Math 6330 Homework 3

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1.89

Let $\mathbb{T} = \mathbf{Z}$, and define $f(t) = g(t) = t^2$ for $t \in \mathbb{T}$. Viewing f as a function on $\mathbf{R}$, we have $f'(t) = 2t$. As we have calculated before, $g^{\Delta}(t) = 2t + 1$. Lastly, $(f \circ g)(t) = t^4$, so

$$(f \circ g)^{\Delta}(t) = (t+1)^4 - t^4 = 4t^3 + 6t^2 + 4t + 1.$$

According to Theorem 1.87, there exists $c \in [2, \sigma(t)] = [2, 3]$ such that

$$(f \circ g)^{\Delta}(2) = f'(g(c))g^{\Delta}(2).$$

Using the formulas for $(f \circ g)^{\Delta}$, f', and g^{Δ}, this means that

$$4 \cdot 2^3 + 6 \cdot 2^2 + 4 \cdot 2 + 1 = 2c^2 \cdot (2 \cdot 2 + 1)$$

or

$$65 = 10c^2 \implies c = \pm\sqrt{\frac{13}{2}}.$$

Since $c \in [2, 3]$, it follows that $c = \sqrt{\frac{13}{2}}$. Note that $\sqrt{\frac{13}{2}} \in [2, 3]$, as promised, because

$$8 \leq 13 \leq 18 \implies 4 \leq \frac{13}{2} \leq 9 \implies 2 \leq \sqrt{\frac{13}{2}} \leq 3.$$