Math 5604 Homework 4

Jacob Hauck

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Problem 1.

Consider

$$y' = \left(1 - 2t^3\right) y^2, \quad t > 0; \qquad y(0) = 1. \tag{1}$$

(a) In order to apply the second-order Taylor series method, we need to use the ODE (1) to find y'' in terms of y:

$$y'' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\left(1 - 2t^3\right) y^2\right] = -6t^2y^2 + 2\left(1 - 2t^3\right) yy' = -6t^2y^2 + 2\left(1 - 2t^3\right)^2 y^3.$$

Then the second-order Taylor series method is given by

$$\begin{cases} y^{n+1} = y^n + k\left(1 - 2t_n^3\right)\left(y^n\right)^2 + \frac{k^2}{2}\left[-6t_n^2\left(y^n\right)^2 + 2\left(1 - 2t_n^3\right)\left(y^n\right)^3\right], & n = 0, 1, 2 \ldots \\ y^0 = 1. \end{cases}$$

This method is implemented in `ts2.m`.

(b) The recursive rule for the two-step Adams-Bashforth method is given by

$$y^{n+1} = y^n + k\left[\frac{3}{2}f\left(t_n, y^n\right) - \frac{1}{2}f\left(t_{n-1}, y^{n-1}\right)\right], \quad n \geq 0,$$

where, in our case, $f(t, y) = \left(1 - 2t^3\right) y^2$. We use the forward Euler method to obtain y^1, as the forward Euler method has second-order local truncation error. Thus, our scheme is

$$\begin{cases} y^{n+1} = y^n + k\left[\frac{3}{2}\left(1 - 2t_n^3\right)\left(y^n\right)^2 - \frac{1}{2}\left(1 - 2t_{n-1}^3\right)\left(y^{n-1}\right)^2\right] & n = 1, 2, 3, \ldots \\ y^1 = y^0 + k\left(1 - 2t_0^3\right)\left(y^0\right)^2 \\ y^0 = 1. \end{cases}$$

This method is implemented in `ab2.m`.

(c)

Problem 2.