Math 5601: Introduction to Numerical Analysis
Homework assignment 1

Dr. Xiaoming He*

Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show. For each numerical experiment, all the .m files of your Matlab code should be electronically submitted to hex@mst.edu together with a .txt file which copies all the information in the Matlab command window when you run the code to obtain the numerical results.

Problem #1: Consider the bisection method applied to $f(x) = arctan(x)$ with initial interval $[a, b] = [-4.9, 5.1]$.

(a) Program bisection method in Matlab.

(b) Use the code to solve $f(x) = 0$ with $\varepsilon = 10^{-2}$, $\varepsilon = 10^{-4}$, $\varepsilon = 10^{-8}$. For each ε, (1) output the numbers of iterations and the final numerical solutions; (2) compute the k at which the algorithm should stop with pencil and paper and compare it with the numerical results.

Problem #2: Consider $g(x) = x - arctan(x)$.

(a) Program the fixed point method $x_{k+1} = g(x_k)$ in Matlab. Starting with $x_0 = 5, -5, 1, -1, 0.1$, perform 10 iterations and output all the numerical solutions at each step.

(b) What behavior do you observe for the numerical solutions? Explain it in terms of the theory of the fixed point method.

Problem #3: Let $G = [0, 2]$ and

$$g(x) = \frac{1}{3}\left(\frac{x^3}{3} - x^2 - \frac{5}{4}x + 4\right).$$

Use the contraction mapping theorem to prove that if $x_0 \in G$, then the sequence defined by $x_{k+1} = g(x_k)$ $(k = 0, 1, \cdots)$ converges to the unique fixed point $z \in G$.

*Department of Mathematics and Statistics, Missouri University of Science and Technology, Rolla, MO 65409, hex@mst.edu