Exponential Functions

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Math 6010

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Objectives

- Define exponential functions ($f(x) = C \cdot a^x$)

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- Understand basic properties of exponential functions

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- Define exponential functions ($f(x) = C \cdot a^x$)
- Understand basic properties of exponential functions
- Graph exponential functions

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Approximate any number x by *finitely many* of its decimal digits.

$$a^\pi \approx a^{3.14} = a^{\frac{314}{100}} = \sqrt[100]{a^{314}}$$

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The more digits we use, the closer we get to the true value of a^π

2^3	$2^{3.1}$	$2^{3.14}$	$2^{3.141}$	$2^{3.1415}$	$\cdots$	2^π
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Better still, we get to keep all the properties of exponents that we already know:

Exponent Properties

If x, y are real numbers, and $a, b > 0$, then

- $a^x \cdot a^y = a^{x+y}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x \cdot b^x$
- $1^x = 1$
- $a^{-x} = \dfrac{1}{a^x} = \left(\dfrac{1}{a}\right)^x$
- $a^0 = 1$

Exponential Functions

Now that we know what a^x means for *any* x, we can define an exponential function.

Exponential Function

An **exponential function** is a functiton f such that

$$f(x) = C \cdot a^x,$$

where

- $C \neq 0$ is called the **initial value**, and
- $a > 0$, $a \neq 1$ is called the **growth factor**.

Exponential Functions

Consider the exponential function $f(x) = 5 \cdot 2^x$. Let's make a table of values

x	$f(x)$
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1	10
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- $f(0) = 5 = C$.

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-3	0.625
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3	40
4	80
5	160

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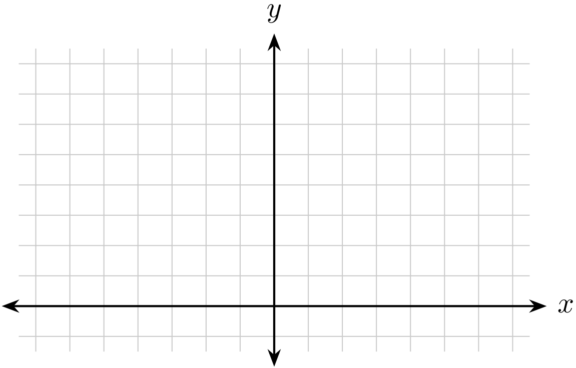
It seems that $f(x) \to 0$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$

Graphing Exponential Functions

Connect the points continuously and use the asymptotic behavior noted



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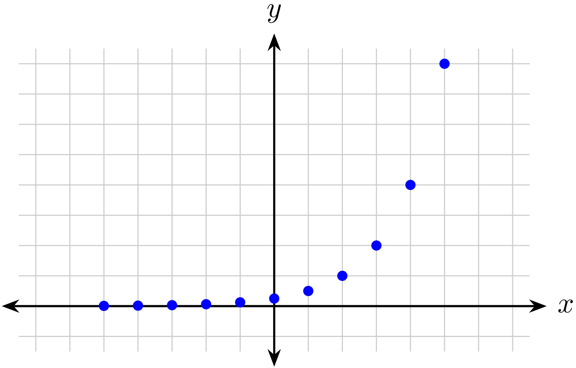

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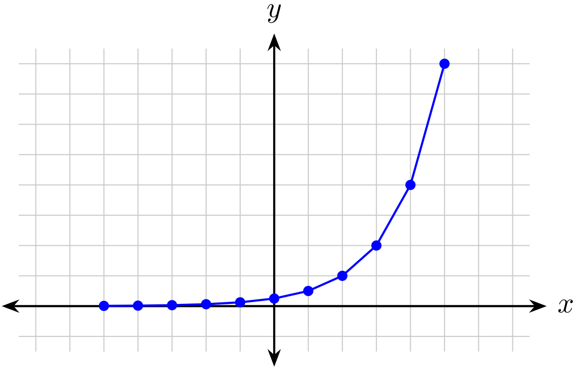

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