Jacob Moore Writing Sample

The writing sample below is an excerpt from my working paper, "Urban-Rural Productivity Spillovers: Theory and Evidence from Colorado." In this paper, I investigate how growth in total factor productivity (TFP) in an urban core associates with future employment growth in proximate rural communities. I develop a quantitative spatial general equilibrium model that highlights the interactions between a city and rural town in its hinterland. I estimate the reduce-form derived from model via confidential establishment-level data on a core-periphery system in the state of Colorado from 2001 to 2017. I find TFP growth in an urban core correlates with lower future employment growth in its rural periphery: a standard deviation increase in the TFP of Colorado's urban core over three years (roughly 3.4 percentage points) is associated with a 1.3 percentage point decrease in employment growth in surrounding rural ZIP codes over the following three years.

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2 Theoretical Framework

I consider a simple quantitative spatial general equilibrium model that identifies the key mechanics determining the movements between urban TFP growth and rural employment growth. I study a region comprised of two locations: a city c and rural town r. The locations exogenously differ in production technology, productivity, amenities, and housing stock. The city and rural town are connected by costly trade of the differentiated rural and urban goods and a mobile homogeneous labour supply with costly commuting options. I identify sufficient conditions for the existence and uniqueness of a regular spatial equilibrium. I then conduct a comparative static analysis, evaluating how equilibrium rural employment responds to (small) proportional changes in urban TFP.

This model evokes the spirit of the canonical New Economic Geography (NEG) core-periphery model developed by Krugman (1991) and refined by Fujita, Krugman, and Venables (1999), but differs in two central ways. First, instead of investigating a model urban hierarchy formation by assuming locations are ex-ante identical and studying how the uneven distribution of economic activity across space arises ex-post, I assume the core-periphery designation pre-established and study the system's adjustment to exogenous shocks. Second, unlike the stylised NEG core-periphery model, this model draws on the empirically tractable structure of quantitative spatial models proposed in more contemporary literature (Allen and Arkolakis, 2014; Redding, 2016; Allen, Arkolakis, and Takahashi, 2020). As such, this model emits reduced-form specifications relating rural employment growth to urban TFP shocks which inform the empirical specification.

2.1 Set-Up

Consumption. The region's economy is populated by $\overline{L} > 0$ identical, intraregionally mobile, and risk neutral workers who live in either the city or rural town and supply one unit of labour inelastically at their chosen location of employment. A worker who lives in i and works in i', where $i, i' \in \{c, r\}$, earns income $w(i, i')$ and has preferences that depend on goods consumption $Q(i, i')$, housing consumption $h(i, i')$, and exogenous location-specific residential amenities B_i:

$$U(i,i') = B_i \left(\frac{Q(i,i')}{\delta}\right)^{\delta} \left(\frac{h(i,i')}{1-\delta}\right)^{1-\delta}, \qquad 0 < \delta < 1 \tag{2.1}$$

where δ is the share of a income allocated to goods consumption. The consumption index $Q(i, i')$ is a Constant Elasticity of Substitution (CES) aggregator defined over consumption of the good produced in the city $q_c(i, i')$ and consumption of the good produced in the rural town $q_r(i, i')$:

$$Q(i,i') = \left[q_c(i,i')^{\frac{\sigma-1}{\sigma}} + q_r(i,i')^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad 1 < \sigma$$

where σ is the elasticity of substitution between the two goods. The total housing stock in location i, H_i, is exogenously determined.

Define r_i to be the per unit housing price in location i and $p_{ii'}$ to be the per unit price of the good produced in i but consumed in i'. The type-(i, i') worker chooses her goods and housing consumption to maximise equation (2.1) subject to her budget constraint:

$$w(i,i') = p_{ci}q_c(i,i') + p_{ri}q_r(i,i') + r_i h(i,i')$$

After taking first order conditions and substituting the resulting Marshallian demands into the worker's utility function (see Online Appendix A.1 for full derivation), the type-(i, i') worker's indirect utility is:

$$V(i,i') = \frac{B_i w(i,i')}{P_i^{\delta} r_i^{1-\delta}} \tag{2.2}$$

where $P_i \equiv (p_{ci}^{1-\sigma} + p_{ri}^{1-\sigma})^{\frac{1}{1-\sigma}}$ is the usual CES price index that measures the cost of a unit of goods consumption utility for workers living in i (i.e. the "true" cost of goods consumption in i).

Production. Denote L_i as the total labour supply in i and let j_i index a firm located in i, where $j_i \in \{1, \ldots, J_i\}$. Perfectly competitive firms that use labour as their sole production factor produce final goods with technology based on their location of operation. Firm j_c

located in the city produces y_{j_c} units of the urban good using ℓ_{j_c} units of the total urban labour supply L_c (i.e., $\sum_{j_c=1}^{J_c} \ell_{j_c} = L_c$) according to:

$$y_{j_c} = A_c L_c^{\alpha} \ell_{j_c}$$

where $A_c > 0$ is city-specific TFP common to urban firms and $\alpha \in (0,1)$ is an agglomeration parameter that determines the extent to which individual firm output is affected by the density of workers in the city. Defining Y_c to be total output in the city, it follows that:

$$Y_c = \sum_{j_c=1}^{J_c} y_{j_c} = \sum_{j_c=1}^{J_c} A_c L_c^{\alpha} \ell_{j_c} = A_c L_c^{1+\alpha} \tag{2.3}$$

Therefore, the agglomeration externality prompts increasing returns to scale in the city.

Using ℓ_{j_r} units of the total rural labour supply L_r, firm j_r in the rural town produces y_{j_r} units of the rural good via constant returns to scale technology:

$$y_{j_r} = A_r \ell_{j_r}$$

where $A_r > 0$ is town-specific TFP common to rural firms. Aggregate rural output is then

$$Y_r = \sum_{j_r=1}^{J_r} y_{j_r} = \sum_{j_r=1}^{J_r} A_r \ell_{j_r} = A_r L_r \tag{2.4}$$

The ex-ante production differences between the city and rural town capture inherent sectoral composition asymmetries present in mature core-periphery systems, with sectors that feature increasing returns, such as manufacturing, tending to sort into cities while constant returns to scale industries, like agriculture or natural resource extraction, locate in the hinterland.

Define p_i to be the mill price of the good produced in i and w_i to be the wage paid to workers by firms in i. Perfect competition implies the zero profit condition holds in equilibrium. Thus, in equilibrium firm j_i chooses labour input ℓ_{j_i} that set profits to zero, which implies the price in i is the wage in i divided by the marginal product of labour for workers at firm j_i:

$$p_i = \frac{w_i}{(\partial y_{j_i} / \partial \ell_{j_i})}, \qquad \forall j_i \in \{1, \ldots, J_i\}$$

In the city, the marginal product of labour for workers at firm j_c is:

$$\frac{\partial y_{j_c}}{\partial \ell_{j_c}} = A_c L_c^\alpha + \ell_{j_i} \frac{\partial (A_c L_c^\alpha)}{\partial \ell_{j_i}}$$

and since in perfect competition J_c is large, the individual firm's labour choice has a negligible effect on the extent of agglomeration economies, L_c^α. This implies $\partial(A_c L_c^\alpha)/\partial \ell_{j_c}$ above can be set to zero. In the rural town, the marginal product of labour for workers at firm j_r is $\partial y_{j_r}/\partial \ell_{j_r} = A_r$. Thus, the equilibrium prices for the urban and rural goods are $p_c = w_c/(A_c L_c^\alpha)$ and $p_r = w_r/A_r$, respectively.

Commuting and Trade. Movement of workers and goods between locations is costly. A worker living in i who works in $i' \neq i$ faces a commuting cost equal to $(1 - \frac{1}{\kappa_{ii'}})\%$ of the wage paid by firms in i', where $\kappa_{ii'} > 1$ is of the iceberg form. Thus, a type-(i, i') worker earns $w(i, i') = \frac{w_{i'}}{\kappa_{ii'}}$ while noncommuting type-(i, i) workers face no commuting cost and earn wage $w(i, i) = w_i$.

Goods trade costs take an iceberg form as well. For a unit of the good made in i to arrive in i', $\tau_{ii'} > 1$ units must be shipped, implying the price faced by workers in i' for the good made in i is $p_{ii'} = \tau_{ii'} p_i$ while workers living in i pay the mill price for the locally produced good. That is, $p_{ii} = p_i$.

These spatial trade and commuting frictions $\kappa_{ii'}$ and $\tau_{ii'}$ introduce wedges in total goods demand. Let X_i be total demand for the good produced in i. Then, X_i equals the demand for the i-good by each worker type times the total number of workers of that type:

$$\begin{aligned} X_i &= q_i(i,i')L_i(i,i) + q_i(i,i')L_{i'}(i,i') + q_i(i',i')L_{i'}(i',i') + q_i(i',i)L_i(i',i) \\ &= \delta \left[\frac{p_i^{-\sigma}}{P_i^{1-\sigma}} \left(w_i L_i(i,i) + \frac{w_{i'}}{\kappa_{ii'}} L_{i'}(i,i') \right) + \frac{(\tau_{ii'} p_i)^{-\sigma}}{P_{i'}^{1-\sigma}} \left(w_{i'} L_{i'}(i',i') + \frac{w_i}{\kappa_{i'i}} L_i(i',i) \right) \right] \end{aligned} \tag{2.5}$$

where $i \neq i'$ and $L_{i'}(i, i')$ is the number of total workers in i' that commute from i.

2.2 Equilibrium

This model is closed by specifying market clearing and stipulating equilibrium utility must achieve some exogenous reservation level of utility, $\overline{V}$. Assuming the core-periphery system is nested in a large domestic economy, $\overline{V}$ is the level of utility that renders a worker indifferent between staying in the core-periphery system or moving elsewhere in the domestic economy. Choosing the price of the urban good to be the numéraire ($p_c \equiv 1$), letting trade costs be bilaterally symmetric ($\tau_{ii'} = \tau_{i'i} = \tau$), and assuming there exists no incentive for urban residents to commute to work in the rural town (i.e., $L_r(c,r) = 0$ and denote $\kappa_{rc} = \kappa$ for simplicity), a regular spatial equilibrium is formally defined as the following.[1]

Definition 1. *Given the set of exogenous parameters,*[2] *a* ***regular spatial equilibrium*** *is characterised by an 8-tuple* $(p_r^*, r_r^*, r_c^*, w_c^*, w_r^*, L_c(c,c)^*, L_c(r,c)^*, L_r(r,r)^*)$ *of strictly positive, continuous values that satisfy:*

1. ***Goods market clearing:*** *the total supply of goods produced in the city and rural town equal the total demand of both goods,* $Y_c + Y_r = X_c + X_r$.
2. ***Labour market clearing:*** *the total demand for labour in the city equals the total supply,* $L_c = L_c(c,c) + L_c(r,c)$*, the total demand for labour in the rural town equals the total supply,* $L_r = L_r(r,r)$*, and the positive supply of labour across all locations equals the total regional population,* $\overline{L} = L_c(c,c) + L_c(r,c) + L_r(r,r)$ *where* $L_c(c,c), L_c(r,c), L_r(r,r) > 0$.
3. ***Housing market clearing:*** *the total demand for housing in each region equals the total supply in that region,* $H_c = h(c,c)L_c(c,c)$ *and* $H_r = h(r,r)L_r(r,r) + h(r,c)L_c(r,c)$.
4. ***No spatial arbitrage:*** *a worker cannot improve her utility by choosing a new location in which to live, choosing a new location in which to work, or leaving the region all together,* $V(c,c) = V(r,c) = V(r,r) = \overline{V}$.

Substituting and combining Marshallian demands for goods and housing as well as the results in equations (2.2) through (2.5) into the above equilibrium requirements (see Online Appendix A.2 for full derivation) yields a single equation that is a function of a single unknown, the urban wage w_c:

$$\overline{V}^{\frac{1}{1-\delta}} \left(\frac{1-\delta}{\delta}\right) \left[\frac{A_r \overline{L}}{w_c^{\sigma}} + \frac{w_c^{\frac{1}{\alpha}-\sigma}}{A_c^{\frac{1}{\alpha}}}(w_c - A_r)\right] = \frac{B_c^{\frac{1}{1-\delta}} H_c (1 + (\frac{\tau w_c}{\kappa A_r})^{-\sigma})}{\left(w_c^{\sigma-1} + (\frac{\kappa A_r}{\tau})^{\sigma-1}\right)^{\frac{\sigma(1-\delta)-1}{(\sigma-1)(1-\delta)}}} + \frac{B_r^{\frac{1}{1-\delta}} H_r (\tau^{-\sigma} + (\frac{w_c}{\kappa A_r})^{-\sigma})}{\kappa^{\frac{1}{1-\delta}} \left((\frac{w_c}{\tau})^{\sigma-1} + (\kappa A_r)^{\sigma-1}\right)^{\frac{\sigma(1-\delta)-1}{(\sigma-1)(1-\delta)}}} \tag{2.6}$$

Lemma 1 specifies exogenous parameter conditions under which there exist (strictly positive values of) w_c that solve equation (2.6) and stricter conditions that guarantee a unique solution.

[1] A spatial equilibrium implies markets clear and utilities are equalised over space, but it need not be the case all locations are inhabited or that workers live and work in disparate locations. A spatial equilibrium is said to be *regular* if all locations are inhabited and, since there is commuting in this model, a positive number of commuters (Allen and Arkolakis, 2014).

[2] This set is comprised of $\{\sigma, \delta, \alpha, \tau, \kappa, \overline{V}, \overline{L}, B_i, A_i, H_i\}$ for $i \in \{c, r\}$.

Lemma 1 (**Existence and uniqueness of solutions to Equation 2.6**). *Let H_r be sufficiently large such that it satisfies the following inequality:*

$$H_r > \left(\frac{B_c}{B_r}\frac{\kappa}{\tau}\right)^{\frac{1}{1-\delta}} \left[\left(\frac{1-\delta}{\delta}\right)\left(\frac{\overline{L}^{1-\delta}}{A_r^{\delta}}\frac{\overline{V}}{B_c}\frac{\tau}{\kappa}\right)^{\frac{1}{1-\delta}} - H_c\right] \tag{2.7}$$

Then, if

(i) $\frac{1}{1-\delta} \leq \sigma < \frac{1+\alpha}{\alpha}$*, there exists at least one* $w_c^* \in \mathbb{R}_{++}$ *that solves equation (2.6).*

(ii) $\frac{1}{1-\delta} = \sigma \in (\frac{1}{\alpha}, \frac{1+\alpha}{\alpha})$*, there exists a unique* $w_c^* \in \mathbb{R}_{++}$ *that solves equation (2.6).*

Proof. See Online Appendix A.3.[3] ∎

Existence of solutions to equation (2.6), and by extension the uniqueness of solutions (since the conditions for uniqueness are a special case of those for existence), depend on curvature restrictions dictated by the elasticities of consumption (δ), substitution (σ), and agglomeration (α) as well as the rural housing stock. Effectively, Lemma 1 states that for a (unique) solution to exist, agents 1) must not have an excessive preference for housing (δ large, implying $1/(1-\delta)$ small) such that the goods market plays a role, 2) the differentiated goods must not be overly substitutable (σ not too large) so there is reason for trade between locations, 3) the city must not have extreme agglomeration forces (α small, implying $(1+\alpha)/\alpha$ large) which would otherwise remove economic incentives to live and work in the rural town, and 4) the rural housing stock must be sufficiently large to accommodate a rural population and keep housing prices low relative to the city.

Satisfaction of Lemma 1 is not sufficient to guarantee the existence and uniqueness of a regular spatial equilibrium. A w_c^* resulting from parameterisations that obey Lemma 1 is a *candidate* equilibrium, as such a w_c^* may not achieve the required equilibrium spatial distribution of labour that makes a spatial equilibrium regular. For instance, a value of w_c^* too large results in all workers living and working in the city (corner solution). Lemma 2 identifies restrictions on the exogenous parameters that ensure the existence of bounds on the urban wage which guarantee that 1) both regions are inhabited and 2) there are a positive number of commuters.

[3] The proof hinges upon showing these conditions result in appropriate curvature and limit behaviour. Inequality (2.7) and the elasticity bounds in part (i) ensure satisfaction of Bolzano's intermediate value theorem, which in turn guarantees existence of a solution. The stronger bounds in part (ii) imply strict monotonicity, which in combination with Bolzano's intermediate value theorem guarantees there can be no more than one solution. I sketch what these solutions can look like in Online Appendix Figure A.1.

Lemma 2 (**Existence of all worker types**). *Define* $\Omega \equiv \left((1-\delta)\,\overline{L}/H_c\right)^{\frac{1-\delta}{\delta}} \left(\overline{V}/B_c\right)^{\frac{1}{\delta}}$. *If* $\overline{L}$ *and* A_c *are such that:*

$$\overline{L} > \left(\frac{H_c}{1-\delta}\right)\left(\frac{\kappa A_r}{\tau}\right)^{\frac{\delta}{1-\delta}}\left(\frac{B_c}{\overline{V}}\right)^{\frac{1}{1-\delta}} \tag{2.8}$$

$$A_c < \overline{L}^{-\alpha}\left(\Omega^{\sigma-1} - \left(\frac{\kappa A_r}{\tau}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{2.9}$$

then there exists a set $S \subset (0, A_r\overline{L}^{\alpha})$ *where for all* $w_c \in S$*, the labour allocations* $L_c(c,c)$*,* $L_c(r,c)$*, and* $L_r(r,r)$ *are strictly positive in equilibrium.*

Proof. See Online Appendix A.4.[4] ■

As long as the region's total population is sufficiently large and the city's TFP is not excessively large, Lemma 2 states there must exist some set of urban wages for which all worker types exist in equilibrium. The intuition is that if the total population is too small, urban agglomeration forces dominate and all workers reside and work in the city. Likewise, if the city is too productive, there would not be any incentive for commuters to exist, as they would be better off living and working in the city.

Together, Lemmas 1 and 2 specify sufficient conditions for existence and uniqueness of regular spatial equilibria, which is formally stated in Proposition 1.

Proposition 1 (**Existence and uniqueness of a spatial equilibrium**). *Consider a core-periphery system with instantaneous equilibrium summarised by equation (2.6) and where Lemmas 1(i) and 2 hold. If a solution* w_c^* *to equation (2.6) guaranteed by Lemma 1(i) is an element of the set* S *that exists under Lemma 2, then that* w_c^* *is a regular spatial equilibrium. If instead Lemmas 1(ii) and 2 hold and the unique solution* $w_c^* \in S$*, then* w_c^* *is a unique regular spatial equilibrium.*[5]

[4] Proof is established by first identifying the unique bounds on w_c within which $L_c(c,c)$ and $L_r(r,r)$ are strictly positive in equilibrium assuming 1) $\overline{L}$ is sufficiently large and 2) A_c is sufficiently small (conditions 2.8 and 2.9, respectively). Specifically, $L_c(c,c)$ and $L_r(r,r)$ are strictly positive if $w_c \in (0, A_r\overline{L}^{\alpha})$. The condition that A_c is sufficiently small (condition 2.9) guarantees 1) $L_c(r,c) > 0$ at the upperbound on w_c (i.e., $L_c(r,c) > 0$ when $w_c = A_r\overline{L}^{\alpha}$) and 2) the existence of a set of values $\widetilde{W} \subset (0, A_r\overline{L}^{\alpha})$ where for each $\widetilde{w}_c \in \widetilde{W}$, $L_c(r,c)(\widetilde{w}_c) = 0$. The supremum of $\widetilde{W}$ is strictly less than $A_r\overline{L}^{\alpha}$ and since $L_c(r,c) > 0$ at $A_r\overline{L}^{\alpha}$, any urban wage greater than $\sup \widetilde{W}$ and less than $A_r\overline{L}^{\alpha}$ results in a positive number of commuters. Defining the set $S = (\sup \widetilde{W}, A_r\overline{L}^{\alpha})$, it follows that for any $w_c \in S$, all worker types exist in equilibrium. I sketch what set S looks like in Online Appendix Figure A.2.

[5] I sketch the unique equilibrium implied by Proposition 1 in Online Appendix A.5.

2.3 Rural Employment Adjustments to Proximate Urban TFP Shocks

To identify this model's predictions on peripheral employment growth in response to a positive TFP shock in the core and derive a baseline empirical model, I study the comparative statics of the unique equilibrium special case. To enrich this model without threatening the existence and uniqueness of an equilibrium, I incorporate the empirical evidence on urban TFP shock migration effects that influence local wage and employment changes identified by Hornbeck and Moretti (2021). I assume that the region's total population is an increasing function of the levels of urban and rural TFP. I define $\overline{L}$ to be a function $\overline{L}(A_c, A_r)$ where $\partial\overline{L}/\partial A_c$, $\partial\overline{L}/\partial A_r > 0$.[6] This implies the region's total population is partially determined by both A_c and A_r, the intuition being more productivity regions attract larger labour forces from elsewhere in the domestic economy in which the region is nested. Crucially, depending on the elasticity of $\overline{L}$ to changes in A_c, this migration effect will influence how the local system adjusts to urban TFP shocks. By exogenising extra-regional in-migration, this model isolates the rural-urban margin that may contribute to the larger systems-of-cities urban TFP shock adjustments identified by Hornbeck and Moretti (2021).

Assume the sufficiency conditions in Proposition 1 hold for the existence of a unique equilibrium. Let $\mu_c = L_c/\overline{L}$ represent the urban regional employment share, $\mu_r = L_r/\overline{L}$ represent the rural regional employment share, and $\beta_{\overline{L}A_c}$ be the elasticity of the total population with respect to urban TFP,[7] Moreover, let $\eta = (1+\alpha)/\alpha - 1/(1-\delta)$ represent the gap between the sensitivity of urban good supply and urban good demand in response to a change in w_c.[8] Small values of η imply similar wage elasticities on the supply and demand sides, while larger values imply disparate responses to changes in the wage. Note that by Lemma 1, $\eta > 0$. Log-linearising equation (2.6) about the unique equilibrium (full derivation in Online Appendix A.6) reveals the reduced-form relationship of interest between rural

[6] Hornbeck and Moretti (2021) find urban manufacturing TFP growth stimulates domestic migration effects, with new workers to the area blunting the wage gains of incumbent workers in response to TFP growth, while increasing employment.

[7] $\beta_{\overline{L}A_c} = (\partial\overline{L}/\partial A_c)(A_c/\overline{L})$.

[8] Set $p_c = 1$ and write equation (2.3) in terms of w_c. Substituting the resulting expression for L_c into equation (2.3), partially differentiating with respect to w_c, and multiplying both sides by w_c/Y_c yields $(\partial Y_c/\partial w_c)(w_c/Y_c) = (1+\alpha)/\alpha$, which is the wage elasticity of the urban good supply. Rewriting total equilibrium demand for the urban good (equation 2.5 for $i = c$) as a function of exogenous parameters and the urban wage, partially differentiating with respect to w_c, and multiplying both sides by w_c/X_c reveals $(\partial X_c/\partial w_c)(w_c/X_c) = 1/(1-\delta)$, the income elasticity of demand for the urban good.

employment and an urban TFP shock:

$$\widehat{L_r} = \underbrace{\Theta_1 \left[\frac{\Theta_2 - (A_c L_c^{\alpha}/A_r)}{(A_c L_c^{\alpha}/A_r) - \Theta_3} \right]}_{\equiv \beta_{L_r A_c}} \widehat{A_c} \tag{2.10}$$

where $\widehat{x} = dx/x$ represents a (small) proportional change in variable x and

$$\begin{aligned} \Theta_1 &= \frac{(1/(\alpha^2\eta) - 1)\mu_c - \beta_{\overline{L}A_c}}{\mu_r} \\ \Theta_2 &= \left(\frac{1-\alpha}{\alpha^2}\right)\left(\frac{(1+\alpha)\mu_c + \alpha\beta_{\overline{L}A_c}}{(1/(\alpha^2\eta) - 1)\mu_c - \beta_{\overline{L}A_c}}\right) \\ \Theta_3 &= \frac{1}{\eta} \end{aligned}$$

Ultimately, equation (2.10) is the reduced-form that I take to data to empirically evaluate this relationship. The mechanics that dictate the sign and magnitude of the parameter $\beta_{L_r A_c}$ are largely governed by the degree of spatial asymmetries in the distribution of productivity. Recall that $A_c L_c^{\alpha}$ and A_r are the marginal product of labour in the city and rural town, respectively. As such, $A_c L_c^{\alpha}/A_r$ can be interpreted as a spatial marginal productivity of labour ratio, which takes on large values if there are large disparities between urban and rural productivity. Equation (2.10) highlights that the size of this ratio, and therefore the size of the productivity gap between the urban core and rural periphery, dictates rural employment adjustments to urban TFP shocks within this model. For urban TFP shocks to contribute to the weak rural employment growth described in the introduction, it must be that $\beta_{L_r A_c} < 0$, which in turn occurs under the conditions set forth in Proposition 2.

Proposition 2 (Urban TFP growth reduces rural employment growth). *Assume Proposition 1 holds. Provided that in the ex-ante equilibrium:*

$$\beta_{\overline{L}A_c} < (1/(\alpha^2\eta) - 1)\mu_c \tag{2.11}$$

$$A_c L_c^{\alpha}/A_r > \max\{\Theta_2, \Theta_3\} \tag{2.12}$$

rural employment will decrease in response to a positive urban TFP shock, ceteris paribus.

Proof. See Online Appendix A.7.[9] ■

[9] Proof is established by showing that Proposition 1 and the restriction in inequality (2.11) ensure Θ_1, Θ_2, and Θ_3 to be strictly positive. The restriction in inequality (2.12) ensures the numerator in $\beta_{L_r A_c}$ is strictly negative, whilst the denominator is strictly positive. Combined with the fact that $\Theta_1 > 0$, it follows that $\beta_{L_r A_c} < 0$.

Proposition 2 is the key theoretical result. It states that long as the extra-regional in-migration stimulated by a shock to the urban core's TFP is not excessively large and the marginal productivity of labour gap is sufficiently large, this model predicts rural employment growth will decline in response to a positive urban TFP shock (i.e., $\beta_{L_r A_c} < 0$). The intuition is that rural workers will be encouraged to respond to the TFP shock by commuting or moving to the city if the region is not flooded by migrants to the region responding to the local shock and workers are more productive in the city relative to the rural town, thereby ensuring that there are jobs available to former rural employees in the city.

Simple back of the envelope calibration using expenditure and population data (see Online Appendix A.8) suggest the upper bound on $\beta_{\overline{L}A_c}$ (equation 2.11) to be larger than the in-migration elasticities identified by Hornbeck and Moretti (2021). Moreover, the calibrated values for Θ_2 and Θ_3 imply the marginal product of labour in the city needs to be only 31% higher than that in the rural town for the inequality in Equation 2.12 to be satisfied. Moretti (2011) finds substantial county-level manufacturing TFP heterogeneity across the U.S., reporting that the most productive county in their sample is 2.9 times more productive than the least productive county. If this heterogenity holds in urban-rural settings, it seems likely that Proposition 2 will be satisfied when taken to data.

Using confidential, establishment-level employment and wage data from the state of Colorado in combination with public data on output and capital stocks from 2001 to 2017, I construct a novel dataset on ZIP-code level revenue TFP. I then estimate equation (2.10) and empirically verify Proposition 2 holds.

References

Allen, T. and Arokolakis, C. 2014. “Trade and Topography of the Spatial Economy.” *Quarterly Journal of Economics* 129(3): 1085–1140. `https://doi.org/10.1093/qje/qju016`.

Allen, T., Arokolakis, C., and Takahashi, Y. 2014. “Universal Gravity.” *Journal of Political Economy* 128(2): 393–433. `https://doi.org/10.1086/704385`.

Bureau of Labor Statistics. 2013-2020. “CAINC1 Personal Income Summary: Personal Income, Population, Per Capita Personal Income.” United States Department of Commerce. [Link] (accessed November 5, 2021).

Bureau of Labor Statistics. 2013-2019. “Average annual expenditures and characteristics of all consumer units, Consumer Expenditure Surveys.” United States Department of Labor. [Link] (accessed November 5, 2021).

Fujita, M., Krugman, P., and Venables, A. 1999. *The Spatial Economy: Cities, Regions, and International Trade*. Cambridge: The MIT Press.

Hornbeck, R. and Moretti, E. 2021. “Estimating Who Benefits From Productivity Growth: Local and Distant Effects of City TFP Shocks on Wages, Rents, and Inequality.” [Link]

Jones, R. 1965. “The Structure of Simple General Equilibrium Models.” *The Journal of Political Economy* 6: 557-572. `https://doi.org/10.1086/259084`.

Krugman, P. 1991. “Increasing Returns and Economic Geography.” *The Journal of Political Economy* 99: 483-499. `https://doi.org/10.1086/261763`.

Redding, S. 2016. “Goods trade, factor mobility and welfare.” *Journal of International Economics* 101: 148-167. `https://doi.org/10.1016/j.jinteco.2016.04.003`.

Moretti, E. 2011. “Local Labor Markets.” In *Handbook of Labor Economics, Volume 4*, edited by Ashenfelter, O. and Card, D.: 1237-1313. Amsterdam: Elsevier. `ttps://doi.org/10.1016/S0169-7218(11)02412-9`.

Rosenthal, S. and Strange, W. 2004. “Evidence on the Nature and Sources of Agglomeration Economies.” In *Handbook of Regional and Urban Economics, Volume 4*, edited by Henderson, V. and Thisse, J.: 2119-2171. Amsterdam: Elsevier. `https://doi.org/10.1016/S1574-0080(04)80006-3`.

Economic Research Service. 2020. “2013 Rural-Urban Continuum Codes.” United States Department of Agriculture. [Link] (accessed November 5, 2021).