Problem 1: (Dividend payment) In unprofitable times corporations sometimes suspend dividend payments. Suppose that after a dividend has been paid the next one will be paid with probability .9, while after a dividend is suspended, the next one will be suspended with probability 0.6. In the long run, what is the fraction of dividends that will be paid?

Problem 2: (Income classes) From one generation to the next, families change their income group Low, Middle or High according to the following Markov chain with transition probability

$$\mathbf{P} = \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.3 & 0.6 \end{pmatrix};$$

where the columns and rows are in order L, M and H.
Find the limiting fractions of the population in the three income classes.

Problem 3: (A working machine) At the beginning of each day, a piece of equipment is inspected to determine its working condition, which is classified as state 1=new, 2, 3, or 4=broken. We assume the state is a Markov chain with the following transition matrix:

$$\mathbf{P} = \begin{pmatrix} 0.95 & 0.05 & 0.0 & 0 \\ 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0.875 & 0.125 \end{pmatrix};$$

where the order of rows and columns is 1,2,3 and 4.
(a) Suppose that the broken machine requires three days to fix. To incorporate this into the Markov chain we add states 5 and 6 and suppose that $p_{45} = 1$, $p_{56} = 1$ and $p_{61} = 1$. Find the fraction of time that the machine is working.
(b) Suppose now that they have an option of performing preventive maintenance when the machine is in state 3, and that this maintenance takes one day and returns the machine to state 1. This change the transition probability to

$$\mathbf{P} = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0 & 0.9 & 0.1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Find the fraction of time the machine is working under this new policy.

Problem 4: A criminal named Xavier and a policeman named Yakov move between three possible hideouts according to a Markov chains X_n and Y_n with transition probabilities

$$\mathbf{P}_{\mathbf{Xavier}} = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{pmatrix}; \quad \mathbf{P}_{\mathbf{Xavier}} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}.$$

At time $T = \min\{n : X_n = Y_n\}$, the game is over and the criminal is caught. Suppose that $X_0 = i$, and $Y_0 = j$, with $i \neq j$. Find the expected value of T.
(b) Suppose the two players generalize their strategies to

$$\mathbf{P}_{\mathbf{Xavier}} = \begin{pmatrix} 1-2p & p & p \\ p & 1-2p & p \\ p & p & 1-2p \end{pmatrix}; \quad \mathbf{P}_{\mathbf{Yakov}} = \begin{pmatrix} 1-2q & q & q \\ q & 1-2q & q \\ q & q & 1-2q \end{pmatrix}.$$

If Yakov uses $q = 0.5$ as he did in (a), what value of p should Xavier uses to maximize the expected time to get caught? How about with $q = 3^{-1}$.

Problem 5: Markov chain simulation Consider a Markov chain $\{X_n : n \geq 0\}$ with $\mathcal{S} = \{1, 2, 3, 4\}$. Take the initial distribution to be $X_0 \sim \pi_0 = (0.2, 0.25, 0.25, 0.3)$. Simulate the Markov chain up to time 5.

Do the following in the textbook:
14 Page 285, 21 Page 286; 23 Page 279; 31 Page 288; 49 Page 292; 60 Page 292