Math 5601: Introduction to Numerical Analysis
Homework assignment 10

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Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show.

Problem #1: Consider

$$A = \begin{pmatrix} 5 & -2 \\ -4 & 7 \end{pmatrix}.$$

Find $\|A\|_1$, $\|A\|_\infty$, $\|A\|_2$. Verify that $\rho(A) \le \|A\|_1$, $\rho(A) \le \|A\|_\infty$, and $\rho(A) \le \|A\|_2$.

Problem #2: Let A be the $n \times n$ tridiagonal matrix with

$$a_{ij} = \begin{cases} 4, & if\ i = j, \\ -1, & if\ i = j+1\ or\ i = j-1, \\ 0, & otherwise, \end{cases}$$

Prove that the Jacobi method converges for this matrix by using the following theorem from the lecture slides of Chapter 3.

Theorem: Consider the iterative method

$$\overrightarrow{x}^{(k+1)} = B\overrightarrow{x}^{(k)} + \overrightarrow{c},\ k = 0, 1, 2, \cdots$$

The following are equivalent:

- (a) The iterative method is convergent.
- (b) $\rho(B) < 1$.
- (c) There exists a matrix norm $\|\cdot\|$ such that $\|B\| < 1$.

Problem #3: Consider

$$\begin{aligned} x_1 + x_3 &= 0, \\ -x_1 + x_2 &= 0, \\ x_1 + 2x_2 - 3x_3 &= 0. \end{aligned}$$

Explain the observation for the numerical results of Problem #3 of HW9, based on the the following theorem from the lecture slides of Chapter 3.

Theorem: Consider the iterative method

$$\overrightarrow{x}^{(k+1)} = B\overrightarrow{x}^{(k)} + \overrightarrow{c},\ k = 0, 1, 2, \cdots$$

The following are equivalent:

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- (a) The iterative method is convergent.
- (b) $\rho(B) < 1$.
- (c) There exists a matrix norm $\|\cdot\|$ such that $\|B\| < 1$.