

Math 5601: Introduction to Numerical Analysis

Homework assignment 3

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Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show.

Problem #1: Consider $f(t) = \sin(t)$. (Simplify your solutions as much as possible)

(a) Without using the general inner product expression of the best approximation, find the best approximation $p(t) \in P^3[-1, 1]$ to $f(t) \in C[-1, 1]$ with respect to the L^2 norm.

(b) Find the Taylor polynomial approximation $p(t)$ of degree 3 at $t = 0$.

(c) Find the Lagrange polynomial approximation $p(t)$ of degree 3 that interpolates $f(t)$ at $t = -1, -\frac{1}{3}, \frac{1}{3}, 1$.

Problem #2: Based on $u_1 = 1, u_2 = x, u_3 = x^2$, use Gram-Schmidt orthogonalization process to compute the three polynomials $w_1(x), w_2(x)$, and $w_3(x)$ which are orthonormal on the interval $[0, 1]$ with respect to the inner product $(f, g) = \int_0^1 f(x)g(x) dx$. Using these polynomials, find the best approximation in $P_2[0, 1]$ for $f(x) = x^{\frac{1}{2}}$.

Problem #3: Find the quadratic polynomial $p(t)$ that satisfies $p(0) = f(0), p(2) = f(2), p'(2) = f'(2)$.

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