Math 5601: Introduction to Numerical Analysis
Homework assignment 9

Dr. Xiaoming He*

Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show. For each numerical experiment, all the .m files of your Matlab code should be electronically submitted to hex@mst.edu together with a .txt file which copies all the information in the Matlab command window when you run the code to obtain the numerical results.

Problem #1: Consider the matrix $A^{(2)}$ for the second step of Gauss elimination of a non-singular matrix A in the lecture slides. Prove that there exists a s such that $a_{s2}^{(2)} \neq 0$ and $2 \leq s \leq n$. (Hint: If the fundamental row operations are applied to a non-singular matrix, then the resulted matrix is still non-singular.)

Problem #2: Prove that the SOR method

$$x_i^{(k+1)} = (1-\sigma)x_i^{(k)} + \frac{\sigma}{a_{ii}} \left\{ b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right\}, \ i = 1, \cdots, n, \ k = 0, 1, 2, \cdots$$

is an iterative method with $M = L + \frac{1}{\sigma}D$ and $N = -\left(U + D - \frac{1}{\sigma}D\right)$. That is

$$\begin{aligned} \overrightarrow{x}^{(k+1)} &= M^{-1}N\overrightarrow{x}^{(k)} + M^{-1}\overrightarrow{b} \\ &= -\left(L + \frac{1}{\sigma}D\right)^{-1}\left(U + D - \frac{1}{\sigma}D\right)\overrightarrow{x}^{(k)} + \left(L + \frac{1}{\sigma}D\right)^{-1}\overrightarrow{b}, \ k = 0, 1, 2, \cdots \end{aligned}$$

Problem #3: Consider

$$\begin{aligned} &x_1 + x_3 = 0, \\ &-x_1 + x_2 = 0, \\ &x_1 + 2x_2 - 3x_3 = 0. \end{aligned}$$

(a) Program both Jacobi and Gauss-Seidel methods. Starting from the initial guess $\overrightarrow{x}^{(0)} = (1, 1, 1)^T$, Solve the above system numerically.

(b) What behavior do you observe?

*Department of Mathematics and Statistics, Missouri University of Science and Technology, Rolla, MO 65409, hex@mst.edu