Bifurcation analysis of a discrete-time prey-predator model

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Outline

- Description and interpretation of a discrete-time predator-prey model
- Determination of fixed points
- Bifurcation analysis
  - Period-doubling bifurcation
  - Neimarck-Sacker bifurcation
- Numerical investigations
  - Bifurcation diagram of period-doubling bifurcation
  - Phase portrait changes at Neimark-Sacker bifurcation

Model Description

$$x_p(n+1) = x_p(n)\left[1 + r\left(1 - \frac{x_p(n)}{k}\right) - a y_p(n)\right]$$

$$y_p(n+1) = y_p(n)\left[1 - b + \frac{c x_p(n)}{y_p(n)}\right]$$

- $n \in \mathbb{Z}$: discrete time step
- $x_p(n)$: number of prey; $y_p(n)$: number of predators
- $r > 0$: intrinsic growth rate of prey
- $k > 0$: carrying capacity of prey
- $a > 0$: predation rate
- $b > 0$: death rate of predators
- $c > 0$: conversion rate (of prey into predators)

Fixed Points

$(0, 0)$ is a fixed point, but not an interesting one.

Ecologically, the important fixed points occur when $x_p > 0$, and $y_p > 0$, when predator and prey are in equilibrium.

There is one such fixed point:

$$\mathcal{P}_* = \left(\frac{rkb}{ack + br}, \frac{crk}{ack + br} \right).$$

Period-doubling Bifurcations

On the time scale $\mathbb{Z}$, fixed points are also 1-periodic solutions. In general, if $x(n)$ is a solution of

$$x(n+1) = f(x(n))$$

such that $x(n+p) = x(n)$, where p is the smallest integer that makes this true, then $x_0 = x(0)$ is called a **periodic point of minimal period** p.

A **period-doubling bifurcation** occurs when the stability of a fixed point changes and a pair of periodic points of minimal period 2 emerge.

See Section 3.4 of *Dynamics and Bifurcations*.

Period-doubling Bifurcation in the Predator-Prey Model

Neimarck-Sacker Bifurcations

Neimarck-Sacker Bifurcation in the Predator-Prey Model

Period-Doubling Bifurcation Diagram

Phase Portraits Near the Neimarck-Sacker Bifurcation