problem, $\theta$, the angle in the cross product and the angle in the z component of $d\vec{B}$. They are not the same.*

$$\frac{\mu_0 I\,d\theta}{4\pi} \cdot \frac{R}{(z^2+R^2)^{3/2}}$$

$dB_z =$

c) Integrate $dB_z$ over the ring to find the net magnetic field at point P

$B =$ $\quad \frac{\mu_0}{4\pi}\,\frac{2\pi R^2 I}{(z^2+R^2)^{3/2}} \qquad \frac{\mu_0 R^2 I}{2(z^2+R^2)^{3/2}}$

d. Start with your answer to (c) and show that if $z \gg R$, the magnetic field can be approximated as $B \approx \mu_0 IA/2\pi z^3$ where $A$ is the area of the current loop. Show all steps.

$A = \pi R^2$

$$\lim_{z+R \to z} \frac{\mu_0 R^2 I}{2(z^2+R^2)^{3/2}} = \quad = \quad \frac{\mu_0 R^2 I}{2(z^2)^{3/2}} = \boxed{\frac{\mu_0 I A}{2\pi z^3}}$$

*The quantity $\mu = IA$ is called the dipole moment $\mu$ of the current loop. The magnetic field along the axis of any current loop is as $B \approx \mu_0 IA/(2\pi z^3)$ for any shape current loop (not necessarily a circle) as long as z is much larger than the size of the loop.*