Math 5604 Homework 1

Jacob Hauck

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Problem 1.

Consider the IVP

$$y' = 3 + e^{-t} - y, \quad t > 0; \qquad y(0) = 1. \tag{1}$$

1.1) Multiplying both sides by the integrating factor e^t gives

$$y'e^t + ye^t = 3e^t + 1. \tag{2}$$

The left-hand side is $(ye^t)'$, so integrating on both sides gives

$$ye^t = 3e^t + t + C, \tag{3}$$

for some constant C, so $y(t) = 3 + (t + C)e^{-t}$. The initial condition $y(0) = 1$ implies that $C = -2$, so

$$y(t) = 3 + (t - 2)e^{-t}. \tag{4}$$

1.2)

Problem 2.

Consider the IVP

$$y' = \frac{3t^2 + 10t + 1}{2(y + 1)}, \quad t > 0; \qquad y(0) = -2. \tag{5}$$

2.1) Multiplying both sides by $2(y + 1)$ gives

$$2(y + 1)(y + 1)' = 3t^2 + 10t + 1. \tag{6}$$

The left-hand side is $\left((y + 1)^2\right)'$, so integrating on both sides gives

$$(y + 1)^2 = t^3 + 5t^2 + t + C \tag{7}$$

for some constant C. The initial condition $y(0) = -2$ implies that $C = 1$. Therefore,

$$y(t) = -1 \pm \sqrt{t^3 + 5t^2 + t + 1}. \tag{8}$$

The initial condition forces us to choose a negative sign after taking the square root; thus,

$$y(t) = -1 - \sqrt{t^3 + 5t^2 + t + 1}. \tag{9}$$