Math 5601: Introduction to Numerical Analysis
Homework assignment 7

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Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show.

Problem #1: Find the Gaussian quadrature of three nodes for $\int_{-1}^{1} f(x)\ dx$ by requiring it to be exact for $f(x) = 1, x, x^2, x^3, x^4, x^5$. If needed, you may use the matlab command "solve" to obtain the solution of a system of nonlinear equations.

Problem #2: Find the Gaussian quadrature of three nodes for $\int_{-1}^{1} f(x)\ dx$ by using orthogonal polynomials.

Problem #3: Suppose that $I(h)$ is an approximation to $\int_a^b f(x)dx$ where h is the width of the uniform subdivision of $[a, b]$. Suppose that the error satisfies

$$I(h) - \int_a^b f(x)dx = c_1 h + c_2 h^2 + O(h^3)$$

where c_1 and c_2 are constants independent of h. Suppose $I(h)$, $I(\frac{h}{2})$, $I(\frac{h}{3})$ have been calculated. Find an $O(h^3)$ approximation to $\int_a^b f(x)dx$.

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