Exam 1 Study Guide

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Theorem

If n is an integer, then $n^2 + 3n + 2$ is an even integer.

Lemma

(a) An even number multiplied by an odd number is even.

(b) A given integer that is even or odd, remains even or odd when summed with an even integer.

Proof

We start by factoring $n^2 + 3n + 2$ to get $n(n+3) + 2$.

Suppose n is even, then $(n+3)$ must be odd, as

$$(n+3) = (2k+3) : k \in \mathbb{Z} = 2k+1 \tag{1}$$

For $n \in 2\mathbb{Z}$ the expression $n(n+3)+2$ is then an even number by lemma a and b.

Now suppose n is odd, then $(n+3)$ must be even as

$$(n+3) = (2k+1+3) : k \in \mathbb{Z} = 2k \tag{2}$$

For $n \in 2\mathbb{Z}+1$ the expression $n(n+3)+2$ is then an even number by lemma a and b.

We have proven that the expression is even when n is even and n is odd.

The union of the two sets is the integer set, so we conclude the following.

If n is an integer, then $n^2 + 3n + 2$ is an even integer.

$\square$