Math 5601 Homework 7

Jacob Hauck

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Problem 1.

Let x_0, x_1, x_2 and w_0, w_1, w_2 be the nodes and weights of the the three-point Gaussian quadrature for $\int_{-1}^{1} f(x)\,\mathrm{d}x$. Then the quadrature must be exact for $f(x) = x^n$, $n \in \{0, 1, 2, 3, 4, 5\}$. That is,

$$\int_{-1}^{1} x^n\,\mathrm{d}x = \sum_{j=0}^{3} w_j x_j^n, \qquad n \in \{0, 1, 2, 3, 4, 5\}. \tag{1}$$

Since

$$\int_{-1}^{1} x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1}\Big|_{-1}^{1} = \begin{cases} \frac{2}{n+1} & n \text{ even} \\ 0 & n \text{ odd}, \end{cases} \tag{2}$$

we obtain the following system of six equations in the six unknowns x_0, x_1, x_2 and w_0, w_1, w_2:

$$\begin{aligned} 2 &= w_0 + w_1 + w_2 & 0 &= w_0 x_0 + w_1 x_1 + w_2 x_2 \\ \frac{2}{3} &= w_0 x_0^2 + w_1 x_1^2 + w_2 x_2^2 & 0 &= w_0 x_0^3 + w_1 x_1^3 + w_2 x_2^3 \\ \frac{2}{5} &= w_0 x_0^4 + w_1 x_1^4 + w_2 x_2^4 & 0 &= w_0 x_0^5 + w_1 x_1^5 + w_2 x_2^5. \end{aligned}$$

Using the following `solve` command in MATLAB gives the solution of this nonlinear system of equations.