

2.6 Exponential Functions

Math 6010

9-20-2023

Objectives

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- ▶ Graph exponential functions
- ▶ Understand basic properties of exponential functions

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Approximate any number x by *finitely many* of its decimal digits.

$$a^\pi \approx a^{3.14} = a^{\frac{314}{100}} = \sqrt[100]{a^{314}}$$

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The more digits we use, the closer we get to the true value of a^π

2^3	$2^{3.1}$	$2^{3.14}$	$2^{3.141}$	$2^{3.1415}$	\dots	2^π
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Better still, we get to keep all the properties of exponents that we already know:

Exponent Properties

If x, y are real numbers, and $a, b > 0$, then

$$\begin{array}{lll} \bullet a^x \cdot a^y = a^{x+y} & \bullet (a^x)^y = a^{xy} & \bullet (ab)^x = a^x \cdot b^x \\ \bullet 1^x = 1 & \bullet a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x & \bullet a^0 = 1 \end{array}$$

Exponential Functions

Now that we know what a^x means for *any* x , we can define an exponential function.

Exponential Function

An **exponential function** is a function f such that

$$f(x) = C \cdot a^x,$$

where

- ▶ $C \neq 0$ is called the **initial value**, and
- ▶ $a > 0, a \neq 1$ is called the **growth factor**.

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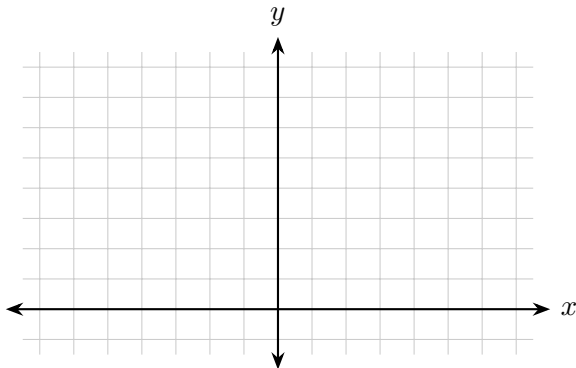
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It seems that $f(x) \rightarrow 0$ as $x \rightarrow -\infty$, and $f(x) \rightarrow \infty$ as $x \rightarrow \infty$

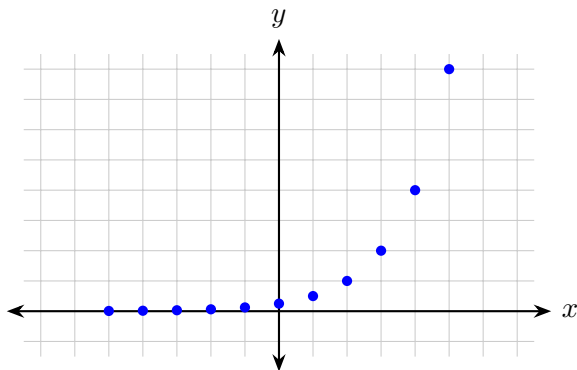
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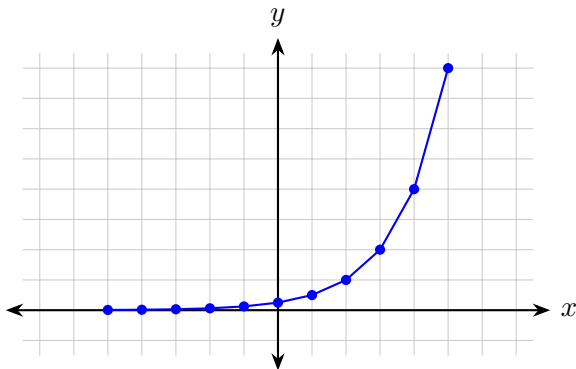
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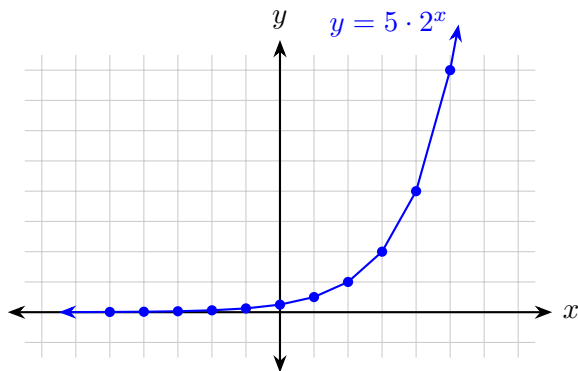
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$$f(x) = C \cdot a^x = C \cdot \left(\frac{1}{a}\right)^{-x} = g(-x)$$

That is, g is the reflection of f over the y -axis.