Exam 1 Study Guide

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1 Chapter 5. Proof Techniques

1.1 Problem 5.3

Theorem

If n is an integer, then $n^2 + 3n + 2$ is an even integer.

Lemma

(a) An even number multiplied by an odd number is even.

(b) A given integer that is even or odd, remains even or odd when summed with an even integer.

Proof

We start by factoring $n^2 + 3n + 2$ to get $n(n + 3) + 2$.

Suppose n is even, then $(n + 3)$ must be odd, as

$$(n + 3) = (2k + 3) : k \in \mathbb{Z} = 2k + 1 \tag{1}$$

For $n \in 2\mathbb{Z}$ the expression $n(n + 3) + 2$ is then an even number by lemma a and b.

Now suppose n is odd, then $(n + 3)$ must be even as

$$(n + 3) = (2k + 1 + 3) : k \in \mathbb{Z} = 2k \tag{2}$$

For $n \in 2\mathbb{Z} + 1$ the expression $n(n + 3) + 2$ is then an even number by lemma a and b.

We have proven that the expression is even when n is even and n is odd.

The union of the two sets is the integer set, so we conclude through proof by cases the following.

If n is an integer, then $n^2 + 3n + 2$ is an even integer.

$\square$

1.2 Problem 5.13

Theorem

$sin^2(x) \leq |sin(x)|$ for all $x \in \mathbb{R}$

Definition

(a) $sin(x) \in \{-1 \leq y \leq 1 : y \in \mathbb{R}\}$

Lemma

(a) $x^2 \leq |x|$ for $-1 \leq x \leq 1$ where $x \in \mathbb{R}$

Proof

We start by proving Lemma a.

Note that the range of x^2 and $|x|$ for $-1 \leq x \leq 1$ is limited to $[0, 1]$.

A real number x where $0 < x \leq 1$ can be represented by $x = \frac{1}{a}, a \in \{\mathbb{N} > 0\}$

Observe that $x^2 = (\frac{1}{a})^2 = \frac{1}{a^2}$.

Since $a^2 \geq a$ for any integer greater than 0, $(\frac{1}{a})^2 \leq \frac{1}{a}$.

The above is equivalent to saying $x^2 \leq x$ for $x \in \mathbb{R}$ where $0 < x \leq 1$.

This leaves the case where x = 0.

Well if $x = 0$, then $x^2 = 0$ as well.

So $x^2 \leq |x|$ for $-1 \leq x \leq 1$ where $x \in \mathbb{R}$.

□

Note that the range of $sin^2(x)$ and $|sin(x)|$ is $[0, 1]$.

We can then conclude by Lemma a the following.

$$sin^2(x) \leq |sin(x)| \text{ for all } x \in \mathbb{R} \tag{1}$$

□

Theorem

TODO

Lemma

(a) TODO

(b) TODO

Proof

TODO

$\square$

Theorem

TODO

Lemma

(a) TODO

(b) TODO

Proof

TODO

$\square$