Labor-Market Frictions and Endogenous Production-Network Formation

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Abstract

This paper presents a model of endogenous production-network formation in which firms face a labor-market featuring worker-firm matching frictions. The model demonstrates the role of labor mobility across sectors in determining production-network centrality. The model also shows how endogenous production-networks determine the distribution of workers' employment searches across sectors. These two effects are inseparable due to the feedback from one effect to another. That is, endogenous production-network formation leads to labor-market shifts, which lead to further changes in the production network, driving another change in the distribution of employees across sectors in a continuing evolution. Failing to account for labor-market mobility in a model of endogenous production-network formation, may considerably misstate the rate at which variables respond to trade shocks and in the case of some variables can even change the sign of the response.

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1 Introduction

Firms combine inputs and workers to produce output. The firm must maintain a costly relationship with its suppliers to ensure that it is able to access the inputs it needs to reach its production goals. At the same time, the firm must also source workers from a labor-market that is not competetive. labor-market frictions affect the way that firms form their production network. In a 2009 survey of managers conducted by the Economist Intelligence Unit, 20% of managers reported that a labor dispute had affected their supply chain over the previous year. The efficiency of the supply chain also determines the distribution of workers searching for employment across sectors. Workers choose to search for employment in a given sector based off the expected wage in that sector. If the sector features a highly integrated supply chain then the probability of finding employment in the sector, the number of workers that look for employment in the sector, and total employment in that sector will be higher. Little is known about the interaction between labor-market frictions and production-network frictions and how they respond to economic shocks. This interaction is potentially important given that the share of labor compensation in GDP is about one half, and that firm expenditure on and sales of intermidiate inputs account for about half of firms' costs and revenues respectively.

In this paper I present a model of endogenous production-network formation in which workers and firms face labor-market matching-frictions.Recent papers by Lim [1] and Huneeus [2] have demonstrated how shocks, and more specifically trade shocks, have changed production networks. However, these models rely on the assumption of a frictionless perfectly competitive input market, facing all firms (they refer to this as labor, but it could be any type of homogeneous input). Allowing for frictions in the sourcing of the homogeneous input will change how production networks shift in response to shocks. As the expected wages in a given sector change due to a trade shock, the endogenous labor supply available to each sector will change, systematically affecting the attractiveness of firms in each sector as potential suppliers or customers to other firms. In turn, as firms are able to match with more or fewer partner firms, their profitability and the expected wages they pay will change, leading to yet another change in the labor supply available to each sector.

This implies that the size of the sectoral wage changes due to shocks will also be determined by the ability of firms in the sector to find trading partners with whom to build their production network.

This paper studies the following questions: What is the effect of labor-market frictions on the endogenous formation of production networks? To what extent does endogenous production-network formation affect total employment? How do workers migrate across sectors in response to shocks and to what extent does this migration drive changes in production-network formation? Finally, how do endogenous changes in the production network drive changes in expected wages, and thus the endogenous labor supply across sectors? In short, I investigate how endogenous production networks affect the endogenous labor supply available across sectors, and vice versa, in a feedback mechanism.

Prior work on endogenous production-network formation has mostly focused on the role of firm-level productivities and network centrality in determining the density of the production network (Lim [1] and Bernard and Moxnes [3]). Here I present a new mechanism for driving density differences across production networks, labor-market friction differences across sectors.This paper builds a model that points out that sectors with lower levels of labor-market frictions will be more central in the production-network. This reflects the fact that firms in sectors with lower levels of labor-market frictions are more profitable for other firms to trade with since they produce more output (making them more attractive as a potential customer) and operate at lower unit costs (making them more attractive as a potential supplier).

Prior work on the responsiveness of production networks to shocks such as Huneeus [2], Baqaee and Farhi [4], Bernard, Moxnes, and Saito [5], Gabaix [6], also have ignored the local migration of employees across sectors, focusing only on how shocks propagate throughout the economy based on firm-level productivities. While this effect is still present in my model, the addition of labor-market frictions induces two new effects. First, some workers migrate out of the sector that is relatively harmed by the shock.This harms the ability of firms in that sector to form production-network linkages. The workers that left the relatively harmed sector will now search for employment in the sector that is relatively less affected by the shock.This first effect suggests that the models above overstate the rate at which production networks respond to trade shocks, and thus the rate at which shocks propogate throughout the economy.The second effect has to do with the feedback mechanism.As workers move into the sector that relatively benefits from the shock, firms in this sector will now find it profitable to supply to more firms (this includes firms in the relatively harmed sector). This increases the liklihood that firms in the relatively harmed sector find it profitable to supply to other firms, counteracting the first effect.The net effect of labor-market frictions on the rate at which procduction networks respond to trade shocks depends on the size of these two effects.

Finally my analysis contributes to the literature on structural labor models that focus on how trade shocks, offshoring, and outsourcing affect labor-market outcomes (Helpman and Itskhoki [7], Helpman et al. [8], Egger and Kreickemeier [9], Autor, Dorn, and Hanson [10], Acemoglu et al. [11], Caliendo, Dvorkin, and Parro [12], and Grossman and Helpman [13]) I derive a model to look at how employees are affected by shocks through changes in firm abilities to source inputs through endogenous production-networks. Previous papers have considered how labor markets respond to shocks when they are exposed through an exogenous production-network, for example, through input-output linkages. However, these models ignore how the production networks themselves might shift and thereby change the level of exposure to these shocks. As Huneeus [2] points out, production networks must change in response to economic shocks, affecting the rate at which these shocks propagate throughout the economy. In this paper I will demonstrate how changes in endogenous production networks lead to changes in the labor market that systematically vary across sectors.

In what follows I first present a model of endogenous production-network formation, similar to that of Lim [1]. However I also include labor-market frictions as in Helpman and Itskhoki [7]. The model not only demonstrates how endogenous production-network formation affects labor-market outcomes, but also shows how these labor-market frictions determine the production-network centrality of sectors in the economy. In the following section I present the predictions of the model and simulate the effects of the home countrty unilaterally imposing a tariff on a particular sector in the foreign country. After discussing the baseline model, I compare it to other models featuring trade and labor-market frictions. In order to demonstrate the effect of labor-market frictions on endogenous production-network formation, I compare the model presented in Section 2 to an analogous extension of Lim [1] that includes an immobile labor supply.To demonstrate the effect of endogenous production-network formation on labor-market outcomes I compare my model to an extension of [7] that includes exogenous sectoral linkages in production, similar to that of Caliendo, Dvorkin, and Parro [12] and Jones [14].

2 A Double Sided Matching Model of Production

In this section I build a model of endogenous production-network formation with labor-market frictions in which firms must pay a fixed cost to match with each customer firm, and they must also pay a cost to post employment vacancies.The economy consists of multiple regions and sectors, each of which has a continuum of firms and a fixed labor supply. Sectors and regions vary by the average cost of matching with customers, and the cost of posting a vacancy. The model consist of a collection of household preferences, product and labor-market structures, and determinants of network of firm linkages, wages, and profits.In what follows in this section, I describe each of these components in detail, discuss the equilibrium conditions of the model, and finally discuss the predictions of the model with respect to within sector and region wage inequality.

2.1 Basic Environment

Suppose the economy consists of multiple sector-region pair indexed by s and r, respectively. Within each location there is a continuum of firms, each with a firm specific labor augmenting technology parameter, indexed by ϕ_ℓ. Firms are indexed by the triplet $\phi = (\phi_\ell, s, r)$. The market is monopolistically competitive with each firm producing their own unique variety that it can sell to both households and other firms. Firm production takes place by obtaining intermediate varieties produced by other firms in the economy and combining them with labor that the firm is able to employ given the labor-market frictions. In order to hire workers, firms must post vacancies and must pay a cost per each vacancy posted. The firms cannot costlessly adjust their labor supply which induces workers and firms to engage in wage bargaining and generating wage differences within sectors and regions.

In what follows I present the model in detail by first considering the household's problem and then examining the firm's problem. I then present definitions of the equilibrium conditions of the model.

2.1.1 The Household's Problem

Within each sector-region (q, z) the representative household supplies $L_{s,r}$ units of labor to the economy by searching for employment.The household has love of variety preferences given by:

$$U_{q,z} = \left[\sum_{s,r} \alpha^H_{s,r} \int [x_{qz}(\phi)]^{\frac{\sigma_{H,s}-1}{\sigma_{H,s}}} dF_{\phi_\ell}(\phi_\ell) \right]^{\frac{\sigma_{H,s}}{\sigma_{H,s}-1}}$$

where $\alpha^H_{s,r}$ governs the degree to which households value inputs from sector-region pair s, r, $\sigma_{H,s} > 1$ is the elasticity of substitutions between firm specific varieties in sector s.Total demand for each firm's brand by the representative consumer in sector region q, z is given by the following:/

$$x^H_{q,z}(\phi) = \frac{I_{q,z}}{P^H_{q,z}} \left(\frac{\alpha^H_{s,r} P^H_{q,z}}{\tau^q_{s,r} p^H_{q,z}(\phi)} \right)^{\sigma_{H,s}} \tag{2.1}$$

where

$$P^H_{q,z} \equiv \left(\sum_{s,r} (\alpha^H_{s,r})^{\sigma_{H,s}} \int [\tau^z_{s,r} p^H_{q,z}(\phi)]^{1-\sigma_{H,s}} dF_{\phi_\ell}(\phi_\ell) \right)^{\frac{1}{1-\sigma_{H,s}}} \tag{2.2}$$

is the price index of the representative consumer. $I_{q,z}$ is the total income of the household in q, z. The parameter $\tau^z_{s,r}$ is an iceberg trade cost term that determines how costly it is to ship sector-s goods from region r to region z. $p^H_{q,z}(\phi)$ is the price charged to households in q, z by firm ϕ.

Each firm can sell to all regions and sectors in the economy. This implies the total demand that firm ϕ faces from households in the economy is given by:

$$x^H(\phi) = \sum_{q,z} x^H_{q,z}(\phi) \tag{2.3}$$

2.1.2 Firm Production and Firm-to-Firm Matching

Firm Production Each firm produces its own variety of the differentiated product via a CES production technology using the intermediates it obtains from other firms and the amount of labor that it is able to hire given the labor-market frictions. The production function is given by the following:

$$X(\phi) = \left(\alpha_s \phi_\ell [\ell(\phi)]^{\frac{\sigma_{\ell,s}-1}{\sigma_{\ell,s}}} + (1-\alpha_s)[x(\phi)]^{\frac{\sigma_{\ell,s}-1}{\sigma_{\ell,s}}}\right)^{\frac{\sigma_{\ell,s}}{\sigma_{\ell,s}-1}}$$

where the employment of labor by the firm is given by the term $\ell(\phi)$. The firm combines intermediates from other sectors via CES aggregator that is denoted by $x(\phi)$, which will be discussed shortly.α is a parameter that governs the input suitability for the industry of the firm, s. $\sigma_{\ell,s}$ is the elasticity of substitution between workers and aggregate intermediates.

Given the assumption that the firm cannot costlessly adjust their labor supply, the firm only retains only a share of the surplus that it generates by hiring workers. This share is given by $1/(1+\beta)$, where β is the bargaining power of the workers, thus the firm only bears this share of the non-labor costs that it incurs[1]. Given this assumption, the demand for labor and aggregated intermediates are:

$$\begin{aligned} \ell(\phi) &= \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,r}]^{-\sigma_{\ell,s}} X(\phi) \eta(\phi)^{\sigma_{\ell,s}} \\ x(\phi) &= (1-\alpha_S)^{\sigma_{\ell,s}} \left[\frac{P(\phi)}{1+\beta}\right]^{-\sigma_{\ell,s}} X(\phi)\eta(\phi)^{\sigma_{\ell.s}} \end{aligned} \tag{2.4}$$

where the unit cost function of the firm is denoted by $\eta(\phi)$ and is given by:

$$\eta(\phi) \equiv \left(\alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,r}]^{1-\sigma_{\ell,s}} + (1-\alpha_s)^{\sigma_{\ell,s}} \left[\frac{P(\phi)}{1+\beta}\right]^{1-\sigma_{\ell,s}}\right)^{\frac{1}{1-\sigma_{\ell,s}}} \tag{2.5}$$

[1]The firm's complete top level problem is discussed in detail in the appendix

The cost of hiring one more worker in sector region-s,r is denoted by $b_{s,r}$ and $P(\phi)$ is the cost the firm must pay to increase its CES aggregator of intermediates ($x(\phi)$) by one unit.

The firm combines intermediates across sectors using the following CES aggregator:

$$x(\phi) = \left(\sum_{s'} \alpha_{s,s'} [x_{s'}(\phi)]^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}}$$

The parameter $\alpha_{s,s'}$ governs the input suitability of sector s' varieties that are used in the production of sector s goods. σ_s is the elasticity of substitution between sectors used in production by sector $s.x_{s'}(\phi)$ is a CES aggregator of the firm's demand for sector s' varieties.The demand for aggregate intermediates from sector-$s'$by the firm is given by:

$$x_{s'}(\phi) = (\alpha_{s,s'})^{\sigma_s} [P_{s'}(\phi)]^{-\sigma_s} x(\phi) P(\phi)^{\sigma_s} \tag{2.6}$$

The firm's cost of increasing the aggregator of total intermediates is defined as:

$$P(\phi) \equiv \left(\sum_{s'} (\alpha_{s,s'})^{\sigma_s} [P_{s'}(\phi)]^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}} \tag{2.7}$$

Where $P_{s'}(\phi)$ is the firm's unit cost of increasing it's sector s' CES aggregator by one unit.

Within it's demand for inputs from a given sector, the firm aggregates across regions and varieties.Before the production function and input demand is presented however it is useful to pause to discuss the production-network framework.

As in Lim [1] there are relationship frictions between firms such that firm ϕ' can only sell to firm ϕ if firm ϕ' pays a fixed cost [2]. This will occur with probability $m(\phi, \phi')$, which I refer to as the *firm-to-firm matching function*. The *firm-to-firm matching function* will be solved for endogenously later in the model but for now is taken as given. This function will completely

[2] I assume that this fixed cost does not vary with the customer firm, only the supplying firm. This implies that conditional on matching with a supplier, a customer firm's demand for a supplier's specific variety is not distorted. In the appendix of [1], the author discusses a model in which firms bargain over the surplus of their match. The outcomes of the model do not differ in a meaningful way.

determine the extensive margin of trade within the model.I assume that the fixed cost of firm-to-firm matching is paid in terms of labor within the model.

Given the matching function, the firm combines intermediate varieties within each sector according to the following CES-aggregator:

$$x_{s'}(\phi) = \left(\sum_{r'} \int m(\phi, \phi')[x(\phi, \phi')]^{\frac{\sigma_{s,s'}-1}{\sigma_{s,s'}}} dF_{\phi_\ell}(\phi'_\ell | s') \right)^{\frac{\sigma_{s,s'}}{\sigma_{s,s'}-1}}$$

where $x(\phi, \phi')$ is the total sector s' variety demanded by the firm and $\sigma_{s,s'}$ is the elasticity of substitution that governs how sector s firms substitute between sector s' varieties.Given this structure, the conditional demand for variety ϕ' by firm ϕ is given by:

$$x(\phi, \phi') = [\tau^r_{r',s'} p(\phi, \phi')]^{-\sigma_{s,s'}} x_{s'}(\phi) P_{s'}(\phi)^{\sigma_{s,s'}} \tag{2.8}$$

where the firm's unit cost of increasing it's sector s' CES aggregator, $P_{s'}(\phi)$, is defined as:

$$P_{s'}(\phi) = \left(\sum_{r'} \int m(\phi, \phi')[\tau^r_{r',s'} p(\phi, \phi')]^{1-\sigma_{s,s'}} dF_{\phi_\ell}(\phi'_\ell | s') \right)^{\frac{1}{1-\sigma_{s,s'}}} \tag{2.9}$$

where $\tau^r_{r',s'}$ is the cost of shipping sector s' goods from region r' to region r and $p(\phi, \phi')$ is the price charged by firm ϕ' when selling to firm ϕ.

Firm Pricing and Firm-to-Firm Matching Given that the price elasticity of demand for a firm's variety only varies across sectors, within each sector pair, a firm does not find it optimal to price discriminate. Firms will find it optimal to price discriminate across sectors, but not within a given sector they are selling to. This implies the standard CES markup over unit cost for each firm:

$$p(\phi, \phi') = \mu_{s,s'} \eta(\phi') \tag{2.10}$$

where $\mu_{s,s'} \equiv \sigma_{s,s'}/(\sigma_{s,s'}-1) > 1$, is the markup over the unit cost of production that is charged by all sector s' firms when selling to sector s. Firm ϕ' similarly charges a markup over unit costs when selling to the representative household employed in q, z that is given by:

$$p^H_{q,z}(\phi) = \mu_{H,s}\eta(\phi) \tag{2.11}$$

where $\mu_{H,s} \equiv \sigma_{H,s}/(\sigma_{H,s}-1)$.

Given the firm's optimal pricing rule, we can now calculate the profit the firm would earn from selling to households in any given sector region.

$$\pi^H_{q,z}(\phi) = \frac{1}{1+\beta}(\mu_{H,s}-1)[\mu_{H,s}]^{-\sigma_{H,s}}[\tau^q_{s,r}\eta(\phi)]^{1-\sigma_{H,s}} I_{q,z}[\alpha^H_{s,r}]^{\sigma_{H,s}}[P^H_{q,z}]^{\sigma_{H,s}-1} \tag{2.12}$$

Aggregating over household demand from all sectors and regions gives:

$$\pi_H(\phi) = \sum_{q,z} \pi^H_{q,z}(\phi) \tag{2.13}$$

Likewise it is also straightforward to calculate the potential profit that firm ϕ' would realize from selling to firm ϕ. Conditional on being able to match with firm ϕ, firm ϕ' would earn a profit of

$$\pi(\phi, \phi') = \frac{1}{1+\beta}(\mu_{s,s'}-1)[\mu_{s,s'}]^{-\sigma_{s,s'}}[\tau^r_{r',s'}\eta(\phi')]^{1-\sigma_{s,s'}} x_{s'}(\phi)[P_{s'}(\phi)]^{\sigma_{s,s'}} \tag{2.14}$$

Given the assumption that the selling firm must pay a fixed cost for each customer firm they take on the firm will agree to sell to a customer if the profit of making the sell is greater than the fixed cost of obtaining the customer.

As in Lim [1] I assume that the relationship fixed cost for each customer obtained by a firm in sector region s, r is equal to $f_{s,r}\xi$. I assume that this fixed cost must be paid in terms of labor. ξ is a random variable that is distributed according to the cumulative distribution function F_ξ, with unit mean. The assumption that firms pay this fixed cost in terms of labor means the firm must hire an additional $f_{s,r}\xi$ workers for each customer firm they choose to match with. Thus for each match with customer firms, the selling firm must pay $b_{s,r} f_{s,r} \xi$ in matching costs. Given these assumptions, firm ϕ' will find it profitable to sell to firm ϕ with probability:

$$m(\phi, \phi') = F_{\xi}\left[\frac{\pi(\phi, \phi')}{b_{s',r'} f_{s',r'}}\right] \tag{2.15}$$

Following with Lim [1] I assume that ξ takes on a Weibull distribution. Therefore the total labor employed by a firm to facilitate matching with all other firms can be calcualted as:

$$FC(\phi) = f_{s,r} \sum_{r',s'} \int \mathbb{E}_{\xi}[\xi_{max}(\phi', \phi)] dF_{\phi_\ell}(\phi'_\ell) \tag{2.16}$$

Where $\xi_{max}(\phi', \phi)$ is the highest level of ξ at which firm ϕ would be willing to sell to firm ϕ'. Note that this is given by the following amount:

$$\xi_{max}(\phi', \phi) = \frac{\pi(\phi', \phi)}{b_{s,r} f_{s,r}}$$

Given the parametric assumption that ξ is distributed according to the weibul distribution, the partial expectation of ξ_{max} is given by the following:

$$\begin{aligned}
\mathbb{E}_{\xi}[\xi_{max}(\phi', \phi)] &= \int_0^{\xi_{max}(\phi', \phi)} \xi dF_{\xi}(\xi) \\
&= \int_0^{\frac{\pi(\phi', \phi)}{b_{s,r} f_{s,r}}} \xi \frac{k}{\lambda} (\frac{\xi}{\lambda})^{k-1} e^{-(\xi/\lambda)^k} dF_{\xi}(\xi)
\end{aligned}$$

where k and λ are the shape and scale parameters, respectively, of the Weibull distribution.

2.2 Labor Market

The labor market features frictions such that firms cannot costlessly find workers to hire as in Stole and Zwiebel [15], and must match with workers via a Cobb-Douglass matching as in Helpman and Itskhoki [7]. Firms must match with enough labor to use in production and to additionally pay their fixed costs of matching with other firms. This implies that the fixed cost of matching can be thought of as a communication cost between the customer and supplier. Real-world examples of this fixed cost include time spent communicating with or finding customers, customization of the production process, etc... Firms post employment vacancies and must pay a cost for each of these vacancies that they post. The cost of posting these employment vacancies translates into a cost of each worker that is hired by the firm.The cost of posting a vacancy differs across regions and sectors leading to comparative advantages between sectors and regions. The varying cost of posting vacancies therefore determines the rate at which shocks propagate throughout the economy. Due to the labor-market frictions, after a firm has matched with its total labor demand the two sides bargain over the surplus generated by their match. Thus the workers are paid a share of variable profits from the firm they are assigned to.

Within each region there is a family of workers, with labor supply denoted by $\bar{L}_r$. The family allocates workers across sectors to maximizes its profits. For there to be employment in each sector in the economy it mus be the case that within each region, expected wages must be equalized. Let the worker and firm matching function be defined as a Cobb-Douglass homogeneous of degree one matching function:

$$H_{s,r} = m_{s,r}^L (V_{s,r})^{\chi} (L_{s,r})^{1-\chi} \tag{2.17}$$

$\chi \in (0,1)$. Where $m_{s,r}^L$ is a parameter that determines the efficiency of labor-market matching in each sector region. $H_{s,r}$ is the total number of labor-market matches in sector region s,r. $V_{s,r}$ is the number of vacancies posted by firms in sector region s,r. $L_{s,r}$ is the number of workers the region r family has chose to allocate to sector s .

In order to solve for the optimal number of vacancies that each firm chooses to post, it will be useful to recall demand for labor used in production by each firm. Restating equation 2.6:

$$\ell(\phi) = \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,r}]^{-\sigma_{\ell,s}} X(\phi) \eta(\phi)^{\sigma_{\ell,s}} \tag{2.18}$$

The firm's demand for labor to be used in production depends on the firm's fundamental labor-productivity, the cost of posting vacancies, the labor share of production, and a firm specific scale

$X(\phi)\eta(\phi)^{\sigma_{\ell,s}}$.The total amount of labor demanded by each firm is given by the labor the firm uses in production and the labor the firm uses to pay their fixed cost of matching with customer firms:

$$L(\phi) = \ell(\phi) + FC(\phi) \tag{2.19}$$

The probability that a region r worker searches in sector s and finds employment is simply $H_{s,r}/L_{s,r}$. Since the total payroll of workers employed at a firm is a share of profits equal to $\beta/(1+\beta)$, the expected wage of searching in sector s is given by:

$$\mathbb{E}(w_{s,r}) = \frac{H_{s.r}}{L_{s,r}} \frac{\beta}{1+\beta} \int \frac{\pi(\phi) - FC(\phi)}{L(\phi)} dF_{\phi_\ell}(\phi_\ell | s, r) \tag{2.20}$$

for there to be employment in all sectors it must be the case that for each sector pair s and s' in all regions, we must have:

$$\mathbb{E}(w_{s,r}) = \mathbb{E}(w_{s',r}) \tag{2.21}$$

To finish describing the household's problem the total number of workers searching across sectors in each region must sum up to equal the total population of workers in the region $r$family:

$$\bar{L}_r = \sum_s L_{s,r} \tag{2.22}$$

To describe the firm's problem, the total number of vacancies posted in each sector region is given by summing across the number of vacancies posted by each firm:

$$V_{s,r} = \int V(\phi) dF_{\phi_\ell}(\phi_\ell | s, r) \tag{2.23}$$

where $V(\phi)$ is the total number of vacancies posted by each firm. Secondly, the total cost of posting vacancies must equal the total cost of hiring workers for each firm.

$$\nu_{s,r} V(\phi) = b_{s,r} L(\phi) \tag{2.24}$$

Where $\nu_{s.r}$ is the cost of posting each employment vacancy.

Finally, the labor market must clear. That is the number of labor-market matches in a given sector region must equal total employment by firms in the same sector region:

$$H_{s,r} = \int L(\phi) dF_{\phi_\ell}(\phi_\ell | s, r) \tag{2.25}$$

2.3 Competitive Equilibrium

Solving the model requires the inclusion of two more sets of equations. First, the goods market must clear. Second the income of households in each sector region must be calculated.

There exists a goods market clearing condition for each variety. Firms sell their output to households and any other firm in any sector region that they agree to match with by paying their fixed cost of matching. Thus the total output of the firm must equal their total sales:

$$X(\phi) = x^H(\phi) + \sum_{s'r'} \int m(\phi', \phi) x(\phi', \phi) dF_{\phi_\ell}(\phi'_\ell) \tag{2.26}$$

This equation (equation 2.1) and the unit cost equation (equation 2.5) classify every firm's problem as each firm specific variable in the problem can be written in terms of the two variables these equations define, given the other sector region specific variables.

The total income of households in each sector region is calculated by aggregating over the total payments to workers by firms.Since the workers receive a share of profits equal to $\beta/(1+\beta)$ this implies that total income in each sector region must be equal to a share of total sector region profits.The total variable profit can be calculated by summing across the variable profit each firm makes from selling to households and each of its customers:

$$\pi(\phi) = \pi_H(\phi) + \sum_{s',r'} \int m(\phi', \phi) \pi(\phi', \phi) dF_{\phi_\ell}(\phi'_\ell) \tag{2.27}$$

This implies that the total income of workers in sector region s, r is given by:

$$I_{s,r} = \frac{\beta}{1+\beta} \int \pi(\phi) - FC(\phi) dF_{\phi_\ell}(\phi_\ell | s, r) \tag{2.28}$$

Having closed the model we can now define a competitive equilibrium of the model:

Definition 2.1. Given a set of parameters and a firm-level distribution of labor augmenting productivities, a *competitive equilibrium* consists of a set of variables that maps from the binary Cartesian power of the set of all firms,

$$\{x(\phi, \phi'), p(\phi, \phi'), \pi(\phi, \phi'), m(\phi, \phi')\}_{\forall \phi, \phi'}$$

, a set of variables that maps from the Cartesian product of the set of all firms and the set of sector regions, $\{x^H_{q,z}(\phi), p^H_{q,z}(\phi), \pi^H_{q,z}(\phi)\}_{\forall \phi, q, z}$, a set of variables that maps from the Cartesian product of the set of all firms and the set of all sectors: $\{P_{s'}(\phi), x_{s'}(\phi)\}_{\forall \phi, s'}$, a set of variables that maps from the set of all firms,

$$\{x^H(\phi), X(\phi), \ell(\phi), x(\phi), \eta(\phi), P(\phi), \pi_H(\phi), FC(\phi), L(\phi), V(\phi), \pi(\phi)\}_{\forall \phi}$$

and a set of variables that maps from the set of all sectors and regions,

$$\{L_{s,r}, H_{s,r}, V_{s,r}, I_{s,r}, b_{s,r}, P^H_{s,r}\}_{\forall s, r}$$

such that equations 2.1 through 2.28 are satisfied for all firms, sectors, and regions.

3 Simulation

In this section I begin by presenting the results of simulations of the model outlined in section 2. I first look at the role of labor mobility across sectors and how this leads to changes in the

Parameter	Description	Assumed Value
β	Household Bargaining Power	0.5
α_ℓ	Employment Share of Production	0.5
σ_H	Household's Elasticity of Substitution Across Varieties	5.0
σ_ℓ	Firm's Elasticity of Substitution Between Inputs and Workers	4.0
σ_s	Firm's Elasticity of Substitution Across Sectors	6.0
σ_{ss}	Firm's Elasticity of Substitution Across Varieties, within a sector	8.0
m	Firm–Worker Matching Function Scale Parameter	1.0
χ	Weight of Vacancies in Firm-Worker Matching Function	0.5
μ_ϕ	Mean of Firm Labor Augmenting Productivities	0.0
σ_ϕ	Variance of Firm Labor Augmenting Productivities	1.0
λ_ξ	Scale Parameter of Distribution of Stochastic Firm to Firm Match Cost	1.0
k_ξ	Shape Parameter of Distribution of Stochastic Firm to Firm Match Cost	0.25

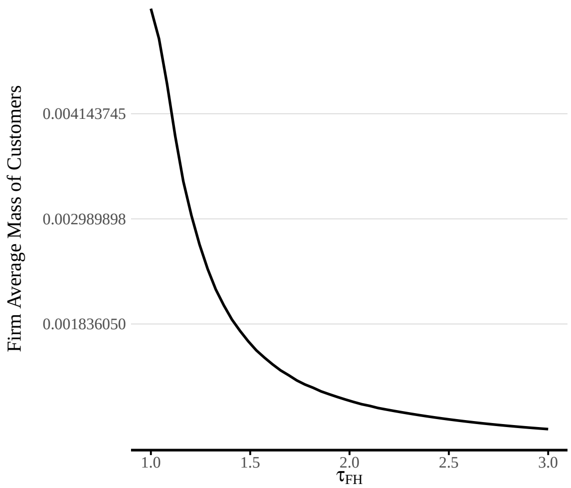
Table 1: Parameters for Simulation

production-network formation in response to changes in trade costs.I then highlight the role that endogenous firm to firm matching plays in how aggregate outcomes change.

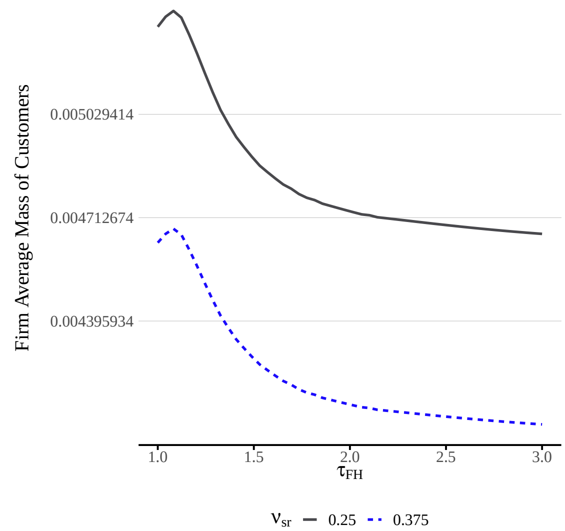
3.1 Baseline Model Simulation

I simulate the effect of a tariff in model assuming the parameter values given in table 1.I assume that the firm specific labor augmenting productivities, ϕ_ℓ are distributed Log-Normal, the mean and variance of their distribution are presented in table 1. In addition to the parameters listed in table 1, I allow ν to vary across sectors and regions. This parameter will not only drive changes in the cost of hiring workers and thus employment in response to changes in trade costs, but it will also govern how firm specific production networks will change in response to the imposition of a tariff.

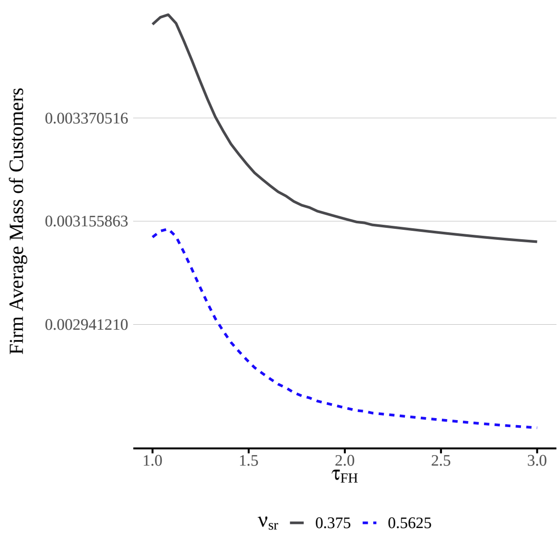
In what follows I assume that there are 2 countries (Home and Foreign), and that Home unilaterally imposes a tariff on imports from one specific sector in Foreign. This tariff applies to sales to households and firms in Home.



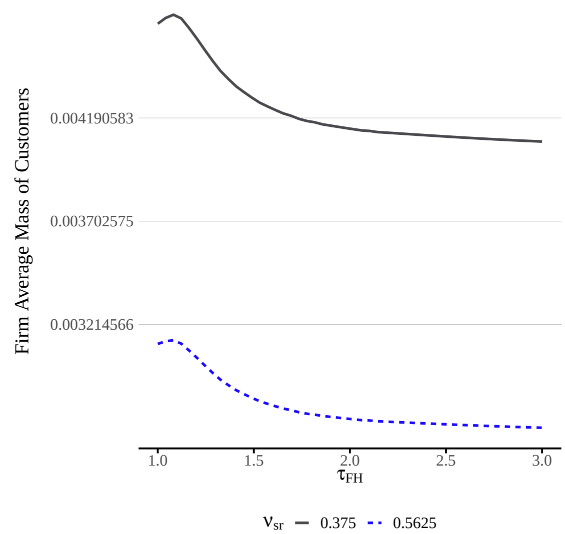
(a) Foreign sector to which the tariff is applied



(b) Import competing sector in Home, across regions



(c) Non-import competing sectors in Home, across regions



(d) Non-import competing sectors in Home, within regions

Figure 3.1: Within sector-region average firm-level mass of customers

Figure 3.1 presents the average mass of customer firms, across all firms within a given sector-region pair as a share of the total mass of firms in the economy, from the model outlined in section §2.The numbers on the y-axis represented in figure 3.1 are small, however they represent the weighted average across the set of firms within each region. Due to the assumption of the Log Normal distribution of labor augmenting productivities, there are many small firms who are unable to match with any customer firms whatsoever.figure 3.2 presents the results from figure 3.1b presenting the mass of customers for only the largest firm in each region.Note that in free trade the most productive firm in the import competing sector with the lowest cost of posting employment vacancies sales to around 9 percent of all firms in the economy.

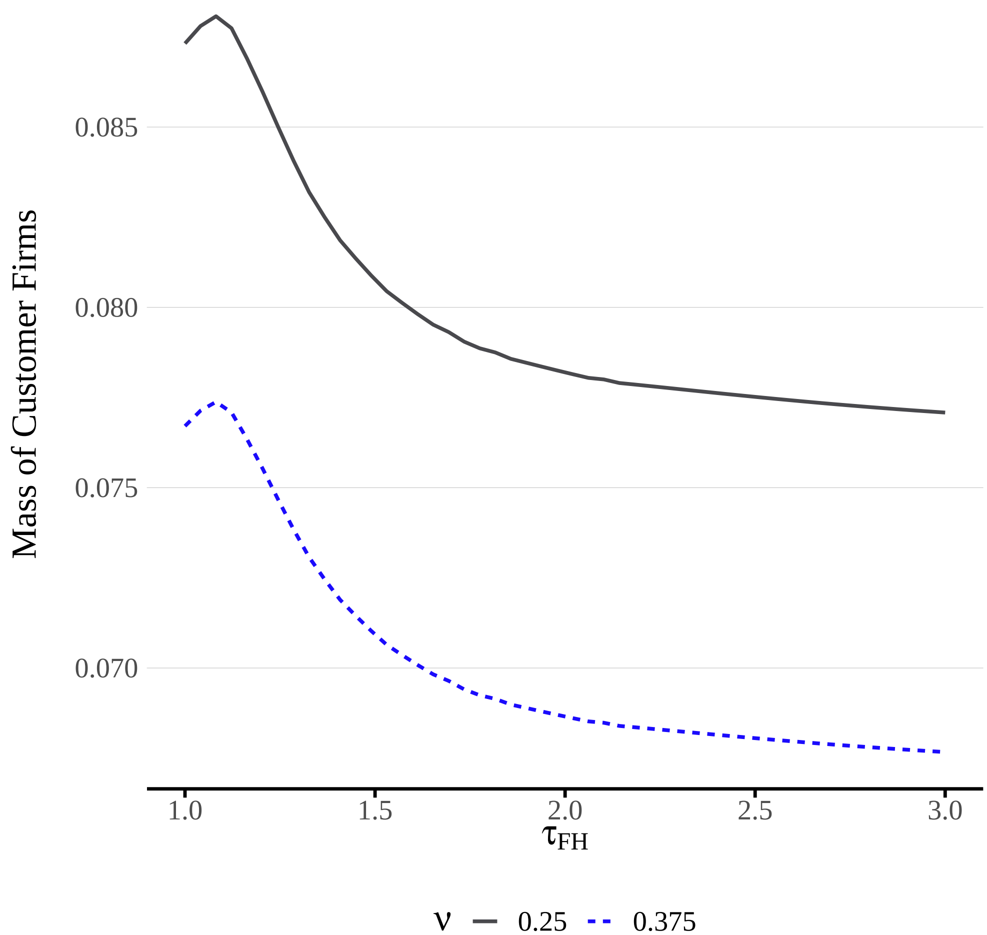
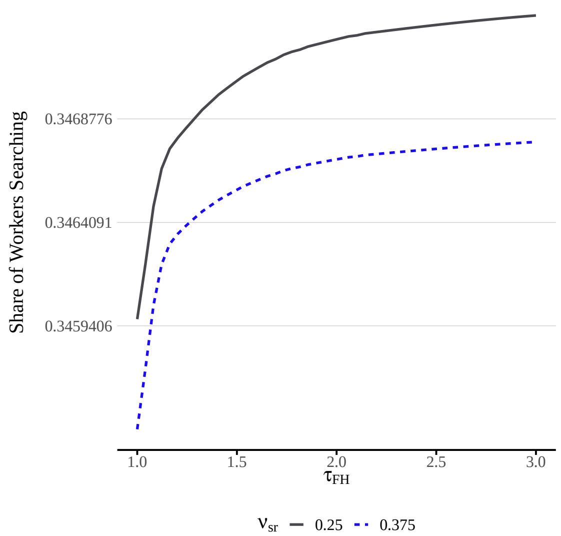


Figure 3.2: Import competing sector in Home across regions, values represent the most productive firm in each sector-region pair.

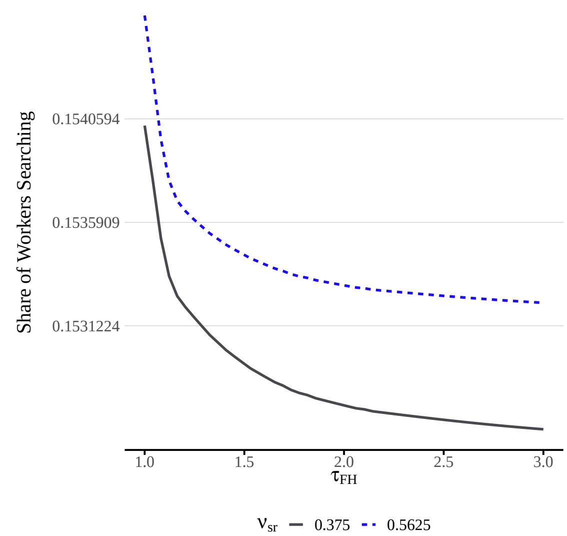
A few results pop out immediately from figure 3.1. First, sector-region pairs with a higher cost of posting employment vacancies feature a lower firm-level average mass of customer firms.This result is due to two mechanisms. The first the fact that in sector-region pairs with a higher cost of posting employment vacancies the probability of a worker choosing to search in one of these sectors and matching with an employer is lower, due to the fact that firms will post less vacancies since it is more expensive. Since less workers search for employment in these high job-vacancy-posting-cost sectors, due to the low probability firms will in general hire less workers and thus be able to produce less. When firms produce less, they are less attractive as customers to other firms limiting their access to inputs and causing their unit cost to rise.When unit costs rise the firm will find it less profitable to sell to other firms due to higher unit costs of production, making it much more likely that the firm doesn't sell to firms.The second mechanism is due to the assumption that workers must pay their fixed cost of matching in terms of labor, implying for each match with a customer-firm that the selling-firm undertakes they must hire an additional unit of labor, implying the firm must post employment vacancies. This limits the relative number of firm-to-firm-matches that firms in high vacancy-cost sector-regions can have.

The second result from figure 3.1 is that the shape of the response to an increase in the trade costs is highly non–linear. For Foreign firms, the tariff causes the mass of customer firms to decay quickly as trade costs move away from free trade. This is due to the decrease in potential profits from selling to firms in Home. Home firms first see an increase in their mass of customer firms as trade costs move away from free trade. This is because as the tariff increases, Home workers are more likely to search for employment in the import competing sector, lowering the firm's unit cost through lower employee-hiring costs, as shown in figure 3.3.This means that at first, firms in the import competing sector are able to match with more customer firms due to their lower unit costs.But as trade costs continue to rise the loss of Foreign suppliers takes over, leading to higher unit costs and a decrease in the mass of customers for firms in the import competing sector industry.Firms in non-import competing sectors follow a similar, but less pronounced, pattern. This is due to the fact that as workers search more in the import competing sector, the cost of hiring workers in the non-import competing sectors increases. However, these firms benefit from the import competing sector's initial growth through the production network.As the gains from the tariff in the import competing sector are eliminated, so to are the lesser gains in the non-import competing sector.

Finally firms in sector-regions with lower costs of posting employment vacancies are more



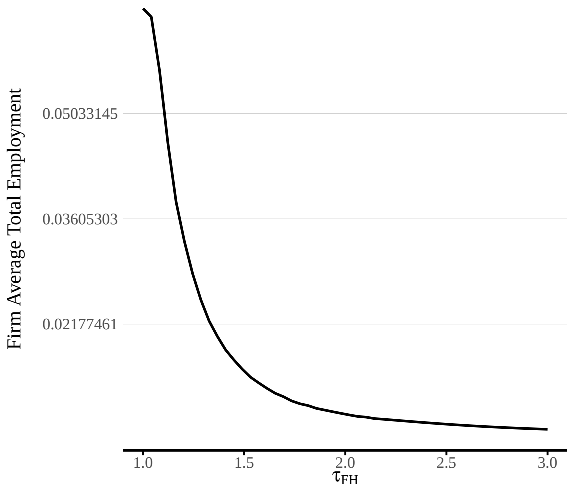
(a) Import competing sector, across regions in Home



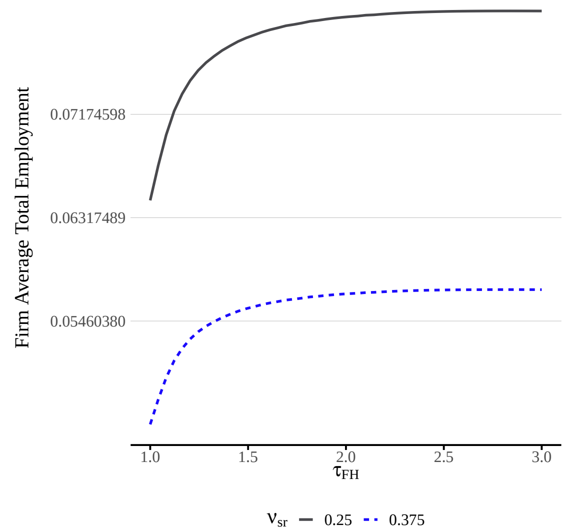
(b) Non-import competing sector across regions in Home

Figure 3.3: Workers Searching For Employment Across Sectors

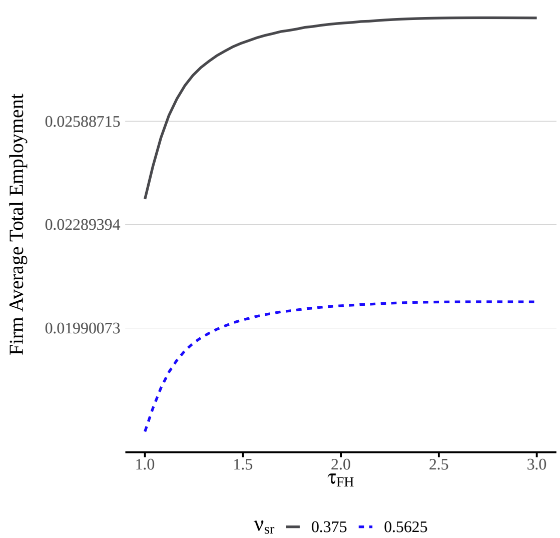
sensitive to changes in the tariff. This is due to the fact that these firms are more integrated with foreign firms, due to their attractiveness as suppliers and customers to other firms.The contrast in responses to trade shocks between sector-regions with different costs of posting employment vacancies is the most pronounced within regions among "non-import competing" sectors, as in figure 3.1d.As trade costs increase, within all regions, each sector sells less due to their increased unit costs (from a lack of access to foreign suppliers).Since low ν firms are more integrated into the international economy their relative total sales decrease and their relative unit costs increase in response to an increase in the tariff rate.These effects counter act one-another in the labor demand equation. Initially as the tariff increase takes the economy away from free trade the firms demand more workers since unit costs increase by more than sales fall. This effect is more pronounced among high ν firms due to their greater integration in the production network.



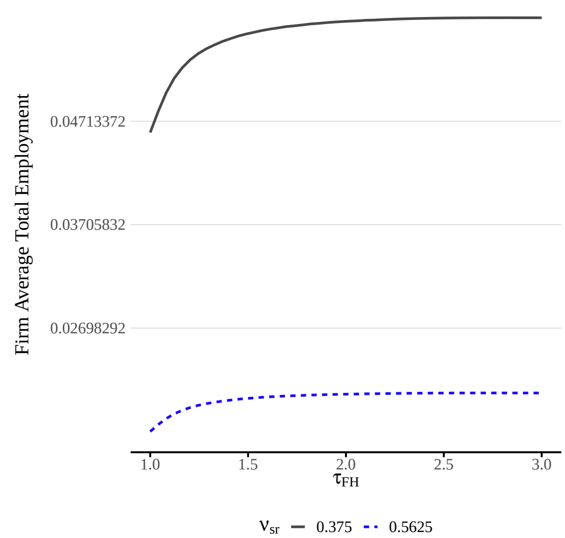
(a) Foreign sector to which the tariff is applied



(b) Import competing sector in Home, across regions



(c) Non-import competing sectors in Home, across regions



(d) Non-import competing sectors in Home, within regions

Figure 3.4: Within sector-region average firm-level total employment

The effect of firm-to-firm matching on sector-region aggregate labor-market variables occurs through two channels, labor demand for production and labor demand for paying the fixed cost of matching between firms with the former dominating the latter. The amount of fixed cost paid in terms of labor by each firm in a sector region, resembles the firm's mass of customers presented in figure 3.1 and is negligible in magnitude.The effect of firm-to-firm production on labor demand is two-fold.First, since labor demand is increasing in total sales, the inclusion of endogenous production networks lowers the amount of labor demanded by each firm due to the fact that firms are not able to sell to all other firms.Secondly, the inclusion of endogenous production-networks makes labor demand less responsive to the change in the tariff. This is because the inclusion of endogenous production network mutes the effect the tariff relative to a model where firms can freely trade without paying a matching cost.

4 Comparison to Different Models

In this section I compare the model to several other models of international trade to emphasize the crucial interaction between mobile labor supply and endogenous production-network formation. First in order to demonstrate how labor-market-mobility affects production networks, I present the results of a model with endogenous firm to firm matching that does not include labor-market mobility across sectors (analogous to Lim [1] and Huneeus [2]). Presenting these models next to my own highlites the importance of labor-market frictions in determining the production network. I then present two models with labor-market frictions that do not feature endogenous production networks. The first of the two has no connections across firms (analogous to that of Helpman and Itskhoki [7]). The second allows firms to be connected to all other firms in the economy through an exogenous production network such that there are linkages that are identical across sector pairs (as in Jones [14] or Caliendo, Dvorkin, and Parro [12]).Contrasting against these models stresses the role that endogenous production networks play in labor-supply shifts and other labor-market variables.

4.1 Comparison to Other Models of Endogenous Production-Network Formation

In the first comparison I emphasize the effect of the reallocation of labor supply across sectors on endogenous production networks by comparing the model to one in which each sector is endowed

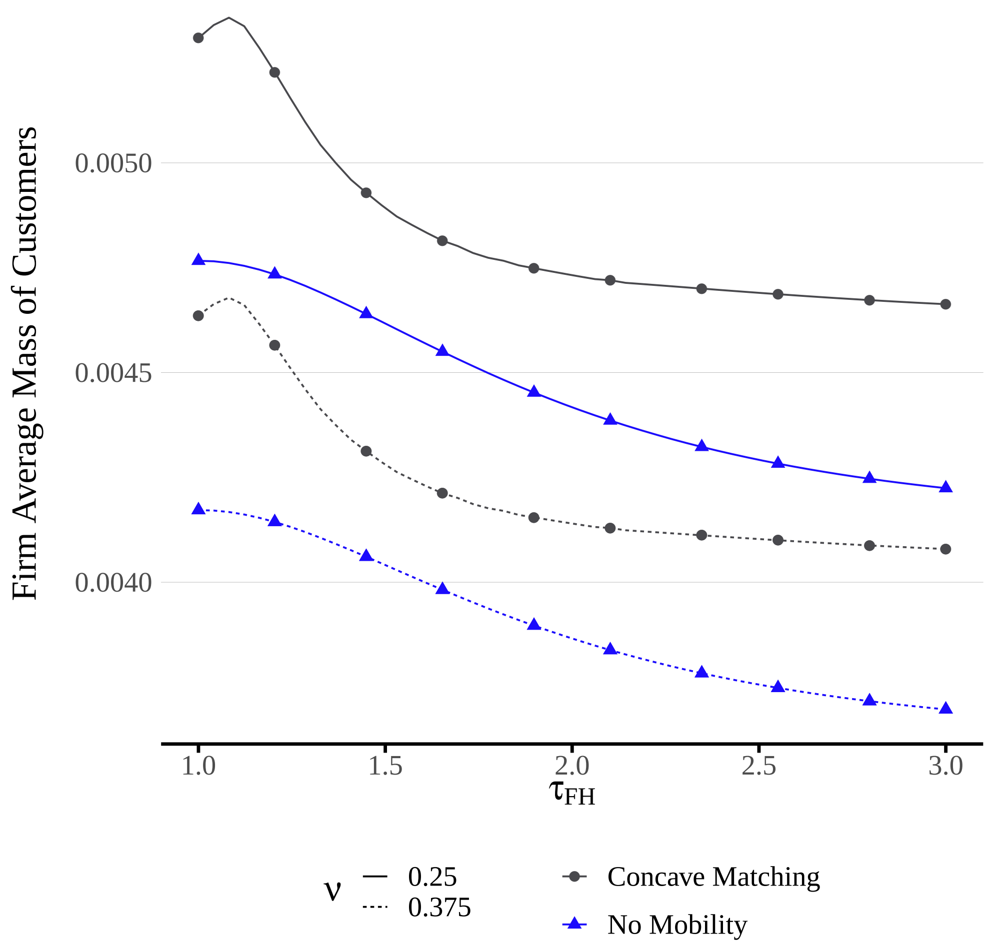


Figure 4.1: Mass of Customers for Home Import Competing Sectors.

with an immobile labor supply. The model is not identical to that presented by Lim [1] and Huneeus [2], however it features the same labor-market setting as the model presented above without the conditions given in equation (2.21) and equation (2.22). The amount of labor supply available to each sector is simply given by $L_{s,r} = \bar{L}_{s,r}$, where $\bar{L}_{s,r}$ is a simple constant that is equal across sectors and regions. This assumption shuts down the migration of labor across sectors, allowing the comparison of this first counterfactual model to the one presented in section §2 to be interpreted as the effect of labor migration on endogenous production networks.

figure 4.1 presents the results of the counterfactual model along-side the results from figure 3.1b. Failing to include labor-market mobility in the endogenous production-network model leads to a different response to the tariff across all sectors, with the slope of the response (and even the sign of the slope) differing for import competing sectors[3]. The mobility of labor increases the labor supply of workers available to the import competing sector in response to the tariff.This gives the firms in the import competing sector access to cheaper labor, and hence lower unit costs increasing the set of firms they are willing to sell to.

4.2 Comparison to a Model with Exogenous Input-Output Linkages Across Sectors

In the second comparison I emphasize the importance of endogenous production-network formation in determining labor-market outcomes, by comparing the model to one that features an exogenous production-network as in Jones [14] or Caliendo, Dvorkin, and Parro [12]. Once again, the comparison model is not identical to either of the ones mentioned, however the spirit of the model is similar. All firms within a given sector pair are linked via the input-suitability parameter α_s. More specifically this second counterfactual model assumes that $m(\chi, \chi') = 1 \ \forall \chi, \chi'$, so that α_s governs the size of linkages across sectors. This assumption preserves all labor-market features laid out in section §2 and maintains the assumption that all firms are connected. The key difference between this counterfactual model and the baseline model is the assumption that the production-network does not change in response to the tariff as it does in the baseline model.

figure 4.2 presents the total employment by firms across non-import competing sectors in Home. The baseline model is more responsive to the tariff than the counterfactual model of exogenous input-output linkages. This is due to the fact that in the input-output model the amount of inputs available to each drastically falls in response to the tariff, whereas in the endogenous production-network model even in free trade only some firms are exposed to the Foreign market

[3]This is also the case for non-import competing sectors.

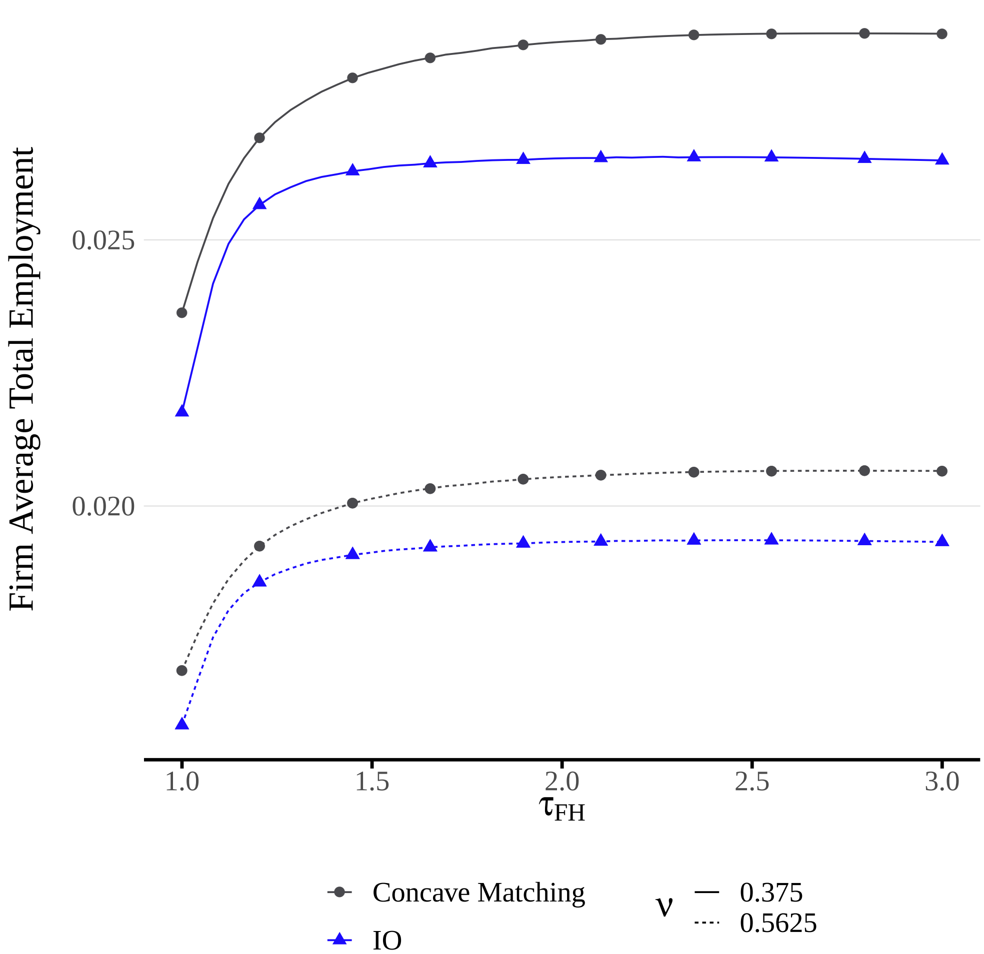


Figure 4.2: Labor Demand in Non-Import Competing Sectors in Home

and implementing the tariff only affects some firms. As the tariff continues to increase, the labor demand in IO model flattens out while labor demand in the endogenous matching model continues to increase. This is due to the changing relationships between other domestic firms. As the import competing sector benefits from the tariff, this spills over into the non-import competing sector through more connections being formed in the endogenous model . This effect is absent in the input-output model since the network between all firms at home are left unaffected.

5 Conclusion

This paper highlights the fact that models of endogenous production-network formation are inseparable from models of labor-market frictions that feature labor mobility across sectors by presenting a model of endogenous production-network formation and labor-market frictions.Comparing the model to one without labor-market frictions reveals the importance of inter-sectoral labor-market mobility in determining how endogenous production-networks change in response to tariffs. Looking at the model next to one with an exogenous production-network shows that the endogenous production-network drives changes the way in which the labor market changes in response to the tariff.Future work is needed to quantify the empirical importance of these features.

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