Homework 4

October 24, 2023

1 Homework 4 – KMeans Lab

1.1 Exercise 1

Create a function for loading and previewing data.

[ ]:

```
import numpy as np

def load_preview(filename):
    data = np.loadtxt(filename)
    print(f'Loaded array shape: {data.shape}')
    print()

    print('First five rows of data')
    print(data[:5])
    print()

    return data
```

Load and preview data.

[ ]:

```
ex1_data = load_preview('hw_data.txt')
```

```
Loaded array shape: (25000, 3)

First five rows of data
[[  1.        65.78331 112.9925 ]
 [  2.        71.51521 136.4873 ]
 [  3.        69.39874 153.0269 ]
 [  4.        68.2166  142.3354 ]
 [  5.        67.78781 144.2971 ]]
```

Get the first 350 rows and last two columns of data. Note that height in inches is column 1, and weight in lbs is column 2.

[ ]:

```
height_in = ex1_data[:350, 1]
weight_lbs = ex1_data[:350, 2]
```

Define a function to print summary statistics.

[ ]:

```
def print_summary(title, x):
    print(title)
    print(f'Min = {np.min(x):.02f}, Max = {np.max(x):.02f}')
    print(f'Mean = {np.mean(x):.02f}, Var = {np.var(x)}, Std = {np.std(x)}')
    print()
```

Print summary.

[ ]:

```
print_summary('Height (in)', height_in)
print_summary('Weight (lbs)', weight_lbs)
```

```
Height (in)
Min = 63.19, Max = 73.90
Mean = 67.95, Var = 3.6033967936014766, Std = 1.8982615187590661

Weight (lbs)
Min = 84.36, Max = 158.96
Mean = 126.85, Var = 145.00038262018663, Std = 12.041610466220314
```

Create function to standardize data.

[ ]:

```
def standardize(x):
    return (x - np.mean(x, axis=0, keepdims=True)) / np.std(x, axis=0,␣
  ↪keepdims=True)
```

Standardize weight and height.

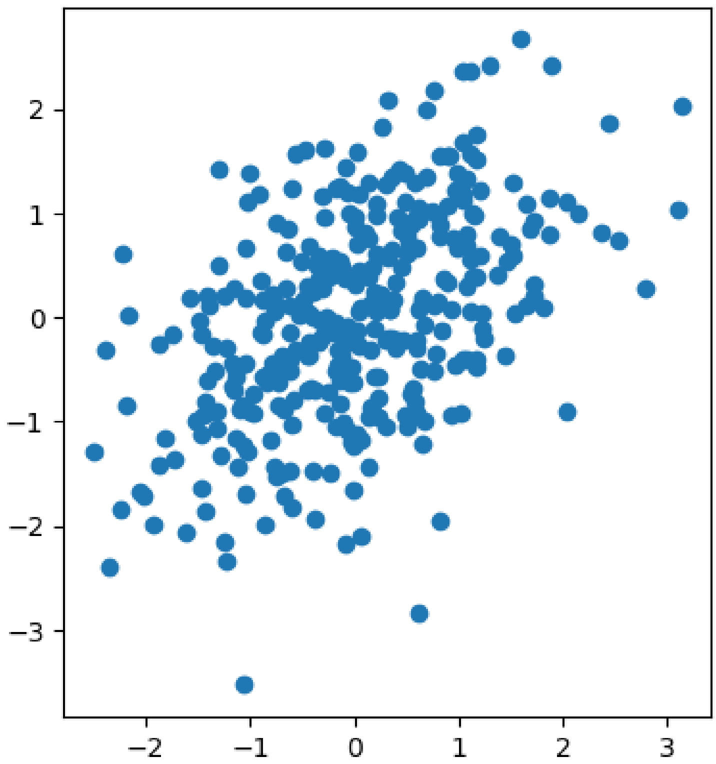
[ ]:

```
height_std = standardize(height_in)
weight_std = standardize(weight_lbs)
```

Plot standardized weights and heights.

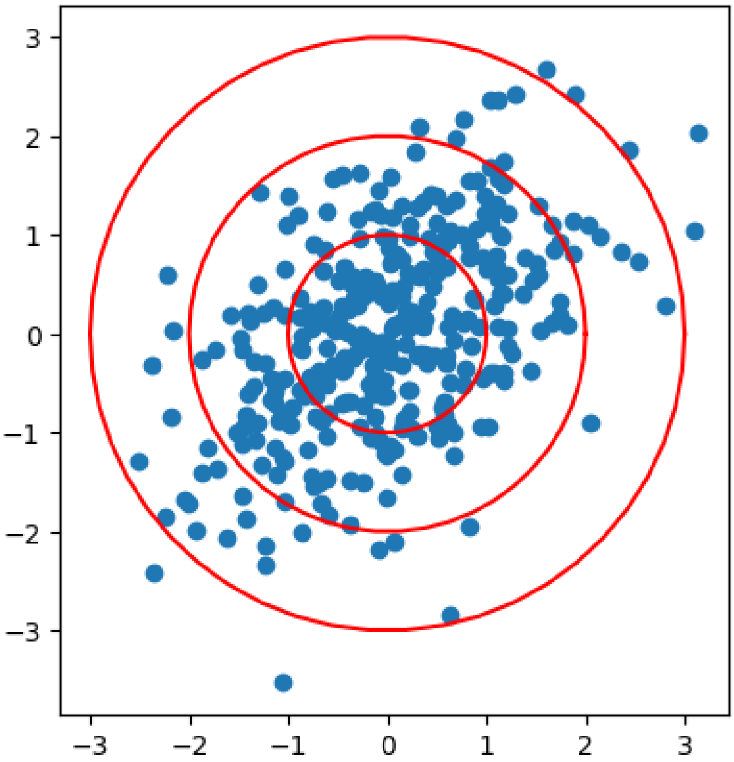
[ ]:

```
import matplotlib.pyplot as plt

_, ax = plt.subplots()
ax.scatter(height_std, weight_std)
ax.set_aspect(1.0)
plt.show()
```



Plot standardized weights and heights with red circles indicating 1, 2, and 3 standard deviations away from the mean. Save the result in `exercise1.jpg`.

[ ]:
```
_, ax = plt.subplots()
ax.scatter(height_std, weight_std)

t = np.linspace(0, 2 * np.pi, 51)
x, y = np.cos(t), np.sin(t)

for r in (1, 2, 3):
    ax.plot(r * x, r * y, c='red')

ax.set_aspect(1.0)
plt.savefig('exercise1.jpg')
plt.show()
```



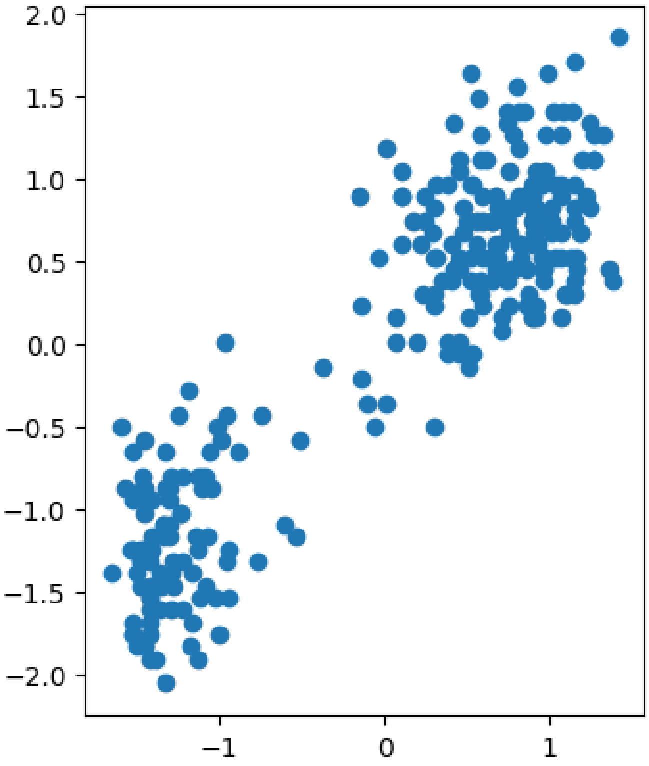
1.2 Exercise 2

Load and preview data, print summary statistics, standardize, and plot standardized data. Note that eruption time is column 0, and wait time is column 1.

```
[ ]: ex2_data = load_preview('faithful_data.txt')

     eruption_time_s = ex2_data[:, 0]
     wait_time_s = ex2_data[:, 1]

     print_summary('Eruption time (s)', eruption_time_s)
     print_summary('Wait time (s)', wait_time_s)

     eruption_time_std = standardize(eruption_time_s)
     wait_time_std = standardize(wait_time_s)

     _, ax = plt.subplots()
     ax.scatter(eruption_time_std, wait_time_std)
     ax.set_aspect(1.0)
     plt.show()
```

```
Loaded array shape: (272, 2)
```

```
First five rows of data
[[ 3.6    79.   ]
 [ 1.8    54.   ]
 [ 3.333 74.   ]
 [ 2.283 62.   ]
 [ 4.533 85.   ]]

Eruption time (s)
Min = 1.60, Max = 5.10
Mean = 3.49, Var = 1.2979388904492861, Std = 1.139271210225768

Wait time (s)
Min = 43.00, Max = 96.00
Mean = 70.90, Var = 184.14381487889273, Std = 13.569960017586371
```



Perform KMeans with $k = 1$ and $k = 2$ and print energy.

[ ]:

```
from sklearn.cluster import KMeans

data2 = np.column_stack([eruption_time_std, wait_time_std])
```

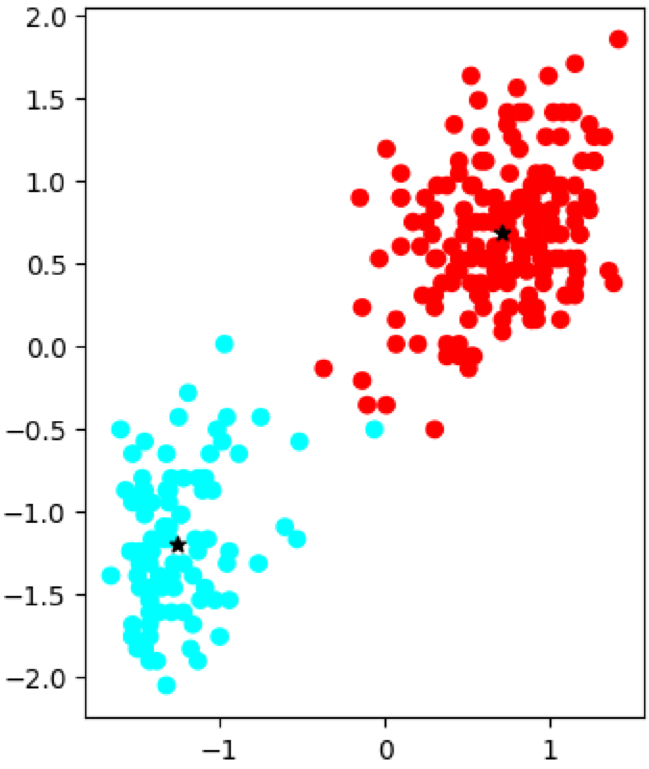
```
kmeans = {}
for k in (1, 2):
    kmeans[k] = KMeans(k, n_init='auto').fit(data2)
    print(f'k = {k}, energy = {kmeans[k].inertia_}')
```

```
k = 1, energy = 544.0
k = 2, energy = 79.57595948827702
```

Plot the clusters with $k = 2$ and mark the centers with a star. Save the plot in `exercise2.jpg`.

[ ]:

```
_, ax = plt.subplots()
c = np.array(['red', 'cyan'])[kmeans[2].labels_]
ax.scatter(eruption_time_std, wait_time_std, c=c)

for center in kmeans[2].cluster_centers_:
    ax.scatter(*center, c='black', marker='*')

ax.set_aspect(1.0)
plt.savefig('exercise2.jpg')
plt.show()
```



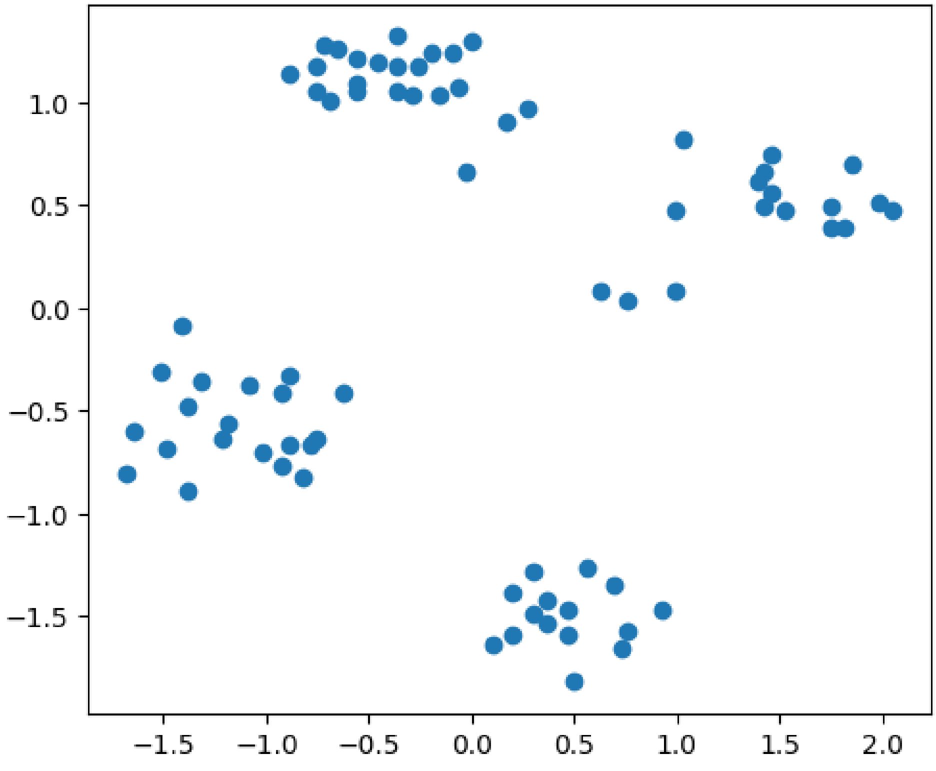
1.3 Exercise 3

Load, preview, and standardize data. Plot column 0 (x) versus column 1 (y).

[ ]:

```
ex3_data = load_preview('ruspini_data.txt')
ex3_data_std = standardize(ex3_data)

_, ax = plt.subplots()
ax.scatter(ex3_data_std[:, 0], ex3_data_std[:, 1])
ax.set_aspect(1.0)
plt.show()
```

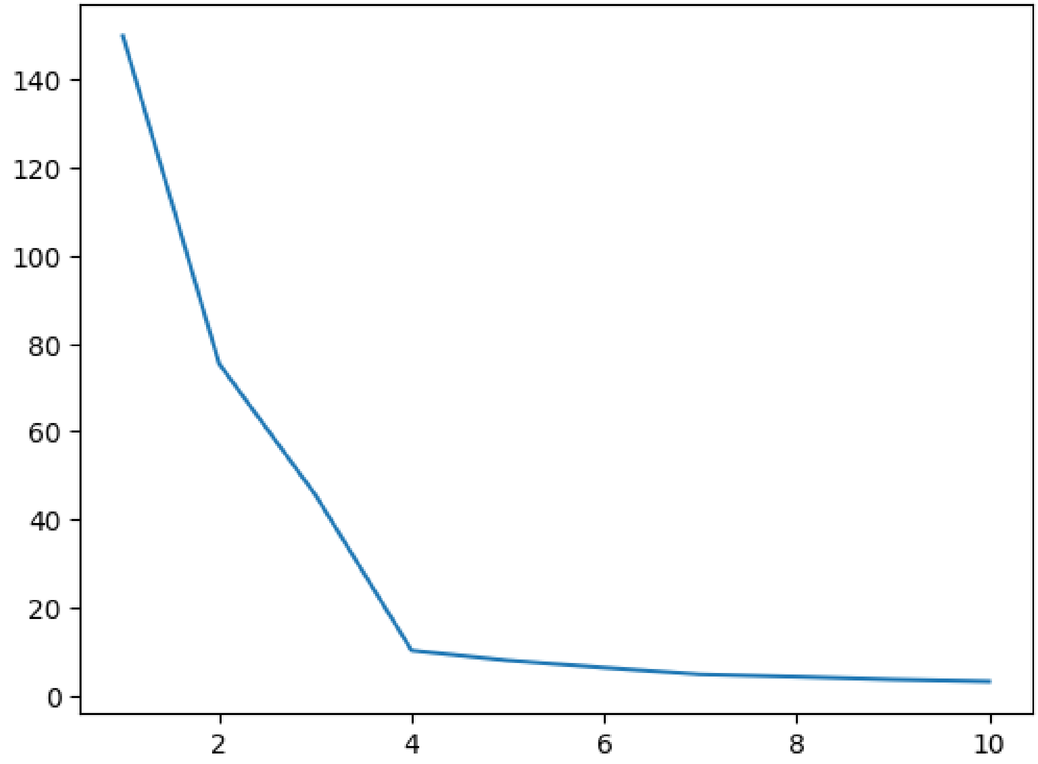
```
Loaded array shape: (75, 2)

First five rows of data
[[ 4. 53.]
 [ 5. 63.]
 [10. 59.]
 [ 9. 77.]
 [13. 49.]]
```



Run KMeans with $1 \leq k \leq 10$. Store and plot the energies.

```
[ ]: from sklearn.cluster import KMeans

     E = []
     for k in range(1, 11):
         E.append(KMeans(k, n_init='auto').fit(ex3_data_std).inertia_)
         print(f'k = {k}, Energy = {E[k-1]}')

     _, ax = plt.subplots()
     ax.plot(range(1, 11), E)
     plt.show()
```

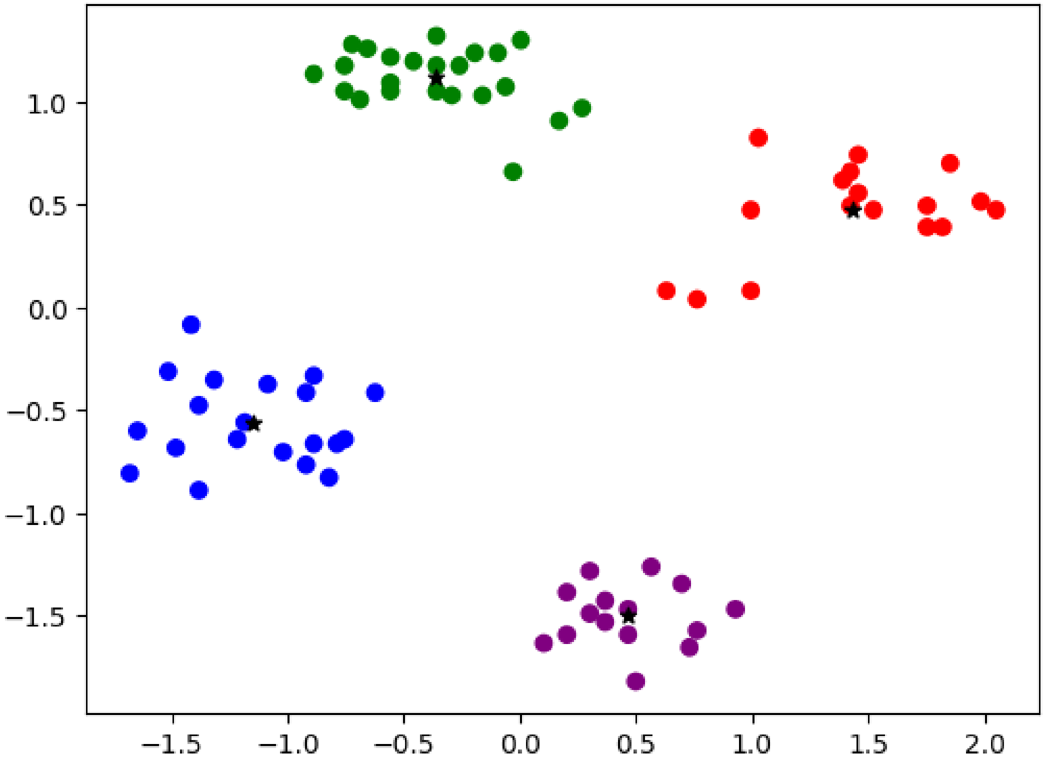
```
k = 1, Energy = 150.0
k = 2, Energy = 75.45304292711121
k = 3, Energy = 45.633562920522536
k = 4, Energy = 10.224126812968091
k = 5, Energy = 7.9651988940254235
k = 6, Energy = 6.312361201725671
k = 7, Energy = 4.7856380992803365
k = 8, Energy = 4.229066995763582
k = 9, Energy = 3.6162664316372406
k = 10, Energy = 3.188491498249629
```



It looks like we should use $k = 4$ clusters. Run KMeans with $k = 4$, show plot of clusters, and save plot to `exercise3.jpg`.

[ ]:

```
good_kmeans = KMeans(4, n_init='auto').fit(ex3_data_std)

_, ax = plt.subplots()
c = np.array(['red', 'blue', 'green', 'purple'])[good_kmeans.labels_]
ax.scatter(ex3_data_std[:, 0], ex3_data_std[:, 1], c=c)

ax.scatter(
    good_kmeans.cluster_centers_[:, 0],
    good_kmeans.cluster_centers_[:, 1],
    c='black', marker='*'
)
plt.savefig('exercise3.jpg')
plt.show()
```



1.4 Exercise 4

Read the image and display it.

[ ]:

```
import imageio
```

```
image = imageio.v3.imread('swim.jpg')
plt.imshow(image)
plt.show()
```

Get array of colors. Make sure to convert to `float`.

[ ]:
```
colors = np.array(image, dtype=float).reshape(-1, image.shape[-1])
print(f'Color array shape: {colors.shape}')
```

```
Color array shape: (1920000, 3)
```

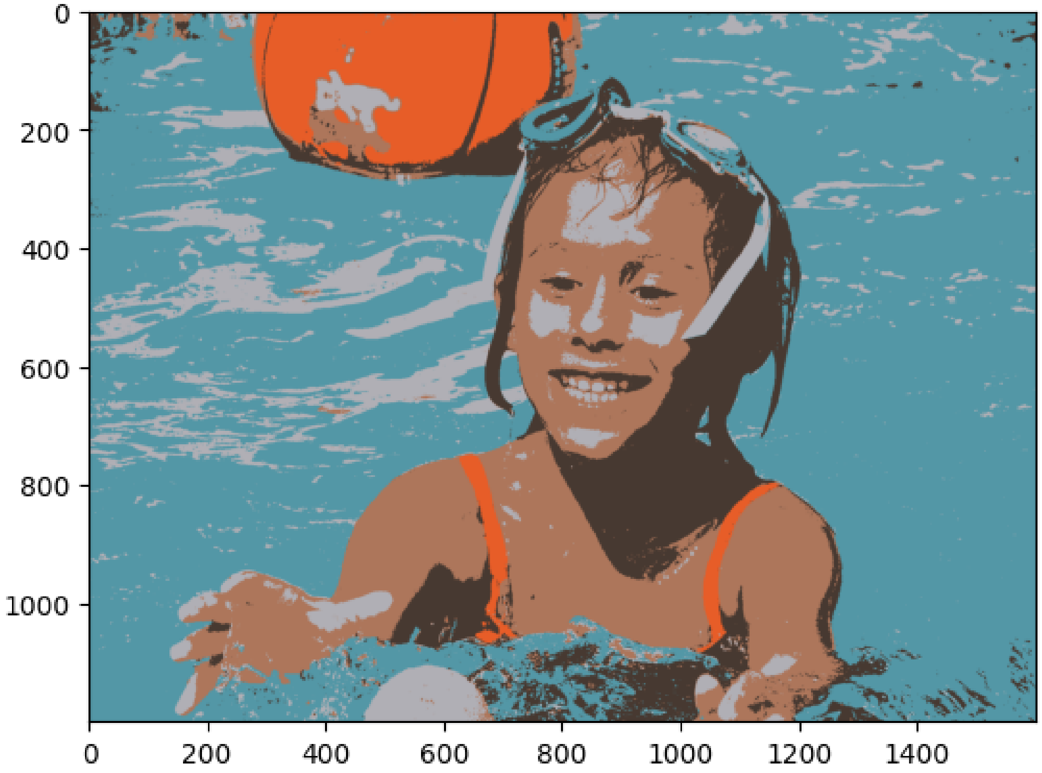
Run KMeans on the color array. Let's use $k = 5$ clusters.

[ ]:
```
kmeans = KMeans(5, n_init='auto').fit(colors)
```

Reconstruct the image, replacing each pixel color by its cluster center. Display reconstructed image and save in `exercise4.jpg`.

[ ]:
```
k_color_image = kmeans.cluster_centers_.astype(np.uint8)[kmeans.labels_].
  ↪reshape(image.shape)

plt.imshow(k_color_image)
plt.show()
```

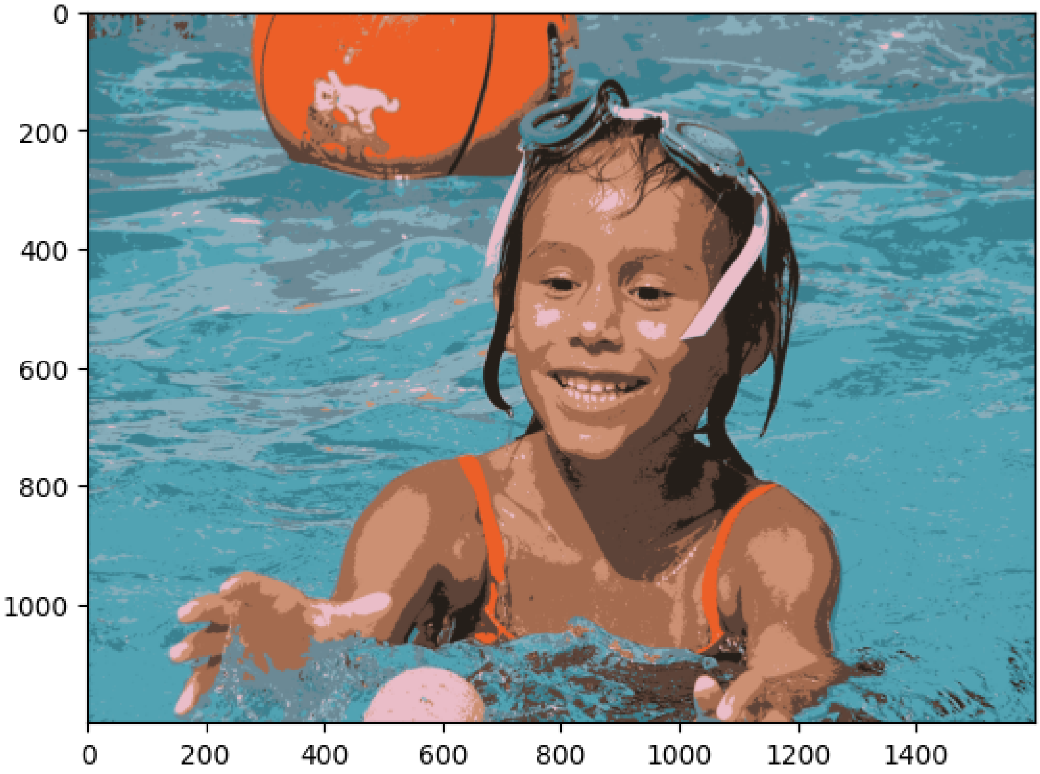
```
imageio.v3.imwrite('exercise4.jpg', k_color_image)
```



We can also try $k = 10$ colors for fun.

[ ]:

```
kmeans = KMeans(10, n_init='auto').fit(colors)

k_color_image = kmeans.cluster_centers_.astype(np.uint8)[kmeans.labels_].
  ↪reshape(image.shape)

plt.imshow(k_color_image)
plt.show()
```



Or $k = 20$.

[ ]:
```
kmeans = KMeans(20, n_init='auto').fit(colors)

k_color_image = kmeans.cluster_centers_.astype(np.uint8)[kmeans.labels_].
  ↪reshape(image.shape)

plt.imshow(k_color_image)
plt.show()
```

0
200
400
600
800
1000
0
200
400
600
800
1000
1200
1400