

# Math 5601 Homework 2

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## Problem 1.

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Let  $R > 0$ , and define  $f(x) = 1 - \frac{1}{Rx}$  for  $x > 0$ . Then clearly  $f(x) = 0$  if and only if  $x = \frac{1}{R}$ , so calculating a zero of  $f$  is equivalent to calculating the reciprocal of  $R$ .

Let  $\{x_k\}$  be the sequence of approximate solutions of  $f(x) = 0$  obtained by using Newton's method. Then, by definition,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \left(1 - \frac{1}{Rx_k}\right) \cdot Rx_k^2 = x_k - Rx_k^2 + x_k = x_k(2 - Rx_k) \quad (1)$$

## Problem 2.

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See `newton.m`; also included here for convenience.

```
1 function result = newton(f, f_prime, x0, epsilon, epsilon_f, epsilon_f_prime, max_it)
2     x_next = x0;
3
4     for k = 0:max_it
5         xk = x_next;
6         fk = f(xk);
7         f_primek = f_prime(xk);
8
9         % check f_prime not zero *before* dividing by it
10        if abs(f_primek) <= epsilon_f_prime
11            fprintf("Failed. f_prime too small.\n");
12            break;
13        end
14
15        % now we can update x_next and compute Cauchy error
16        x_next = xk - fk / f_primek;
17        cauchy_error = abs(x_next - xk);
18
19        fprintf(...
20            ['k = %d, x_k = %.5g, Cauchy error = %.5g, '...
21            'f(x_k) = %.5g, f''(x_k) = %.5g\n'], ...
22            k, xk, cauchy_error, fk, f_primek ...
23        );
24
25        if cauchy_error < epsilon || abs(fk) < epsilon_f
26            break;
27        end
28    end
29
30    result = xk;
```

$x_0$	Converged
0.5	✓
1	✓
1.3	✓
1.4	✗
1.35	✓
1.375	✓
1.3875	✓
1.39375	✗
1.390625	✓
1.3921875	✗

Table 1: Convergence of Newton's method for  $f(x) = \tan^{-1}(x)$ 

The iteration appears to converge (quickly) for some starting values and diverge for others. See Table 1 for a summary of the results. The full outputs from the MATLAB console can be found in `outputs.txt`. In particular, there seems to be a cutoff  $c \approx 1.391$  such that the method converges if  $|x_0| < c$  and diverges if  $|x_0| > c$ .

### Problem 3.

See `secant.m`; also included here for convenience. Outputs from the MATLAB console can be found in `outputs.txt`.

```

1 function result = secant(f, x0, x1, epsilon, epsilon_f, max_it)
2     xk = x0;
3     fk = f(xk);
4     x_next = x1;
5
6     for k = 1:max_it
7         xkml = xk;
8         xk = x_next;
9
10        fkml = fk;
11        fk = f(xk);
12
13        cauchy_error = abs(xk - xkml);
14        x_next = xk - fk * (xk - xkml) / (fk - fkml);
15
16        fprintf( ...
17            ['k = %d, x_{k-1} = %.05g, xk = %.05g, Cauchy error = %.05g, ' ...
18            'f(xk) = %.05g\n'], ...
19            k, xkml, xk, cauchy_error, fk ...
20        );
21
22        if cauchy_error < epsilon || abs(fk) < epsilon_f
23            break;
24        end
25    end
26
27    result = xk;

```