Math 5601 Homework 9

Jacob Hauck

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Problem 1.

Let A be a nonsingular matrix, and let $A^{(2)}$ be the matrix from the lecture slides in the second step of Gaussian elimination. Then there exists $s \geq 2$ such that $a_{2s}^{(2)} \neq 0$.

Proof. Suppose on the contrary. By the Gaussian elimination process, we know that $a_{21}^{(2)} = 0$. If there is no $s \geq 2$ such that $a_{2s}^{(2)} \neq 0$, then the whole second row of $A^{(2)}$ is zero. Hence, expanding by cofactors along the second row, we see that the determinant of $A^{(2)}$ is

$$\det\left(A^{(2)}\right) = 0 \cdot \det(B_1) + 0 \cdot \det(B_2) + \cdots + 0 \cdot \det(B_n) = 0, \tag{1}$$

where B_i is the cofactor corresponding to $a_{2i}^{(2)}$. Then $A^{(2)}$ is singular.

This is a contradiction because $A^{(2)}$ was obtained from A by elementary row operations, and A was nonsingular, and applying row operations to a nonsingular matrix must result in a nonsingular matrix. $\square$

Problem 2.