Math 5604 Homework 1

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1. This homework is due by **11:59PM, Thursday (02/01/2024)**
2. Present your schemes, results/discussion, and codes (in order) in a single PDF file.
3. Create a folder including (1) your source codes with necessary description; (2) PDF file in Item 2.
4. Compress the folder in Item 3 into a ZIP file with name "FirstName-LastName-HW1.zip" (e.g., John-Wang-HW1.zip) and upload it via Canvas.
5. **No late homework is accepted.**

Problem 1. Consider the initial value problem

$$y' = 3 + e^{-t} - y, \quad \text{for } t > 0; \qquad y(0) = 1. \tag{0.1}$$

1. **[3 points]** Find the analytical solution of the initial value problem (0.1).
2. **[10 points]**
   (a) Discretize (0.1) by the forward Euler method. Write down the detailed discretization schemes and implement it into computer code.
   (b) What is the numerical value of $y(2)$ when time step is $k = 1/32$?
   (c) Plot the numerical result for time step $k = 1/16$ and $1/64$ as well as the exact solution in the same plot, for time $t \in [0, 2]$.
3. **[10 points]** Redo Part 2 by using the backward Euler method.
4. **[5 points]** Compare your numerical results with the analytical results and compute the numerical errors in the Table 1, where $\hat{y}$ represents the numerical solution at $t = 2$. What's your observations/conclusions from Table 1?

Problem 2. Consider the initial value problem

$$y' = \frac{3t^2 + 10t + 1}{2(y+1)}, \quad \text{for } t > 0; \qquad y(0) = -2. \tag{0.2}$$

1. **[3 points]** Find the analytical solution of the initial value problem (0.2).
2. **[10 points]** Discretize (0.2) by the backward Euler method. Write down the detailed discretization schemes and implement it into computer code.
3. **[2 points]** What is the numerical value of $y(1)$, when time step is $k = 1/32$ and the tolerance for Newton's iteration as $\epsilon = 0.1$, i.e., $|x_{m+1} - x_m| < \epsilon$?

Time step k	Forward Euler method	Backward Euler method
	$\lvert y(2) - \hat{y} \rvert$	$\lvert y(2) - \hat{y} \rvert$
$1/4$		
$1/8$		
$1/16$		
$1/32$		
$1/64$		
$1/128$		
$1/256$		
$1/512$		

Table 1: Numerical errors at $t = 2$.

4. **[2 points]** What is the numerical value of $y(1)$, when time step is $k = 1/32$ and the tolerance for Newton's iteration as $\epsilon = 10^{-8}$?

5. **[5 points]** Compare your numerical results with the analytical results and compute the numerical errors at $t = 1$ in Table 2. What's your observations?

Time step k	$\epsilon = 0.1$	$\epsilon = 10^{-3}$	$\epsilon = 10^{-8}$
	error	error	error
$1/4$			
$1/8$			
$1/16$			
$1/32$			
$1/64$			
$1/128$			
$1/256$			
$1/512$			

Table 2: Numerical errors at $t = 1$.