Labor-Market Frictions and Endogenous Production-Network Formation *

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Abstract

This paper studies the quantitative implications of labor-market frictions in a setting where firms source intermediate inputs through an endogenously formed global production network. I build a model to show that decreases in relative labor-market frictions across sectors can cause the sector-level average firm's relative number of suppliers to double in magnitude. The model also shows how the second moment of the firm-to-firm matching function determines the effect of a trade shock on wage inequality. I then simulate the effect of a protectionist tariff by the Home country. The model predicts that a protectionist tariff leads to increases in long-run unemployment and real wages, for all sectors in the Home country. The latter result is driven by the reallocation of Foreign labor across sectors in response to the tariff. This reallocation leads to an increase in demand for Home varieties by firms in Foreign. The demand increase in Foreign, offsets the losses of Foreign inputs for firms in the Home country.

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1 Introduction

Trade in intermediates account for about half of firm sales, with cross-border trade in production networks playing a key role. And yet, the number of upstream and downstream linkages between firms can be small. Among Japanese firms, for example, the typical number of customers and suppliers is both only three, pointing to significant costs that are incurred to establish production networks. [1] Furthermore, firms produce combining intermediates with labor services. The interaction between firms' intermediate sourcing decisions as they form and dissolve production networks and their hiring decisions is important in understanding how changes in the costs of networking and trade affect hiring and the level of economic activity, inequality between workers, and the structure of the economy inside and outside of production networks. In this paper, I quantify how labor market outcomes and production-networks change in response to economic shocks, specifically trade shocks. More specifically, this paper asks the following two questions: How do changes in the production-network after a shock, drive changes in labor-market outcomes, such as employment, unemployment, and wage-inequality, across sectors? How does the reallocation of labor across sectors in response to a trade shock, change the production-network and cross-border trade?

In this paper I present a model of endogenous production-network formation in which workers and firms face labor-market frictions. Previous models of endogenous production-network formation have assumed that all firms use labor in production, which they must source from a friction-less competitive market. Allowing for frictions in the sourcing of labor will change how production-networks form and shift in response to shocks. This change will be different from an adjustment of firm productivities or network centrality. While an increase in the productivity of one sector will benefit other sectors as well (through an increase in the output and lowering of the unit cost of production for firms in all sectors), an increase in the labor-market efficiency of one sector will lead to a reallocation of labor, which will benefit one sector at the expense of the other. The reallocation of labor that occurs in response to a shock implies that network centrality will shift across sectors in response to a shock. The implications for this are critical for understanding

[1] See Bernard et al. (2019).

how welfare, employment, and wages respond to trade shocks differently across sectors.

Allowing for the endogenous formation of production-networks will also change how labor-market outcomes respond to shocks. Relative to a model that treats the production-network as exogenous, a model featuring endogenous production-network formation will lead to a change in the responsiveness of labor-market outcomes to trade costs. The sign of this relative change depends on the sensitivity of the production-network to trade shocks. If there is a large (small) change in the number of firm relationships in response to a trade shock, then there will also be a relatively large (small) change in labor market outcomes, relative to a model featuring exogenous production-networks.

Prior work on endogenous production-network formation has mostly focused on the role of firm-level productivities and network centrality in determining the density of the production network (Lim (2018), Bernard and Moxnes (2018)). Here I present a new mechanism for driving density differences across production networks, the difference in labor-market frictions across sectors. This paper builds a model that points out that sectors with relatively lower labor-market frictions will be more central in the production-network. This reflects the fact that firms in sectors with relatively lower labor-market frictions are more profitable for other firms to trade with. Firms in sectors with relatively lower labor-market frictions will produce more output (making them more attractive as a potential customer) and operate at lower unit costs (making them more attractive as a potential supplier).

Prior work on the responsiveness of production networks to shocks such as Huneeus (2018), Bernard et al. (2019), Gabaix (2011), also have ignored the sectoral reallocation of employees across sectors, focusing only on how shocks propagate throughout the economy based on firm-level productivities. While this effect is still present in my model, the addition of labor-market frictions induces a new effect. With the introduction of labor-market frictions, some workers will now be reallocated across sectors in response to the shock. This will affect the ability of firms in each sector to form production-network linkages differently. The reallocation of labor across sectors will lead to firms in one sector being able form production-network linkages more easily, and in the other sector it will now be more difficult for firms to form production-network linkages. This suggests that models of production-network formation which do not take into account labor-market reallocation misstate the rate at which production networks respond to trade shocks, and thus misstate the rate at which shocks propagate throughout the economy.

My analysis also contributes to the literature that investigates how trade shocks, offshoring, and outsourcing affect labor-market outcomes (Acemoglu et al. (2014), Autor et al. (2013), Caliendo et al. (2019), Grossman and Helpman (2005), Helpman and Itskhoki (2010)). I derive a model to examine how employees are affected by shocks through changes in firm-level abilities to source inputs through an endogenously-formed production-networks. Previous papers have considered how labor markets respond when they are exposed to economic shocks through an exogenous production-network, for example, through input-output linkages. However, these models ignore how the production-network itself might shift and thereby change the level of exposure to these shocks. Production networks must change in response to economic shocks, affecting the rate at which these shocks propagate throughout the economy. Shocks will change the labor-market outcomes across all sectors in the economy, and the way that shocks propagate across sectors will be determined by the way that production-network linkages respond to the shocks. In this paper I demonstrate how changes in endogenous production networks lead to changes in the labor market that systematically vary across sectors, and by production-network centrality.

Finally, this paper contributes to the literature on how wage inequality responds to trade shocks. Costinot and Vogel (2010), Egger and Kreickemeier (2009), Helpman et al. (2017) and Helpman et al. (2018), all explore how trade between countries drives wage inequality. These papers explore how wage-inequality will change in response to trade shocks, given firms are operating in an environment featuring worker-firm matching, or fair wage conditions. This paper contributes to this literature by exploring how firm-to-firm matching drives changes in within- and across-sector wage-inequality, in response to trade shocks. I show how the concavity of the firm-to-firm matching function determines the sign of the response of within-sector wage-inequality to trade shocks. This implies that trade liberalization could actually actually lead to a reduction in within sector wage inequality if the firm-to-firm matching function is concave for the average firm pair.

In what follows I first present a model of endogenous production-network formation, similar to that of Lim (2018), that includes labor-market frictions as in Helpman and Itskhoki (2010). The model demonstrates not only, how endogenous production-network formation affects labor-market outcomes, but also how labor-market frictions determine the production-network centrality of sectors in the economy. In the following section I present the model and discuss the predictions of the model. I then simulate the model in Section 3. I then simulate the effects of globalization, and the effects of the Home country unilaterally imposing a tariff on a particular sector in the foreign country. I then compare the simulation results to other models featuring trade and labor-market frictions. In order to demonstrate the effect of labor-market frictions on endogenous production-network formation, I compare the model presented in Section 2 to an analogous extension of Lim (2018) that includes an immobile labor supply. To demonstrate the effect of endogenous production-network formation on labor-market outcomes I compare my model to an extension of Helpman and Itskhoki (2010) that includes exogenous sectoral linkages in production, similar to that of Caliendo et al. (2019) and Jones (2013).

2 A Double Sided Matching Model of Production

In this section I build a model of endogenous production-network formation with labor-market frictions in which firms must pay a fixed cost to match with each customer firm, and they must also pay a cost to post employment vacancies.The economy consists of two countries and two sectors, each of which has a continuum of firms and a fixed labor supply. Sectors and countries vary by the average cost of matching with customers, and the cost of posting a vacancy. The model consist of a collection of household preferences, product and labor-market structures, and determinants of network of firm linkages, wages, and profits.In what follows in this section, I describe each of these components in detail, discuss the equilibrium conditions of the model, and finally discuss the predictions of the model with respect to within sector and country wage inequality.

2.1 Basic Environment

Suppose the economy consists of two countries, Home (H) and Foreign (F) consisting of two sectors, indexed by s. Within each country there is a continuum of firms, each with a firm specific labor augmenting technology parameter, indexed by ϕ_ℓ. Firms are indexed by the triplet $\phi = (\phi_\ell, s, c)$, where $c \in \{H, F\}$. The market is monopolistically competitive with each firm producing its own unique variety that it can sell to both households and other firms. Firm production takes place by obtaining intermediate varieties produced by other firms in the economy and combining them with labor that the firm is able to employ given the labor-market frictions. In order to hire workers, firms must post vacancies and must pay a cost per each vacancy posted. The firms cannot costlessly adjust their labor supply which induces workers and firms to engage in wage bargaining and generating wage differences across firms within sectors.

In what follows I present the model in detail by first considering the household's problem and then examining the firm's problem. I then present definitions of the equilibrium conditions of the model. I then discuss how changing the labor market parameters lead to changes in the formation of each firm's production-network. I discuss how labor market outcomes change depending on the production-network efficiency. Finally I discuss the implication of changes in trade costs and discuss the predictions of the model in response to a protectionist tariff.

2.1.1 The Household's Problem

Within each sector for a given country, (s, c) the representative household supplies $L_{s,c}$ units of labor to the economy by searching for employment. The household has love of variety preferences given by:

$$U_{s,c} = \left[\sum_{s',c'} \alpha_{s'}^{HH} \int [x_{s,c}^{HH}(\phi')]^{\frac{\sigma^{HH}-1}{\sigma^{HH}}} dF_{\phi_\ell}(\phi'_\ell) \right]^{\frac{\sigma^{HH}}{\sigma^{HH}-1}}$$

where $\alpha_{s,c}^{HH}$ governs the degree to which households value inputs from sector-country pair s, c, $\sigma^{HH} > 1$ is the elasticity of substitutions between firm specific varieties. Total demand for each firm's brand by the representative consumer in sector country s, c is given by the following:

$$x^{HH}_{s,c}(\phi') = \frac{I_{s,c}}{P_{s,c}}\left(\frac{\alpha^{HH}_{s'}P_{s,c}}{\tau^{c}_{s',c'}p^{HH}(\phi')}\right)^{\sigma^{HH}} \tag{2.1}$$

where

$$P_{s,c} \equiv \left(\sum_{s',c'}(\alpha^{HH}_{s'})^{\sigma^{HH}}\int[\tau^{c}_{s',c'}p^{HH}(\phi')]^{1-\sigma^{HH}}dF_{\phi_\ell}(\phi'_\ell)\right)^{\frac{1}{1-\sigma^{HH}}} \tag{2.2}$$

is the price index of the representative consumer. $I_{s,c}$ is the total income of the household in s, c. The parameter $\tau^{c}_{s',c'}$ is an iceberg trade cost term that determines how costly it is to ship sector-s' goods from country c' to country c. [2] $p^{HH}(\phi)$ is the price charged to households by firm ϕ. Total demand that firm ϕ faces from households in the economy is given by:

$$x^{HH}(\phi) = \sum_{s',c'}x^{HH}_{s',c'}(\phi) \tag{2.3}$$

2.1.2 Firm Production and Firm-to-Firm Matching

Firm Production Each firm produces its own variety of the differentiated product via a CES production technology using the intermediates it obtains from other firms and the amount of labor that it is able to hire given the labor-market frictions. The production function is given by the following:

$$Y(\phi) = \left(\alpha_s\phi_\ell[\ell(\phi)]^{\frac{\sigma_{\ell,s}-1}{\sigma_{\ell,s}}} + (1-\alpha_s)[x(\phi)]^{\frac{\sigma_{\ell,s}-1}{\sigma_{\ell,s}}}\right)^{\frac{\sigma_{\ell,s}}{\sigma_{\ell,s}-1}}$$

where the employment is given by the term $\ell(\phi)$. The firm combines intermediates from other sectors via CES aggregator that is denoted by $x(\phi)$, which will be discussed shortly.α is a parameter that governs the input suitability for the industry of the firm, s. $\sigma_{\ell,s}$ is the elasticity of substitution between workers and aggregate intermediates.

Given the assumption that the firm cannot costlessly adjust their labor supply, the firm only retains only a share of the surplus that it generates by hiring workers. This share is given by

[2] In the current version I assume that households can only purchase from domestic firms. This is akin to assuming $\tau^{c}_{s',c'} = \infty$, $\forall c \neq c'$

$1/(1+\beta)$, where β is the bargaining power of the workers, thus the firm only bears this share of the non-labor costs that it incurs. Given this assumption, the demand for labor and aggregated intermediates are:

$$\ell(\phi) = \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,r}]^{-\sigma_{\ell,s}} Y(\phi) \eta(\phi)^{\sigma_{\ell,s}} \tag{2.4}$$
$$x(\phi) = (1-\alpha_S)^{\sigma_{\ell,s}} \left[\frac{P(\phi)}{1+\beta}\right]^{-\sigma_{\ell,s}} Y(\phi)\eta(\phi)^{\sigma_{\ell,s}}$$

where the unit cost function of the firm is denoted by $\eta(\phi)$ and is given by:

$$\eta(\phi) \equiv \left(\alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,r}]^{1-\sigma_{\ell,s}} + (1-\alpha_s)^{\sigma_{\ell,s}} \left[\frac{P(\phi)}{1+\beta}\right]^{1-\sigma_{\ell,s}}\right)^{\frac{1}{1-\sigma_{\ell,s}}} \tag{2.5}$$

The cost of hiring one more worker in sector country-s, c is denoted by $b_{s,r}$ and $P(\phi)$ is the cost the firm must pay to increase its CES aggregator of intermediates ($x(\phi)$) by one unit.

The firm combines intermediates across sectors using the following CES aggregator:

$$x(\phi) = \left(\sum_{s'} \alpha_{s,s'} [x_{s'}(\phi)]^{\frac{\sigma_S - 1}{\sigma_S}}\right)^{\frac{\sigma_S}{\sigma_S - 1}}$$

The parameter $\alpha_{s,s'}$ governs the input suitability of sector s' varieties that are used in the production of sector s goods. σ_s is the elasticity of substitution between sectors used in production by sector s. $x_{s'}(\phi)$ is a CES aggregator of the firm's demand for sector s' varieties. The demand for aggregate intermediates from sector-$s'$by the firm is given by:

$$x_{s'}(\phi) = (\alpha_{s,s'})^{\sigma_s} [P_{s'}(\phi)]^{-\sigma_s} x(\phi) P(\phi)^{\sigma_s} \tag{2.6}$$

The firm's cost of increasing the aggregator of total intermediates is defined as:

$$P(\phi) \equiv \left(\sum_{s'} (\alpha_{s,s'})^{\sigma_s} [P_{s'}(\phi)]^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} \tag{2.7}$$

Where $P_{s'}(\phi)$ is the firm's unit cost of increasing it's sector s' CES aggregator by one unit.

Within it's demand for inputs from a given sector, the firm aggregates across countries and varieties. However, before the production function and input demand is presented it is useful to pause to discuss the production-network framework.

As in Lim (2018) there are relationship frictions between firms such that firm ϕ' can only sell to firm ϕ if firm ϕ' pays a fixed cost. [3] This will occur with probability $m(\phi, \phi')$, which I refer to as the *firm-to-firm matching function*. The *firm-to-firm matching function* will be solved for endogenously later in the model but for now is taken as given. This function will completely determine the extensive margin of trade within the model. I assume that the fixed cost of firm-to-firm matching is paid in terms of labor.

Given the matching function, the firm combines intermediate varieties within each sector according to the following CES-aggregator:

$$x_{s'}(\phi) = \left(\sum_{c' \in \{H,F\}} \int m(\phi, \phi')[x(\phi, \phi')]^{\frac{\sigma_{s,s'}-1}{\sigma_{s,s'}}} dF_{\phi_\ell}(\phi'_\ell | s') \right)^{\frac{\sigma_{s,s'}}{\sigma_{s,s'}-1}}$$

where $x(\phi, \phi')$ is the total sector s' variety demanded by the firm and $\sigma_{s,s'}$ is the elasticity of substitution that governs how sector s firms substitute between sector s' varieties. Given this structure, the conditional demand for variety ϕ' by firm ϕ is given by:

$$x(\phi, \phi') = [\tau^c_{c',s'} p(\phi, \phi')]^{-\sigma_{s,s'}} x_{s'}(\phi) P_{s'}(\phi)^{\sigma_{s,s'}} \tag{2.8}$$

where the firm's unit cost of increasing it's sector s' CES aggregator, $P_{s'}(\phi)$, is defined as:

$$P_{s'}(\phi) = \left(\sum_{c' \in \{H,F\}} \int m(\phi, \phi')[\tau^c_{c',s'} p(\phi, \phi')]^{1-\sigma_{s,s'}} dF_{\phi_\ell}(\phi'_\ell | s') \right)^{\frac{1}{1-\sigma_{s,s'}}} \tag{2.9}$$

[3] I assume that this fixed cost is not paid by the customer firm, only the supplying firm. This implies that conditional on matching with a supplier, a customer firm's demand for a supplier's specific variety is not distorted. In the appendix of Lim (2018), the author discusses a model in which firms bargain over the surplus of their match. The outcomes of the model do not differ in a meaningful way.

where $\tau^{c}_{c',s'}$ is the cost of shipping sector s' goods from country c' to country c and is assumed to take the form:

$$\tau^{c}_{c',s'} \begin{cases} 1 & \text{if } c = c' \\ \\ 1 + t_{s'} & \text{otherwise} \end{cases} \tag{2.10}$$

where $t_{s'} > 0$. Also note that $p(\phi, \phi')$ is the price charged by firm ϕ' when selling to firm ϕ.

Firm Pricing and Firm-to-Firm Matching Given that the price elasticity of demand for a firm's variety only varies across sectors, within each sector pair, a firm does not find it optimal to price discriminate. Firms will find it optimal to price discriminate across sectors, but not within a given sector they are selling to. This implies the standard CES markup over unit cost for each firm:

$$p(\phi, \phi') = \mu_{s,s'} \eta(\phi') \tag{2.11}$$

where $\mu_{s,s'} \equiv \sigma_{s,s'}/(\sigma_{s,s'} - 1) > 1$, is the markup over the unit cost of production that is charged by all sector s' firms when selling to sector s. Firm ϕ' similarly charges a markup over unit costs when selling to the representative household employed in any sector or country is given by:

$$p^{HH}(\phi) = \mu^{HH}_{s} \eta(\phi) \tag{2.12}$$

where $\mu^{HH} \equiv \sigma^{HH}/(\sigma^{HH} - 1)$.

Given the firm's optimal pricing rule, we can now calculate the profit the firm would earn from selling to households in any given country-sector pair.

$$\pi^{HH}_{s',c'}(\phi) = \frac{(\mu^{HH} - 1)}{1 + \beta} \frac{I_{s',c'}}{P_{s',c'}} \left[\frac{\alpha^{HH}_{s} P_{s',c'}}{\mu^{HH}} \right]^{\sigma^{HH}} [\tau^{c'}_{s,c} \eta(\phi)]^{1-\sigma^{HH}} \tag{2.13}$$

Aggregating over household demand from all sectors and countries gives:

$$\pi^{HH}(\phi) = \sum_{s',c'} \pi^{HH}_{s',c'}(\phi) \tag{2.14}$$

Likewise it is also straightforward to calculate the potential profit that firm ϕ' would realize from selling to firm ϕ. Conditional on being able to match with firm ϕ, firm ϕ' would earn a profit of

$$\pi(\phi, \phi') = \frac{(\mu_{s,s'} - 1)[\mu_{s,s'}]^{-\sigma_{s,s'}}}{1+\beta}[\tau^{r}_{r',s'}\eta(\phi')]^{1-\sigma_{s,s'}}x_{s'}(\phi)[P_{s'}(\phi)]^{\sigma_{s,s'}} \tag{2.15}$$

Given the assumption that the selling firm must pay a fixed cost to match with each customer firm they take on as a customer, the selling firm will agree to sell to a customer-firm if the profit of making the sell is greater than the fixed cost of obtaining the customer.

As in Lim (2018) I assume that the relationship fixed cost for each customer obtained by a firm in sector customer s, c is equal to $f_{s,c}\xi$. I assume that this fixed cost must be paid in terms of labor. ξ is a random variable that is distributed according to the cumulative distribution function F_{ξ}, with unit mean. The assumption that firms pay this fixed cost in terms of labor means the firm must hire an additional $f_{s,r}\xi$ workers for each customer firm they choose to match with, which imposes additional costs. Thus for each match with a customer firms, the selling firm must pay $b_{s,c}f_{s,c}\xi$ in matching costs. Given these assumptions, firm ϕ will find it profitable to sell to firm ϕ' with probability:

$$m(\phi', \phi) = F_{\xi}\left[\frac{\pi(\phi', \phi)}{b_{s,c}f_{s,c}}\right] \tag{2.16}$$

Following with Lim (2018) I assume that ξ takes on a Weibull distribution. Therefore the total labor employed by a firm to facilitate matching with all other firms can be calculated as:

$$FC(\phi) = f_{s,c}\sum_{c',s'}\int \mathbb{E}_{\xi}[\xi_{max}(\phi', \phi)]dF_{\phi_{\ell}}(\phi'_{\ell}) \tag{2.17}$$

Where $\xi_{max}(\phi', \phi)$ is the highest level of ξ at which firm ϕ would be willing to sell to firm ϕ'. Note that this is given by the following amount:

$$\xi_{max}(\phi', \phi) = \frac{\pi(\phi', \phi)}{b_{s,r}f_{s,r}}$$

Given the parametric assumption that ξ is distributed according to the weibul distribution, the partial expectation of ξ_{max} is given by the following:

$$\mathbb{E}_\xi[\xi_{max}(\phi', \phi)] = \int_0^{\xi_{max}(\phi', \phi)} \xi dF_\xi(\xi)$$
$$= \int_0^{\frac{\pi(\phi', \phi)}{b_{s,r} f_{s,r}}} \xi \frac{k}{\lambda} (\frac{\xi}{\lambda})^{k-1} e^{-(\xi/\lambda)^k} dF_\xi(\xi)$$

where k and λ are the shape and scale parameters, respectively, of the Weibull distribution.

2.2 Labor Market

The labor market features frictions such that firms cannot costlessly find workers to hire as in Stole and Zwiebel (1996), and must match with workers via a Cobb-Douglass matching as in Helpman and Itskhoki (2010). Firms must match with enough labor to use in production and to additionally pay their fixed costs of matching with other firms. This implies that the fixed cost of matching can be thought of as a communication cost between the customer and supplier or as an upkeep cost of each transactions. Real-world examples of this fixed cost include time spent communicating with or finding customers, customization of the production process, billing expenses etc... Firms post employment vacancies and must pay a cost for each of these vacancies that they post. The cost of posting these employment vacancies translates into a cost of each worker that is hired by the firm. The cost of posting a vacancy differs across countries and sectors leading to comparative advantages between sectors and countries. The varying cost of posting vacancies therefore determines the rate at which shocks propagate throughout the economy. Due to the labor-market frictions, after a firm has matched with enough employees to fulfill its total labor demand, the two sides bargain over the surplus generated by their match. Thus the workers are paid a share of variable profits from the firm they are assigned to.

Within each country there is a representative household of workers, with labor supply denoted by $\bar{L}_r$. The household allocates workers across sectors to maximizes its profits. For there to be employment in each sector in the economy it must be the case that within each country expected wages must be equalized. Let the *worker-firm matching function* be defined as a homogeneous of degree one Cobb-Douglass function:

$$H_{s,c} = m_{s,c}^{L}(V_{s,c})^{\chi}(L_{s,c})^{1-\chi} \tag{2.18}$$

$\chi \in (0,1)$. Where $m_{s,c}^{L}$ is a parameter that determines the efficiency of labor-market matching in each sector country pair. $H_{s,c}$ is the total number of labor-market matches in sector-country s, c. $V_{s,c}$ is the number of vacancies posted by firms in sector-country s, c. $L_{s,c}$ is the number of workers the country c household has chose to allocate to, and search for employment in, sector s.

In order to solve for the optimal number of vacancies that each firm chooses to post, it will be useful to recall demand for labor used in production by each firm. Restating equation 2.6:

$$\ell(\phi) = \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,c}]^{-\sigma_{\ell,s}} Y(\phi)\eta(\phi)^{\sigma_{\ell,s}} \tag{2.19}$$

The firm's demand for labor to be used in production depends on the firm's fundamental labor-productivity, the cost of posting vacancies, the labor share of production, and a firm specific scale $Y(\phi)\eta(\phi)^{\sigma_{\ell,s}}$.The total amount of labor demanded by each firm is given by the labor the firm uses in production and the labor the firm uses to pay their fixed cost of matching with customer firms:

$$L(\phi) = \ell(\phi) + FC(\phi) \tag{2.20}$$

The probability that a country c worker searches in sector s and finds employment is simply $H_{s,c}/L_{s,c}$. Since the total payroll of workers employed at a firm is a share of profits equal to $\beta/(1+\beta)$, the expected wage of searching in sector s is given by:

$$\mathbb{E}(w_{s,c}) = \frac{H_{s,c}}{L_{s,c}} \frac{\beta}{1+\beta} \int \frac{\pi(\phi) - FC(\phi)}{L(\phi)} dF_{\phi_\ell}(\phi_\ell | s, c) \tag{2.21}$$

for there to be employment in all sectors it must be the case that for each sector pair s and s' in all countries, it must be the case that:

$$\mathbb{E}(w_{s,c}) = \mathbb{E}(w_{s',c}) \tag{2.22}$$

To finish describing the household's problem, the total number of workers searching across sectors in each country must sum up to equal the total population of workers in the country:

$$\bar{L}_c = \sum_s L_{s,c} \tag{2.23}$$

To describe the firm's employment problem, the total number of vacancies posted in each sector-country is given by summing across the number of vacancies posted by each firm:

$$V_{s,c} = \int V(\phi) dF_{\phi_\ell}(\phi_\ell | s, c) \tag{2.24}$$

where $V(\phi)$ is the total number of vacancies posted by each firm. Secondly, the total cost of posting vacancies must equal the total cost of hiring workers for each firm.

$$\nu_{s,c} V(\phi) = b_{s,c} L(\phi) \tag{2.25}$$

Where $\nu_{s,c}$ is the cost of posting each employment vacancy.

Finally, the labor market must clear. That is the number of labor-market matches in a given sector-country must equal total employment by firms in the same sector-country:

$$H_{s,c} = \int L(\phi) dF_{\phi_\ell}(\phi_\ell | s, c) \tag{2.26}$$

2.3 Competitive Equilibrium

Solving the model requires the inclusion of two more sets of equations. First, the goods market must clear and second, the income of households in each country-sector pair must be calculated.

There exists a goods market clearing condition for each firm's variety. Firms sell their output to households and any other firm in any sector country that they agree to match with (by paying their fixed cost of matching). Thus the total output of the firm must equal their total sales:

$$Y(\phi) = x^H(\phi) + \sum_{s'r'} \int m(\phi', \phi)x(\phi', \phi)dF_{\phi_\ell}(\phi'_\ell) \tag{2.27}$$

This equation (equation 2.1) and the unit cost equation (equation 2.5) classify every firm's problem as each firm specific variable in the problem can be written in terms of the two variables these equations define, given the other sector-country specific variables.

The total income of households in each country-sector pair is calculated by aggregating over the total payments to workers by firms. Since the workers receive a share of profits equal to $\beta/(1+\beta)$ this implies that total income in each sector-country must be equal to a share of total sector-country profits. The total variable profit can be calculated by summing across the variable profit each firm makes from selling to households and each of its customers:

$$\pi(\phi) = \pi_H(\phi) + \sum_{s',c'} \int m(\phi', \phi)\pi(\phi', \phi)dF_{\phi_\ell}(\phi'_\ell) \tag{2.28}$$

This implies that the total income of workers in sector-country s, c is given by:

$$I_{s,c} = \frac{\beta}{1+\beta} \int \pi(\phi) - FC(\phi)dF_{\phi_\ell}(\phi_\ell|s,c) \tag{2.29}$$

Having closed the model we can now define a competitive equilibrium of the model:

Definition 2.1. Given a set of parameters and a firm-level distribution of labor augmenting productivities, a *competitive equilibrium* consists of a set of variables that maps from the binary Cartesian power of the set of all firms,

$$\{x(\phi, \phi'), p(\phi, \phi'), \pi(\phi, \phi'), m(\phi, \phi')\}_{\forall \phi, \phi'}$$

, a set of variables that maps from the Cartesian product of the set of all firms and the set of country-sector pairs, $\{x^{HH}_{s',c'}(\phi), p^{HH}(\phi), \pi^{HH}_{s',c'}(\phi)\}^{c'\in\{H,F\}}_{\forall\phi,s'}$ a set of variables that maps from the Cartesian product of the set of all firms and the set of all sectors: $\{P_{s'}(\phi), x_{s'}(\phi)\}_{\forall\phi,s'}$, a set of variables that maps from the set of all firms,

$$\{x^H(\phi), X(\phi), \ell(\phi), x(\phi), \eta(\phi), P(\phi), \pi_H(\phi), FC(\phi), L(\phi), V(\phi), \pi(\phi)\}_{\forall\phi}$$

and a set of variables that maps from the set of all sectors and countries,

$$\{L_{s,c}, H_{s,c}, V_{s,c}, I_{s,c}, b_{s,c}, P^H_{s,c}\}^{c\in\{H,F\}}_{\forall s}$$

such that equations 2.1 through 2.29 are satisfied for all firms and sectors in H and F.

2.3.1 The Role of Labor Market Frictions in Production Network Formation

Relative labor market frictions across sectors within each country are a determinant of comparative advantage. The relative efficiency of each sector at matching job searchers with firms will determine the distribution of job searchers across sectors. From equations 2.26 and 2.25 a sector with a large labor supply will in general have a a lower cost of hiring workers (b_s).

When the cost of hiring workers changes there are 3 effects on the extensive margin of trade between firms:

1. *Output Effect*: change in labor demand for production by each firm, $\ell(\phi)$. This effect is made obvious in equation 2.19. Since labor and aggregate inputs are related in the production process, changes in the cost of hiring labor will lead to changes in the demand for aggregate inputs by the firm. This change will affect the firm's extensive margin of trade through the intensive margin. If the firm's aggregate demand for inputs increases then the firm will be more profitable for supplying firms to match with, see 2.15. This would lead to an increase in the cardinality of the firm's set of suppliers.

2. *Efficiency Effect*: change in unit costs of production for the firm, $\eta(\phi)$. This effect appears in

equation 2.5. If the cost of hiring a worker falls this leads to a lower unit cost of production, which ultimately leads to an increase in a firm's set of suppliers. This drives changes in the extensive margin of through the intensive margin of trade between the firms. If the selling firm's unit cost falls then the potential profit of the match increases for the customer firm, again see equation 2.15. This would lead to an increase in the cardinality of the firm's set of customer-firms.

3. *Direct Network Effect*: change in the matching function for the firm, $m(\phi', \phi)$ due to a change in the firm's hiring costs. This effect is made evident by equation 2.16. If a firm's cost of hiring one worker decreases then the firm will be able to match with more customer firms, using the same amount of labor. In contrast to the previous two effects, this effect occurs only along the extensive margin of firm-to-firm trade.

To illustrate each of these effects mathematically I differentiate equations 2.16 and 2.15:

$$\frac{\partial m(\phi', \phi)}{\partial b_{s,c}} = \frac{1}{f_{s,c} b_{s,c}} \left(\frac{\partial \pi(\phi', \phi)}{\partial b_{s,c}} \right) - \underbrace{\frac{\pi(\phi', \phi)}{f_{s,c}(b_{s,c})^2}}_{\substack{\text{Direct Network} \\ \text{Effect}}} \tag{2.30}$$

$$\frac{\partial \pi(\phi', \phi)}{\partial b_{s,c}} = A_{s,c}^{s',c'} \left(\underbrace{[\eta(\phi)]^{1-\sigma_{s',s}} \frac{\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}}{\partial b_{s,c}}}_{\text{Output Effect}} + \underbrace{\frac{\partial [\eta(\phi)]^{1-\sigma_{s',s}}}{\partial b_{s,c}} x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}}_{\text{Efficiency Effect}} \right) \tag{2.31}$$

where

$$A_{s,c}^{s',c'} \equiv \frac{(\mu_{s',s} - 1)[\mu_{s',s}]^{-\sigma_{s',s}} [\tau_{c,s}^{c'}]^{1-\sigma_{s',s}}}{1 + \beta}$$

The direct-network-effect comes from the standard assumption that the fixed cost of matching must be paid in terms of labor. It is a negative effect, increasing hiring costs leads to less downstream matches.

The efficiency-effect reflects the fact that firms use labor in the production process. This is also a negative effect, increasing the cost to hire one more worker causes firm's unit costs to rise, and since there are constant markups, the price charged by the firm rises. This leads to less downstream matches, in addition to lowering the intensive margin of each potential downstream relationship.

The output effect is less obvious than the efficiency- or direct-network-effect. Suppose $b_{s,H}$ were to rise, how would this effect a sector-s'-firm's demand-shifter for sector-s inputs ($x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}$)? When $b_{s,H}$ increases this leads to workers move out of sector-s and into sector-s'. The greater labor supply available to sector-s' will change the demand for inputs by sector-s'-firms.

The sign of the output effect depends on the elasticity of substitution between aggregate inputs and labor, and whether or not the firm-to-firm match is occurring across or within sectors. If aggregate inputs are substitutes for labor, then the output effect is negative across sectors. That is, increasing $b_{s,H}$ will decrease the demand for aggregate inputs by sector-s'-firms, including sector-s-firms. This implies that the sector-s'-firm's demand-shifter for sector-s inputs would decrease in response to an increase in $b_{s,H}$. Likewise if aggregate inputs are complements for labor, then the output effect is positive across sectors.

Proposition 2.1. If $\sigma_{\ell,s} > 1$, and $s' \neq s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} < 0$ and the output effect is negative. And if $\sigma_{\ell,s} < 1$, and $s' \neq s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} > 0$ and the output effect is positive.

When the match occurs within a sector, the sign of the output-effect is the opposite of what it would be if the match occurred across sectors. If aggregate inputs are substitutes for labor, then as $b_{s,H}$ increases and workers move out of sector-s, a sector-s-firm's demand-shifter for all inputs (including sector-s inputs) would increase. Therefore if aggregate inputs are substitutes then the output effect is positive within a sector. Likewise, if aggregate inputs are complements then the output effect will be negative across sectors.

Proposition 2.2. If $\sigma_{\ell,s} > 1$, and $s' = s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} > 0$ and the output effect is positive. And if $\sigma_{\ell,s} < 1$, and $s' = s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} < 0$ and the output effect is negative.

Consider the role of labor market frictions in the cardinality of the set of customer-firms and the cardinality of the set of suppliers for a given firm. The output effect matters for a firm's set of suppliers while the efficiency effect and the direct-network effect matters for a firm's set of customer-firms.

The cardinality of the set of customers for firm-ϕ is given by:

$$|C(\phi)| = \sum_{s',c'} \int m(\phi', \phi) dF_{\phi}(\phi') \tag{2.32}$$

while the cardinality of the set of suppliers for firm-ϕ is given by:

$$|S(\phi)| = \sum_{s',c'} \int m(\phi, \phi') dF_{\phi}(\phi') \tag{2.33}$$

I will aggregate these sectors and treat these as sector level measures of production-network "centrality", denoted by $|S_s|$ and $|C_s|$.

An increase in the relative cost of posting an employment vacancy, $\nu_{1,H}/\nu_{2,H}$, leads to $b_{1,H}$ increasing and a decrease in $b_{2,H}$. When inputs and labor are substitutes, this leads to identical increases in $|C_1|$ and $|C_2|$ while $|S_1|$ will fall and $|S_2|$ will rise. This is due to the aggregation of the output effect, efficiency effect, and the direct network effect across all firms and sectors.

The direct network effect, output effect, and efficiency effect summarize how a change in hiring costs lead to changes in the extensive margin of trade between firms. When there is a change in one sector's labor market frictions, this changes the relative labor market frictions and induces workers to move from one sector and into the other sector. This effects the ability of firms to form firm-to-firm linkages that differs across sectors. This stands in contrast to an increase in the parameter that governs the sector-level average matching ability, $f_{s,c}$, which leads to an increase in the matching ability of all sectors.

2.3.2 The Effect of Network Frictions on Employment

When there is an increase in sector level network frictions there is only an effect along the extensive margin of trade. When there is a relative increase in $f_{s,c}$ across sectors expected increase in profits (and thus wages) in all sectors of the economy. However this increase will be smaller in sector-s', causing the reallocation of workers across sectors. This mechanism is similar to that in Helpman and Itskhoki (2010), when there is a reduction in the fixed cost of of exporting to a foreign country, however it differs in two critical ways.

First it differs by the structure of trade, in Helpman and Itskhoki (2010) once a firm pays a fixed cost to export, the firm can link with all customers in the foreign economy. In their framework an adjustment of the exporting friction will lead to a less smooth response in the expected wage across sectors than in the model presented in this paper. In the model presented in this paper, the network friction is per firm match. Adjustment of the friction will lead to smoother labor market adjustment than if the fixed cost was paid to access all firms in another market.

Secondly there are spillovers due to the network adjustments that occur across sectors. When $f_{s,c}$ increases sector-s-firms find it more profitable to match with firms across all sectors. This effect occurs due to two competing factors that can be seen from revisiting the labor demand equation 2.19.

$$\ell(\phi) = \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,c}]^{-\sigma_{\ell,s}} Y(\phi) \eta(\phi)^{\sigma_{\ell,s}}$$

The first mechanism is a scale effect, that is captured by changes in $Y(\phi)$. As firms are able to make more sales, labor demand increases due to increases in total output of the firm. The second effect is a gains from suppliers effect and occurs through changes in each firm's unit cost, $\nu(\phi)$. As firms gain access to more suppliers, $\nu(\phi)$ falls due to the gains from variety in production.

The increase in $f_{s,c}$ leads to a relative increase in labor demanded for production by all firms in sector-s. This is because the scale effect dominates the gains from suppliers. An increase in $f_{s,c}$ increases the firm's mass of suppliers and customers for firms in all sectors. However the increase in $f_{s,c}$ will lead to a greater increase in the mass of customers for sector-s firms, relative to sector-s'-firms. Therefore, when $f_{s,c}$ increases this leads to a relative increase in labor demand for use in production in sector-s.

Wage Inequality and Network Efficiency This model can also be used to study within sector wage inequality. Workers are matched with firms randomly within a given sector. The distribution of firm-level labor augmenting productivities, and the wage bargaining leads to a distribution of wages within each sector. To understand how the firm-level labor augmenting productivity (ϕ_ℓ) drives changes in wages, note that the unit-cost equation, 2.5, is decreasing in ϕ_ℓ. This implies that firms with a high level of ϕ_ℓ will find it profitable to match with more customer-firms, than firms with a lower level of ϕ_ℓ. Since firms only match when it is profitable, then more matches with customer-firms implies greater profits.

When $f_{s,c}$ increases, firms with lower levels of ϕ_ℓ will now find it profitable to match with customers. This implies average profits and wages will increase for all firms, but the greatest increase will be realized by firms in the left half of the distribution of firms.

The exact response of wage inequality depends on the assumptions placed on the distribution of the firm matching shocks. However the intuition of this result holds without going into exact detail. Firms with a high level of ϕ_ℓ will find it beneficial to sale to a large customer base, independent of $f_{s,c}$. These firms have a low cost of production, so their firm-relationships are not as sensitive to the parameter $f_{s,c}$. Relationships with customer-firms for suppliers with lower levels of ϕ_ℓ are more sensitive to the parameter $f_{s,c}$.

This result suggests that governments can decrease wage inequality in their economy by investing in infrastructure, professional conferences, etc... Investment in anything that reduces the cost of face to face interaction for firms, or simply reduces the costs of doing business for firms can lead to an increase in average wages and a decrease in wage inequality.

2.4 Globalization and Tariffs

I will model Globalization through a decrease in international shipping costs and international matching costs. This decrease will be symmetric across sector and country pairs. A change in shipping costs will lead to a different result from a change in matching costs. Therefore I separate these effects in my discussion of globalization.

2.4.1 A Reduction in Firm Matching Costs

When there is a symmetric change in international matching costs between countries, it is easier for all firms in Home to find more customer-firms and suppliers. However it is possible for one sector in the home country to become relatively more “central" in response to a symmetric reduction in these international matching costs. This result depends on two factors: the sector-level comparative advantage for countries, determined by their labor market efficiency, and the concavity of the matching function for the average firms in each sector.

First, it must be the case that the Home country has a comparative advantage in one of the sectors. Helpman and Itskhoki (2010) show how relative labor market frictions determine comparative advantage for countries. They show that if sector-s in Home has a more efficient labor market, relative to sector-s', than sector-s in Foreign, then Home will be a net exporter of sector-s varieties. In my model this feature still holds. However, combining it with the concavity of the matching function will yield predictions about how employment and unemployment will change in response to a reduction in firm-matching costs.

When the matching function for the average firm pairs in each sector is concave a change in the parameters or variables that determine the likelihood of a match will be less than if the matching function were convex. That is, when the matching function is convex firm to firm matches a more sensitive.

Proposition 2.3. Suppose Home has a comparative advantage in sector-s, if there is a reduction in international firm matching frictions:

1. If the firm-matching function is convex, Home will increase its net-exports of sector-s varieties and Foreign will increase its net-exports of sector-s' varieties.

2. If the firm matching function is concave, Home will decrease its net-exports of sector-s varieties and Foreign will decrease their net exports of sector-s'

The intuition of this result is as follows. If Home has a comparative advantage in sector-s varieties, then the average firm in sector-s has more matches with customer-firms than its sector-

s' counterparts. If the matching function is convex and the fixed cost of firm matches falls, then sector-s primes will find it more profitable to match with even more foreign customers. The average sector-s' firm will also find it more profitable to match with more foreign customers, but this increase will be less than the increase in sector-s.

These gains in exports will cause workers to reallocate across sectors in response to a global reduction in international firm-matching costs. If the matching function is concave, globalization as represented by a decrease in international matching costs will lead to a reallocation of labor out the net-exporting sector and into the net-importing sector.

This reduction in firm-matching costs will also have effects on wage inequality within and across each sector. Each sector will benefit from more matches, allowing more firms to export and thus reducing wage inequality within each sector. Again this effect depends on sector level comparative advantage and the concavity of the average firm's firm-matching function.

The mechanism driving this is the same as the mechanism laid out in proposition 2.3. If the matching function is concave, globalization as represented by a decrease in international matching costs will lead to an increase in relative average wages in the net-importing sector. This is due to convergence in profits across sectors. This convergence implies that if the firm-matching function is concave, then a reduction in network frictions will lead to a reduction in wage inequality across sectors.

However, if the firm-matching function is convex for the average firm in each sector, then a decrease in the firm matching frictions leads to greater gains in profits for the net-exporting industry. This implies that there is an increase in wage inequality across sectors if the firm-matching function is convex.

Within sector wage inequality will depend on the concavity of the matching function. Sector pay will become more unequal as firm profits diverge, due to the Nash-bargaining that occurs over firm profits. If the firm-matching function is concave, firm profits will increase by more on the left hand side of the distribution in response to the reduction in the firm matching fixed costs. This implies that a fall in firm matching fixed costs will lead to a reduction in within sector wage inequality. Likewise if matching costs are convex, then within sector wage inequality will rise in response to a fall in firm matching costs.

2.4.2 A Reduction in Iceberg Trade Costs

I now consider the effects of globalization as represented by a symmetric reduction in international trade costs. Changes in iceberg trade costs will yield different predictions than a change in international matching frictions. This is due to the fact that international matching frictions will directly effect the extensive margin of trade, and indirectly effect the intensive margin of trade. A change in iceberg trade costs will directly impact both the intensive margin of trade and the extensive margin of trade.

The set of results for sector-level outcomes in each country will be similar for both a symmetric reduction in iceberg and a symmetric reduction in matching frictions. The results for iceberg trade costs will simply be amplified relative to changes that would occur from a reduction in network costs. This is due adjustments that occur along the intensive margin of trade.

Revisiting equation 2.15 it is obvious that a reduction in iceberg trade costs will directly impact $\pi(\phi, \phi')$ for all firms such that $c \neq c'$. This implies, not only that the number of matches will increase for firms, but also that the average profit per match will increase for all firms.

Therefore it follows that:

Proposition 2.4. Suppose Home has a comparative advantage in sector-s, if there is a reduction in international iceberg trade costs, and if the firm matching function is...:

1. Convex, Home will increase its net-exports of sector-s varieties and Foreign will increase its net-exports of sector-s' varieties.

2. Concave, Home will decrease its net-exports of sector-s varieties and Foreign will decrease their net exports of sector-s'

However since there is a direct adjustment along the extensive margin in 2.4, in addition to the adjustment described in proposition 2.3, the following holds:

Proposition 2.5. The magnitude of the changes in net-exports described in 2.4 is greater than the magnitude of change in net-exports described in 2.3, for a identical percentage change in $\overline{\pi(\phi, \phi')/f_{s',c'}b_{s',c'}}$. [4]

This implies that a reduction in iceberg trade costs will have larger implications for the distribution of economic activity across sectors, relative to a similar reduction in firm-matching frictions.

A reduction in international matching costs and iceberg trade costs will lead to an increase in average real wages for employees in all sectors. However the effects on the distribution of wages will differ within and across sectors, depending on the shape of the firm-matching function.

2.4.3 A Protectionist Tariff

A protectionist tariff will: reduce the number of potential suppliers to all firms in Home and lead to a reallocation of labor across sectors in Home and Foreign. The effects of a protectionist tariff will differ from a change in symmetric iceberg trade costs due to the distortion effect created by the reallocation of labor across sectors in Foreign. A protectionist tariff in this setting can lead to an increase in real income in Home.

When Home imposes a tariff on imports from sector-s, firms located in Home will lose access to potential suppliers. This causes the unit cost of firms in Home to increase. From equation 2.4, as unit costs ($\eta(\phi)$) increase labor demanded will increase as well. This effect will lead firms to post more employment vacancies in an effort to hire more labor for use in production.

Each firm in Home will demand more aggregate inputs as well. This effect is driven by the increase in unit costs, $\eta(\phi)$, from the decrease in suppliers. This is can also be seen in equation 2.4. This implies that the loss in suppliers leads to an increase in labor demand in Home, and a “thickening" of the domestic production-network, firms will demand more inputs from one another and trade more within the Home country.

At the same time, when Home places a tariff on imports from a specific sector in Foreign, Foreign job-searchers move out of the sector that is facing the tariff and into the sector that is not.

[4] Where $\overline{x}$ denotes the average across all firm pairs for variable x.

This leads to a reduction in unit costs in the non-taxed tariff in foreign, This makes the firms in the non-tariff sector more viable as exporters, and offsets the loss in suppliers from the tariff, leading to a slight increase in demand for labor and inputs.

This reallocation of Foreign labor also offsets losses in the household price index. Foreign firms in the non-taxed sector experience a reduction in unit costs, due to a reduction in the cost of hiring workers. This savings is passed on to domestic firms and households. These effects are enough such that a protectionist tariff in one sector can lead to an increase in nominal and real income. Furthermore it can lead to a reduction in the household price index.[5]

3 Model Simulations

In this section I present the results from several simulations of the model. I first lay out the assumptions I place on parameters in the model. I then lay out the set of counterfactual experiments that I simulate for the model. I simulate a relative increase in the efficiency of the sector-1 labor market Home. I then simulate a relative increase in the average production network efficiency for sector-1 firms in Home. I use these simulations to further discuss the propositions outlined in section 2.3.1. To simulate the effect of globalization I simulate a reduction in firm average production network frictions. Finally I simulate the effects of a protectionist tariff for sector-1 in the Home country.

3.1 Simulation Setup

I assume there are two countries, Home and Foreign, each consisting of two sectors, sector-1 and sector-2. I take the price charged by the sector-2 firm with the lowest productivity draw (ϕ_ℓ) in free trade to be the numeraire of the model. I simulate the model using parameters guided by calibrations from Lim (2018) and parameter choices made by Helpman and Itskhoki (2010).

Following both Lim (2018) and Helpman and Itskhoki (2010), I assume that the firm-level

[5] I want to emphasize that this effect is a long run effect, if workers are not able to move across sectors in the foreign sector (as is typically the case in the short run) then there will be an increase in the domestic price index. Furthermore, this model does not take into account retaliatory tariffs which would erase any gains from the protectionist tariff.

distribution of labor-augmenting productivities is distributed according to a log-normal distribution. I assume the distribution has mean and standard deviation given by μ_{ϕ_ℓ} and σ_{ϕ_ℓ} respectively. I follow Lim (2018) by assuming that the random component of the firm-matching cost is distributed according to the Gumbel distribution with shape and scale parameters given by λ and k respectively.

Parameter	Description	Value
μ_{ϕ_ℓ}	Mean of Log-Normal distribution of firm productivities	0
σ_{ϕ_ℓ}	Standard Deviation of Log-Normal distribution of firm productivities	1
λ	Shape parameter for distribution of the random component of firm-matching costs (ξ)	1
k	Scale parameter for distribution of the random component of firm-matching costs (ξ)	1
α_s^{HH}	Sector-level consumption suitability parameter	0.25
σ^{HH}	Household elasticity of substitution	3.1
α_s	Production function weight of aggregate inputs	0.5
$\sigma_{\ell,s}$	elasticity of substitution between inputs and labor	3.1
β	Worker's bargaining power	0.5
σ_s	Firm's elasticity of substitution between sectors	3.1
$\alpha_{s,s'}$	Input suitability of sector-s' inputs for use in sector-s production.	0.25
$\sigma_{s,s'}$	Elasticity of substitution for sector-s' varieties for use in production by sector-s firms	0.25
$m_{s,c}^L$	Scale of firm-worker matching function	0.95
χ	Weight of employment vacancies in firm-worker matching function	0.25
$\bar{L}_c$	Total mass of workers in country-c	1

Table 1: Parameters Used in Simulation

In order simplify the model, I remove one dimension of non-linearities in the model by assuming that the elasticity of production is identical at all stages of the firm production function. Huneeus (2018) and Baqaee and Farhi (2017) explore the implications of these non-linearities, so I simplify the model by ignoring them.

Another feature of note is that the input suitability parameters do not vary by sector pairs. I will explore this feature in a future paper when the model is extended to a dynamic framework and estimated using firm-level data.

Notably absent from table are the costs of posting an employment vacancy ($\nu_{s,c}$), and the average value of the firm-matching parameter ($f_{s,c}$). These parameters will take on multiple values through the course of the simulations. I simulate the model multiple times where each of these parameters takes a value over the interval $(0, 1)$. These simulations will be discussed in the following sections.

3.2 Relative Changes in Labor Market Efficiency

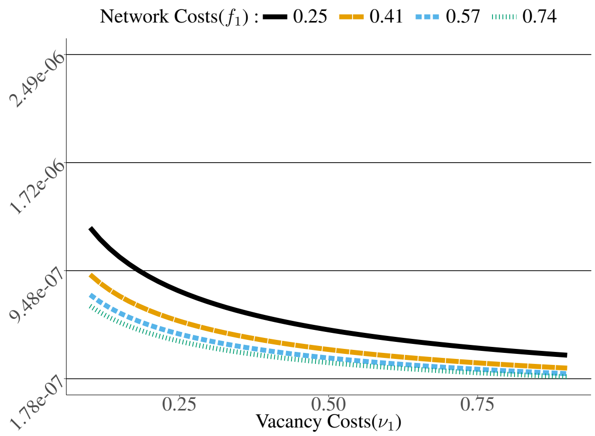
In this section I simulate an change in the relative labor market efficiency to show how this leads to a reallocation of labor across sectors. I then show how this effect leads to changes in firm-linkages.

I simulate an change in the cost of posting an employment vacancy for sector-1 in Home ($nu_{1,H}$) over the interval $(0, 1)$. I hold all other sectors costs of positing an employment vacancy equal to $1/2$. When $\nu_{1,H} < 1/2$, this implies that Home has a comparative advantage in sector 1. As it becomes relatively more expensive for firms to post an employment vacancy in sector 1, job-searchers in sector-1 will move into sector-2.

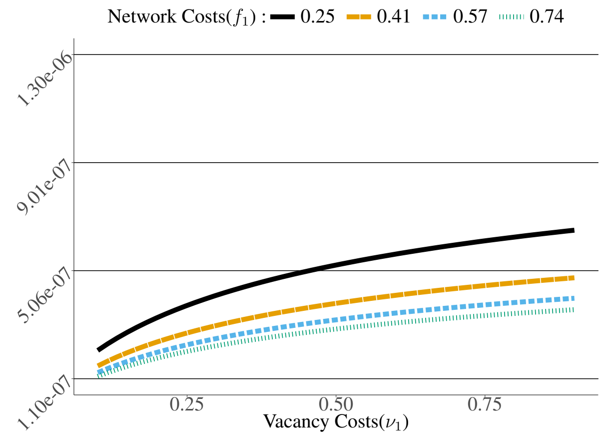
3.1 presents the results of the simulated increase in $\nu_{1,H}$. 3.1 shows the mass of customer- and supplier-linkages, for the average firm in each sector. As the cost of posting an employment vacancy increases for sector-1, the average firm's mass of customer-linkages decreases for both sectors. However the average firm's mass of supplier-linkages is decreasing for sector 1, and increases for sector-2.

As the cost of posting a vacancy increases, workers reallocate out of sector-1 and into sector-2 due to a decreasing probability of finding a match in sector-1. The sector-1 cost of hiring a worker in Home, $b_{1,H}$, increases and the cost of hiring a worker in sector-2 in Home, $b_{2,H}$ falls as nu_1 increases.[6] This is confirmed in figure 3.2. When this occurs, the *direct network effect* leads to a decrease in the mass of customer-linkages for sector-1 firms, and an increase for sector-2 firms. Adding to this, the *efficiency effect* predicts that unit costs will fall in sector-2 and increase in sector-1, further increasing the mass of customers-linkages for sector-2, and depressing it for

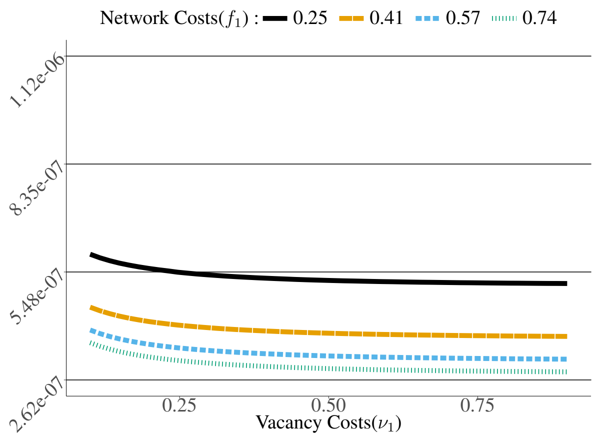
[6]The increase in $b_{1,H}$ is larger than the decrease in $b_{2,H}$. This is because $b_{1,H}$ rises due to 2 factors, the increased cost of posting vacancies, and the lower labor supply in sector-1, as job searchers move from sector-1 into sector-2. In comparison, $b_{2,H}$ only increases due to a greater number of job searchers.



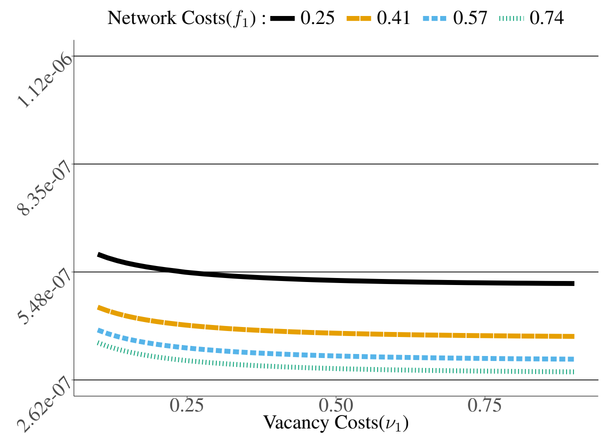
(a) Sector 1 Average Mass of Supplier-Linkages



(b) Sector 2 Average Mass of Supplier-Linkages



(c) Sector 1 Average Mass of Customer-Linkages



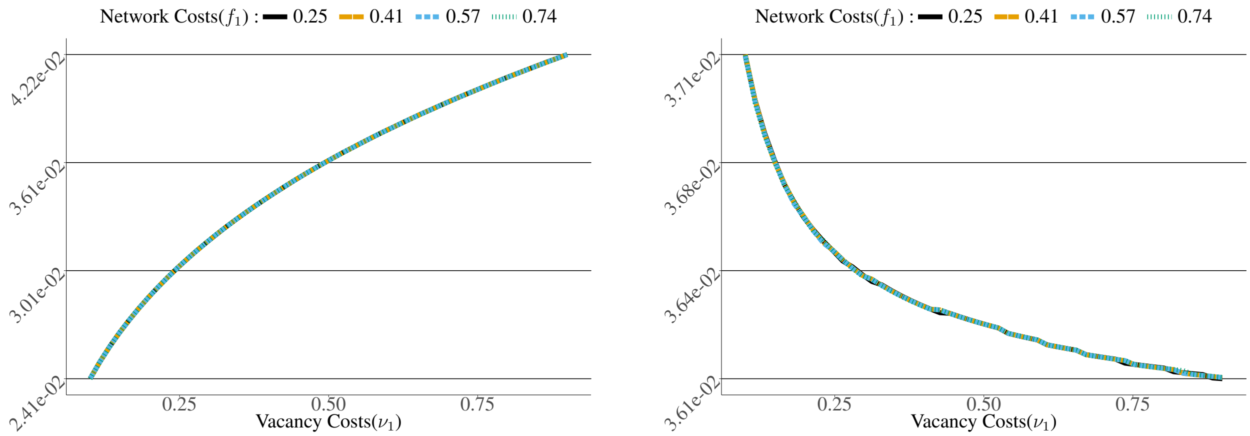
(d) Sector 2 Average Mass of Customer-Linkages

Figure 3.1: Relative Labor Market Frictions and the Production-Network

sector-1.

This would suggest that the mass of customer-linkages should increase for sector-2 firms, and decrease for sector-1. However, labor and inputs are assumed to be substitutes, since $\sigma_{\ell,s} > 1$. When the price of labor rises in sector-1, $b_{s,H}$, sector-1 firms increase their demand for aggregate inputs from all sectors. Likewise, as the price of labor falls in sector-2, firms decrease their demand for aggregate inputs. Proposition 2.1 gives this result.

For sector-2 it is the case that the *output effect* is dominated by the *direct network effect* and the *efficiency effect*, leading to an increase in the average-firm's mass of suppliers. Likewise is sector-1, the *output effect* is dominated by the *direct network effect* and the *efficiency effect*.



(a) Sector 1 Cost of Hiring

(b) Sector 2 Cost of Hiring

Figure 3.2: Cost of Posting a Vacancy and the Cost of Hiring

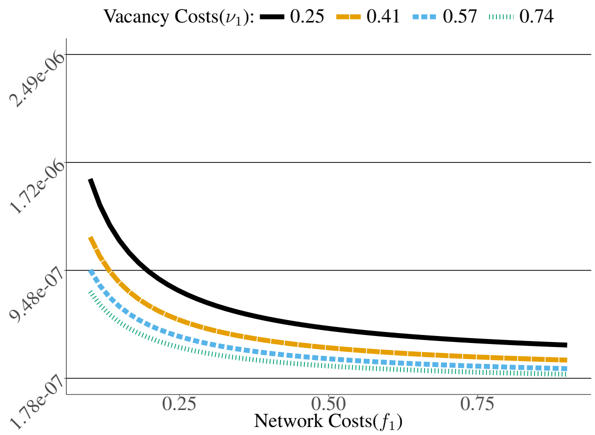
3.3 The Production Network Frictions and Network Centrality

In this section I simulate the effects of an increase in the firm-network frictions for sector-1 in Home, $f_{1,H}$. This effect differs from a relative increase in $\nu_{1,H}$, because there is minimal realloca-tion of labor when only $f_{1,H}$ changes.

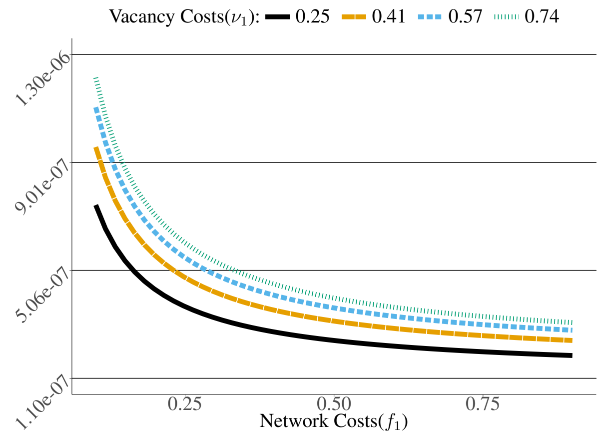
To understand how this model contributes to the literature on production network formation it is useful to compare figure 3.3b and figure 3.1b. Increasing the average cost of matching for sector-1 firms, decreases the mass of supplier-linkages for sector-2 firms. Compare this to the increase in the mass of supplier-linkages for sector-2 firms, as the Average cost of posting an employment vacancy rises in sector-1. This difference is due to the larger reallocation of labor that occurs in the latter. This reallocation of labor effects firms demand for inputs, and leads to a differential impact on the number of suppliers to each sector.

3.4 Globalization Simulation

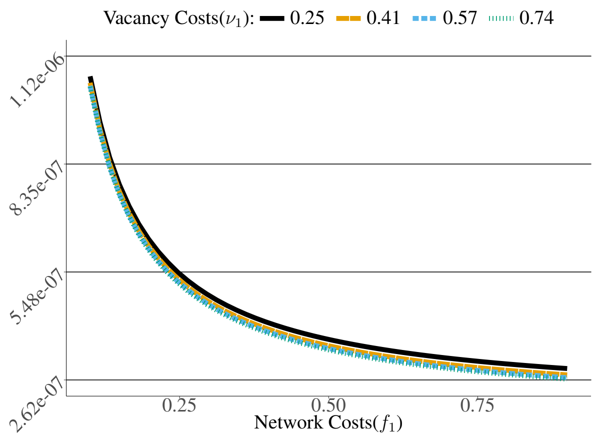
Globalization and Wage Inequality Results Coming Soon



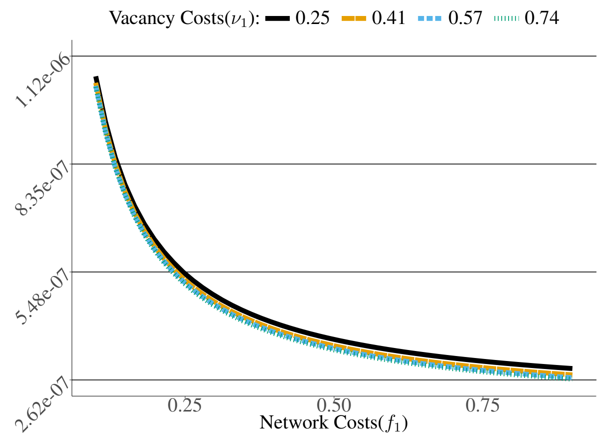
(a) Sector 1 Average Mass of Supplier-Linkages



(b) Sector 2 Average Mass of Supplier-Linkages



(c) Sector 1 Average Mass of Customer-Linkages



(d) Sector 2 Average Mass of Customer-Linkages

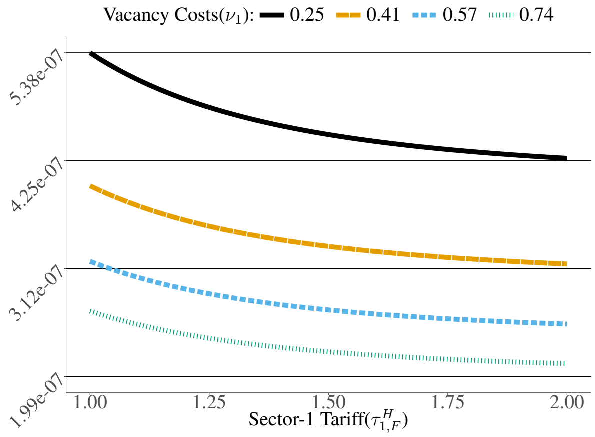
Figure 3.3: Relative Production Network Frictions and the Production-Network

3.5 A Simulated Protectionist Tariff

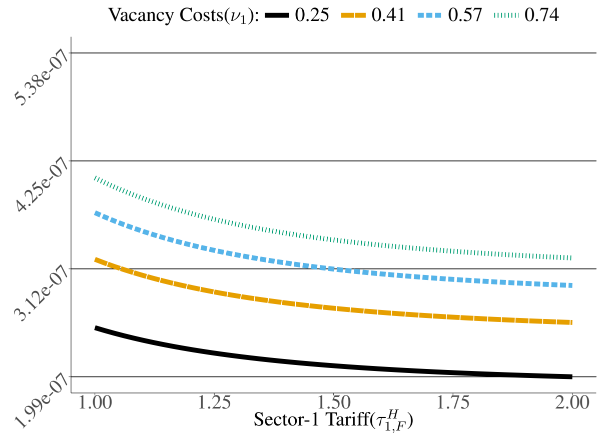
In this section I simulate the effects of a protectionist tariff for sector-1 by the Home country. The tariff leads to a reduction of cheap inputs for Home and a movement of Foreign labor out of sector-1 and into sector-2 (as shown in figures 3.4a and 3.4b). This reallocation of Foreign labor is consequential for consumers, employment, and real income in Home.

Firms in both sectors in Home form more linkages with customer-firms. This effect is a substitution effect, as firms substitute away from Foreign sector-1 varieties towards others. This effect is a “thickening" of the domestic production-network. This effect is shown in figures 3.4c and 3.4d.

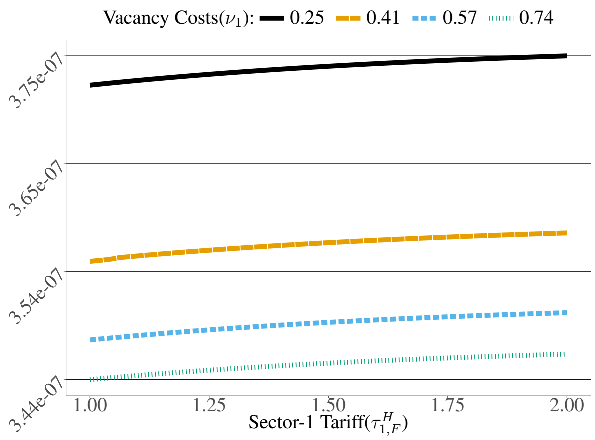
At the same time, labor reallocates out of the Foreign taxed sector and into the sector that is not



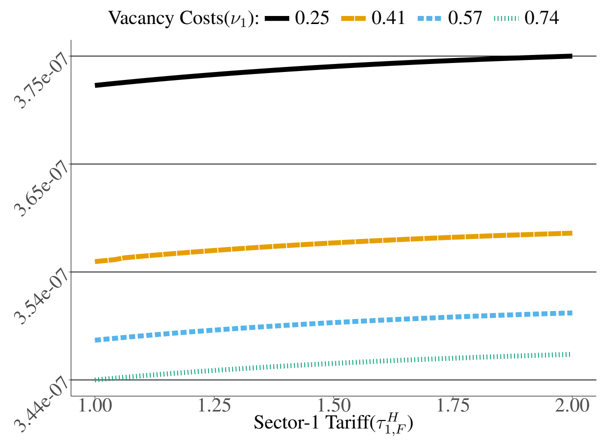
(a) Sector 1 Average Mass of Supplier-Linkages



(b) Sector 2 Average Mass of Supplier-Linkages



(c) Sector 1 Average Mass of Customer-Linkages



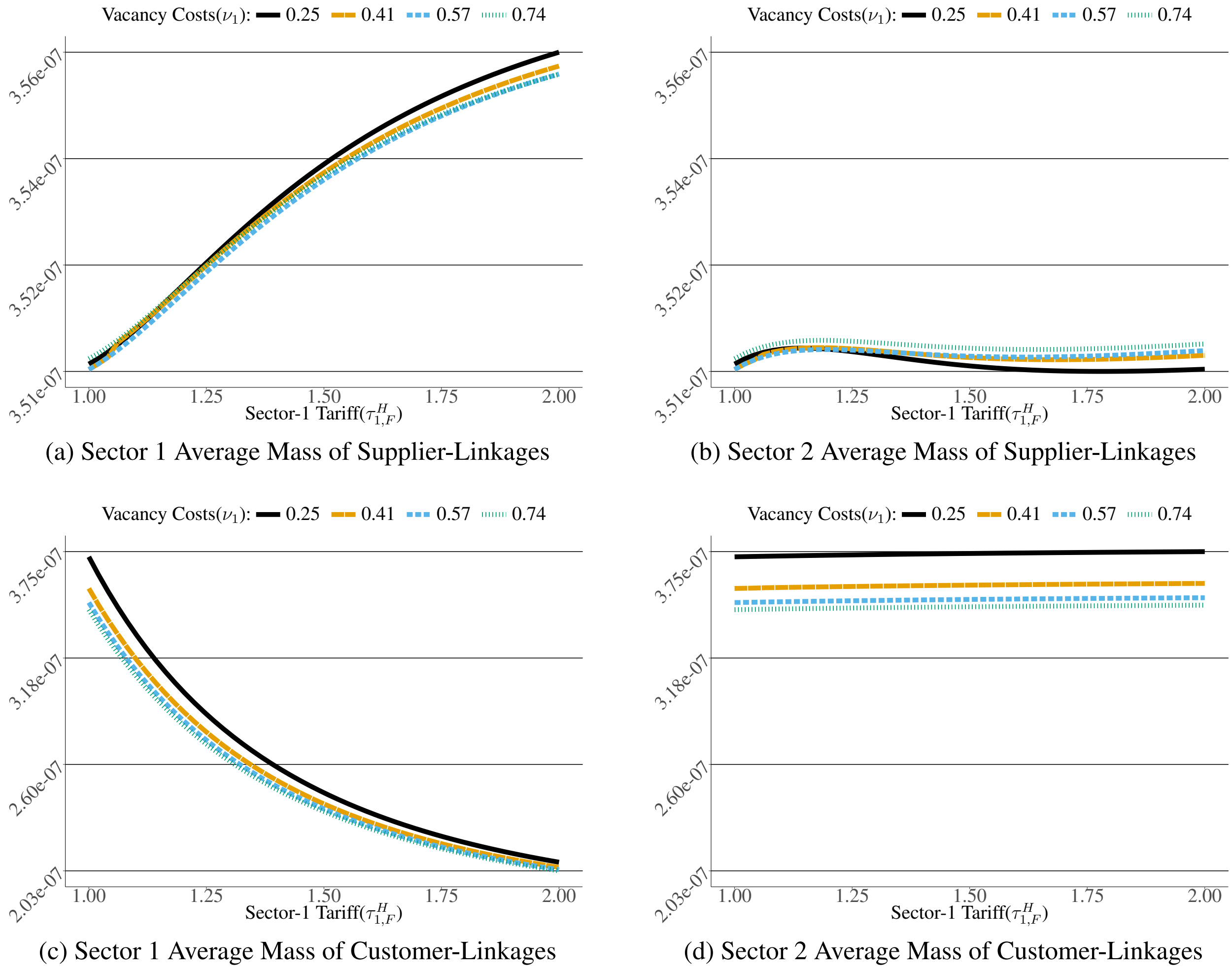
(d) Sector 2 Average Mass of Customer-Linkages

Figure 3.4: A Protectionist Tariff and Home's Production-Network

facing the tariff. Since labor and inputs are substitutes, this induces firms in the foreign taxed sector to increase their demand for inputs. This induces more firms in home to export to Foreign. Note that if labor and inputs were instead complements, the sector not facing the tariff would increase their demand for workers, leading to similar results for Home.

This “thickening" of the domestic production-network, lowers production costs of firms across all sectors in Home, leading to a decrease in the household price index in Home. At the same time Home-firms export more and make more sales to Home-firms, this leads to an increase in nominal wages. These effects combine to increase real wages in the Home

I want to again re-emphasize that this paper does not support a protectionist tariff as a method of



(a) Sector 1 Average Mass of Supplier-Linkages

(b) Sector 2 Average Mass of Supplier-Linkages

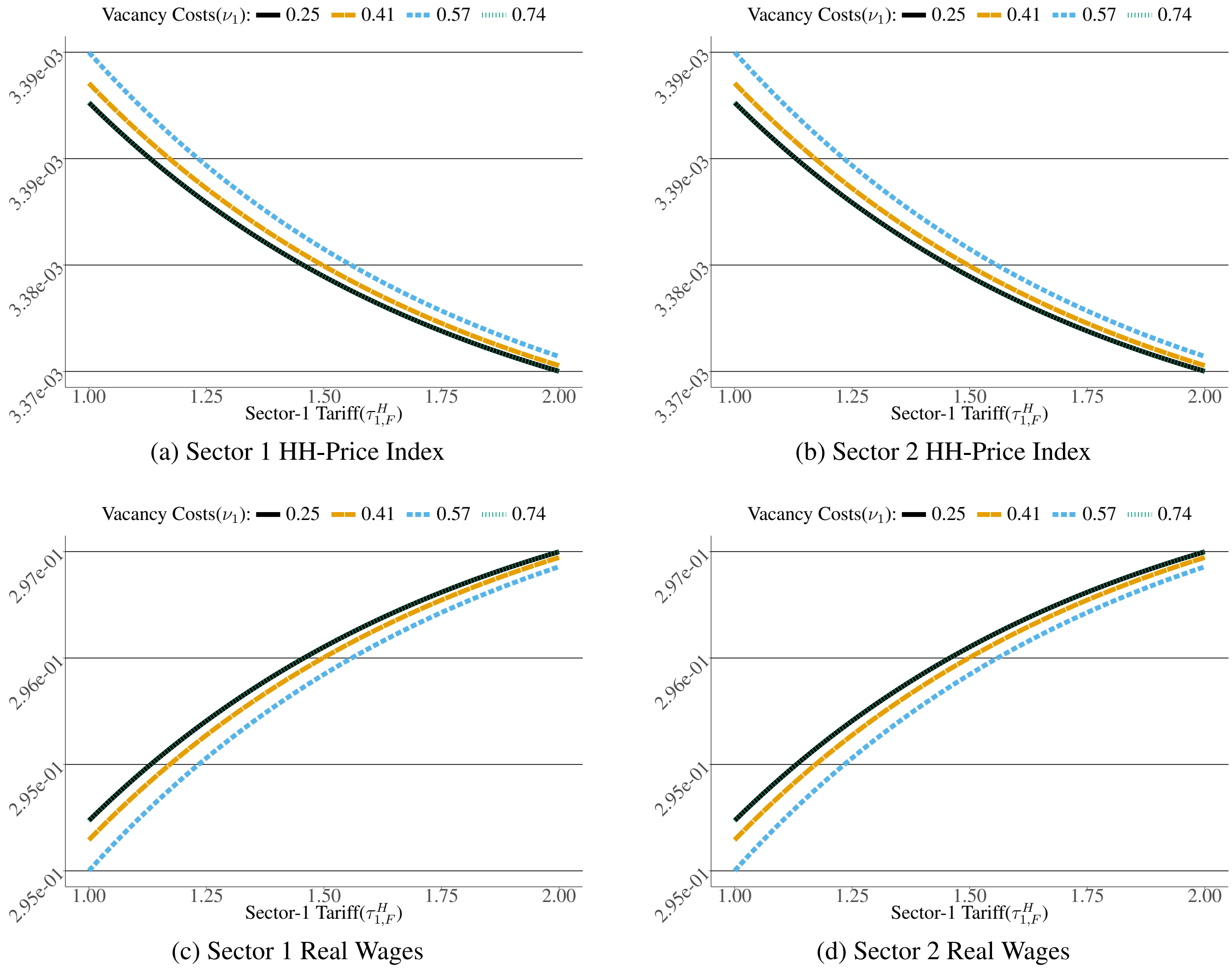
(c) Sector 1 Average Mass of Customer-Linkages

(d) Sector 2 Average Mass of Customer-Linkages

Figure 3.5: A Protectionist Tariff and Foreign's Production-Network

lowering household prices. This model does not include a channel for strategic interaction between countries. Nor does this model allow for sticky firm-to-firm relationships, or employment. Either of these realistic features would erase the gains from Foreign's sectoral reallocation of labor. These features would imply that costs of production would increase for firms in Home and that real wages would fall.

These simulations demonstrate the importance of sectoral reallocation in determining production network centrality, firm profits, and labor market outcomes. Any policy that changes relative wages across sectors will induce a reallocation of workers across sectors. This reallocation of labor will have consequences for how firms choose to form their production-networks.



(a) Sector 1 HH-Price Index

(b) Sector 2 HH-Price Index

(c) Sector 1 Real Wages

(d) Sector 2 Real Wages

Figure 3.6: A Protectionist Tariff and Home's Labor Market

4 Conclusion

This paper presented a model that demonstrates how endogenous production-network formation and relative labor market frictions interact. The model generates a rich set of predictions about how of the production-network to changes in response to sector-level relative labor market frictions, and how labor markets change in response to globalization and tariffs.

Future work should be done to empirically study the model in a dynamic setting. Changes in the economy in a setting featuring sticky -employment and -production-networks will lead to meaningful transitions in economic outcome. The larger benefit of extending the model to a dynamic setting is to allow for the credible structural estimation of the model to determine the importance of labor-market frictions in production-network formation, and the importance of production network frictions in determining labor market outcomes.

Understanding how production network formation has changed in response to globalization is critical to understanding how workers have been impacted by trade. This model takes a step in this direction by analyzing the role of labor market institutions and frictions are intertwined with production-network formation. The model demonstrates how worker outcomes change in response to globalization. when the intensive and extensive margins of trade are allowed to change.

The model also demonstrates how the reallocation of workers across sectors leads to long run changes in the production network. When trade policy, or local labor market institutions, change in a way that one sector is made relatively more attractive for workers there will be changes in production-network outcomes.

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