Math 5601: Introduction to Numerical Analysis
Homework assignment 6

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Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show. For each numerical experiment, all the .m files of your Matlab code should be electronically submitted to hex@mst.edu together with a .txt file which copies all the information in the Matlab command window when you run the code to obtain the numerical results.

Problem #1: Use Newton-Cotes formula to derive the two point open rule with $x_0 = a + \frac{b-a}{3}$ and $x_1 = a + \frac{2(b-a)}{3}$:

$$J(f) = \int_a^b f(x)\ dx \quad \approx \quad Q(f) = \frac{b-a}{2} f(a + \frac{b-a}{3}) + \frac{b-a}{2} f(a + \frac{2(b-a)}{3}).$$

Problem #2: Assume the numerical quadrature for $\hat{f}(\hat{x})$ on $[0,1]$ is

$$\hat{J}(\hat{f}) = \int_0^1 \hat{f}(\hat{x})\ d\hat{x} \approx \hat{Q}(\hat{f}) = \sum_{j=0}^{m} \hat{\alpha}_j \hat{f}(\hat{x}_j).$$

Derive the numerical quadrature of $J(f) = \int_a^b f(x)\ dx$.

Problem #3: Use the following Newton-Cotes formulas to approximate $J(f) = \int_1^2 \frac{cos\left(\frac{\pi}{4}x\right)}{sin^2\left(\frac{\pi}{4}x\right)}\ dx$. Compare the numerical results obtained from your Matlab code (or by hand) with the analytic solution $-\frac{4}{\pi} + \frac{4}{\pi}\sqrt{2} \approx 0.527393087579050$.

(a) Middle point rule.

(b) Two point open rule.

(c) Simpson's rule.

(d) $\int_0^1 f(x)\ dx \approx \frac{1}{8} f(0) + \frac{3}{8} f(\frac{1}{3}) + \frac{3}{8} f(\frac{2}{3}) + \frac{1}{8} f(1)$.

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