Math 5601: Introduction to Numerical Analysis
Homework assignment 5

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Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show. For each numerical experiment, all the .m files of your Matlab code should be electronically submitted to hex@mst.edu together with a .txt file which copies all the information in the Matlab command window when you run the code to obtain the numerical results.

Problem #1: Show that the IVP $y'(t) = \frac{t}{9}cos(2y) + t^2$ with $y(0) = 1$ has a unique solution for $|t| \leq 10$.

Problem #2: Consider $y'(t) = y^{\frac{1}{3}}$ with $y(0) = 0$ and $0 \leq t \leq 2$.

(a) Program forward Euler method in Matlab.

(b) Use the code to solve the above initial value problem. What behavior do you observe? Explain why.

Problem #3: Consider the forward Euler method

$$y_{j+1}^h = y_j^h + hf(x_j, y_j^h) \ (j = 0, 1, 2, \cdots, N)$$

for approximating $y'(x) = f(x, y)$ with $y(0) = \alpha$. Assume that

$$y_j^h - y(x_j) = c_1 h + c_2 h^2 + c_3 h^3 + \cdots = \sum_{m=1}^{\infty} c_m h^m$$

where c_m $(m = 1, 2, 3 \cdots)$ are independent of h. Suppose that y_j^h, $y_j^{\frac{h}{2}}$, and $y_j^{\frac{h}{3}}$ have been calculated. Find an approximation to $y(x_j)$ that has third order accuracy.

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