

# Math 6330 Homework 5

Jacob Hauck and Rajrani Gupta

February 29, 2024

## Bifurcation diagram for $\dot{x} = 1 + dx - x^3$

---

### 2.4

---

Let  $f(x) = kx - cx^2 - hx$ , where  $k$ ,  $c$ , and  $h$  are positive. Consider the model  $\dot{x} = f(x)$ , where  $x$  represents the size of a population.

Then  $f(x) = 0$  gives the equilibrium points as  $x_1 = 0$  and  $x_2 = \frac{k-h}{c}$ . Thus, if  $k < h$ , then  $x_2 < 0$ . Moreover,  $f(x) = -(h-k)x - cx^2 < 0$  for all  $x > 0$ , so solutions starting with  $x_0 > 0$  approach  $x_1 = 0$ . That is, the population will always be exterminated.

If  $k = h$ , then  $x_1 = x_2$ , and there is only one equilibrium point. In this case,  $f(x) < 0$  for all  $x > 0$ , so the population will still be exterminated.

If  $k > h$ , then, because  $f$  is a downward parabola, we have  $f(x) > 0$  for  $x < x_2$  and  $f(x) < 0$  for  $x > x_2$ . Therefore, all solutions with  $x_0 > 0$  approach the equilibrium  $x_2 > 0$ .

### 2.11

---

(a)

(b)