

Exponential Functions

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Math 6010

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Objectives

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- ▶ Understand basic properties of exponential functions
- ▶ Graph exponential functions

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$$3 = 3, \quad \frac{2}{5} = 0.4, \quad \pi = 3.14159265\dots, \quad \sqrt{2} = 1.41421356\dots$$

Approximate any number x by *finitely many* of its decimal digits.

$$a^\pi \approx a^{3.14} = a^{\frac{314}{100}} = \sqrt[100]{a^{314}}$$

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The more digits we use, the closer we get to the true value of a^π

2^3	$2^{3.1}$	$2^{3.14}$	$2^{3.141}$	$2^{3.1415}$	\dots	2^π
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Better still, we get to keep all the properties of exponents that we already know:

Exponent Properties

If x, y are real numbers, and $a, b > 0$, then

$$\begin{array}{lll} \bullet a^x \cdot a^y = a^{x+y} & \bullet (a^x)^y = a^{xy} & \bullet (ab)^x = a^x \cdot b^x \\ \bullet 1^x = 1 & \bullet a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x & \bullet a^0 = 1 \end{array}$$

Exponential Functions

Consider the exponential function $f(x) = 5 \cdot 2^x$. Let's make a table of values

x	$f(x)$
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0	5
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- ▶ $f(0) = 5 = C$.

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$$\frac{f(x+1)}{f(x)} = \frac{C \cdot a^{x+1}}{C \cdot a^x} = \frac{a^{x+1}}{a^x} = a^{x+1-x} = a$$

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3	40
4	80
5	160

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It seems that $f(x) \rightarrow 0$ as $x \rightarrow -\infty$, and $f(x) \rightarrow \infty$ as $x \rightarrow \infty$

Graphing Exponential Functions

Connect the points continuously and use the asymptotic behavior noted

